Quantum transport of few photons in waveguides: many-body, strong and ultrastrong effects

Eduardo Sánchez-Burillo

Instituto de Ciencia de Materiales de Aragón
Universidad de Zaragoza

March 7, 2015

The crew

This work has been done in collaboration with:

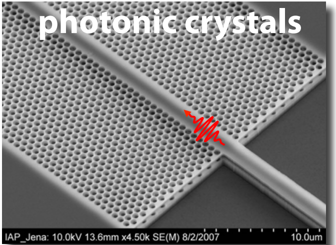
ICMA(CSIC-Unizar), Zaragoza **IFF(CSIC), Madrid**

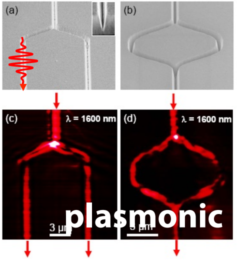
Outline

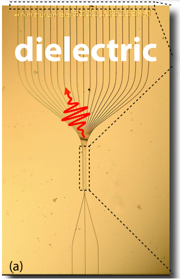
Introduction

- In the last years, a lot of effort has been devoted to **light-matter** interactions in **1D** quantum systems, (both **experimentally** and **theoretically**).
- Nowadays it is possible to control interaction between **few photons** and **few qubits** (few=1, 2, 3,. . .).
- **Strong coupling** has been achieved (coupling, and not losses, dominates the dynamics).
- Performing tasks with **minimum power** is the goal: single-photon transistor and detector, spectroscopy, quantum gates, etc.
- Generating matter-mediated photon-photon interaction (or photon-mediated qubit-qubit interaction).

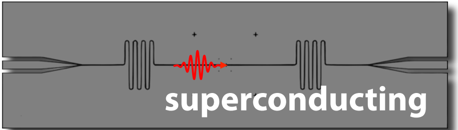
Waveguides





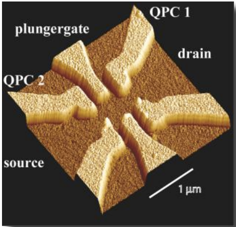


waveguides



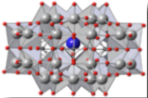
Qubits

qubits / emitters



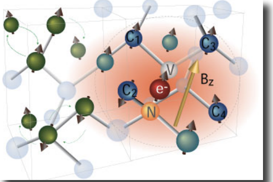
Q-dots

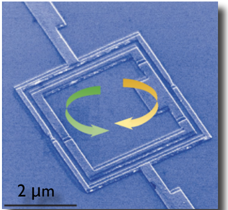
molecules



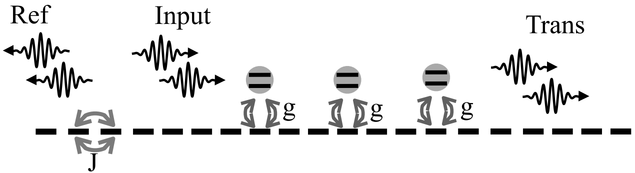
superconducting

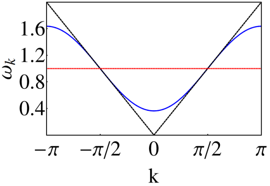
NV-centers





- Discretisation of a 1D continuous waveguide coupled to M qubits with field-dipole interaction.





$$H = H_{wg} + H_{qb} + H_{int} = \sum_n \left(\varepsilon a_n^\dagger a_n - J(a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n) \right) + \sum_{i=1}^{M} \Delta_i \sigma_i^+ \sigma_i^- + H_{int}$$

$$[a_n, a_m^\dagger] = \delta_{nm} \qquad [\sigma_i^-, \sigma_j^+] = \delta_{ij}(1 - 2\sigma_i^+ \sigma_i^-)$$

Dipole-field interaction (just for a qubit)

- $H_{int} = -\vec{d} \cdot \vec{E} = g(\sigma^- + \sigma^+)(a_0 + a_0^\dagger)$, with g the coupling constant.

Dipole-field interaction (just for a qubit)

- $H_{int} = -\vec{d} \cdot \vec{E} = g(\sigma^- + \sigma^+)(a_0 + a_0^\dagger)$, with g the coupling constant.
- Splitting H_{int}:

$$H_{int} = g(\sigma^+ a_0 + \sigma^- a_0^\dagger) + g(\sigma^+ a_0^\dagger + \sigma^- a_0) =: H_{RW} + H_{CR} \quad (1)$$

Dipole-field interaction (just for a qubit)

- $H_{int} = -\vec{d} \cdot \vec{E} = g(\sigma^- + \sigma^+)(a_0 + a_0^\dagger)$, with g the coupling constant.
- Splitting H_{int}:

$$H_{int} = g(\sigma^+ a_0 + \sigma^- a_0^\dagger) + g(\sigma^+ a_0^\dagger + \sigma^- a_0) =: H_{RW} + H_{CR} \qquad (1)$$

- "Small" coupling ($g/\omega < 0.1$), Rotating Wave Approximation, RWA $\Rightarrow$ Jaynes-Cummings model:

$$H_{int} = H_{RW} + \cancel{H_{CR}}$$

Dipole-field interaction (just for a qubit)

- $H_{int} = -\vec{d} \cdot \vec{E} = g(\sigma^- + \sigma^+)(a_0 + a_0^\dagger)$, with g the coupling constant.
- Splitting H_{int}:

$$H_{int} = g(\sigma^+ a_0 + \sigma^- a_0^\dagger) + g(\sigma^+ a_0^\dagger + \sigma^- a_0) =: H_{RW} + H_{CR} \qquad (1)$$

- "Small" coupling ($g/\omega < 0.1$), Rotating Wave Approximation, RWA $\Rightarrow$ Jaynes-Cummings model:

$$H_{int} = H_{RW} + \cancel{H_{CR}}$$

- If RWA, **the number of particles is conserved**; otherwise, it is not.

Dipole-field interaction (just for a qubit)

- $H_{int} = -\vec{d} \cdot \vec{E} = g(\sigma^- + \sigma^+)(a_0 + a_0^\dagger)$, with g the coupling constant.
- Splitting H_{int}:

$$H_{int} = g(\sigma^+ a_0 + \sigma^- a_0^\dagger) + g(\sigma^+ a_0^\dagger + \sigma^- a_0) =: H_{RW} + H_{CR} \qquad (1)$$

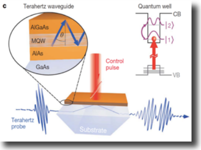
- "Small" coupling ($g/\omega < 0.1$), Rotating Wave Approximation, RWA $\Rightarrow$ Jaynes-Cummings model:

$$H_{int} = H_{RW} + \cancel{H_{CR}}$$

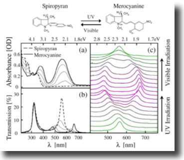
- If RWA, **the number of particles is conserved**; otherwise, it is not.
- We sometimes consider arbitrary values for g. In such a case we shall take the full Hamiltonian (1); this regime is known as **ultra-strong**.

Ultra-strong coupling in the literature

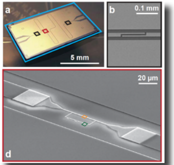
Gunter 2009 (g/ω =0.2 → g/ω=0.58)



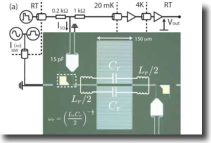
Schwartz 2011 (g/ω=0.16)



Experiments



Niemczyk 2010 (g/ω=0.12)



Forn Díaz 2010 (g/ω=0.1)

Not so fast... we are trying to solve a hard problem

- **Several photons:** high nonlinearities, correlated states and so on.
- **Ultrastrong regime:** the ground state is not trivial, the number of particles is not conserved, etc.

Some approaches

- Analytical works: Fan, Sun, Baranger, Sørensen, Lukin, Nori, Roy...

Some approaches

- Analytical works: Fan, Sun, Baranger, Sørensen, Lukin, Nori, Roy...
- Numerically: **Brute force?** No way... **Exponentially** huge Hilbert space

$$\dim(\mathcal{H}) = d^{N_{cav}},$$

if $d = 2$, $N_{cav} = 100$, $\dim(\mathcal{H}) \simeq 10^{30}$.

Some approaches

- Analytical works: Fan, Sun, Baranger, Sørensen, Lukin, Nori, Roy...
- Numerically: **Brute force?** No way... **Exponentially** huge Hilbert space

$$\dim(\mathcal{H}) = d^{N_{cav}},$$

if $d = 2$, $N_{cav} = 100$, $\dim(\mathcal{H}) \simeq 10^{30}$.

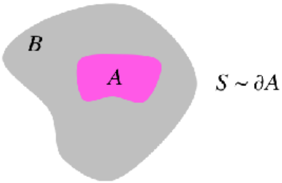
- What about a systematic tool?

Area law

- Given a bipartite system $A \cup B$, the entropy:

$$S = -\mathrm{Tr}(\rho_A \log \rho_A),$$

with $\rho_A = \mathrm{Tr}_B(|\Psi\rangle\langle\Psi|)$, measures the quantum entanglement between A and B. Given a general state, S increases with the physical volume of the system.



- **Area law:** S increases with the area separating both subsystems, not with the volume, for low energy states of many-body systems with local interactions.

Solution: MPS technique

- Point: how to parametrise low entangled states.

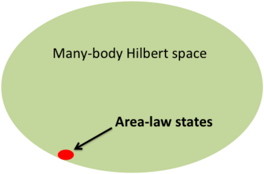


Figure: Orús, arXiv, 2013

Solution: MPS technique

- Point: how to parametrise low entangled states.

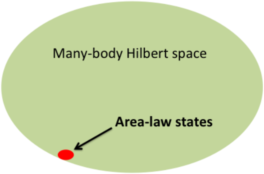


Figure: Orús, arXiv, 2013

- For 1D many-body systems this parametrisation is achieved with **Matrix Product States** (MPS) (Cirac, Verstraete, Östlund, Rommer, Ripoll, Orús...).

Singular value decomposition

- Given a bipartite system with Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$ and $\dim(\mathcal{H}_i) = d$:

$$|\Psi\rangle = \sum_{i_A, i_B = 1}^{d} c_{i_A, i_B} |i_A, i_B\rangle \qquad d^2 \text{ parameters}$$

Singular value decomposition

- Given a bipartite system with Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$ and $\dim(\mathcal{H}_i) = d$:

$$|\Psi\rangle = \sum_{i_A, i_B = 1}^{d} c_{i_A, i_B} |i_A, i_B\rangle \qquad d^2 \text{ parameters}$$

- c accepts singular value decomposition (SVD):

$$c = U\, W\, V^{\dagger}$$

with U and V unitary matrices and W diagonal with $w_\alpha := W_{\alpha,\alpha} \geq 0$.

Singular value decomposition

- Given a bipartite system with Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$ and $\dim(\mathcal{H}_i) = d$:

$$|\Psi\rangle = \sum_{i_A, i_B = 1}^{d} c_{i_A, i_B} |i_A, i_B\rangle \qquad d^2 \text{ parameters}$$

- c accepts singular value decomposition (SVD):

$$c = U\, W\, V^\dagger$$

  with U and V unitary matrices and W diagonal with $w_\alpha := W_{\alpha,\alpha} \geq 0$.

- For area-law states, w_α decreases exponentially, so we can severely truncate the matrix and take the χ largest values of w_α.

Exponential decay of w_α: example

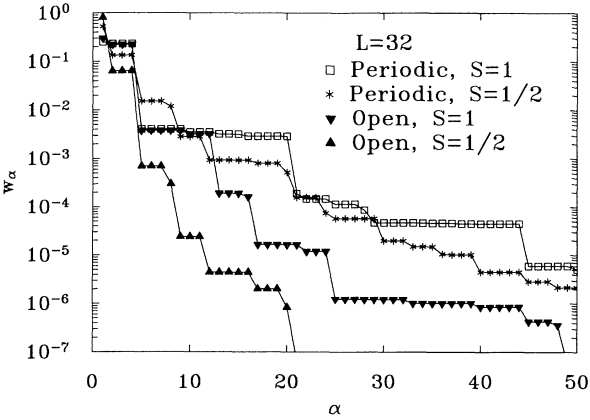


Figure: Taken from S. White, PRB (1993)

An efficient way of parametrising a low energy state

- Iterating SVD in a many-body state we have MPS:

$$|\Psi\rangle = \sum_{\{i_n\}} c_{i_1,i_2,\ldots,i_L}|i_1,i_2,\ldots,i_L\rangle = \sum_{\{i_n\}} \mathrm{Tr}(A_1^{i_1}A_2^{i_2}\ldots A_L^{i_L})|i_1,i_2,\ldots,i_L\rangle,$$

with $A_n^{i_n}$ a $\chi \times \chi$ matrix.

An efficient way of parametrising a low energy state

- Iterating SVD in a many-body state we have MPS:

$$|\Psi\rangle = \sum_{\{i_n\}} c_{i_1,i_2,\ldots,i_L}|i_1,i_2,\ldots,i_L\rangle = \sum_{\{i_n\}} \mathrm{Tr}(A_1^{i_1}A_2^{i_2}\ldots A_L^{i_L})|i_1,i_2,\ldots,i_L\rangle,$$

  with $A_n^{i_n}$ a $\chi \times \chi$ matrix.
- $d^{N_{cav}}$ vs $N_{cav} d\chi^2$ parameters; in general, $\chi = \mathcal{O}(\exp(N_{cav}))$, but, for slightly entangled states, $\chi = \mathcal{O}(\mathrm{poly}(N_{cav}))$.

An efficient way of parametrising a low energy state

- Iterating SVD in a many-body state we have MPS:

$$|\Psi\rangle = \sum_{\{i_n\}} c_{i_1,i_2,\ldots,i_L}|i_1,i_2,\ldots,i_L\rangle = \sum_{\{i_n\}} \mathrm{Tr}(A_1^{i_1}A_2^{i_2}\ldots A_L^{i_L})|i_1,i_2,\ldots,i_L\rangle,$$

  with $A_n^{i_n}$ a $\chi \times \chi$ matrix.
- $d^{N_{cav}}$ vs $N_{cav} d\chi^2$ parameters; in general, $\chi = \mathcal{O}(\exp(N_{cav}))$, but, for slightly entangled states, $\chi = \mathcal{O}(\text{poly}(N_{cav}))$.
- In our computations, χ is at most 10. Example: 2 photons ($d = 3$), $N_{cav} = 100$ cavities and $\chi = 10 \Rightarrow \simeq 3000$ parameters.

An efficient way of parametrising a low energy state

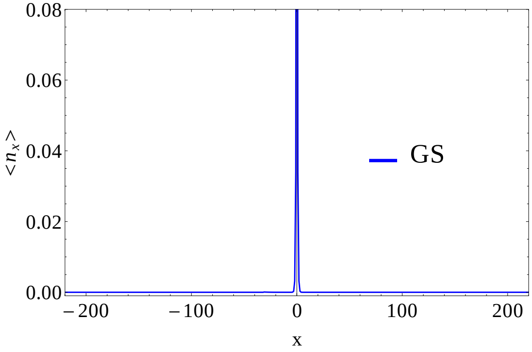
- Iterating SVD in a many-body state we have MPS:

$$|\Psi\rangle = \sum_{\{i_n\}} c_{i_1,i_2,\ldots,i_L}|i_1,i_2,\ldots,i_L\rangle = \sum_{\{i_n\}} \mathrm{Tr}(A_1^{i_1}A_2^{i_2}\ldots A_L^{i_L})|i_1,i_2,\ldots,i_L\rangle,$$

  with $A_n^{i_n}$ a $\chi \times \chi$ matrix.
- $d^{N_{cav}}$ vs $N_{cav} d\chi^2$ parameters; in general, $\chi = \mathcal{O}(\exp(N_{cav}))$, but, for slightly entangled states, $\chi = \mathcal{O}(\mathrm{poly}(N_{cav}))$.
- In our computations, χ is at most 10. Example: 2 photons ($d = 3$), $N_{cav} = 100$ cavities and $\chi = 10 \Rightarrow \simeq 3000$ parameters.
- It allows to initialise **initialise simple states**, compute **eigenstates**, **mean values** and **dynamics**.

Outline of the simulations

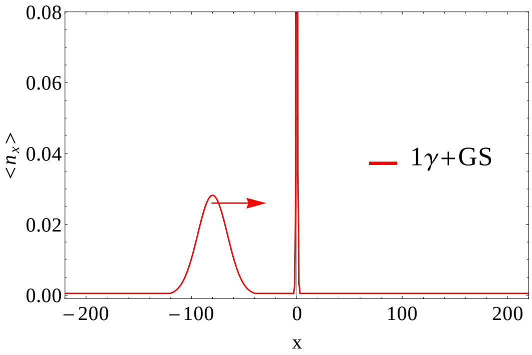
- Ground state
  ($g/\Delta = 0.70$).



Outline of the simulations

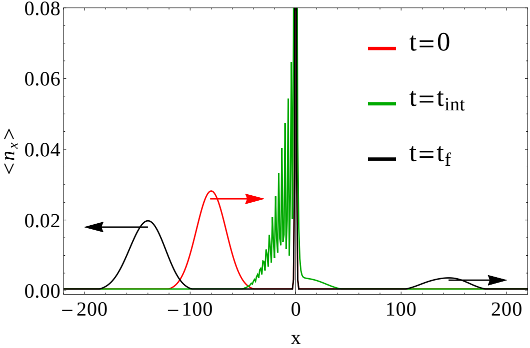
- Ground state
  $(g/\Delta = 0.70)$.

- Flying photon
  $(\omega_{in}/\Delta = 0.70)$.



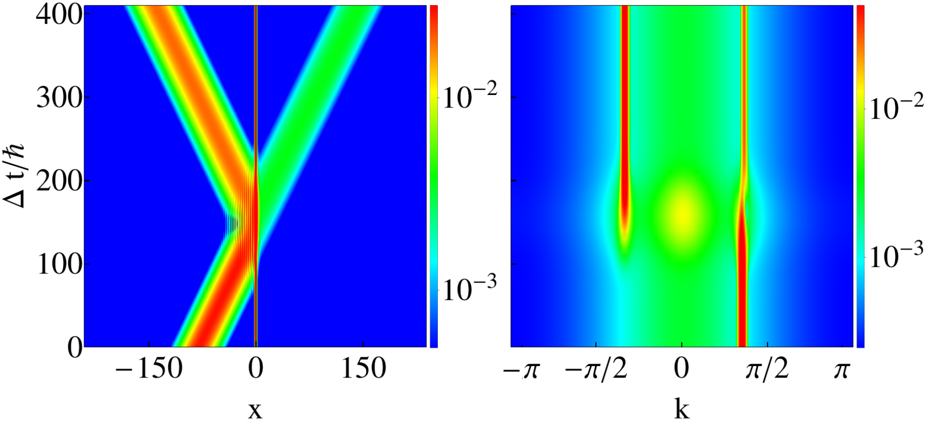
Outline of the simulations

- Ground state
  $(g/\Delta = 0.70)$.
- Flying photon
  $(\omega_{in}/\Delta = 0.70)$.
- Long-time dynamics.



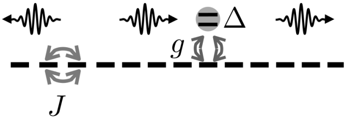
Example

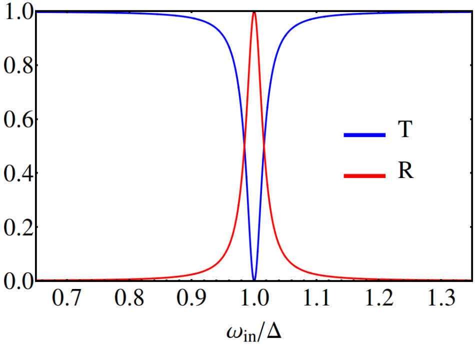
- 1 photon vs 1 qubit: number of photons in position and momentum space.



1 photon vs 1 qubit in RWA

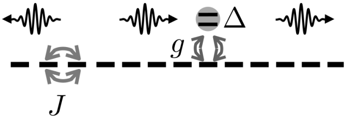
- Analytically solvable problem: resonance at $\omega_{in} = \Delta$ (Shen and Fan, PRL, 2005; Zhou, Gong, Liu, Sun and Nori, PRL, 2008).

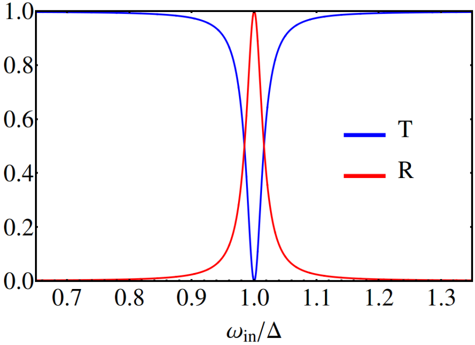




1 photon vs 1 qubit in RWA

- Analytically solvable problem: resonance at $\omega_{in} = \Delta$ (Shen and Fan, PRL, 2005; Zhou, Gong, Liu, Sun and Nori, PRL, 2008).





- Application: **single-photon transistor** (*D.E. Chang , A.S. Sørensen , E.A. Damler and M.D. Lukin*, Nature Physics, Vol 3, 807-812 (2007)).

Qubit implies nonlinearity

- A photon impinges on a qubit.

Qubit implies nonlinearity

- A photon impinges on a qubit.

- The photon gets absorbed.

Qubit implies nonlinearity

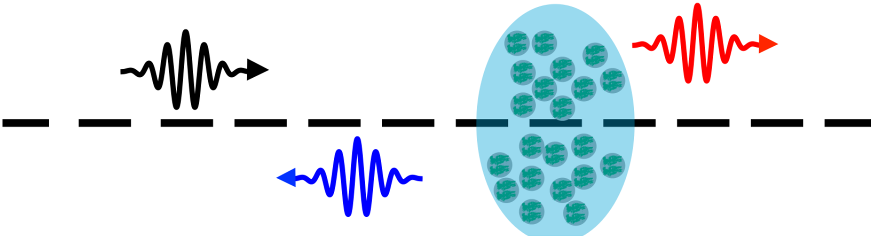
- A photon impinges on a qubit.

- The photon gets absorbed.

- A new photon impinges on it... but it cannot absorb this, it is **saturated**.

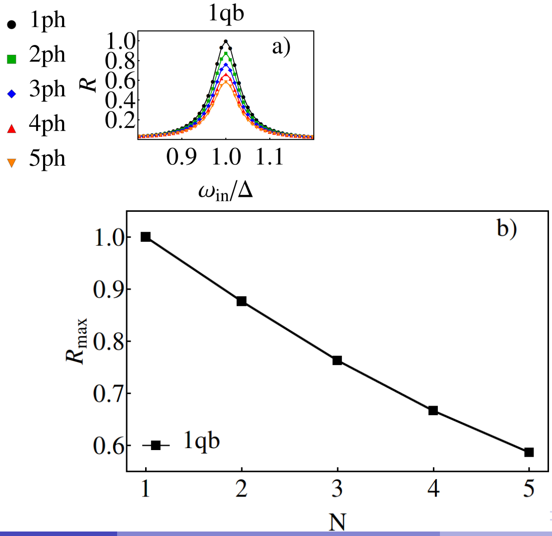
Few photons versus few qubits

- Linear-nonlinear crossover versus number of photons N and number of qubits M.
- Simulations: we send N photons vs M identical qubits in resonance.



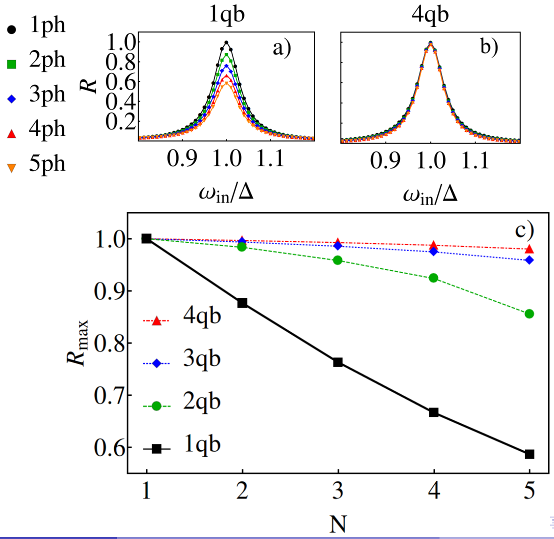
Reflection: N photons M qubits

$g = 0.15$



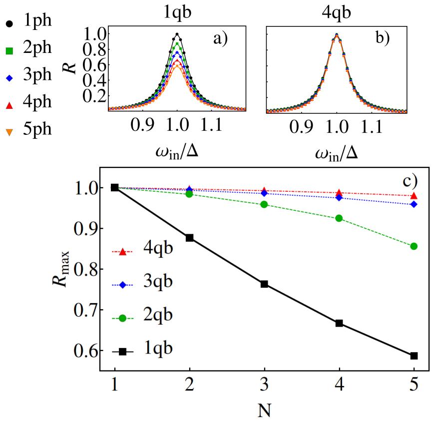
Reflection: N photons M qubits

$$g\sqrt{M} = 0.15$$



Reflection: N photons M qubits

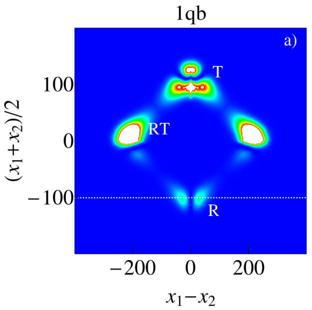
$$g\sqrt{M} = 0.15$$



- The system behaves nonlinearly as $N/M \gtrsim 1$.

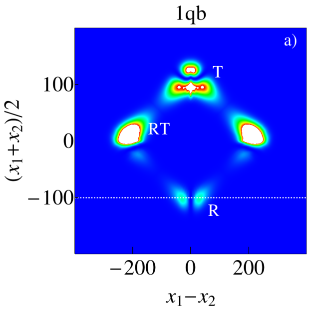
Correlations: 2 photons vs M qubits

- Known result: two photon vs one qubit gives antibunching in reflection component in resonance (Shen and Fan, PRA, 2007).



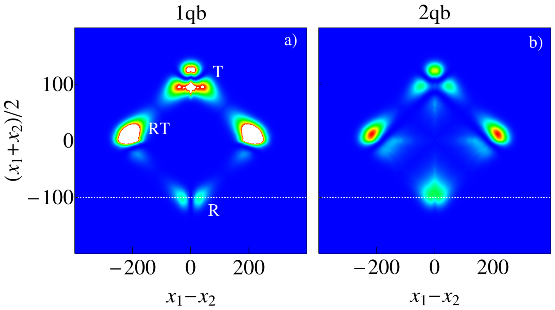
Correlations: 2 photons vs M qubits

- Known result: two photon vs one qubit gives antibunching in reflection component in resonance (Shen and Fan, PRA, 2007).
- What if we add more qubits? We study the correlations $g_2(x_1, x_2) = \langle n_{x_1} n_{x_2} \rangle - \delta_{x_1,x_2} \langle n_{x_1} \rangle$ vs M, for fixed effective coupling $g\sqrt{M} = 0.10$.



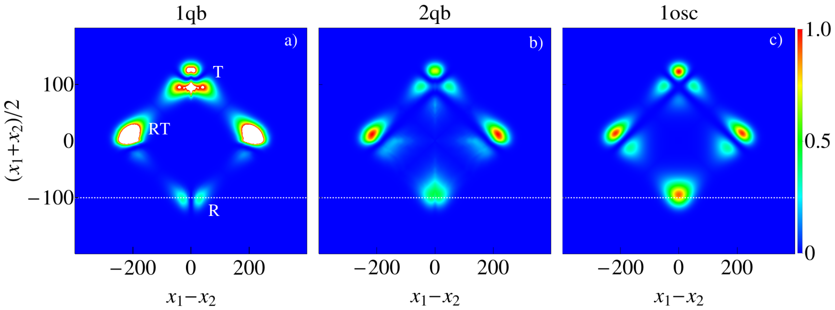
Correlations: 2 photons vs M qubits

- Known result: two photon vs one qubit gives antibunching in reflection component in resonance (Shen and Fan, PRA, 2007).
- What if we add more qubits? We study the correlations $g_2(x_1, x_2) = \langle n_{x_1} n_{x_2} \rangle - \delta_{x_1,x_2} \langle n_{x_1} \rangle$ vs M, for fixed effective coupling $g\sqrt{M} = 0.10$.

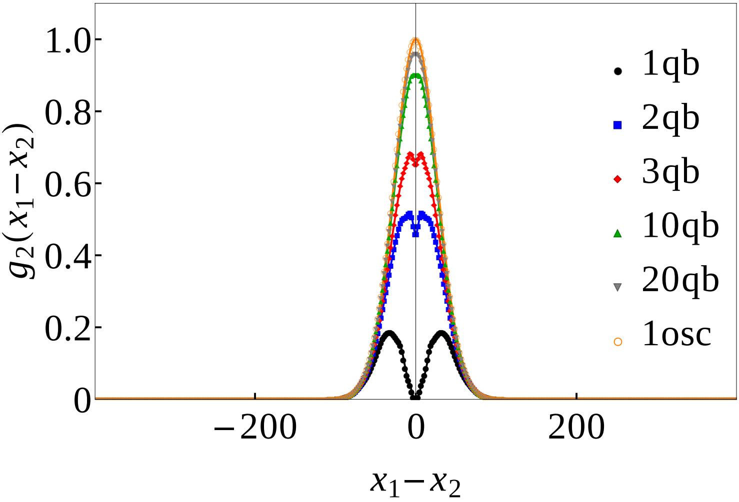


Correlations: 2 photons vs M qubits

- Known result: two photon vs one qubit gives antibunching in reflection component in resonance (Shen and Fan, PRA, 2007).
- What if we add more qubits? We study the correlations $g_2(x_1, x_2) = \langle n_{x_1} n_{x_2} \rangle - \delta_{x_1,x_2} \langle n_{x_1} \rangle$ vs M, for fixed effective coupling $g\sqrt{M} = 0.10$.

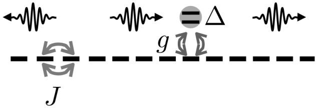


Antibunching: Nonlinear-linear

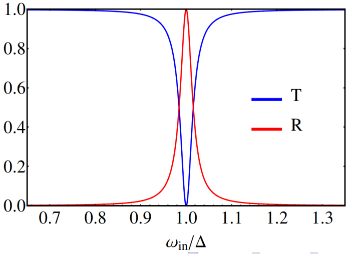


- Now nonlinearities become evident even for $N/M = 2/20$!!!

One photon vs one qubit in ultrastrong regime

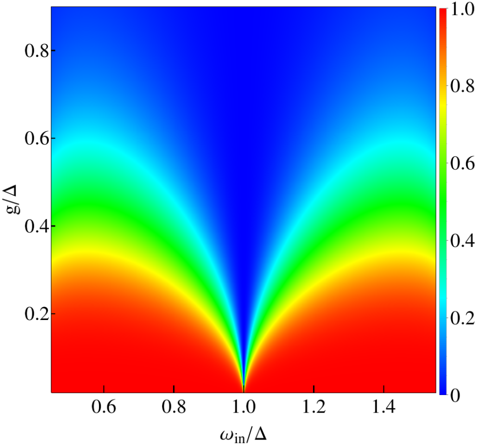


- Remember... resonance as $\omega_{in} = \Delta$ in RWA.
- What happens in the ultrastrong regime?

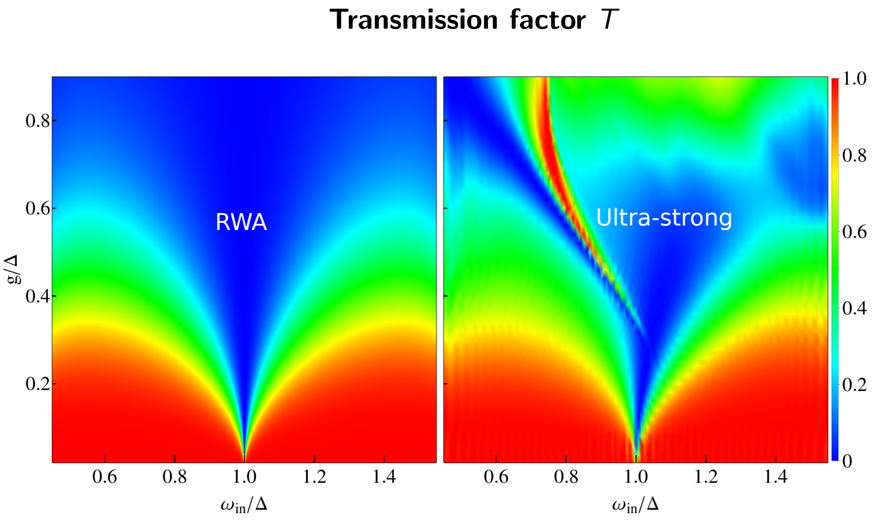


Transmission factor: RWA

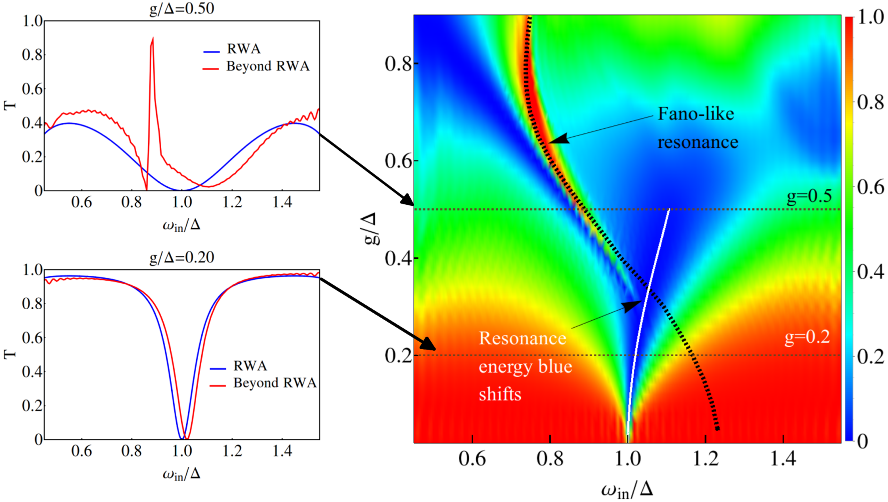
Transmission factor T



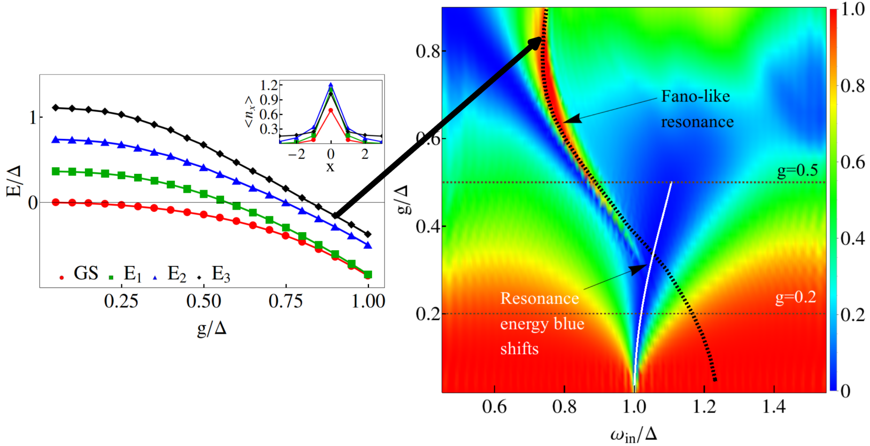
Transmission factor: comparing to RWA case



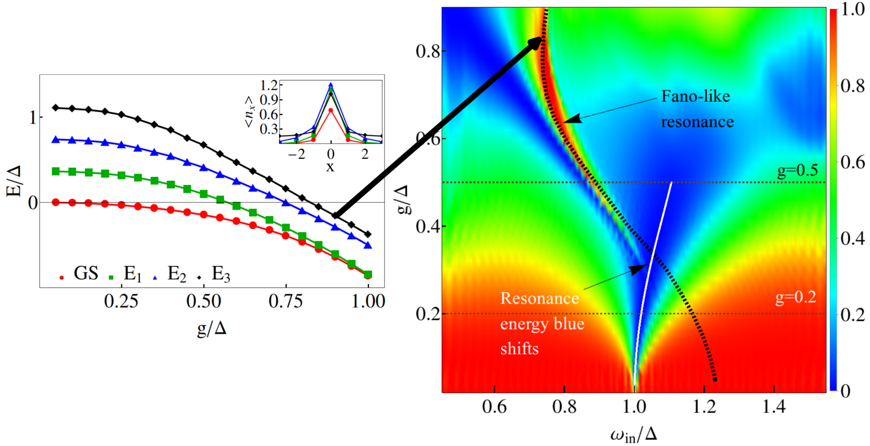
Analysing the transmission factor T



Analysing the transmission factor T

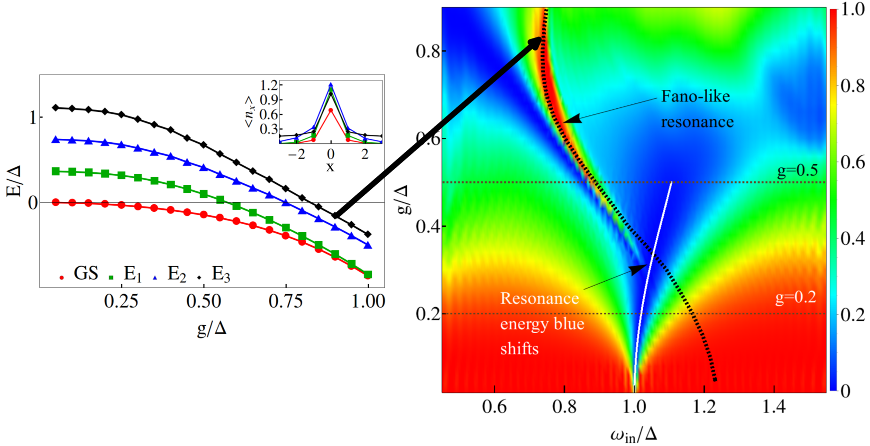


Analysing the transmission factor T



- Resonance tunneling as $\omega_{in} = E_3 - E_{GS}$.

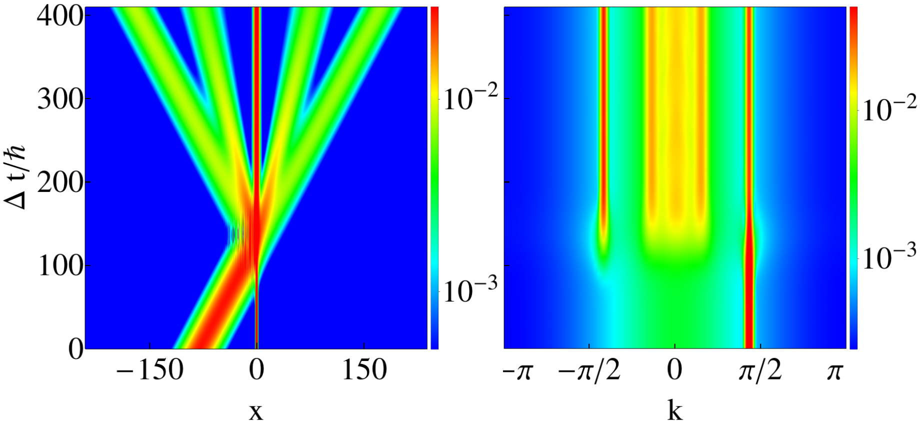
Analysing the transmission factor T



- Resonance tunneling as $\omega_{in} = E_3 - E_{GS}$.
- $\langle N \rangle_3 = 3$ for small $g \Rightarrow$ Impossible in RWA.

Inelastic scattering: example

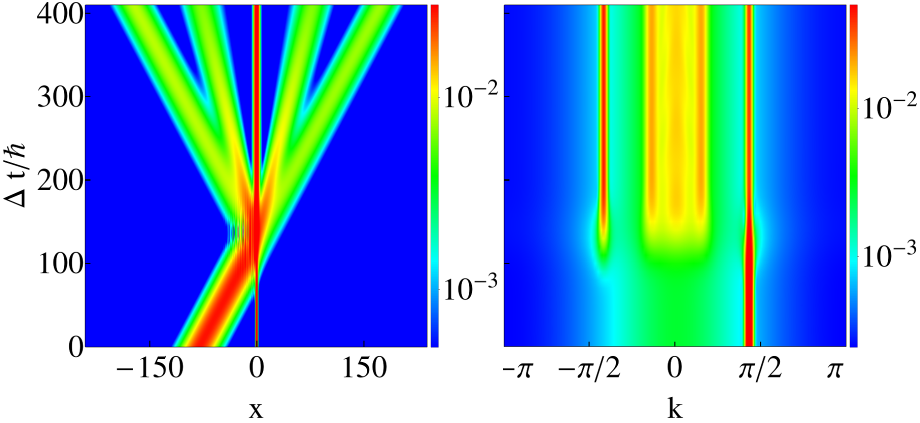
$g = 0.70$, $\omega_{in} = 0.85$



- A new photon with a different momentum appears.

Inelastic scattering: example

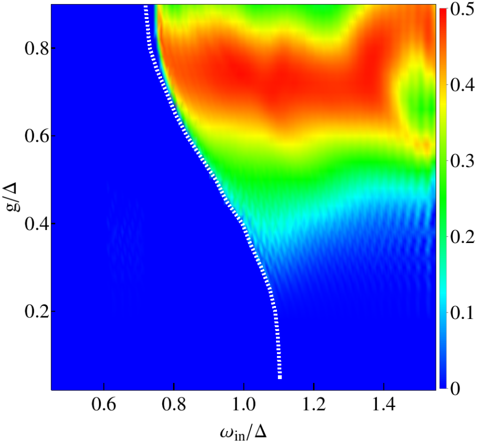
$g = 0.70$, $\omega_{in} = 0.85$



- A new photon with a different momentum appears.
- The photonic cloud around the qubit changes.

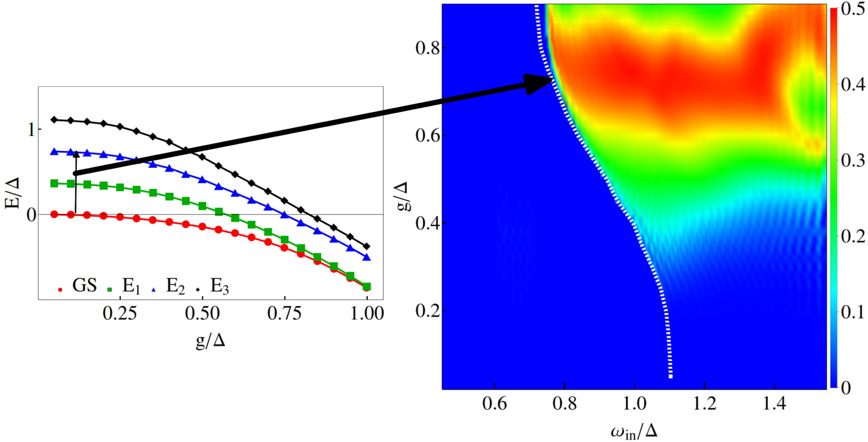
Inelastic scattering probability

Probability of having inelastic scattering: $1 - (R + T)$



Inelastic scattering probability

- Process: $\gamma_{in} + GS \rightarrow \gamma_{out} + E_2$
- $\omega_{in} + E_{GS} = \omega_{out} + E_2$: **Raman scattering** with E_2.



Summarising...

- We have applied the MPS technique for the first time to scattering problems.
- Linear-nonlinear crossover has been explored: depending on the quantity (total reflection or antibunching), nonlinearities are more or less clear as N/M decreases.
- Scattering beyond RWA has been solved: RWA resonance blue shifts, Fano-like resonance, highly efficient Raman scattering.

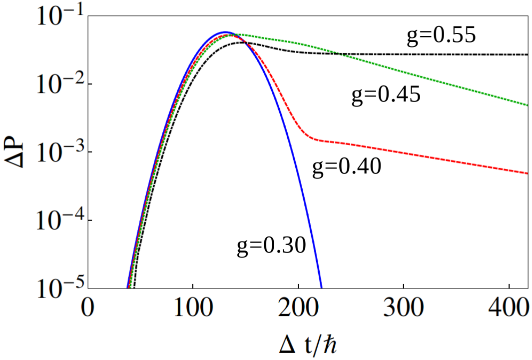
E. Sanchez-Burillo, D. Zueco, J. J. Garcia-Ripoll and L. Martin-Moreno, *Scattering in the ultrastrong regime: nonlinear optics with one photon*, PRL **113**, 264604 (2014)

E. Sanchez-Burillo, J. J. Garcia-Ripoll, L. Martin-Moreno and D. Zueco, *Nonlinear quantum optics in the (ultra)strong light-matter coupling*, Faraday Discussions 178, DOI: 10.1039/C4FD00206G (2015)

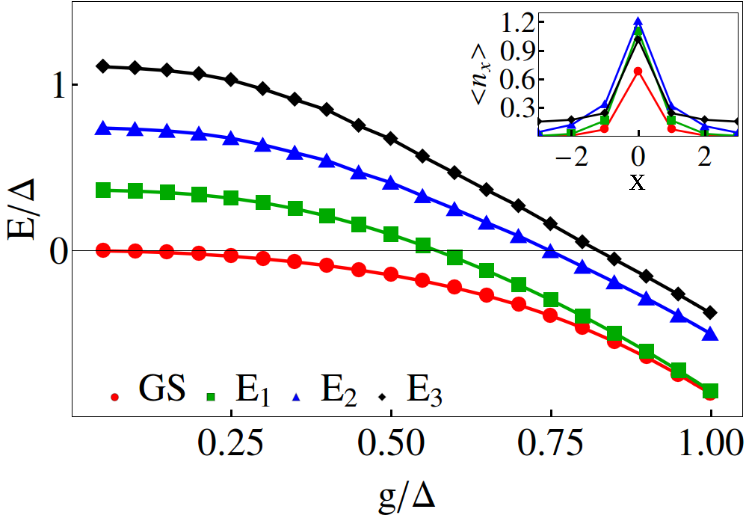
Thank you very much!!! Any question?

Qubit dynamics

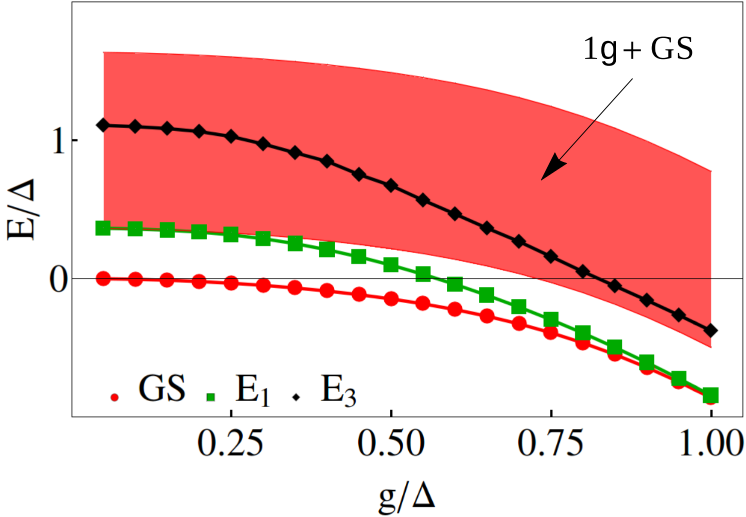
- We fix $\omega_{in} = 0.90$, take different values of g and see the dynamical behaviour of $\Delta P := P(t) - P_{GS}$ ($P = \langle \sigma^+ \sigma^- \rangle$).
- $g = 0.30$: standard behaviour; $g = 0.40, 0.45$: slow dynamics (Fano-like resonance) and $g = 0.55$: decay to a different state (Raman scattering).



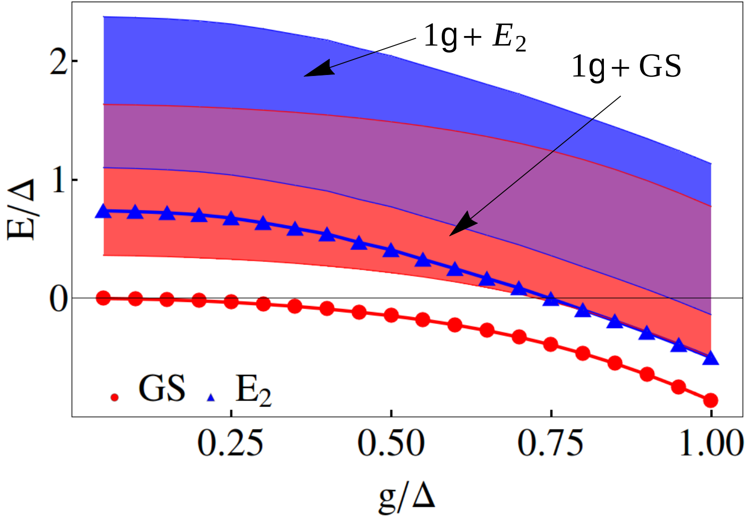
Bound states



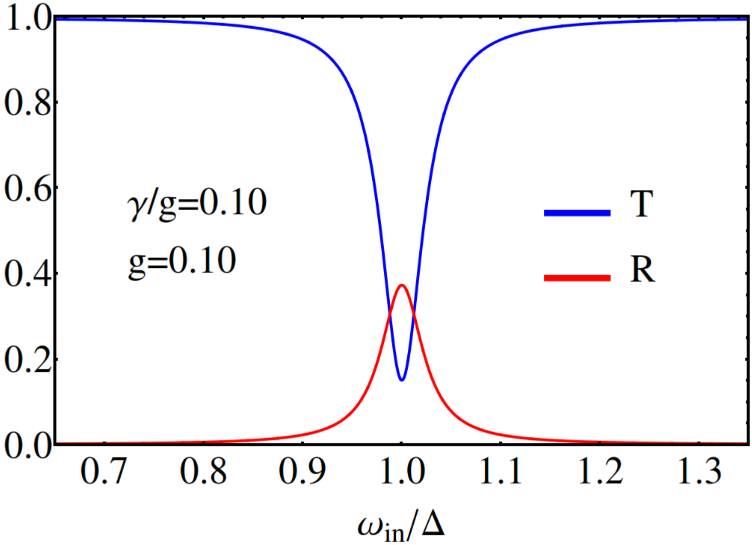
Bound states: Fano resonance



Bound states: Raman scattering



Losses


1.0
0.8
0.6
0.4
0.2
0.0
0.7
0.8
0.9
1.0
1.1
1.2
1.3
ω_{in}/Δ
γ/g=0.10
g=0.10
T
R

Losses: experiments

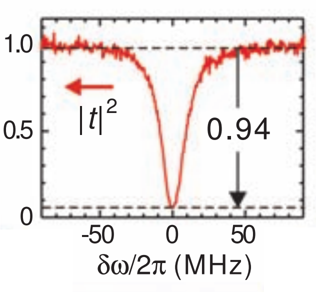


Figure: Astafiev et al., Nature Physics, 2010

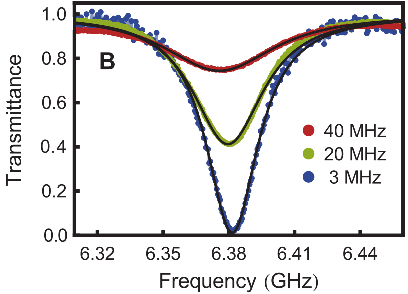
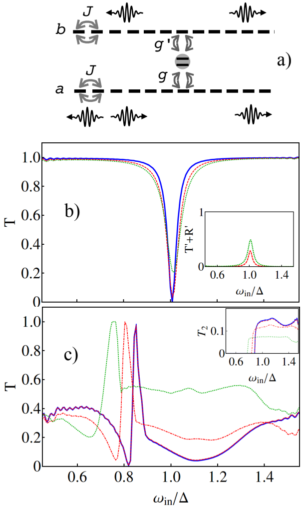


Figure: van Loo et al., Science, 2013

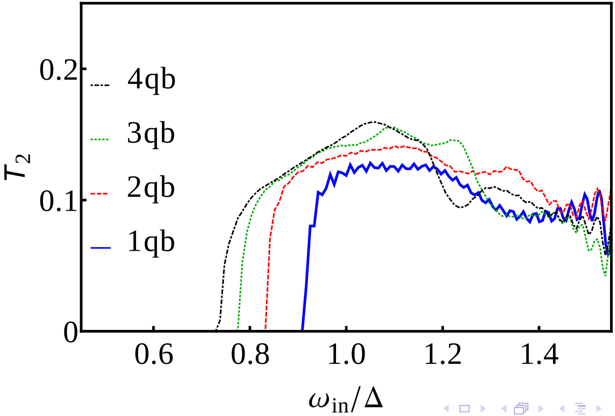
Losses: ultrastrong



Raman scattering: bosonic limit

- **Theorem:** If we replace the qubit by a harmonic oscillator, there is not Raman scattering in ultrastrong coupling.
- Smooth crossover by adding qubits? Not so easy...

$g\sqrt{M} = 0.50$



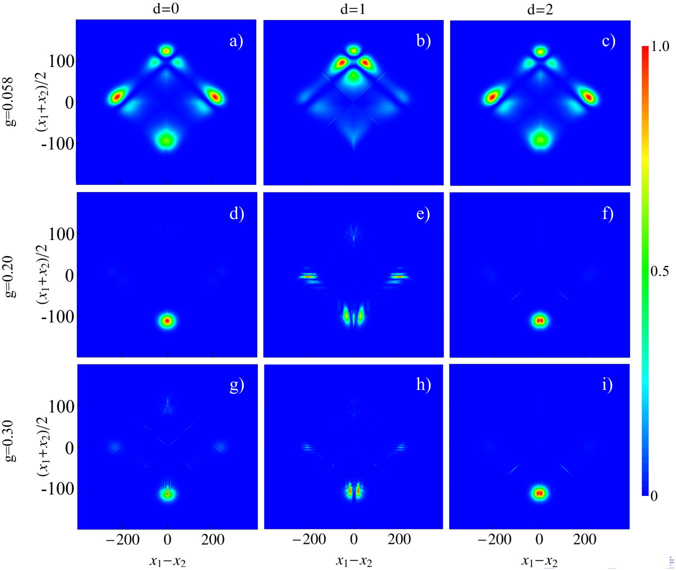
Distance: periodicity

- Effective qubit-qubit interaction and decay rate vary by modifying qubit-qubit distance.

- Markovian regime:

$$g_{qb-qb} \sim \sin(kd) \qquad \gamma \sim \cos(kd)$$

Distance (and ultrastrong): 3 qubits



Distance: 3 qubits