BLACK HOLE SOLUTIONS IN MASSIVE GRAVITY

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OUTLINE

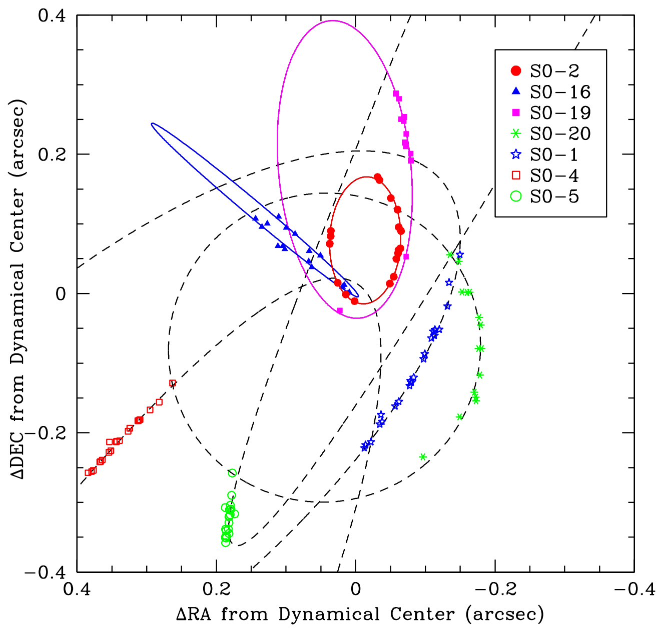
- Motivation
- The model
- Schwarzschild black hole
- Kerr black hole
- Discussion

MOTIVATION

The existence and properties of the black holes is the most non-trivial prediction of the general relativity.

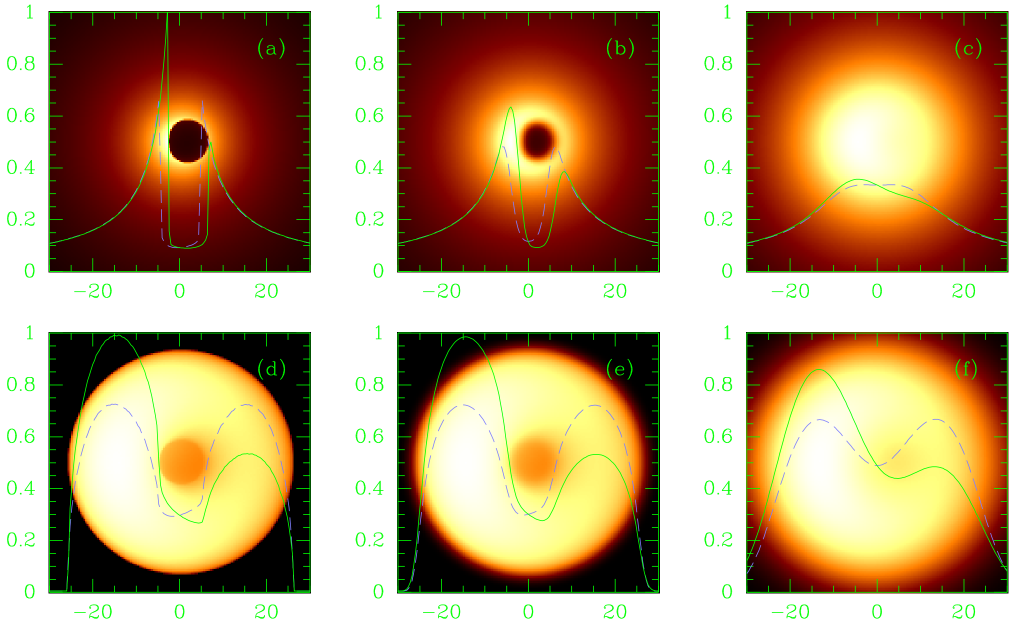
Black holes are believed to be a very common object. They are found in the centers of many galaxies including Milky Way.

One may even observe the individual star orbits in the gravitational field of the black hole.



Ghez et al, Astrophys.J. 620 (2005) 744

The detailed properties of the black holes may become observationally accessible in the near future. For instance, one will be able to measure the black hole spin.



Falcke et al, ApJ lett, 528, L13 (2000)

Can black hole properties serve as a discriminator between different models of gravity?

The answer is not so obvious as black hole solution seems to be "universal": for instance, in the Brans-Dicke model the black hole solution is identical to that of the Einstein theory, even though the gravitational law is different. This is a consequence of the "no-hair" theorems.

Bekenstein PRD **5** (1972) 1239; PRD **5** (1972) 2403.

$\Longrightarrow$ It would be nice to have an alternative model of gravity where the black hole solution is *different* from the standard one.

We address this question in the context of massive gravity.

THE MODEL

Arkani-Hamed, Georgi, Schwartz 2003; Arkani-Hamed, Cheng, Luty & Mukohyama 2004;

The model we consider is the generalization of the ghost condensate. It involves gravity and 4 scalar fields ϕ_0, ϕ_i which couple to gravity by the derivative coupling. The action reads

Dubovsky 2004

$$S = \int d^4x\sqrt{-g}\Big\{-M_{Pl}^2 R + \Lambda^4 F(X, W^{ij})\Big\}$$

where

$X = g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^0$
$V_i = g^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_i$
$W_{ij} = g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_j - V_iV_j/X$
$F(X, W^{ij})$ — an arbitrary function
Λ — cutoff scale

There exists a Minkowski vacuum solution:

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} \\ \phi^0 &= a\,t \\ \phi^i &= b\,x^i \end{aligned}$$

It breaks Lorentz invariance, but leaves rotational symmetry unbroken.

For this configuration $T_{\mu\nu}$ may be set to 0 by the choice of a and b (discussed in detail below).

General classification of graviton mass terms:

Rubakov, 2004
Dubovsky 2004

The most general rotationally-invariant graviton mass term is

$$L_m = M_{Pl}^2 \left\{ m_0^2 h_{00}^2 + 2 m_1^2 h_{0i}^2 - m_2^2 h_{ij}^2 + m_3^2 (h_{ii})^2 - 2 m_4^2 h_{00} h_{ii} \right\}$$

$h_{\mu\nu}$ — metric perturbations

Various cases:

Feirz-Pauli model: $m_0^2 = 0$; $m_1^2 = m_2^2 = m_3^2 = m_4^2$.

Lorentz invariance: $m_0^2 = \alpha^2 + \beta^2$; $m_1^2 = m_2^2 = -\alpha^2$; $m_3^2 = m_4^4 = \beta^2$.

Ghost condensate: $m_1^2 = m_2^2 = m_3^2 = m_4^2 = 0$.

Rubakov's proposal: $m_0^2 = 0$.

We consider the case $m_1^2 = 0$.

Note: this relation can be ensured by imposing the symmetry $x^i \to x^i + \xi^i(t)$. This case is phenomenologically very interesting.

One can show that in this case the resulting low-energy effective theory has the following properties:

* No ghosts or UV instabilities
* No VDVZ discontinuity

Dubovsky 2004

* The cutoff scale is $\Lambda = \sqrt{m M_{Pl}}$

Small perturbations

"Unitary gauge": perturbations of Goldstone fields are zero. What remains is:

$$\begin{aligned} \delta g_{00} &= 2\varphi; \\ \delta g_{0i} &= S_i - \partial_i B; \\ \delta g_{ij} &= -h_{ij} - \partial_i F_j - \partial_j F_i + 2(\psi\delta_{ij} - \partial_i\partial_j E), \end{aligned}$$

∗ *Tensor perturbations:*

$$(-\partial_0^2 + \partial_i^2 - m_2^2)h_{ij} = 0$$

$\Longrightarrow$ Two massive propagating degrees of freedom with the mass m_2

∗ *Vector perturbations:* Like in GR: no propagating degrees of freedom.

$\ast$ *Scalar perturbations:* Gauge-invariant potentials ψ and $\Phi = \varphi + \partial_0 B - \partial_0^2 E$:

$$\psi = \psi_E$$

Newton's potential:

$$\boxed{\Phi = \Phi_E + \mu^2 \frac{1}{\partial_i^4} \frac{\delta T_{00}}{M_{Pl}^2}}$$

where

$$\mu^2 = m_2^2 \frac{3m_4^4 - m_0^2(3m_3^2 - m_2^2)}{m_4^4 - m_0^2(m_3^2 - m_2^2)}$$

Note:

— μ goes to zero when all masses uniformly go to zero

— the extra term produces "confining" potential which goes like r

* The condition $\mu^2 = 0$ can be imposed by setting

$$F(X, W^{ij}) = F(X^{\gamma} W^{ij}),$$

where γ is a parameter. Then graviton is massive, but Newton's law remains unmodified.

To summarize linear analysis:

* graviton has a finite mass $m \sim \Lambda^2 / M_{\rm Pl}$ (just 2 polarizations!)

* the gravitational potential behaves like $1/r$

SCHWARZSCHILD BLACK HOLE

General spherically symmetric solution is too difficult to find $\Longrightarrow$ ask a sumpler question:

Can one adjust scalar fields ϕ^0 and ϕ^i so that the standard Schwarzshild metric is a solution?

To answer this question one has to find such ϕ_0 and ϕ_i that $T_{\mu\nu} = 0$ in the Schwarzschild background.

Consider general expression for $T_{\mu\nu}$ in the case $F = F(X^\gamma W_{ij})$.

Denote:

$$X^\gamma W_{ij} \equiv Z_{ij}; \qquad \delta F \equiv F_{ij} \delta Z_{ij}$$

Then

$$T_{\mu\nu} = 2F_{ij}(Z)\Big\{\gamma X^{\gamma-1}W_{ij}\,\partial_\mu\phi_0\partial_\nu\phi_0 + \frac{V_iV_j}{X^2}\,\partial_\mu\phi_0\partial_\nu\phi_0$$

$$+X^\gamma\,\partial_\mu\phi_i\partial_\nu\phi_j - \frac{V_i}{X}\,\partial_\mu\phi_j\partial_\nu\phi_0 - \frac{V_i}{X}\,\partial_\nu\phi_j\partial_\mu\phi_0\Big\}$$

$$-g_{\mu\nu}F(Z)$$

where

$$X = g^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0$$

$$V_i = g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_0$$

$$W_{ij} = g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_j - \frac{V_iV_j}{X}$$

How at all this $T_{\mu\nu}$ can be zero?!

To see this let's go back to the vacuum case.

Then

$$g_{\mu\nu} = \eta_{\mu\nu}; \quad \phi_0 = at; \quad \phi_i = bx^i$$

and one has

$$X = a^2; \quad V_i = 0; \quad W_{ij} = -b^2\delta_{ij}; \quad F_{ij} = \tilde{F}\delta_{ij}$$

Therefore,

$$T_{00} = -2\tilde{F}\gamma a^{2\gamma-2}b^2 - F$$
$$T_{ij} = (2\tilde{F}a^{2\gamma}b^2 + F)\delta_{ij}$$

Thus in the vacuum one has $F_{ij} = 0 \qquad F = 0$

$\Longrightarrow$ One has to require that in the black hole metric

$$\boxed{Z_{ij} = X^\gamma W_{ij} = -\delta_{ij}}$$

$\Longrightarrow$ One needs to solve (overdetermined) first order equations.

This can be done for the Schwarzschild metric

$$ds^2 = \left(1 - \frac{R_g}{r}\right) dt^2 - \left(1 - \frac{R_g}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

One finds

$$\phi^0 = t + 2\sqrt{rR_g} - R_g \ln \frac{\sqrt{r} + \sqrt{R_g}}{\sqrt{r} - \sqrt{R_g}}$$

$$\phi^i = x^i$$

These fields solve both the Einstein and field equations.

Note: this is the same field configuration as in the case of the ghost condensate model.

Mukohyama, PRD 71:104019, 2005

KERR BLACK HOLE

This program fails in the case of the rotating black hole.

To see this one notes that in a particular gauge one has

$$Z_{ij} \propto W_{ij} \to (g_{ij})^{-1}$$

$\Longrightarrow$ The requirement $Z_{ij} = -\delta_{ij}$ implies that there exists a frame in which the space part of the metric is conformally flat

Amazingly, precisely this question was addressed by people simulating black hole merging. They have proven a
Theorem. There does not exist conformally flat slicing of a metric which asymptotically goes to the flat space and has non-zero angular momentum.

Garat, Price PRD 61(2000) 124011;
Kroon Cl.Q.Gr.21(2004)3237

$\Longrightarrow$ There do not exist fields ϕ_0, ϕ_i which solve the equations $Z_{ij} = -\delta_{ij}$

$\Longrightarrow$ There does not exist a solution in our model with the metric equal to the Kerr metric

DISCUSSION

A natural question to ask now is: how does the correct solution look like? The answer is not known. There is a number of possibilities:

- a static solution with a different metric and non-zero momentum? – unlikely.

- permanent accretion? – unlikely

- no solution with a non-zero momentum at all? What happens then at the collapse of the $J \neq 0$ matter? Is momentum radiated away?

- something else?...