B decays and mixing

and consequences for the Standard Model

M Wingate, First LatticeNET workshop on challenges in lattice field theory, Benasque, 2022-09-12

Preliminary remarks

Review + emphasis on B mixing

- Advertisement of other talks
  - 11:00 Monday: Alejandro Vaquero, *B → D* l ν and related anomalies*
  - 16:30 Monday: Enrico Lunghi, *B and K physics, role of lattice QCD*
  - 09:30 Tuesday: Chris Bouchard, *B → K l l and D → K l ν form factors and phenomenology*
  - 11:00 Tuesday: Simon Kuberski, *Heavy flavor physics with O(a) improved Wilson quarks*
- Survey of CKM status
- More in-depth summary of B meson mixing calculations

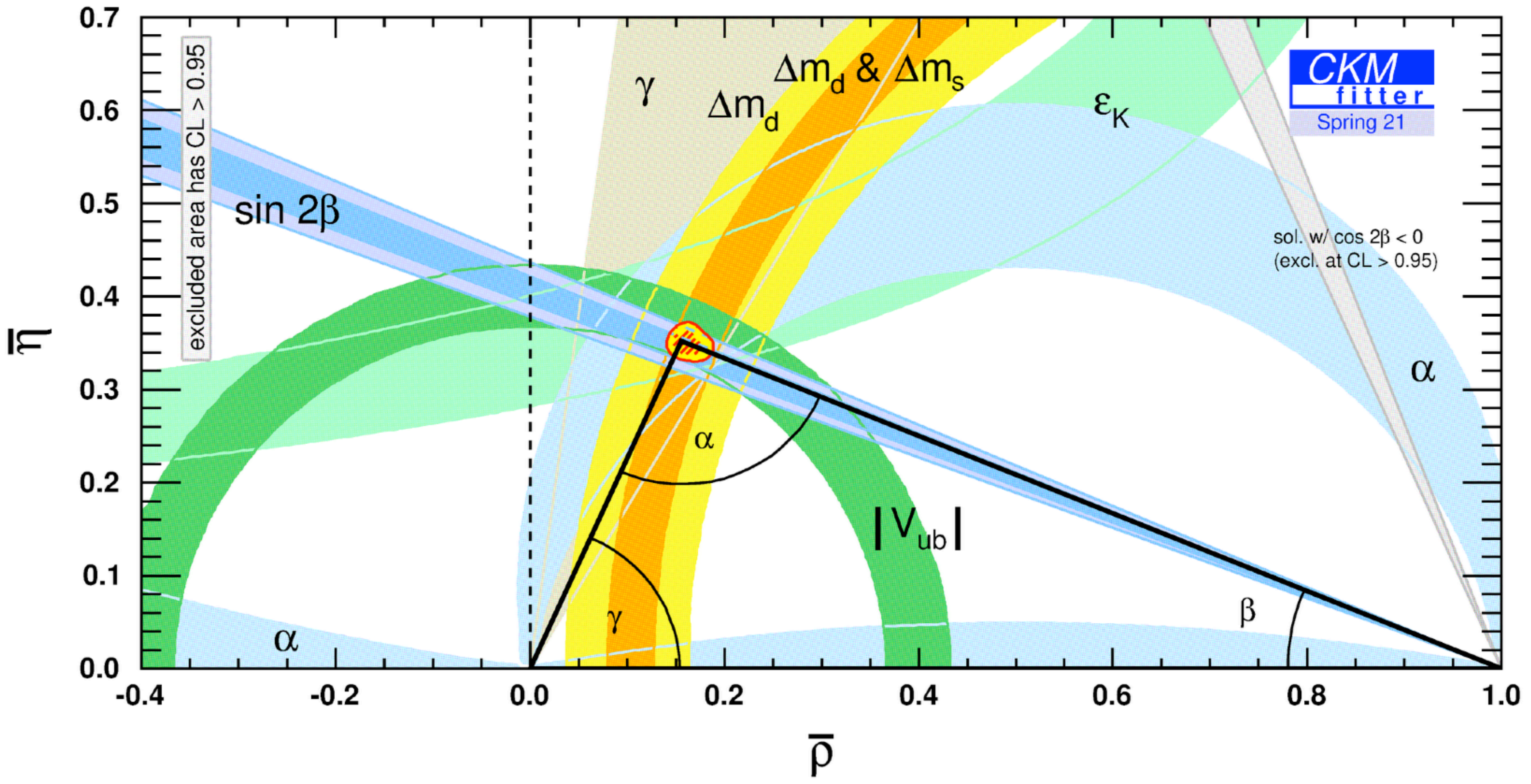
Introduction

Quark flavor in the Standard Model

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

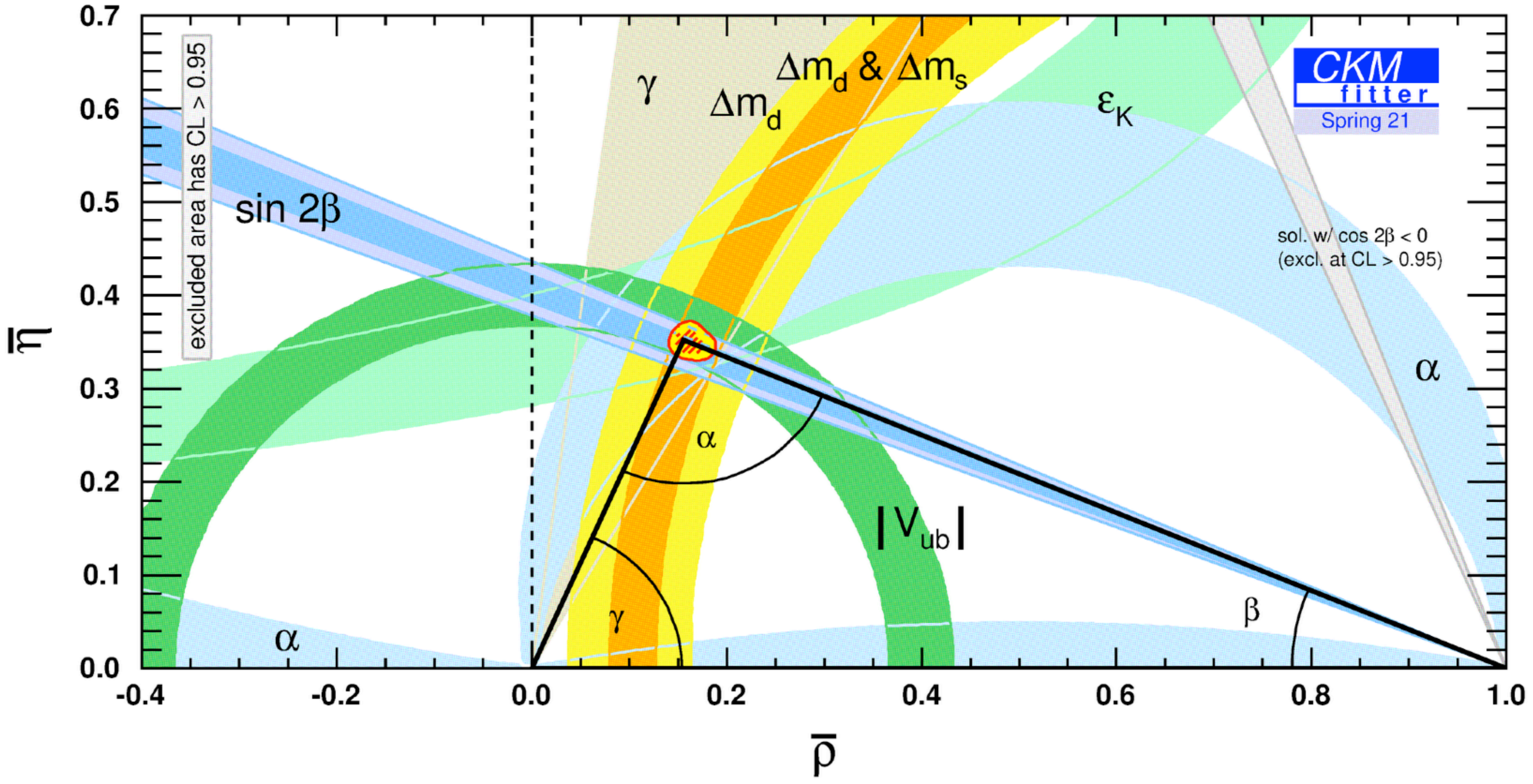
- Are the 4 free parameters in the CKM matrix sufficient to describe all ***quark*** flavor-changing interactions?
- High intensity (& high energy) era:
  LOTS of experimental measurements of ***hadronic*** processes,
  many very precise
- Require similarly precise SM predictions
- Lattice QCD connects hadronic observables to quark parameters

CKM constraints — (ρ, η) plane



CKMfitter, Moriond 2021

CKM constraints — (ρ, η) plane



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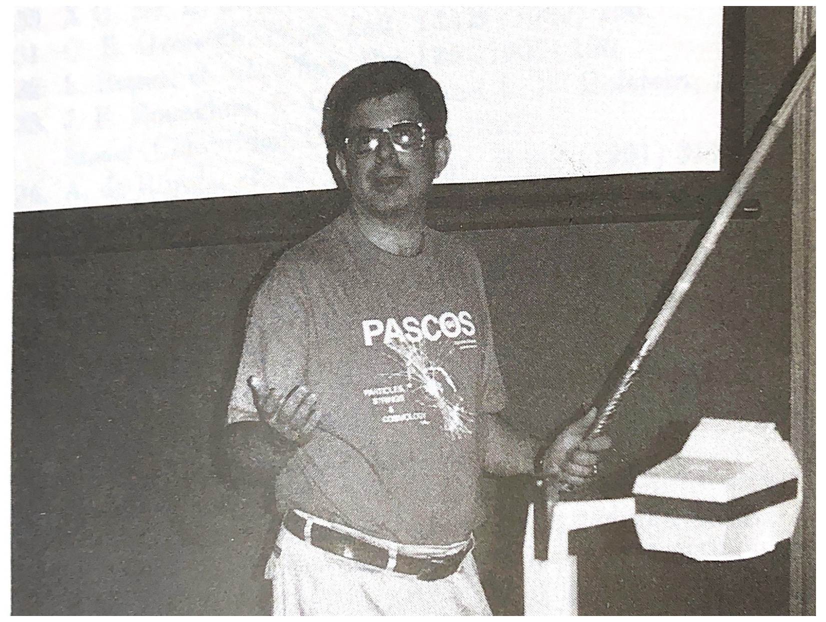
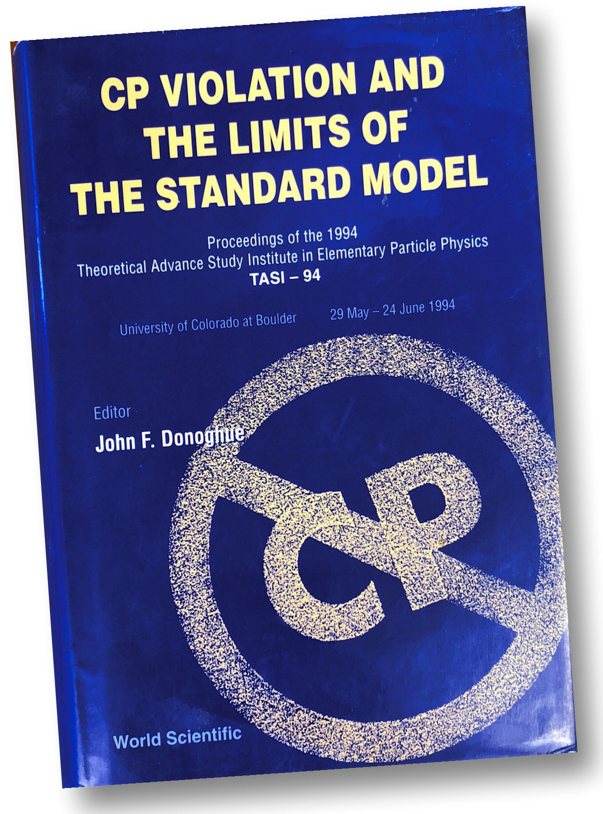




Sheldon Stone, TASI 1994

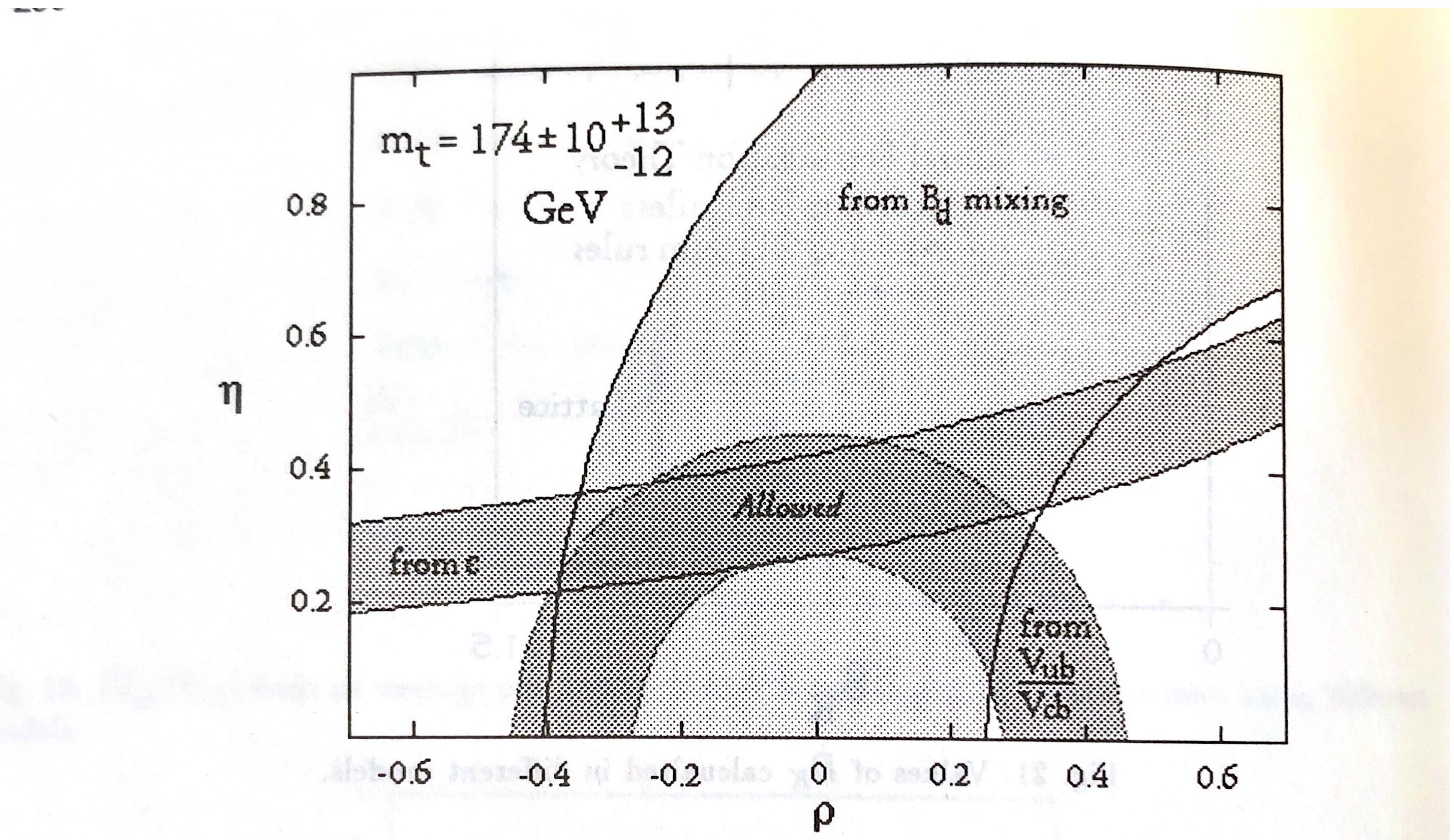


Fig. 22. Constraints in the $\rho - \eta$ plane from $|V_{ub}/V_{cb}|$, $B_d^0 - \bar{B}_d^0$ mixing and CP violation in the K_L^0 system (ϵ). The bands are $\pm 1\sigma$.

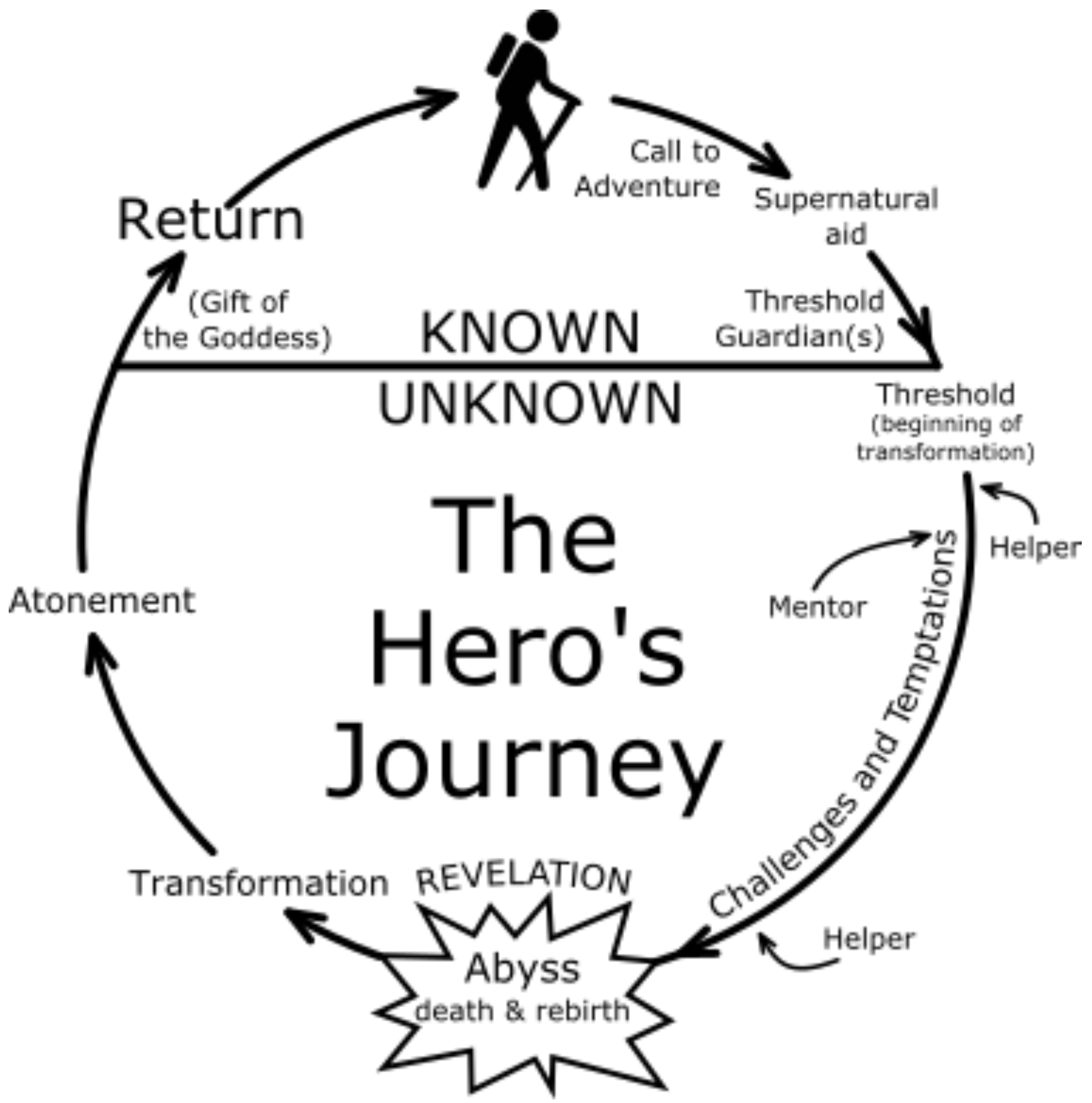
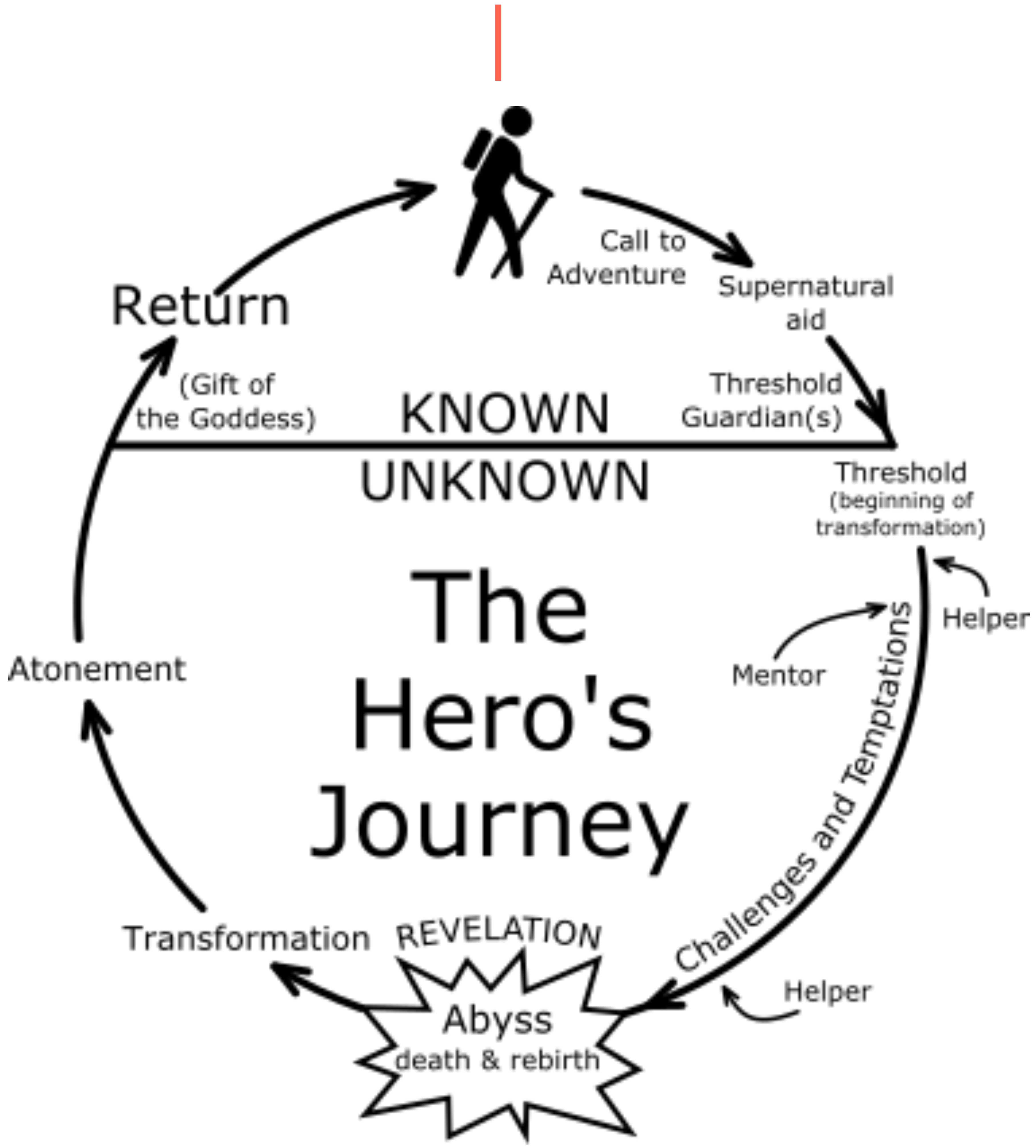


Image source: Wikipedia Commons, public domain

Idea!



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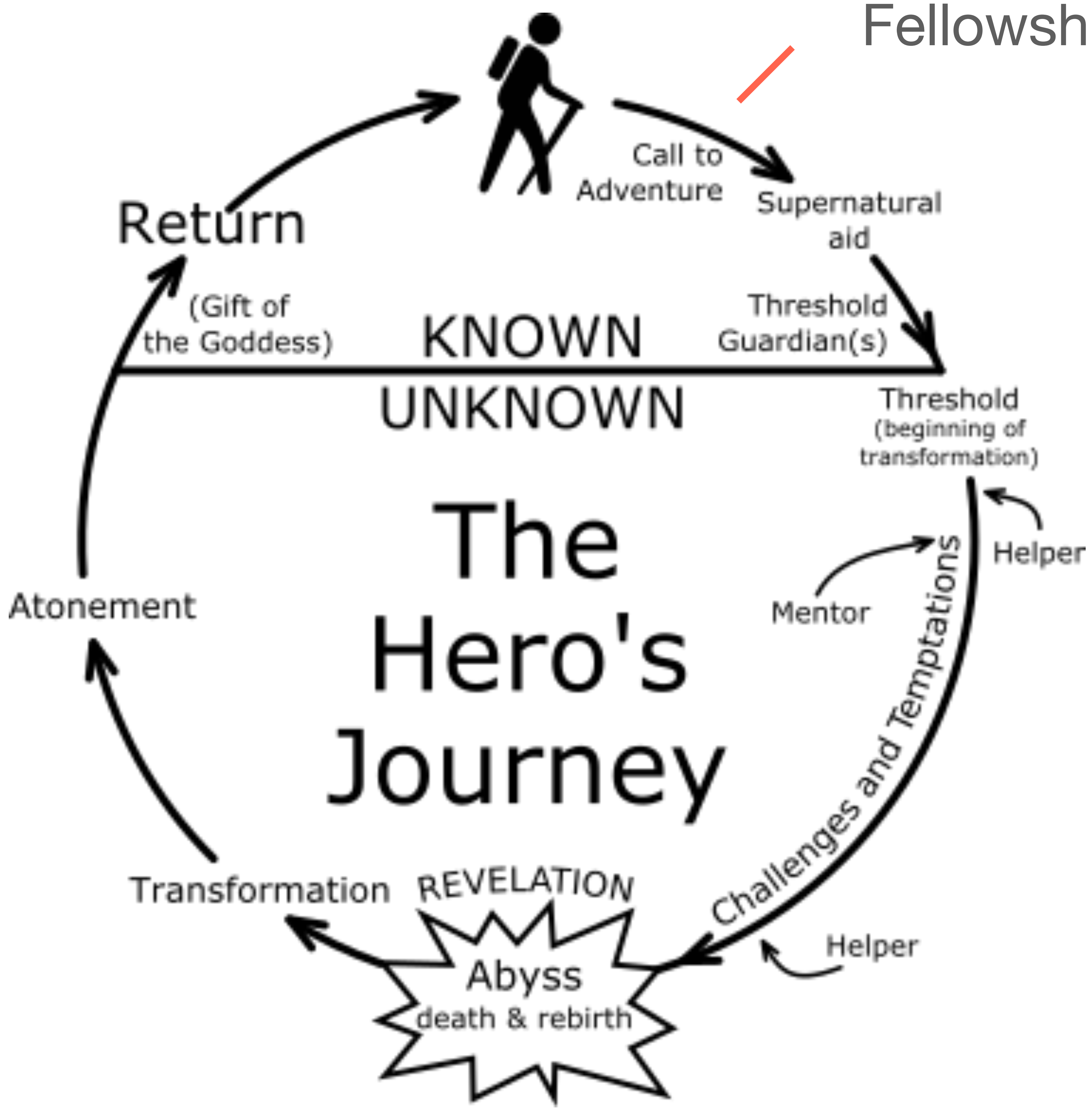
Idea!

Fellowship/collaboration



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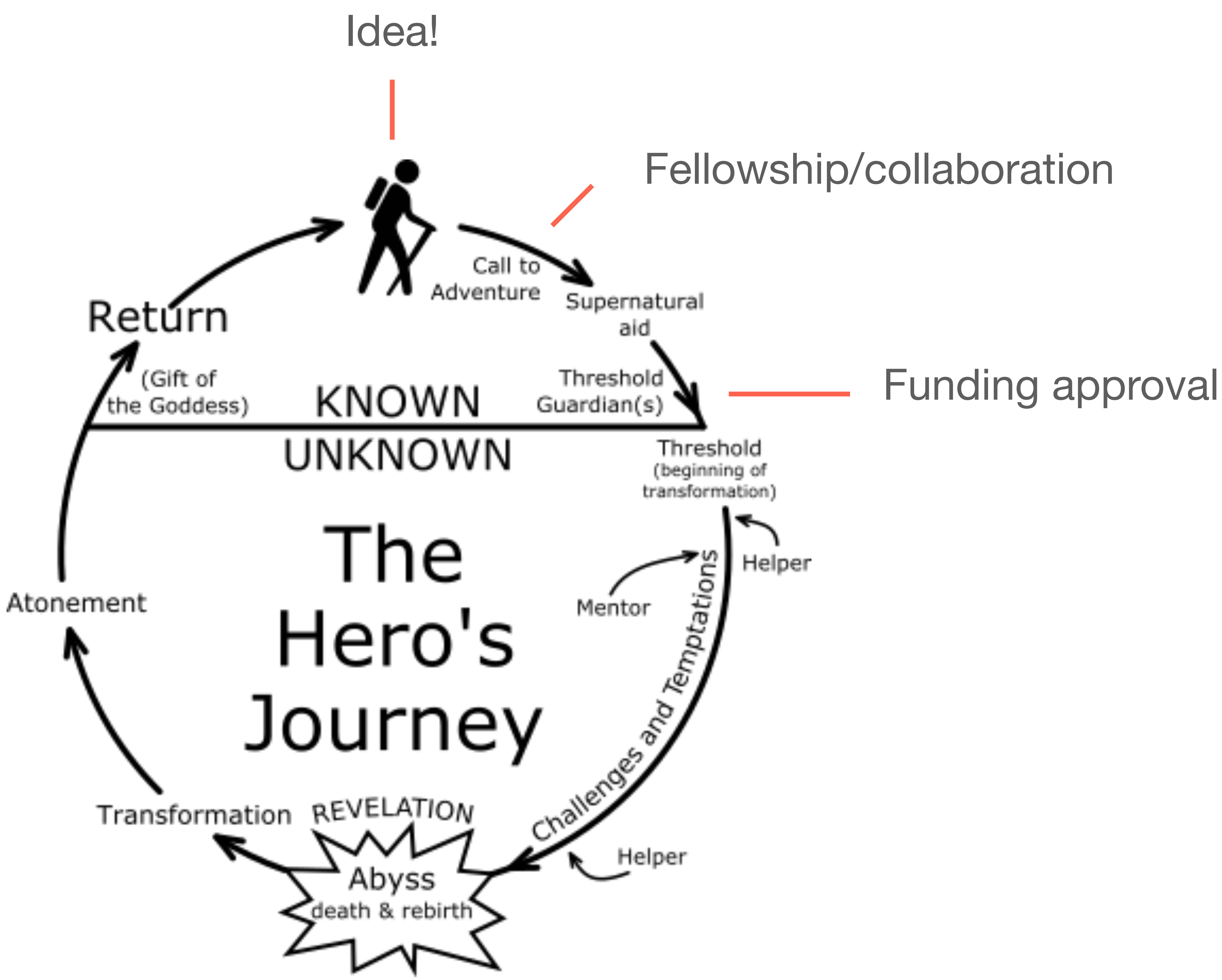

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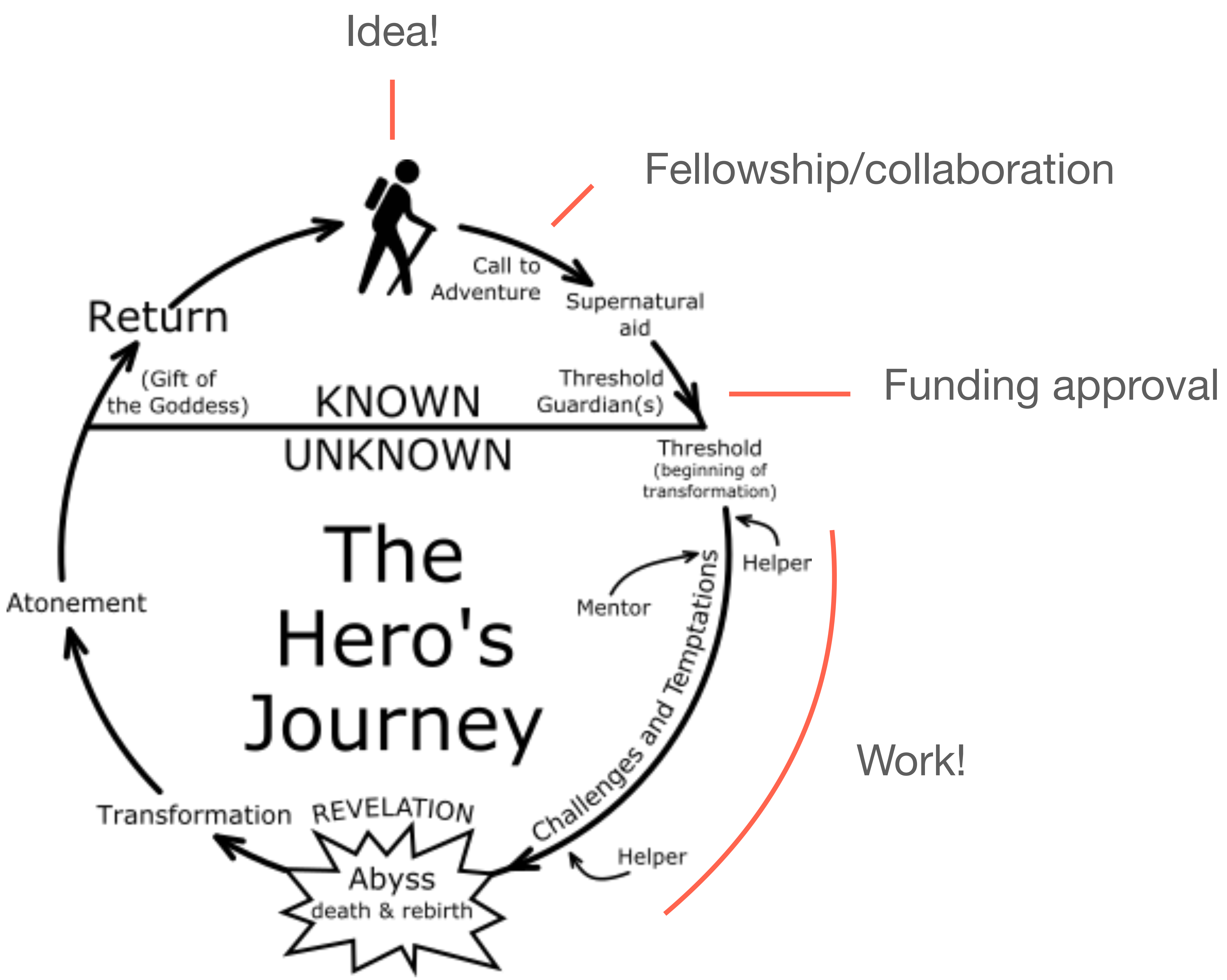

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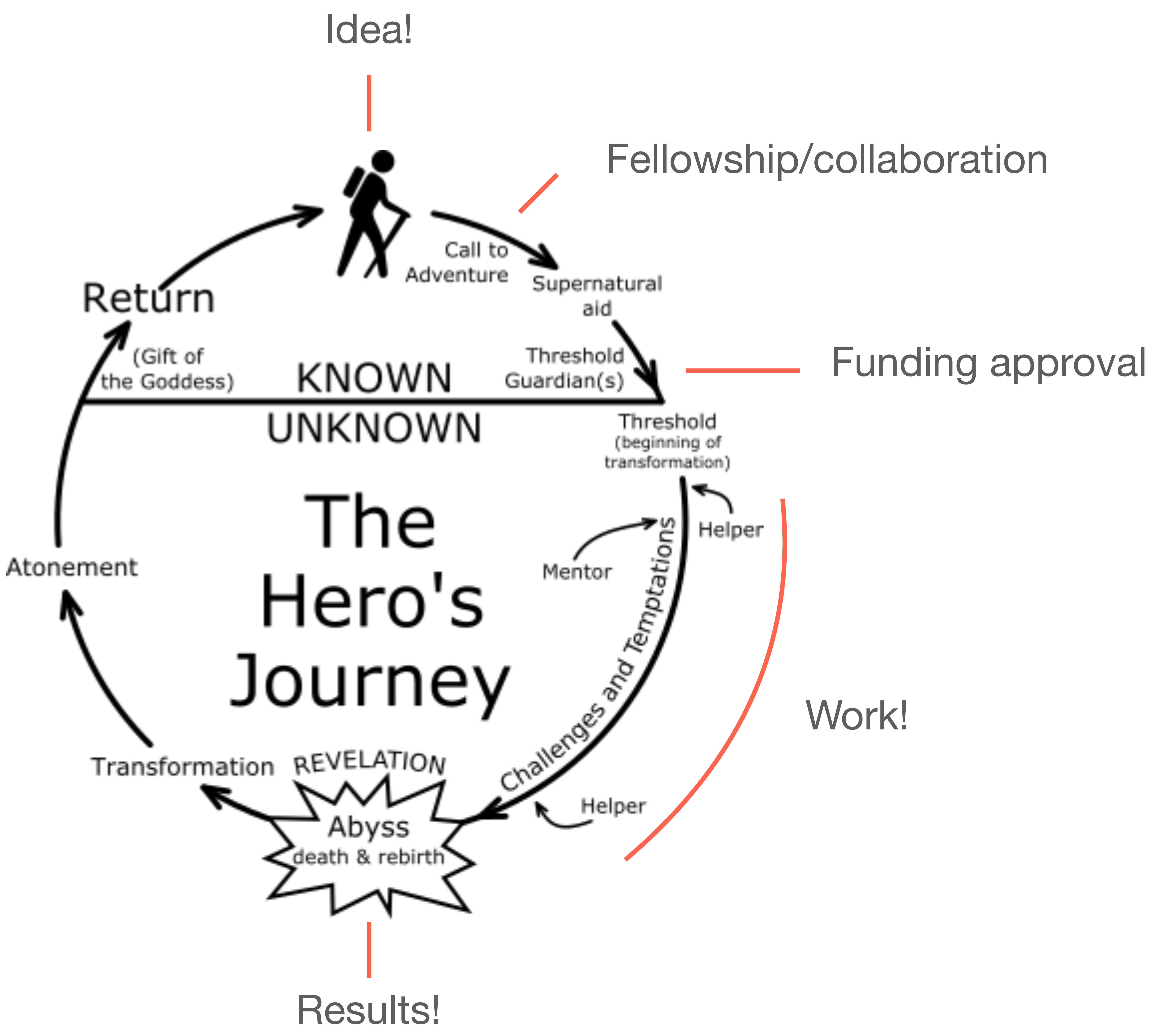

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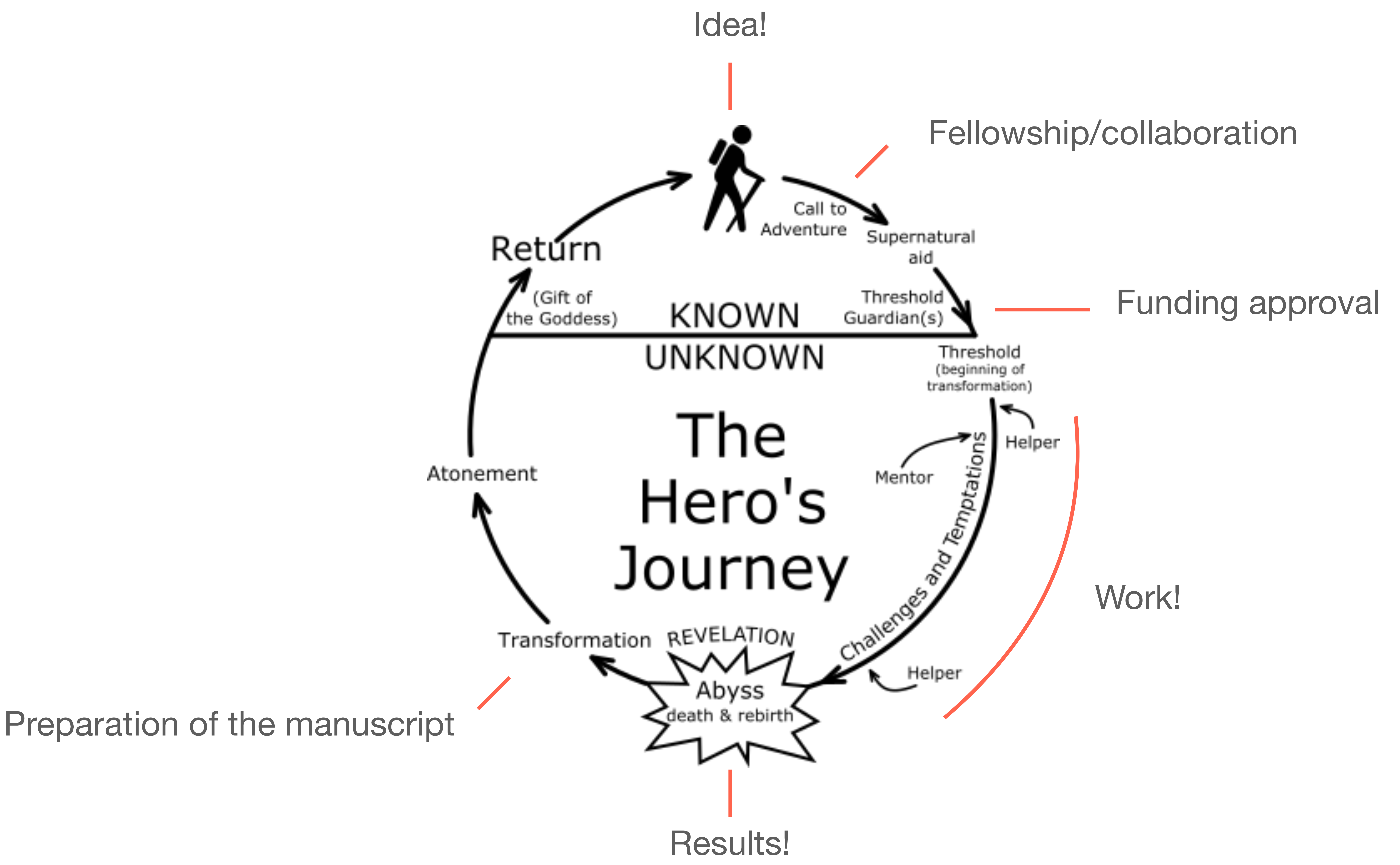

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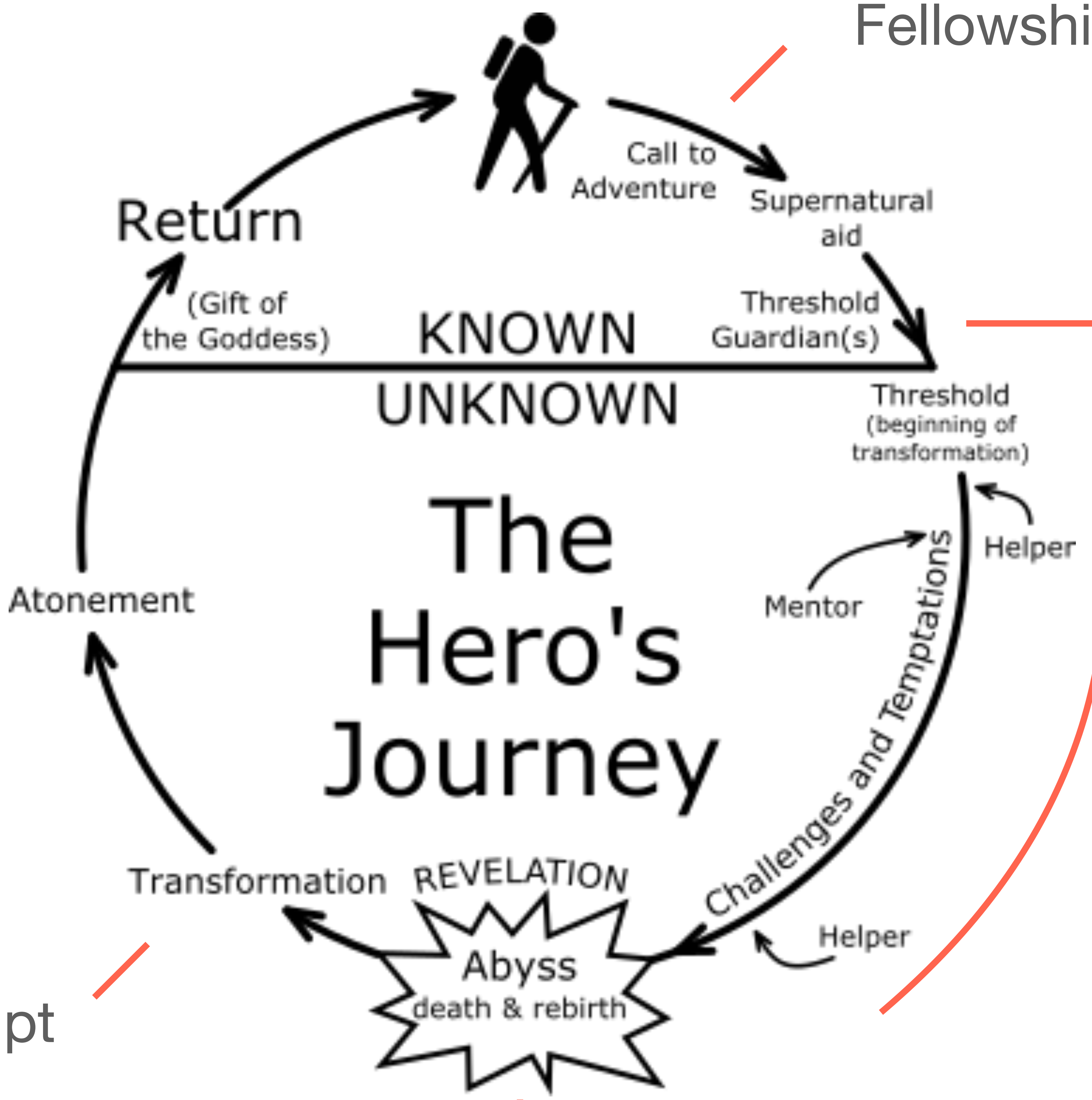
Idea!

Fellowship/collaboration

Funding approval



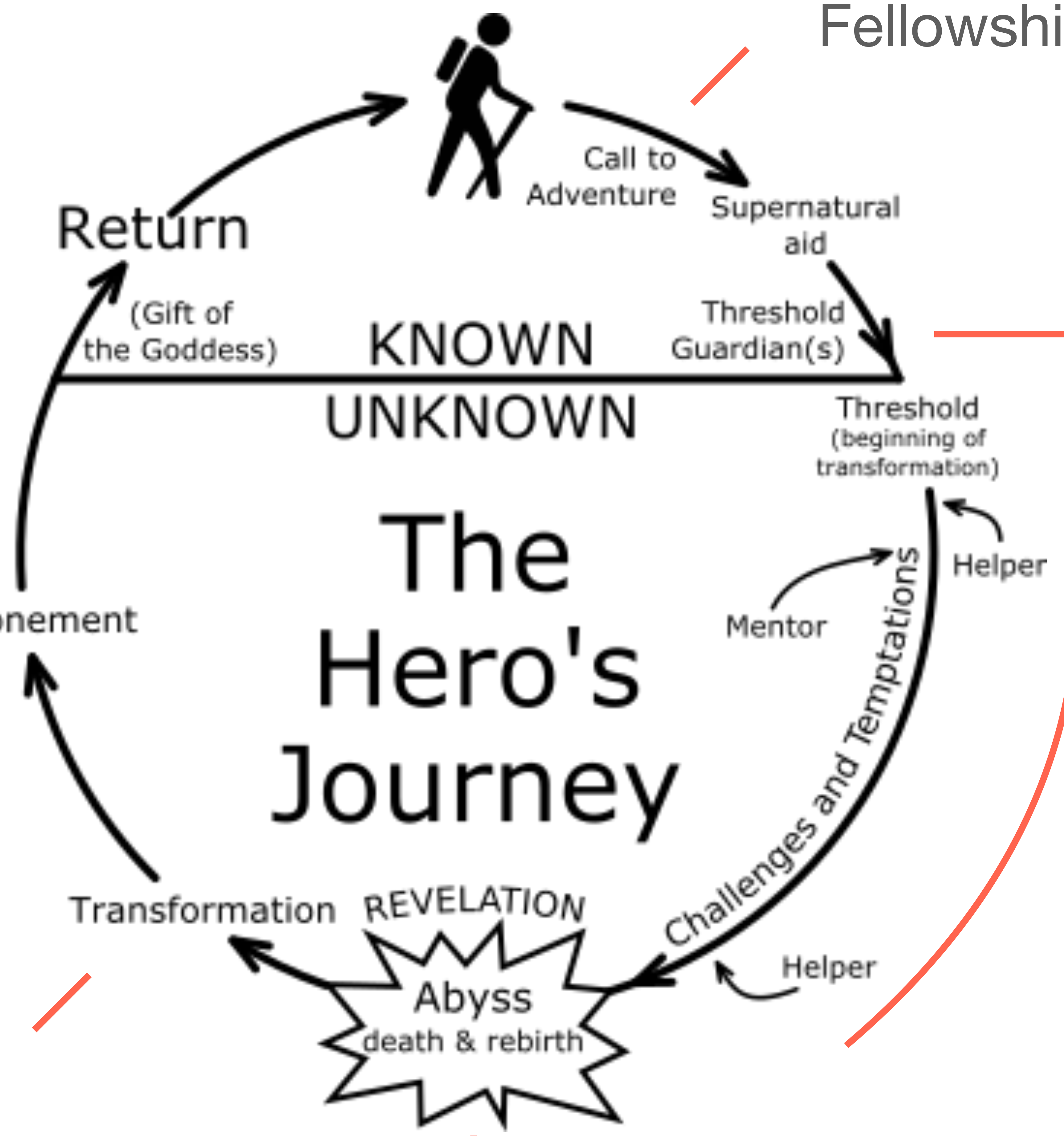
Work!

"We thank the referees for their careful reading of our manuscript…"

Preparation of the manuscript

Results!

Image source: Wikipedia Commons, public domain



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Publication

Image source: Wikipedia Commons, public domain

Idea!

Conferences & seminars

Fellowship/collaboration

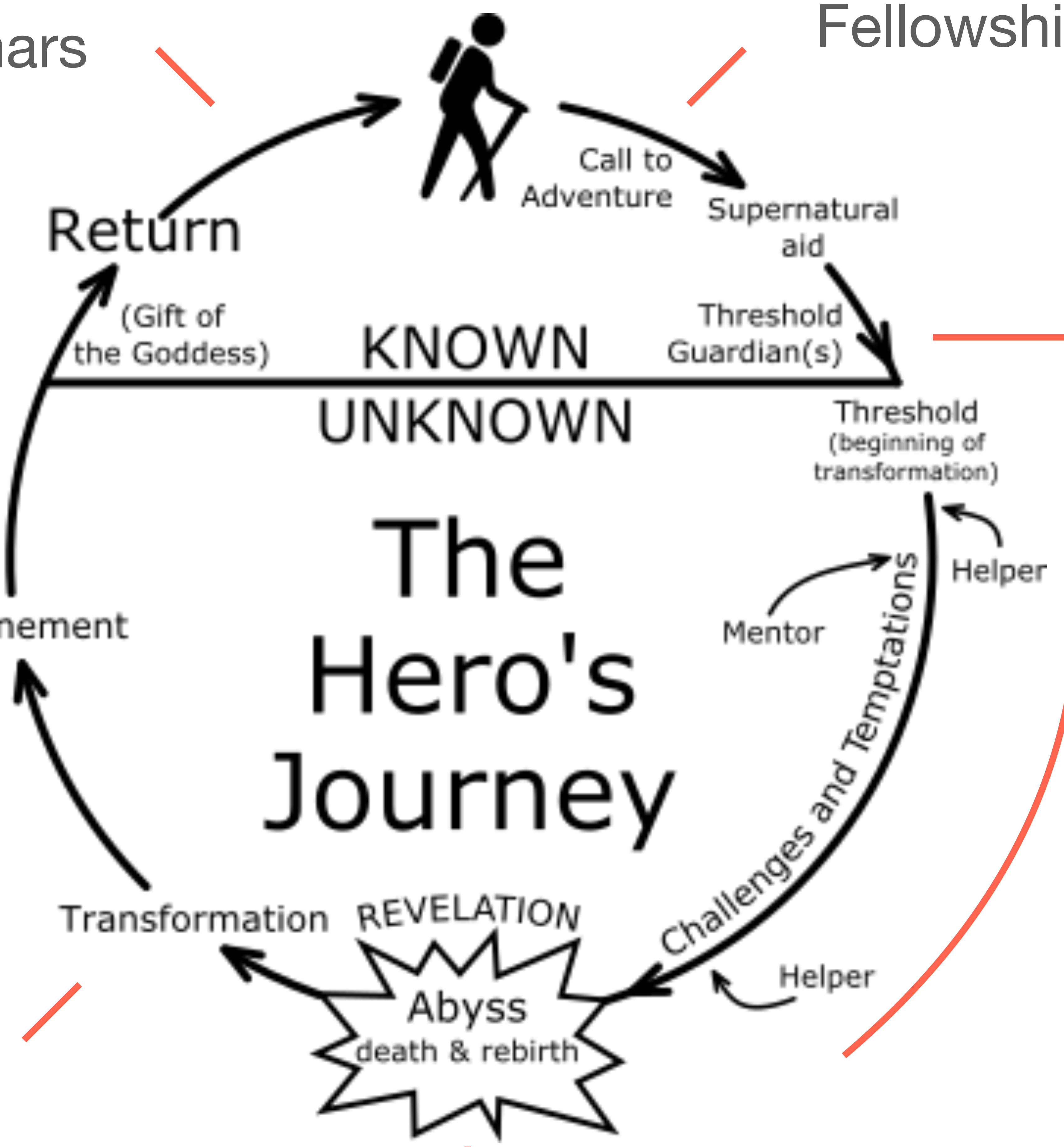

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Image source: Wikipedia Commons, public domain

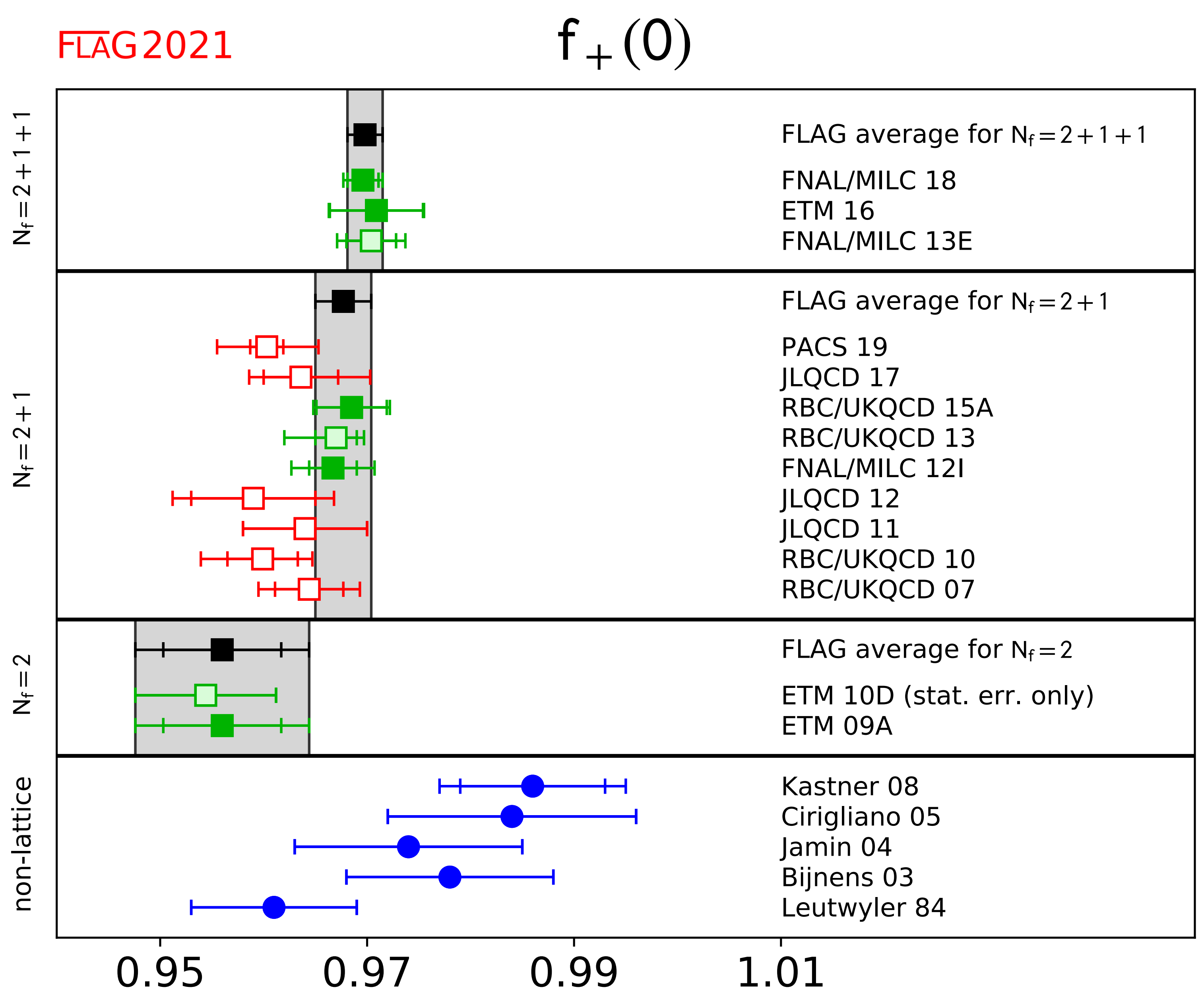
CKM: 1st row

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

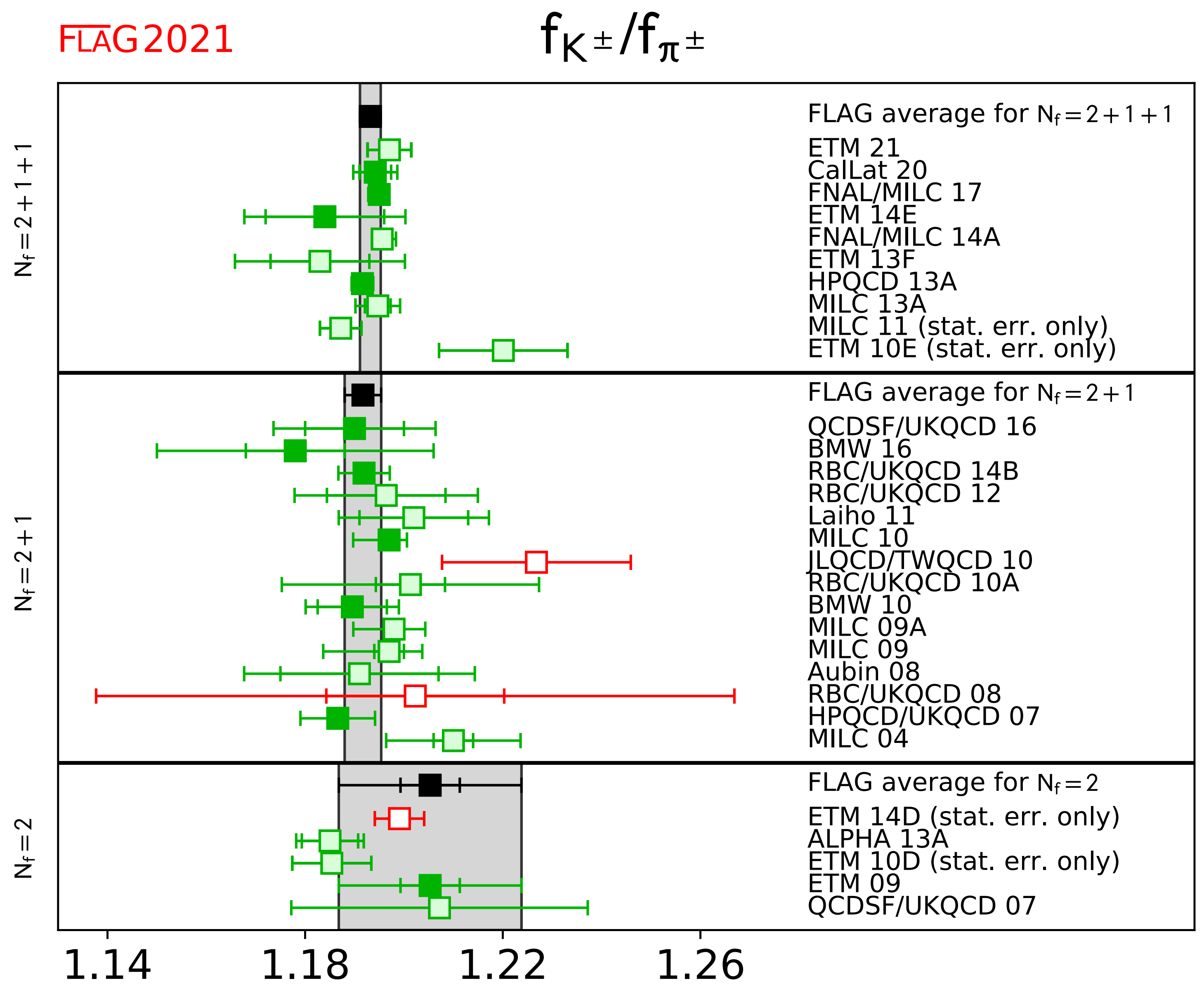
Precision test of actions

Kaon and pion decays

Semileptonic form factor (normalization)



Leptonic decay constants

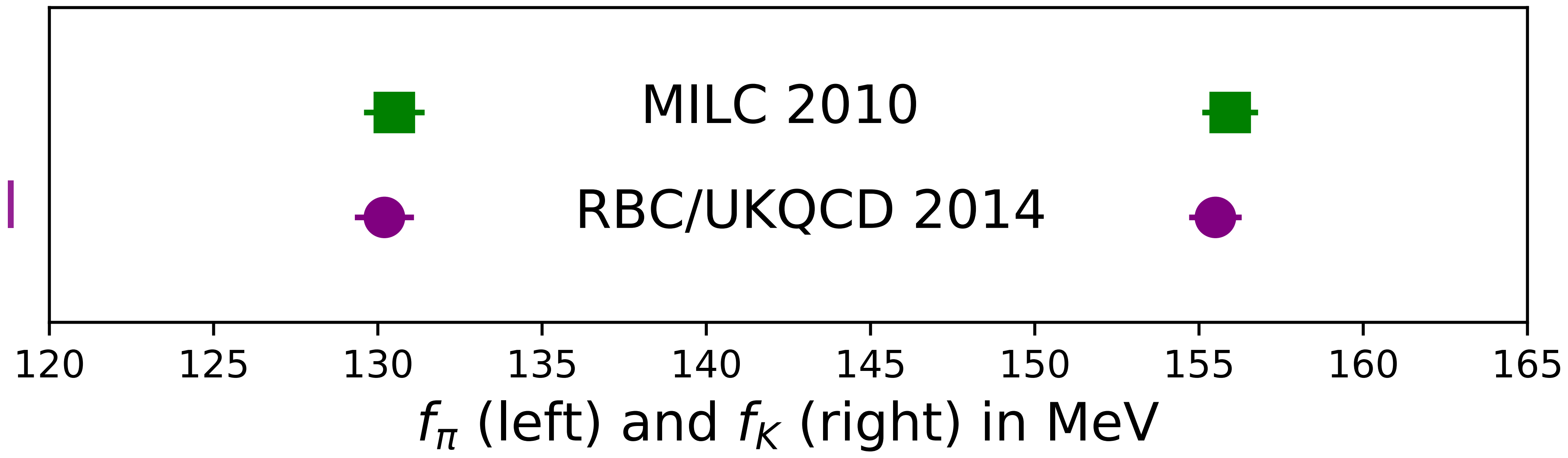


Agreement @ level < 1%

$n_f = 2 + 1$

Staggered

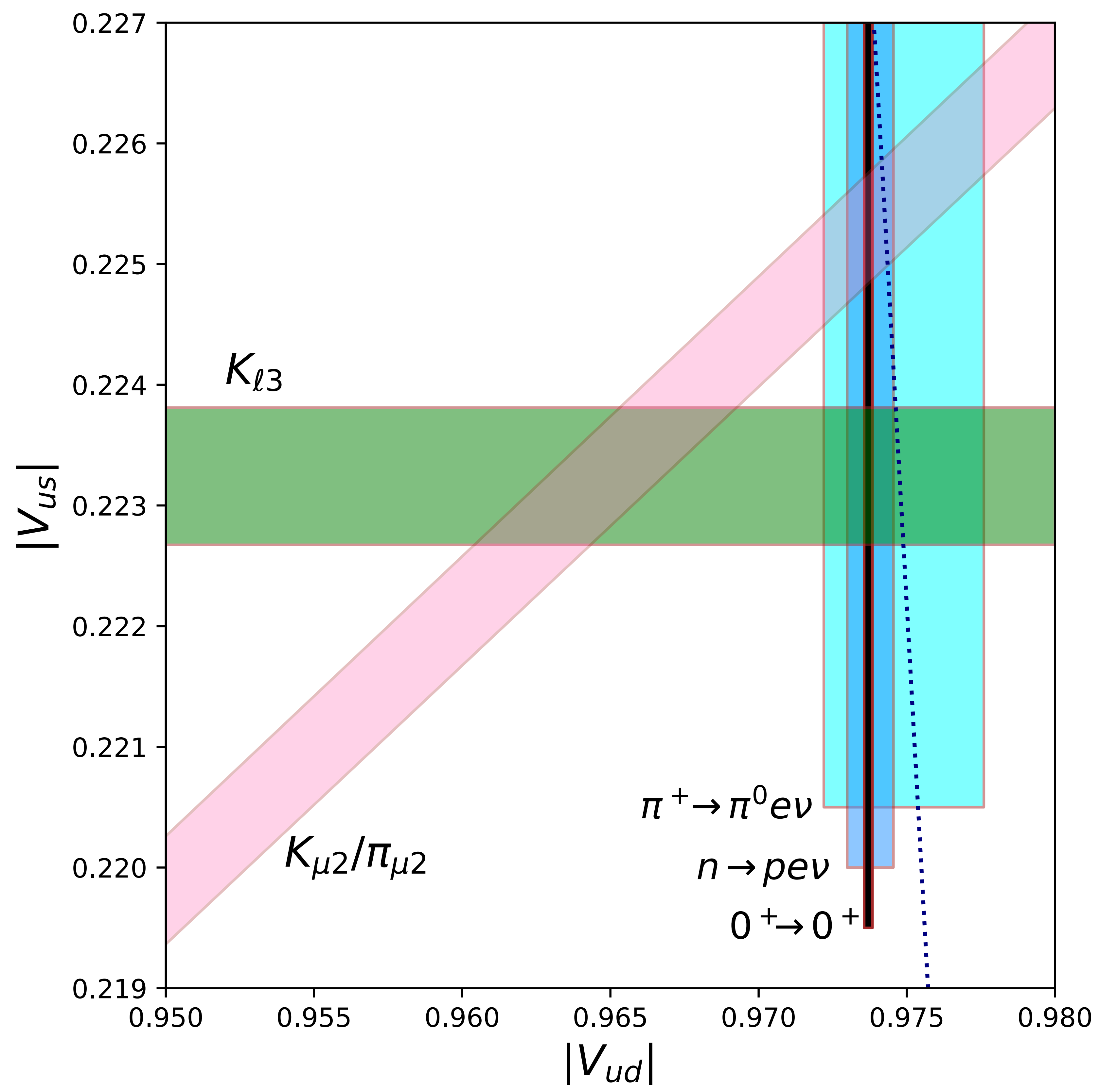
Domain wall



FLAG, arXiv:2111.09849

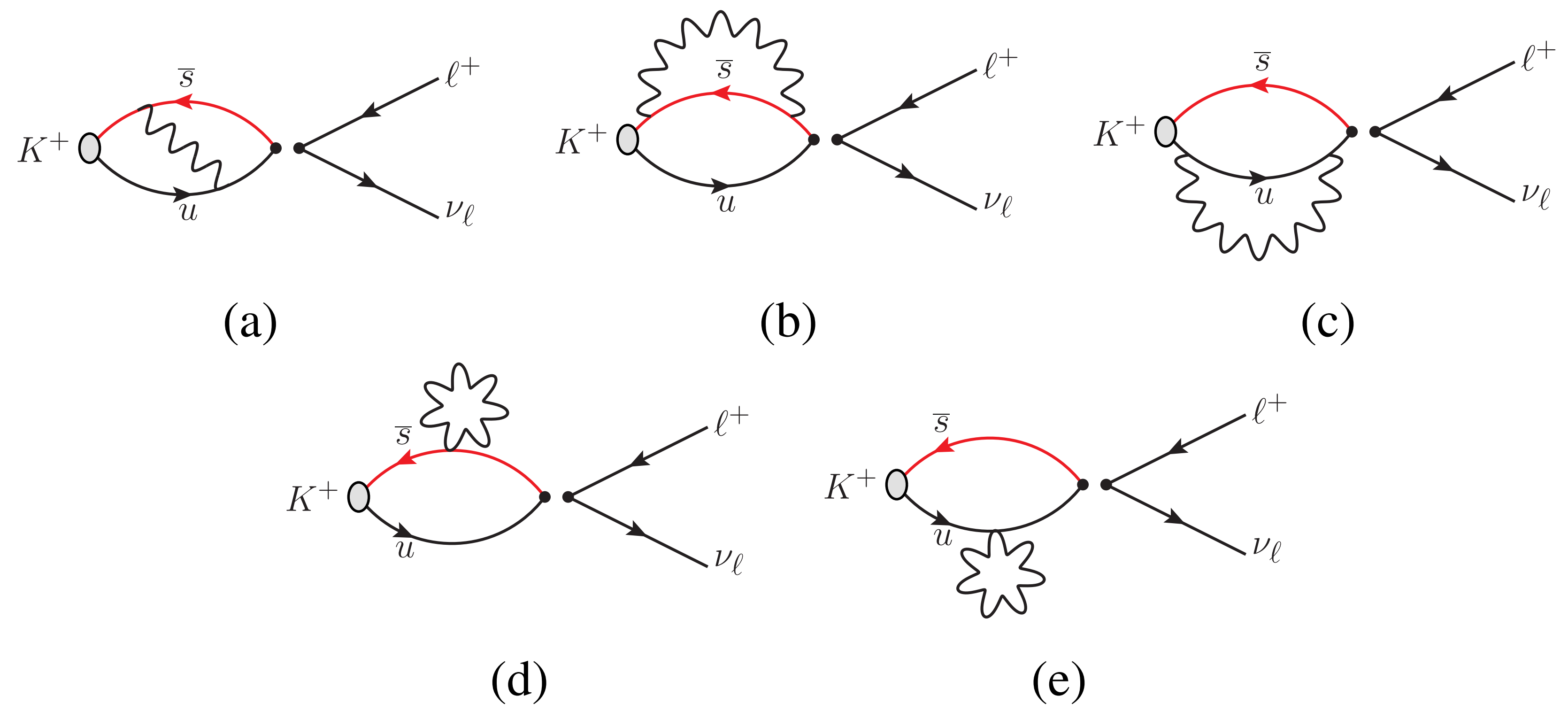
First row, unitarity, "Cabibbo anomaly"

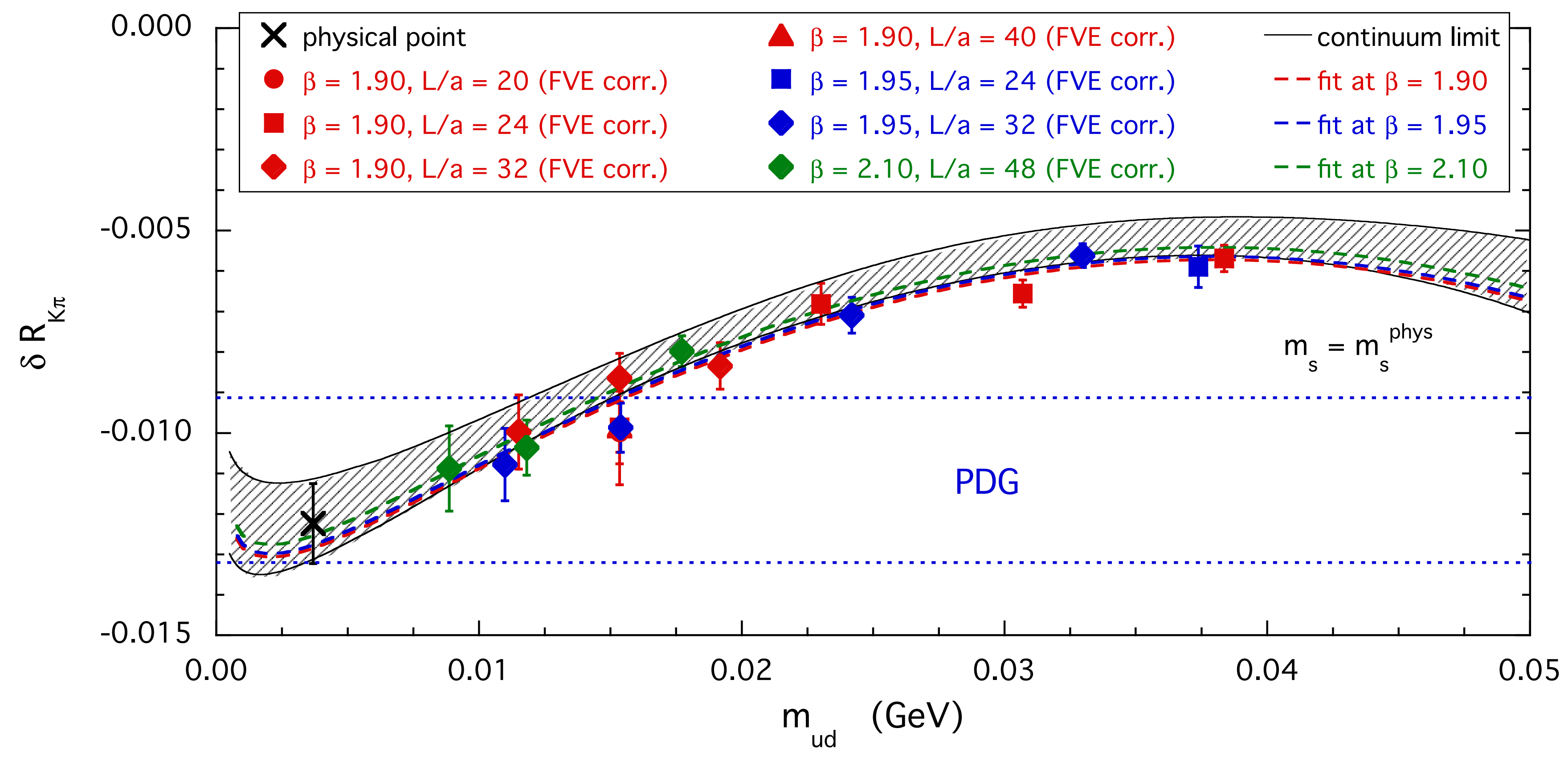
- Note the precision on the axes! Lots of effort to shrink the constraints!
- Dotted line corresponds to $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- Two puzzles:
  — Are 2 parameters enough to explain the 3 classes of constraints?
  — Are the parameters consistent with CKM unitarity?
- Tension with unitarity due to new result from nuclear decays
- Precision of lattice matrix elements now means unequal *u/d* mass and QED effects are required



Including QED effects

- RM123 method to include isospin breaking effects in LQCD determinations of leptonic decays of mesons (must include soft photon in final state)
- Several technical issues to face: finite volume effects, cancellation of IR divergences
- First calculation found good agreement with chiral P.T. predictions
- Continuing progress to extend applicability to larger photon momentum, requires electromagnetic form factors



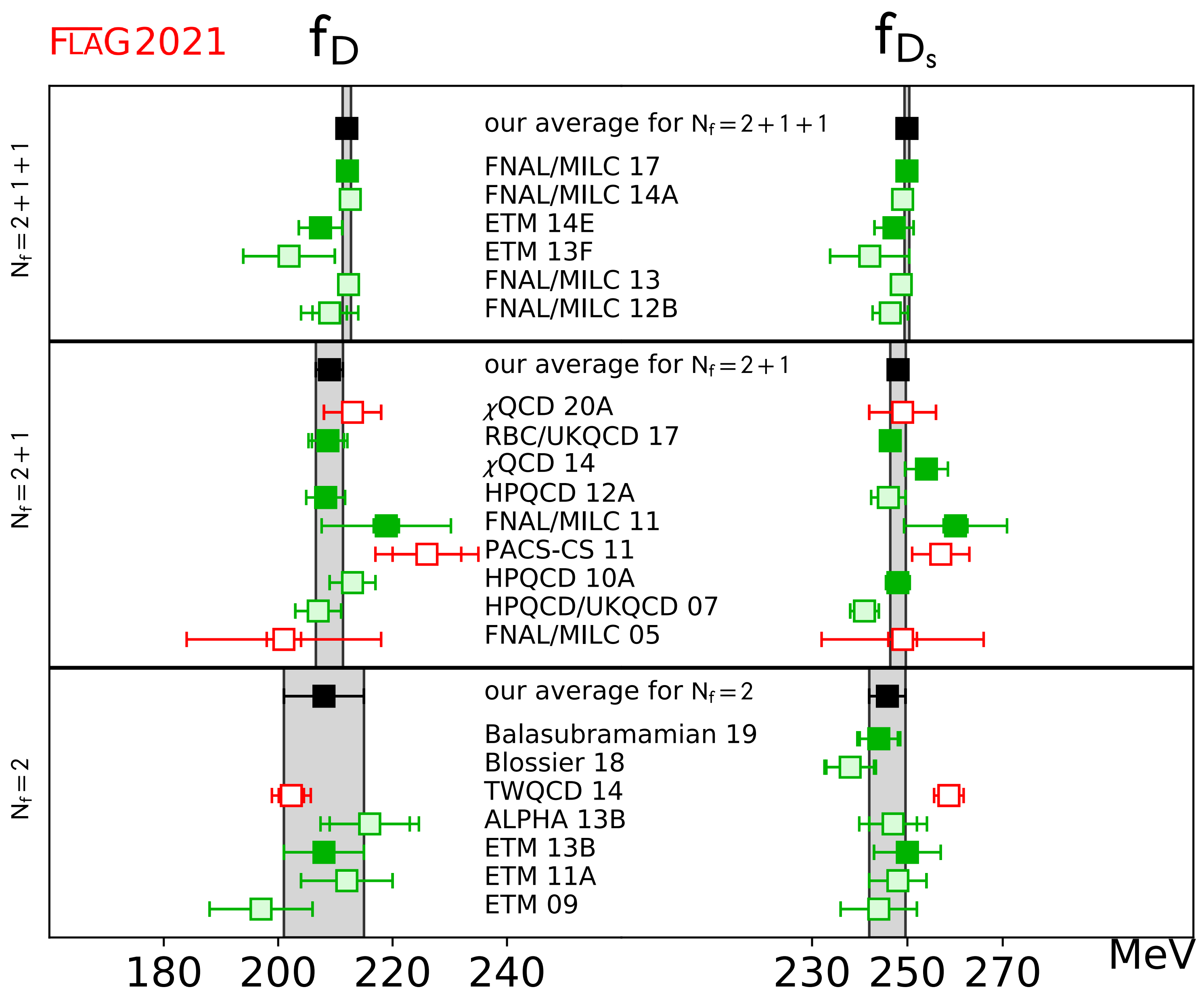


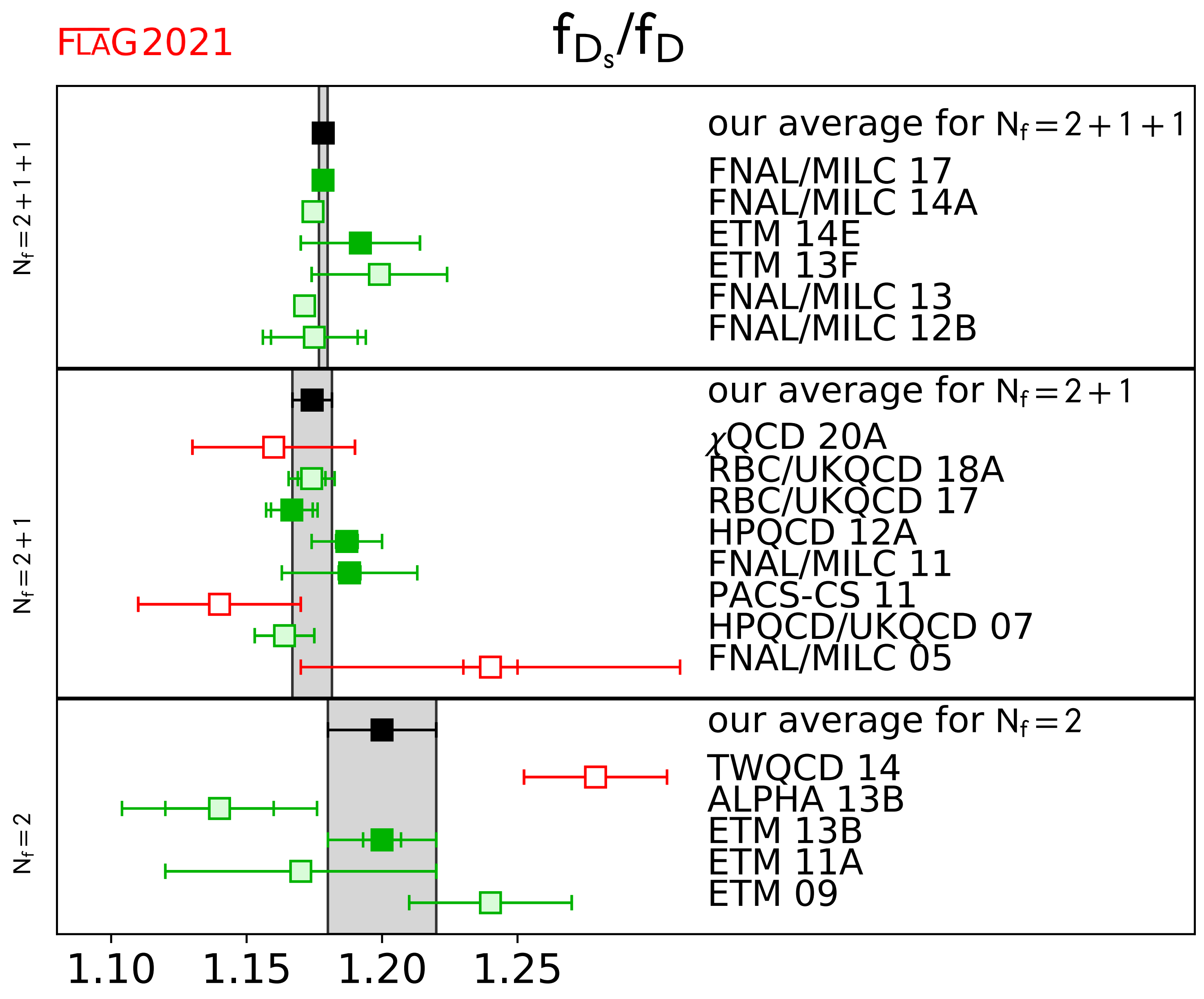
Giusti et al., PRL 120, 072001 (2018)

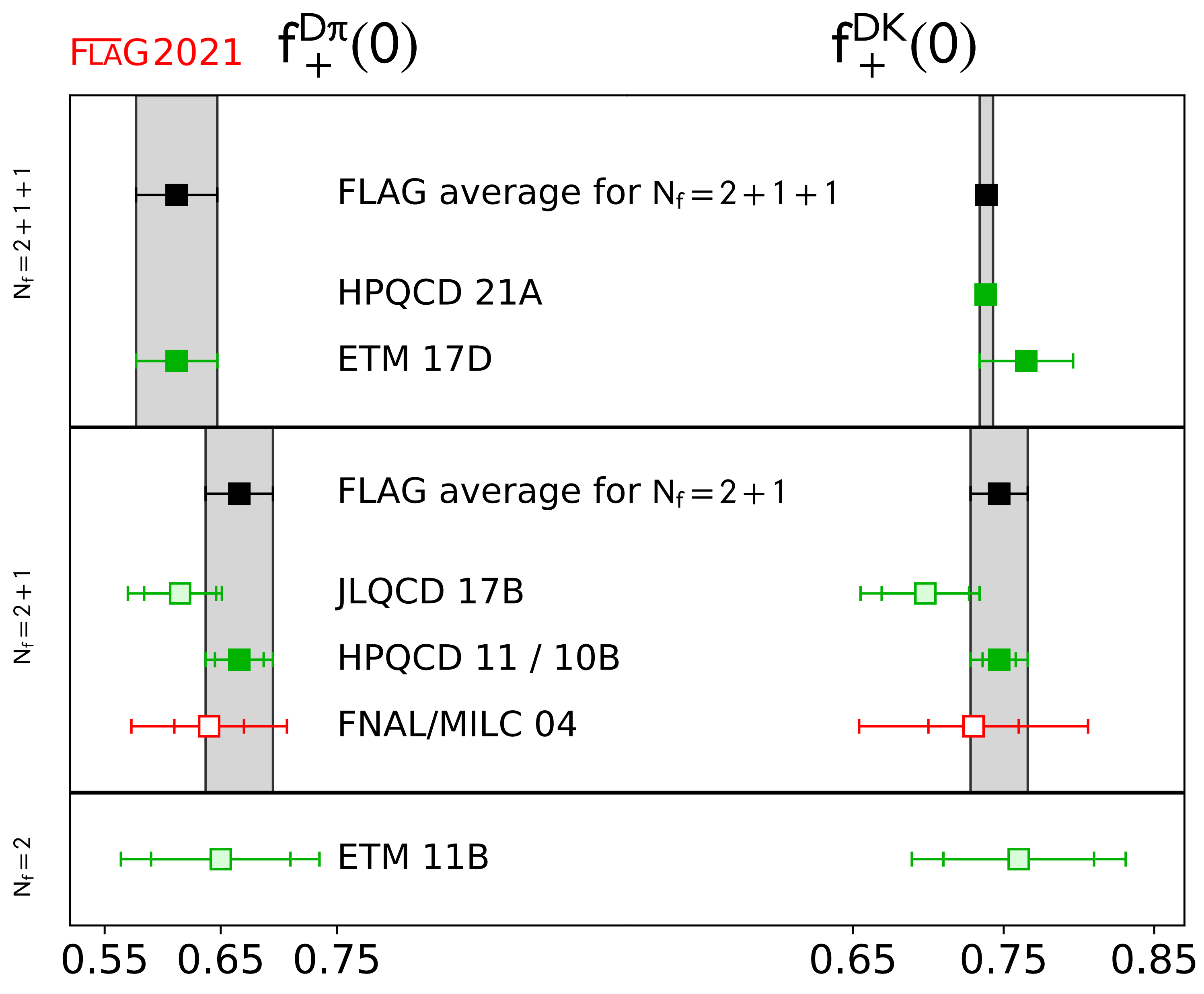
CKM: 2nd row

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

D leptonic and semileptonic decays



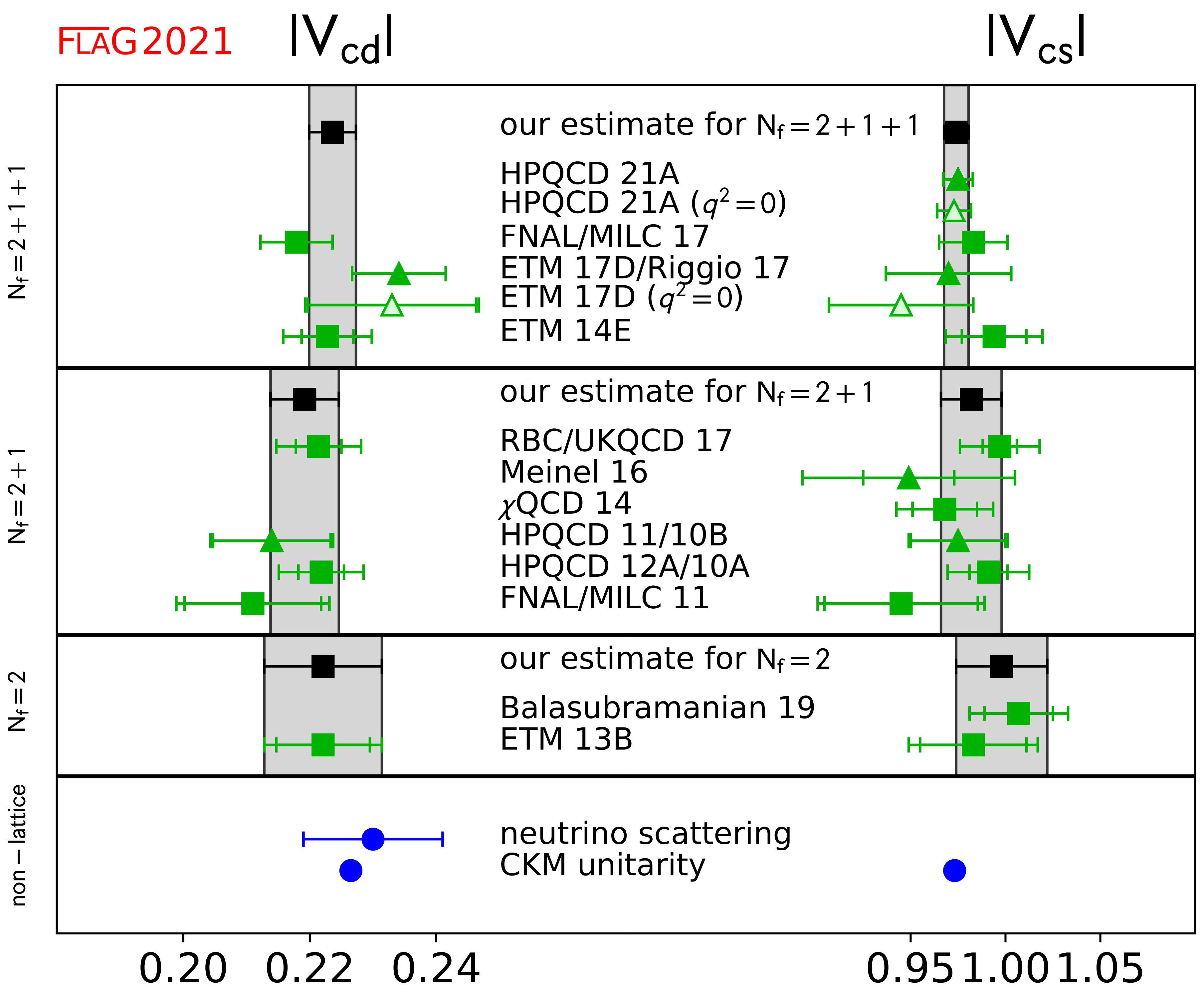


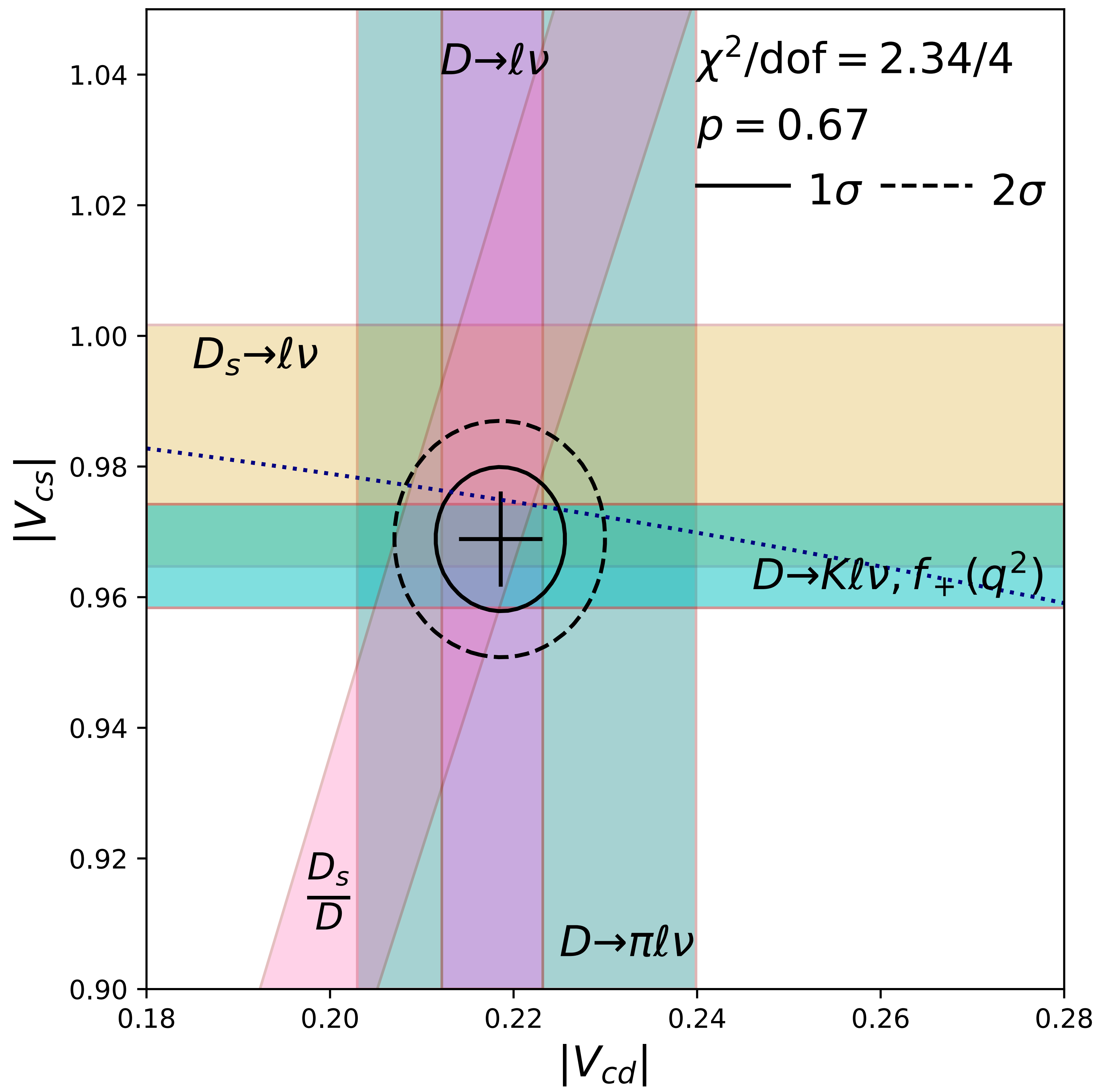


Much more on $D \to K\ell\nu$ from Chris Bouchard, Tues 09:30

FLAG, arXiv:2111.09849

Second row and unitarity

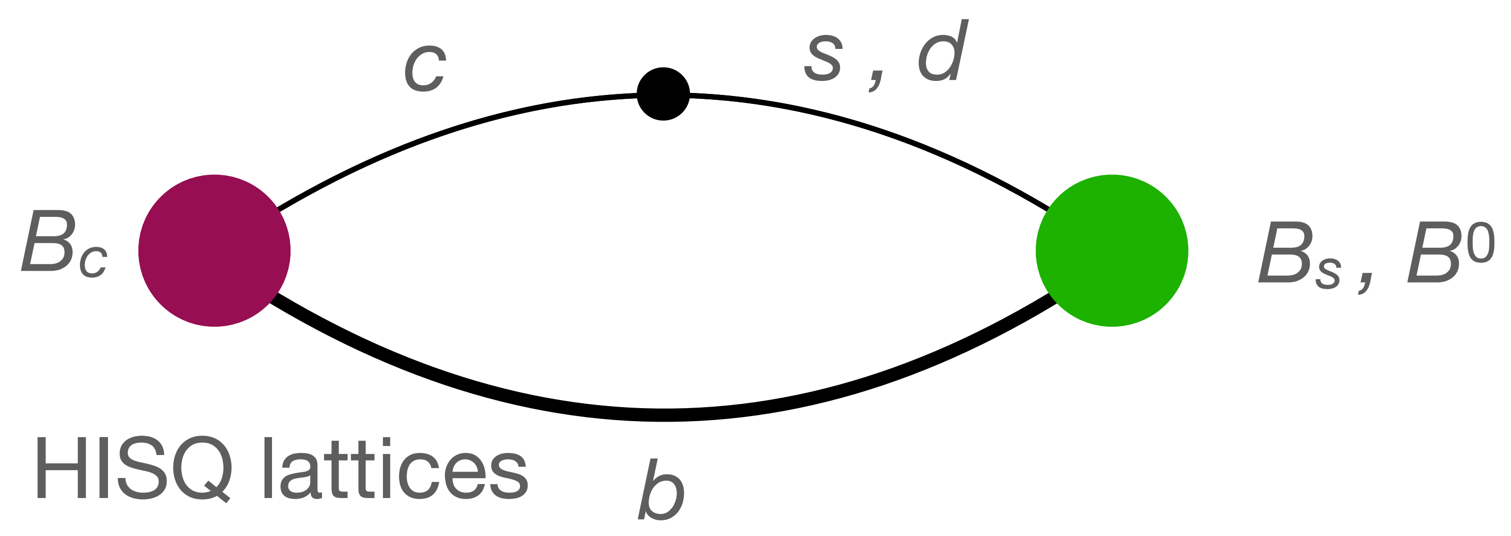




FLAG, arXiv:2111.09849

Fig. from MW, Eur. Phys. J A 57, 239 (2021)

Charm decays in B_c

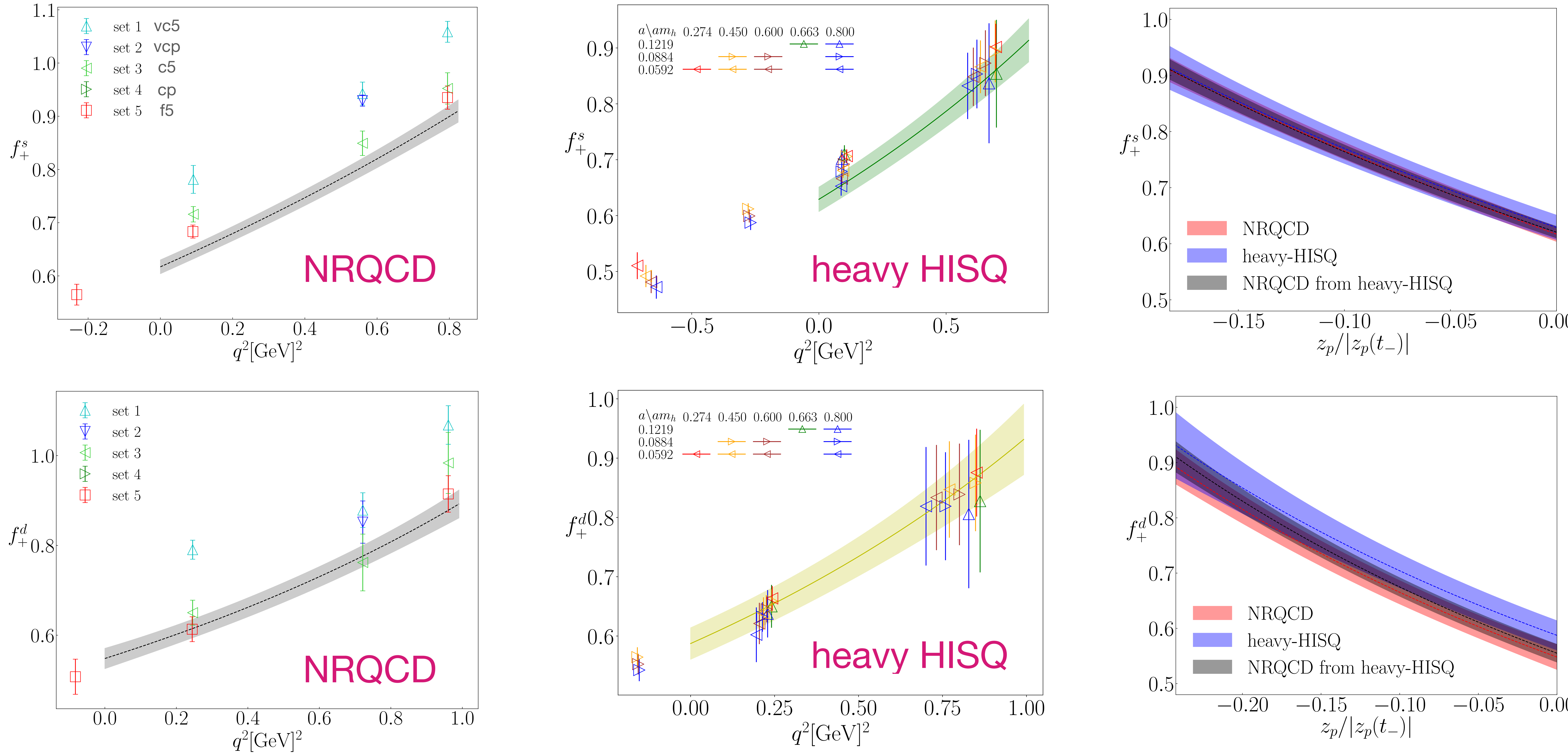


HPQCD: Combined analysis of NRQCD and heavy-HISQ results on MILC 2+1+1 HISQ lattices

Vector form factor f_+

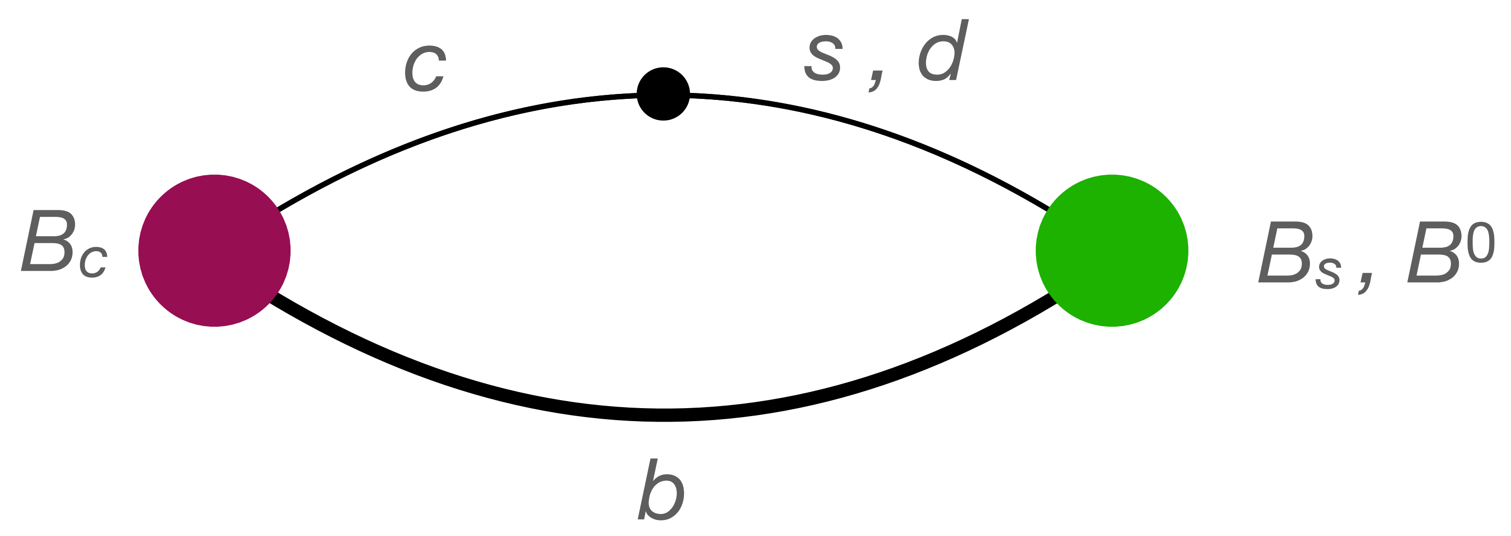
$c \to s$

$c \to d$

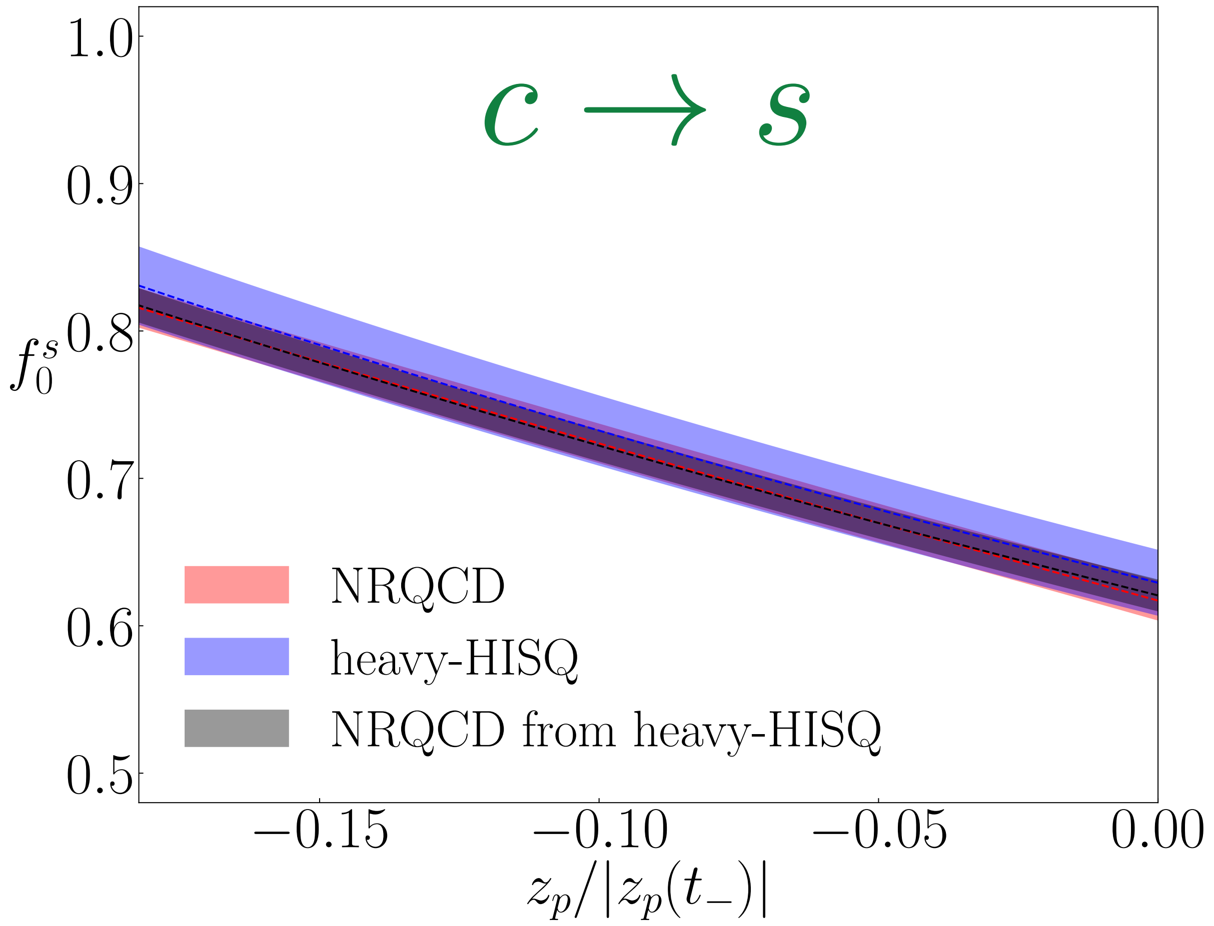


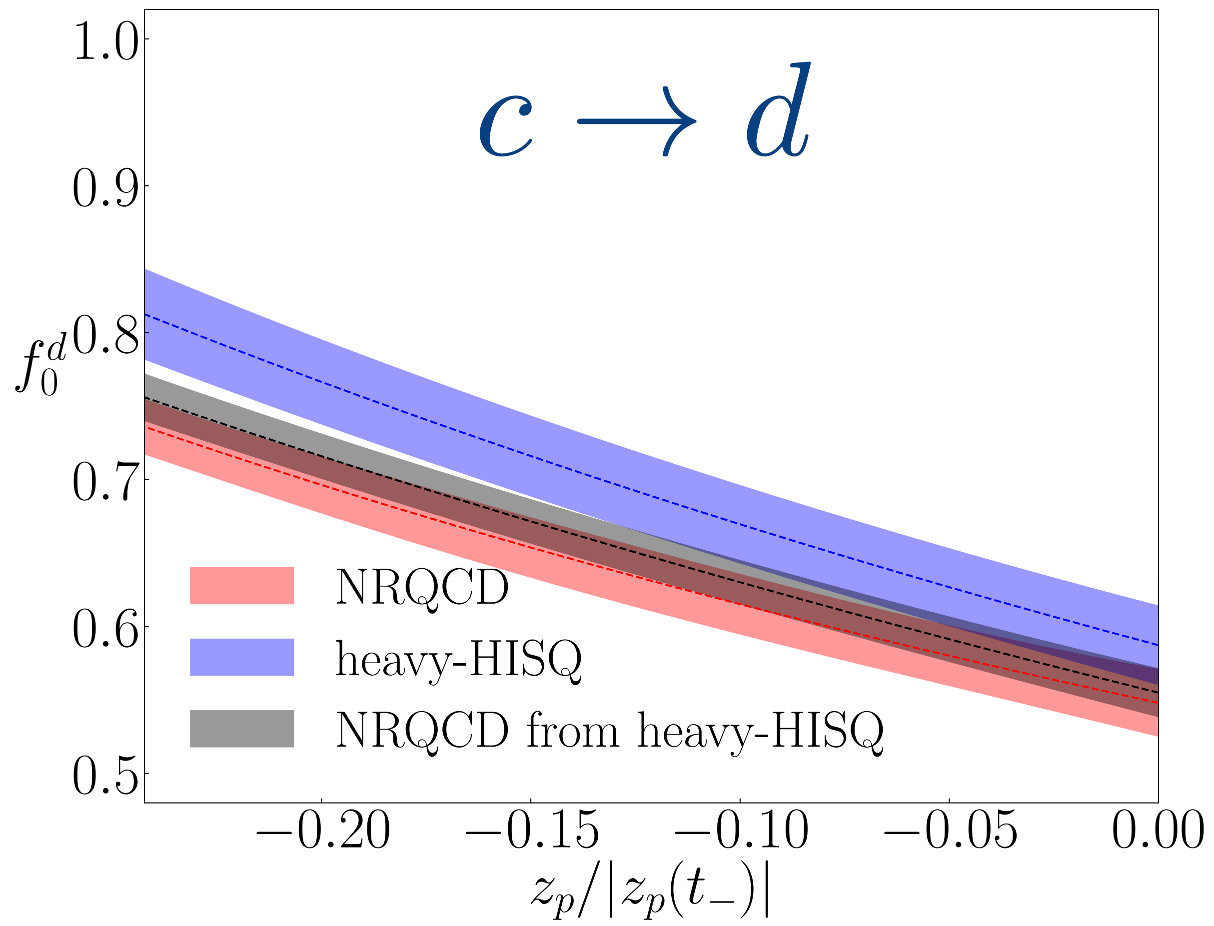
Cooper et al., (HPQCD) Phys. Rev. D 102, 014513 (2020)

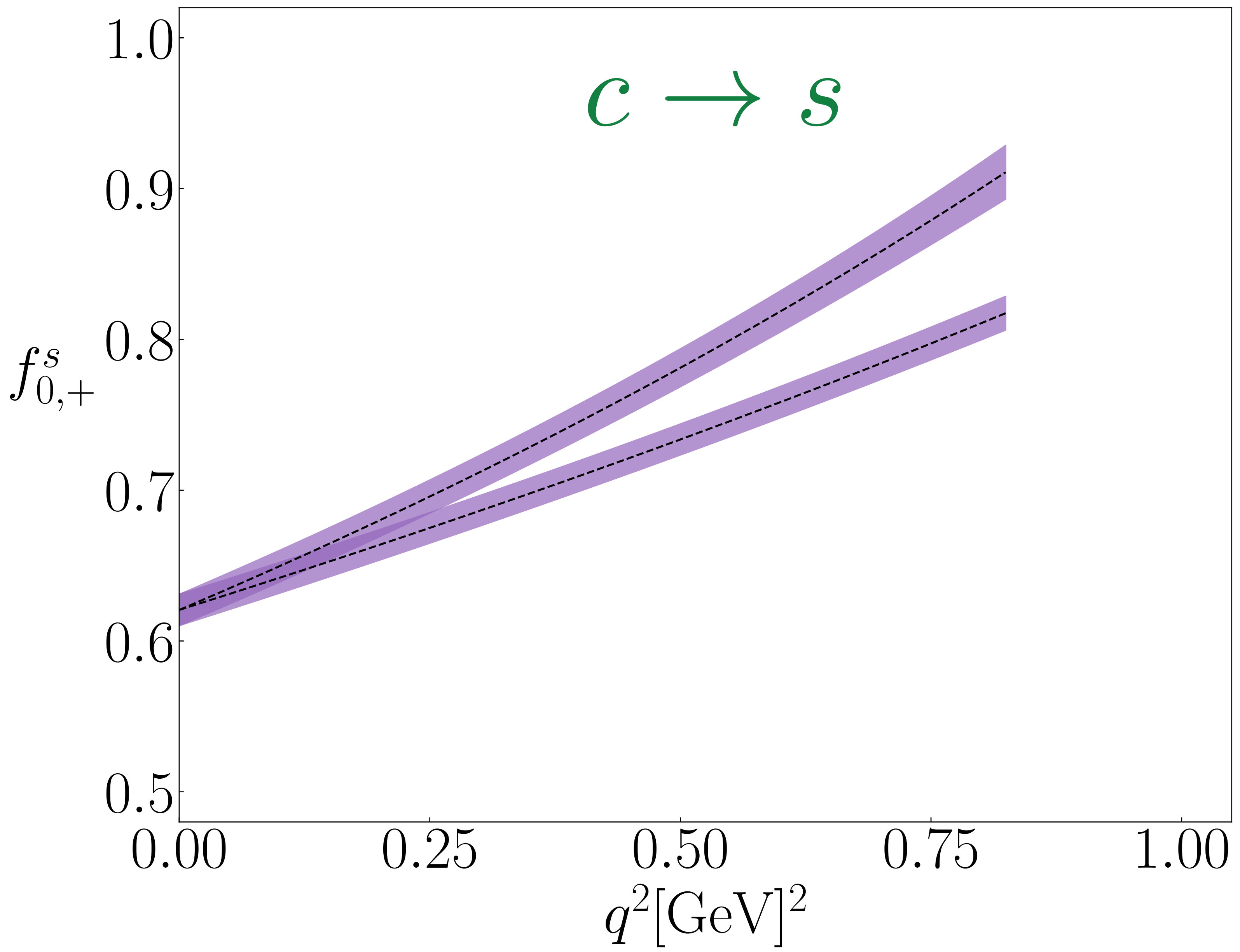
Charm decays in B_c

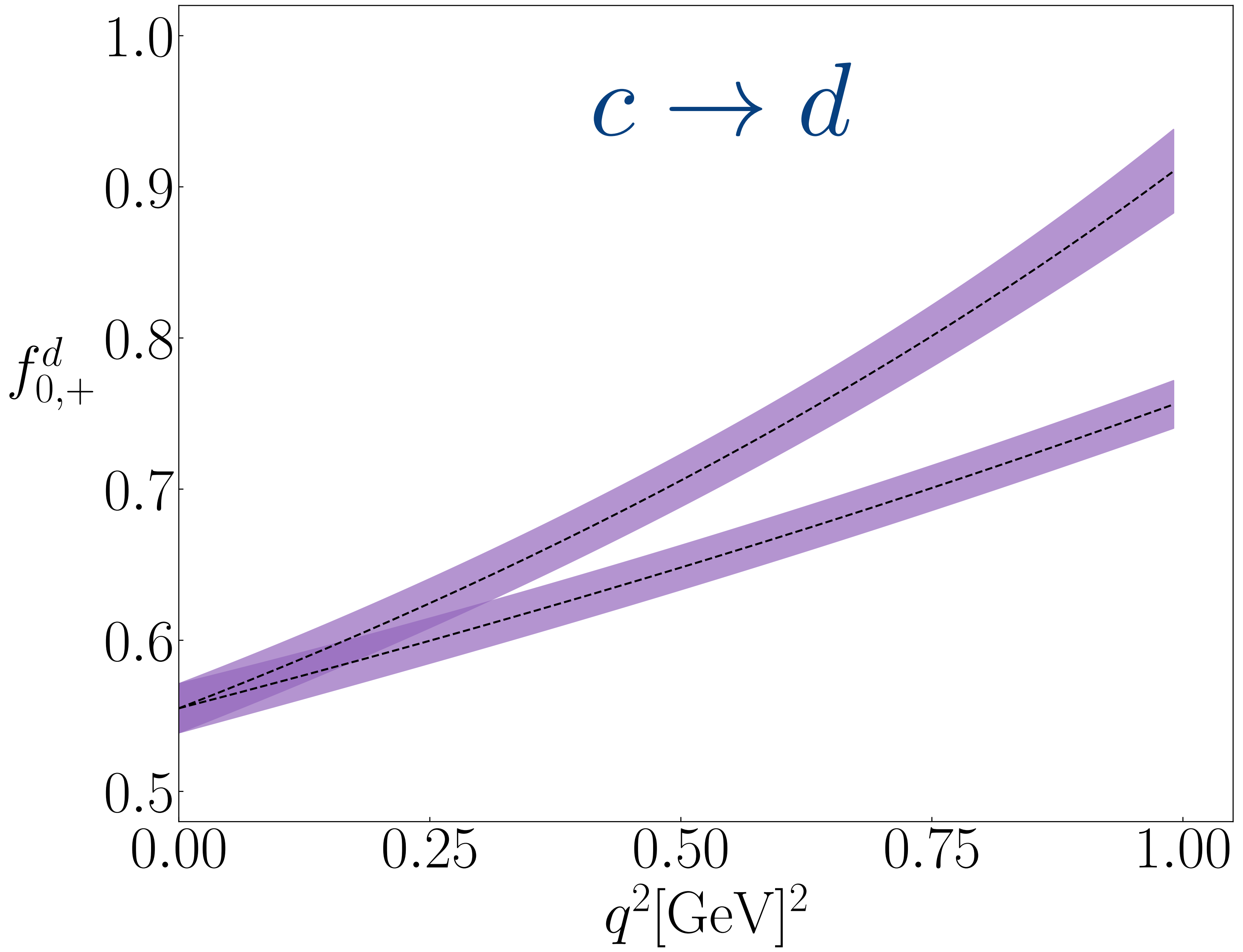


Scalar form factor f_0









SM predictions:

$$\Gamma(B_c^+ \to B_s^0 \bar{\ell}\nu_\ell) = 26.2(1.2) \times 10^9\,\mathrm{s}^{-1}$$

$$\Gamma(B_c^+ \to \bar{B}^0 \bar{\ell}\nu_\ell) = 1.65(10) \times 10^9\,\mathrm{s}^{-1}$$

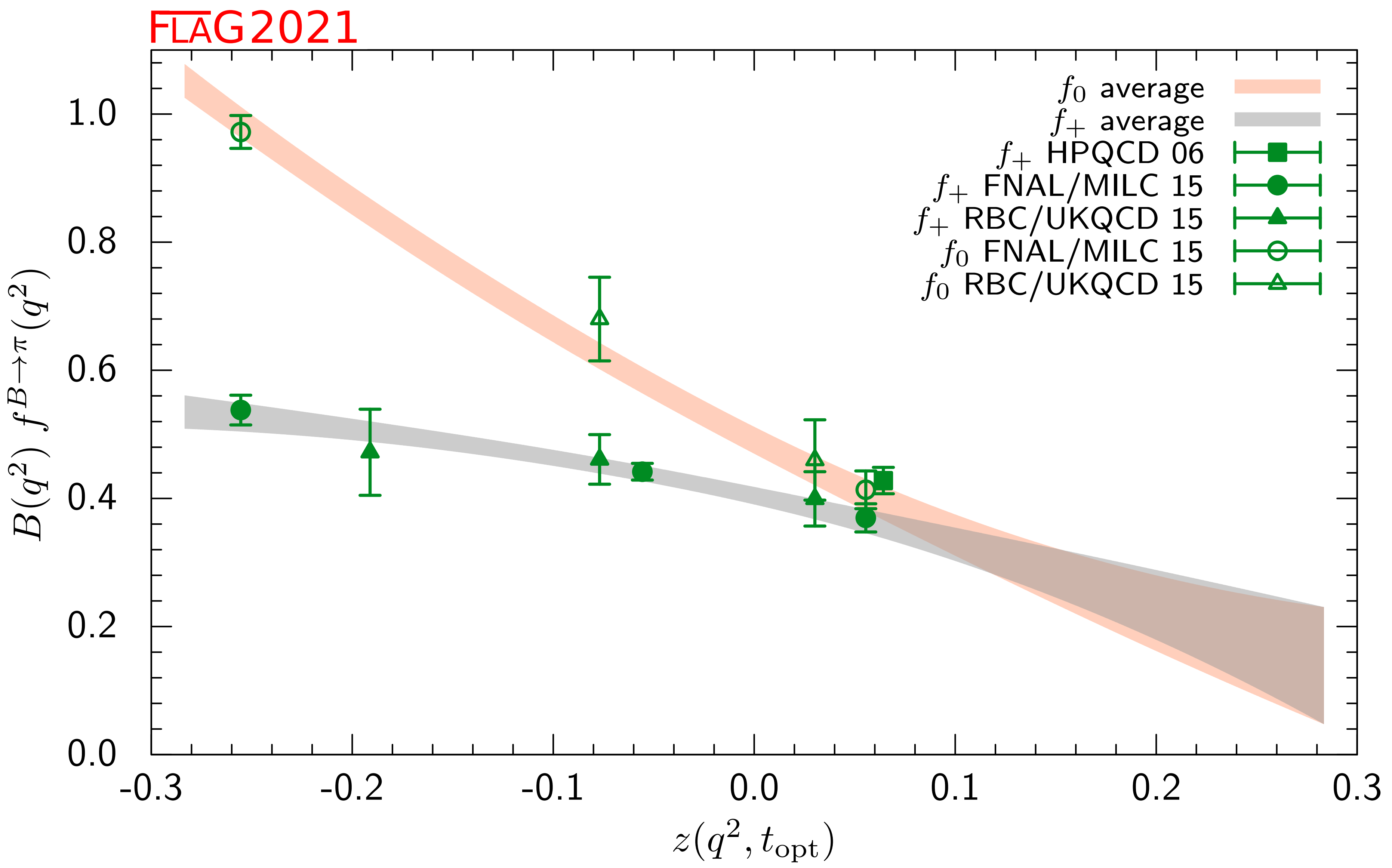
Cooper et al., (HPQCD) Phys. Rev. D 102, 014513 (2020)

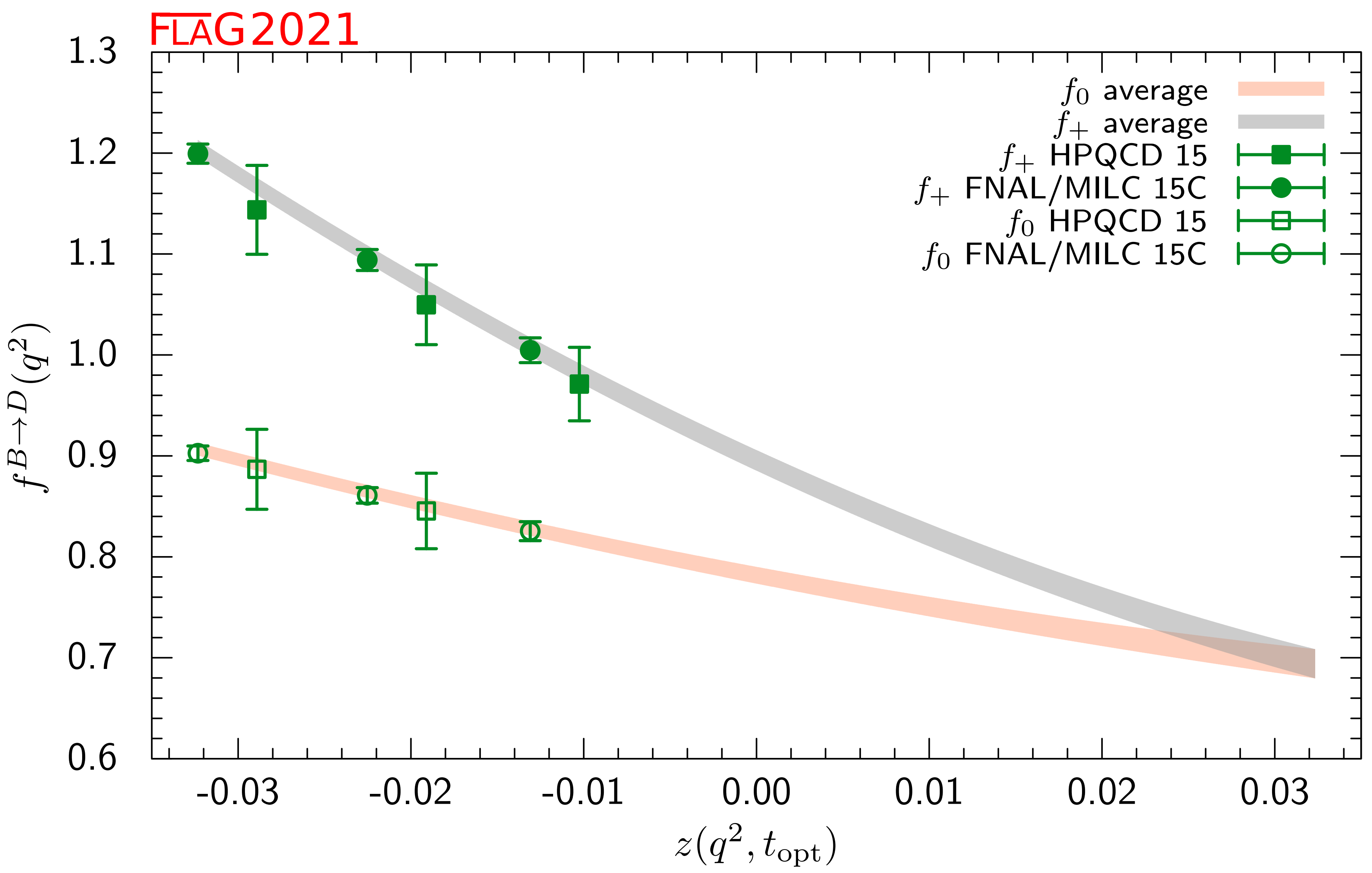
CKM: 3rd column & V_{xb}

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Semileptonic decays
High precision modes

$B \to \pi$ and $B \to D$ form factors awaiting updates

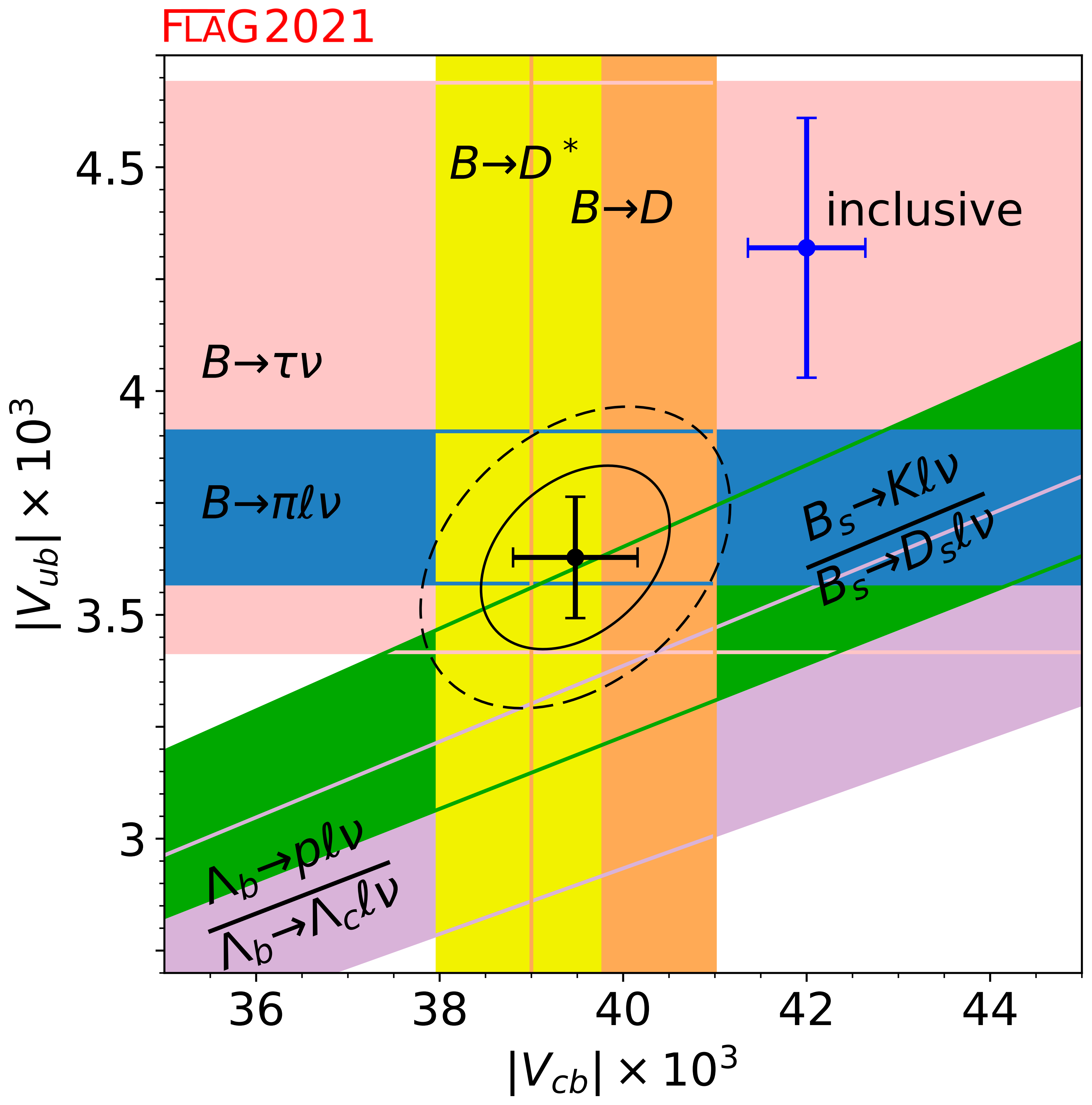




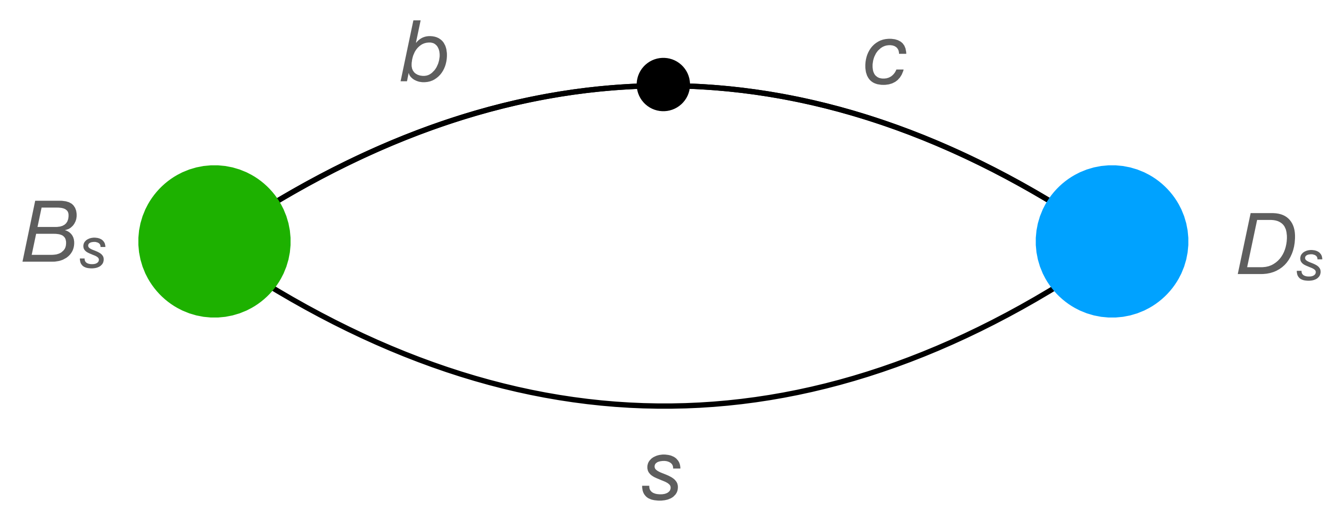
Talk by Alejandro Vaquero next, covering new results for $B \to D^*$ form factors

Third column

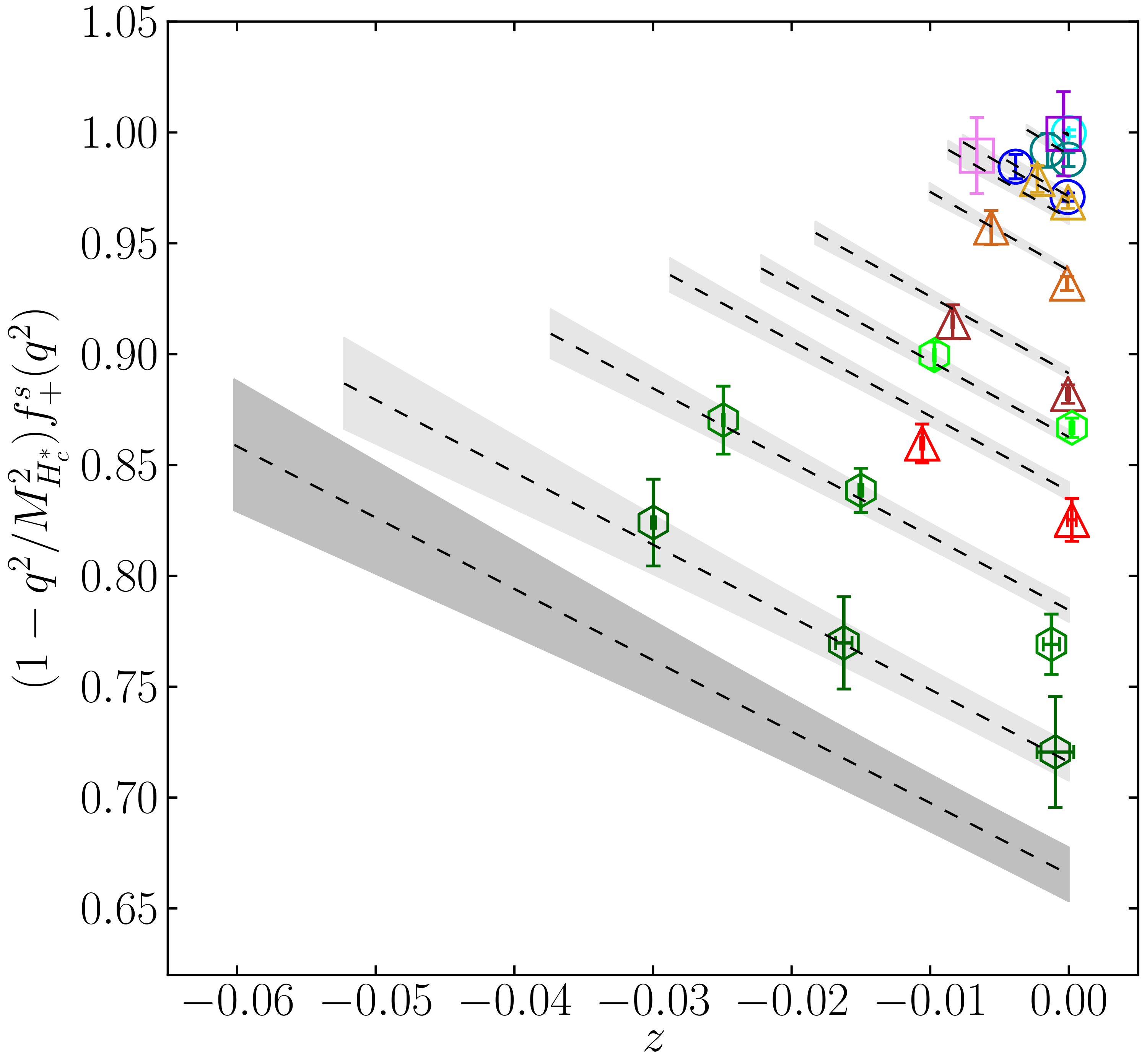
- Persistent tension between inclusive and exclusive determinations
- Much more about $B \to D^* \ell \nu$ in Alejandro Vaquero's talk on Monday at 11:00
- Details of form factor calculations also in Chris Bouchard's talk on Tuesday at 09:30
- More exclusive modes being measured, requiring corresponding form factors

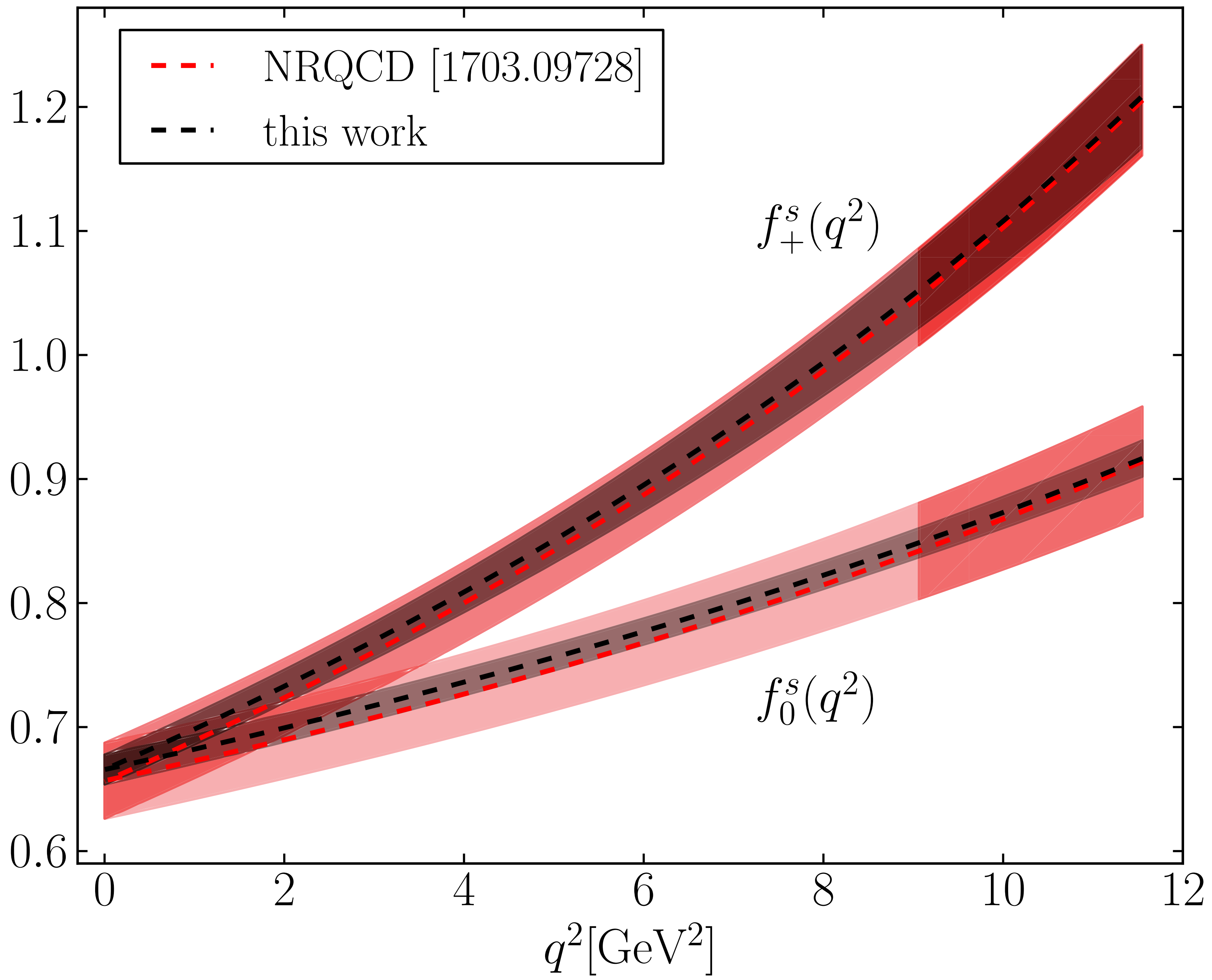


Strange spectator: $b \to c$



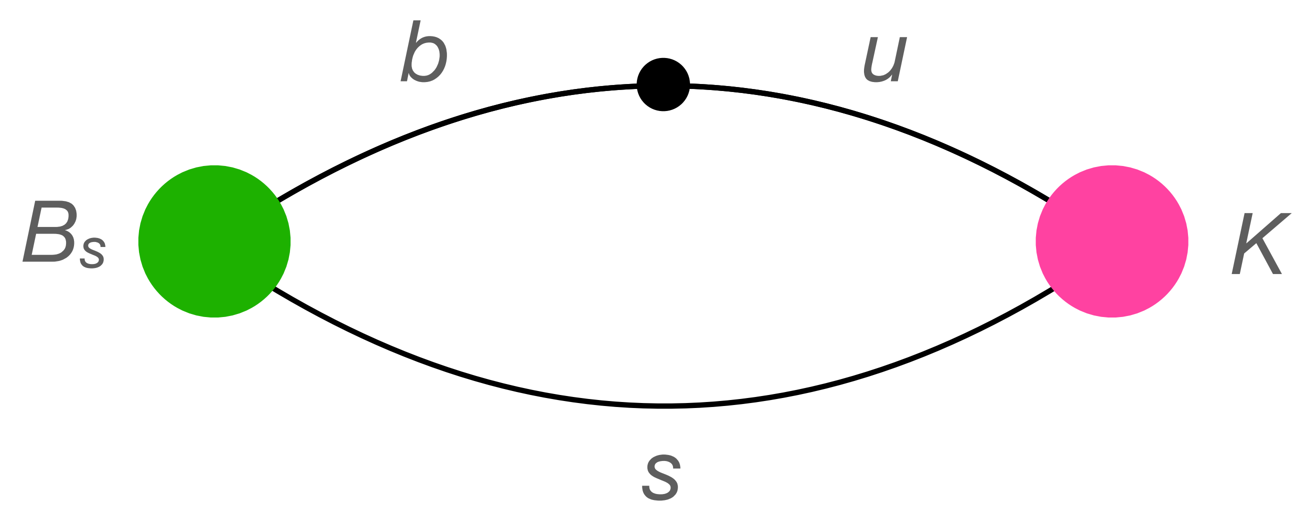
HPQCD: Heavy HISQ on MILC's n_f = 2+1+1 HISQ lattices

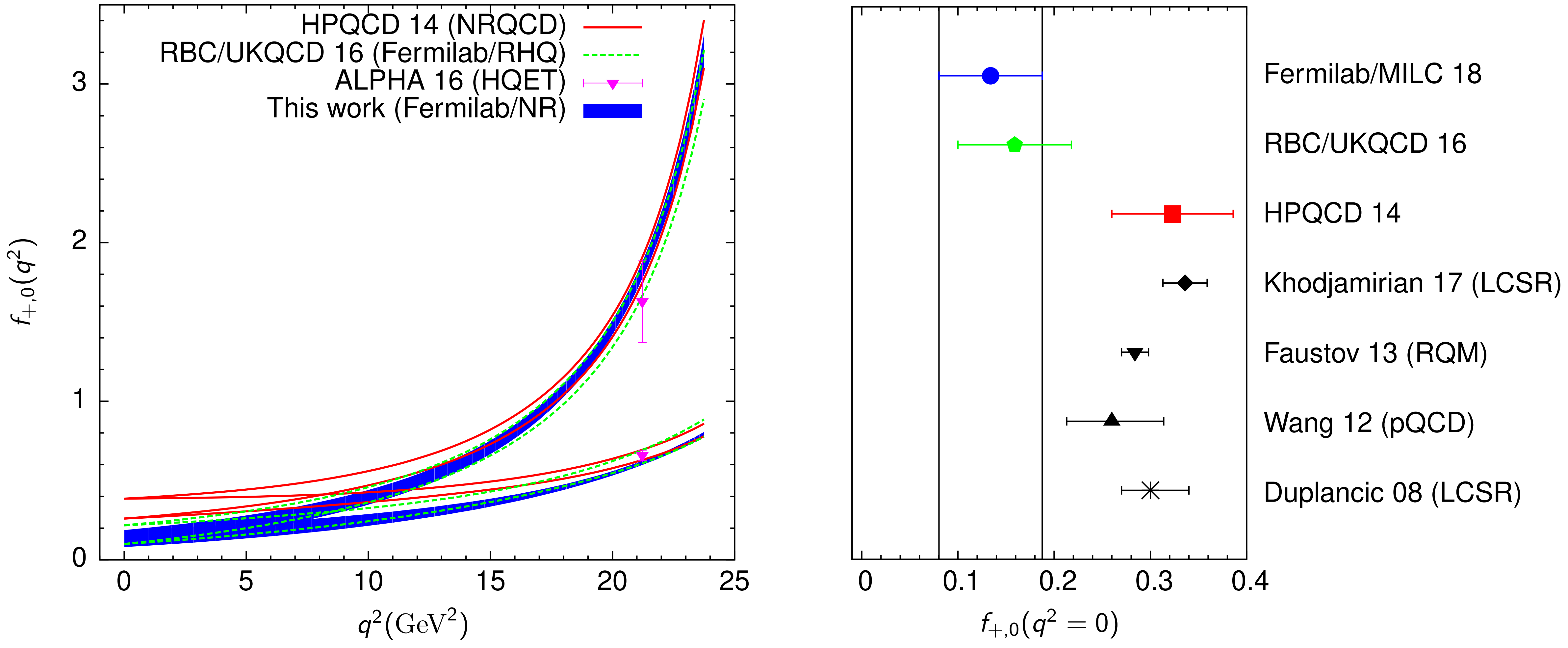




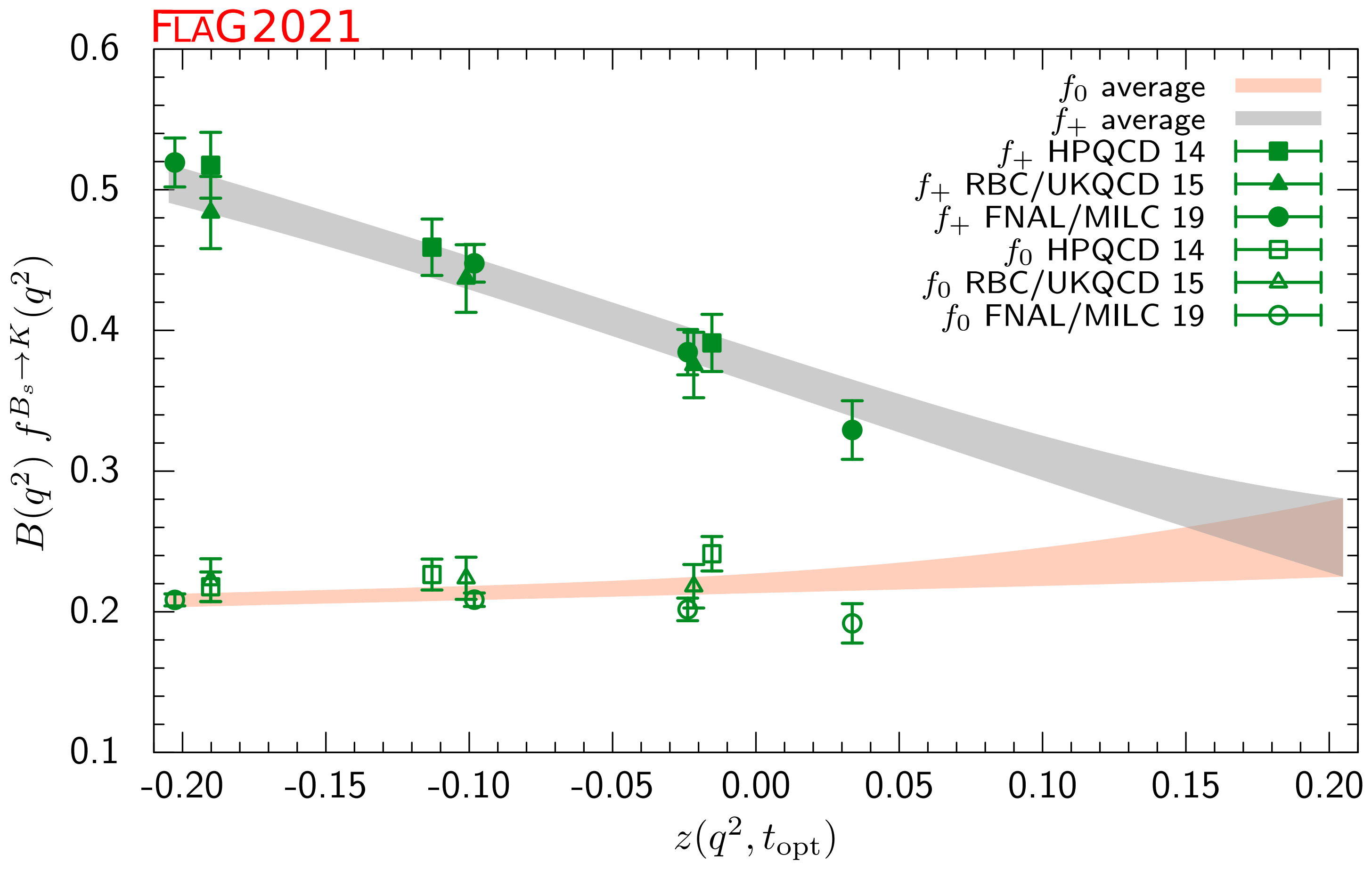
McLean, Davies, Koponen, Lytle (HPQCD), Phys. Rev. D 101, 074513 (2020)

Strange spectator: *b* → *u*

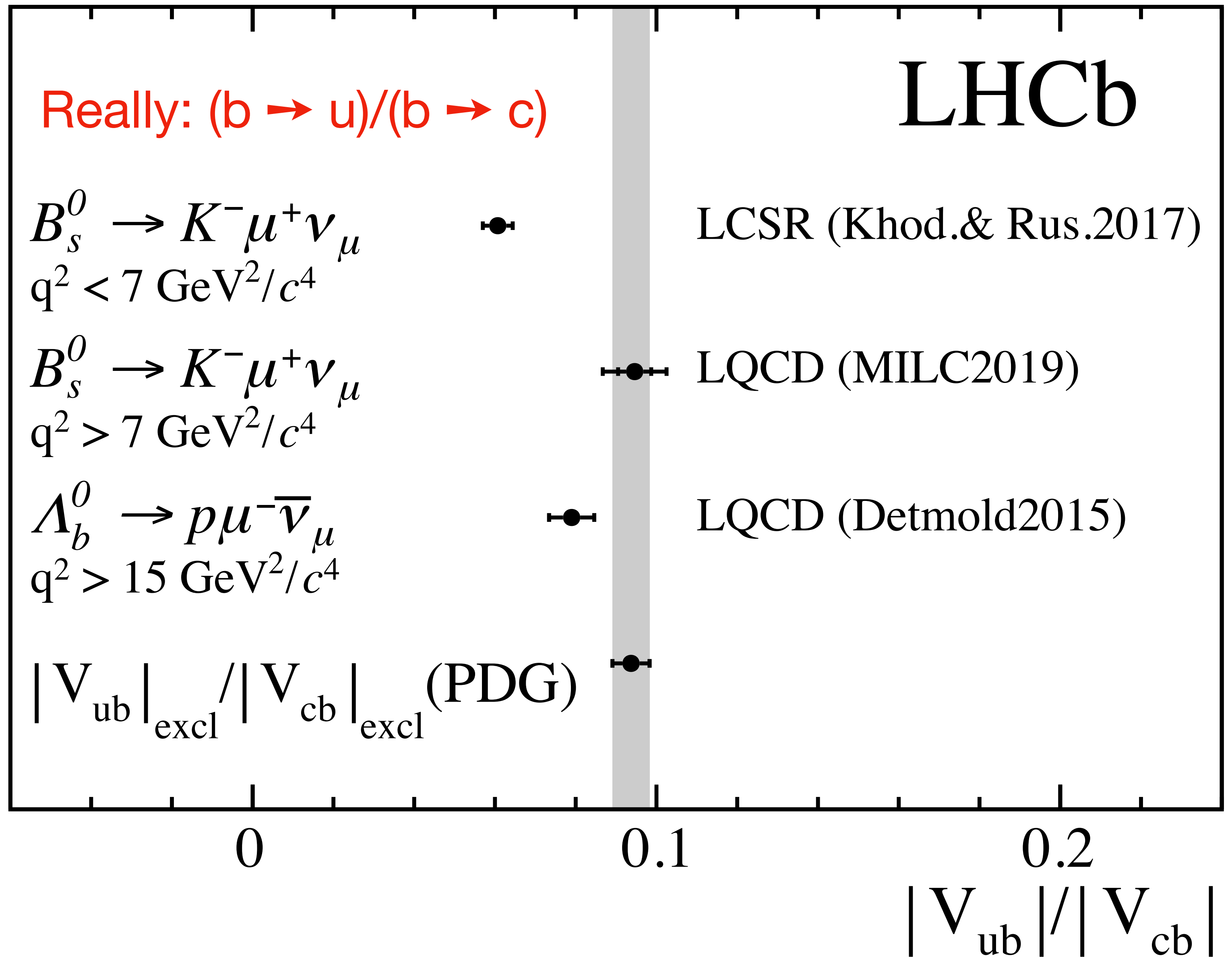




Figures from Bazavov et al., (Fermilab/MILC), Phys. Rev. D 100, 034501 (2019)

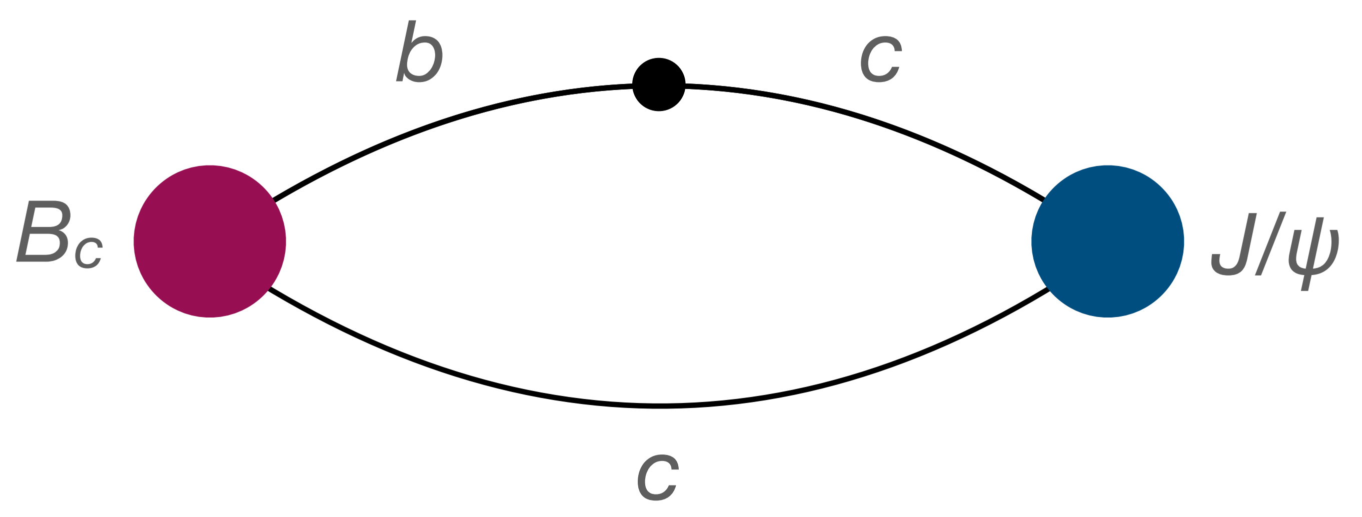


FLAG2021: Fit to $B_s \to K$ form factor data has χ²/d.o.f. = 1.54



LHCb, Phys. Rev. Lett. 126, 081804 (2021)

B_c semileptonic decays: *b* → *c*



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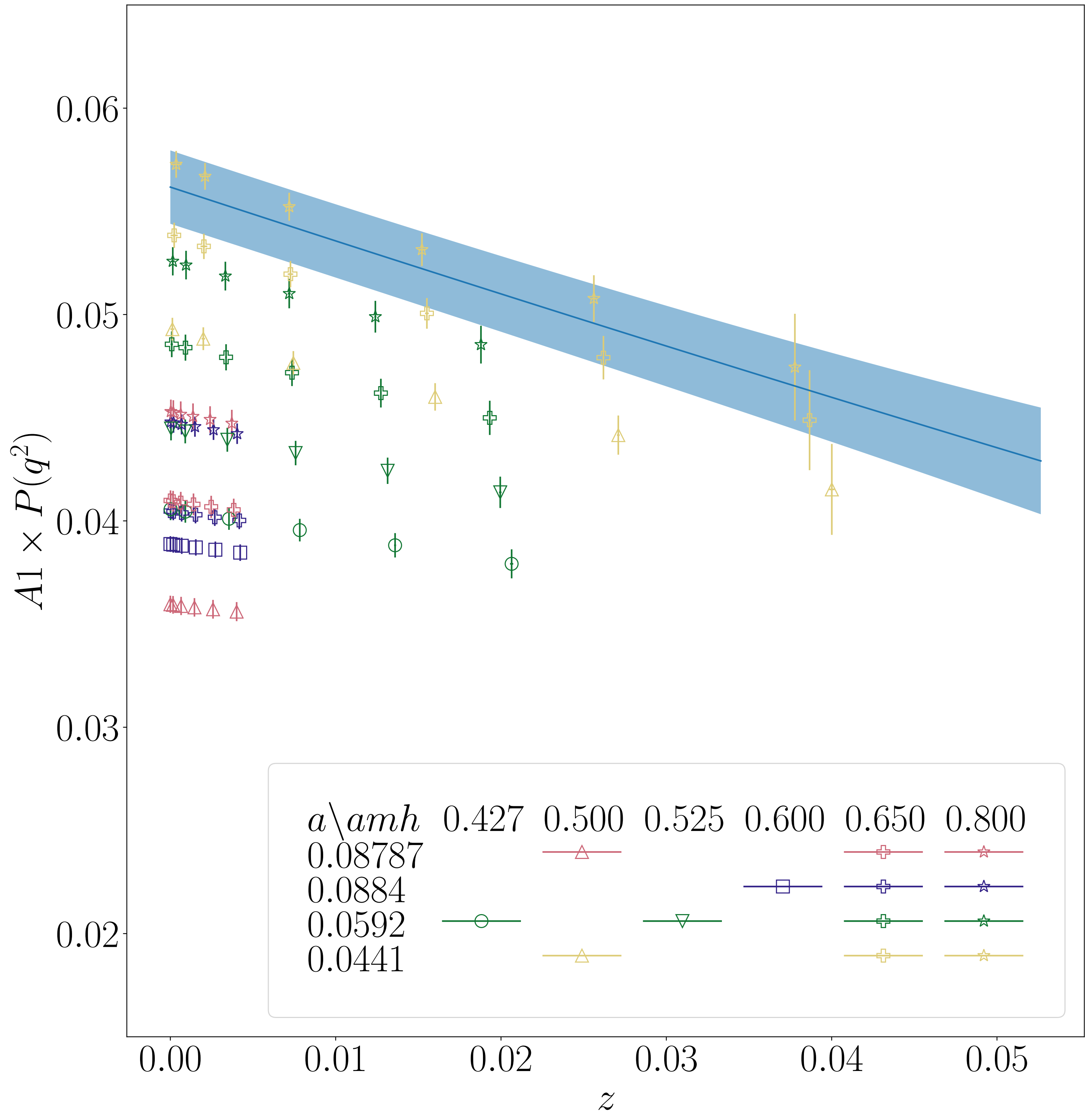
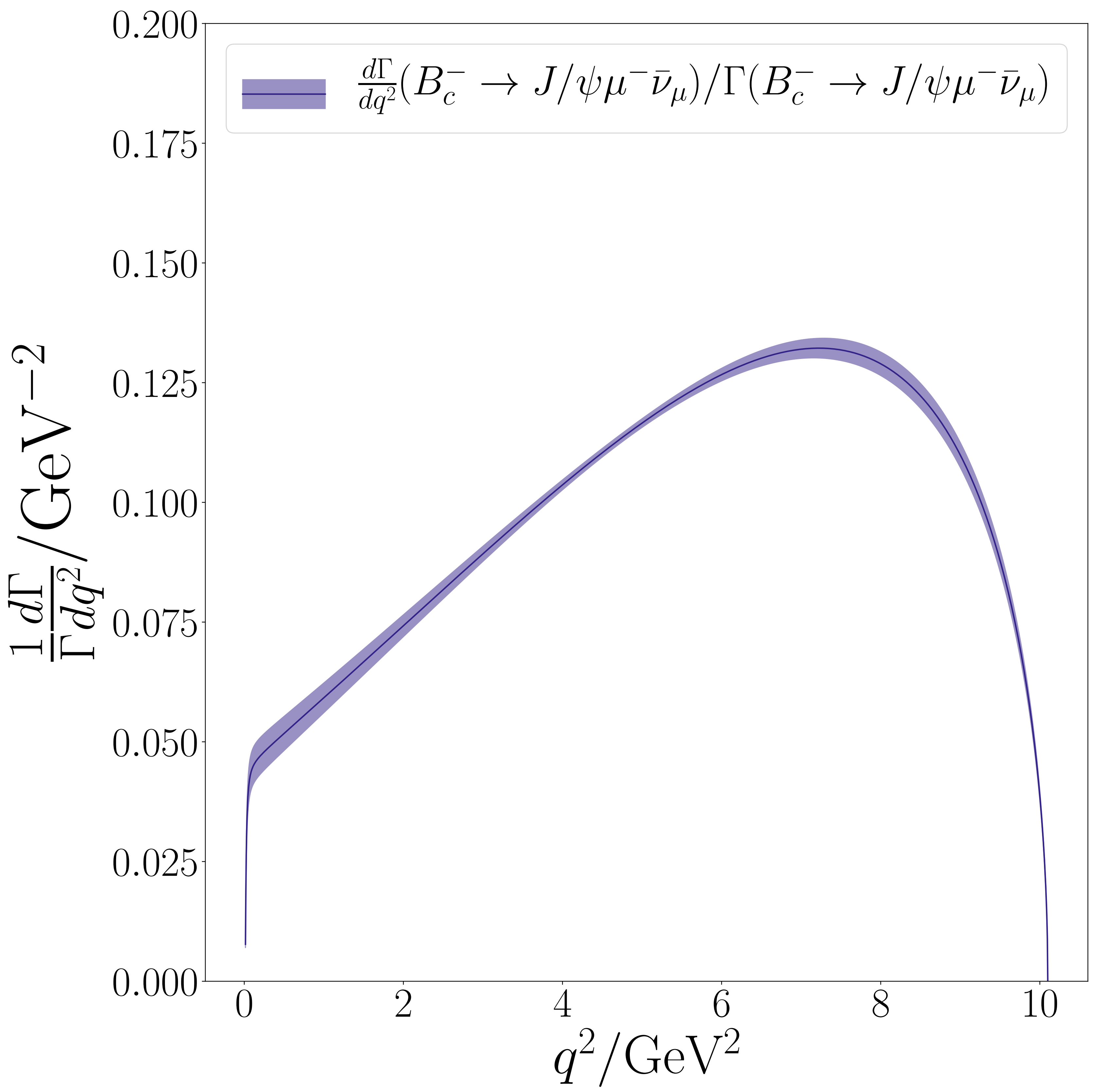
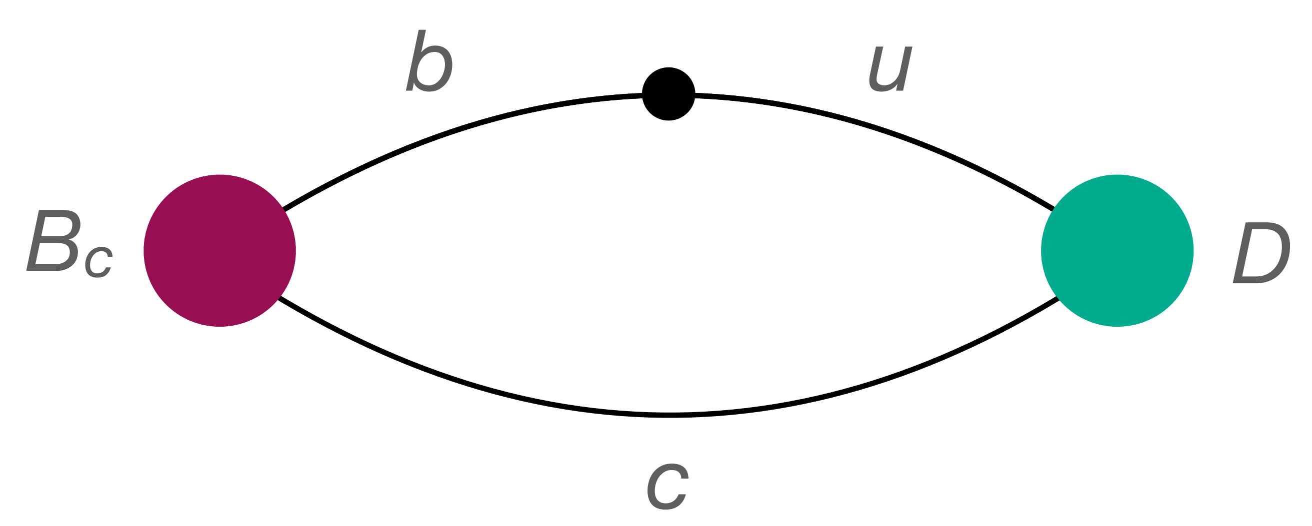




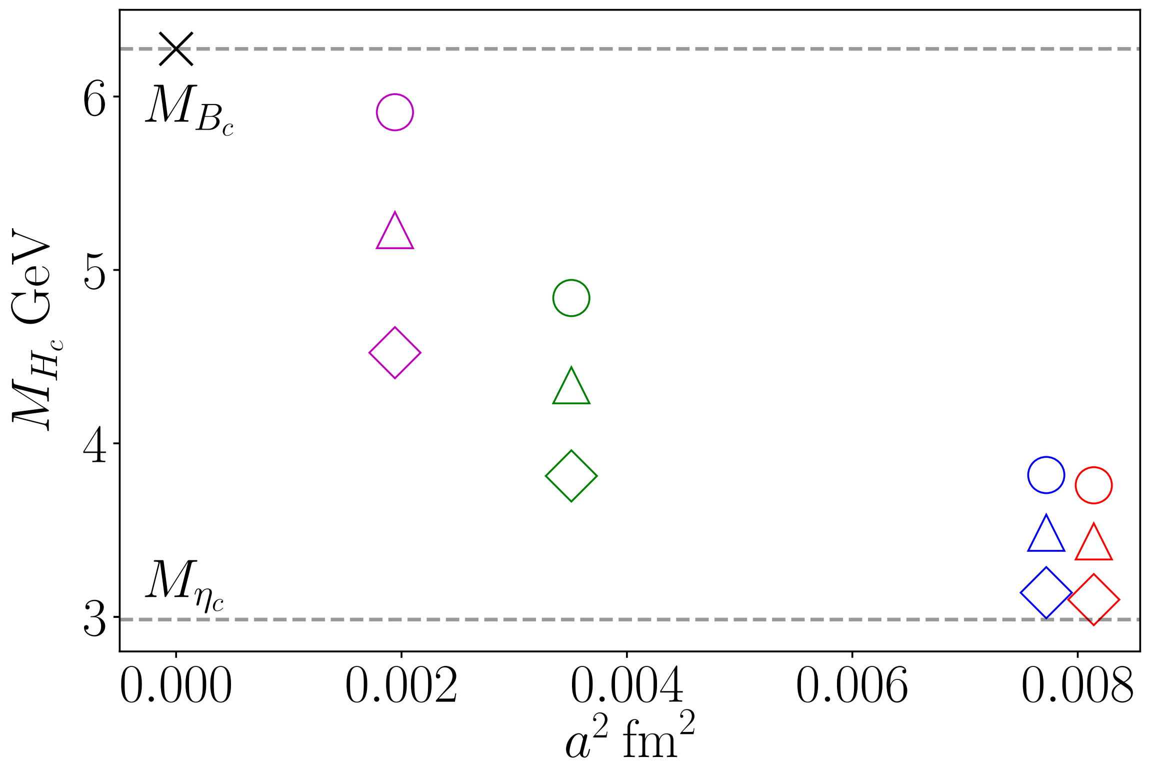
$$R(J/\psi)_{\mathrm{SM}} = 0.2601(36)$$

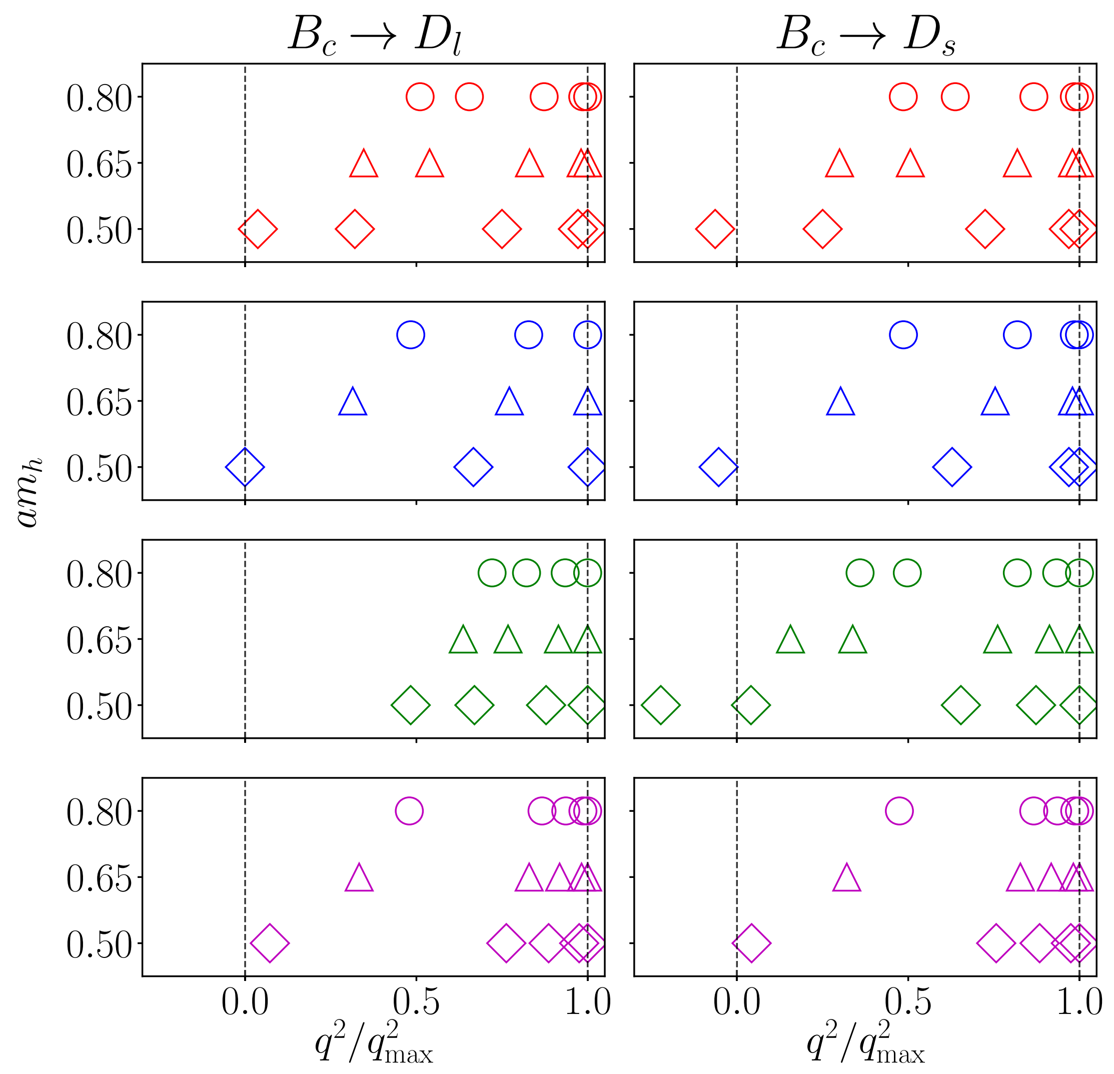
Harrison, Davies, Lytle, (HPQCD), PRD 102, 094518 (2020), PRL 125, 222003 (2020)

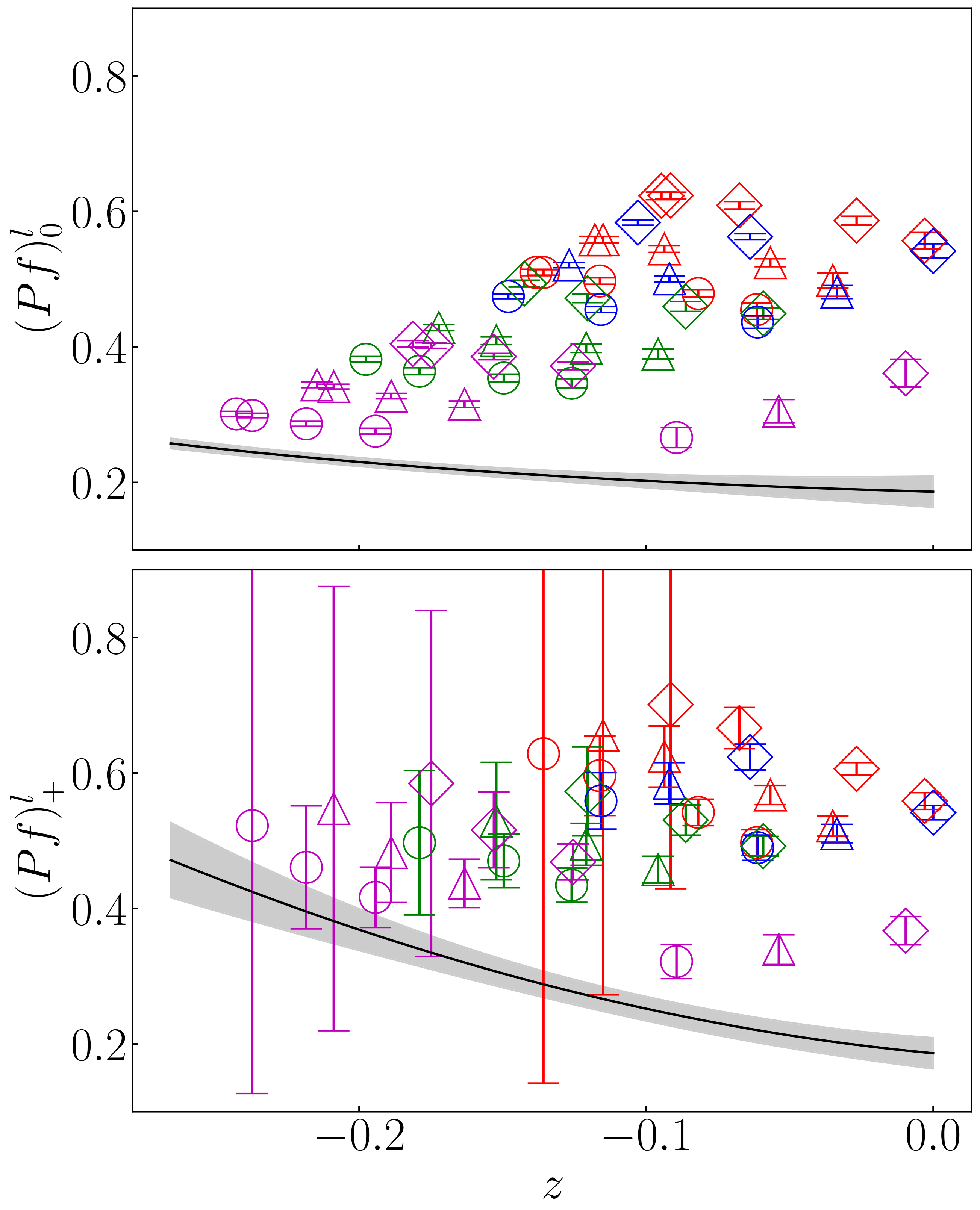
B_c semileptonic decays: $b \to u$



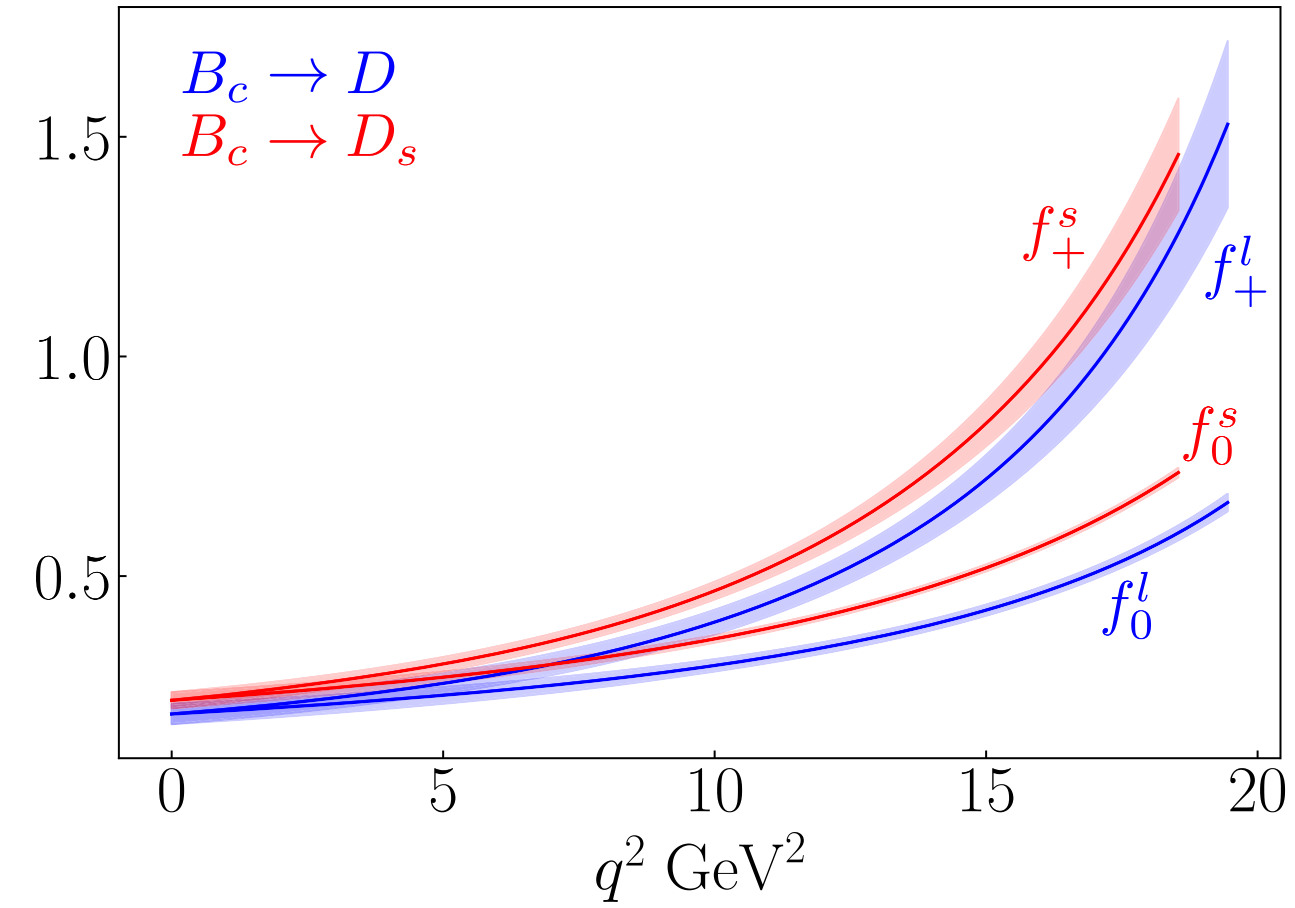
HPQCD: Heavy HISQ on MILC's n_f = 2+1+1 HISQ lattices

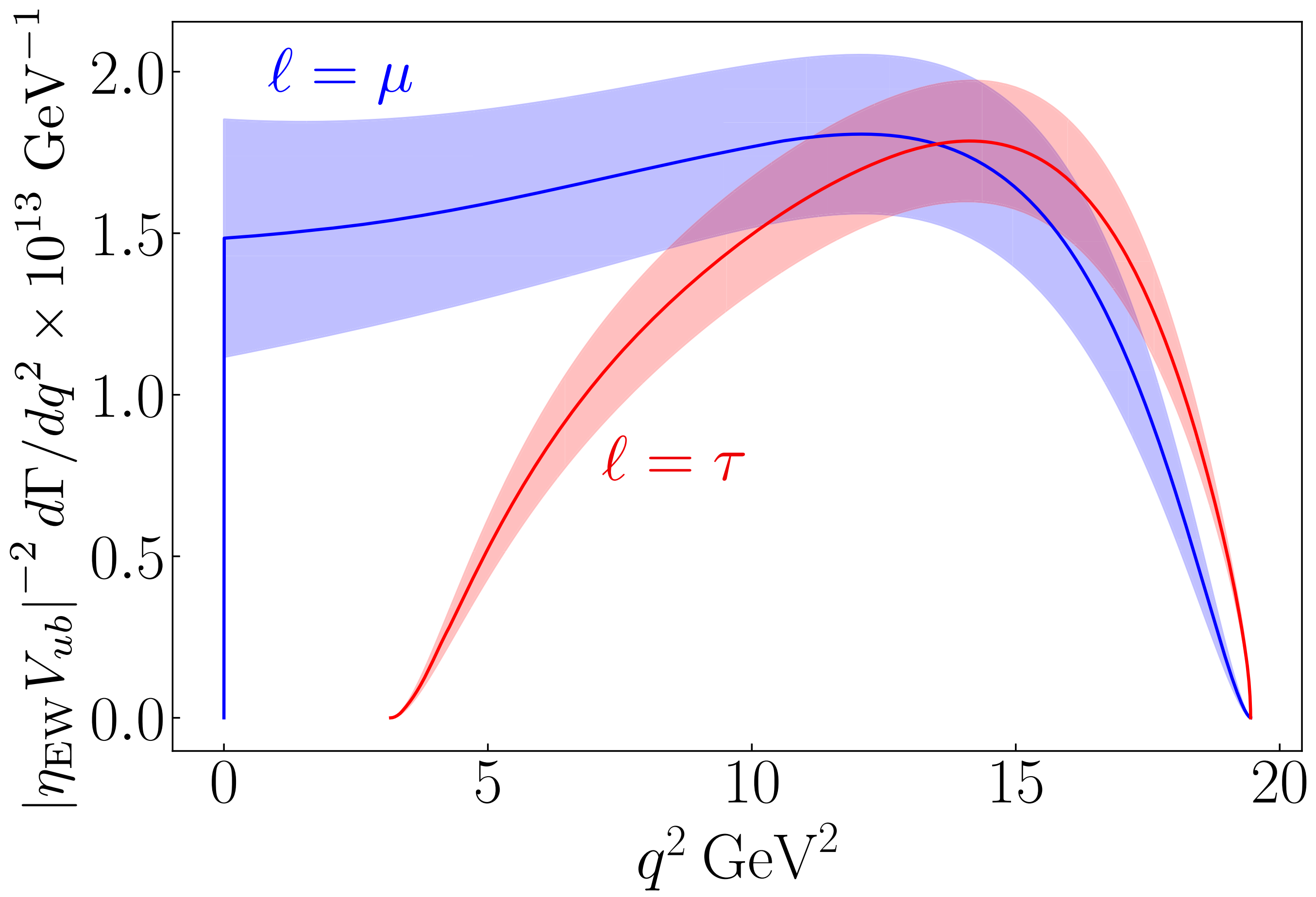






LHCb is working on an analysis of
$\mathscr{B}(B_c \to D^{(*)}\ell\nu)/\mathscr{B}(B_c \to J/\psi\,\ell\nu)$





Cooper, Davies, MW, (HPQCD), Phys. Rev. D 105, 014503 (2022)

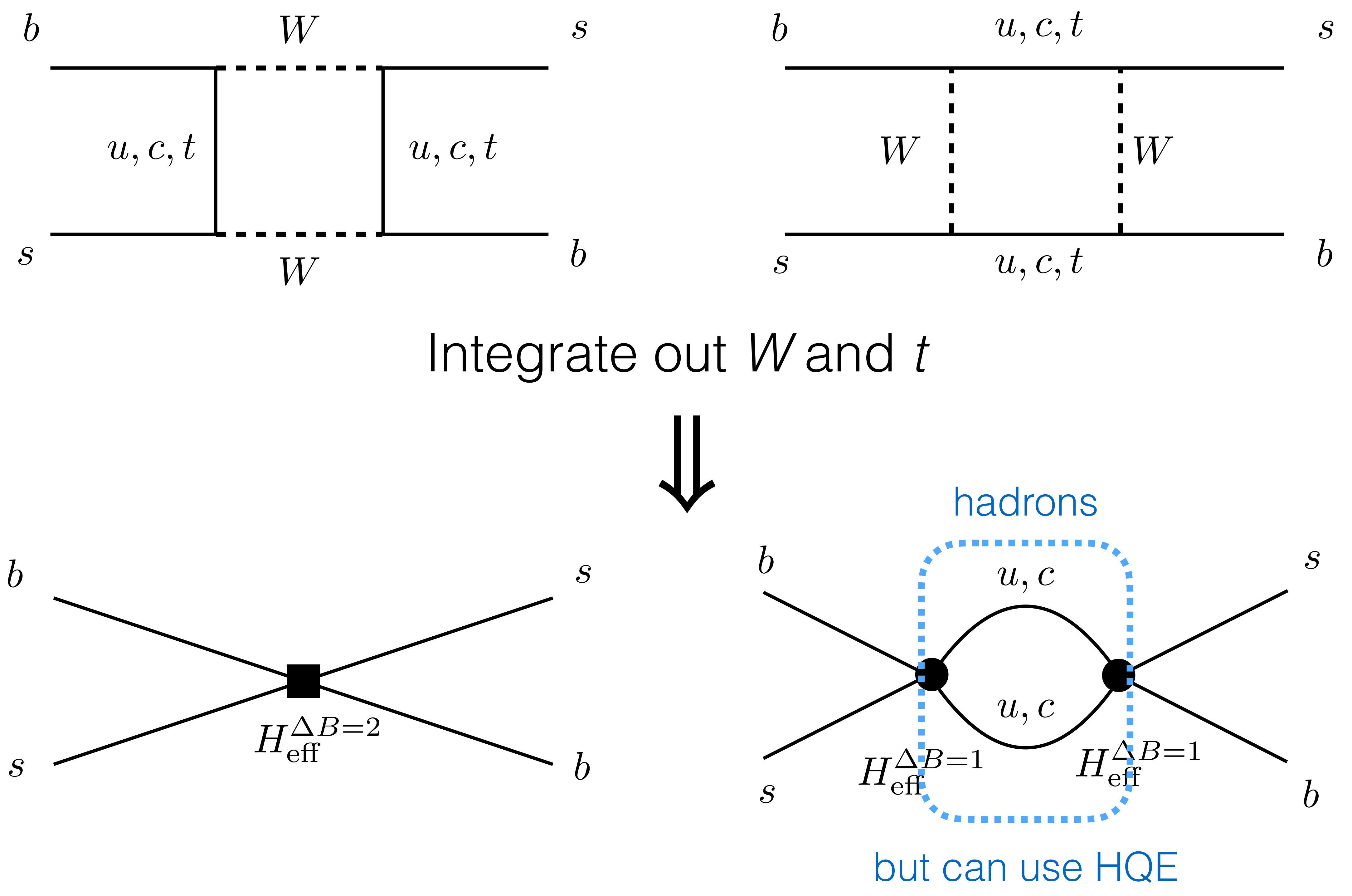
Inclusive decays

- Gambino and Hashimoto (PRL 125, 032001 (2020)) proposal to determine inclusive decay rates from lattice QCD
- Want $$W^{\mu\nu}(p,q) = \sum_{X_c}(2\pi)^3\delta^{(4)}(p-q-r)\frac{1}{2E_{B_s}}\langle\bar{B}_s(\boldsymbol{p})|J^{\mu\dagger}|X_c(\boldsymbol{r})\rangle\langle X_c(\boldsymbol{r})|J^{\nu}|\bar{B}_s(\boldsymbol{p})\rangle$$
- Can compute $$C^{JJ}_{\mu\nu}(t;\boldsymbol{q}) = \sum_{\boldsymbol{x}}\frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2m_{B_s}}\langle\bar{B}_s(\boldsymbol{0})|J^{\dagger}_{\mu}(\boldsymbol{x},t)J_{\nu}(\boldsymbol{0},0)|\bar{B}_s(\boldsymbol{0})\rangle$$
- Can be related to integral over W $$\begin{aligned} M_{\mu\nu}(t;\boldsymbol{q}) &= \int d^3x\,\frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2m_B}\langle\bar{B}(\boldsymbol{0})|J^{\dagger}_{\mu}(\boldsymbol{0},0)e^{-t\hat{H}+i\hat{\boldsymbol{P}}\cdot\boldsymbol{x}}J_{\nu}(\boldsymbol{0},0)|\bar{B}(\boldsymbol{0})\rangle \\ &= \frac{(2\pi)^3}{2m_B}\langle\bar{B}(\boldsymbol{0})|J^{\dagger}_{\mu}(\boldsymbol{0},0)e^{-t\hat{H}}\delta^3(\hat{\boldsymbol{P}}+\boldsymbol{q})J_{\nu}(\boldsymbol{0},0)|\bar{B}(\boldsymbol{0})\rangle \\ &= \int_0^{\infty} d\omega\,W_{\mu\nu}(\omega,\boldsymbol{q})\,e^{-\omega t}\ . \end{aligned}$$
- Progress reported at Lattice 2022 by A. Smecca, A. Barone, R. Kellermann

CKM: 3rd row & B mixing

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

B-mixing in the Standard Model



$$\Delta M_s = \frac{1}{2m_{B_s}} \langle \bar{B}_s | H_{\text{eff}}^{\Delta B=2} | B_s \rangle$$

$$H_{\text{eff}}^{\Delta B=2} = \frac{G_F^2 m_W^2}{4\pi^2} (V_{ts}^* V_{tb})^2 \sum_{i=1}^{5} C_i Q_i$$

$$Q_1 = (\bar{b}^\alpha \gamma^\mu (1-\gamma^5) s^\alpha)(\bar{b}^\beta \gamma_\mu (1-\gamma^5) s^\beta)$$

$$Q_2 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1-\gamma^5) s^\beta)$$

$$Q_3 = (\bar{b}^\alpha (1-\gamma^5) s^\beta)(\bar{b}^\beta (1-\gamma^5) s^\alpha)$$

$$Q_4 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1+\gamma^5) s^\beta)$$

$$Q_5 = (\bar{b}^\alpha (1-\gamma^5) s^\beta)(\bar{b}^\beta (1+\gamma^5) s^\alpha)$$

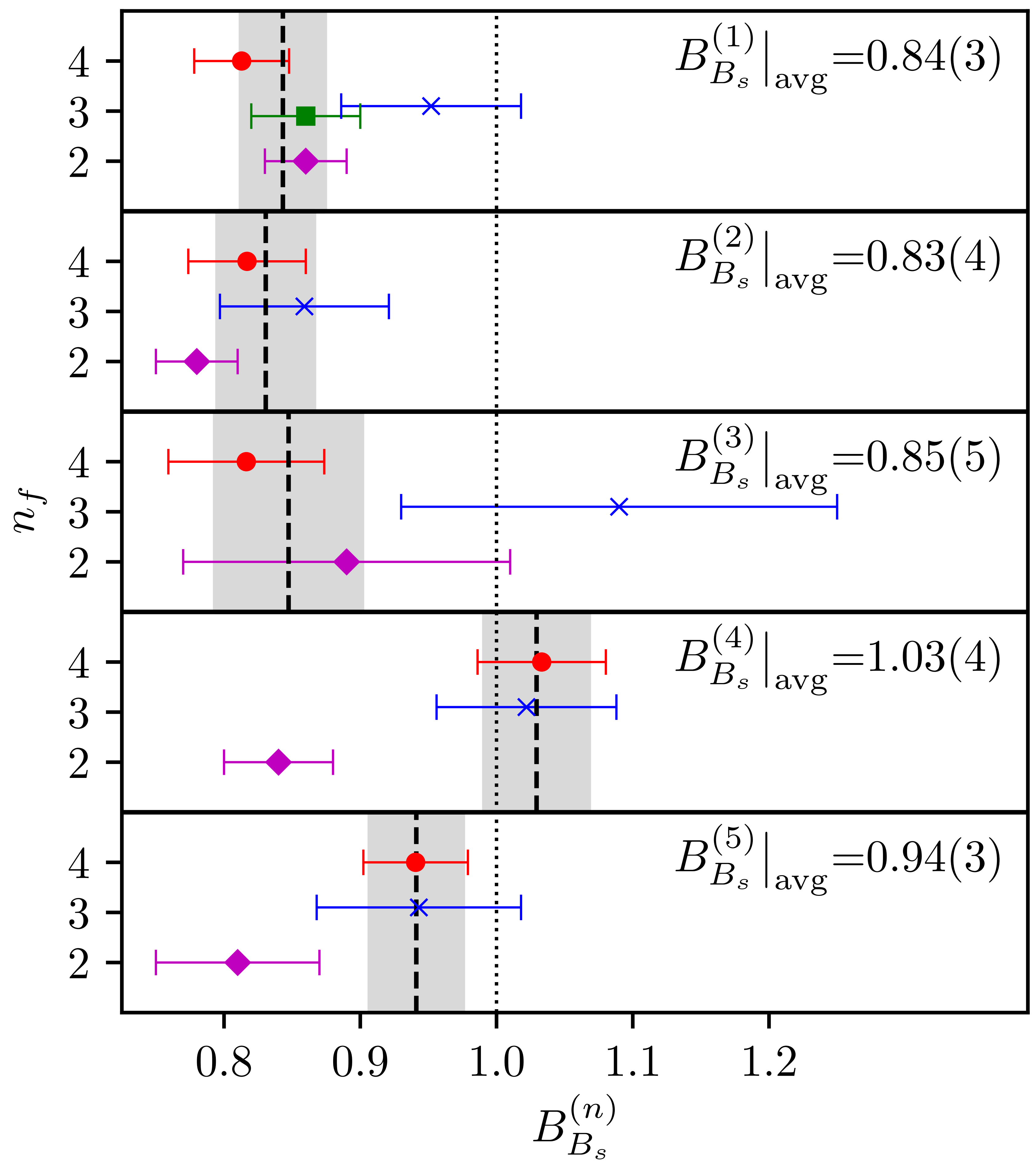
In the SM, only Q_1 enters ΔM_s

B mixing: recent LQCD results

- ETMC (Carrasco et al., JHEP 03 (2014) 016) — ratio method (incl. static limit), twisted mass valance and sea (n_f = 2), RI-MOM
- Fermilab/MILC (Bazavov et al., PRD 93, 113016 (2016)) — Fermilab *b*, AsqTad valance and sea (n_f = 2+1), mostly non-perturbative
- RBC/UKQCD (Boyle et al., arXiv:1812.08791) — domain wall valance (extrapolating to the *b*) and sea (n_f = 2+1), ratios only
- HPQCD (Dowdall et al., PRD 100, 094508 (2019)) — NRQCD b, HISQ valance and sea (n_f = 2+1+1, MILC), perturbative
- Results presented for “bag factors”

$$B^{(i)}_{B_q}(\mu) \equiv \frac{\langle B_q | O^q_i | \bar{B}_q \rangle^{(\mu)}_{\overline{\text{MS}}}}{\eta^q_i(\mu) f^2_{B_q} M^2_{B_q}},$$

Comparisons — bag factors



Legend

- Red circle: HPQCD
- Blue cross: Fermilab/MILC
- Green square: HPQCD
- Purple diamond: ETMC

Legend

- Red circle: HPQCD
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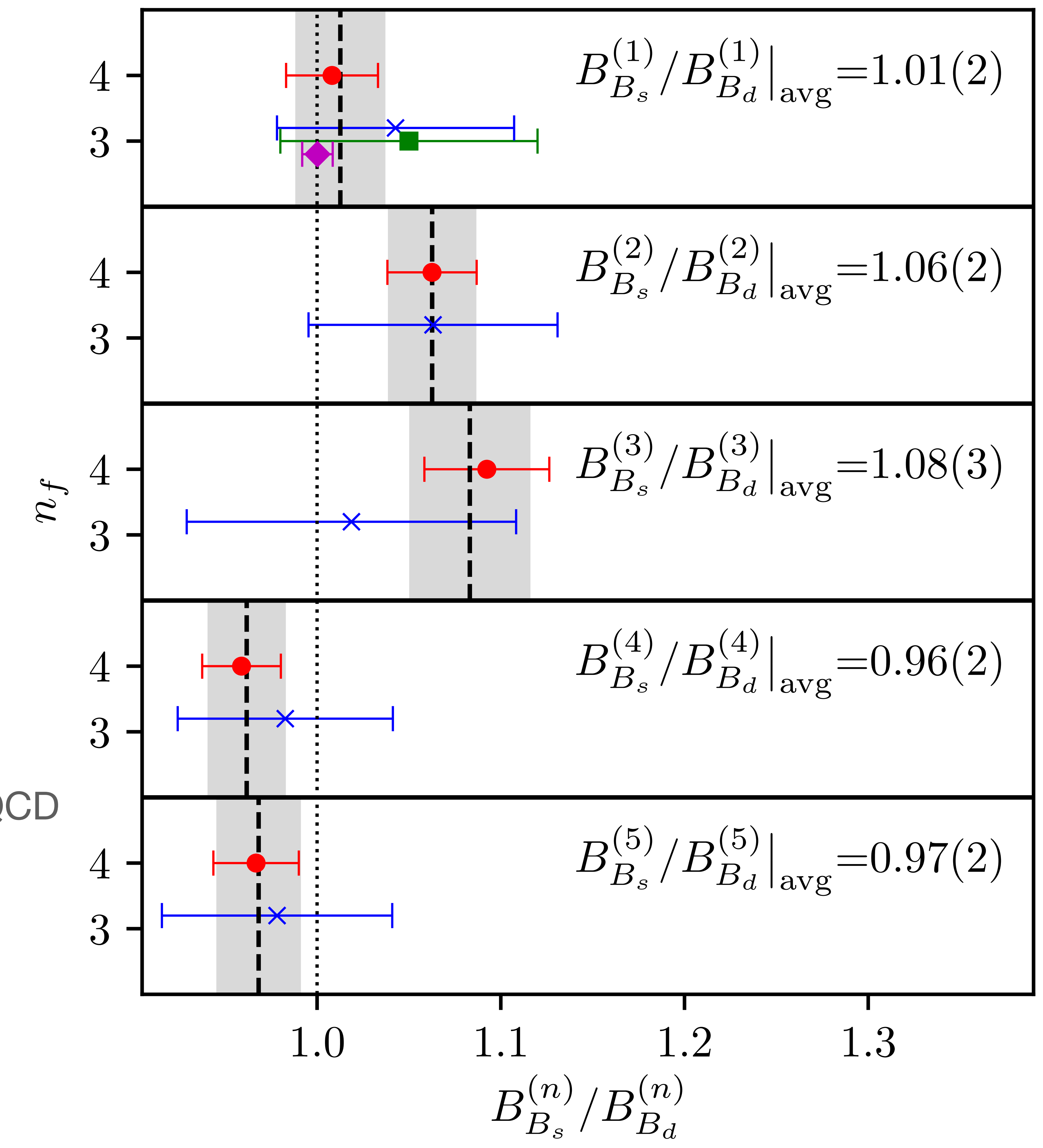
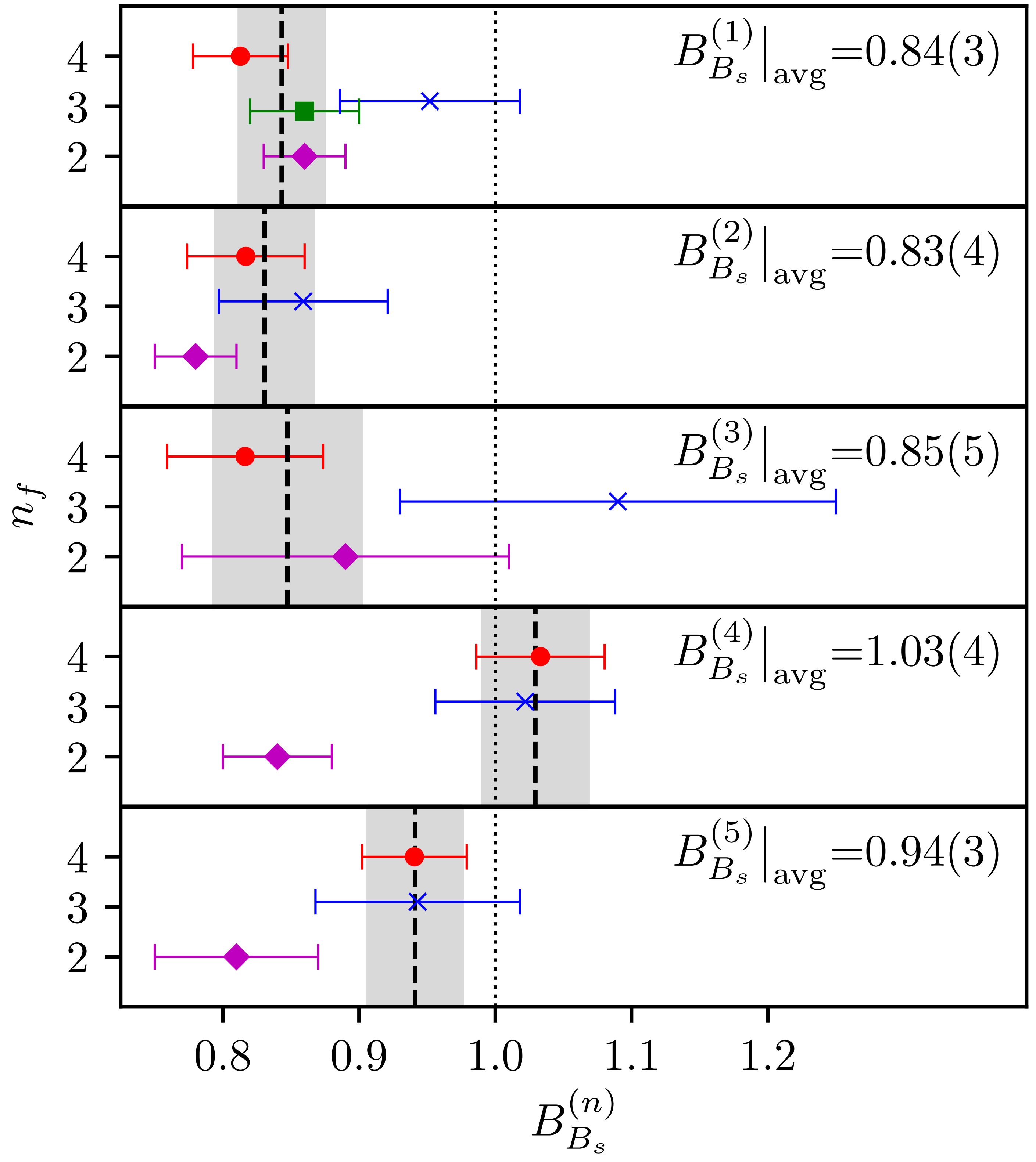
- Green square: HPQCD
- Purple diamond: RBC-UKQCD



Dowdall et al. (HPQCD), Phys. Rev. D 100, 094508 (2019)

Similarities between K and B mixing

B mixing

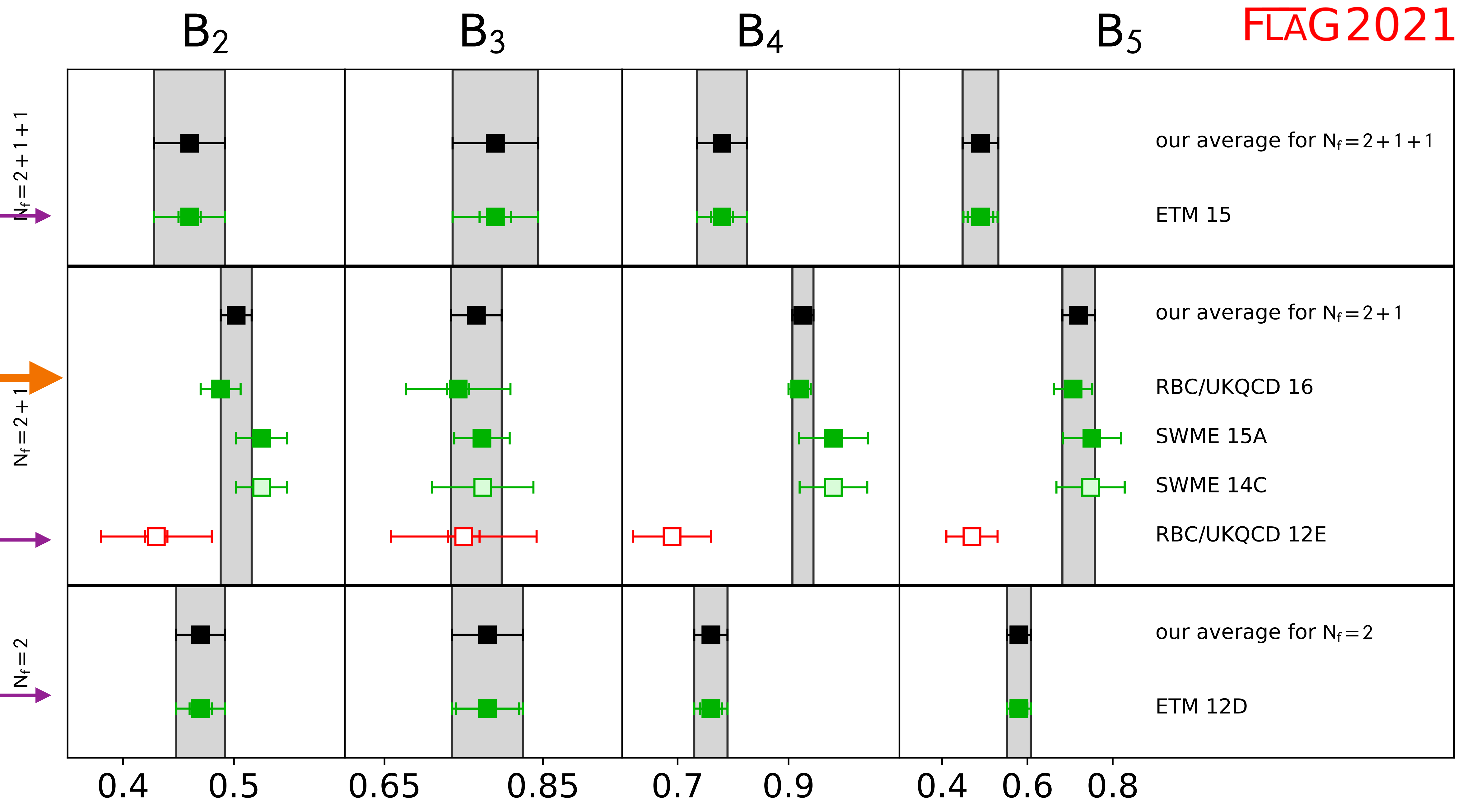


← Purple: RI-MOM

RI-SMOM

Rest: perturbative

K mixing



Role of exceptional momenta in discrepancies in B_4 and B_5 …

FLAG, arXiv:2111.09849

Operator matching

RI-MOM schemes

Calculate (gauge-fixed) quark matrix elements nonperturbatively

Define renormalization constants by imposing conditions at large momentum. The same conditions can be applied in continuum perturbation theory.

$$O(\mu) = Z_O\big(\mu a, g(a)\big)O(a)\,.$$

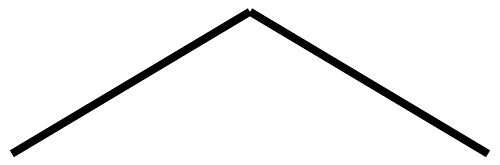
Z_O is found, by imposing the renormalization condition

$$Z_O\big(\mu a, g(a)\big)Z_\psi^{-1}\big(\mu a, g(a)\big)\Gamma_O(pa)|_{p^2=\mu^2} = 1\,,$$

$$\Gamma_O(pa) = \tfrac{1}{12}\mathrm{tr}\left(\Lambda_O(pa)\hat{P}_O\right)\,,$$

where

$$\Lambda_O(pa) = S(pa)^{-1}G_O(pa)S(pa)^{-1}\,.$$



Martinelli et al., NPB 445, 81 (1995)

Must take care not to allow large contributions from low energy

(i) symmetric or nonexceptional momentum configuration:

RI-SMOM

$$p_1^2 = p_2^2 = q^2 = -\mu^2, \qquad \mu^2 > 0,$$
$$q = p_1 - p_2;$$

(ii) asymmetric or exceptional momentum configuration:

RI-MOM

$$p_1^2 = p_2^2 = -\mu^2, \qquad \mu^2 > 0,$$
$$p_1 = p_2, \qquad q = 0,$$

STURM, AOKI, CHRIST, IZUBUCHI, SACHRAJDA, AND SONI

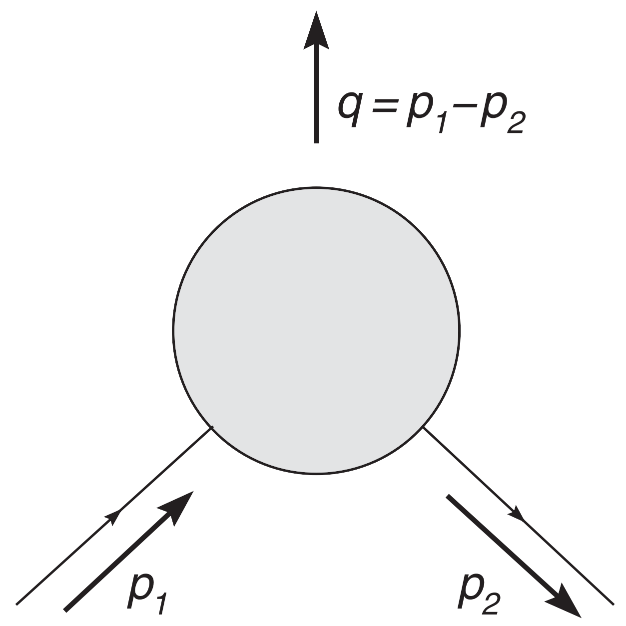
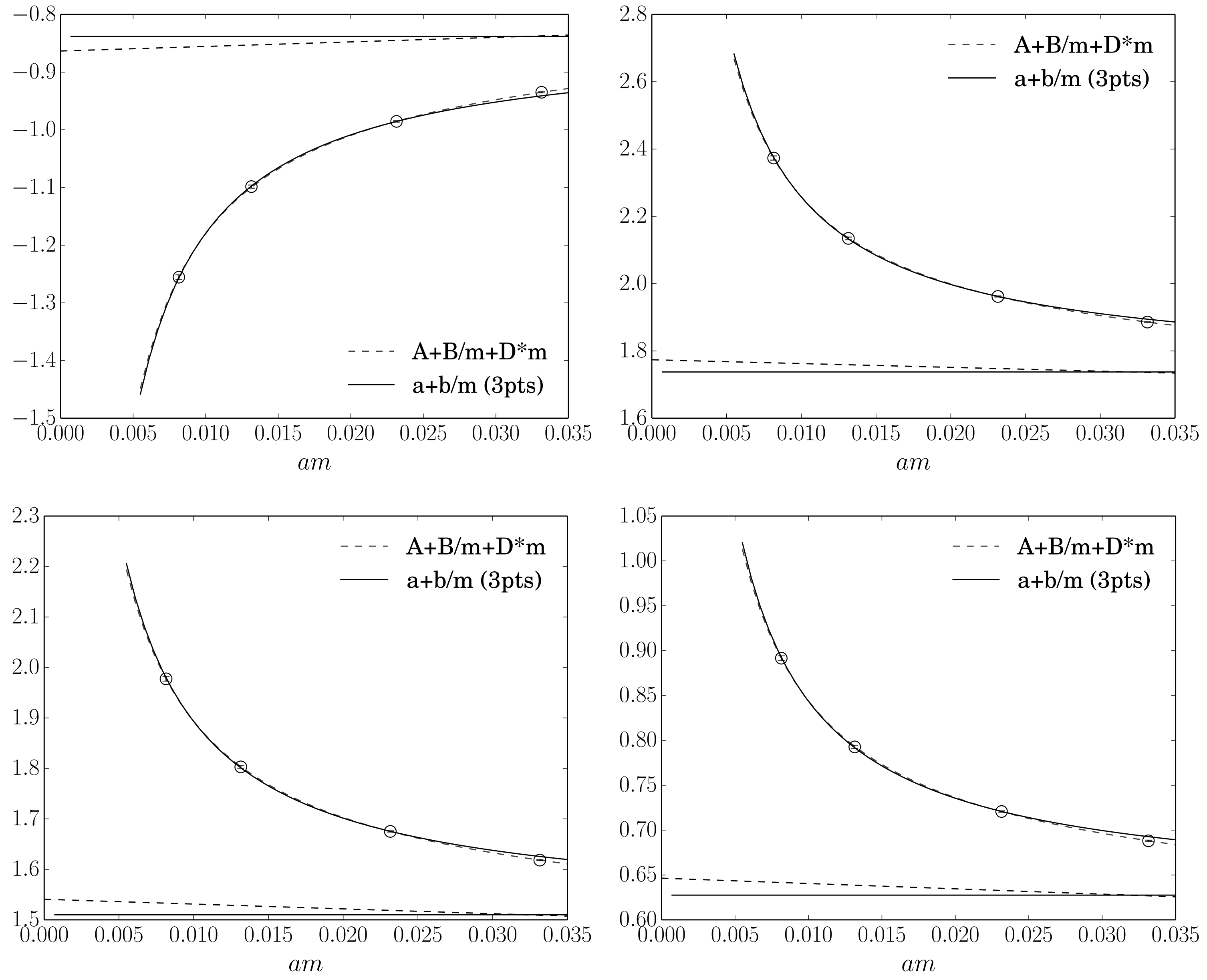


FIG. 1. Momentum flow of a generic diagram required for the renormalization procedure with nonexceptional momenta. The gray bubble stands for an operator insertion and higher order corrections.

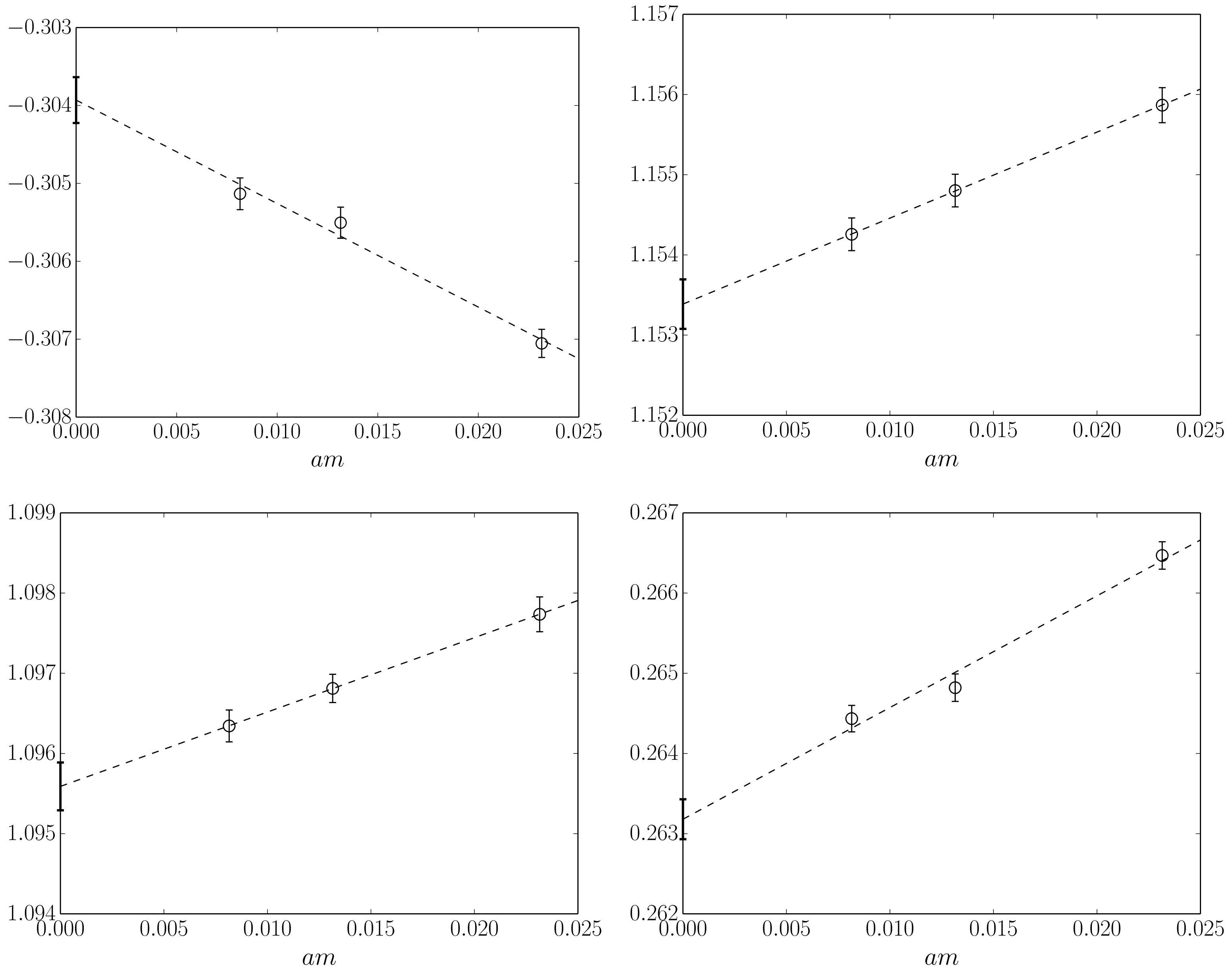
Sturm et al., PRD 80, 014501 (2009)

Avoiding exceptional momenta

RI-MOM

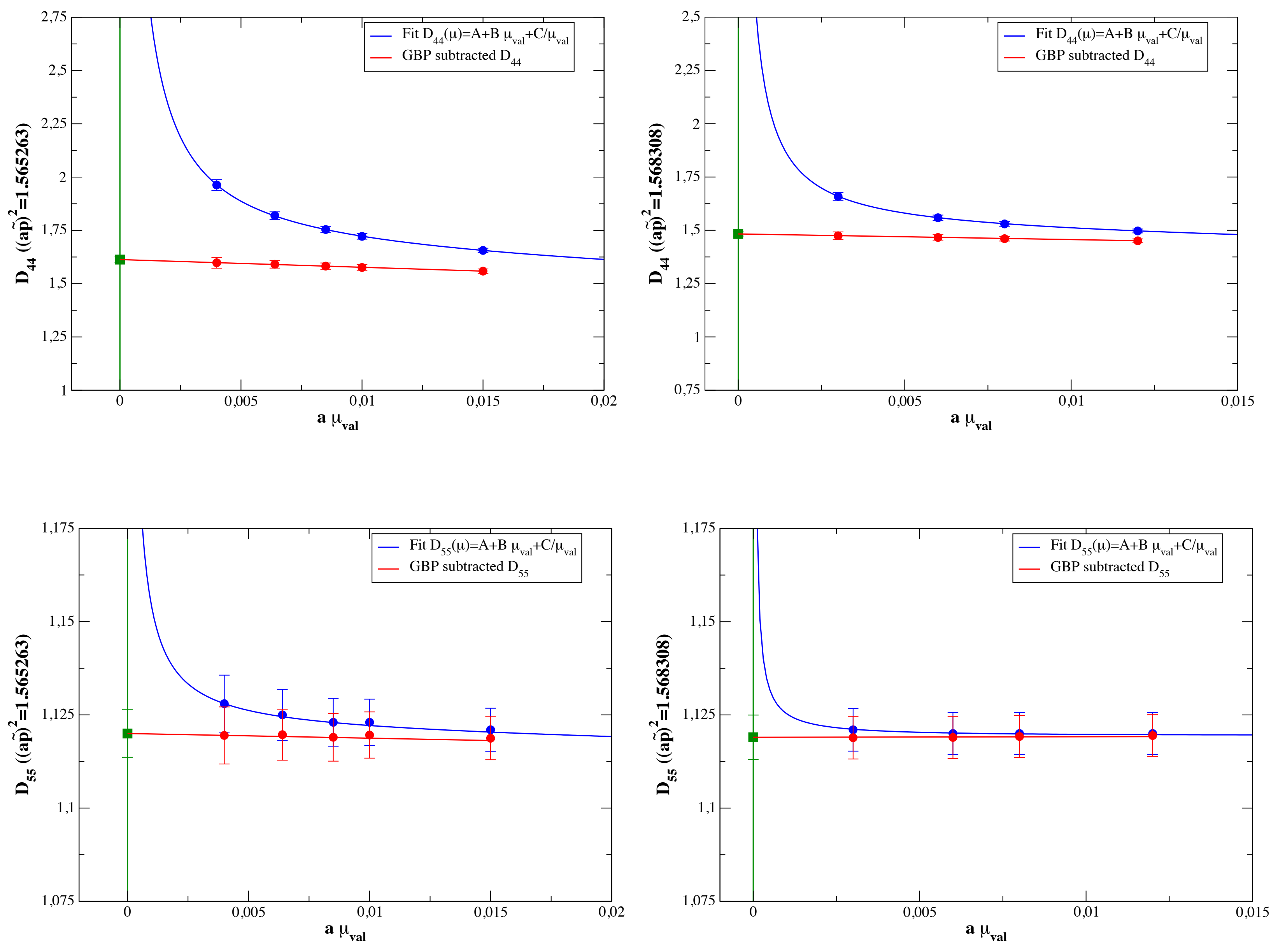


RI-SMOM



Figs. 3 & 4 from Boyle, Garron, Hudspith, Lehner, Lytle, JHEP 10 (2017) 054

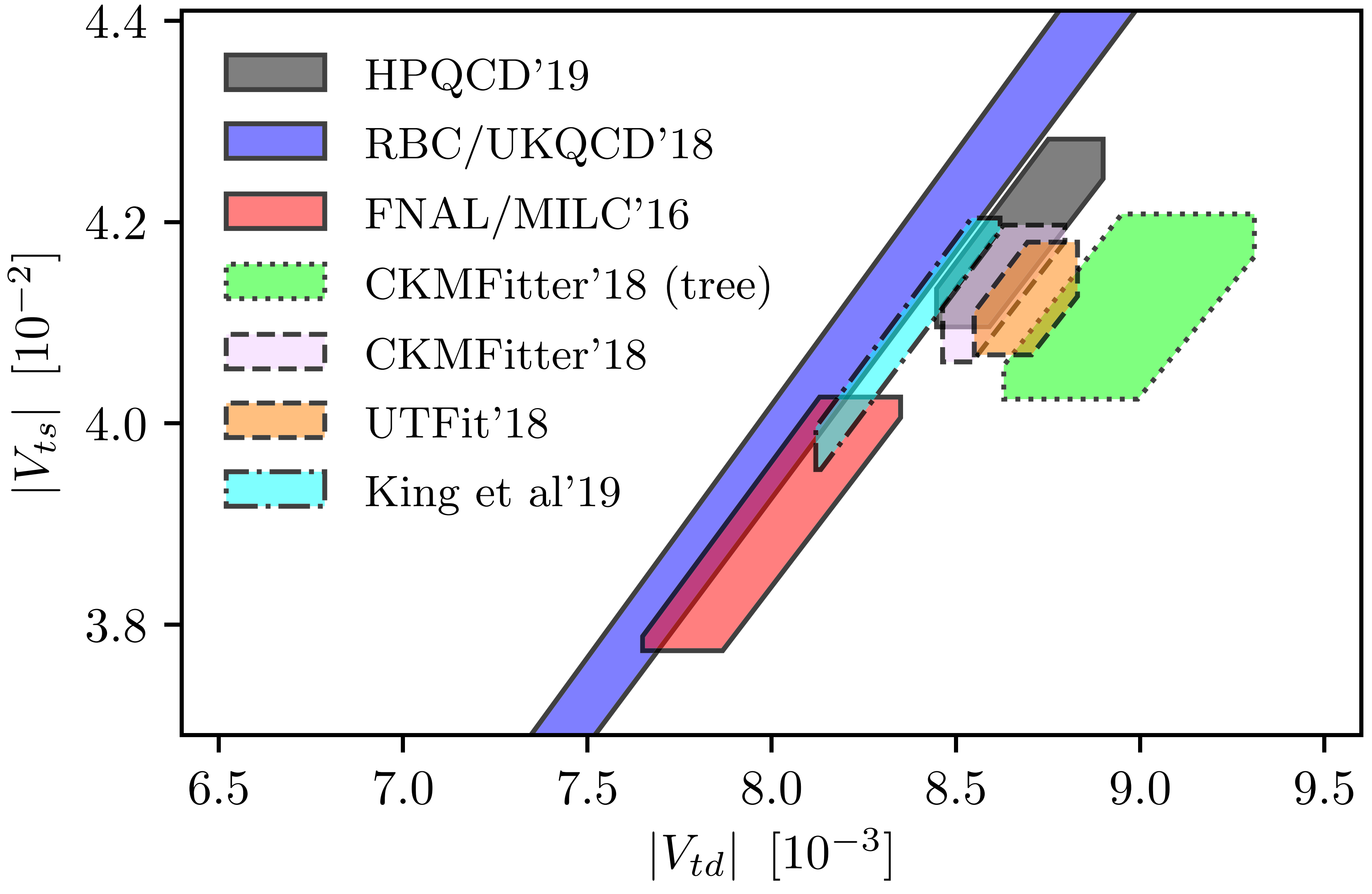
RI-MOM

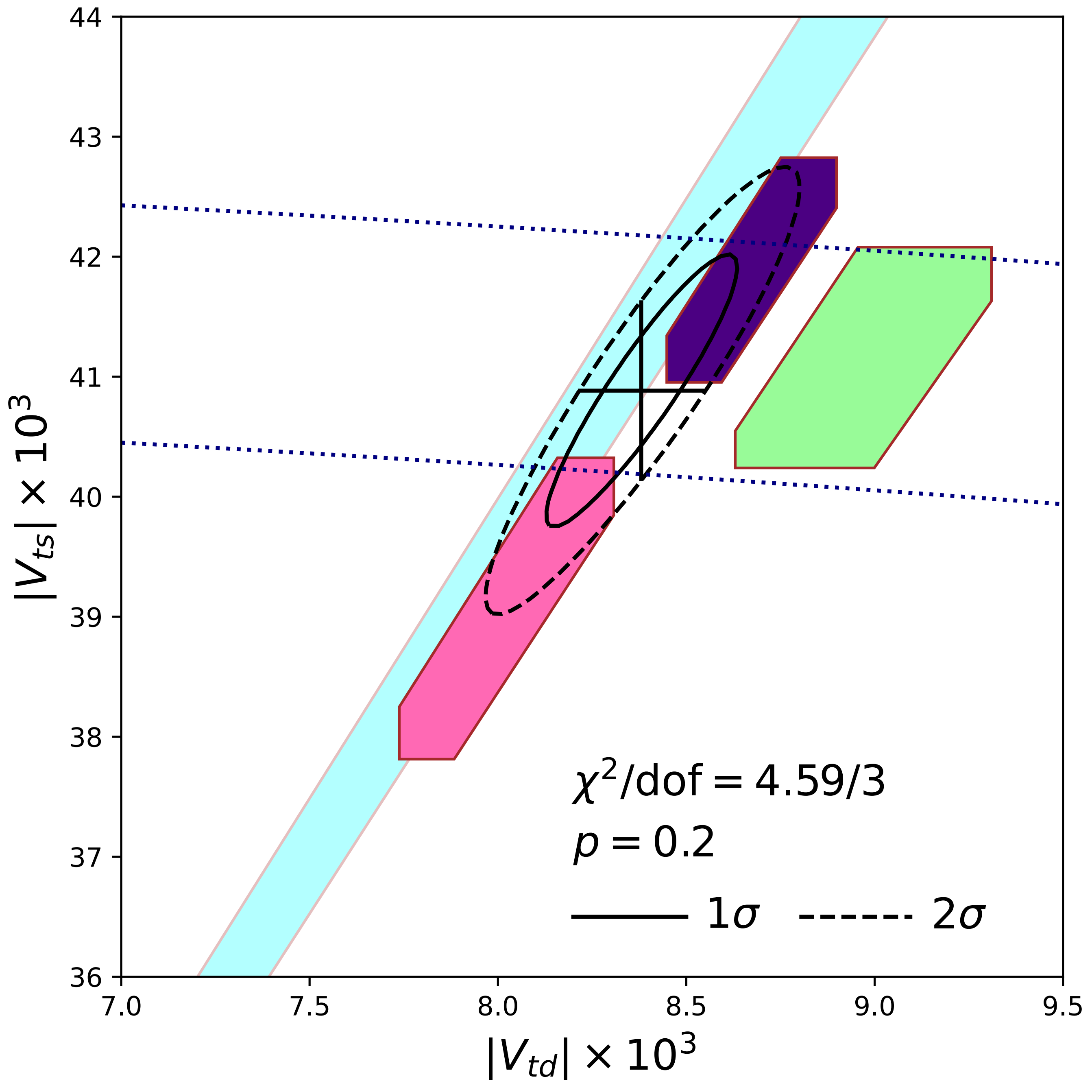


Bertone et al. (ETM), JHEP 03 (2013) 089

Third row constraints from B mixing

1σ hexagons





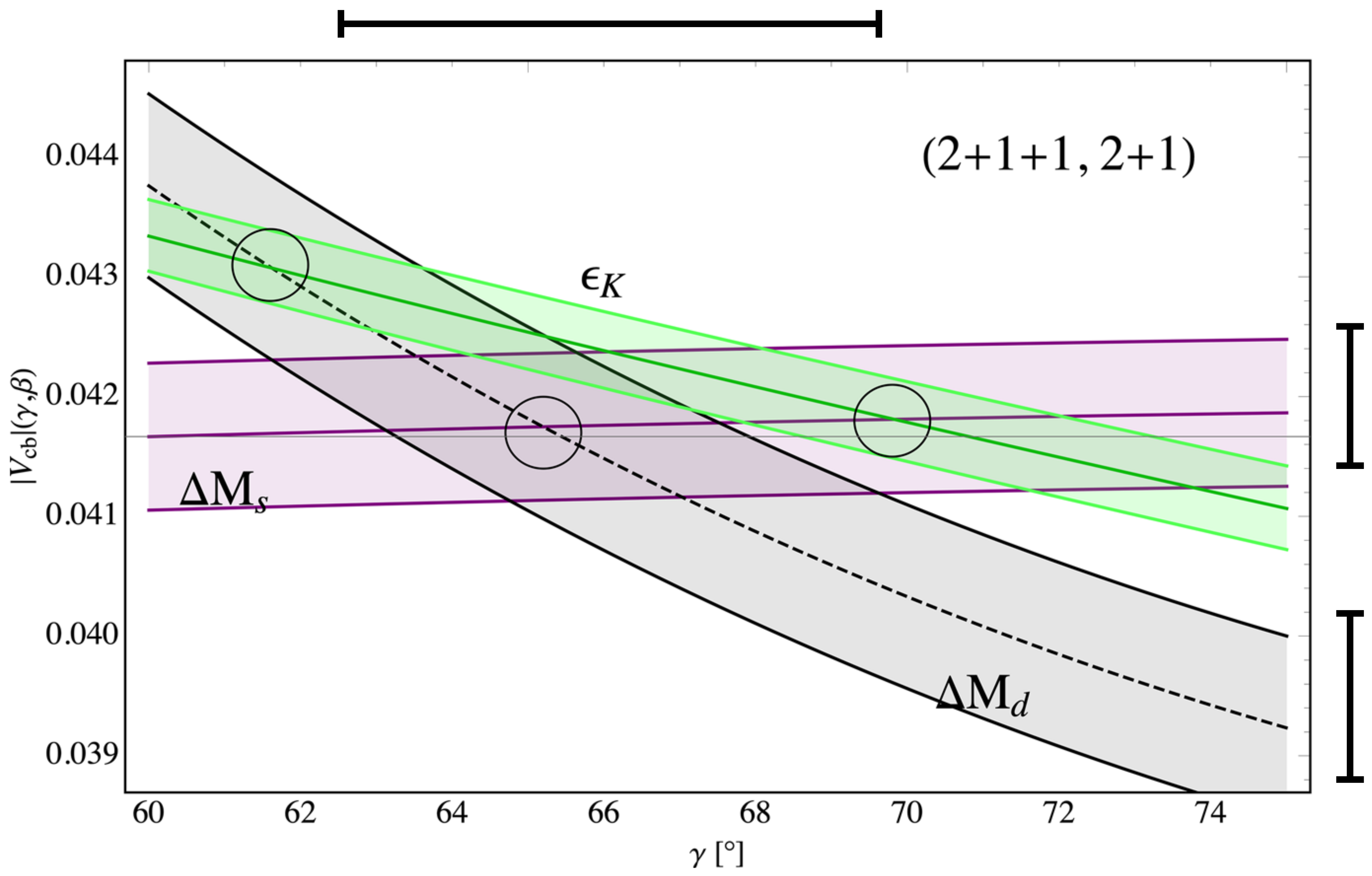
- General agreement of LQCD results, along with sum rules
- Tension with exclusive $|V_{ub}/V_{cb}|$, angle γ, and unitarity

Dowdall et al. (HPQCD), Phys. Rev. D 100, 094508 (2019)

MW, Eur. Phys. J A 57, 239 (2021)

B Mixing, unitarity, and V_{cb}

HFLAV 2021: $66.2^\circ \pm 3.5^\circ$



From inclusive semileptonic B decays
0.0420(6)

From exclusive semileptonic B decays
0.0395(7)

Buras and Venturini, Eur. Phys. J. C 82, 615 (2022)

Conclusions

- LQCD: lots of progress in precision, general agreement between methods
- CKM: persistent inclusive/exclusive tension. Growing tension between exclusive $b \to c$ and B-mixing (+unitarity)
- Future: lots of “next generation calculations” in progress, errors being reduced with a range of formulations
  New ideas for including QED effects, contributing to inclusive decays, improving dispersive constraints on ff shapes
- Advertisement of other talks
  - 11:00 Monday: Alejandro Vaquero, *$B \to D^* l \nu$ and related anomalies*
  - 16:30 Monday: Enrico Lunghi, *B and K physics, role of lattice QCD*
  - 09:30 Tuesday: Chris Bouchard, *$B \to K l l$ and $D \to K l \nu$ form factors and phenomenology*
  - 11:00 Tuesday: Simon Kuberski, *Heavy flavor physics with O(a) improved Wilson quarks*