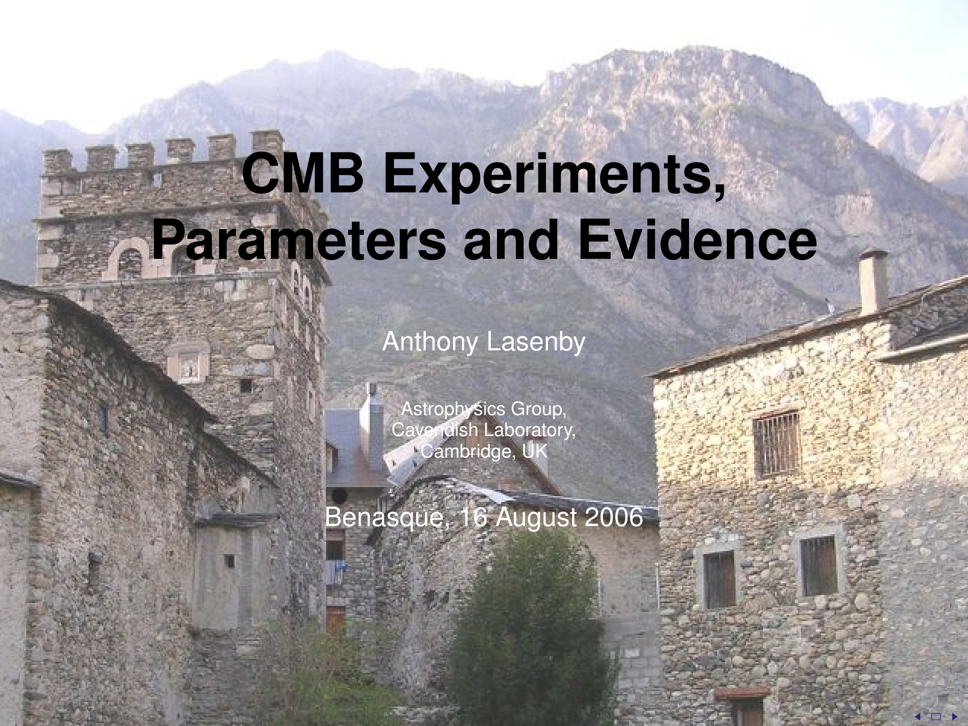


CMB Experiments, Parameters and Evidence

Anthony Lasenby

Astrophysics Group,
Cavendish Laboratory,
Cambridge, UK

Benasque, 16 August 2006

Overview

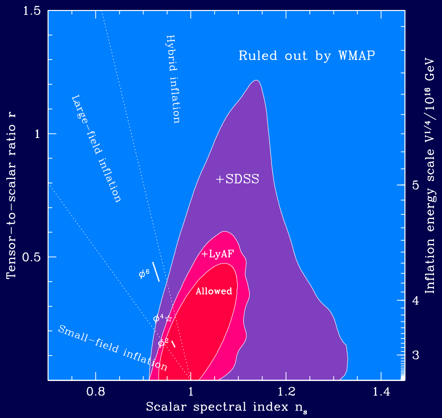
- Will be talking in broad terms about three or four distinct topics
- Firstly, some CMB experiments and prospects for them not yet covered in this meeting
- Secondly, the broad topic of evidence
- This has now become influential both in model selection and as a tool in thinking about experiment design
- Gave a talk on it, with Mike Hobson, at the Ischia meeting, and will repeat some of that here
- Two things we apply it to are
  - The primordial power spectrum
  - Bianchi models
- Re former, will discuss mention a proposal for a slightly closed universe and power spectrum from that
- Re latter, will discuss the extension to Bianchi models with Λ

EXPERIMENTS DISCUSSED

- CLOVER — Cardiff, Cambridge, Oxford B-mode bolometric experiment
- QUIET — USA B-mode heterodyne experiment
- AMI — SZ surveyor at Cambridge
- SZA — SZ surveyor in California
- But first, what are some of the science drivers as regards inflation?
- We know a key feature is that B-mode polarisation is a ‘smoking gun’ for tensor perturbations, and this is now a key goal

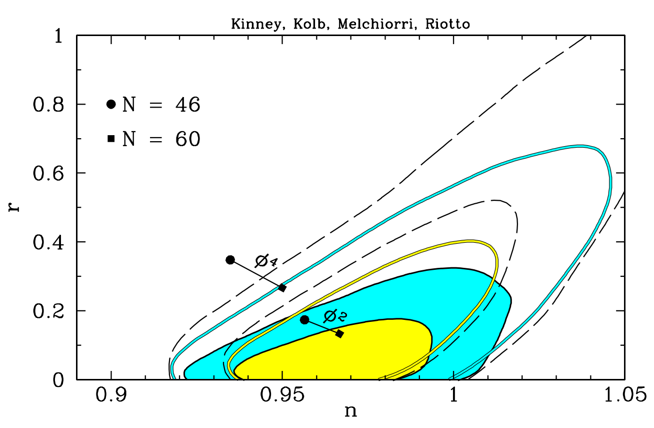
Inflation: r-n_s constraints (pre WMAP3 slide)

- Energy scale totally uncertain: $V^{1/4} < 2.6 \times 10^{16}$ GeV but could be as low as electroweak scale (100 GeV) but theoretical prior not uniform!
- No evidence for dynamics of inflation (data consistent with low-energy, flat potential giving $r \approx 0$ and $n_s \approx 1$)
- Some models already ruled out (e.g. ϕ^6 and ϕ^4)



(Tegmark et al. 2003; Seljak et al. 2004)

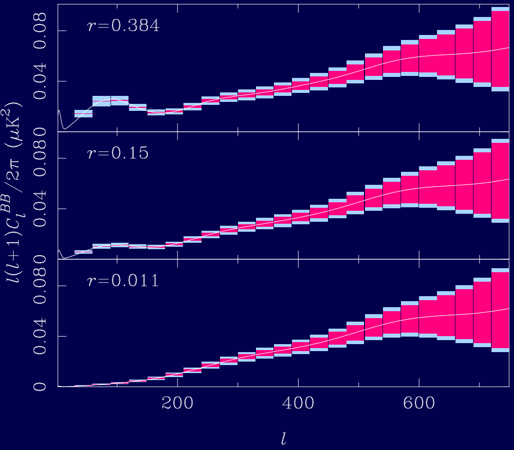
WMAP3 constraints on form of potential



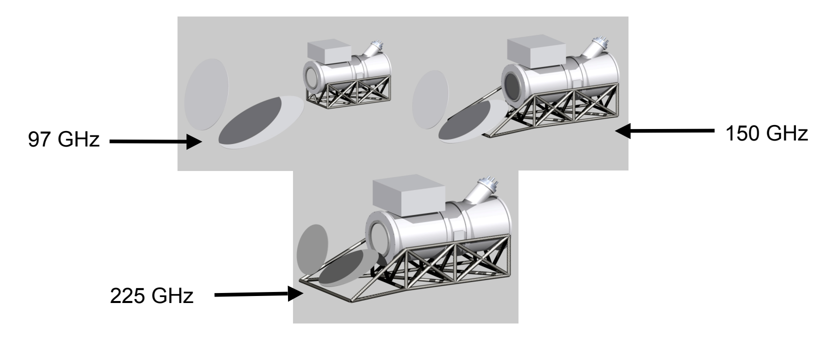
- Thus good evidence now starting to build up against ϕ^4 type theory or plain H-Z, and for ϕ^2
- This is in principle good news for B-mode detections! (typical $r \sim 0.15$ for ϕ^2)

Summary of CLOVER science goals

- Characterise *B*-mode polarization on scales $20 < l < 600$
  - Sufficient thermal sensitivity (magenta) to be limited by sample variance of lensing signal for $l < 200$
- Detect gravity waves if $r > 0.01$ (3σ; c.f. current 95% limit of ~ 0.36)
  - Hence measure energy scale of inflation if $> 1.0 \times 10^{16}$ GeV
- Place tight constraints on dynamics of inflation



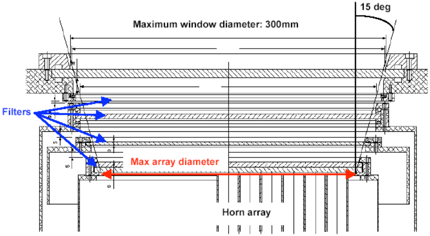
CLOVER EXPERIMENT

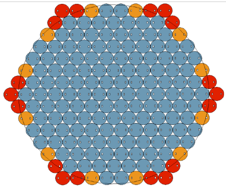


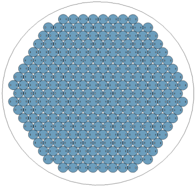
- Collaboration between
  - Cambridge — detectors, software
  - Cardiff — telescope, mount and integration
  - Oxford — optics
- Three independent scaled telescopes: 97, 150, 225 GHz
- 160, 256, 256 pixels, 8-arcmin resolution
- Transition Edge Sensor (TES) detectors at $\sim 100\,\mathrm{mK}$
- Three independent 3-axis mounts

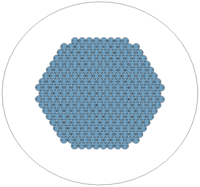
CLOVER Focal Planes

- Filter stack 300mm
- Limits array size



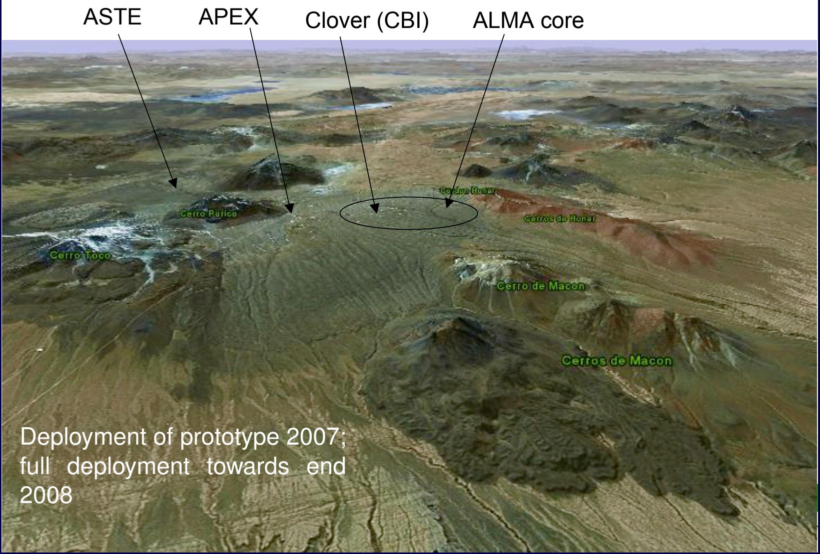






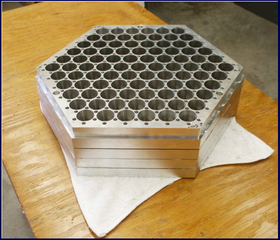
	97 GHz	150 GHz	220 GHz
No. horns:	160	256	256

CLOVER Site


ASTE
APEX
Clover (CBI)
ALMA core
Cerro Purico
Cerro Toco
Cerros de Honar
Cerro de Macon
Cerros de Macon
Deployment of prototype 2007; full deployment towards end 2008

QUIET

- QUIET Heterodyne receiver CMB polarization experiment
- Collaboration between Chicago, JPL, Miami, Princeton Caltech, Columbia, Stanford and Oxford
- Bruce Winstein (Chicago) PI
- Pathfinders: 100-element W-band (90 GHz) array on 1m telescope
- 37-element Q-band (40 GHz) array
- Two optical platforms: Novel 1m-scale telescope on CBI in Chile for large angular scales



QUIET

- Lucent 7m telescope, currently in New Jersey and recently used for CAPMAP, will be moved to Chile for small angular scales (approx 4 arcmin)
- For ultimate instrument, two frequencies at each angular scale: 1000-element W-band arrays; 300-element Q-band arrays
- Operate for 3+ years
- Funding for first stages now agreed


The Arcminute MicroKelvin Imager (Cambridge)

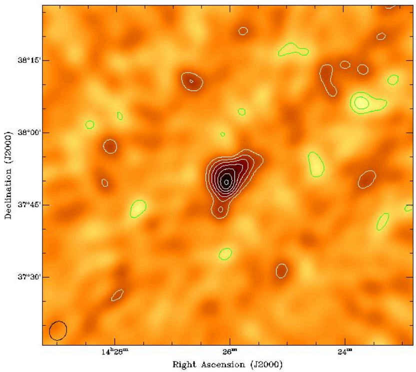
ARCMINUTE MICROKELVIN IMAGER – AMI

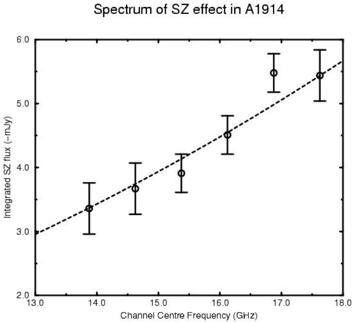
Small Array

Large Array

- Cluster survey instrument looking for SZ imprints
- Sited at Lords Bridge
- Small Array: ten 3.7m dishes
- Large Array: upgraded Ryle Telescope
- Supported on rolling grant until at least Mar 2010

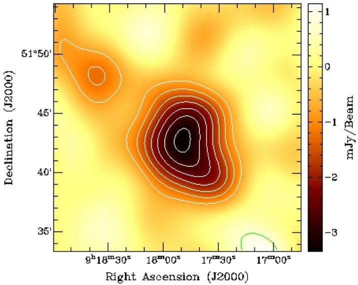
FIRST AMI SZ EFFECT

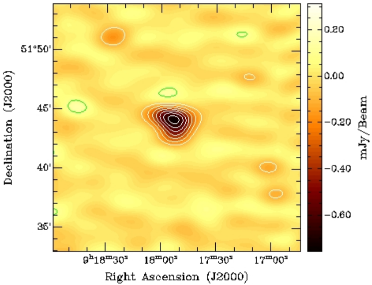




- Commissioning data – just 8 aerials; poor calibration etc.

SZ EFFECT IN A773





6 hour AMI image

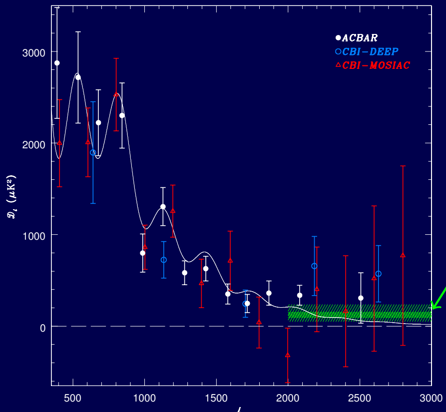
460 hour RT image

- Outer regions of gas now being detected.
- Telescope sensitivity matches theoretical prediction.

$\Rightarrow$ 10^3 improvement in survey speed over RT.

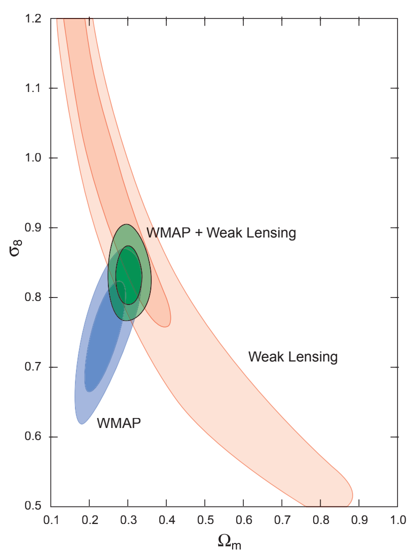
Damping tail and CBI excess

- Photon diffusion suppresses photon density fluctuations below $\sim 3\,\mathrm{Mpc}$ at last scattering; $80\,\mathrm{Mpc}$ width of last scattering surface further washes out projection to ΔT
- Predicted exponential decline seen by CBI (30 GHz) and ACBAR (150 GHz) but ...
  - CBI and BIMA see excess emission at $l > 2000$: interpreted as SZ gives $\sigma_8 \approx 1.0$



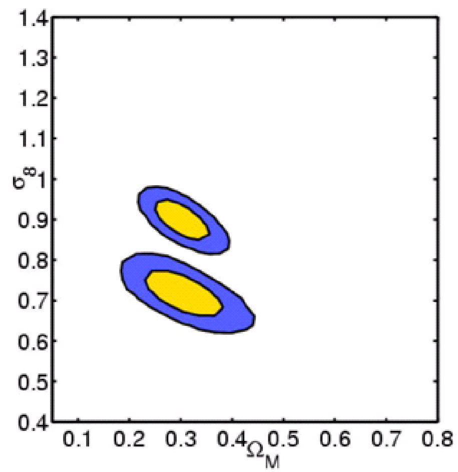
SZ expected at 150 GHz given CBI result (Kuo et al 2004)

WMAP3 versus weak lensing



- A $\sigma_8 \approx 1.0$ would, however, now be a real problem
- Ok (in general) with weak lensing, but not now with WMAP3
- Combination of losing some optical depth and lower $\Omega_{\rm cdm}$ means σ_8 now significantly lower
- $\sigma_8 = 0.92 \pm 0.1$ (WMAP1) now goes to $\sigma_8 = 0.76 \pm 0.05$ (WMAP3)
- This seems to be a real tension between models

Predicted AMI Cosmological Constraints



- 1 year, 100 square degrees AMI survey
- Other currently working experiment aiming at same thing is the SZA

SZA Construction/Integration – winter '04

Photos: Leitch

The SZA

- Chicago, Columbia, Caltech/JPL collaboration
- P.I. John Carlstron
- Eight 3.5 m diameter telescopes
- Like AMI, close-packed configuration for high surface brightness (1.2 diameter spacings)
- 30 GHz Receivers (cluster survey) (cf. AMI 15Ghz)
- Currently taking science data
- SZA to be integrated with OVRO and BIMA telescopes (CARMA) will allow high resolution cluster imaging

Evidence - What is it and what does it do for us?

- Collect a set of N data points D_i ($i = 1, 2, \ldots, N$), which we denote collectively as the data vector $\boldsymbol{D}$.
- Propose some model (or hypothesis) H for the data, depending on a set of M parameters θ_j ($j = 1, \ldots, M$), that we denote by the parameter vector $\boldsymbol{\theta}$.
- Apply Bayes' theorem

$$\Pr(\boldsymbol{\theta}|\boldsymbol{D}, H) = \frac{\Pr(\boldsymbol{D}|\boldsymbol{\theta}, H)\Pr(\boldsymbol{\theta}|H)}{\Pr(\boldsymbol{D}|H)}$$

- The prior $\Pr(\boldsymbol{\theta}|H)$ represents our state of knowledge (or prejudices) about the parameter values before analysing the data
- This is modulated by the likelihood $\Pr(\boldsymbol{D}|\boldsymbol{\theta}, H)$, of the data given a particular set of parameter values
- This gives the posterior $\Pr(\boldsymbol{\theta}|\boldsymbol{D}, H)$

Model selection and Bayesian evidence

- The evidence $\Pr(D|H)$ in

$$\Pr(\theta|D,H) = \frac{\Pr(D|\theta,H)\Pr(\theta|H)}{\Pr(D|H)}$$

  provides normalisation of the posterior
- We can use this to decide which of a set of alternative models best describes the data, using the evidence
- For example, suppose we have two alternative models H_0 and H_1 for describing a data-set D, where H_0 depends on the parameter set θ_0, and H_1 on the set θ_1.
- For H_i $(i = 0, 1)$, the probability density associated with the observed data D is

$$\Pr(D|H_i) = \int \Pr(D|\theta_i, H_i)\Pr(\theta_i|H_i)\,d\theta_i$$

Model selection contd.

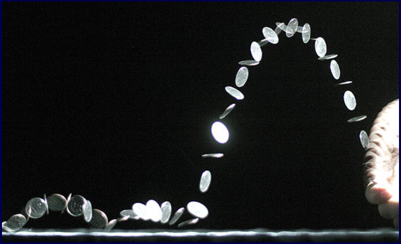
$$\Pr(D|H_i) = \int \Pr(D|\theta_i, H_i)\Pr(\theta_i|H_i)\,d\theta_i$$

- In either case H_0 or H_1, the evidence is the average of the likelihood with respect to the prior

  $\Rightarrow$ a model has a large evidence if more of its allowed parameter space is likely, given the data

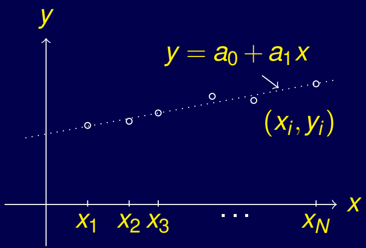
  $\Rightarrow$ a model has a small evidence if there are large areas of its allowed parameter space with low likelihood values
- Hence evidence naturally incorporates Occam's razor: a simpler theory is preferred to a more complicated one, unless latter is significantly better at describing the data
- Thus, the preferred model is that with the largest evidence

A simple example (from David Mackay's book)



- A real case (reported in the newspapers) concerned a one Euro Belgium coin
- In a spinning experiment this had come down heads 140 times, and tails 110
- What is the ratio of evidences for H_1 'it is biased' versus H_0 'it is fair'?
- Clearly can only answer this with a definite form for H_1
- E.g. assume H_1 corresponds to a uniform prior, over $[0, 1]$ for the probability p of heads: so $P(p|H_1) = 1$
- Then
  $P(D|H_1) = \int P(D|p, H_1)dp$
  $= \int_0^1 p^{n_H}(1-p)^{n_T}dp$
  $= \frac{n_H! n_T!}{(n_H+n_T+1)!}$
- Meanwhile, if the coin is fair then $P(D|H_0) = (1/2)^{n_H+n_T}$
- Ratio, for numbers given, is therefore
  $P(D|H_1)/P(D|H_0) =$
  $\frac{2^{250}\, 140!\, 110!}{251!} = 0.48$
- That the coin is fair is favoured by 2 to 1 for this prior!

Another example (more relevant to astronomy)



- Suppose we have data at known sample points and want to know if there is a 'trend' present
- So the models are
  H_1: $y_i = a_0 + a_1 x_i + \varepsilon_i$ and
  H_0: $y_i = a_0 + \varepsilon_i$
- ε is a noise vector belonging to $N(0, \sigma^2)$ say
- We need to specify priors on a_0 and a_1. If let these be uniform (and uncorrelated) over $(-\infty, \infty)$ then can do integrals analytically
- (Actual form of priors not too important if data is telling us something — will see we get a sensible answer)
- Find following:

$$P(D|H_1)/P(D|H_0) = \sqrt{\frac{2\pi\sigma^2}{\sum(x_i - \bar{x})^2}}$$

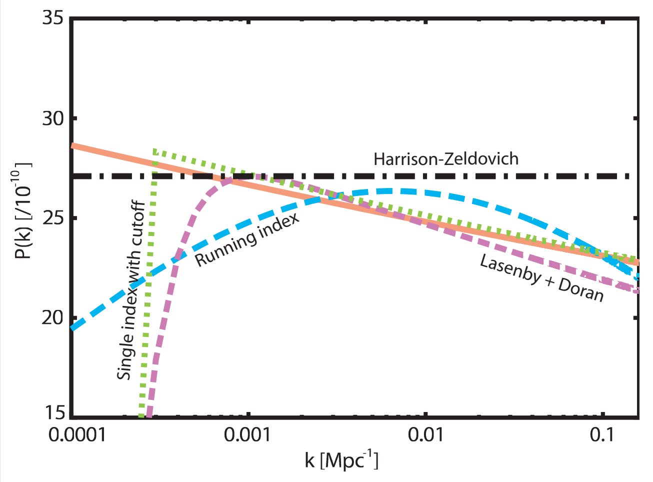
$$\times \exp\left\{ \frac{[\sum(x_i - \bar{x})(y_i - \bar{y})]^2}{2\sigma^2 \sum(x_i - \bar{x})^2} \right\}$$

- So depends on the (positive) exponential of the correlation coefficient squared!

Two Examples in Cosmology

- In general, evaluation of the evidence integral has to be numerical, and is challenging.
- For the remainder of the talk, want to look at two applications of evidence to cosmology which we have been working on recently
- First is quite similar to 'trend' example above: is there evidence for departures from scale invariance in the primordial power spectrum coming out of inflation and if so, which models are preferred (in a proper evidence sense)?
- Evidence for rotation of the universe
- Starting with first, this has been examined in astro-ph/0511573 (Bridges, Lasenby & Hobson) (plus another in last few weeks by same authors for WMAP3)

Evidence for different primordial spectra



- Figure shows some of the different type of spectra that were considered
- 'Lasenby + Doran' is for a particular model leading to a slightly closed universe

Slightly closed models

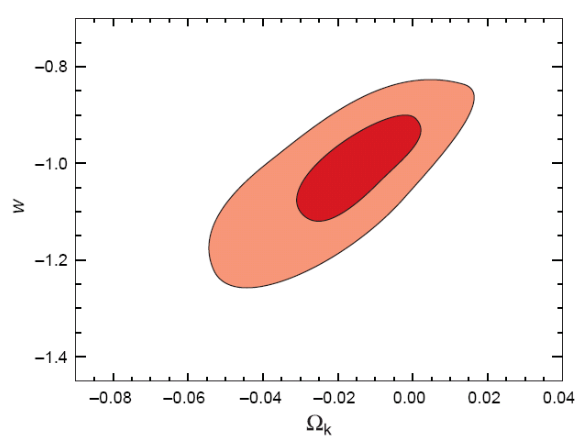
- At this stage, I ought to declare an interest
- With Chris Doran, I have developed a model in which a slightly closed universe (few percent level) emerges naturally
- 'Closure' during inflation naturally gives a low k cutoff in primordial spectrum
- Model has its basis in a conformal geometry approach to understanding Λ
- Works with a simple $m^2\phi^2$ scalar field potential
- Described in Phys.Rev.D **71, (2005) 063502** (Lasenby & Doran)
- The conformal geometry part gives a novel boundary condition at the ***end*** of the universe!
- Also gives natural linkage between Λ and number of e-folds N of inflation

$$\Lambda \sim \exp(-6N) \quad \text{which gives} \quad \sim 10^{-122} \quad \text{in natural units if} \quad N \sim 46$$

- Thus I am very interested in whether universe is indeed just closed, and want $w = -1$, so that Λ can be purely geometrical!

Nature of dark energy

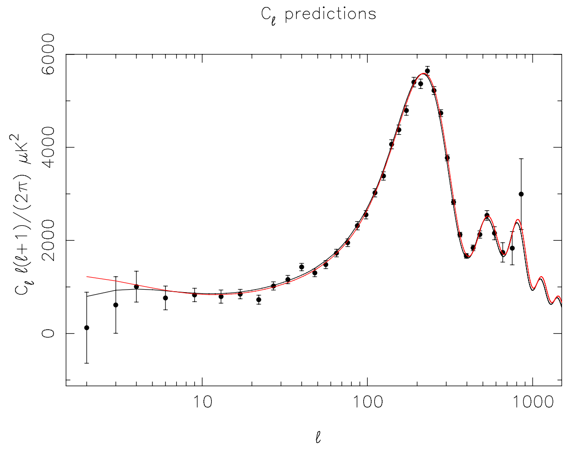
- This, and slight closure of the universe, fit in fine with all current data, e.g. following from Spergel et al WMAP3



This is for full CMB data set+2dfGRS+SDSS+SN

- get $w = -1.062^{+0.128}_{-0.079}$, $\Omega_k = -0.024^{+0.016}_{-0.013}$

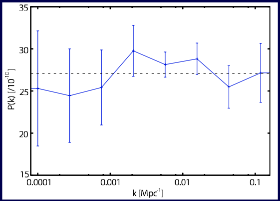
Comparison of L+D model with WMAP1 points



- Predicted CMB power spectrum for a model with $\Omega_{tot} = 1.04$
- Red line is WMAP best fit Λ CDM power law spectrum
- Catch is that our curve is for $H_0 = 60\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$!
- HST value is 72 ± 8, for comparison

Evidence for different primordial spectra (contd.)

Also considered a free-form fit in 8 bins for the power spectrum, plus a 'broken spectrum' with two scale-invariant sections joined by a sloping line

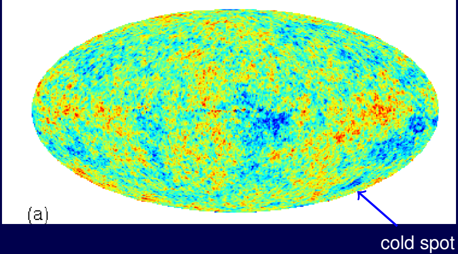


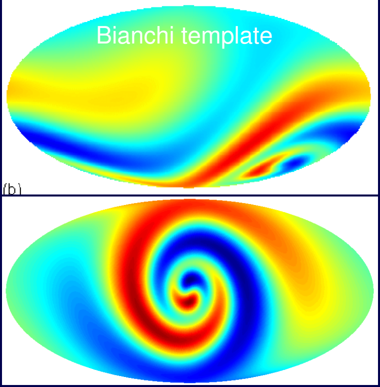
- Some sample evidence results were as in following:

Table: Differences of log evidences (for primordial parameters) for all models with respect to single index model within a current (near) concordance cosmology: $\Omega_0 = 1.024, \Omega_b h^2 = 0.0229, h = 0.61, \Omega_{cdm} h^2 = 0.118$, as compared to the Lasenby & Doran model (treated as a template)

Model	$\ln E_i - \ln E_0$
Constant n	0.0 $\pm$ 0.5
H-Z	-4.4 $\pm$ 0.5
Running	-0.8 $\pm$ 0.6
Cutoff	0.4 $\pm$ 0.5
Broken	-2.7 $\pm$ 0.6
Binned	-6.1$\pm$ 0.6
Lasenby & Doran	4.1 $\pm$ 0.5

A Bianchi Model Universe?





- Several authors have commented on significant North/South asymmetry in the WMAP data, plus strange alignment between low multipoles
- Jaffe et al. (astro-ph/0503213) fitted a Bianchi VIIh template to WMAP sky
- Found a best fit with $\Omega_0 = 0.5$
- Coldest part of template corresponds with a non-Gaussian spot found in Vielva et al. (astro-ph/0310273) and drawn attention to in Cruz et al. (astro-ph/0405341)
- But $\Omega_0 = 0.5$ in conflict with most other astrophysical indicators
- Can one achieve the same in models including Λ?

CMB sky in a Bianchi VIIh universe

- Solution for temperature anisotropy, in a Bianchi VIIh model with background open universe specified by just a matter density Ω_0 can be split into an 'end term', expressing the effects of the velocity of the matter at emission, plus an 'integrated term' which has the accumulated effects on the photon energy over the path from the emission region to observer
- Turns out second is dominant, and harder to compute (could do it all effectively analytically otherwise)
- Following Barrow, Juszkiewicz & Sonoda 1985 (MNRAS, **213**, 917), and using σ_{12} and σ_{13} to express the two independent shear mode amplitudes, find this term (without Λ) is given by:

CMB sky in a Bianchi VIIh universe

$$\frac{\Delta T}{T} = -\int_{\tau_E}^{\tau_0} \frac{h^{1/2}(1-\Omega_0)^{3/2}}{\Omega_0^2} \sin 2\theta \left[\left(\frac{\sigma_{12}}{H}\right)_0 \cos\phi + \left(\frac{\sigma_{13}}{H}\right)_0 \sin\phi \right] \frac{d\tau}{\sinh^4(h^{1/2}\tau/2)}$$

Here τ is conformal time, and θ and ϕ express the direction of the photon, as it goes along a geodesic. The equations for these are:

$$\theta' = -\sqrt{h}\sin\theta, \qquad \phi' = -\cos\theta$$

which look quite innocent, but when integrated become:

$$\tan\left(\frac{\theta}{2}\right) = \tan\left(\frac{\theta_R}{2}\right)\exp(-(\tau-\tau_R)\sqrt{h})$$

$$\phi = \phi_R + \tau - \tau_R - \frac{1}{\sqrt{h}} \ln\left\{ \sin^2\left(\frac{\theta_R}{2}\right) + \cos^2\left(\frac{\theta_R}{2}\right)\exp[2(\tau-\tau_r)\sqrt{h}] \right\}$$

CMB sky in a Bianchi VIIh universe

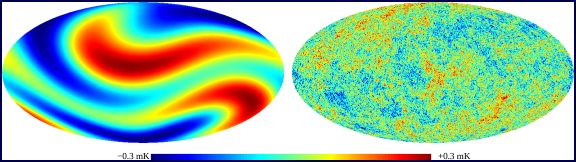
- In a general Λ universe, these geodesic equations are the same, but now the development of scale factor with conformal time is no longer analytic
- Thus have to replace the integrals which find the accumulated effect on the photon motion with linked differential equations, which can solve numerically to find both the scale factor evolution and the photon energy evolution
- Have now done this, and Jason McEwen (student with Mike Hobson and ANL at Cambridge) has now implemented in fast Fortran
- Gives opportunity to fit for parameters of the Bianchi model in a more reasonable cosmological context
- Note Jaffe et al, independently carried out the same extension, working with Hervik

Effects of including Λ

- Movie shows effects of changing Ω_Λ, with fixed other parameters ($h = 0.01$, $\Omega_{\rm m} = 0.26$)
- (Generally, putting in Λ has the effect of shortening conformal time available, and so need more drastic (small) h values in order to get similar smaller scale effects)

Evidence and Bianchi models

- Results (Jaffe et al. second paper (astro-ph/0512433)) are that it's not possible to find a good model in which the Bianchi template cosmology values match those of something which fits rest of data (e.g. acoustic peaks etc.)
- Supported by the full MCMC analysis in Bridges et al (astro-ph/0605325)
- However, still interesting to evaluate evidence for Bianchi VIIh model, just treating it as a template - how much do we really need it in our data??
- E.g., both of these are for a reasonable 'just open' cosmology $\Omega_{\rm m} = 0.3$, $\Omega_\Lambda = 0.69$



- Can simulate different vorticities and see how well evidence can discriminate

The Bianchi versus CMB degeneracies

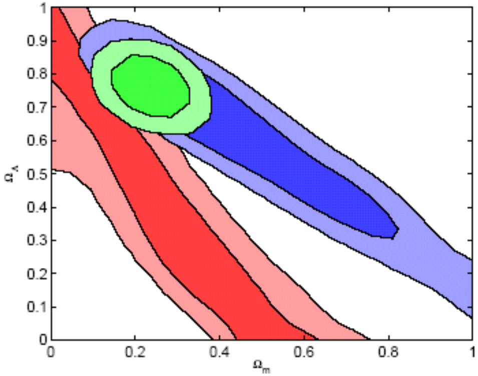
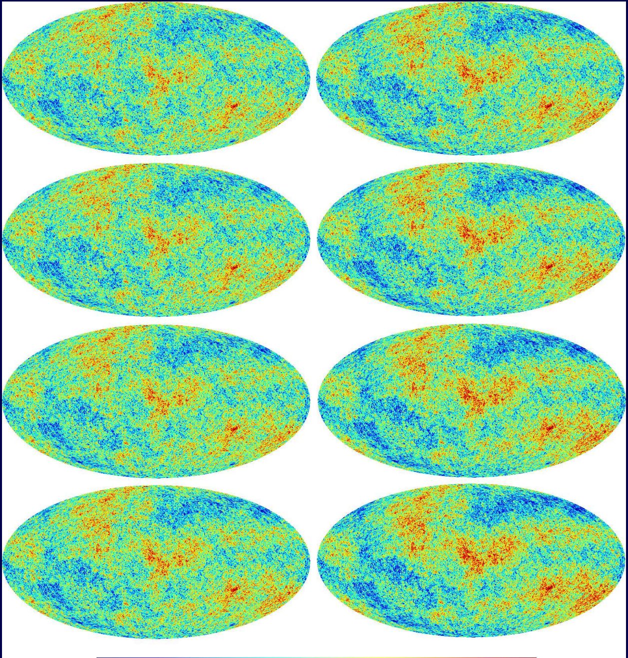


Figure 8. Comparison of the $\Omega_m - \Omega_\Lambda$ Bianchi degeneracy (shaded with 1 and 2σ contours) with the familiar CMB geometric degeneracy from WMAP first (blue) and third year + polarisation (green) data (with 1 and 2σ contours).

From Bridges et al (astro-ph//0605325)

Evidence for a Bianchi template in simulations



$\omega = 3 \times 10^{-10}$
$\Delta \ln E = -5$

$\omega = 7 \times 10^{-10}$
$\Delta \ln E = +10$

$\omega = 4 \times 10^{-10}$
$\Delta \ln E = -3$

$\omega = 8 \times 10^{-10}$
$\Delta \ln E = +7$

$\omega = 5 \times 10^{-10}$
$\Delta \ln E = -1$

$\omega = 9 \times 10^{-10}$
$\Delta \ln E = +11$

$\omega = 6 \times 10^{-10}$
$\Delta \ln E = +1$

$\omega = 10 \times 10^{-10}$
$\Delta \ln E = +18$

Evidence and Bianchi models

- So we start to be able to discriminate, at about the level of the original Bianchi template
- Indeed, considering this (no Λ now), we find that for both the WMAP1 and WMAP3 data such a template is needed by the data by a $\Delta \ln E$ difference which is positive but < 1 (so not definitive)
- So not clear if the data really warrant the introduction of these kinds of large scale features yet.

Conclusions as regards CMB and cosmology

- Basic predictions from CMB now impressively verified:
  - Large-scale Sachs-Wolfe effect and ISW
  - Acoustic peaks and diffusion damping
  - E-mode polarization, correlation with ΔT and reionization in TE
- In the near-future:
  - Better polarization; B-modes from lensing (and possibly gravity waves)
  - Physics of reionization, SZ surveys, defect searches from small-angle CMB)
- Inflation holding up well and just starting to get evidence for dynamics during inflation
  - Character (adiabatic) and statistics (Gaussian) from high sensitivity CMB will be important future probes
  - Gravity waves from inflation should be detectable in B-mode polarization if $V^{1/4} > \mathrm{few} \times 10^{15}\,\mathrm{GeV}$ (lensing, foregrounds, systematics?)
- Unresolved issues on large angles (topology, foregrounds, systematics, chance?)