Discrete Gradient Flows
for a Shape Optimization Problem

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(joint work with P. Morin and R.H. Nochetto)

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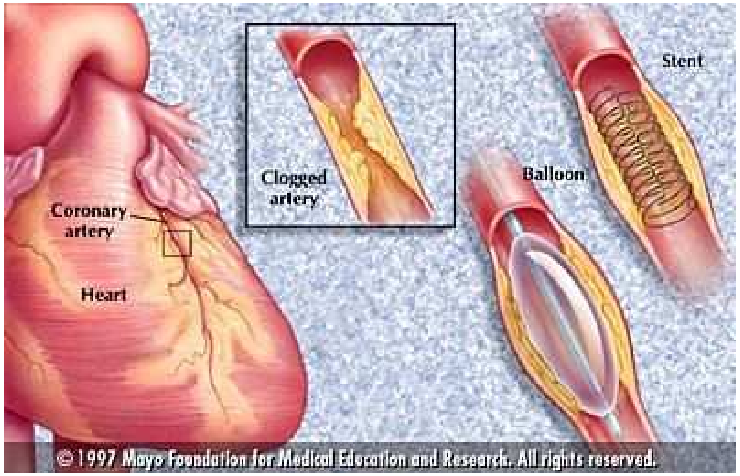
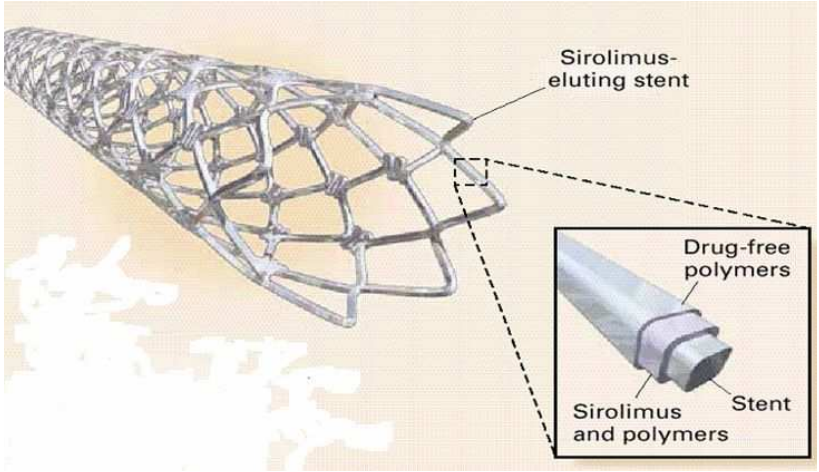
Outline

- Motivation
- Shape Calculus
- Numerical Approximation

Drug Eluting Stents





Abstract Problem

$$\Omega^* \in \mathcal{U}_{ad} : \qquad J(\Omega^*, y(\Omega^*)) = \inf_{\Omega \in \mathcal{U}_{ad}} J(\Omega, y(\Omega)),$$
$$y(\Omega): \qquad Ly(\Omega) = f, \text{ in } \Omega.$$

Mathematical problems:

- Continuous dependence of $y(\Omega)$ with respect to Ω
- Existence (and uniqueness) of Ω^*
- Optimality Conditions
- Algorithms for the numerical approximation of Ω^*
- ...

Toy Problem

$$\min_{\Omega \in \mathcal{U}_{ad}} J(\Omega) := \min_{\Omega \in \mathcal{U}_{ad}} \frac{1}{2} \int_D (y(\Omega) - z_g)^2 dx$$

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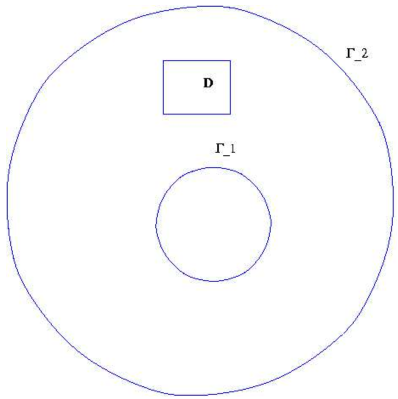
z_g given and $y = y(\Omega)$ solution of

$$\begin{cases} -\triangle y = 0, & \text{in } \Omega \\ y = 0, & \text{on } \Gamma_2 \\ \frac{\partial y}{\partial n} = 1, & \text{on } \Gamma_1, \end{cases}$$



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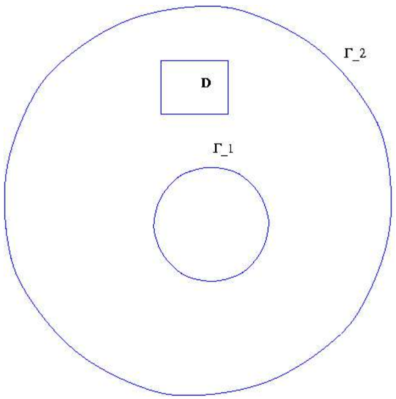
- $\mathcal{U}_{ad}$: fix Γ_2 and move Γ_1 s.t. $\Gamma_1 \cap D = \emptyset$.



Toy Problem

$$\min_{\Omega\in\mathcal{U}_{ad}} J(\Omega) := \min_{\Omega\in\mathcal{U}_{ad}} \frac{1}{2}\int_D (y(\Omega)-z_g)^2 dx + \gamma \int_{\Gamma_1} ds$$

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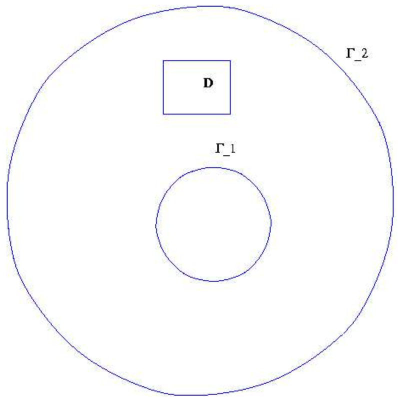

Deformation of Domains: the Speed Method

- $\vec{V}$ given
- Consider the function $X(\cdot,t) : x \in \mathbb{R}^n \longrightarrow X(x,t) \in \mathbb{R}^n$ defined by

$$\frac{dX}{dt} = \vec{V}(X(x,t)), \qquad X(x,0) = x \in \Omega, \quad (\Omega := \Omega_0)$$

- $\Omega_t = \{X(x,t) : \quad x \in \Omega_0\}$
- $\vec{V}$ “regular" $\longrightarrow$ regularity of Ω is preserved for $t > 0$.

[Cea et al, 1974], [Sokolowski and Zolesio, 1992], ...

Shape sensitivity

- Eulerian derivative in direction $\vec{V}$:

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- Shape derivative:

$$y'(\Omega; \vec{V}) = \underbrace{\dot{y}(\Omega; \vec{V})}_{\text{material derivative}} \quad - \quad \underbrace{< \nabla y(\Omega), \vec{V} >}_{\text{space derivative}}$$

Examples I $\quad (V = \vec{V} \cdot \vec{\nu})$

$$J(t) = \int_{\alpha_-(t)}^{\alpha_+(t)} \phi(x,t)dx \Rightarrow \frac{dJ}{dt} = \int_{\alpha_-(t)}^{\alpha_+(t)} \partial_t \phi(x,t)dx \pm \phi(\alpha_\pm(t))\alpha'_\pm(t)$$

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$$J(\Omega) = \int_\Omega \phi(\cdot,\Omega)dx \Rightarrow dJ(\Omega,\vec{V}) = \int_\Omega \phi'(\Omega,\vec{V}) + \int_\Gamma \phi V dx$$

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Shape sensitivity for the toy problem I $(V = \vec{V} \cdot \vec{\nu})$

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$$\begin{cases} -\triangle y' = 0, & \text{in } \Omega \\ y' = -\frac{\partial y}{\partial n} V = 0, & \text{on } \Gamma_2 \ \text{(fix)} \\ \dfrac{\partial y'}{\partial n} = \mathrm{div}_\Gamma(V \ \nabla_\Gamma y) + \kappa V, & \text{on } \Gamma_1 \end{cases}$$

Shape sensitivity for the toy problem II

Let $J(\Omega) = \frac{1}{2}\int_D (y(\Omega) - z_g)^2$ and p s.t.

$$\begin{cases} -\triangle p = \chi_D (y - z_g), & \text{in } \Omega \\ p = 0, & \text{on } \Gamma_2 \\ \frac{\partial p}{\partial n} = 0, & \text{on } \Gamma_1, \end{cases}$$

then

$$\begin{aligned} dJ(\Omega; \vec{V}) &= \int_D (y - z_g) y' = -\int_\Omega \triangle p y' \\ &= \int_{\Gamma_1} \underbrace{(-\nabla_\Gamma y \nabla_\Gamma p + kp)}_{:= G} V \end{aligned}$$

Shape sensitivity for the toy problem II

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Problem 1

Given Ω, how to choose $\vec{V}$ such that $dJ(\Omega; \vec{V}) < 0$?

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Choice of a scalar product

- $L^2(\Gamma)$: $\quad b(V, W) = \int_\Gamma VW$
- $H^1(\Gamma)$: $\quad b(V, W) = \int_\Gamma \alpha \nabla_\Gamma V \nabla_\Gamma W + \beta VW$
- $H^{-1}(\Gamma)$: $\quad b(V, W) = \int_\Gamma (-\triangle_\Gamma)^{-1} VW$

[Cea, Zolesio, ...]

Problem 2

Given Ω^0, how to build $\{\Omega^k\}_k$ such that $J(\Omega^{k+1}) \leq J(\Omega^k)$?

Time Discretization (discrete Gradient Flow)

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[Almgren, Taylor and Wang, 1993], [Luckhaus, 1990], [De Giorgi, 1990], ...

Linearization: Semi-Implicit method

$$b_{n+1}(V^{n+1}, W) = -\int_{\Gamma_1^{n+1}} G^{n+1} W \quad \forall W$$

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- Replace in optimality condition:

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$$G = -\nabla_\Gamma y \nabla_\Gamma p - k(p + \gamma)$$

How to proceed?

$$\Omega^n \longrightarrow \Omega^{n+1}: \qquad \vec{X}^{(n+1)} = \vec{X}^n + \tau \vec{V}^{(n+1)}$$

$$\begin{aligned}
\vec{k}^{(n+1)} + \tau \triangle_\Gamma \vec{V}^{(n+1)} &= -\triangle_\Gamma \vec{X}^n \\
k^{(n+1)} - \vec{k}^{(n+1)} \cdot \vec{\nu}^{(n)} &= 0 \\
\mathcal{B} V^{n+1} &= \nabla_\Gamma y^{(n)} \nabla_\Gamma p^{(n)} - k^{(n+1)} (p^{(n)} + \gamma) \\
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\end{aligned}
$$

$\mathcal{L} = -\mathrm{div}_\Gamma(\alpha \nabla_\Gamma \cdot) + \beta I$

FEM discretization

Let $\mathcal{V}(\Gamma) \subset H^1(\Gamma)$. Find $\vec{V}, \vec{k} \in \vec{\mathcal{V}}(\Gamma)$, $V, k \in \mathcal{V}(\Gamma)$, s.t.

$$< \vec{k}, \vec{\phi} > -\tau < \nabla_\Gamma \vec{V}, \nabla_\Gamma \vec{\phi} >= - < \nabla_\Gamma \vec{X}, \nabla_\Gamma \vec{\phi} >$$

$$< k, \phi > - < \vec{k} \cdot \vec{\nu}, \phi >= 0$$

$$< \alpha \nabla_\Gamma V, \nabla_\Gamma \phi > + < \beta V, \phi > + < kp, \phi >=< \nabla_\Gamma y \nabla_\Gamma p, \phi >$$

$$< \vec{V}, \vec{\phi} > - < V, \vec{\phi} \cdot \vec{\nu} >= 0$$

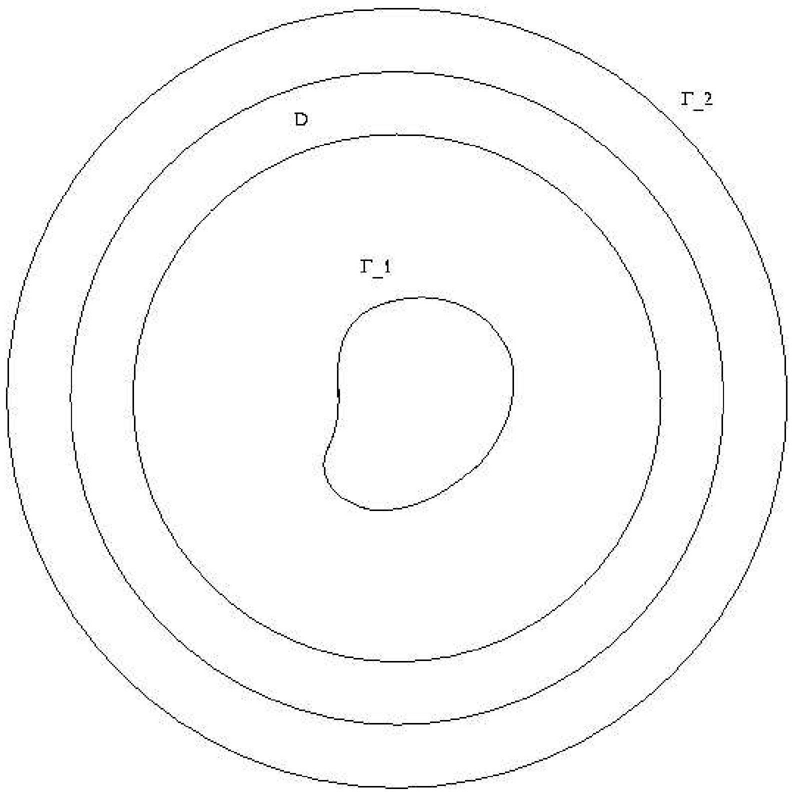
$$\forall \vec{\phi} \in \vec{\mathcal{V}}(\Gamma), \phi \in \mathcal{V}(\Gamma)$$

Schur complement

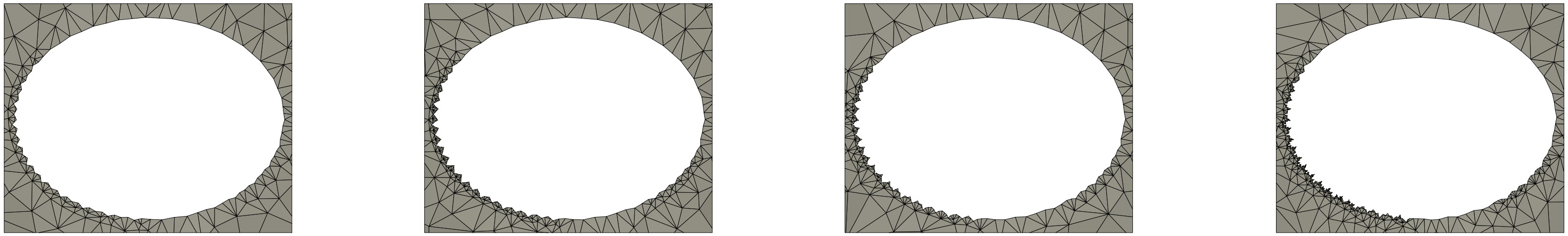
$$(\tau M_p M^{-1} \vec{N} \vec{M}^{-1} \vec{A} \vec{M}^{-1} \vec{N}^T + M + A) V = -f - M_p M^{-1} \vec{N} \vec{M}^{-1} \vec{A} \vec{X}_n$$

Test case

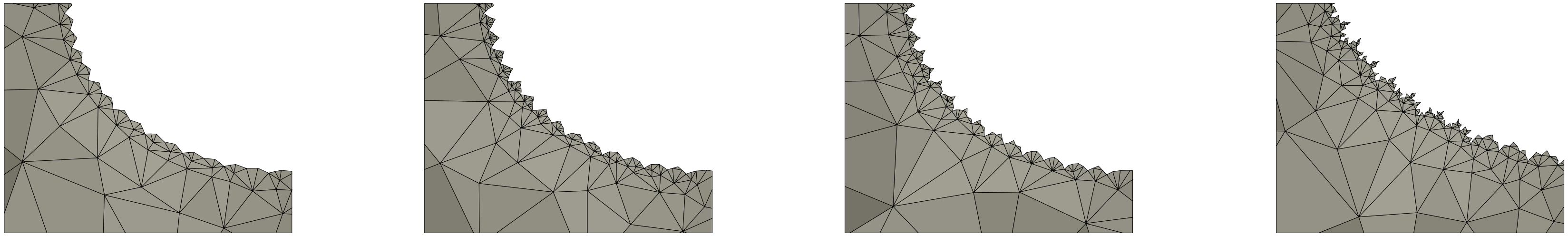
- $\min_{\Omega \in \mathcal{U}_{ad}} \frac{1}{2} \int_D [y(\Omega) - (\log(3) - \log(\rho(x)))]^2$
- $D = \{x \in \mathbb{R}^2 : \quad 2 \leq \rho(x) \leq 2.5\}$
- $\mathcal{U}_{ad}$: Γ_2 fi x and move Γ_1 s.t. $\Gamma_1 \cap D = \emptyset$.



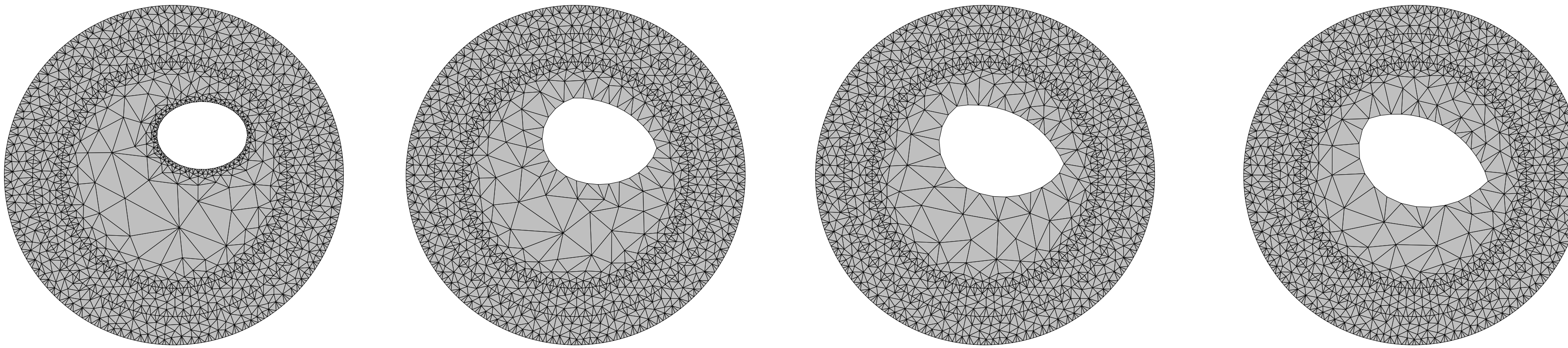
L^2 Flow



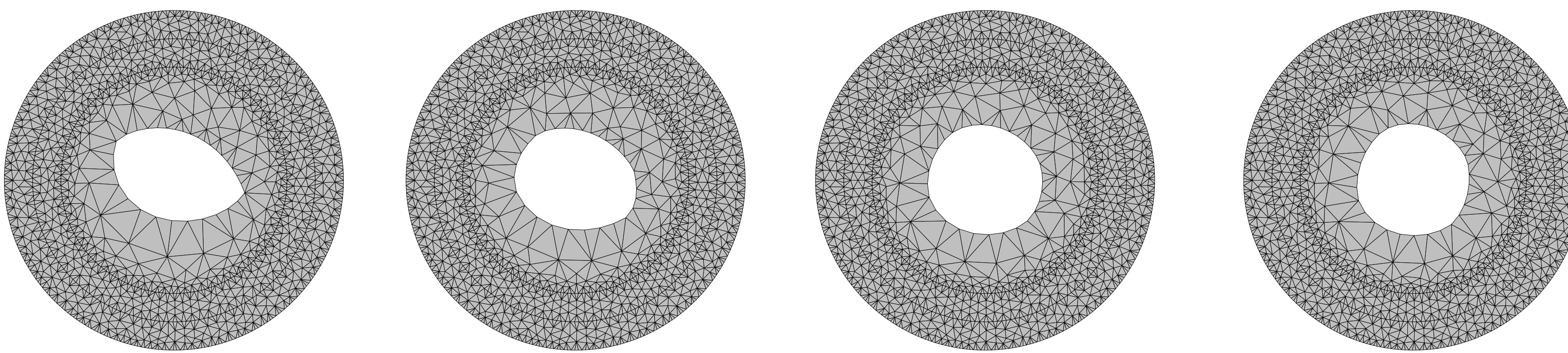
$J(3) = 0.09879199$ $J(6) = 0.08488141$ $J(9) = 0.07282547$ $J(22) = 0.03951210$



H^1_w flow



$J(0) = 0.10257$ $J(20) = 0.05646$ $J(40) = 0.02642$ $J(60) = 0.00973$



$J(80) = 0.00232$ $J(100) = 0.000288$ $J(120) = 0.0000213$ $J(140) = 0.00000137$

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- Possibility of choosing the metric for the descent direction
- Convergence of the algorithm ?
- “Optimal” choice of the metric ?