The Gravitational field

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Winter meeting 2015 Benasque

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Consequences of Special relativity

No instantaneous action at a distance is possible

The source of gravitation must be the energy density

Ergo: Any system with energy produces and is affected by gravity.

No energy can hide from gravity

Combined with the QM fact that the photon has got an energy proportional to its frequency

This implies immediately that the photon has to lose energy to escape from a gravitational field

GRAVITATIONAL REDSHIFT

$$V_g \sim -\frac{GM}{h+R_\oplus} \tag{5.111}$$

$$\frac{\omega(h)}{\omega(0)} \sim 1-\frac{GM}{c^2 R_\oplus}+\frac{GM}{c^2\left(R_\oplus+h\right)} \sim 1-\frac{GM}{c^2 R_\oplus}+\frac{GM}{c^2 R_\oplus}\left(1-\frac{h}{R_\oplus}\right)=$$
$$=1-\frac{GM}{c^2 R_\oplus^2}\, h \tag{5.112}$$

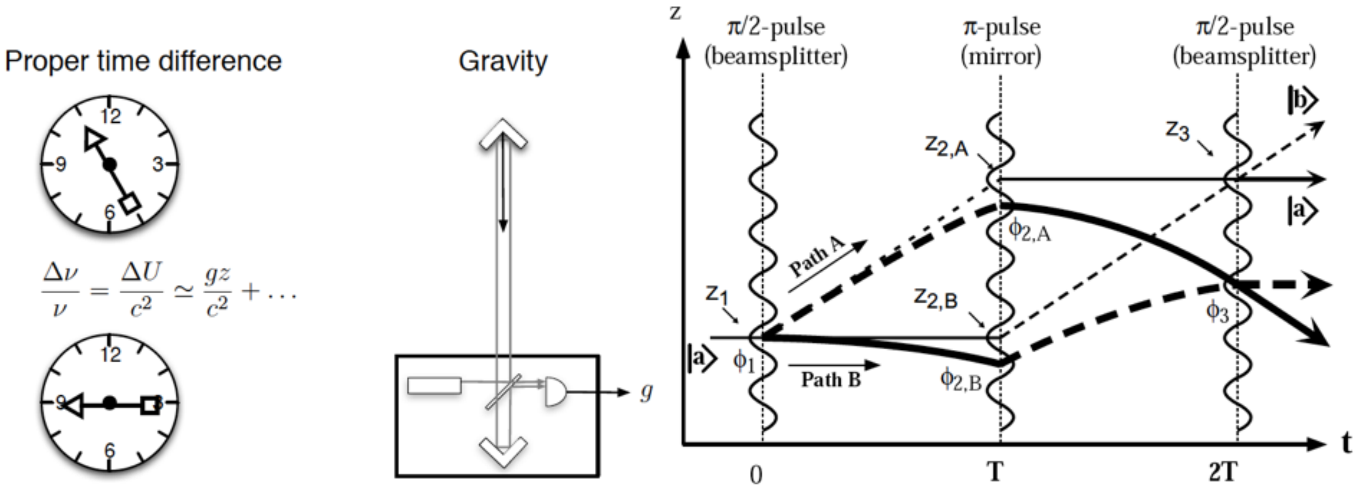


Figure 1. Left: Generic redshift experiment: Two clocks at different locations are compared; the gravitational potential difference ΔU between the two locations is measured by monitoring a geodesic trajectory. **Right:** Mach-Zehnder atom interferometer. A laser-cooled atom encounters three laser pulses that act as beam splitters and mirrors for the matter wave. Each of these pulses transfers the momentum $\hbar k$, where $k = k_1 + k_2$, to the atom The first laser pulse places the atom into a coherent superposition of two quantum states, which physically separate. The second pulse redirects the atom momentum so that the paths merge at the time of the third pulse.

10 E-8 precision claimed!

Michael A. Hohensee, Brian Estey, Francisco Monsalve, Geena Kim, Pei-Chen Kuan, Shau-Yu Lan, and Holger Müller

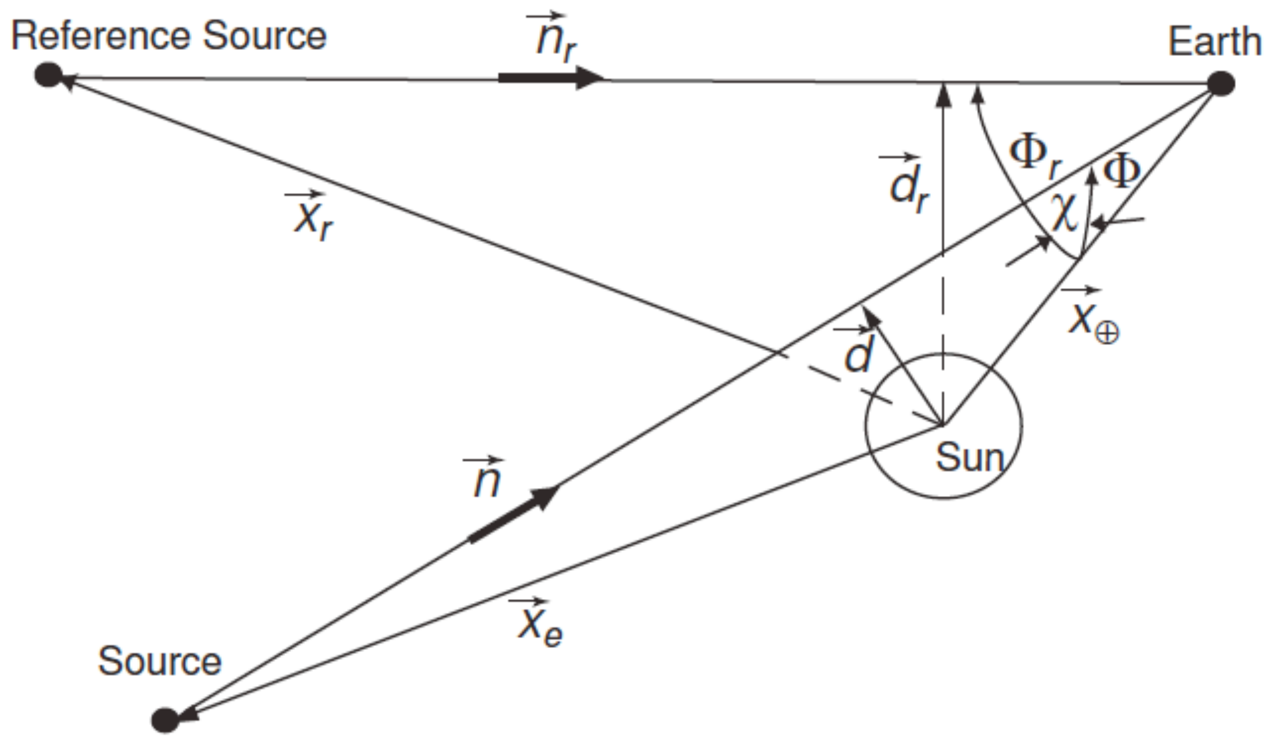
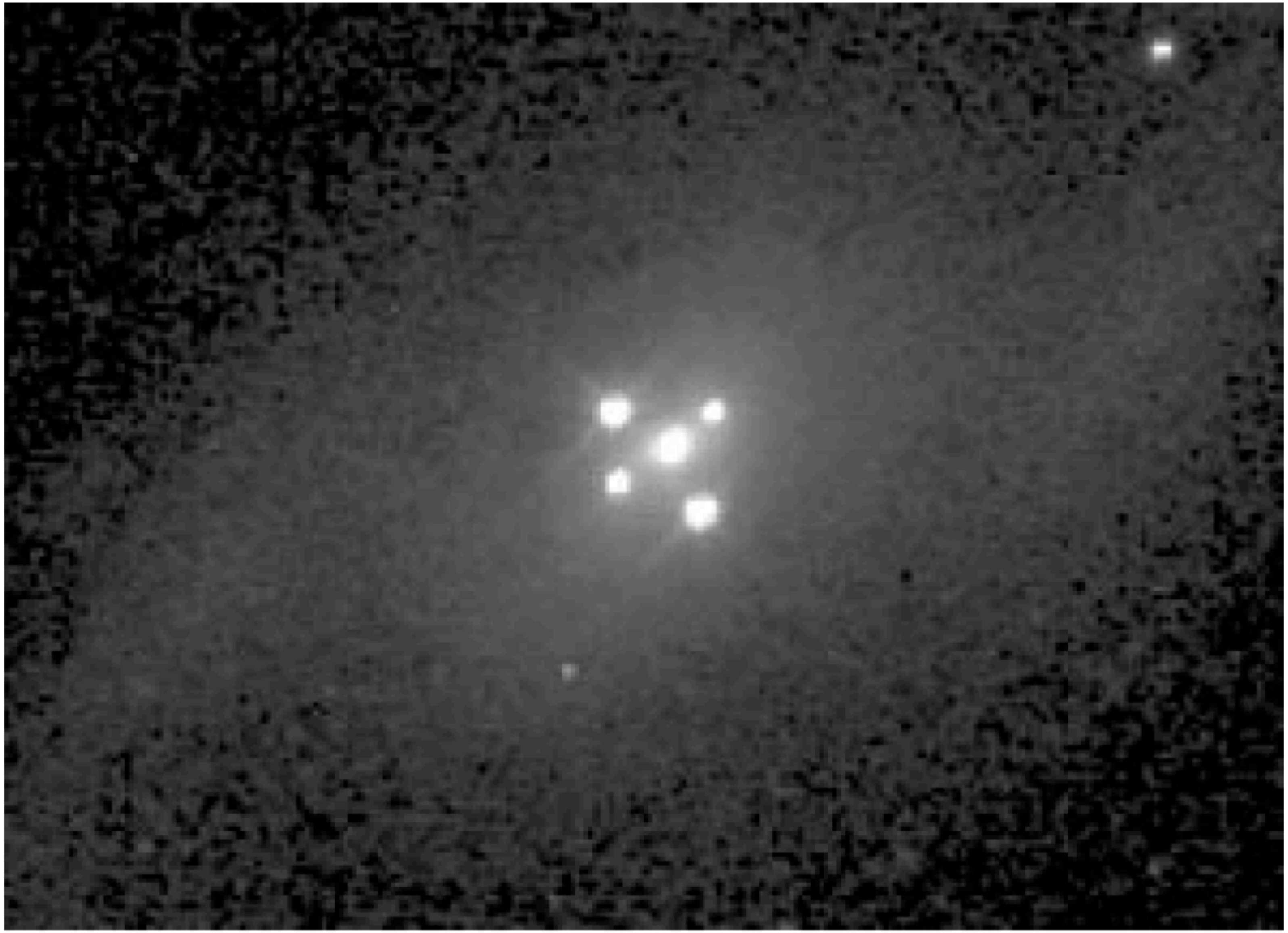


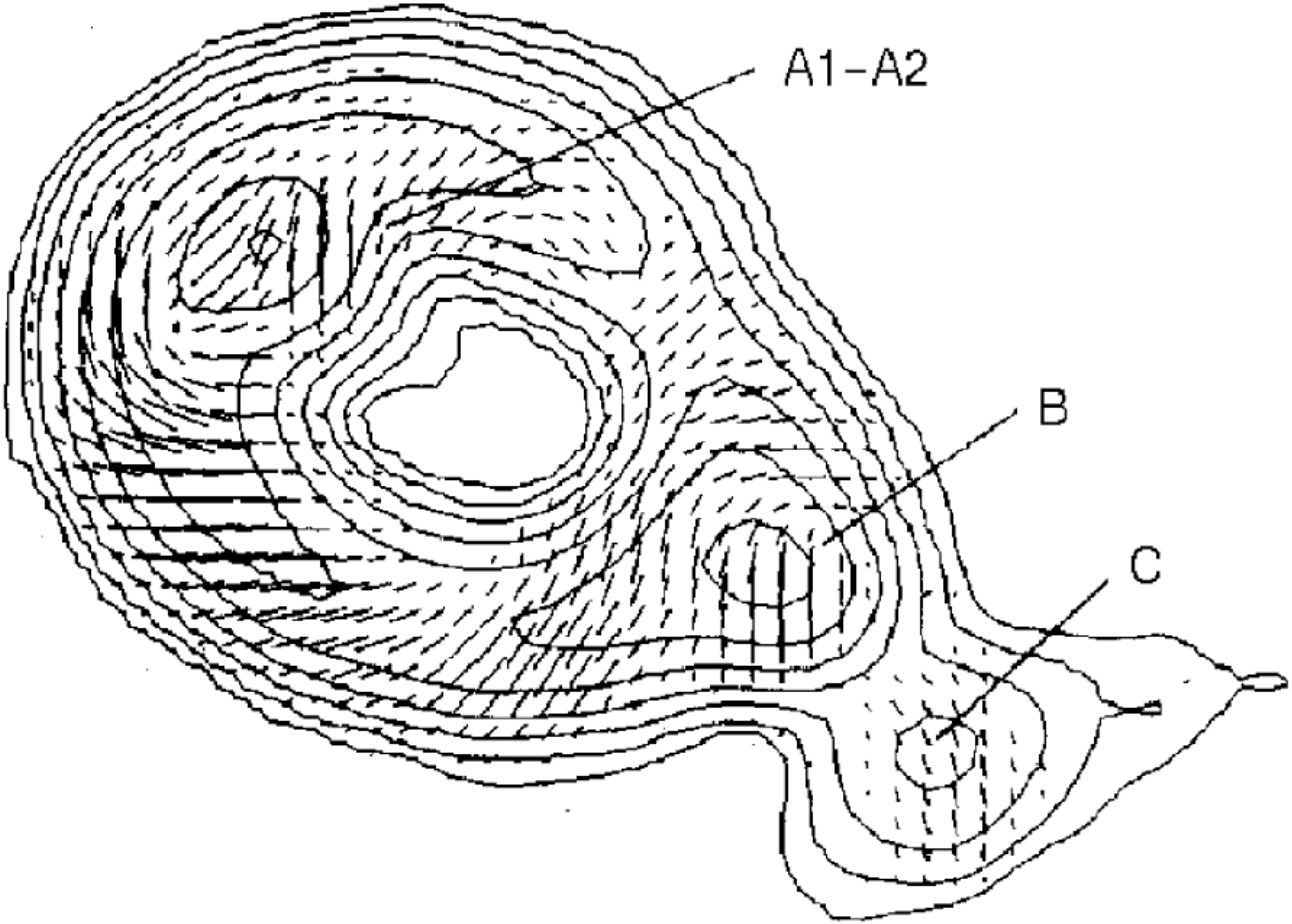
Figure 4: *Geometry of light deflection measurements.*

(Will,2006)



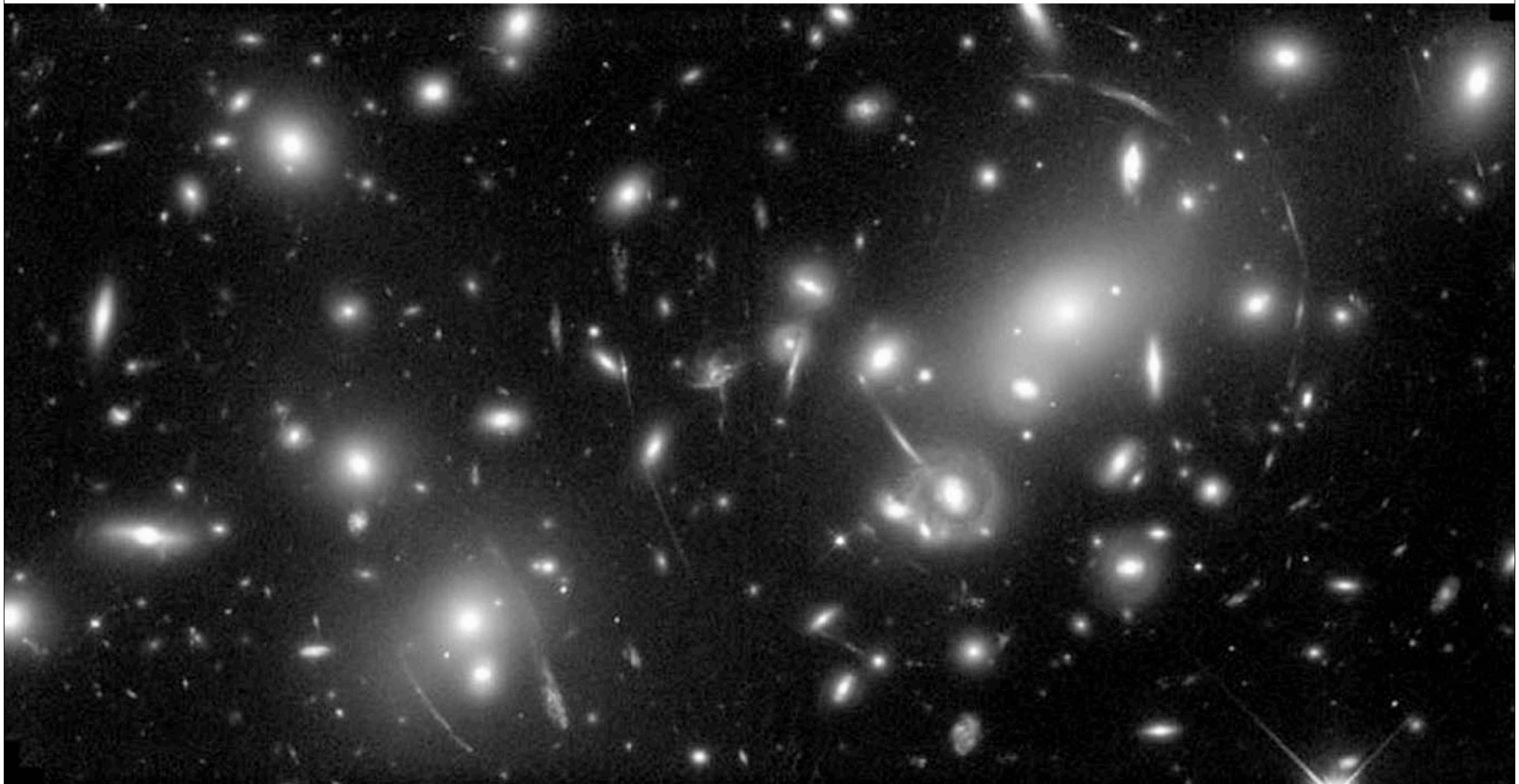
Gravitational lensing

M. Bartelmann, P. Schneider



Einstein rings

M. Bartelmann, P. Schneider



Microlensing arcs

M. Bartelmann, P. Schneider

Poor man's GR (Pauli-Feynman)

Assume there is an intermediate particle (Yukawa)

Spin?

S=0	No bending of light rays
S=1,3,...	Likes repel, unlikes attract

We are led to spin two S=2

The origin of gauge invariance from the massless limit (Veltman)

Massive gravitons have got five polarizations

$$e_i \otimes e_j + e_j \otimes e_i - \frac{2}{3}\left(\sum_k e_k \otimes e_k\right)\delta_{ij}$$

$$\epsilon_1 \equiv e_1 \equiv (0,1,0,0)$$
$$\epsilon_2 \equiv e_2 \equiv (0,0,1,0)$$
$$\epsilon_3 \equiv e_3 \equiv (0,0,0,1)$$

$$k = (1,0,0,1)$$

A massless particle of any spin has only two polarizations (Wigner)

In the massless limit

$$\epsilon_3 \equiv k \otimes e_2 + e_2 \otimes k = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\epsilon_4 \equiv k \otimes e_1 + e_1 \otimes k = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\epsilon_5 \equiv k \otimes k = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\epsilon_1 \equiv e_1 \otimes e_2 + e_2 \otimes e_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\epsilon_2 \equiv e_1 \otimes e_1 - e_2 \otimes e_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The last two rotate amongst themselves under the little group

$$\epsilon_{\alpha\beta} \sim \epsilon_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha$$

Gauge invariance= equivalence classes of polarizations

Quadratic lagrangians from propagators
And
Propagators from unitarity

$$D_{\mu\nu\lambda\sigma} = c_1 \left(\theta_{\mu\nu}\theta_{\lambda\sigma} - \frac{3}{2}\left(\theta_{\mu\lambda}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\lambda}\right) \right)$$

Transverse projector

$$\theta_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

$$S = \int d^4x \left\{ \frac{1}{4}(\partial_\mu h_{\rho\sigma}\partial_\mu h^{\rho\sigma} - \frac{1}{2}\partial_\mu h^{\mu\nu}\partial^\rho h_{\nu\rho} + \frac{1}{2}\partial^\mu h \partial_\rho h^{\mu\rho} - \frac{1}{4}\partial_\mu h \partial^\mu h - \frac{m^2}{4}\left(h_{\alpha\beta}h^{\alpha\beta} - h^2\right) \right\} \tag{4.29}$$

The Fierz-Pauli lagrangian is uniquely determined by unitarity in the massive case.

It is gauge invariant in the massless limit

It can be easily checked that in the massless case, $m = 0$, this is the *unique* lagrangian which is invariant under the gauge symmetry

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \tag{4.30}$$

The FP equations of motion

$$\left(k^2-m^2\right)h_{\mu\nu}-k_\mu k^\rho h_{\rho\nu}-k_\nu k^\rho h_{\mu\rho}+\eta_{\mu\nu}k^\rho k^\sigma h_{\rho\sigma}+k_\mu k_\nu h-\left(k^2-m^2\right)\eta_{\mu\nu}h=0$$

Acting with k $$k^\nu K_{\mu\nu\rho\sigma}h^{\rho\sigma}=-2m^2\left(k^\rho h_{\rho\mu}-k_\mu h\right)=0$$

Ergo $$k^2 h=k_\rho k_\sigma h^{\rho\sigma}$$

Taking the trace $$\eta^{\mu\nu}K_{\mu\nu\rho\sigma}h^{\rho\sigma}=-2(1-n)m^2 h$$

Ergo $$h=k_\mu k_\nu h^{\mu\nu}=0$$

Every component obeys the Klein-Gordon equation

$$\boxed{\left(\Box+m^2\right)h_{\mu\nu}=0}$$

Let us compute the force between two static sources

Harmonic gauge (de Donder)

$$\mathcal{O}^{\mu\nu\rho\sigma} = -\frac{1}{8}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right)\Box + \frac{1}{8}\Box\eta_{\mu\nu}\eta_{\rho\sigma}$$

The force is given by the propagator

$$W \equiv \int d^4 z\, T_{\mu\nu}(x)\, \left(\mathcal{O}^{-1}\right)^{\mu\nu\rho\sigma}(x-y)\, T_{\rho\sigma}(y)$$

In the massless case

$$W = \frac{1}{k^2} \left(T^{(1)}_{\mu\nu} T^{(2)\mu\nu} - \frac{1}{2} T^{(1)} T^{(2)} \right)$$

Whereas in the massive case

$$W = \frac{1}{k^2} \left(3\, T^{(1)}_{\mu\nu} T^{(2)\mu\nu} - T^{(1)} T^{(2)} \right)$$

The massless case is not the massless limit of the massive case

Van Dam-Veltman discontinuity

Self-Coupling and Coupling to matter

If we want the energy density to be the source of gravitation, we need

$$L_{\text{int}} \equiv \kappa h^{\mu\nu} T_{\mu\nu}[\phi] = \kappa h^{\mu\nu} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \left(\partial_\rho \phi \partial^\rho \phi - \frac{m^2}{2} \phi^2 \right) \eta_{\mu\nu} \right)$$

The new EM read $\qquad K_{\mu\nu\rho\sigma} h^{\rho\sigma} = \kappa T_{\mu\nu}$

But the FP operator is transverse $\qquad \partial^\mu K_{\mu\nu\rho\sigma} = 0$

The old EMT is not conserved anymore

$$T^{\rho}_{\mu} \equiv \frac{\partial L}{\partial(\partial_\rho \phi_a)} \partial_\mu \phi_a - L \eta^{\rho}_{\mu}$$

The conserved EMT reads now

$$T^{\text{can}}_{\mu\nu} = T_{\mu\nu} + \kappa \left(\left(h^{\rho}_{\mu} \partial_\rho \phi - h \partial_\mu \phi \right) \partial_\nu \phi - h^{\alpha\beta} T_{\alpha\beta} \eta_{\mu\nu} \right)$$

We should then correct the coupling so that the graviton couples to the conserved EMT.

But this modifies again the EMT….(Feynman's approach)
Nobody suceeded in summing the whole series….

Deser resummation

First order form of Fierz-Pauli

$$L \equiv -\kappa \bar{h}^{\mu\nu}\left[\partial_\mu \Gamma^\rho_{\nu\rho} - \partial_\rho \Gamma^\rho_{\mu\nu}\right] + \eta^{\mu\nu}\left[\Gamma^\lambda_{\mu\rho}\Gamma^\rho_{\nu\lambda} - \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\lambda\rho}\right]$$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\, h\, \eta_{\mu\nu}$$

$$\Gamma^\alpha_{[\beta\gamma]} = 0$$

$$\Gamma_\mu \equiv \Gamma^\lambda_{\lambda\mu}$$
$$\Gamma^\mu \equiv \eta^{\rho\sigma}\Gamma^\mu_{\rho\sigma}$$

The EM read

$$\frac{\delta S}{\delta \bar{h}^{\mu\nu}} = -\partial_\rho \Gamma^\rho_{\mu\nu} + \frac{1}{2}\left(\partial_\mu \Gamma_\nu + \partial_\nu \Gamma_\mu\right)$$

$$\frac{\delta S}{\delta \Gamma^\alpha_{\mu\nu}} = \partial_\alpha \bar{h}^{\mu\nu} - \frac{1}{2}\left(\delta^\nu_\alpha \partial_\sigma \bar{h}^{\mu\sigma} + \delta^\mu_\alpha \partial_\sigma \bar{h}^{\nu\sigma}\right) - \eta^{\mu\nu}\Gamma_\alpha -$$
$$-\frac{1}{2}\left(\Gamma^\mu \delta^\nu_\alpha + \Gamma^\nu \delta^\mu_\alpha\right) + \Gamma^\nu_{\alpha\sigma}\eta^{\mu\sigma} + \Gamma^\mu_{\alpha\beta}\eta^{\nu\beta}$$

$$\partial^\lambda \Gamma_\lambda = \partial_\mu \Gamma^\mu \qquad\qquad \Gamma^\nu = \partial_\sigma \bar{h}^{\nu\sigma}$$

$$\partial_\alpha \bar{h} - \partial_\lambda \bar{h}^\lambda_\alpha - n\Gamma_\alpha - \Gamma^\lambda \eta_{\lambda\alpha} + \Gamma_\alpha + \Gamma_\alpha = 0$$

$$\Gamma_\alpha = -\frac{1}{n-2}\partial_\alpha \bar{h}$$

$$\Gamma_{\nu;\alpha\mu} + \Gamma_{\mu;\nu\alpha} = +\partial_\alpha h_{\mu\nu}$$

The end result is the linear Christoffel

$$\Gamma_{\mu;\nu\alpha} = \frac{1}{2}\left(\partial_\nu h_{\alpha\mu} + \partial_\alpha h_\mu \ - \partial_\mu h_{\nu\alpha}\right)$$

The other EM can be written as

$$R^L_{\mu\nu}[h] = 0$$

(Equivalent to Fierz-Pauli)

Bianchi still holds at the linear level

$$\partial_\mu R_L^{\mu\nu} = \frac{1}{2}\partial^\nu R_L$$

This is actually the reason why the Fierz-Pauli operator is transverse

Now we modify slightly the FO lagrangian

$$L \equiv -\kappa \bar{h}^{\mu\nu} \left[\partial_\mu \Gamma^\rho_{\nu\rho} - \partial_\rho \Gamma^\rho_{\mu\nu} \right] + \left(\eta^{\mu\nu} - \kappa \bar{h}^{\mu\nu} \right) \left[\Gamma^\lambda_{\mu\rho} \Gamma^\rho_{\nu\lambda} - \Gamma^\lambda_{\mu\nu} \Gamma^\rho_{\lambda\rho} \right]$$

$$\eta^{\mu\nu} - \kappa \bar{h}^{\mu\nu} \equiv f^{\mu\nu}$$

$$f_{\alpha\lambda} f^{\lambda\beta} = \delta^\beta_\alpha$$

$$\boxed{L \equiv (f^{\mu\nu} - \eta^{\mu\nu}) \left[\partial_\mu \Gamma^\rho_{\nu\rho} - \partial_\rho \Gamma^\rho_{\mu\nu} \right] + f^{\mu\nu} \left[\Gamma^\lambda_{\mu\rho} \Gamma^\rho_{\nu\lambda} - \Gamma^\lambda_{\mu\nu} \Gamma^\rho_{\lambda\rho} \right]}$$

This is still a flat space lagrangian

Working out the EM:

$$\frac{\delta S}{\delta f^{\mu\nu}} = R_{\mu\nu}\left[\Gamma\right]$$

$$\frac{\delta S}{\delta \Gamma^{\alpha}_{\mu\nu}} = \partial_\alpha f^{\mu\nu} - \frac{1}{2}\left(\delta^\nu_\alpha \partial_\sigma f^{\mu\sigma} + \delta^\mu_\alpha \partial_\sigma f^{\nu\sigma}\right) - f^{\mu\nu}\Gamma_\alpha -$$

$$-\frac{1}{2}\left(\Gamma^\mu f^\nu_\alpha + \Gamma^\nu f^\mu_\alpha\right) + \Gamma^\nu_{\alpha\sigma} f^{\mu\sigma} + \Gamma^\mu_{\alpha\beta} f^{\nu\beta}$$

It follows $$\Gamma^\mu_{\alpha\beta} f^{\alpha\beta} = -\partial_\sigma f^{\sigma\mu}$$

$$\Gamma_\lambda = \frac{1}{n-2} f_{\alpha\beta}\partial_\lambda f^{\alpha\beta} = -\frac{1}{n-2} F^{-1}\partial_\alpha F$$

$$F \equiv \det \ f_{\mu\nu}$$

$$\partial_\alpha f^{\mu\nu} + \frac{1}{n-2} f^{\mu\nu} F^{-1} \partial_\alpha F + \Gamma^\mu_{\sigma\alpha} f^{\nu\sigma} + \Gamma^\nu_{\alpha\sigma} f^{\sigma\mu} = 0$$

$$\partial_\alpha \left(f^{\mu\nu} \ F^{\frac{1}{n-2}} \right) + \Gamma^\mu_{\sigma\alpha} f^{\nu\sigma} \ F^{\frac{1}{n-2}} + \Gamma^\nu_{\alpha\sigma} f^{\sigma\mu} \ F^{\frac{1}{n-2}} = 0$$

Everything depends on the physical metric only

$$\boxed{g^{\mu\nu} \equiv F^{\frac{1}{n-2}} \ f^{\mu\nu}}$$

$$f^{\mu\nu} = \sqrt{g} g^{\mu\nu}$$

Miraculously, the full lagrangian is diff invariant with the measure

$$d(vol) \equiv \sqrt{g}\ d^n x$$

Linear Christoffels grow to real Christoffels

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\lambda} \left(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu} \right)$$

Fierz-Pauli grows into Einstein

$$R_{\mu\nu} \equiv R_{\mu\nu}[g]$$

General Relativity is recovered starting from Fierz-Pauli

Poor man's way is not however Einstein's way

The Einstein way: The equivalence principle

$$m_2^i \ddot{\vec{r}}_2 = G m_1^g m_2^g \frac{\vec{r}_1 - \vec{r}_2}{r_{12}^3}$$

$$\boxed{m^i = m^g}$$

All bodies are subject to the same acceleration

There is a local inertial system
in which gravity is not felt

Free falling= FREFO=LIS =Locally inertial system

Einstein: SEP= Special relativity is valid for FREFOS

Experimental limits on violation of the WEP

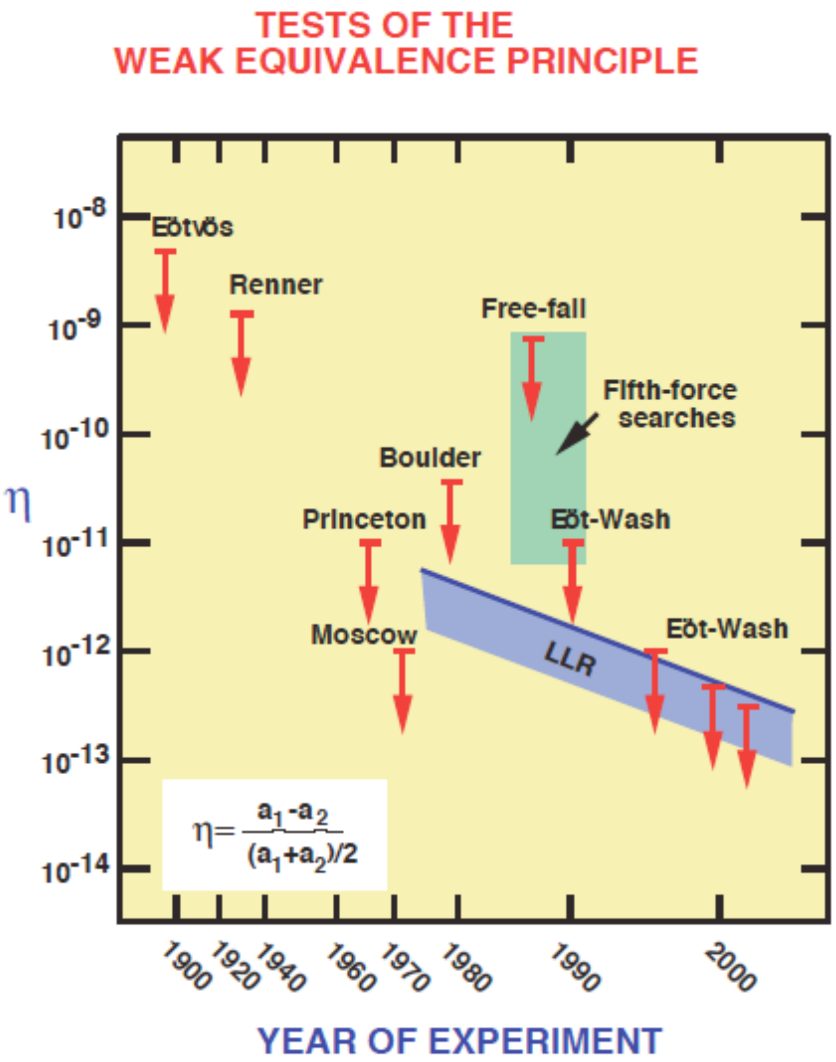
$$(\Delta a/a)_{\mathrm{BeTi}} = (0.3 \pm 1.8) \times 10^{-13} \ ,$$

$$(\Delta a/a)_{\mathrm{BeAl}} = (-0.7 \pm 1.3) \times 10^{-13} \ ,$$

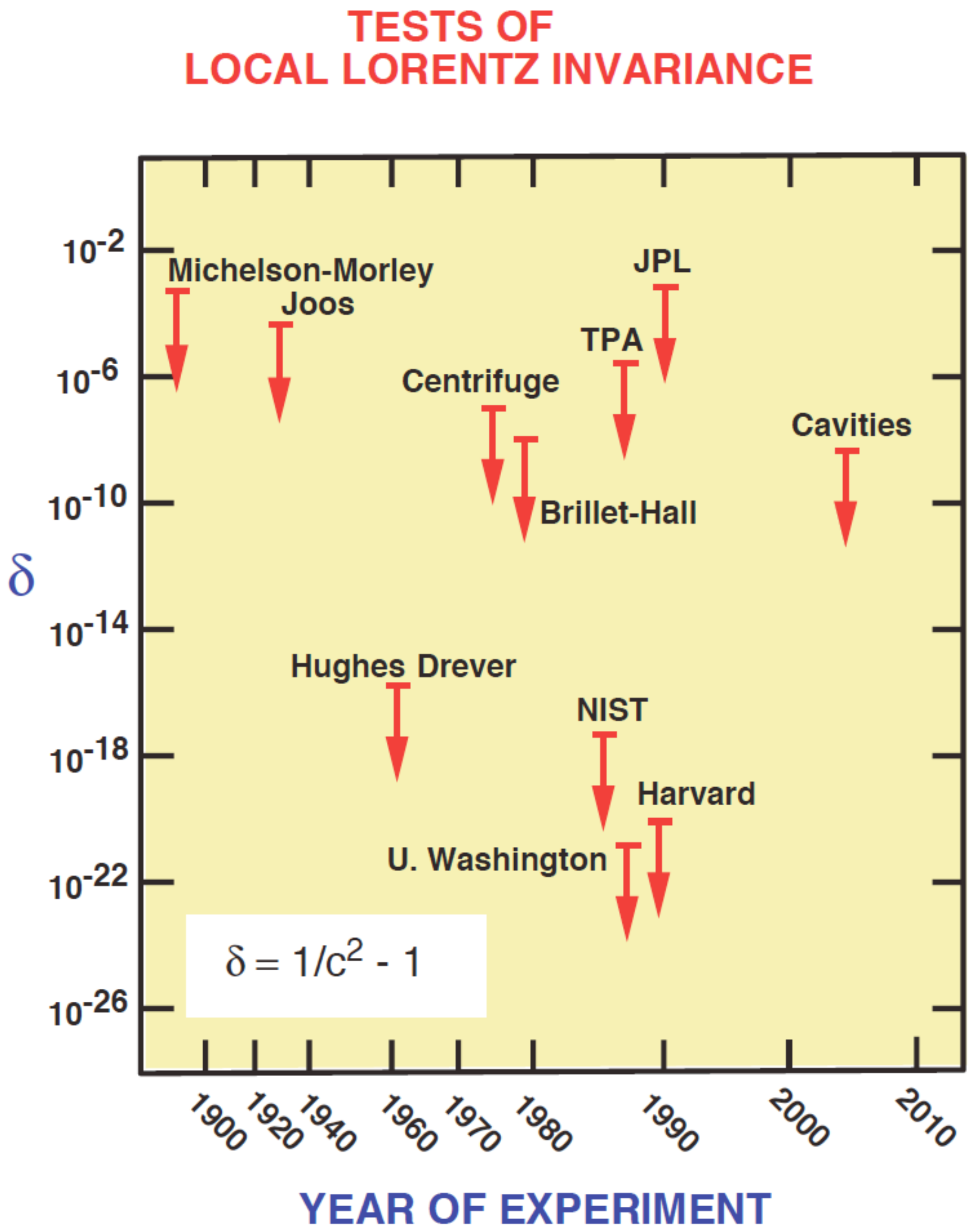
$$(\Delta a/a)_{\mathrm{EarthMoon}} = (-0.8 \pm 1.3) \times 10^{-13} \ .$$

(PDG)

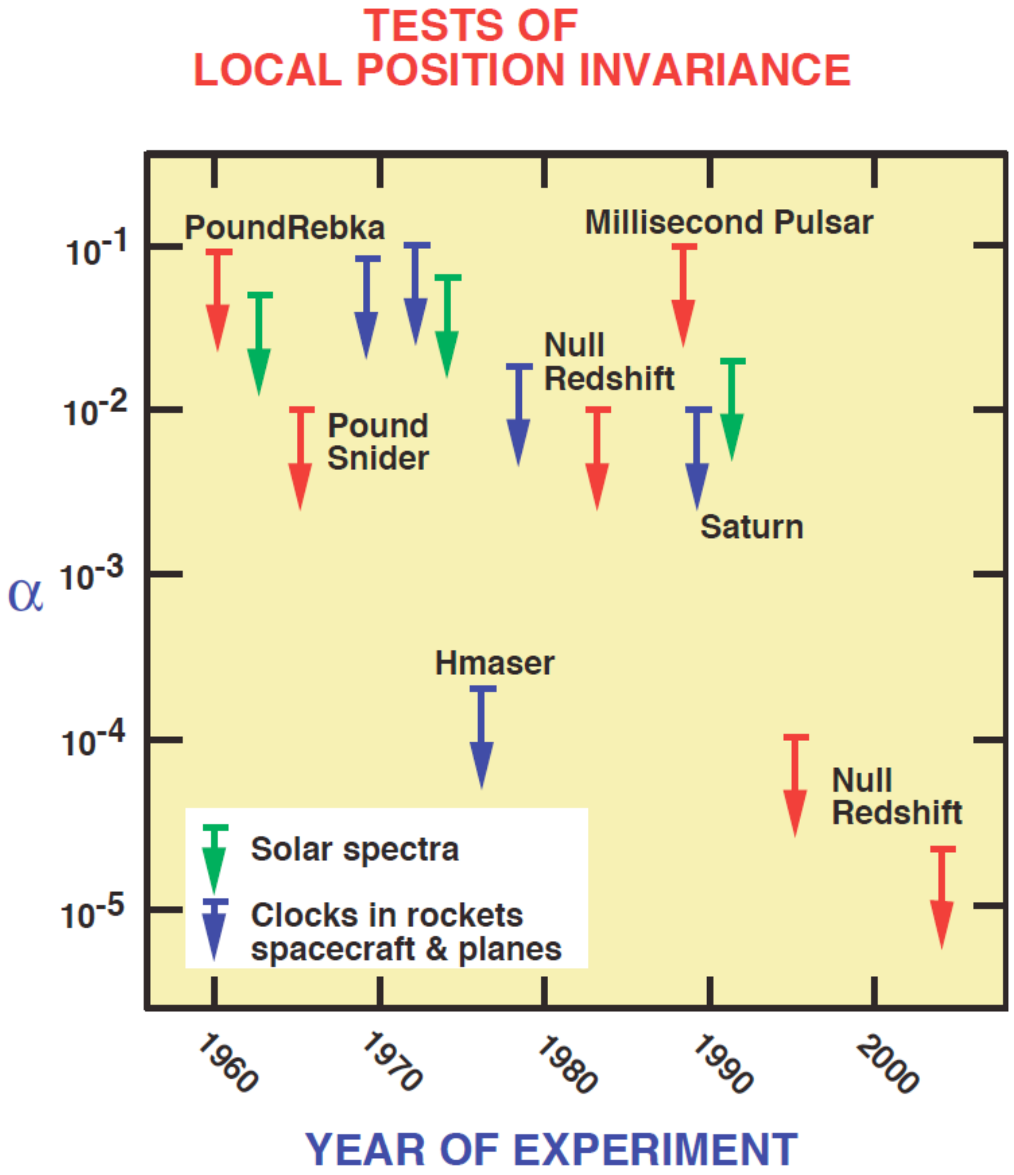
$$\eta \equiv 2\frac{|a_1 - a_2|}{|a_1 + a_2|} = \sum_A \eta^A \left(\frac{E_1^A}{m_1 c^2} - \frac{E_2^A}{m_2 c^2} \right)$$



(Will-2006)


TESTS OF
LOCAL LORENTZ INVARIANCE
Michelson-Morley
Joos
JPL
TPA
Centrifuge
Cavities
Brillet-Hall
Hughes Drever
NIST
Harvard
U. Washington
$\delta = 1/c^2 - 1$
δ
10^{-2}
10^{-6}
10^{-10}
10^{-14}
10^{-18}
10^{-22}
10^{-26}
1900
1920
1940
1960
1970
1980
1990
2000
2010
YEAR OF EXPERIMENT

(Will-2006)



$$\boxed{\Delta\nu/\nu = (1+\alpha)\Delta U/c^2}$$

(Will,2006)

Einstein-Grossmann
GEOMETRIZATION OF PHYSICS

The earth looks flat locally

At each point in the surface of a sphere there is a tangent plane

∴ At each point in the surface of a four-dimensional manifold there is a tangent space= four-dimensional vector space= four dimensional Minkowski space= FREFO

∴ Physical quantities as geometrical entities defined in a spacetime manifold

General coordinate invariance=Diffeomorphism invariance

Wilsonian approach:
Relevant operators

Dimension zero: Cosmological constant

Dimension Two: Scalar Curvature

Dimension four: Quadratic lagrangians

$$S = -\frac{c^3}{16\pi G}\int_V d^n x\sqrt{|g|}\;(R + 2\lambda) + S_{\mathrm{matter}}\,.$$

Einstein-Hilbert action

Einstein's equations

$$S_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2}(R + 2\lambda)\ g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta} \equiv \frac{\kappa^2}{c^4} T_{\alpha\beta}$$

Marble = Timber

Landau: The most beautiful construct of human mind

Their mathematical structure is not yet understood

Much more complicated than Yang-Mills or Navier-Stokes (Millennium Prize problems)

Weak field limit

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

(In order to keep the canonical dimension for h)

$$R_{\mu\nu} = \kappa \left(-\frac{1}{2}\Box h_{\mu} \ + \frac{1}{2}\partial_\mu \partial_\lambda h^\lambda_\mu + \frac{1}{2}\partial_\nu \partial_\lambda h^\lambda_\mu - \frac{1}{2}\partial_\mu \partial_\nu h \right) + O(\kappa^2)$$

Harmonic gauge $\quad \partial_\lambda h^\lambda_\mu = \frac{1}{2}\partial_\mu h$

D'Alembert Wave Equation $\quad \Box h_{\mu\nu} = 0$

Weak field static limit

$$\Delta V_g = 4\pi G\rho$$

$$R^0_0 = \frac{\kappa^2}{2}\rho \qquad\qquad R^0_0 \sim \frac{1}{c^2}\Delta\Phi$$

$$\Delta\Phi = \frac{c^2\kappa^2}{2}\rho$$

Determines the strength of the gravitational coupling

$$\kappa^2 \equiv \frac{8\pi G}{c^2}$$

A few facts

Test particles move on geodesics of spacetime

Vacuum solutions are Ricci-flat spacetimes

Gravitational Energy cannot be a geometrical notion (it vanishes for FREFOS)

Gravitational waves can be shown to carry energy to infinity (Bondi)

Expansion of the universe predicted by Friedmann (1924) five years before Hubble.

The Big Bang is a successful model

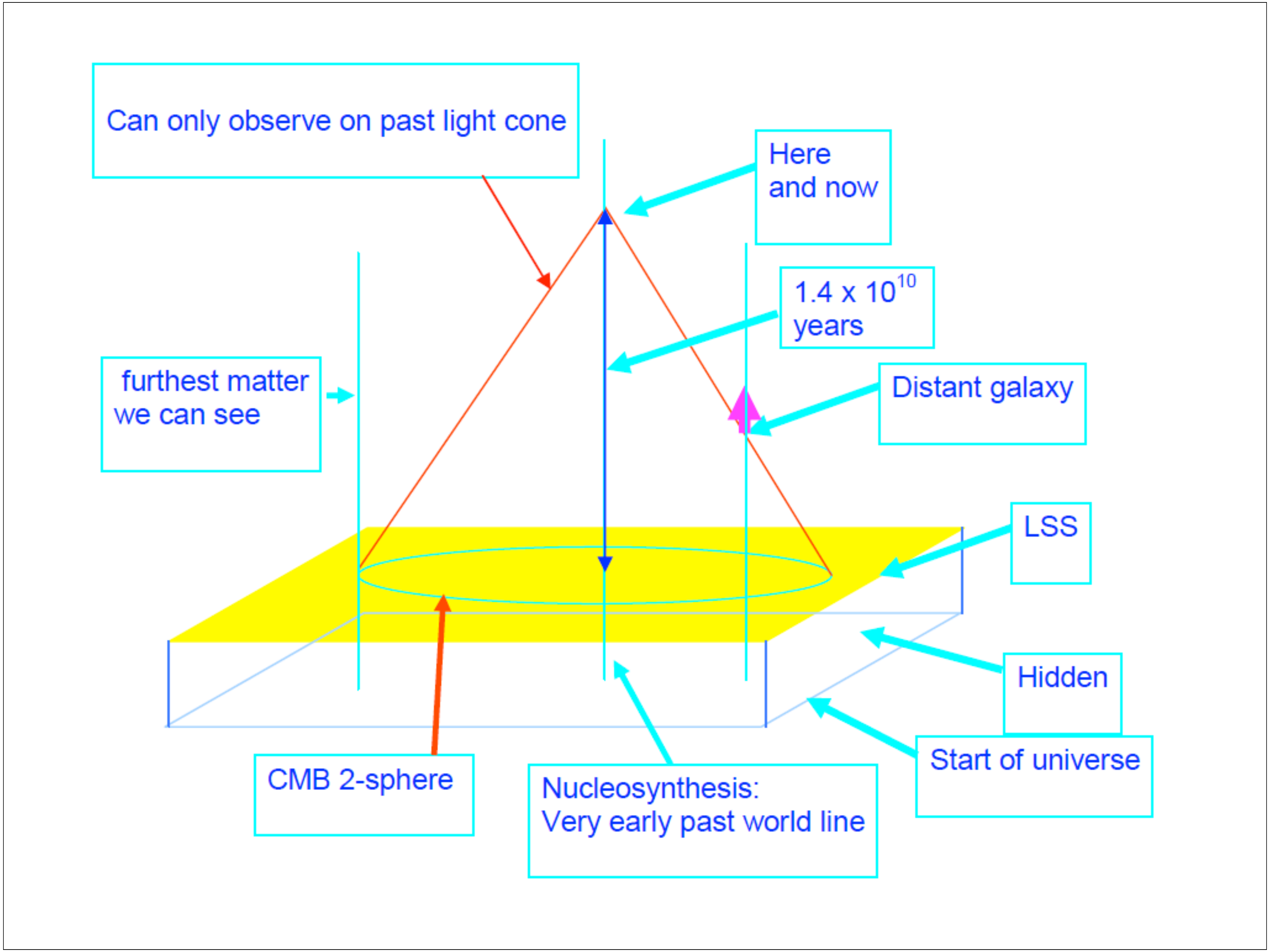
Primordial nucleosynthesis (ABG)

Hubble's law

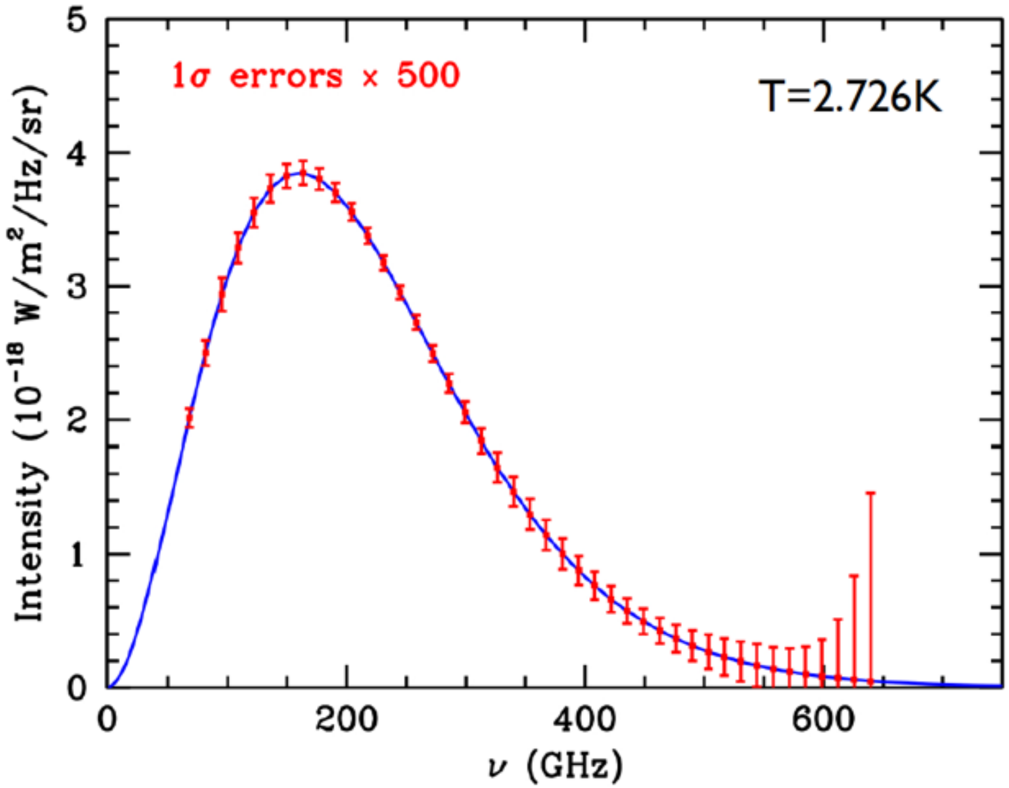
Cosmic microwave background

Dark matter?

Dark energy?

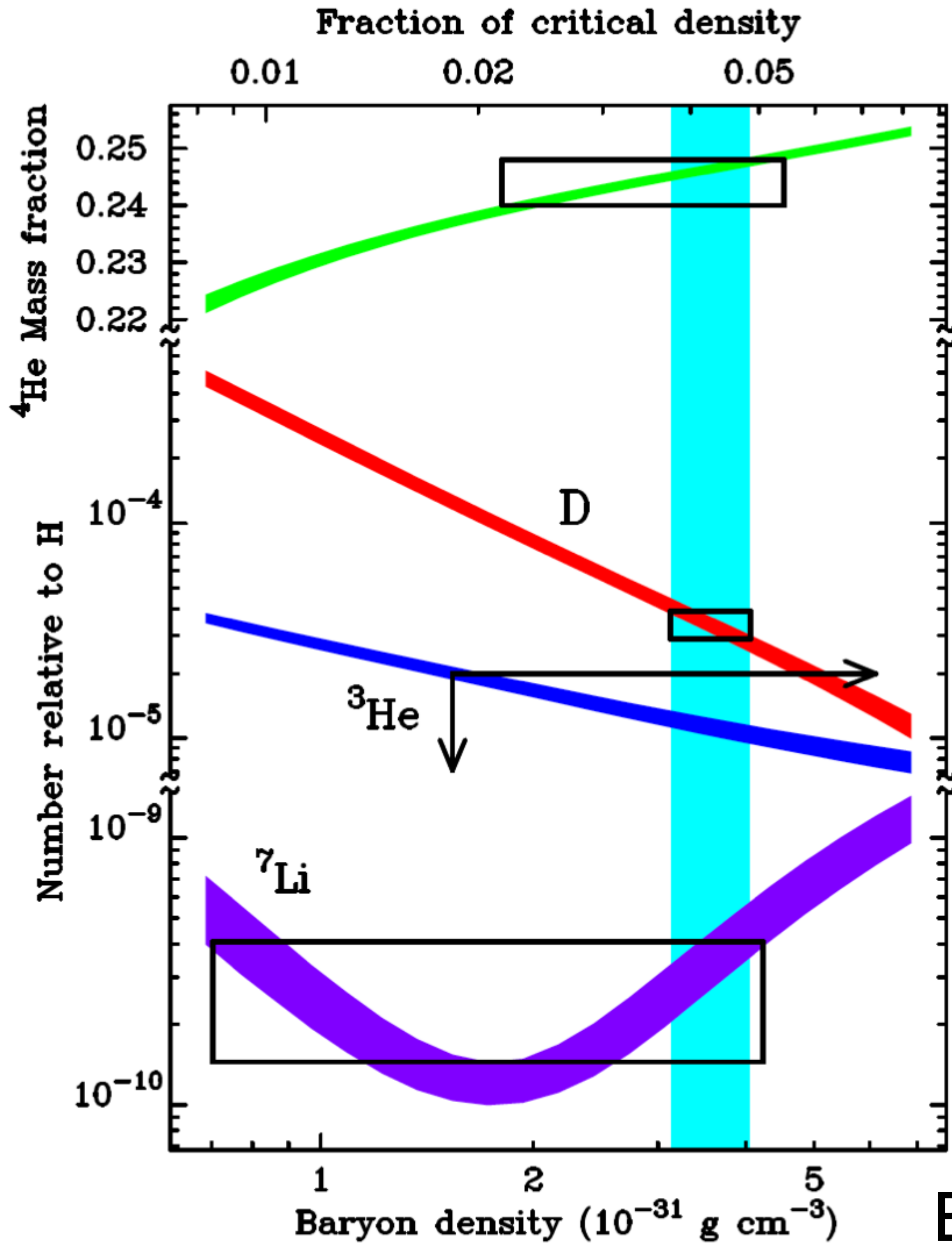

Can only observe on past light cone
Here
and now
1.4 x 10^10
years
furthest matter
we can see
Distant galaxy
LSS
Hidden
Start of universe
CMB 2-sphere
Nucleosynthesis:
Very early past world line

- intensity as measured by FIRAS[1] experiment aboard COBE satellite:

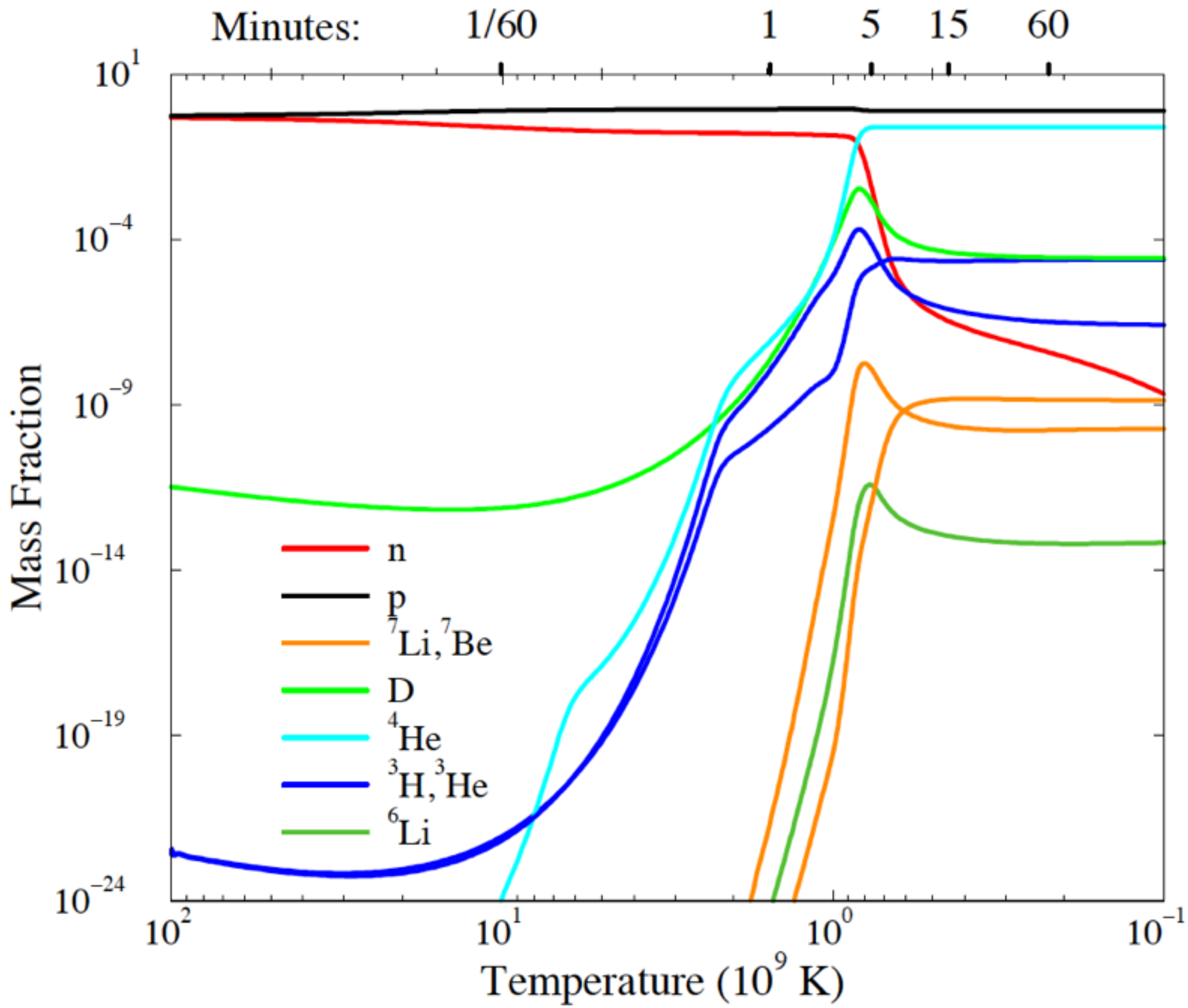


[1] **F**ar-**I**nfra**R**ed-**A**bsolute-**S**pectrophotometer

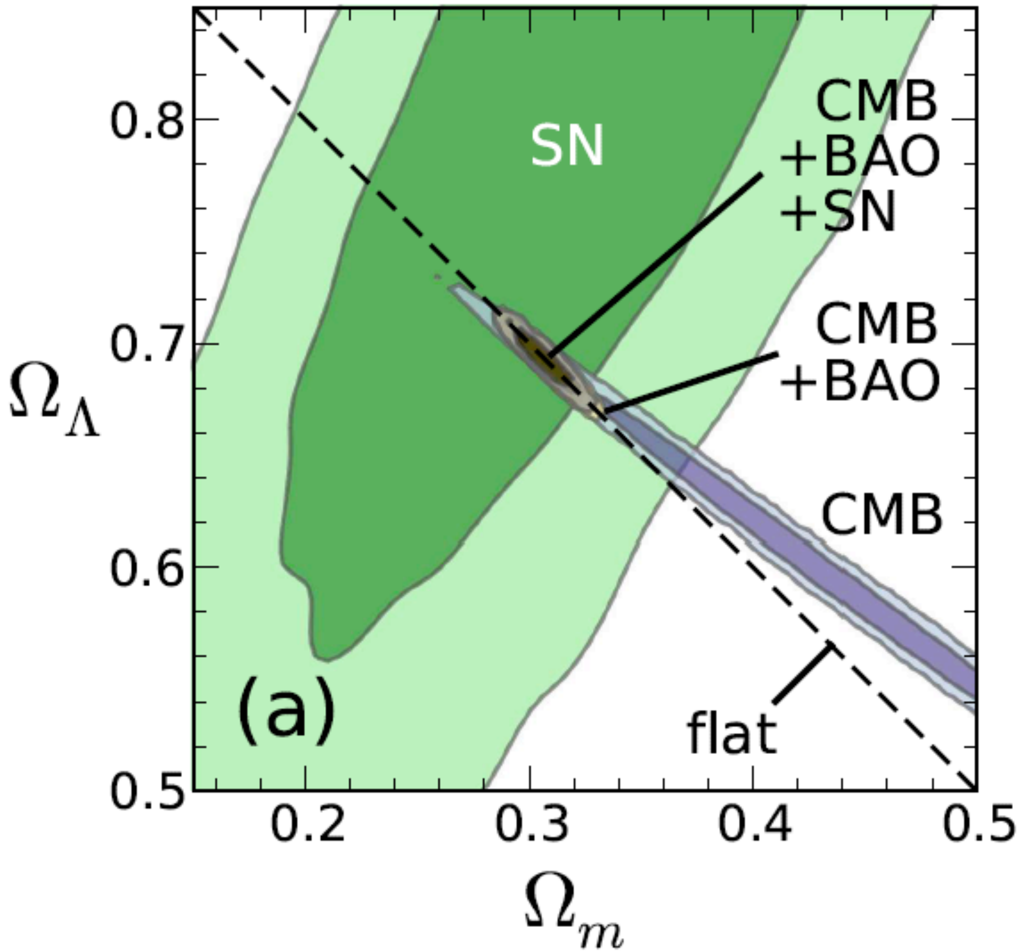
(A.Knebe)


Fraction of critical density
0.01
0.02
0.05
4He Mass fraction
0.25
0.24
0.23
0.22
Number relative to H
10−4
10−5
10−9
10−10
D
3He
7Li
1
2
5
Baryon density (10−31 g cm−3)

Burles et al

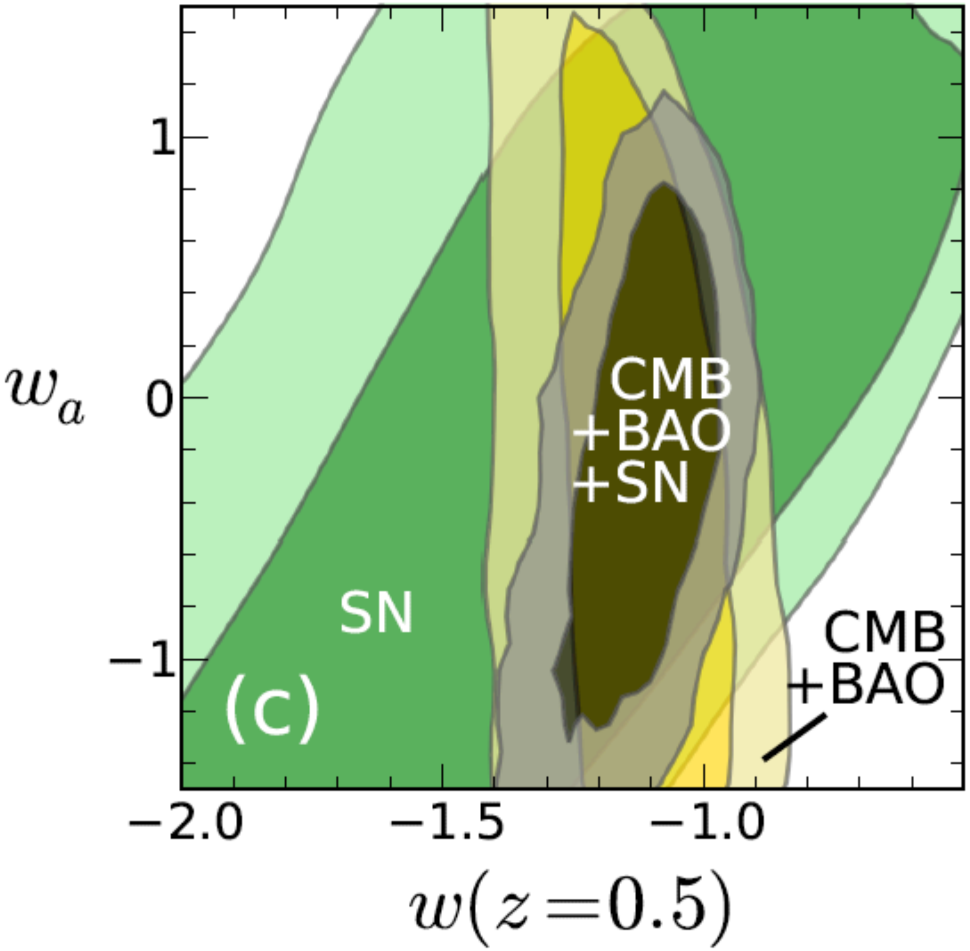

Minutes:
1/60
1
5
15
60
10^1
10^{-4}
10^{-9}
10^{-14}
10^{-19}
10^{-24}
Mass Fraction
n
p
^{7}Li, ^{7}Be
D
^{4}He
^{3}H, ^{3}He
^{6}Li
10^2
10^1
10^0
10^{-1}
Temperature (10^9 K)

Burles et al

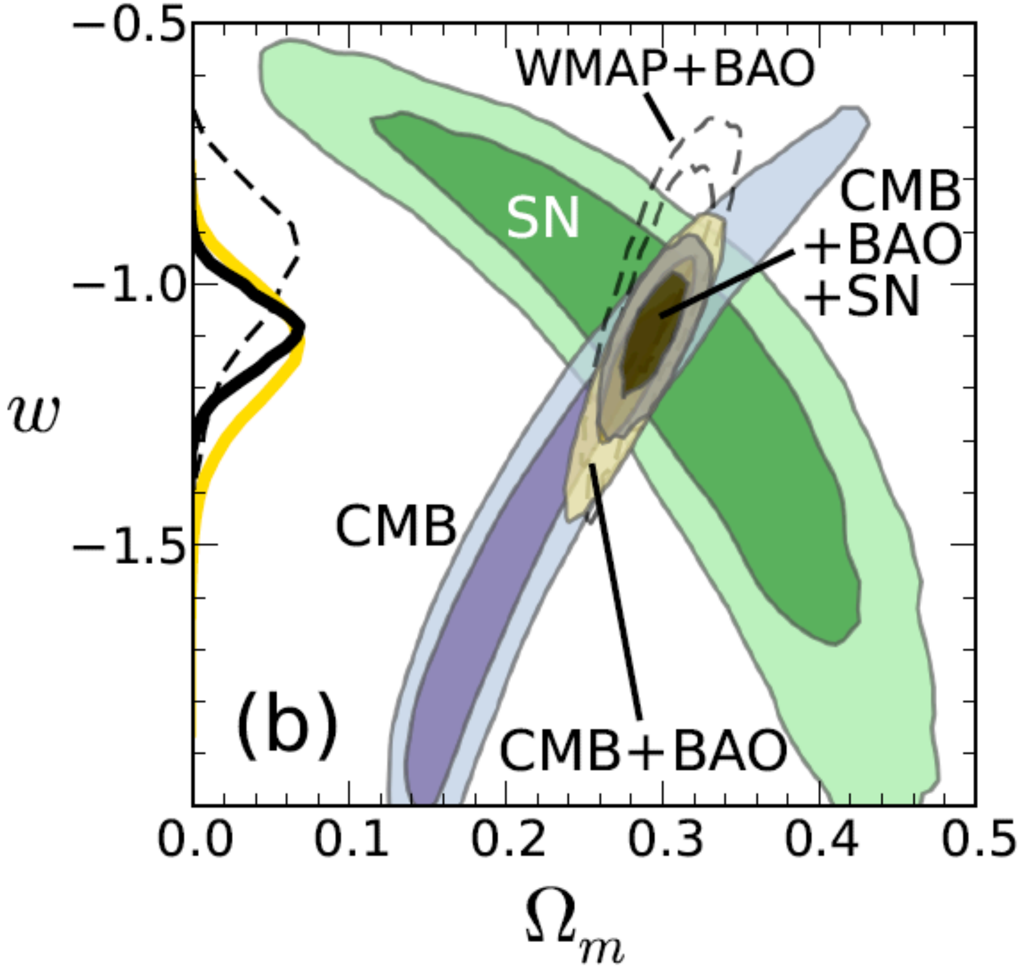


SN
CMB
+BAO
+SN
CMB
+BAO
CMB
(a)
flat
0.8
0.7
0.6
0.5
0.2
0.3
0.4
0.5
Ω_Λ
Ω_m

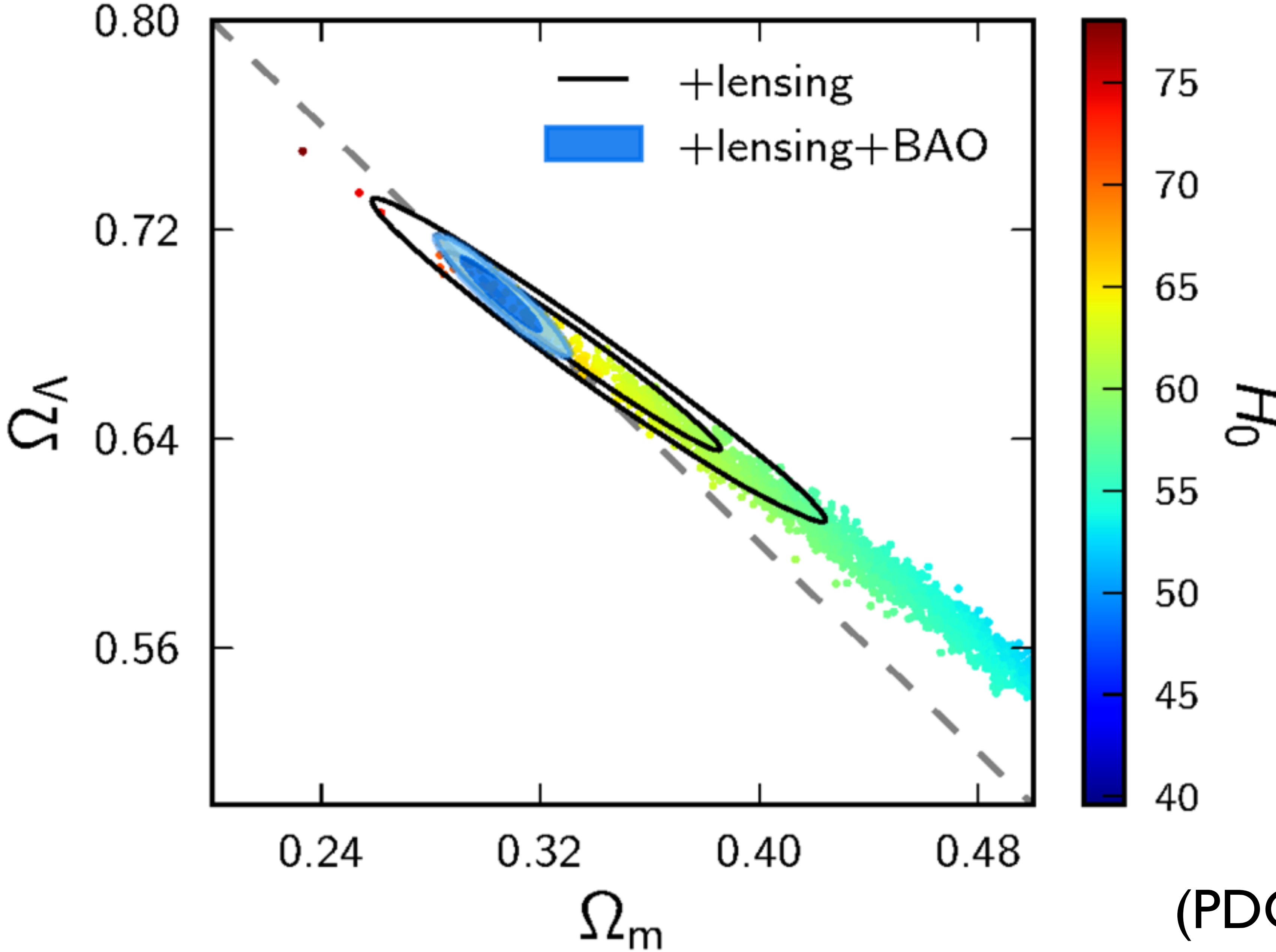
(PDG)



(PDG)


WMAP+BAO
SN
CMB
+BAO
+SN
CMB
CMB+BAO
(b)
-0.5
-1.0
-1.5
w
0.0
0.1
0.2
0.3
0.4
0.5
Ω_m

(PDG)


0.80
0.72
0.64
0.56
Ω_Λ
0.24
0.32
0.40
0.48
Ω_m
+lensing
+lensing+BAO
75
70
65
60
55
50
45
40
H_0
(PDG)

Dirac's theory on variation of constants

Constant k	Limit on $\dot{k}/k$ (yr^{-1})	Redshift	Method
Fine structure constant ($\alpha_{\mathrm{EM}} = e^2/\hbar c$)	$< 30 \times 10^{-16}$	0	Clock comparisons [181, 31, 111, 209]
	$< 0.5 \times 10^{-16}$	0.15	Oklo Natural Reactor [72, 116, 210]
	$< 3.4 \times 10^{-16}$	0.45	^{187}Re decay in meteorites [205]
	$(6.4 \pm 1.4) \times 10^{-16}$	0.2 – 3.7	Spectra in distant quasars [269, 193]
	$< 1.2 \times 10^{-16}$	0.4 – 2.3	Spectra in distant quasars [242, 51]
Weak interaction constant ($\alpha_{\mathrm{W}} = G_{\mathrm{f}} m_{\mathrm{p}}^2 c/\hbar^3$)	$< 1 \times 10^{-11}$	0.15	Oklo Natural Reactor [72]
	$< 5 \times 10^{-12}$	10^9	Big Bang nucleosynthesis [179, 223]
e-p mass ratio	$< 3 \times 10^{-15}$	2.6 – 3.0	Spectra in distant quasars [135]

(Will,2006)

$$d\ln(\alpha_{\rm em})/dt = (-2.5 \pm 2.6) \times 10^{-17}{\rm yr}^{-1},$$

$$d\ln(\mu)/dt = (-1.5 \pm 3.0) \times 10^{-16}{\rm yr}^{-1},$$

$$d\ln(m_q/\Lambda_{QCD})/dt = (7.1 \pm 4.4) \times 10^{-15}{\rm yr}^{-1}.$$

$$|\Delta\mu/\mu| < 1.8 \times 10^{-6}(95\%\ C.L.)\ ,$$

$$\mu = m_p/m_e$$

(PDG)

Method	$\dot{G}/G$ $(10^{-13}\ \mathrm{yr}^{-1})$	Reference
Lunar laser ranging	4 ± 9	[295]
Binary pulsar $1913+16$	40 ± 50	[143]
Helioseismology	0 ± 16	[122]
Big Bang nucleosynthesis	0 ± 4	[65, 21]

(Will,2006)

Parameterized post-newtonian (PPN) framework

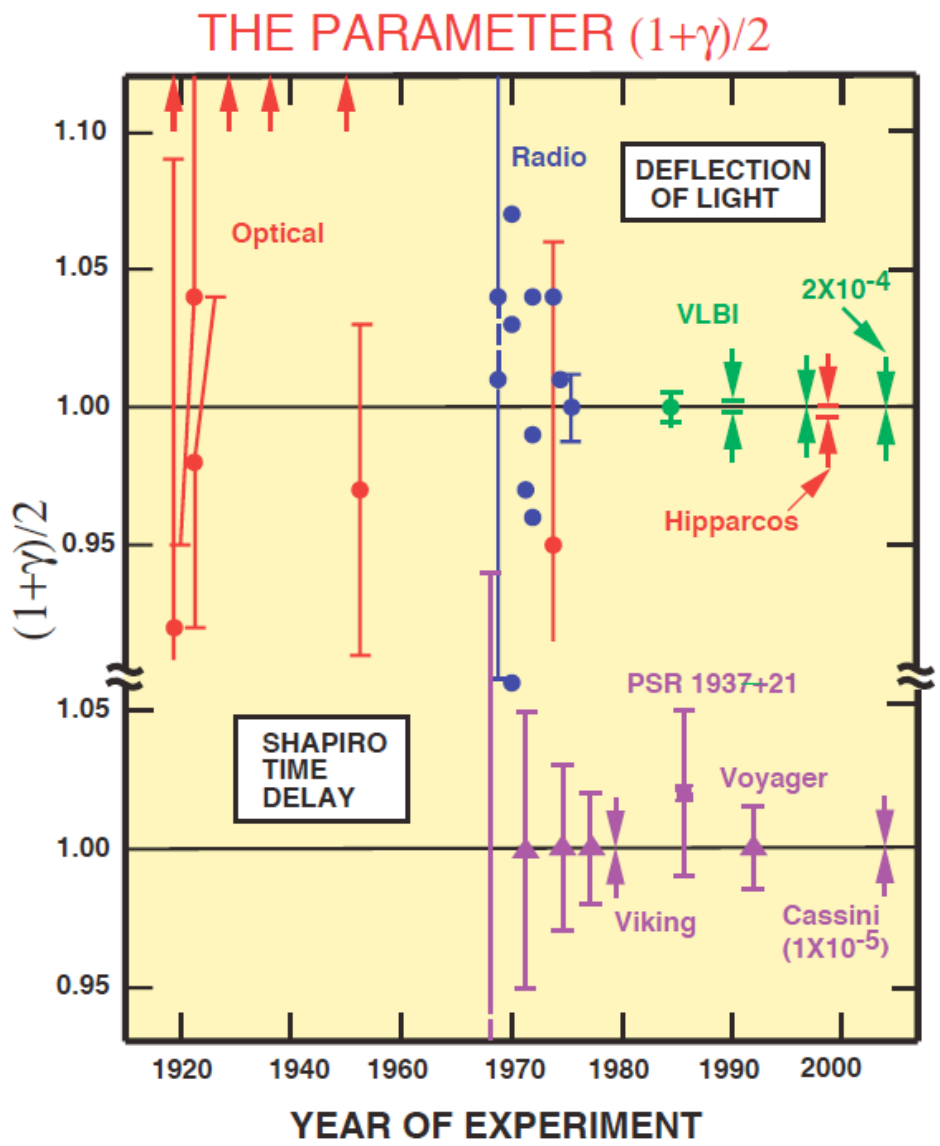
Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation of total momentum?	0	0	0
ζ_1		0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

(Will,2006)

Theory	Arbitrary functions or constants	Cosmic matching parameters	PPN parameters				
			γ	β	ξ	α_1	α_2
General relativity	none	none	1	1	0	0	0
Scalar-tensor							
Brans–Dicke	ω_{BD}	ϕ_0	$\frac{1+\omega_{\mathrm{BD}}}{2+\omega_{\mathrm{BD}}}$	1	0	0	0
General	$A(\varphi)$, $V(\varphi)$	φ_0	$\frac{1+\omega}{2+\omega}$	$1+\Lambda$	0	0	0
Vector-tensor							
Unconstrained	$\omega, c_1, c_2, c_3, c_4$	u	γ'	β'	0	α_1'	α_2'
Einstein-Æther	c_1, c_2, c_3, c_4	none	1	1	0	α_1'	α_2'
Rosen's bimetric	none	c_0, c_1	1	1	0	0	$\frac{c_0}{c_1}-1$

Table 3: *Metric theories and their PPN parameter values ($\alpha_3 = \zeta_i = 0$ for all cases). The parameters γ', β', α_1', and α_2' denote complicated functions of u and of the arbitrary constants. Here Λ is not the cosmological constant Λ_C, but is defined by Equation (37).*

(Will,2006)


THE PARAMETER (1+γ)/2
(1+γ)/2
1.10
1.05
1.00
0.95
1.05
1.00
0.95
Optical
Radio
DEFLECTION OF LIGHT
VLBI
2X10^-4
Hipparcos
SHAPIRO TIME DELAY
PSR 1937+21
Voyager
Viking
Cassini (1X10^-5)
1920
1940
1960
1970
1980
1990
2000
YEAR OF EXPERIMENT

(Will,2006)

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	4×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	2.3×10^{-4}	$\eta_N = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		2×10^{-4}	PSR J2317+1439
α_2	spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	4×10^{-20}	pulsar $\dot{P}$ statistics
η_N	Nordtvedt effect	9×10^{-4}	lunar laser ranging
ζ_1	—	2×10^{-2}	combined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	$\ddot{P}_p$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ_4	—	—	not independent (see Equation (58))

Table 4: *Current limits on the PPN parameters. Here η_N is a combination of other parameters given by* $\eta_N = 4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 + 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$.

(Will,2006)

Gravitational radiation

No direct detection in spite of great efforts yet,

but…

There is a small problem set in Landau's book

$$\dot{r} = -\frac{64k^3}{5c^5}\frac{m_1 m_2 (m_1 + m_2)}{r^3}$$

$$\frac{\dot{P}_{\rm b}^{\rm corr}}{\dot{P}_{\rm b}^{\rm GR}} = 1.0013 \pm 0.0021.$$

The quadrupole-formula controversy

In a remarkable pair of papers published in 1916 and 1918, Einstein calculated the gravitational-wave field and radiated energy of a time-dependent source, such as a rotating dumbbell, for which self-gravity is unimportant. He performed this computation in a slow-motion approximation, using the linearized Einstein equations, and obtained the quadrupole formula (11.55). It is perhaps a slight exaggeration to say that it was all downhill from there, at least until 1979.

It didn't help that Einstein made a calculational error in his 1918 paper, leading to a wrong factor of 2, discovered later by Eddington. Nor did it help that Eddington, concerned about the gauge freedom available in the description of gravitational waves, wondered in 1922 whether aspects of gravitational waves were physically real or purely coordinate artifacts; as he put it, perhaps they "propagate with the speed of thought." Although Eddington understood that the gauge-invariant modes were physical and believed that gravitational waves did exist, his remark, taken out of context, had the effect of making the entire subject seem dubious.

To make matters worse, in 1936 Einstein and his young colleague Nathan Rosen (of Einstein–Podolsky–Rosen paradox fame) submitted a paper to *The Physical Review* with the provocative title "Do gravitational waves exist?". They thought they had found an exact solution of the field equations describing a plane gravitational wave, but because the solution had a singularity, it could not be physically valid, and they concluded that gravitational waves could not exist. *The Physical Review* sent the paper for review, and the report that came back pointed out that the Einstein–Rosen solution in fact described a cylindrical wave, and that the singularity was merely a harmless coordinate singularity associated with the axis. So the solution was perfectly valid, and in fact it supported the existence of gravitational waves. Einstein was so angry that the journal had sent his paper out to be refereed, a practice that was unfamiliar to him, that he withdrew the paper and never published again in that journal. Shortly thereafter, however, Einstein was convinced by another of his assistants, Leopold Infeld (who had been approached by the anonymous referee), that the referee had been perfectly correct. Einstein rewrote the paper with the opposite conclusion and published it under the title "On gravitational radiation" (but not in *The Physical Review*). While there has been plenty of speculation as to the identity of the anonymous referee, it wasn't until 2005 that our friend Daniel Kennefick was allowed access to the records of the journal and revealed conclusively that the referee was the well-known Princeton and Caltech relativist H.P. Robertson (the co-discoverer of the Robertson–Walker metric for cosmology).

This episode did not end the debate over the existence of gravitational waves. Even if one accepts the validity of Einstein's prediction that a rotating dumbbell will radiate gravitational waves, the argument was made that a binary-star system would *not* radiate. After all, each body is moving on a geodesic, and is therefore unaccelerated relative to a local freely falling frame. Without acceleration, the argument went, there should be no radiation. Peter Havas was one of the proponents of this possibility.

Beginning in the late 1940s, numerous attempts were made to calculate the "back reaction" forces that would alter the motion of a binary system in response to the radiation of energy and angular momentum (this is the primary subject of Chapter 12). Yet different workers got different answers.

By 1974, while most researchers in the field accepted the reality of gravitational waves and the validity of the quadrupole formula for slowly moving binary systems, a vocal minority remained skeptical. This "quadrupole-formula controversy" came to a head with the September 1974 discovery of the first binary pulsar by Russell Hulse and Joseph Taylor. It was immediately clear that it would be possible to test the quadrupole formula by exploiting the high-precision timing of the pulsar's radio signals to measure the slow variation in the orbit induced by the loss of orbital energy to radiation.

But in a letter published in the *Astrophysical Journal* in 1976, Jürgen Ehlers, Arnold Rosenblum, Joshua Goldberg, and Peter Havas argued that the quadrupole formula could not be justified as a theoretical prediction of general relativity. They presented a laundry list of theoretical problems that they claimed had been swept under the rug by proponents of the quadrupole formula. Among them were these: people assumed energy balance to infer the reaction of the source to the flux of radiation, but there was no proof that this was a valid assumption; no reliable calculation of the equations of motion that included radiation reaction had (in their opinion) ever been carried out; many "derivations" of the quadrupole formula relied on the linearized theory, which was clearly wrong for binary systems; since higher-order corrections had not been calculated, it was impossible to know if the quadrupole formula was even a good approximation; even worse, higher-order terms were known to be rife with divergent integrals.

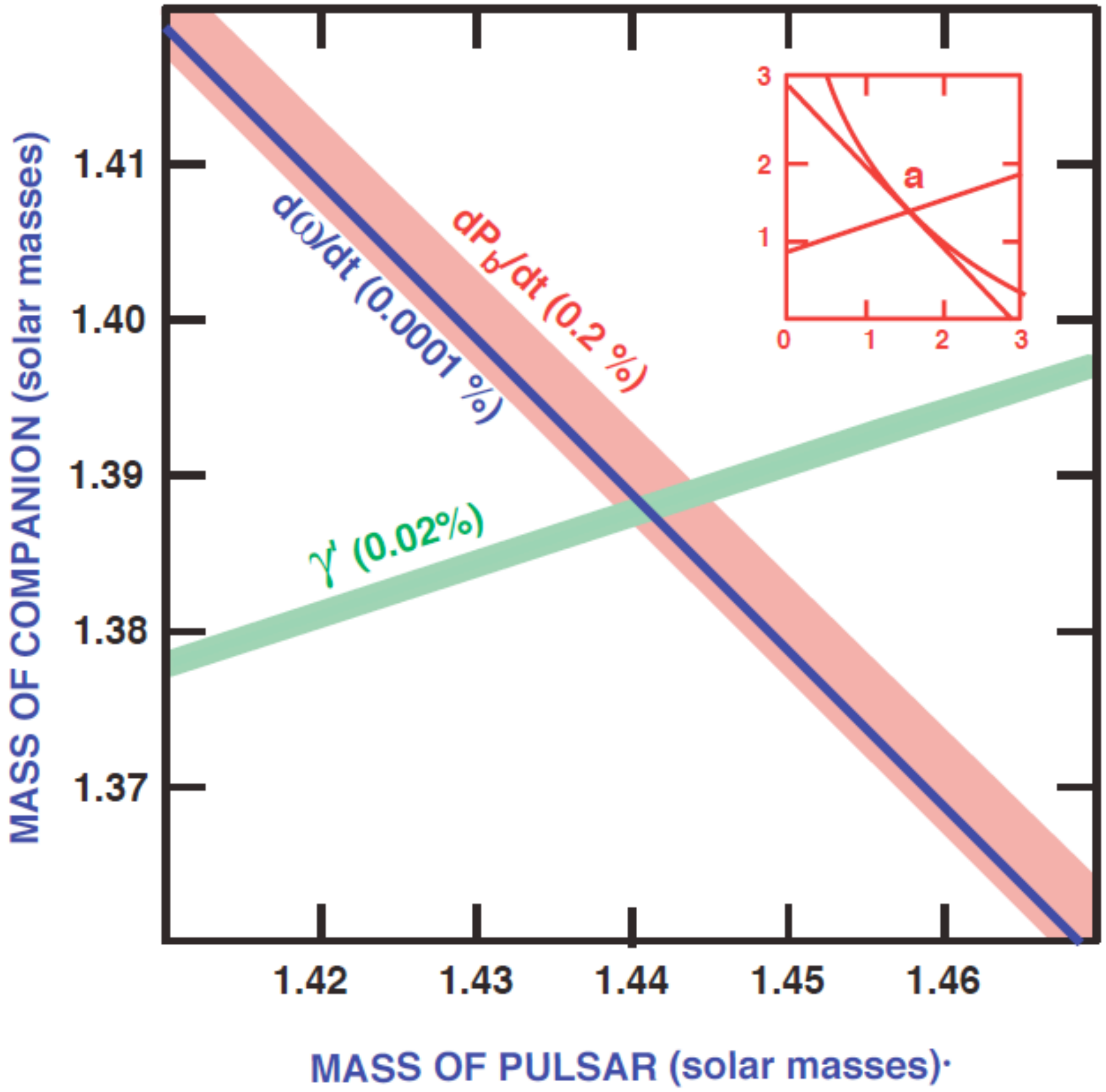
There was considerable annoyance among holders of the "establishment" viewpoint when this paper appeared, mainly because it was realized that its criticisms had considerable merit. As a result many research groups embarked on a program to return to the fundamentals and to develop approximation schemes for equations of motion and gravitational radiation that would not be subject to the flaws that so disturbed Ehlers *et al*. Among the noteworthy outcomes of this major effort was the fully developed post-Minkowskian formalism that forms the heart of this book. Toward the end of his life, Jürgen Ehlers, one of the great relativists of his time, admitted to one of us (after some prodding, to be sure, and only up to a point!) that the justification of the quadrupole formula was in much better shape than it was in 1976.

Experimentally, the situation was not at all controversial. By 1979, Taylor and his colleagues had measured the damping of the binary pulsar's orbit, in agreement with the quadrupole formula to about 10 percent; by 2005, the agreement was at the 0.2 percent level. The formula has also been beautifully confirmed in a number of other binary-pulsar systems.

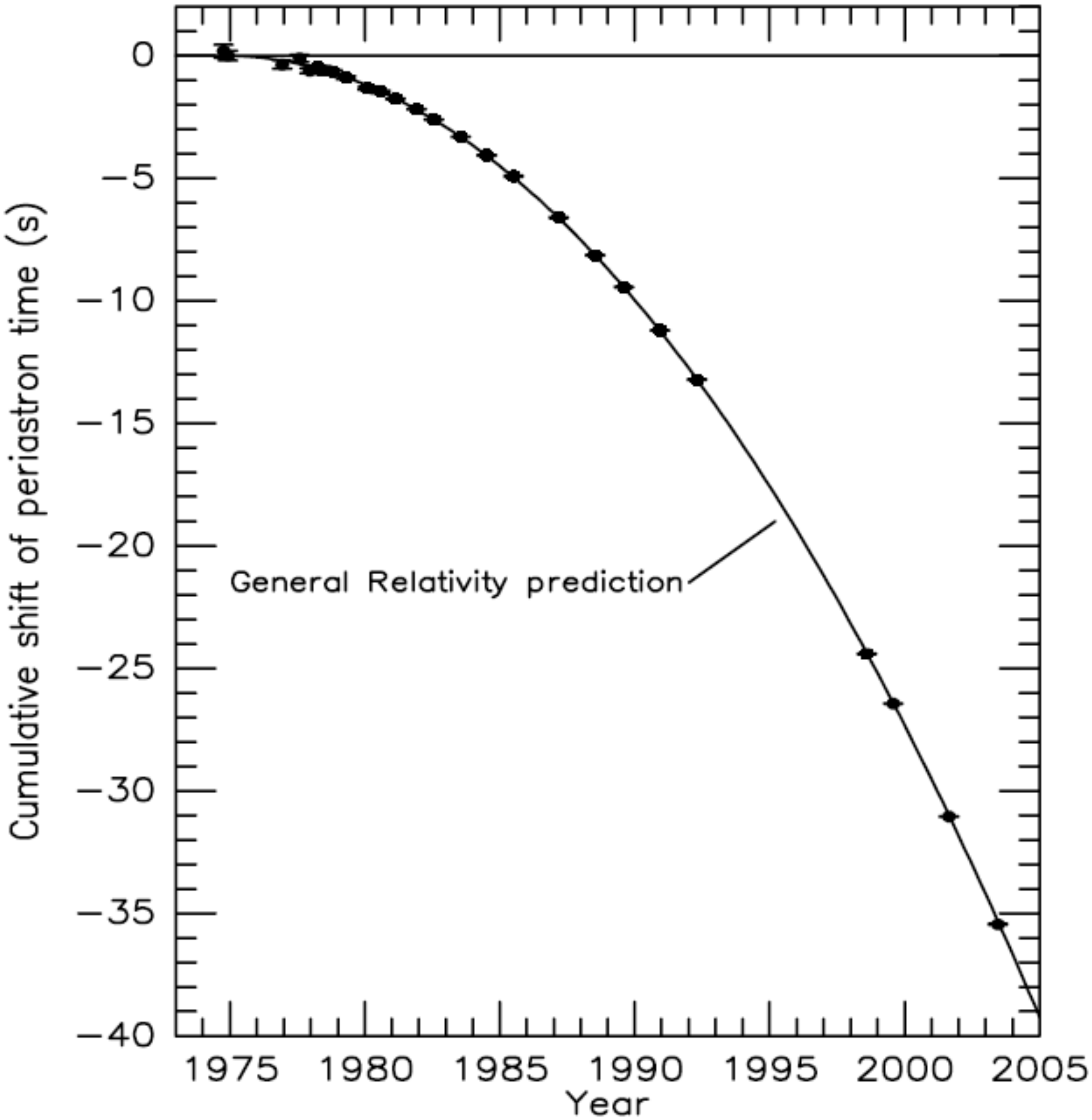
Parameter	B1534+12	B2127+11C	J1141–6545	J0737–3039(A, B)
(i) "Keplerian" parameters:				
$a_{\rm p} \sin i$ (s)	3.7294626(8)	2.520(3)	1.85894(1)	1.41504(2)/1.513(3)
e	0.2736767(1)	0.68141(2)	0.171876(2)	0.087779(5)
$P_{\rm b}$ (day)	0.420737299153(4)	0.335282052(6)	0.1976509587(3)	0.102251563(1)
(ii) "Post-Keplerian" parameters:				
$\langle\dot{\omega}\rangle$ ($^\circ$ yr^{-1})	1.755805(3)	4.457(12)	5.3084(9)	16.90(1)
γ' (ms)	2.070(2)	4.67	0.72(3)	0.382(5)
$\dot{P}_{\rm b}$ (10^{-12})	−0.137(3)	−3.94	−0.43(10)	−1.21(6)
r (μs)	6.7(1.0)			6.2(5)
$s = \sin i$	0.975(7)			0.9995(4)

Table 7: *Parameters of other binary pulsars. References may be found in the text; for an online catalogue of pulsars with reasonably up-to-date parameters, see [18].*

(Will,2006)

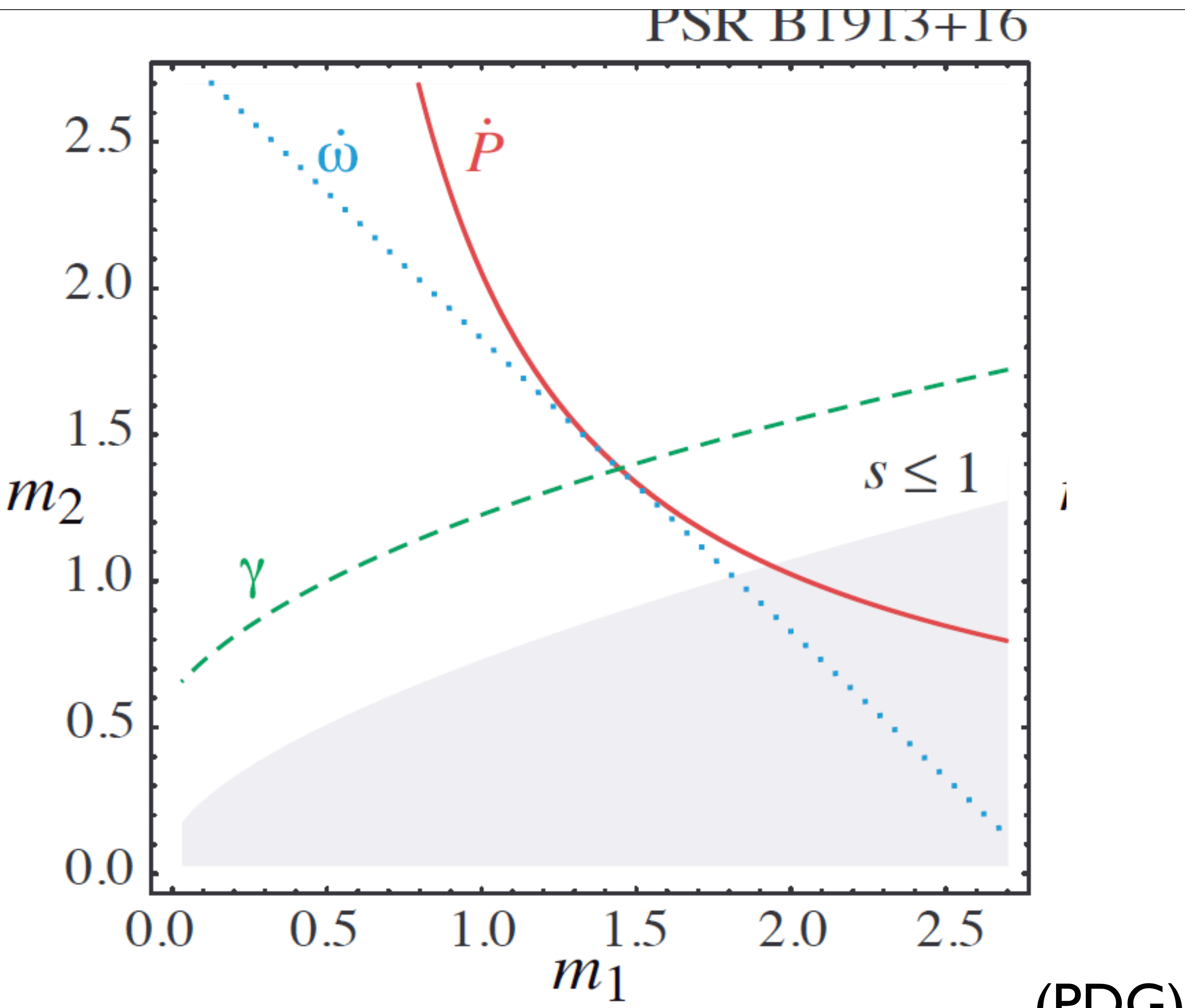

MASS OF COMPANION (solar masses)
1.41
1.40
1.39
1.38
1.37
1.42
1.43
1.44
1.45
1.46
MASS OF PULSAR (solar masses)
dω/dt (0.0001 %)
dP_b/dt (0.2 %)
γ' (0.02%)
3
2
1
0
1
2
3
a

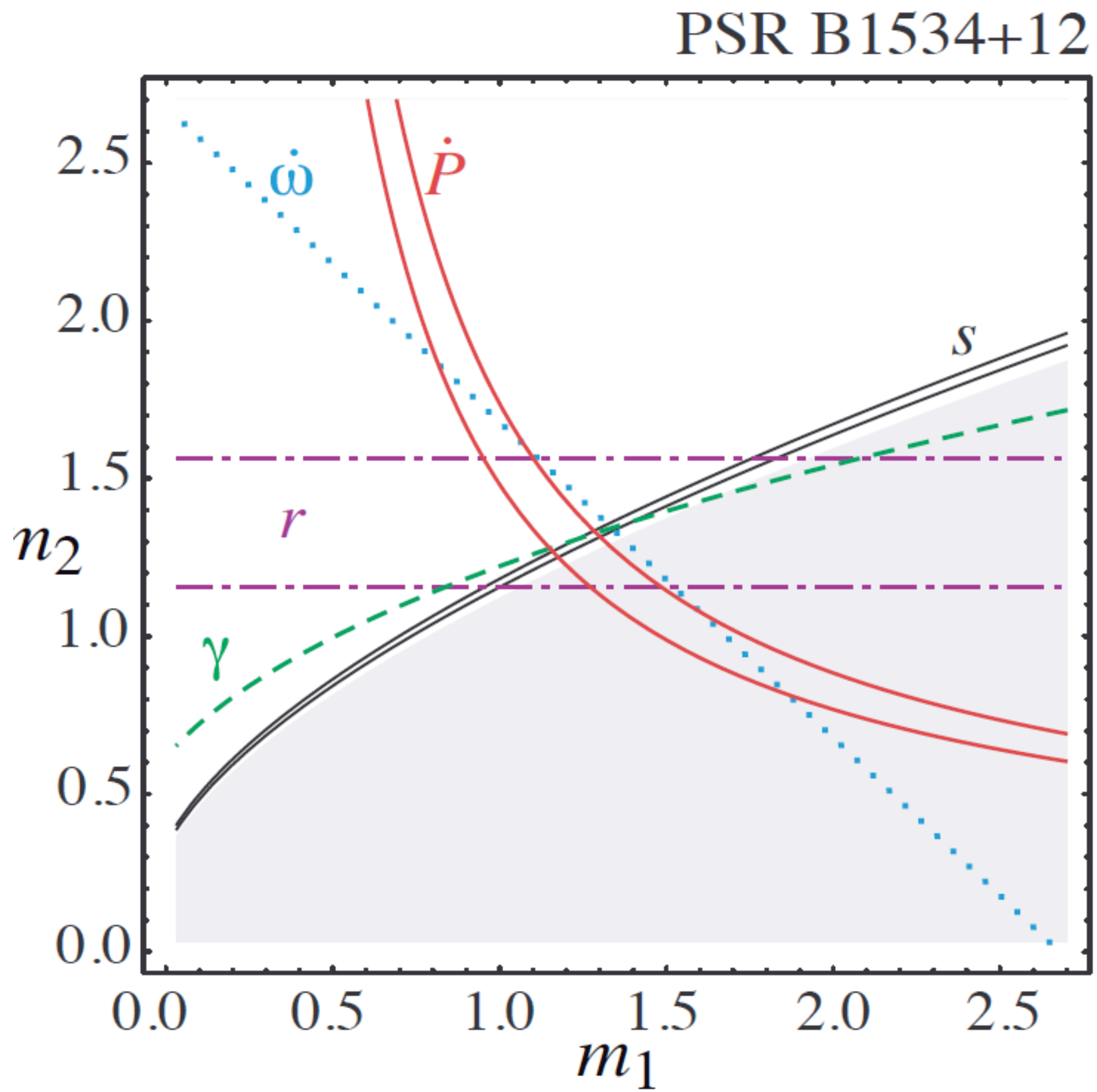
(Will,2006)



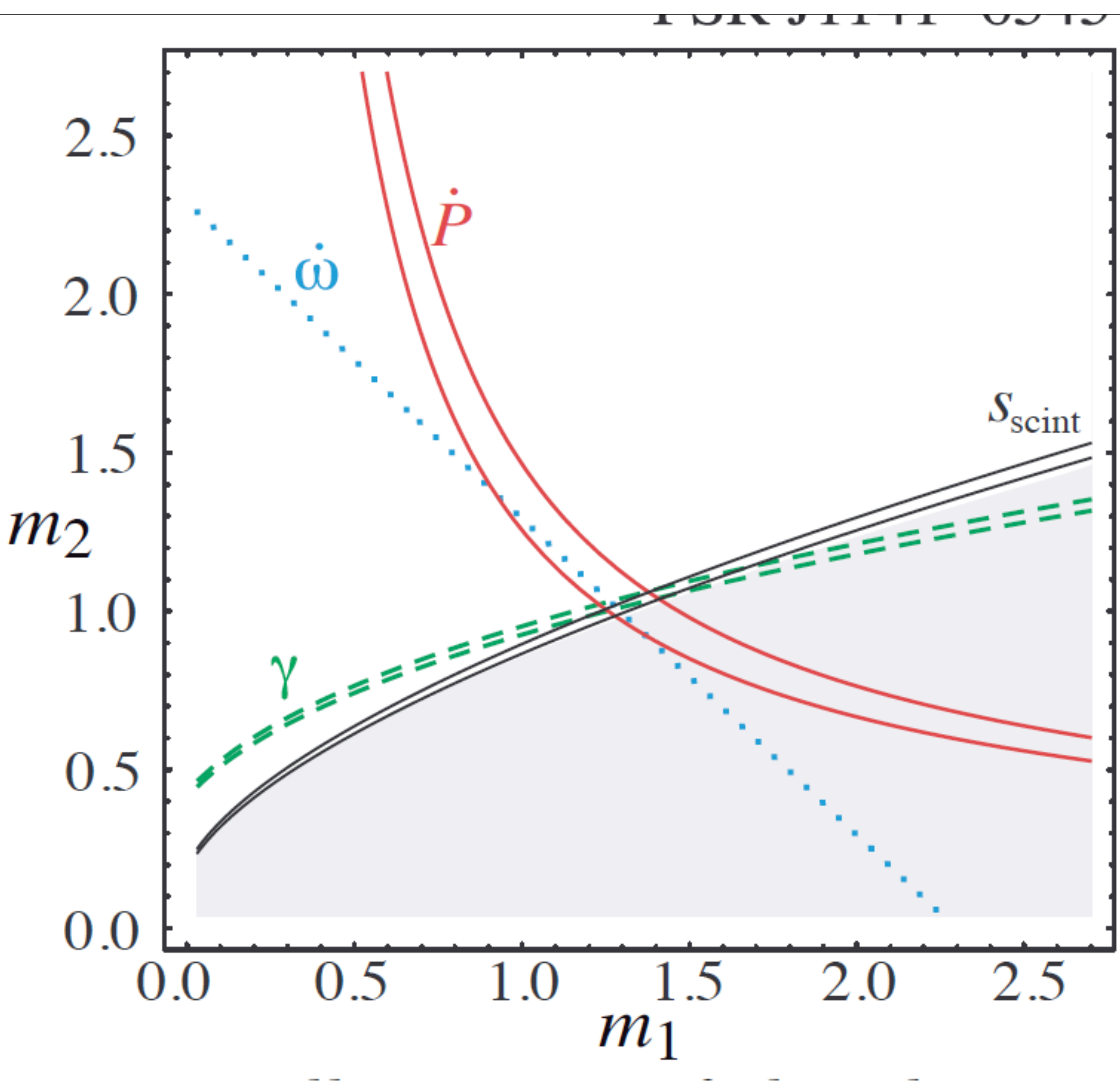
Cumulative shift of periastron time (s)
0
−5
−10
−15
−20
−25
−30
−35
−40
General Relativity prediction
1975
1980
1985
1990
1995
2000
2005
Year

(Will,2006)

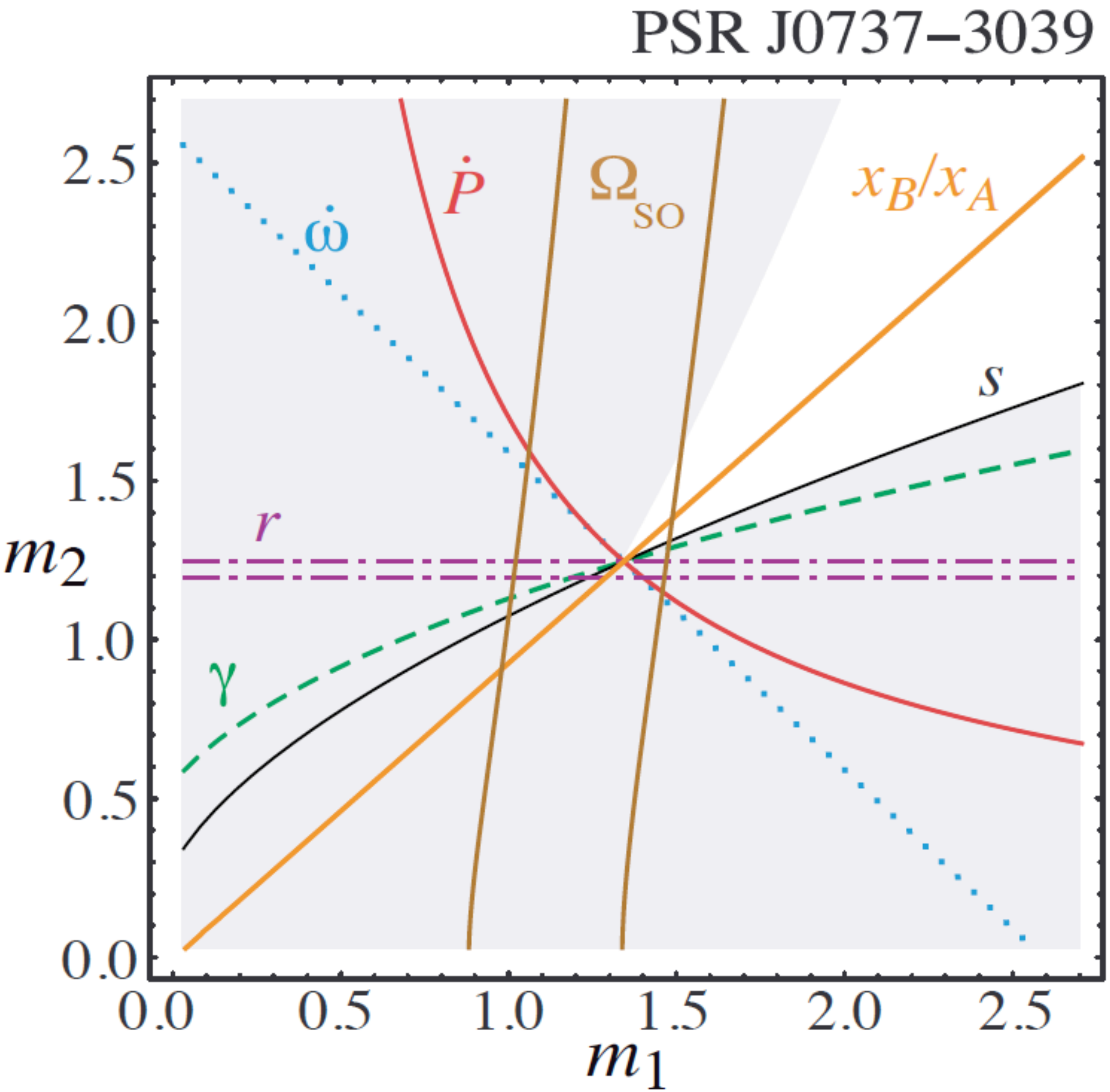

PSR B1913+16
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
m_2
m_1
$\dot{\omega}$
$\dot{P}$
γ
$s \leq 1$
(PDG)



(PDG)


2.5
2.0
1.5
1.0
0.5
0.0
m_2
0.0
0.5
1.0
1.5
2.0
2.5
m_1
$\dot{P}$
$\dot{\omega}$
s_{scint}
γ

(PDG)



PSR J0737−3039
$\dot{\omega}$
$\dot{P}$
Ω_{SO}
x_B/x_A
s
r
γ
m_2
m_1
0.0
0.5
1.0
1.5
2.0
2.5

(PDG)

Conclusions

GR enters the XXI century in a healthy state

Too healthy (cc. etc)?

No idea of the QFT of gravitation

Nevertheless, also here we know there are things missing

Wilsonian effective field theories?

UV/IR connection

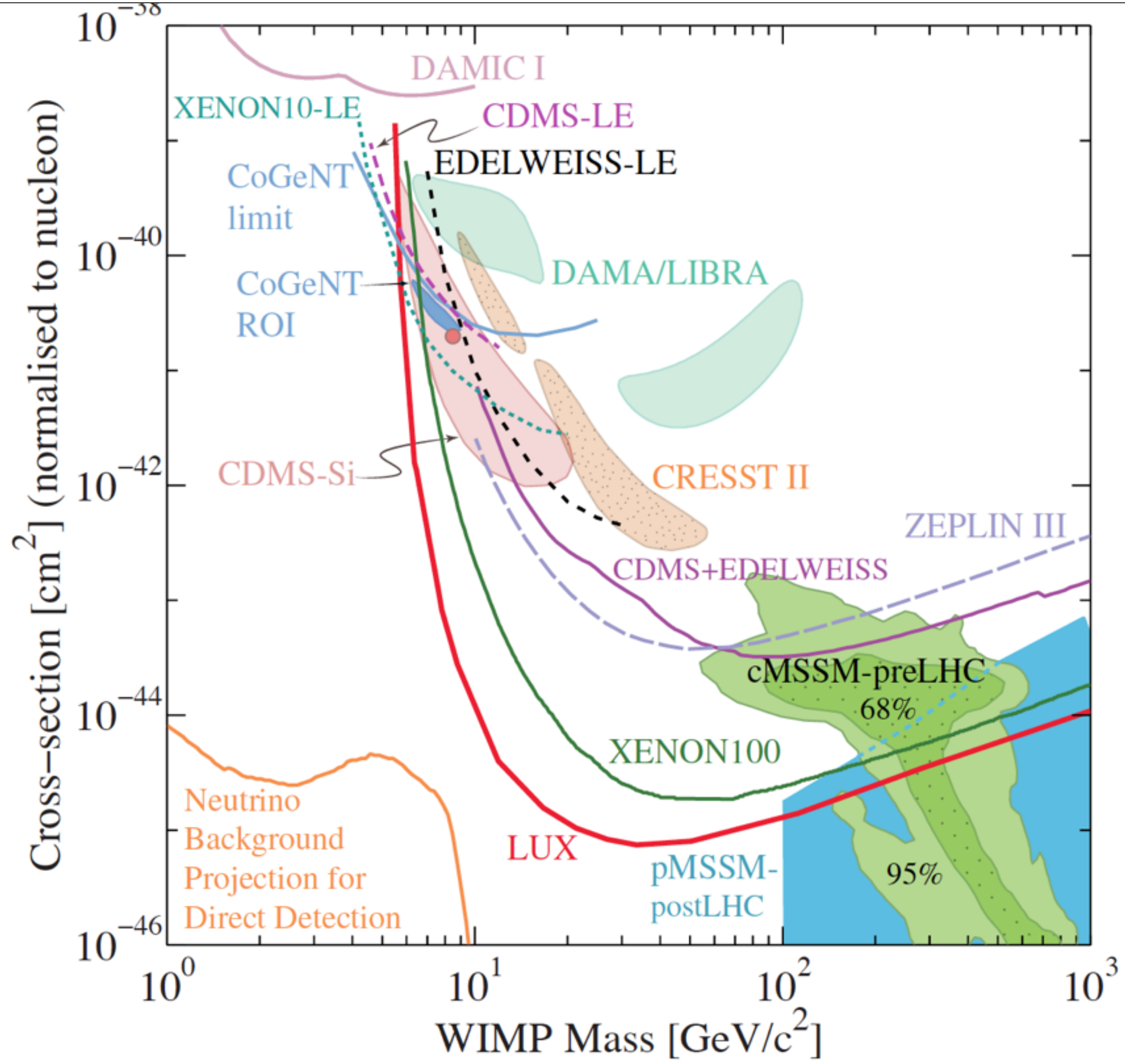
Backup

MODIFIED GRAVITY

Dark matter?

Dark energy?

Almost all extensions of the SM lead to MD



DAMIC I
XENON10-LE
CDMS-LE
EDELWEISS-LE
CoGeNT limit
CoGeNT ROI
DAMA/LIBRA
CDMS-Si
CRESST II
ZEPLIN III
CDMS+EDELWEISS
cMSSM-preLHC
68%
95%
XENON100
LUX
pMSSM-postLHC
Neutrino Background Projection for Direct Detection
Cross-section [cm^2] (normalised to nucleon)
WIMP Mass [GeV/c^2]
10^{-38}
10^{-40}
10^{-42}
10^{-44}
10^{-46}
10^0
10^1
10^2
10^3

(PDG)

Low energy effective theory (AHGS)

$$H_{\mu\nu}(x) \equiv g_{\mu\nu}(x) - \bar{g}^{\pi}_{\mu\nu}(x)$$

We would like to study mass terms of the type

$$S_m \equiv \int d^4x \sqrt{g} g^{\mu\nu} g^{\alpha\beta} \left(a H_{\mu\nu} H_{\alpha\beta} + b H_{\mu\alpha} H_{\nu\beta} \right) + \ldots$$

(Diff invariant owing to the Goldstones)

$$g_{\mu\nu}(x) \equiv \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x)$$

$$H_{\mu\nu} \equiv h_{\mu\nu} + \bar{g}_{\mu\alpha} \bar{\nabla}_{\nu} \pi^{\alpha} + \bar{g}_{\nu\alpha} \bar{\nabla}_{\mu} \pi^{\alpha} + \ldots$$

Longitudinal and transverse goldstones

$$\pi^\alpha \equiv \bar{g}^{\alpha\beta} \left(A_\beta + \partial_\beta \phi \right)$$

There is a fake gauge invariance

$$A_\beta \to A_\beta + \partial_\beta \Lambda$$
$$\phi \to \phi - \Lambda$$

(Dimensions of fields are nonstandard)

$$[\pi] = -1 \quad \therefore \quad [A] = -1 \quad [\phi] = -2$$

Flat background $\bar{g} = \eta$.

$$\int d^4x \ \left(a\left(h + 2\Box\phi\right)^2 + b\left(h_{\mu\rho} + 2\partial_\mu\partial_\rho\phi\right)\left(h^{\rho\mu} + 2\partial^\mu\partial^\rho\phi\right)\right)$$

$$H_{\mu\nu} \sim h_{\mu\nu} + 2\partial_\mu\partial_\nu\phi + ..$$

$$R^{(2)} \sim H^{\alpha\beta}\Box H_{\alpha\beta} + \ldots \sim h^{\alpha\beta}\partial_\alpha\partial_\beta\phi + \phi\Box^3\phi + \ldots$$

$$4\left(a+b\right)\int d^4x\Box\phi\Box\phi \qquad\qquad a+b=0$$

(Fierz-Pauli choice)

$$L = f^4\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)$$

The graviton mass is $$m_g^2 = \frac{f^4}{M^2}$$

When a+b=0 the scalar only gets propagator through the mixing with the graviton

$$4f^4 \left(h\Box\phi - \partial_\alpha\partial_\beta\phi h^{\alpha\beta}\right)$$

Performing a Weyl transformation

$$\int \sqrt{g}R\left(1+m_g^2\phi\right) \sim \int \sqrt{\tilde{g}}\tilde{R}$$

$$M^2\ m_g^4\left(\partial_\mu\phi\right)^2 \sim \frac{f^8}{M^2}\left(\partial_\mu\phi\right)^2$$

What happens when a+b does not vanish?

$$\frac{f^8}{M^2}\, p^2\phi^2 + (a+b)p^4\phi^2$$

Ghosts when $$p^2 \sim \frac{f^8}{(a+b)\ M^2} \sim m_g^2 \frac{f^4}{a+b}$$

$$a+b << f^4$$

Canonically normalized fields

$$\phi_c \equiv M m_g^2 \phi \sim \frac{f^4}{M}\phi \qquad A_c = f^2 A = m_g M\ A$$

Curved space

$$-\int \nabla_\nu \nabla_\mu \phi \nabla^\nu \nabla^\mu \phi = \int \nabla^2 \nabla_\mu \phi \nabla^\mu \phi = \int \nabla^\alpha \nabla_\mu \nabla_\alpha \phi \nabla^\mu \phi =$$

$$\int \left(\nabla_\mu \nabla^2 \phi + \nabla_\lambda \phi \; R^\lambda_\mu \right) \nabla^\mu \phi$$

$$-\frac{2\lambda}{n-2} \int (\nabla \phi)^2$$

Scalar propagator $\frac{f^4}{l^2}$

The canonically normalized scalar is now $\phi = \frac{l}{f^2} \phi_c$

Strong coupling scale

$$\int f^4 \left((\partial^2 \phi)^3 + (\partial^2 \phi)^4 + \partial^2 \phi \partial A \partial A \right)$$

$$\int \left(f^4 = m_g^2 M^2\right) \left(\frac{1}{M^3 m_g^6} (\partial^2 \phi_c)^3 + \frac{1}{M^4 m_g^8} (\partial^2 \phi_c)^4 + \frac{1}{M m_g^2} \frac{1}{M^2 m_g^2} \partial^2 \phi_c \partial A_c \partial A_c \right)$$

The dimension five operator gets important at a scale

$$\Lambda^5 \equiv M m_g^4$$

The scalar does not decouple when the graviton mass vanishes (van Dam-Veltman)

$$\int T^{\mu\nu}\tilde{h}_{\mu\nu} = \int T^{\mu\nu}\left(h_{\mu\nu} + \frac{m_g^2}{n-2}\phi\eta_{\mu\nu}\right) = \int T^{\mu\nu}\left(h_{\mu\nu} + \frac{1}{M}\frac{1}{n-2}\phi_c\eta_{\mu\nu}\right)$$

It does decouple in AdS

$$\int T^{\mu\nu}\left(h_{\mu\nu} + \frac{l}{f^2}\frac{m_g^2}{n-2}\phi_c\eta_{\mu\nu}\right) = \int T^{\mu\nu}\left(h_{\mu\nu} + \frac{l m_g}{(n-2)M}\phi_c\eta_{\mu\nu}\right)$$

F(R)= Scalar-tensor

$$S = C \int d(vol) f(R)$$

$$S = \int \sqrt{|g|} d^n x \left(f(\chi) + f'(\chi) (R - \chi) \right)$$

$$\frac{\delta S}{\delta \chi} = f''(\chi) (R - \chi) \qquad \phi \equiv f'(\chi)$$

$$V(\phi) \equiv \phi \chi(\phi) - f(\chi(\phi))$$

$$S = \int \sqrt{|g|} d^n x \left(\phi R - V(\phi) \right)$$

Jordan frame versus Einstein frame

$$S_J \equiv \int d^n x |e| \left(F(\phi)R - G(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi) + L_{\mathrm{matt}}(e,\psi)\right)$$

Through a Weyl rescaling it is always possible to work in the Einstein frame

$$S = \int d(vol)\left(-M^2 R + L(g_{\mu\nu}, \phi_i)\right)$$

This is a point transformation; so that it is a symmetry even of the path integral

Chameleons and other lizards

$$L = \sqrt{|g|}\left(-\frac{1}{2}M_p^2 R + \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right) + L_m(\psi_i, g_i^{\mu\nu})$$

Peculiar ad-hoc coupling

$$L_m = \frac{1}{2}\, e^{\frac{(n-2)\beta\phi}{M_p}}\ \sqrt{|g|}(\nabla\psi)^2 - e^{\frac{n\beta\phi}{M_p}} W(\psi)$$

$$-\nabla^2\phi - V' + \frac{(n-2)\beta}{M_p}\, e^{\frac{(n-2)\beta\phi}{M_p}}\ (\nabla\psi)^2 - \frac{n\beta}{M_p}\, e^{\frac{n\beta\phi}{M_p}} W(\psi) = 0$$

We are interested in the physical situation where

$$\rho \sim W(\psi)$$

$$V_{\text{eff}} \equiv V(\phi) + \frac{n\beta}{M_p}\, e^{\frac{n\beta\phi}{M_p}} \rho$$

It is easy to tune models in such a way that there is a minimun .

The larger the density of the environment, the larger the chameleon mass

The MOND/AQUAL/TeVeS Idea

$$a_N \equiv \frac{GM}{r^2}$$

Only true if the acceleration itself is bigger that a critical value

$$a_0 \sim 1.2 \times 10^{-10} m s^{-2} \sim 10^{-11} g$$

Otherwise a different formula is postulated

$$a_{MOND} = \sqrt{a_N a_0} = \frac{\sqrt{GMa_0}}{r}$$

TeVeS is quite contrived

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R[g]$$

$$S_V = -\frac{K}{32\pi G}\int \sqrt{-g}\; d^4x \left(\frac{1}{4}\; F^2_{\mu\nu} - 2\frac{\lambda(x)}{K}\left(V^2 - 1\right)\right)$$

$$f_{\mu\nu} \equiv e^{-2\phi}\left(g_{\mu\nu} - V_\mu V_\nu\right) + e^{2\phi}V_\mu V_\nu = e^{-2\phi}g_{\mu\nu} + V_\mu V_\nu \sinh 2\phi$$

$$f^{\alpha\beta} = e^{2\phi}g^{\alpha\beta} - 2V^\alpha V^\beta \sinh 2\phi$$

GR is recovered in the limit

$$K \to 0 \qquad \phi_* \equiv l\phi$$

$$l \to \infty \qquad \sigma_* \equiv \sqrt{k}\ \sigma$$

$$f_{\alpha\beta} = e^{\frac{-2\phi_*}{l}} g_{\alpha\beta} - 2V_\alpha V_\beta \mathrm{sinh}\frac{2\phi_*}{l}$$

$$S_s = -\frac{1}{2k^2 l^2}\int d^4x\sqrt{-g}\left(k\sigma_*^2 h^{\alpha\beta}\partial_\alpha\phi_*\partial_\beta\phi_* + \frac{1}{2}G\sigma_*^4 F(G\sigma_*^2)\right)$$

$$S_s = -\frac{1}{2}\int \sqrt{-g}d^4x\left(\sigma^2 h^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \frac{1}{2}\frac{G}{l^2}\sigma^4 F(kG\sigma^2)\right)$$

$$h^{\alpha\beta} \equiv g^{\mu\nu} + V^\mu V^\nu$$

Matter fields are coupled to the metric f

$$\sqrt{-f} = e^{-2\phi}\,\sqrt{-g}$$