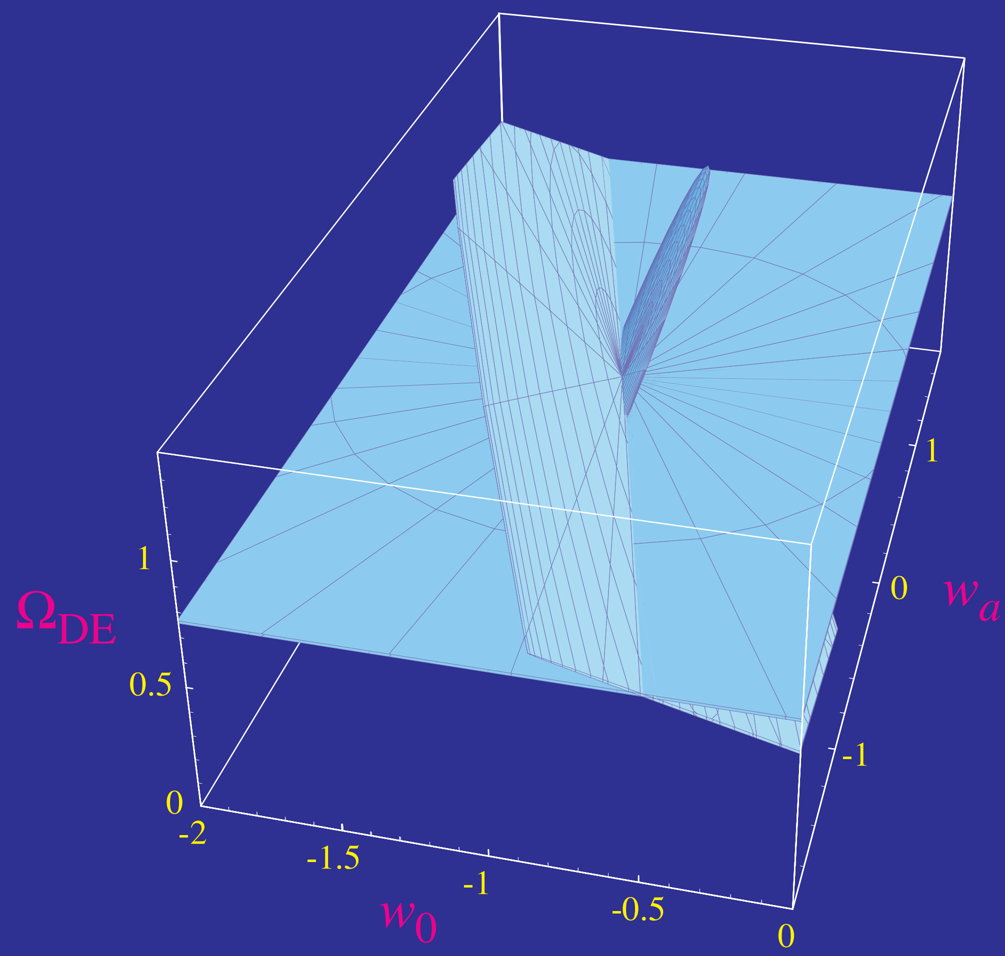
Gravitational Lensing of the CMB



Wayne Hu

Benasque, August 2006

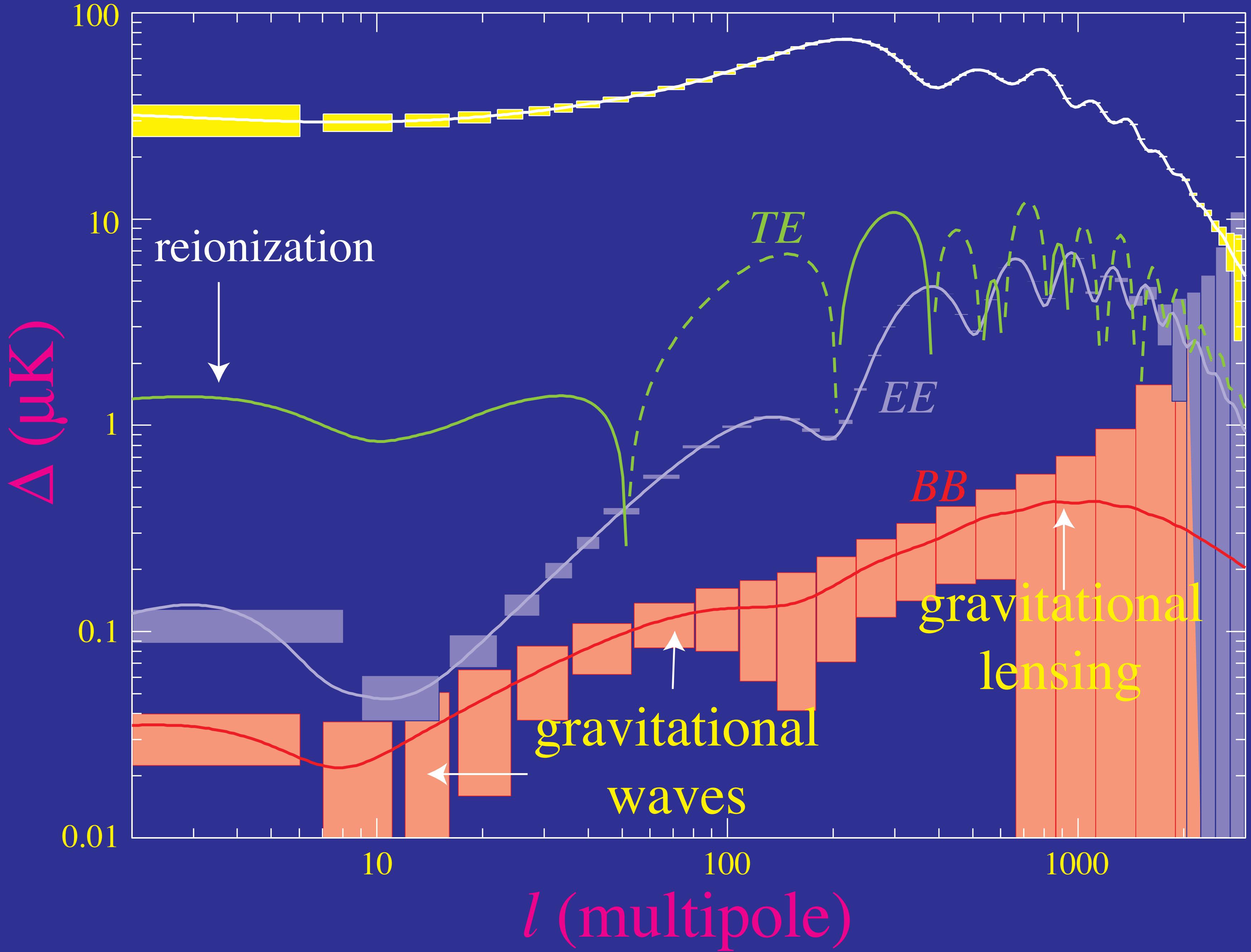
Outline

- Gravitational Lensing of Temperature and Polarization Fields
- Cosmological Observables from Lensed Power Spectra
- Complementarity with Dark Energy Probes
- Direct Mass Reconstruction

Collaborators:

- Dragan Huterer
- Manoj Kaplinghat
- Michael Mortonson
- Yong-Seon Song
- Iggy Sawicki
- Kendrick Smith

Temperature and Polarization Spectra



Lensing of CMB Fields

Gravitational Lensing

- Lensing is a surface brightness conserving remapping of source to image planes by the gradient of the projected potential

$$\phi(\hat{\mathbf{n}}) = 2 \int \frac{dz}{H(z)} \frac{D_A(D_s - D)}{D_A(D)\, D_A(D_s)} \Phi(D_A \hat{\mathbf{n}}, D) \,,$$

such that the fields are remapped as

$$x(\hat{\mathbf{n}}) \to x(\hat{\mathbf{n}} + \nabla\phi) \,,$$

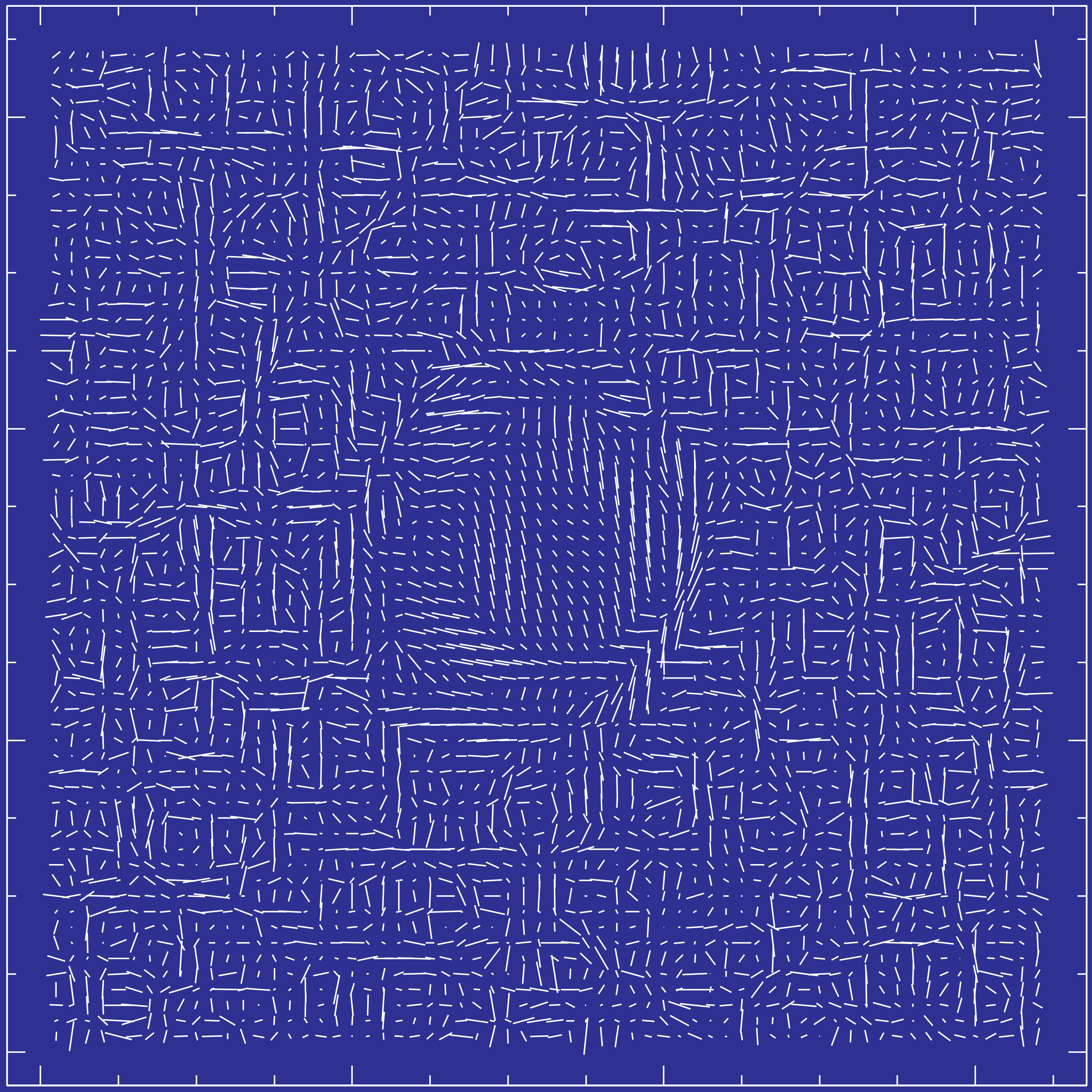
where $x \in \{T, Q, U\}$ temperature and polarization.

- Taylor expansion leads to product of fields and Fourier mode-coupling
- Appears in the power spectrum as a convolution kernel for T and E and an $E \to B$.

Lensing of a Gaussian Random Field

- CMB temperature and polarization anisotropies are Gaussian random fields – unlike galaxy weak lensing
- Average over many noisy images – like galaxy weak lensing

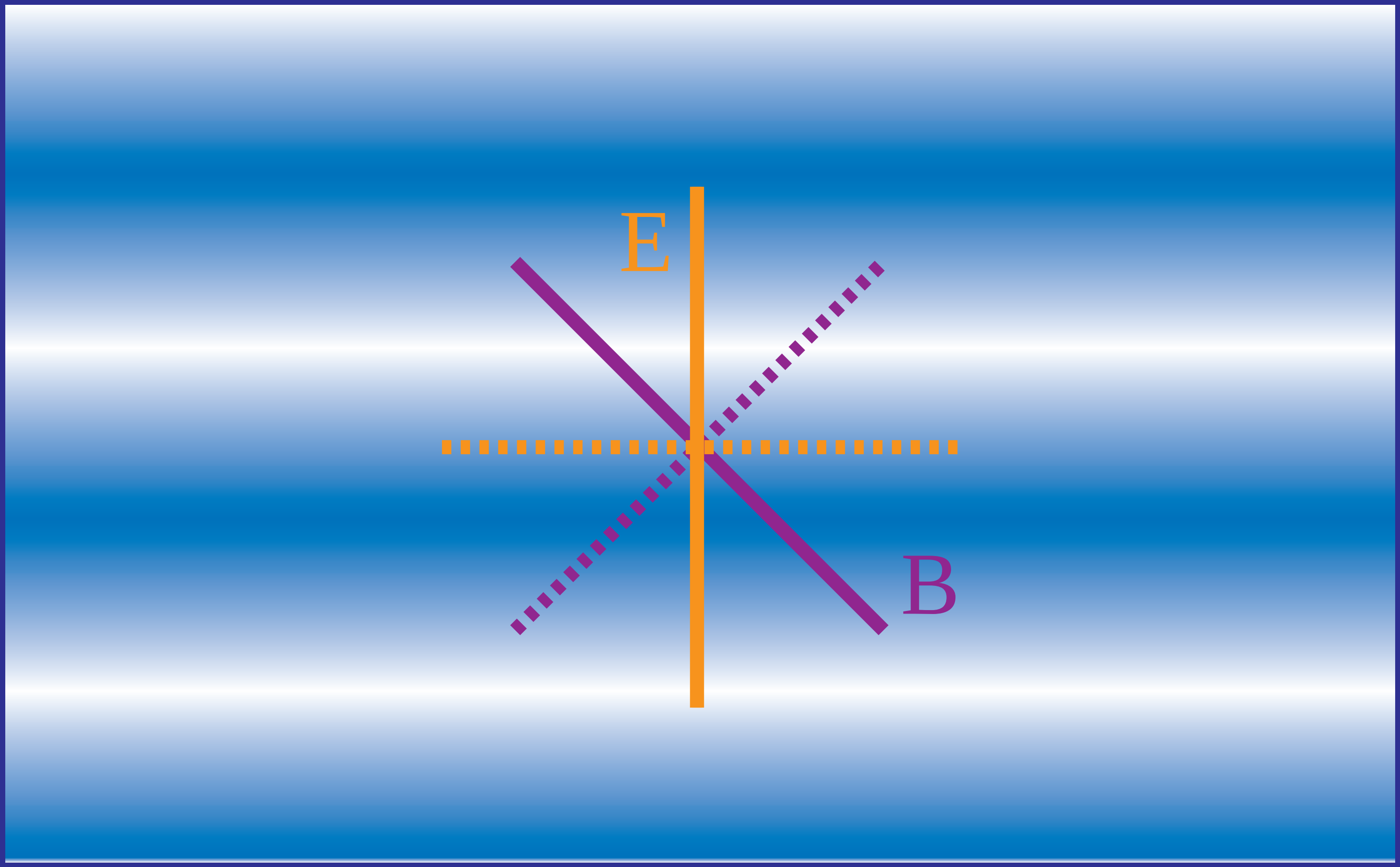
Polarization Lensing



Electric & Magnetic Polarization

(a.k.a. gradient & curl)

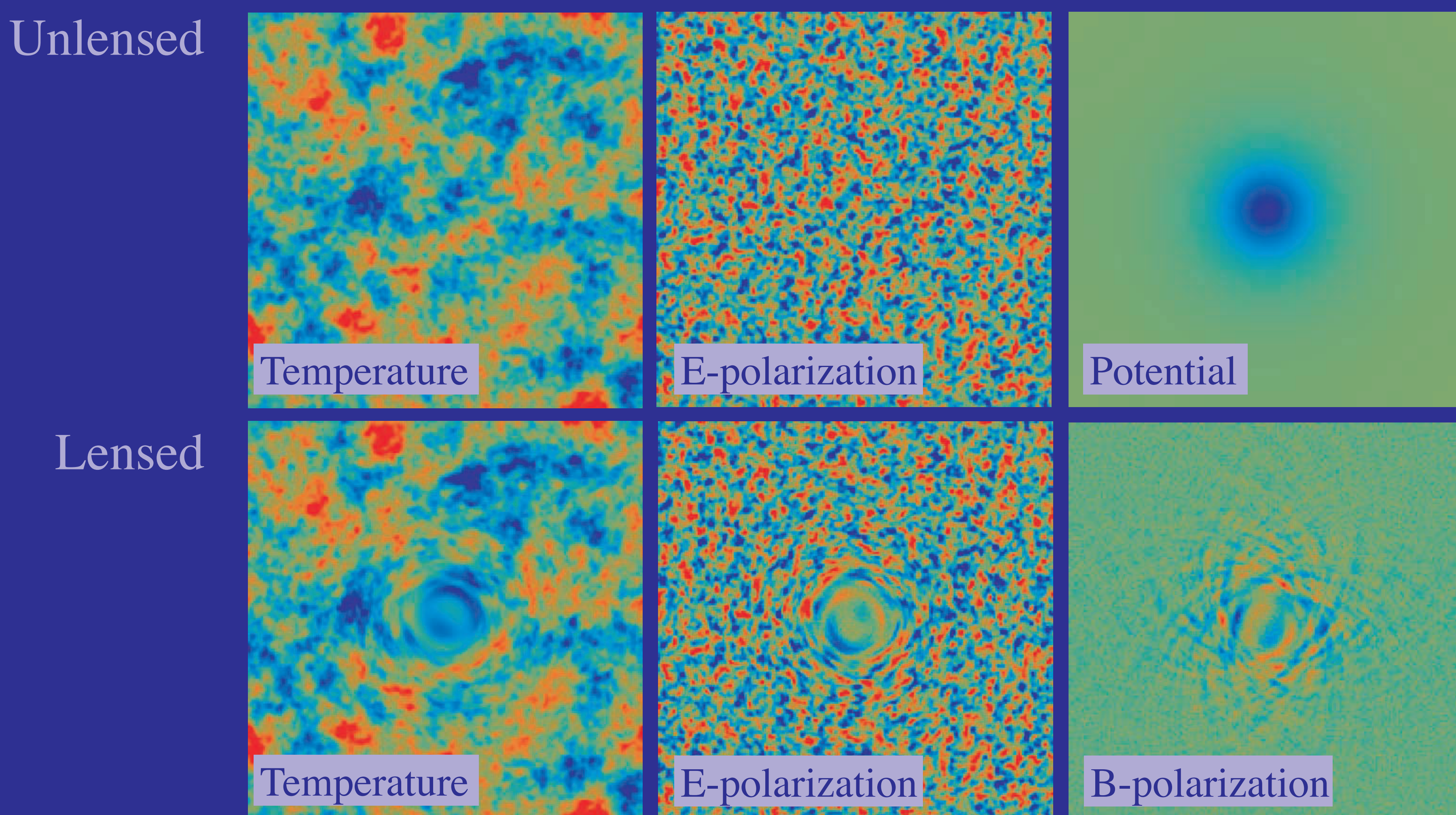
- Alignment of principal vs polarization axes
  (curvature matrix vs polarization direction)



Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)

Temperature & Polarization

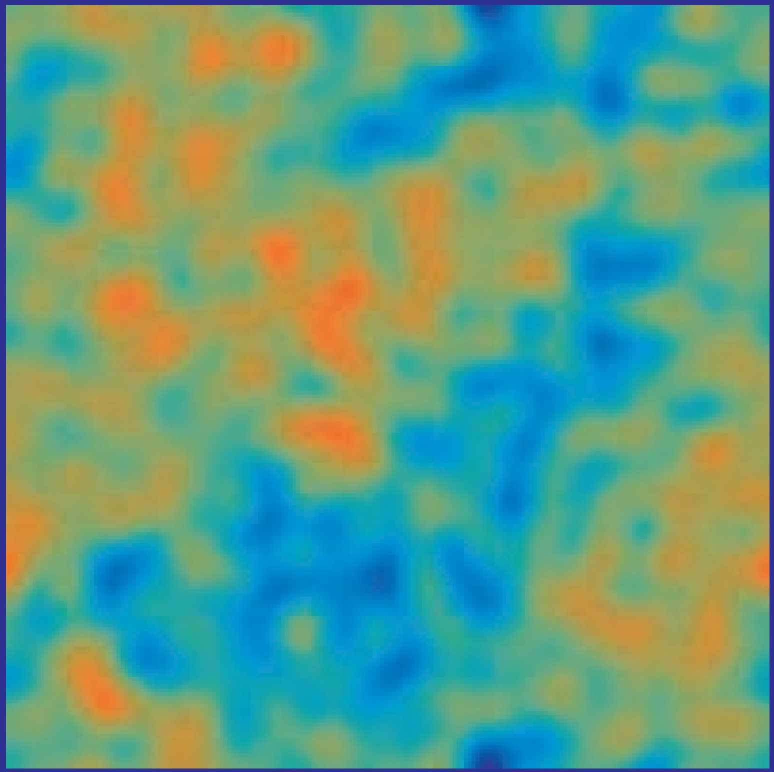
- Warping of the polarization field generates B-modes from E-modes at recombination (100 sq deg.)



Zaldarriaga & Seljak (1999) [figure from Hu & Okamoto (2001)]

Lensing by a Gaussian Random Field

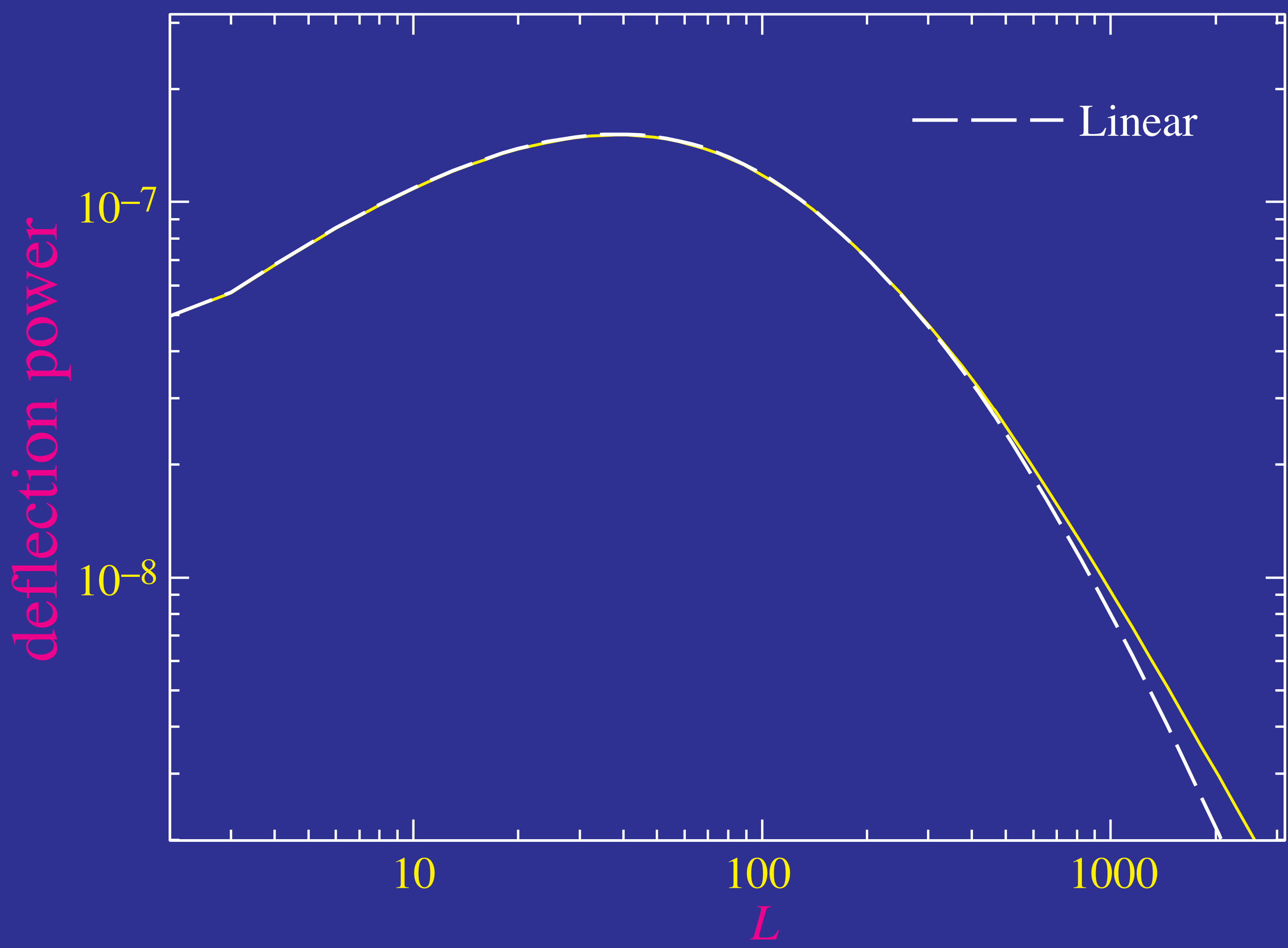
- Mass distribution at large angles and high redshift in in the linear regime
- Projected mass distribution (low pass filtered reflecting deflection angles): 1000 sq. deg



rms deflection
2.6'
deflection coherence
10°

Deflection Power Spectrum

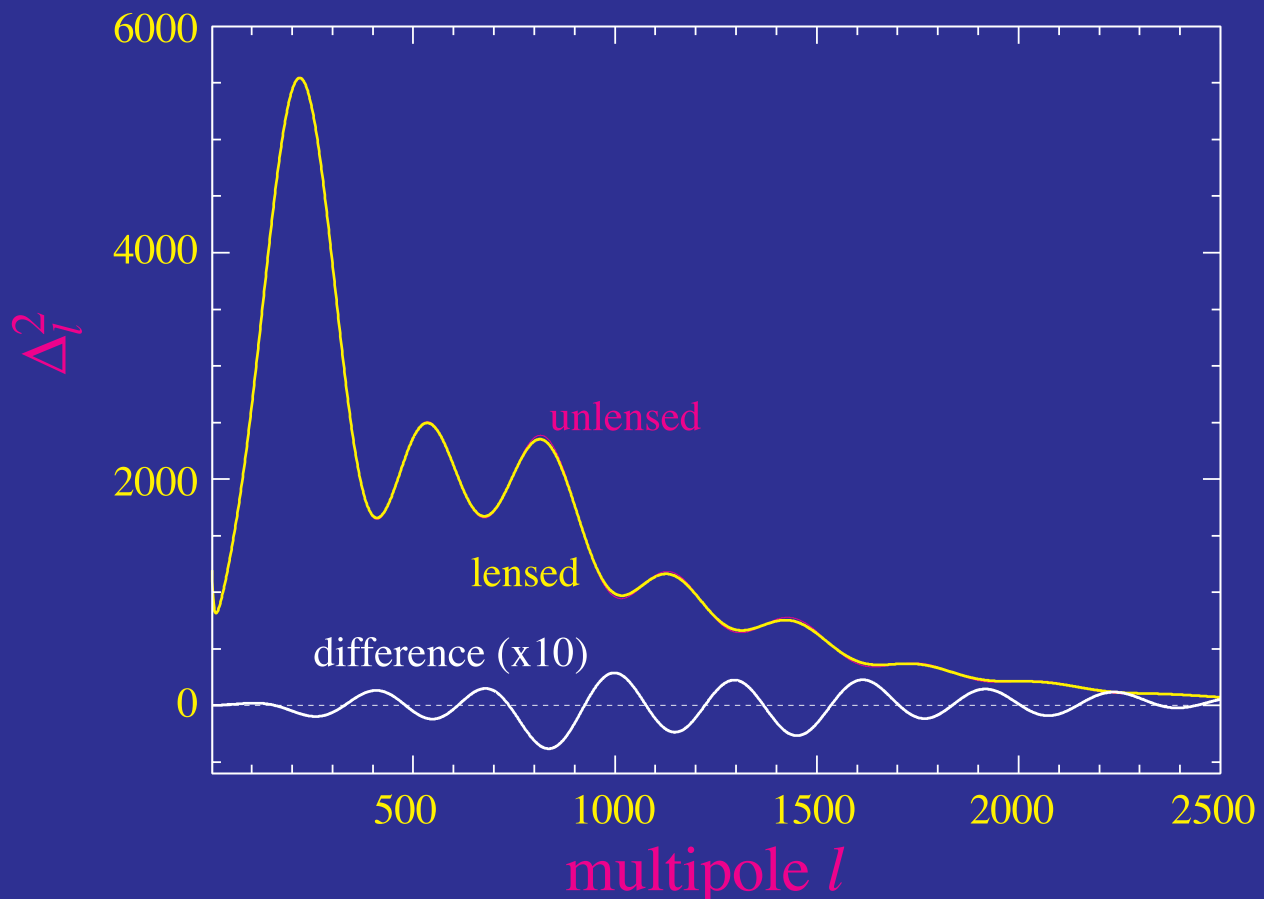
- Fundamental observable is deflection power spectrum (or convergence / l^2)
- Nearly entirely in linear regime



Power Spectrum Observables

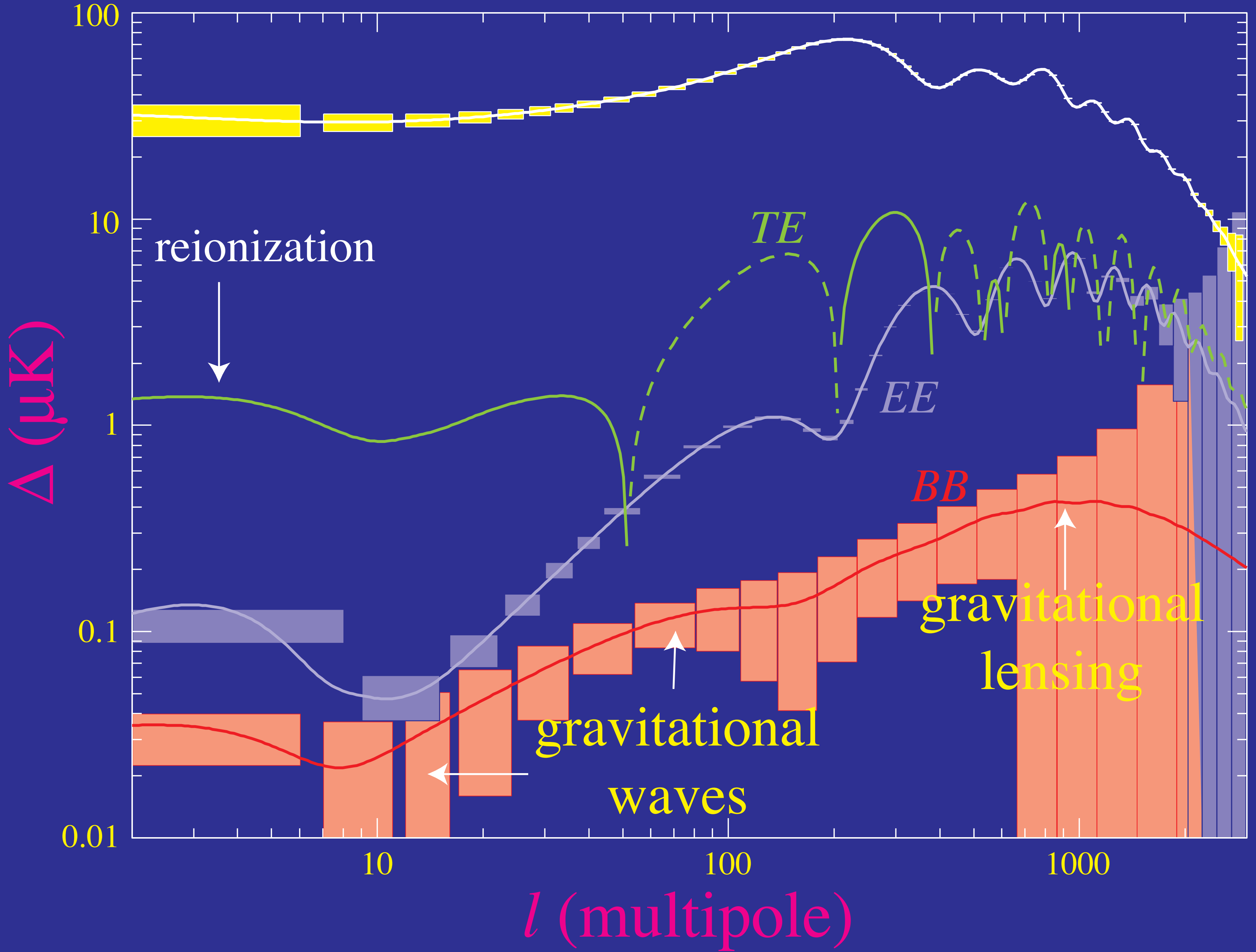
Temperature Power Spectrum

- Lensing acts to smooth the temperature (and E polarization peaks)
- Subtle effect reaches 10% deep in the damping tail
- Statistically detectable at high significance with Planck in the absence of other secondaries and foregrounds



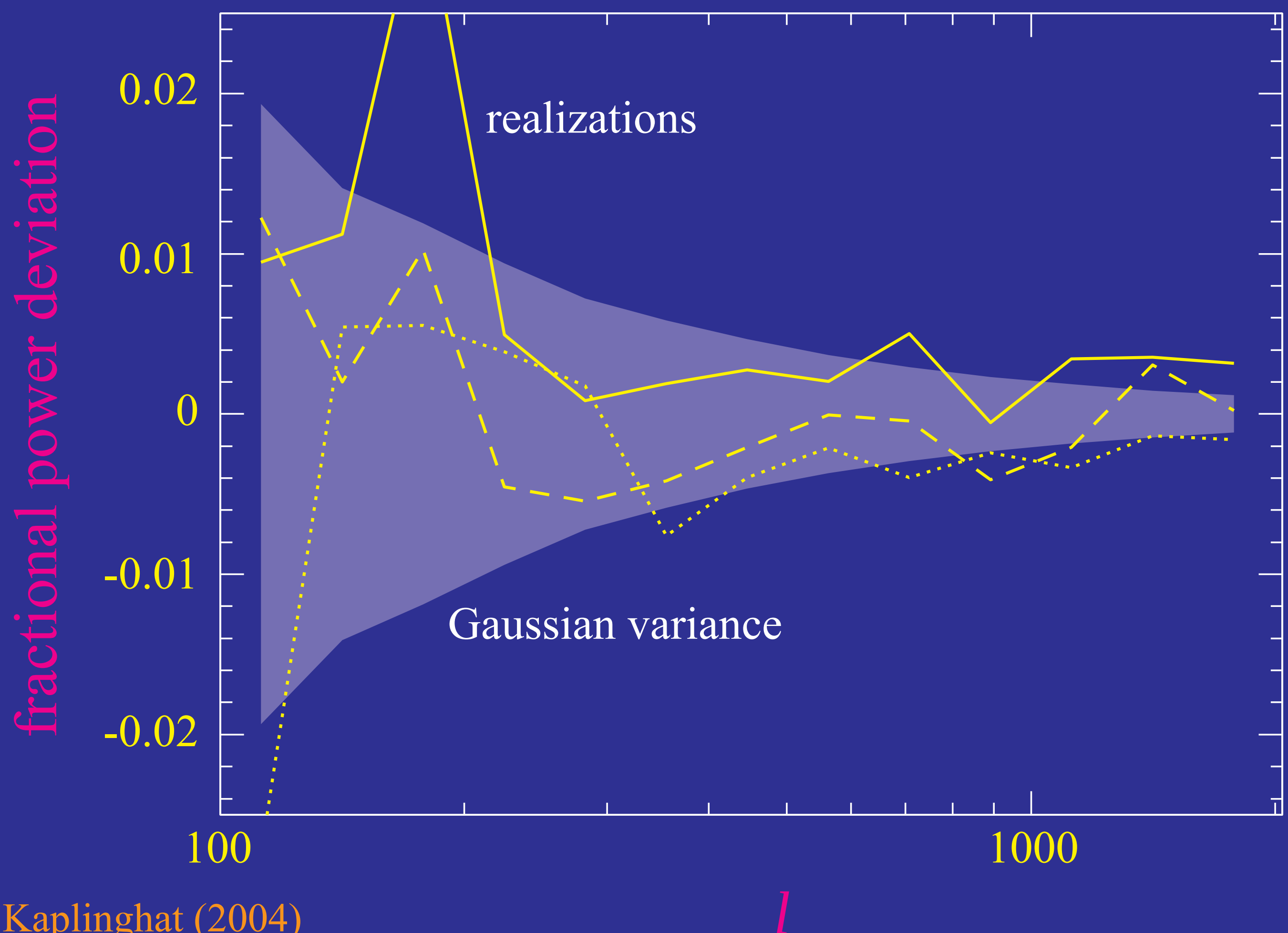
Seljak (1996) [see Challinor & Lewis (2006) for refinements]

Temperature and Polarization Spectra



Power Spectrum Measurements

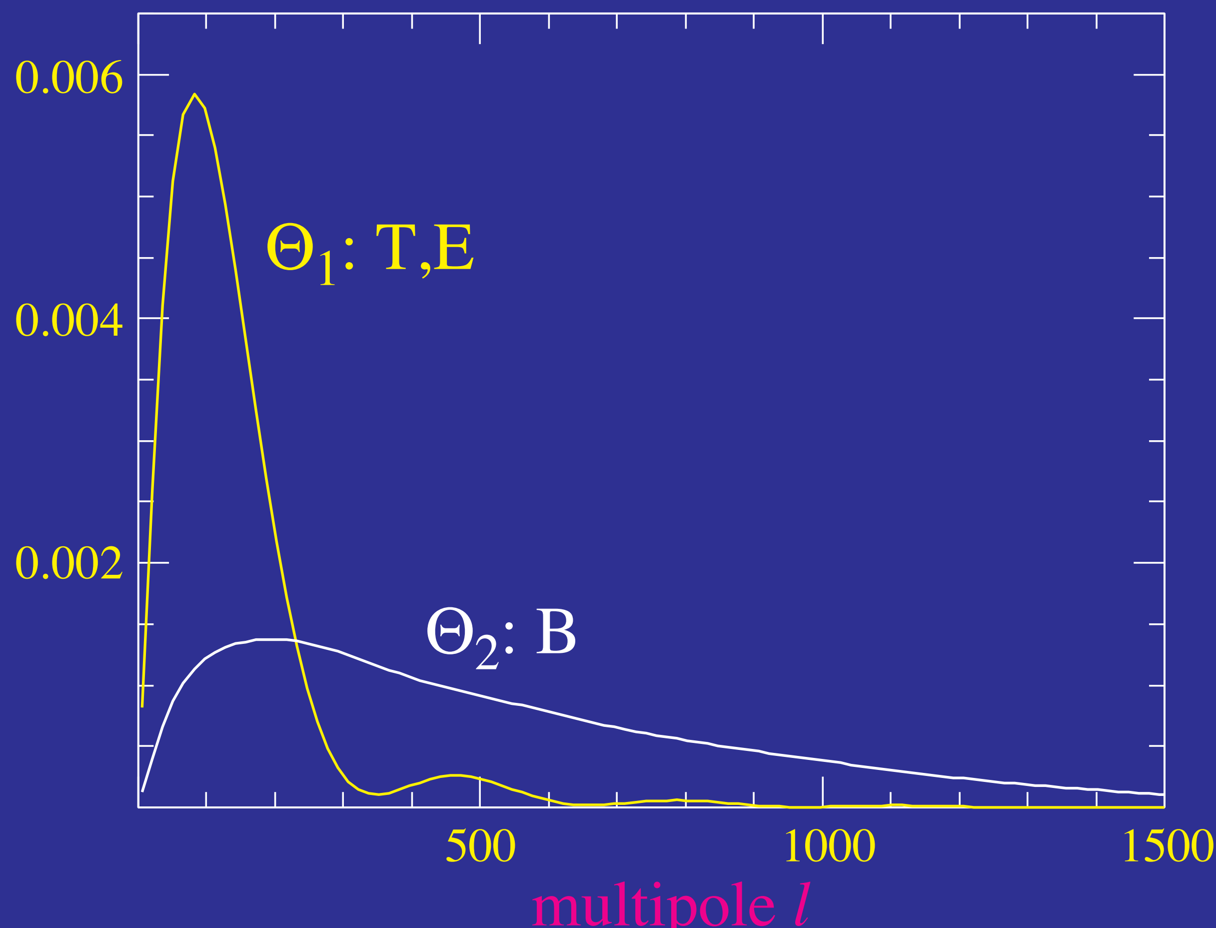
- Lensed field is non-Gaussian in that a single degree scale lens controls the polarization at arcminutes
- Increased variance and covariance implies that 10x as much sky needed compared with Gaussian fields



Smith, Hu & Kaplinghat (2004)

Lensed Power Spectrum Observables

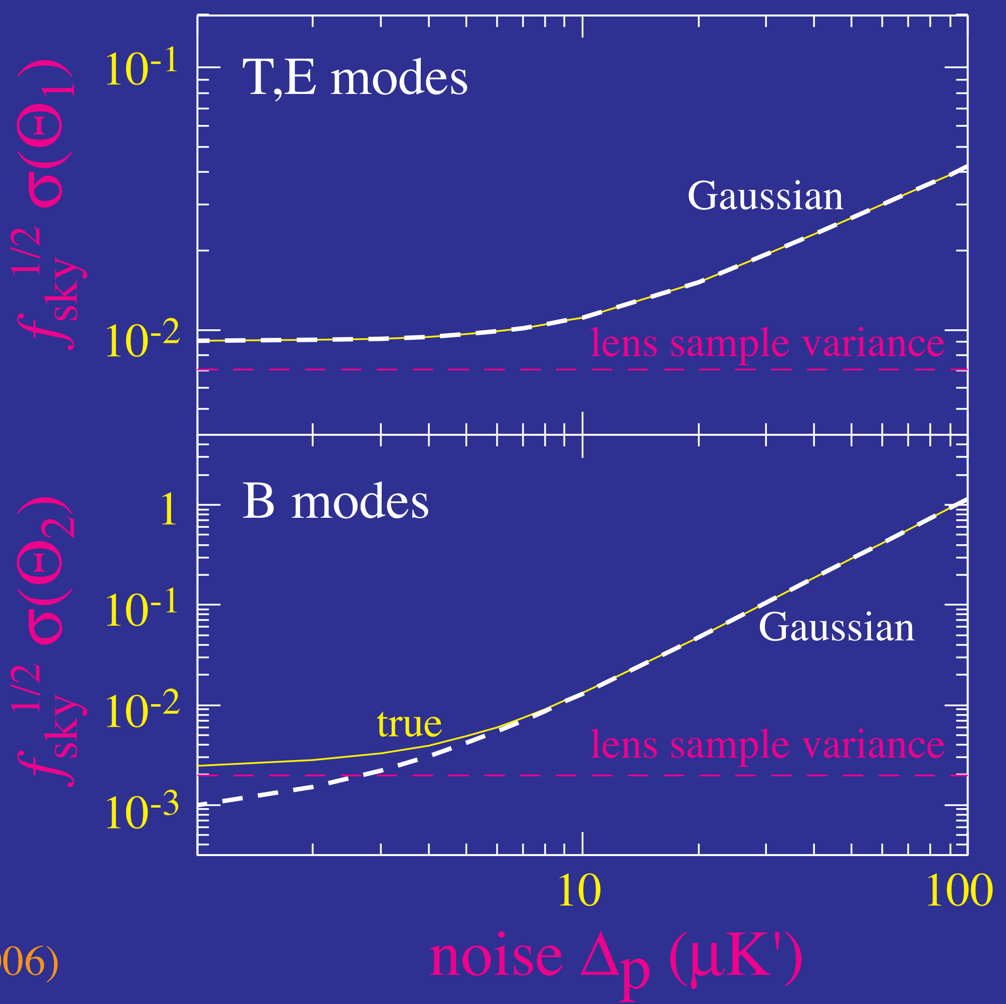
- Principal components show two observables in lensed power spectra
- Temperature and E-polarization: deflection power at l~100
  B-polarization: deflection power at l~500
- Normalized so that observables error = fractional lens power error



Smith, Hu & Kaplinghat (2006)

Constraints on Lensing Observables

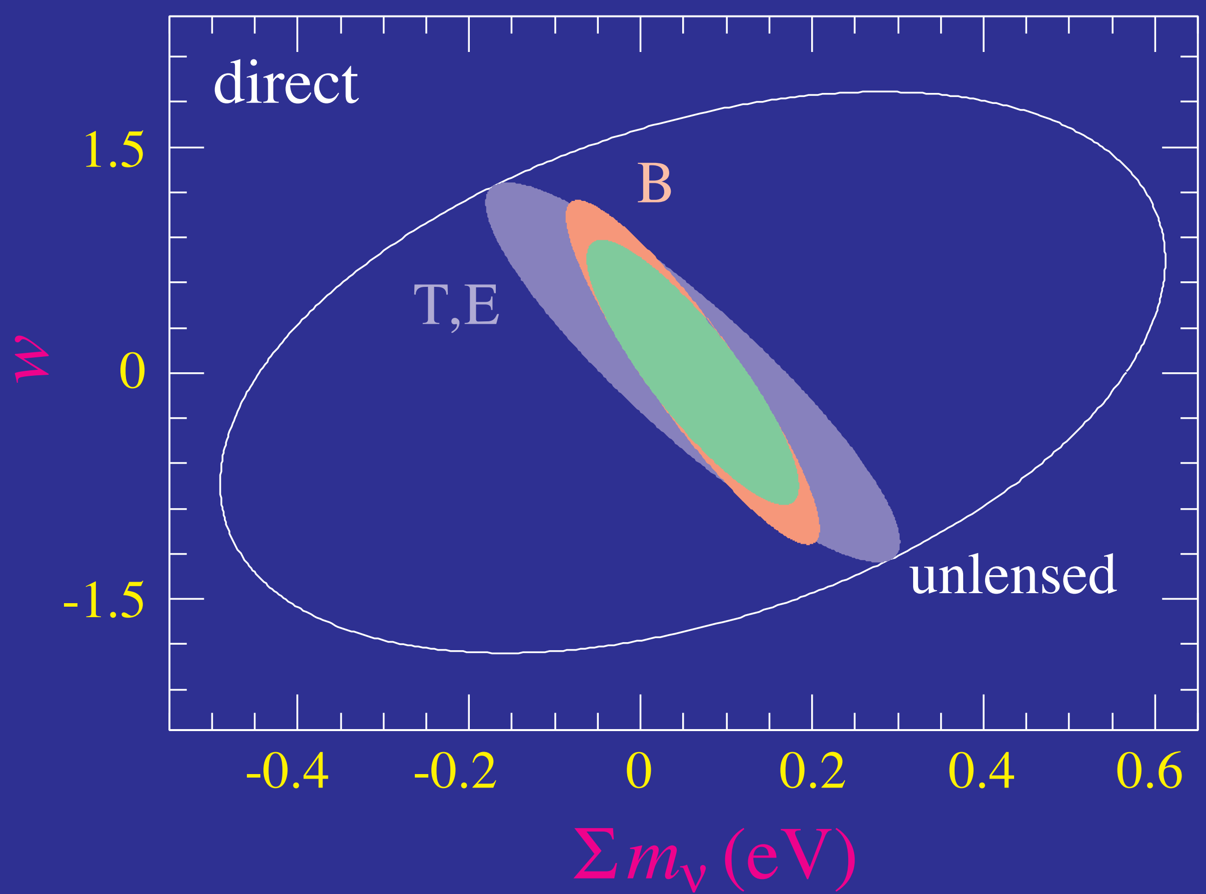
- Lensing observables in T,E are limited by CMB sample variance
- Lensing observables in B are limited by lens sample variance
- B-modes require 10x as much sky at high signal-to-noise or 3x as much sky at the optimal signal-to-noise with Δ_P=4.7uK'



Smith, Hu & Kaplinghat (2006)

Lensing Observables

- Lensing observables provide a simple way of accounting for non-Gaussianity and parameter degeneracies
- Direct forecasts for Planck + 10% sky with noise Δ_P=1.4uK'



Smith, Hu, Kaplinghat (2006)

Lensing Observables

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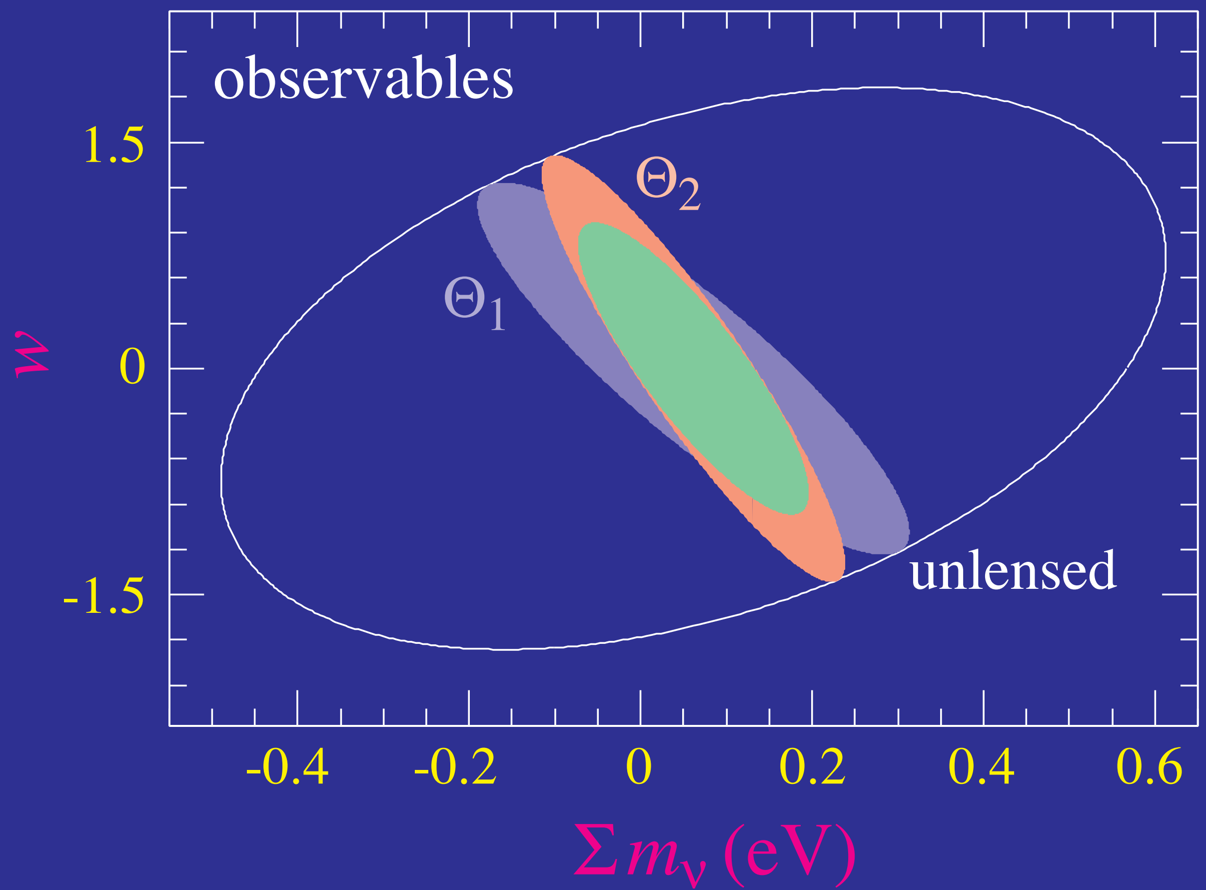


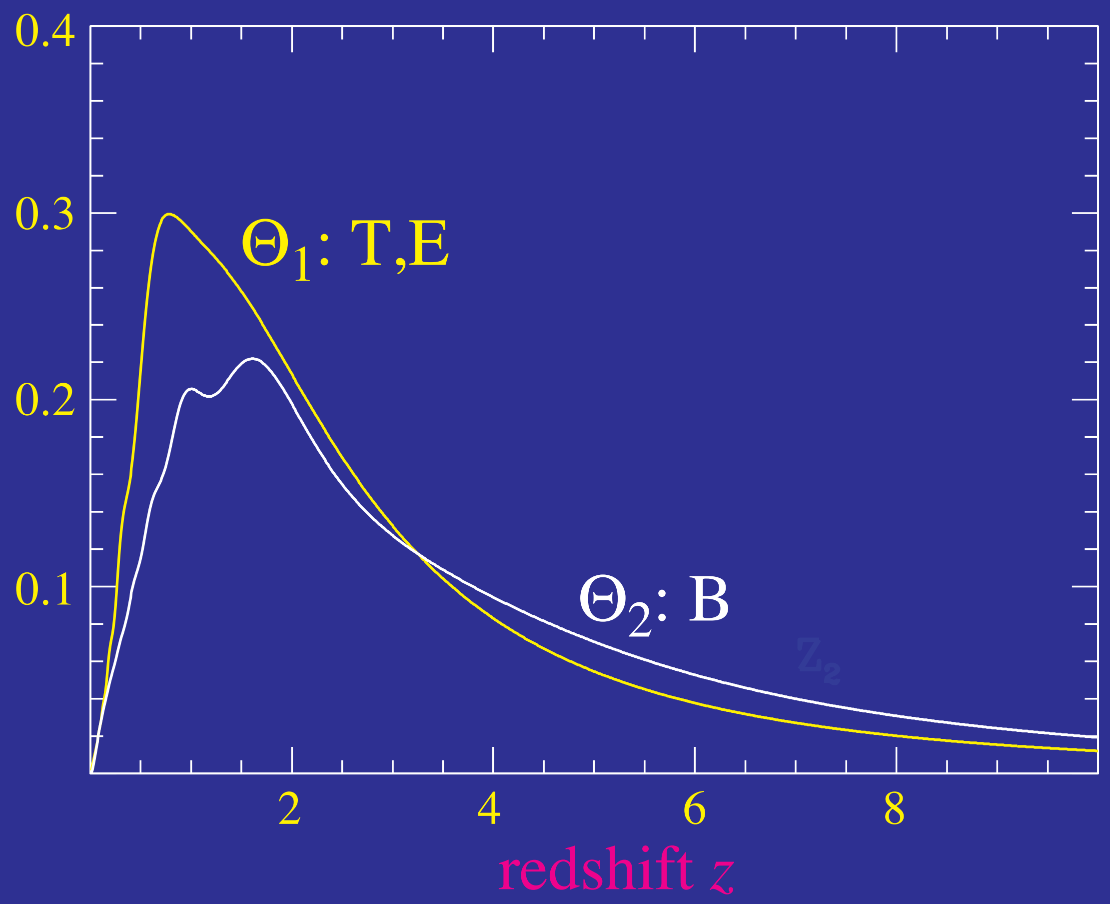
Smith, Hu, Kaplinghat (2006)

Complementarity

Redshift Sensitivity

- Lensing observables probe distance and structure at high redshift

$$\frac{\delta\Theta_i}{\Theta_i} = \left[\left(3 - \frac{d\ln\Delta_m^2}{d\ln k}\right)\frac{\delta D_A}{D_A} - \frac{\delta H}{H} + 2\frac{\delta G}{G} + 2\frac{\delta D_A(D_s - D)}{D_A(D_s - D)}\right]$$



Smith, Hu & Kaplinghat (2006)

What is It Good For

- High redshift weight of CMB lensing: blessing or curse?
- Acceleration/dark energy is mainly a low redshift phenomenon
- Given CMB at recombination, largest deviations in distance and growth is at $z = 0$ - e.g. H_0
- CMB lensing complements low redshift probes by fixing other parameters that confuse measurement - e.g. spatial curvature, neutrinos

Outline:

- why low z probes are good
- why low z probes + 1 or more high z probe are better

Fixed Deceleration Epoch

- CMB determination of matter density controls all determinations in the deceleration (matter dominated) epoch
- Current status: $\Omega_m h^2 = 0.13 \pm \ 0.01 \rightarrow 8\%$
- Distance to recombination D_* determined to $\frac{1}{4} 8\% \quad \approx 2\%$
- Expansion rate during any redshift in the deceleration epoch determined to 8%
- Distance to any redshift in the deceleration epoch determined as

$$D(z) = D_* - \int_z^{z_*} \frac{dz}{H(z)}$$

- Volumes determined by a combination $dV = D_A^2 d\Omega dz / H(z)$
- Structure also determined by growth of fluctuations from z_*
- $\Omega_m h^2$ can be determined to $\sim 1\%$ in the future.

Value of Local Measurements

- With high redshifts fixed, the largest deviations from the dark energy appear at low redshift $z \sim 0$
- By the Friedman equation $H^2 \propto \rho$ and difference between $H(z)$ extrapolated from the CMB $H_0 = 36$ and 73 is entirely due to the dark energy in a flat universe
- With the dark energy density fixed by H_0, the deviation from the CMB observed D_* from the ΛCDM prediction measures the equation of state (or evolution of the dark energy density)

$$p_{\rm DE} = w\rho_{\rm DE}$$

- Intermediate redshift dark energy probes can then test flatness assumption and the evolution of the equation of state: e.g.

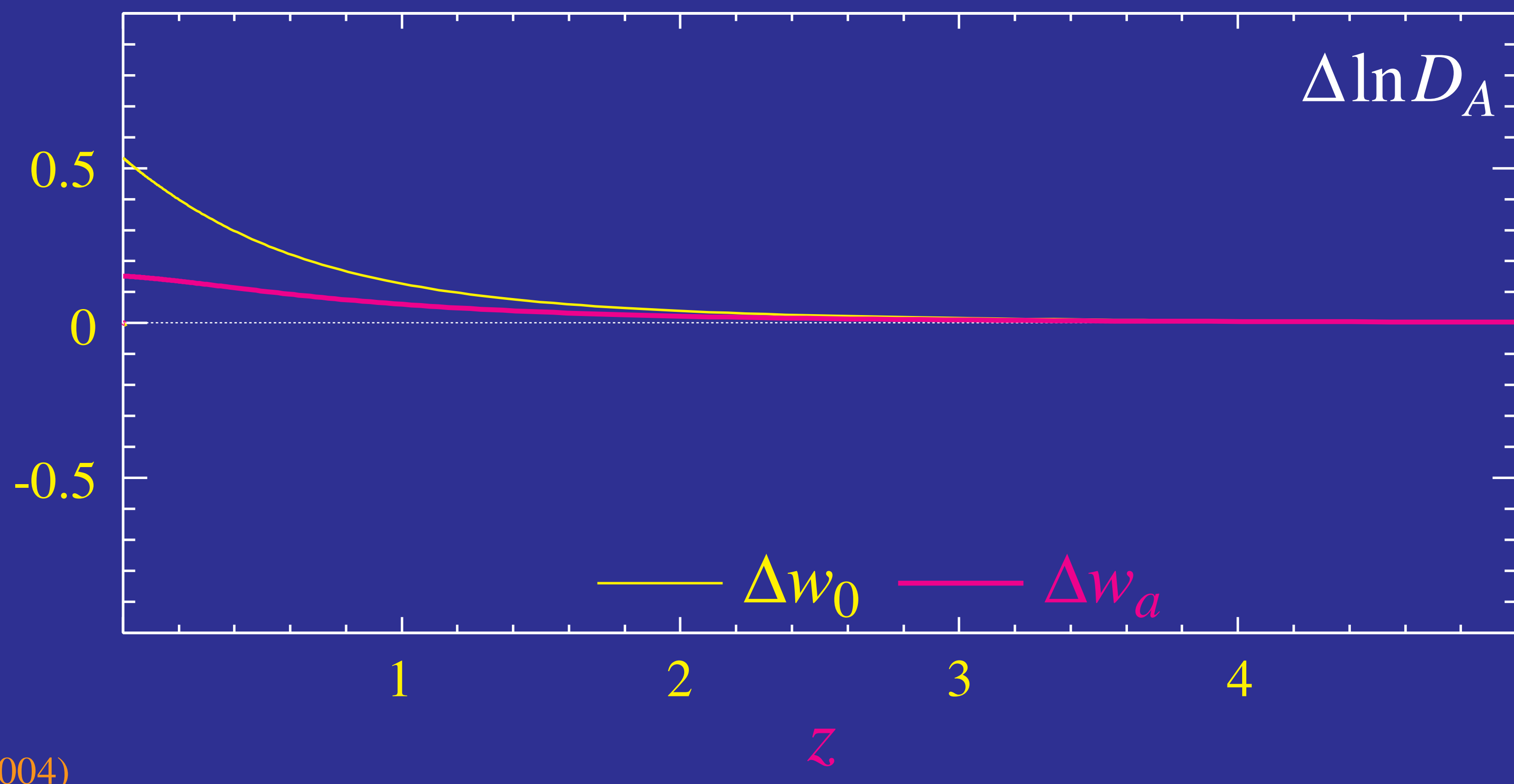
$$w(a) = w_0 + (1-a)w_a$$

Dogma and Heresies

- Pivot $w(a_p) \equiv w_p$ is equation of state at the redshift that is best constrained
- Can be constructed from $w_0 - w_a$ but also equivalent to the first principal component of $w(a)$
- $\sigma(w_p)$ quantifies an experiments ability to test cosmological constant $w = -1$ at all z - also equal to $\sigma(w_0)$ for $w_a = 0$
- w_a acts as second principal component: measures evolution in equation of state around a_p: but is best measured not necessarily most interesting to measure!
- In testing the specific predictions of flat ΛCDM assuming spatial flatness while testing w and $w = -1$ when testing flatness justified
- If deviations from flat ΛCDM are measured then important to distinguish dynamical dark energy from a small spatial curvature

Curvature, H_0 and Dark Energy

- CMB peaks fix distance to recombination and $\Omega_m h^2$
- Deviations from ΛCDM distance at low z indicate dark energy equation of state $w = w_0 - (1-a)w_a \neq -1$ if universe is flat
- Maximal at $z=0$: Hubble constant



Hu (2004)

Redshift Sensitivity

- Lensing observables probe distance and structure at high redshift

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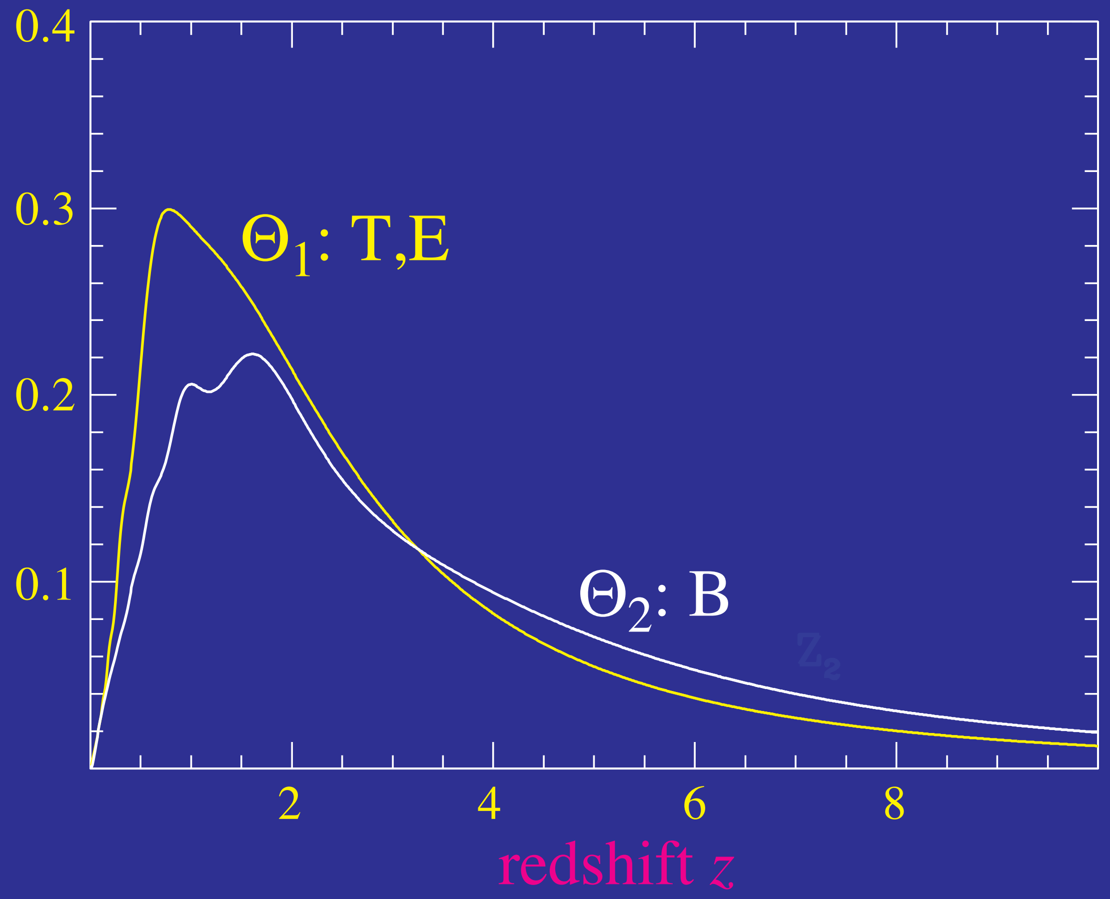
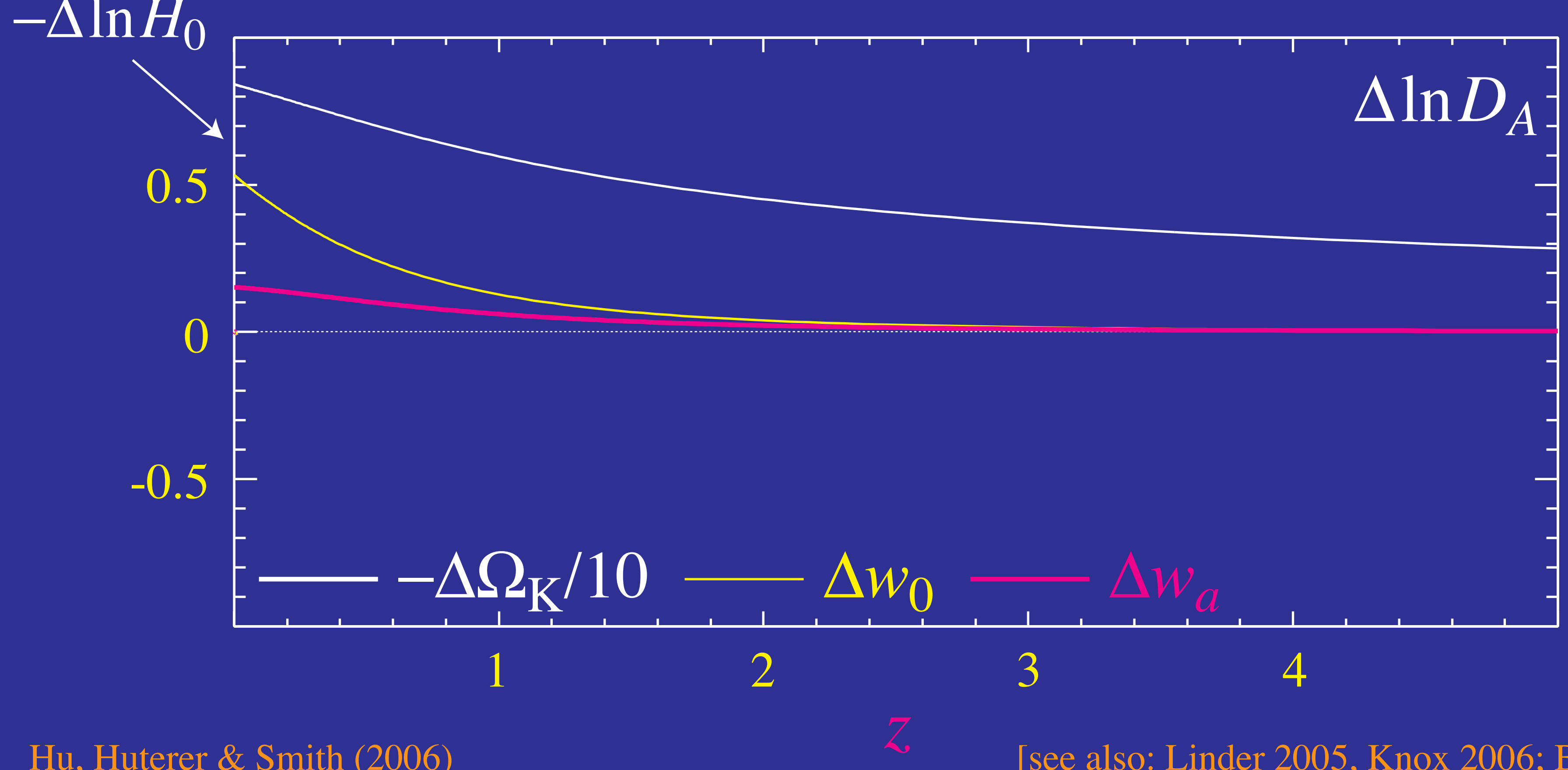


Smith, Hu & Kaplinghat (2006)

Curvature, H_0 and Dark Energy

- CMB peaks fix distance to recombination and $\Omega_m h^2$
- Deviations from ΛCDM distance at low z indicate either spatial curvature or dark energy equation of state $w=w_0-(1-a)w_a\neq-1$
- Allowing H_0 to measure the dark energy

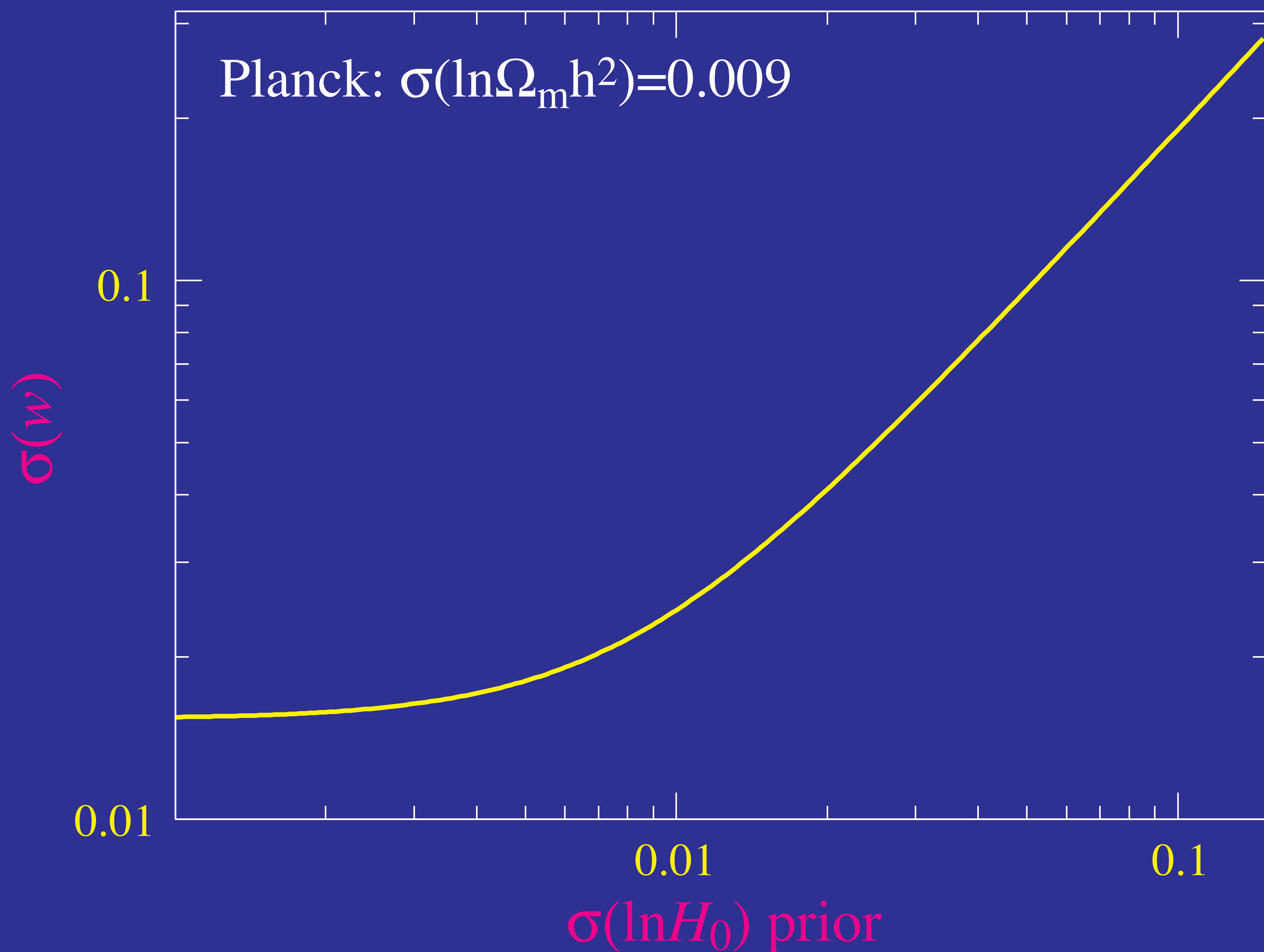


Hu, Huterer & Smith (2006)

[see also: Linder 2005, Knox 2006; Bernstein 2006]

Forecasts for CMB+H_0

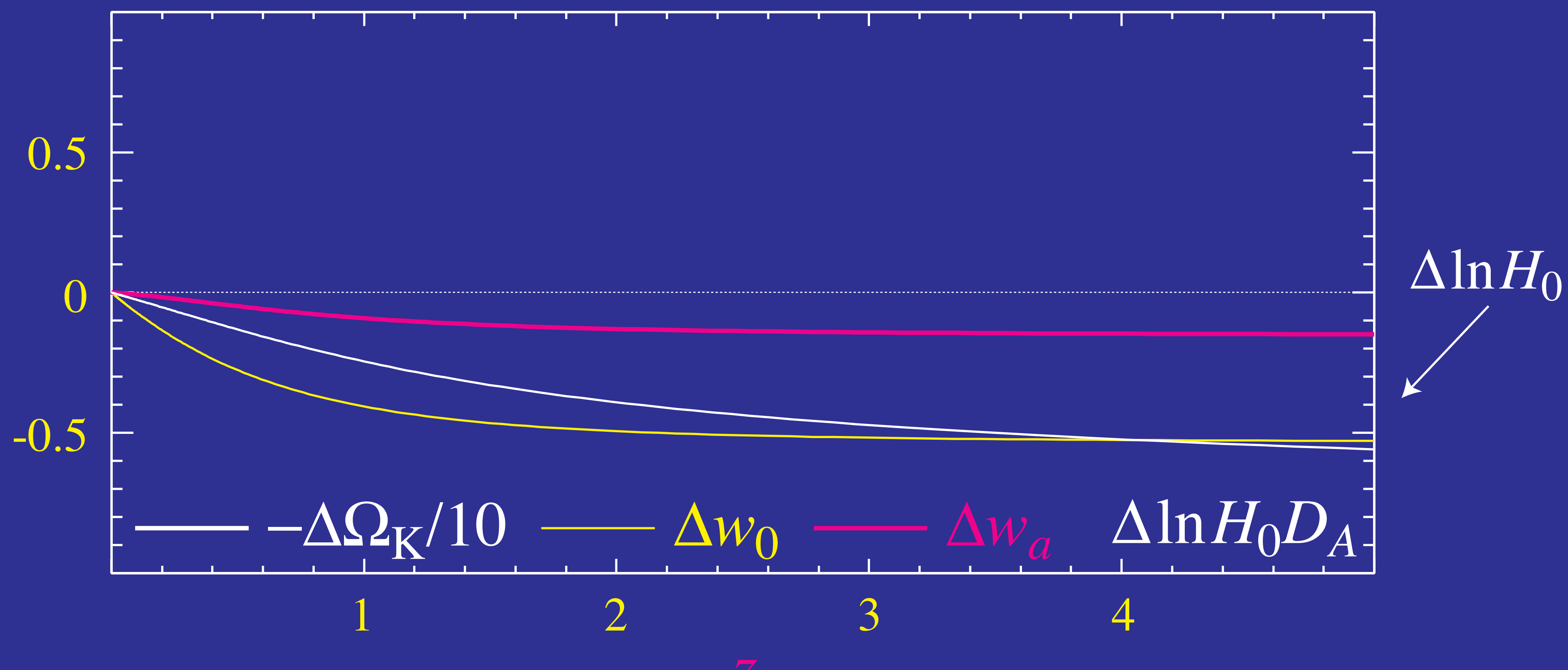
- To complement CMB observations with $\Omega_m h^2$ to 1%, an H_0 of ~1% enables constant w measurement to ~2% in a flat universe



Hu (2004)

Curvature, H_0 and Dark Energy

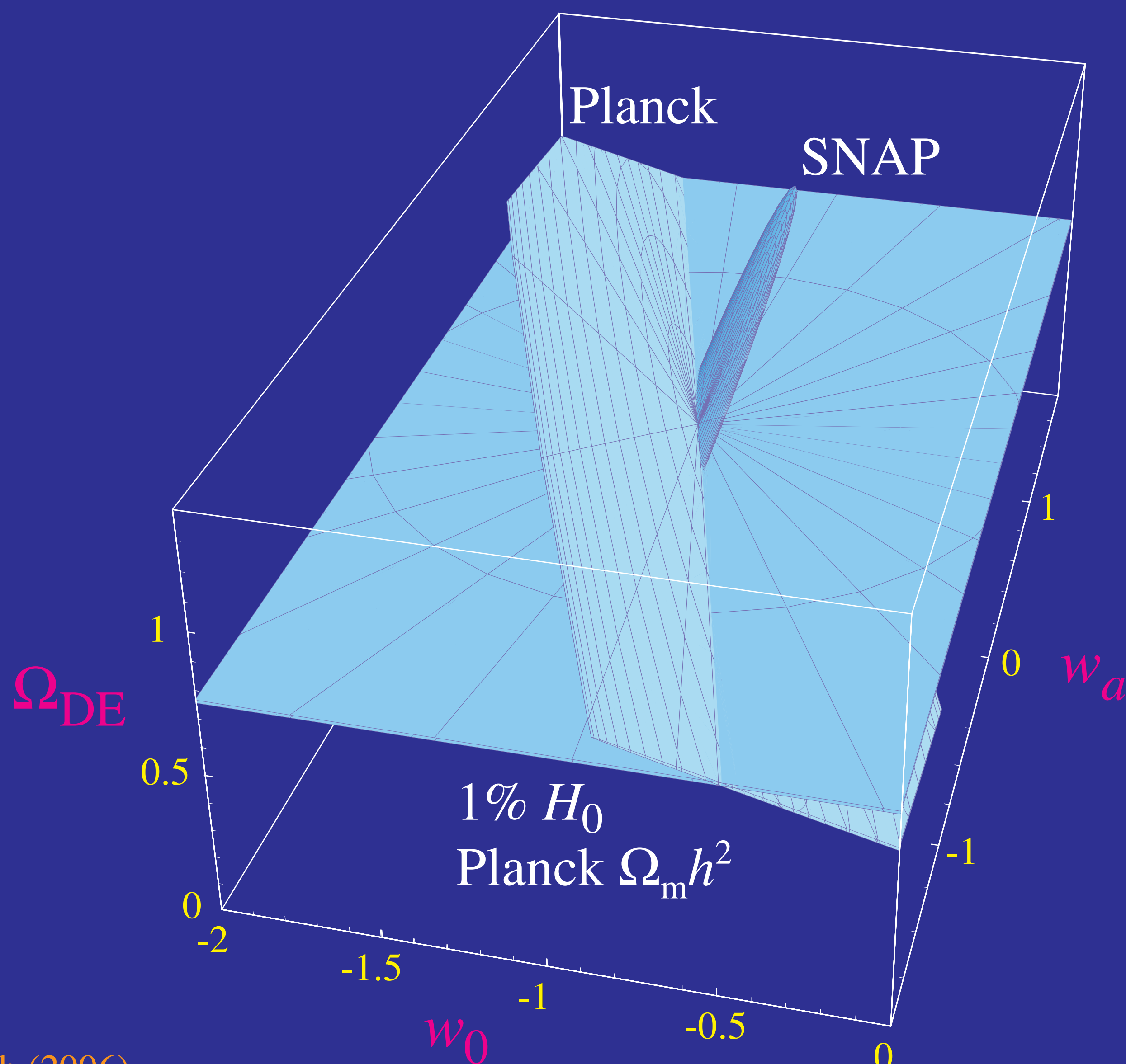
- CMB peaks fix distance to recombination and $\Omega_m h^2$
- Deviations from ΛCDM distance at low z indicate either spatial curvature or dark energy equation of state $w=w_0-(1-a)w_a\neq-1$
- SNIa relative distance measurements to measure (w_0, w_a)
- (Alternately H_0 can remove the curvature degeneracy)



Hu, Huterer & Smith (2006)

Flat Universe Precision

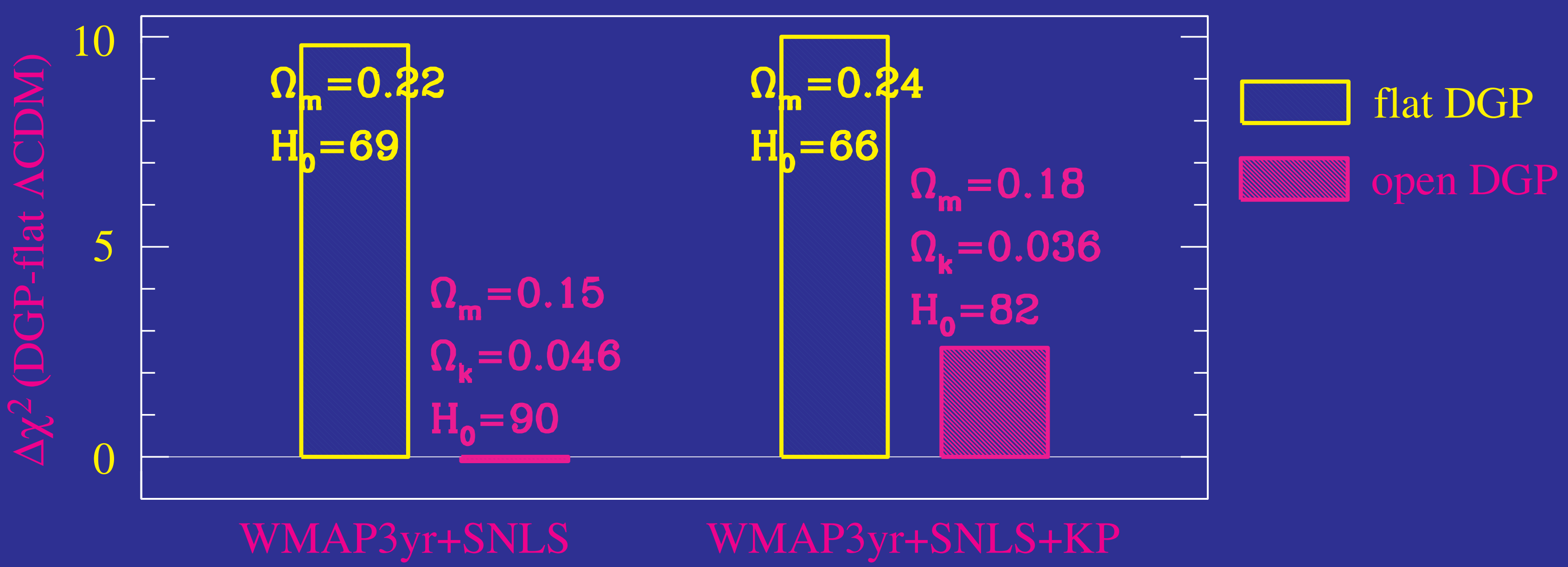
- Planck acoustic peaks, 1% H_0, SNAP SNe to z=1.7 in a flat universe



Hu, Huterer & Smith (2006)

DGP Example

- DGP modified gravity is in tension with distance measures alone: CMB & SNe distances cannot be jointly satisfied in a flat universe
- Even fitting out curvature, Hubble constant is too high for Key Project measurement (and baryon oscillations)
- Joint maximization leads to a poorer fit even with extra curvature parameter



Song, Sawicki & Hu (2006) [also Fairbairn & Goobar 2005, Maartens & Majerotto 2006]

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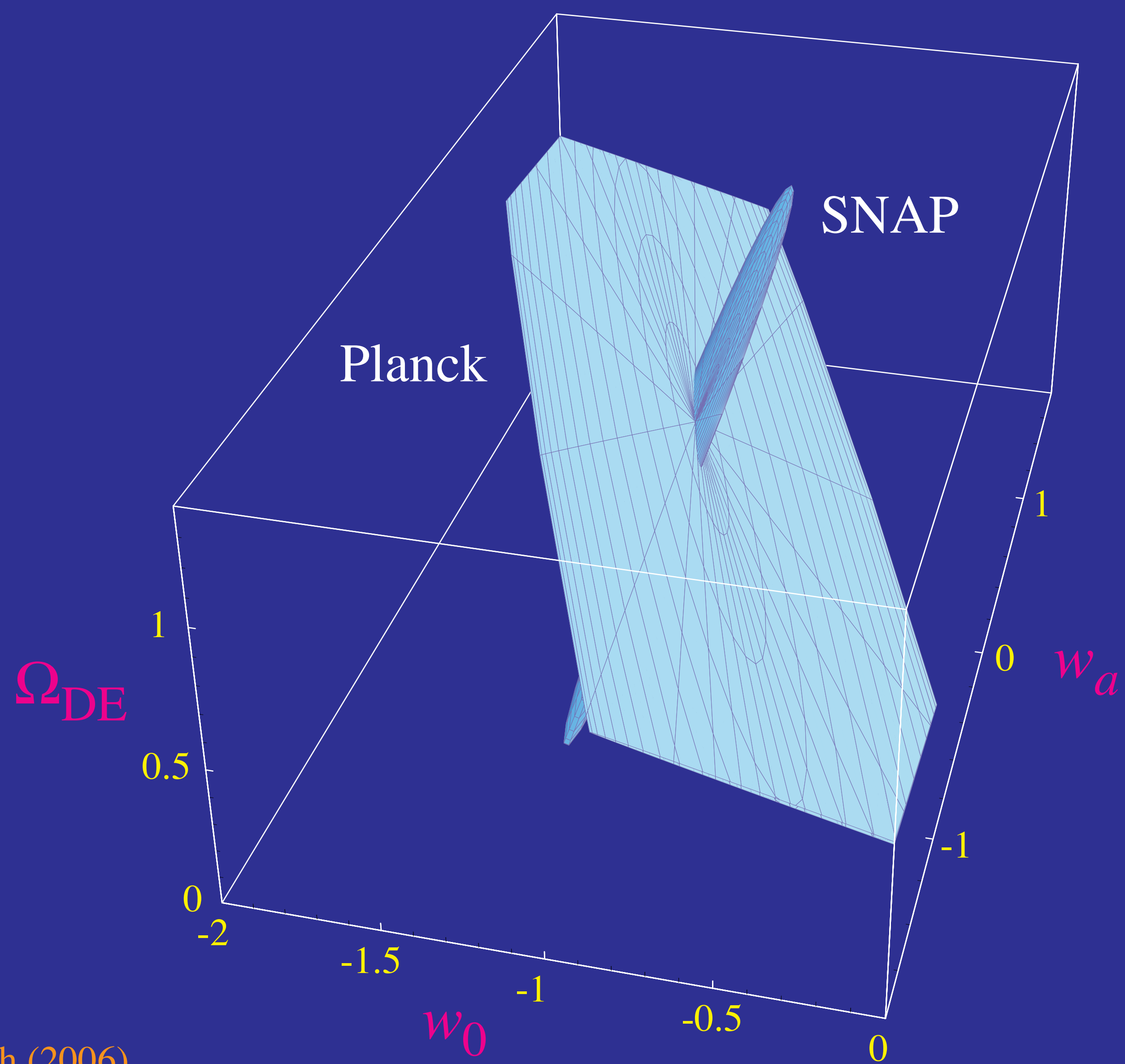
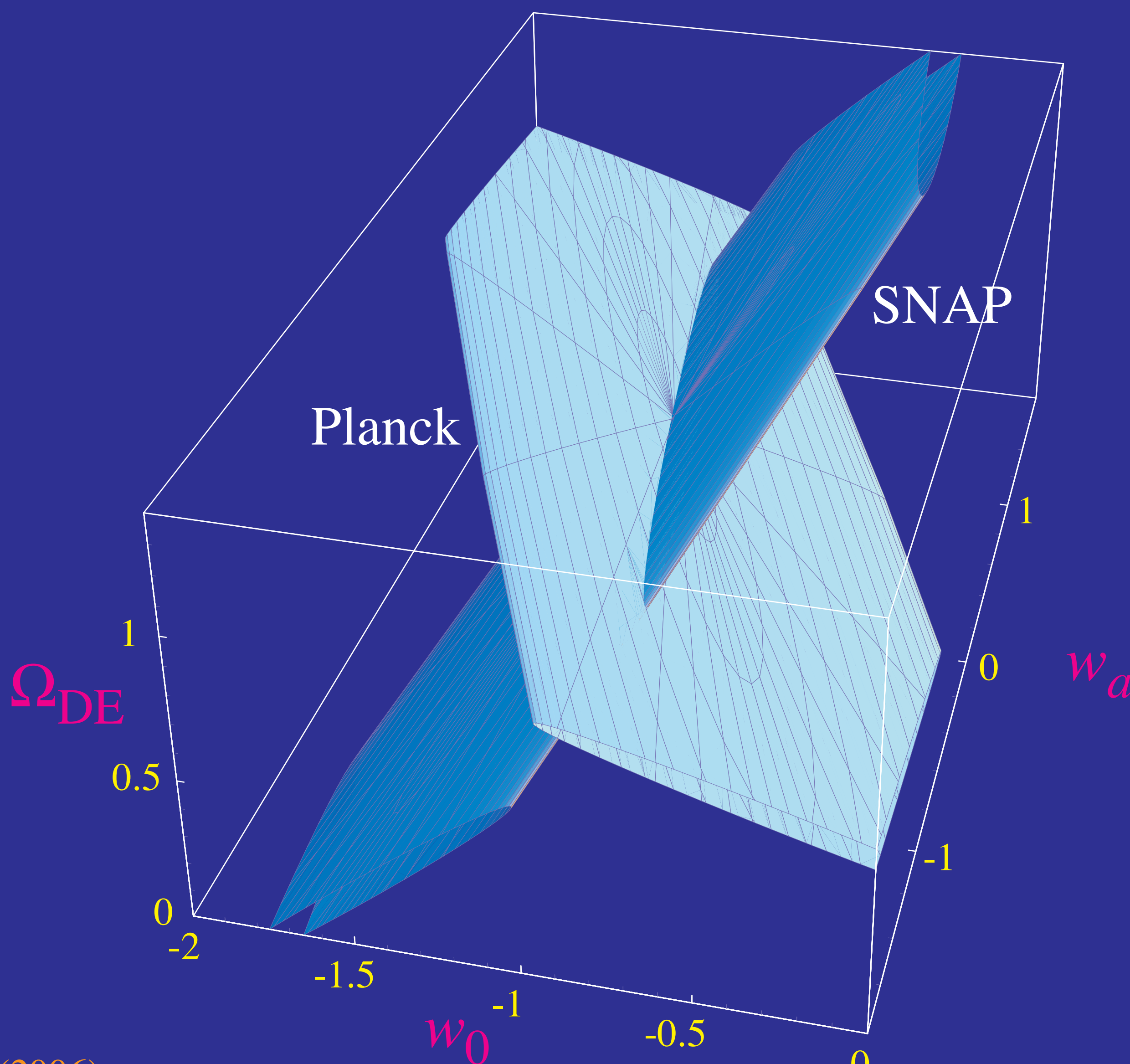


Hu, Huterer & Smith (2006)

Marginalizing Curvature w/o Lensing

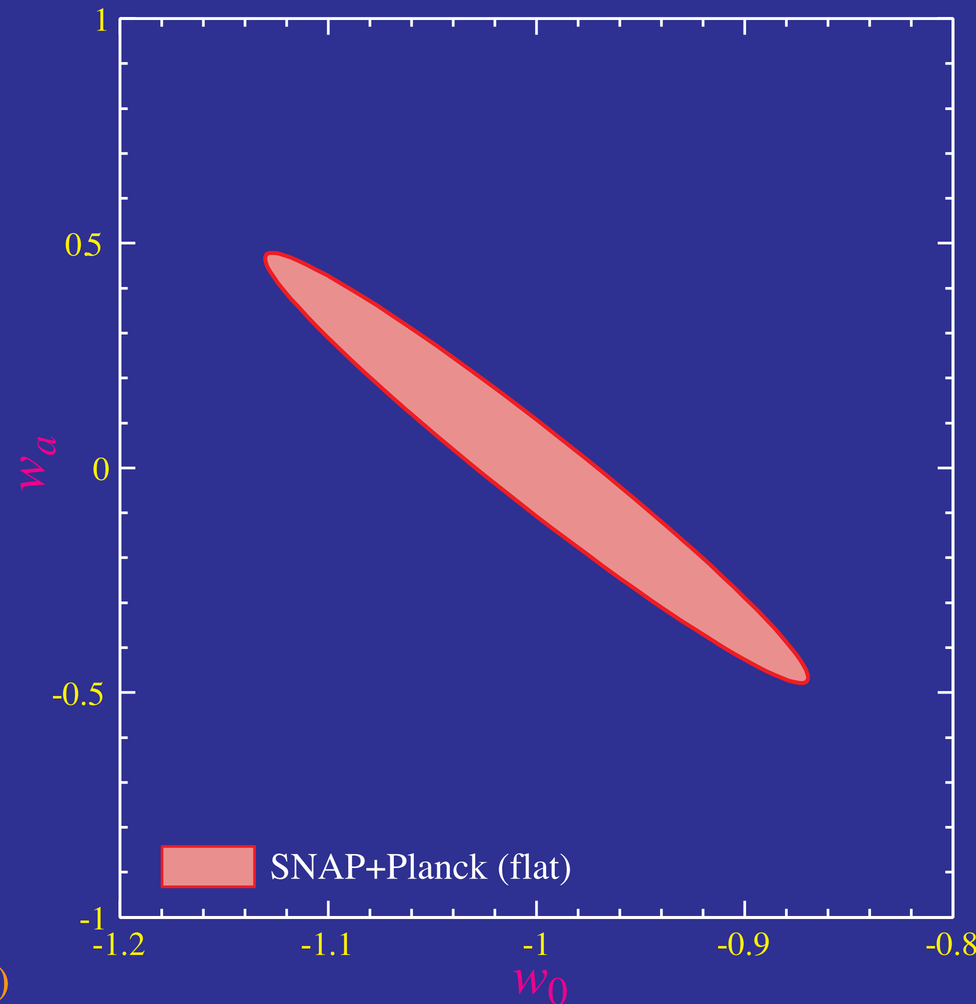
- Marginalizing curvature acts as a superposition of error ellipses



Hu, Huterer & Smith (2006)

Dark Energy Equation of State

- Marginalizing curvature degrades 68% CL area by 4.8
- CMB lensing information from SPTpol (~3% B-mode power) fully restores constraints



Hu, Huterer & Smith (2006)

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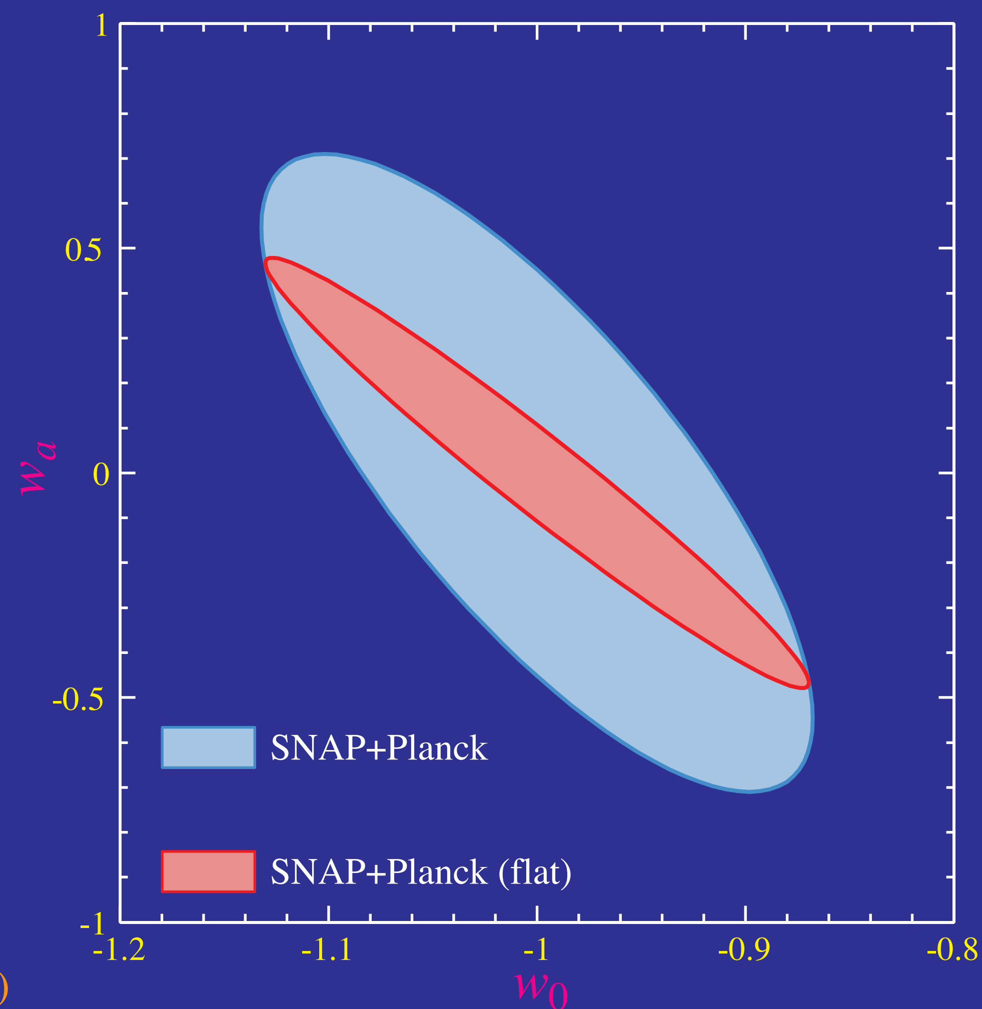

Hu, Huterer & Smith (2006)

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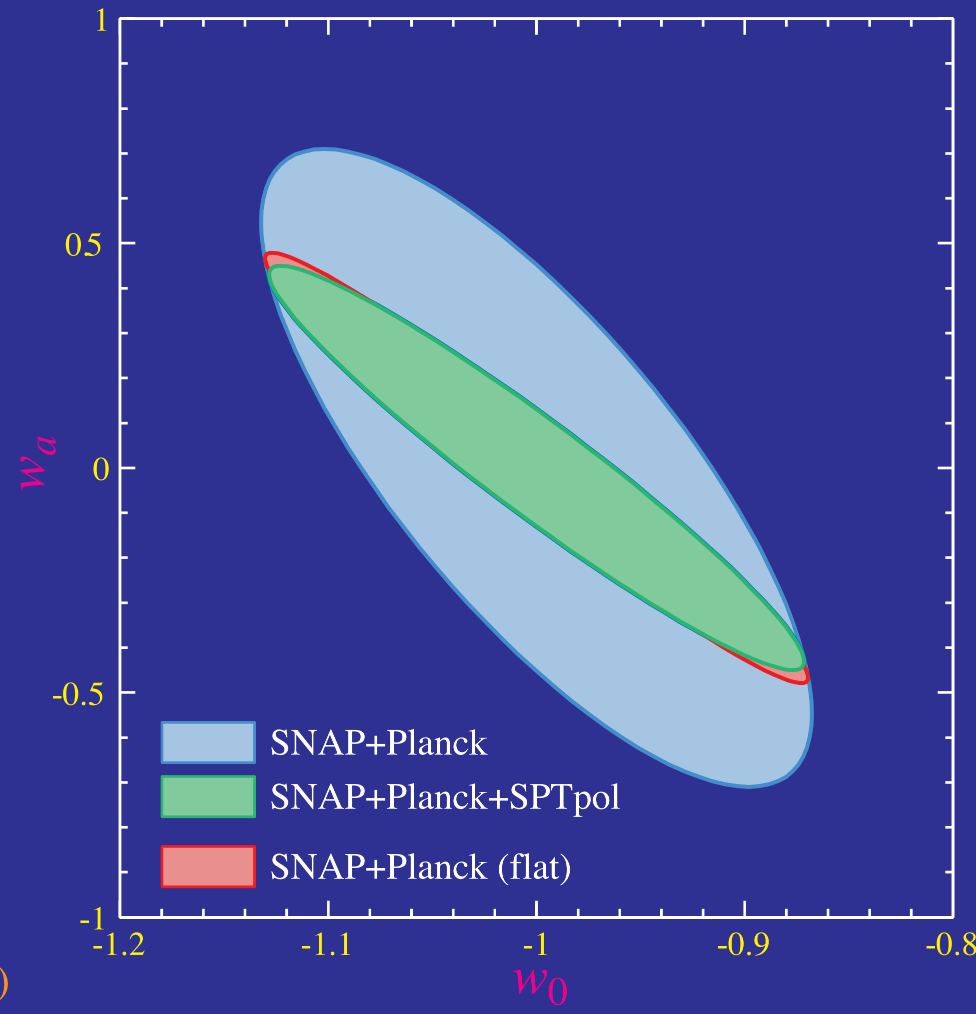
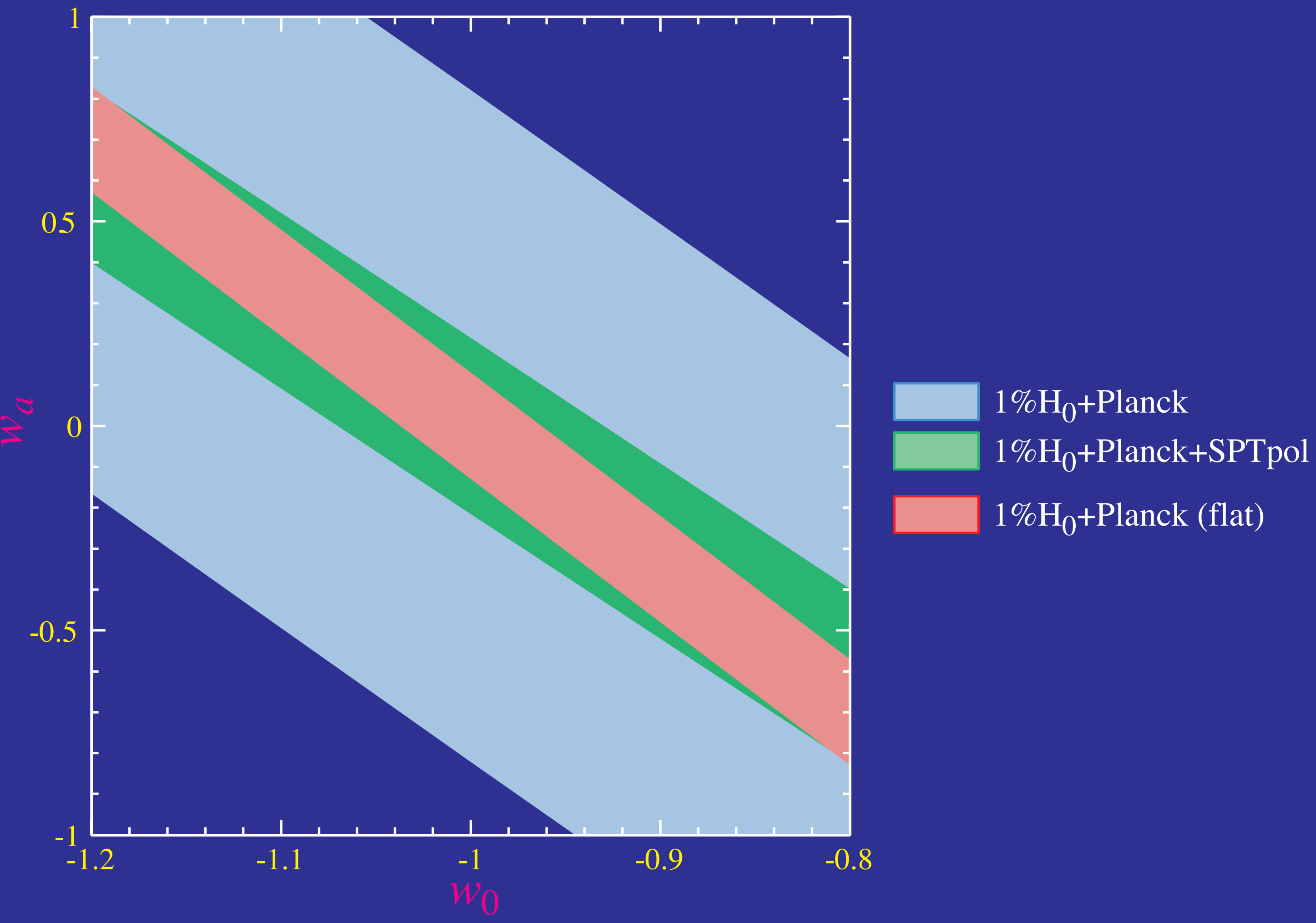


Hu, Huterer & Smith (2006)

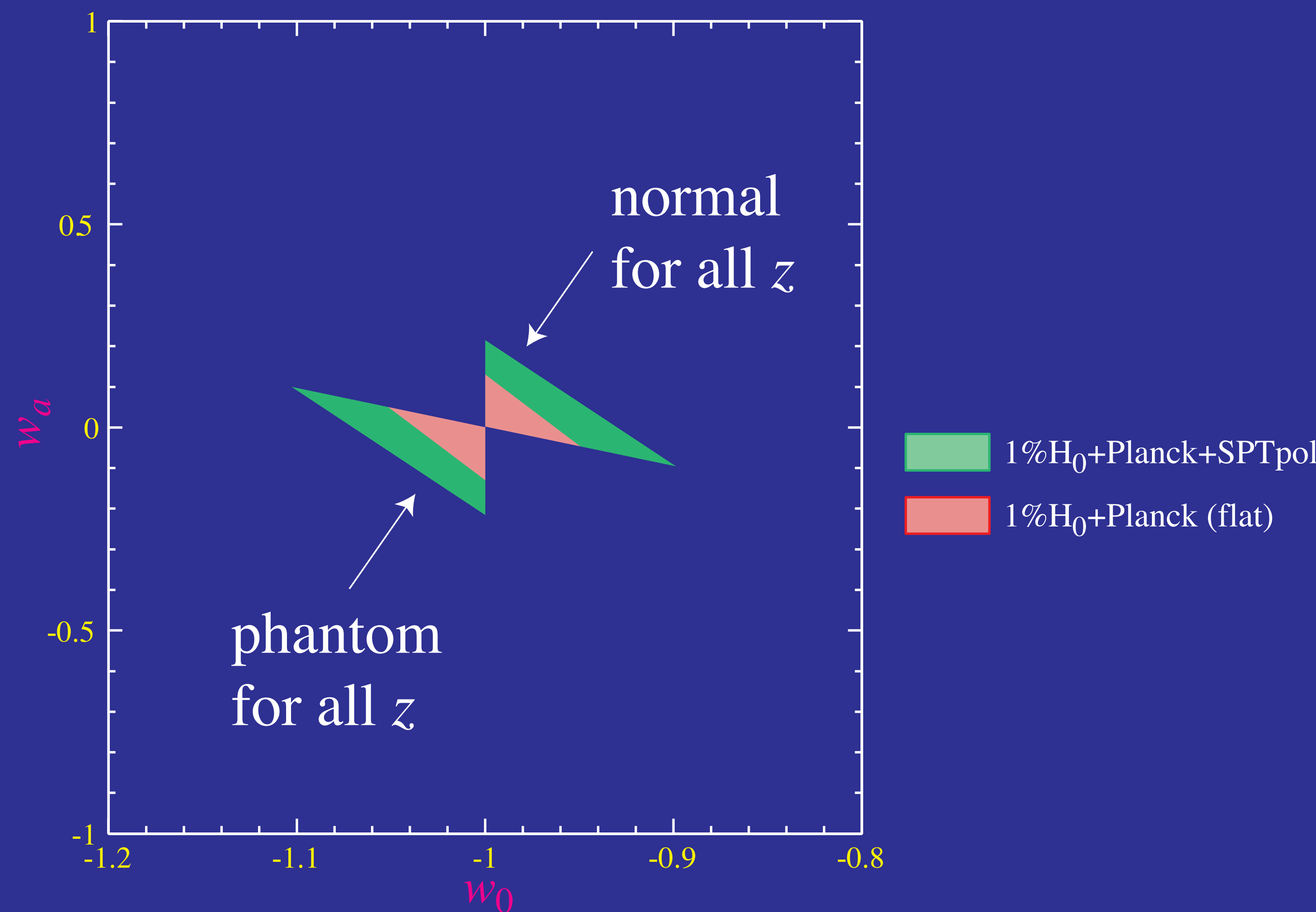
Dark Energy from Percent H_0

- Marginalizing curvature degrades 68% CL area by 7.4
- CMB lensing information from SPTpol (~3% B-mode power) largely restores constraints and yields $\sigma(w_p)$=0.05 vs $\sigma(w_p)$=0.025
- Excellent consistency test for SNe



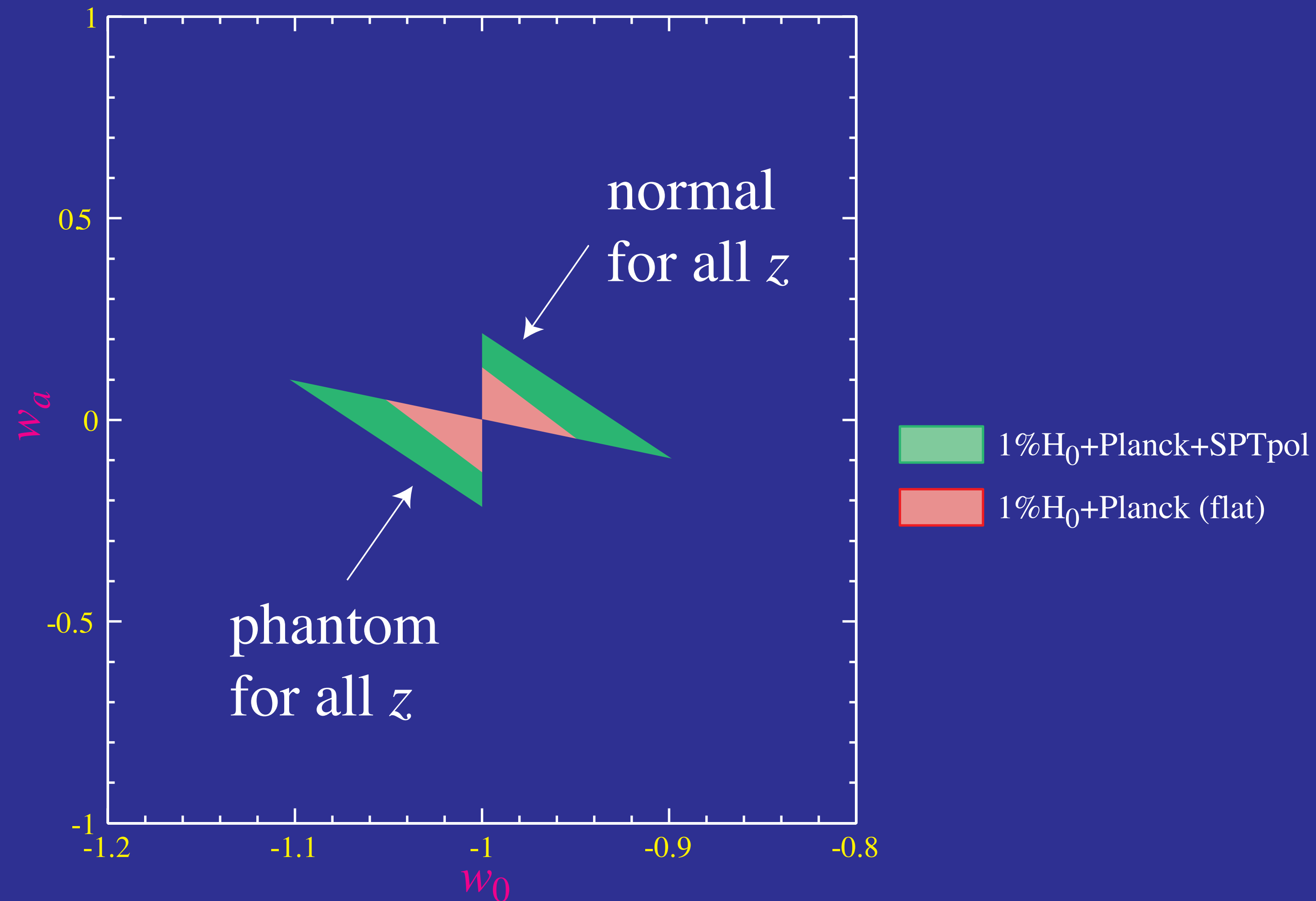
Crossing the Phantom Divide

- If constraints remained consistent with a cosmological constant most of the allowed space requires an evolution across w=-1
- A single scalar field with potential and kinetic degrees of freedom only cannot evolve stably across this divide Hu (2004)



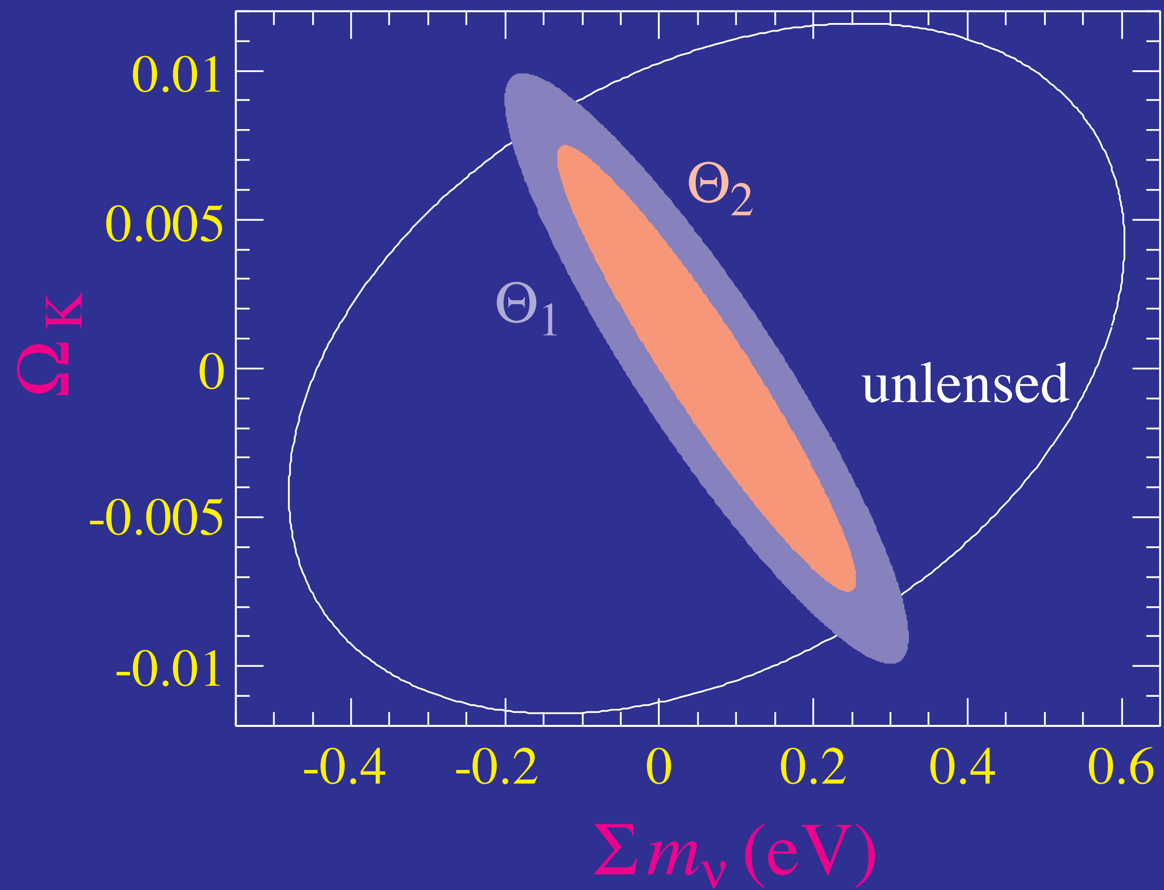
Crossing the Phantom Divide

- For substantial deviations, dark energy has multiple internal degrees of freedom (e.g. multiple fields, higher order derivatives...) or gravity modified
- In a scalar field context, w_0-w_a or low redshift deviations may not be the right figure of merit



Degeneracy with Massive Neutrinos

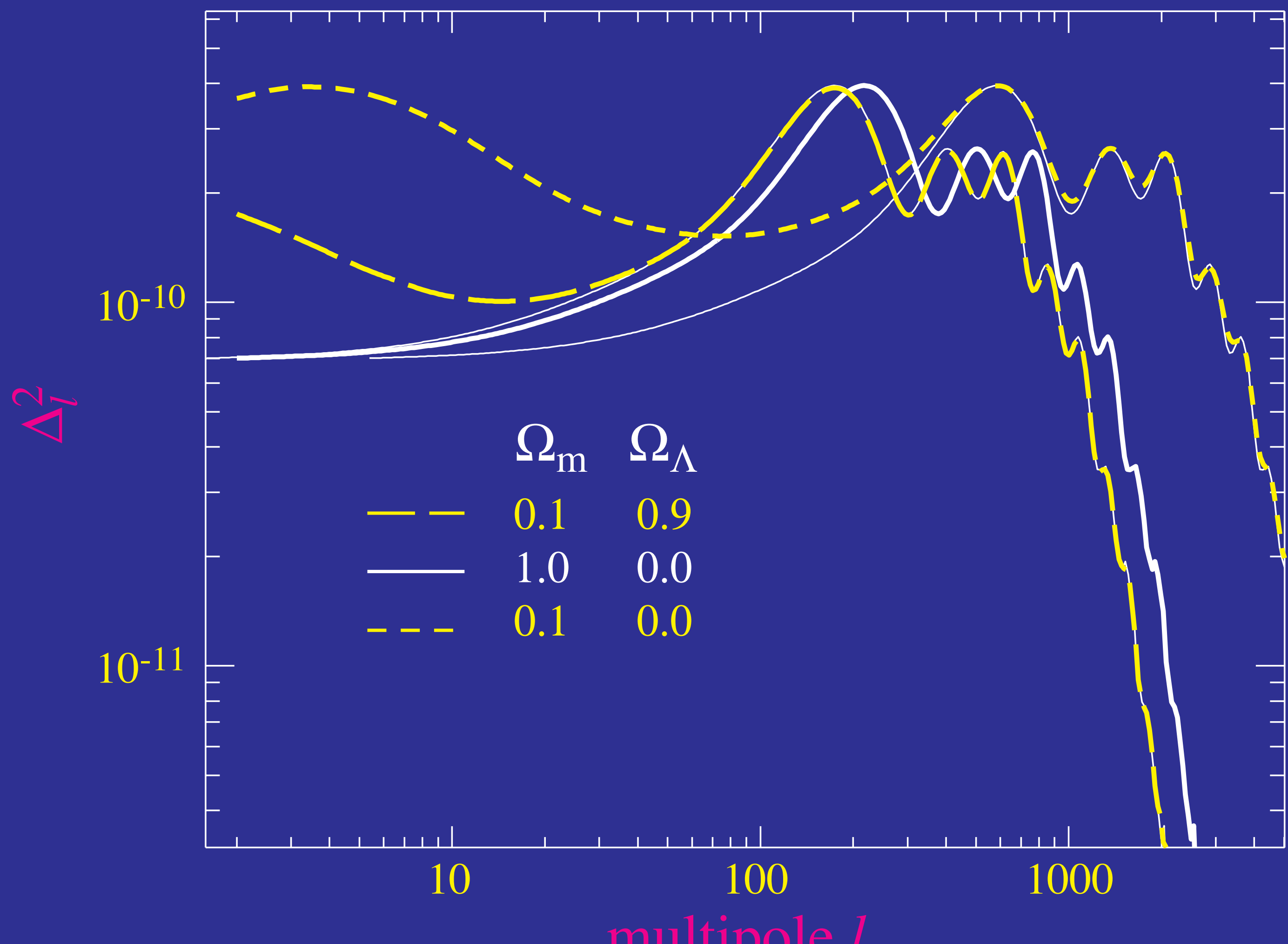
- Lensing observables are nearly degenerate in neutrinos and curvature
- Planck + 10% sky with noise Δ_P=1.4uK'



Smith, Hu, Kaplinghat (2006)

Degeneracy with Massive Neutrinos

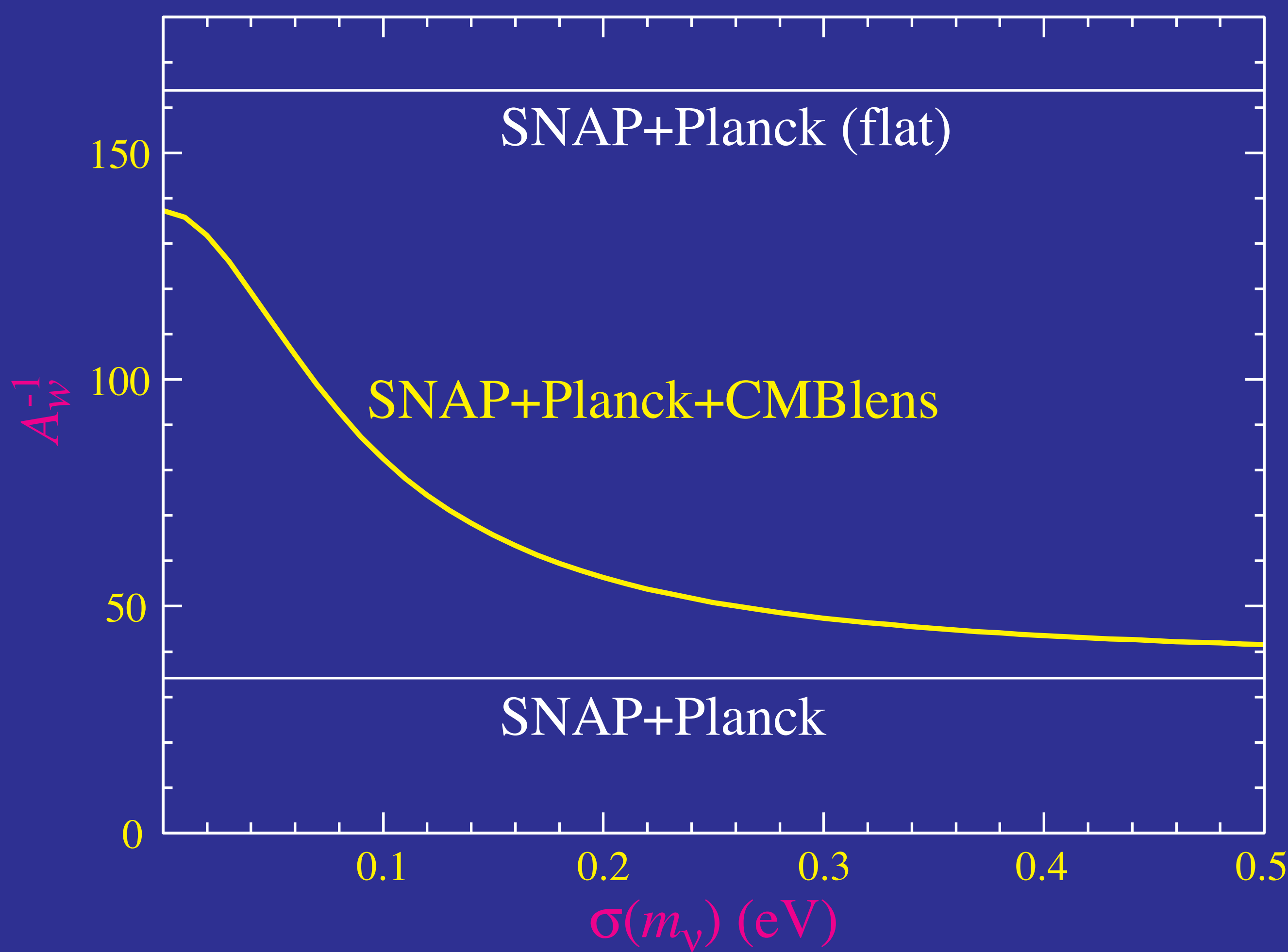
- Degeneracy is partially an accidental cancellation of intrinsic information from curvature across last scattering surface
- Degeneracy potentially removeable beyond power spectra



Hu & White (1996)

Priors on Massive Neutrinos

- In a normal hierarchy and with the lightest neutrino <0.01eV, masses are well enough determined from oscillation experiments already
- More generally, priors on the sum of masses <0.1eV required



inverse area in w_0xw_a

Hu, Huterer & Smith (2006)

Mass Reconstruction

Quadratic Estimator

- Taylor expand mapping

$$T(\hat{\mathbf{n}}) = \tilde{T}(\hat{\mathbf{n}} + \nabla\phi)$$
$$= \tilde{T}(\hat{\mathbf{n}}) + \nabla_i\phi(\hat{\mathbf{n}})\nabla^i\tilde{T}(\hat{\mathbf{n}}) + \ldots$$

- Fourier decomposition $\to$ mode coupling of harmonics

$$T(\mathbf{l}) = \int d\hat{\mathbf{n}}\, T(\hat{\mathbf{n}}) e^{-il\cdot\hat{\mathbf{n}}}$$
$$= \tilde{T}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} (\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1\, \tilde{T}(\mathbf{l}_1)\phi(\mathbf{l}-\mathbf{l}_1)$$

- Consider fixed lens and Gaussian random CMB realizations: each pair is an estimator of the lens at $\mathbf{L} = \mathbf{l}_1 + \mathbf{l}_2$ (Hu 2001):

$$\langle T(\mathbf{l})T'(\mathbf{l}')\rangle_{\rm CMB} \approx \left[\tilde{C}^{TT}_{l_1}(\mathbf{L}\cdot\mathbf{l}_1) + \tilde{C}^{TT}_{l_2}(\mathbf{L}\cdot\mathbf{l}_2)\right]\phi(\mathbf{L}) \quad (\mathbf{l} \neq -\mathbf{l}')$$

Reconstruction from the CMB

- Generalize to polarization: each quadratic pair of fields estimates the lensing potential (Hu & Okamoto 2002)

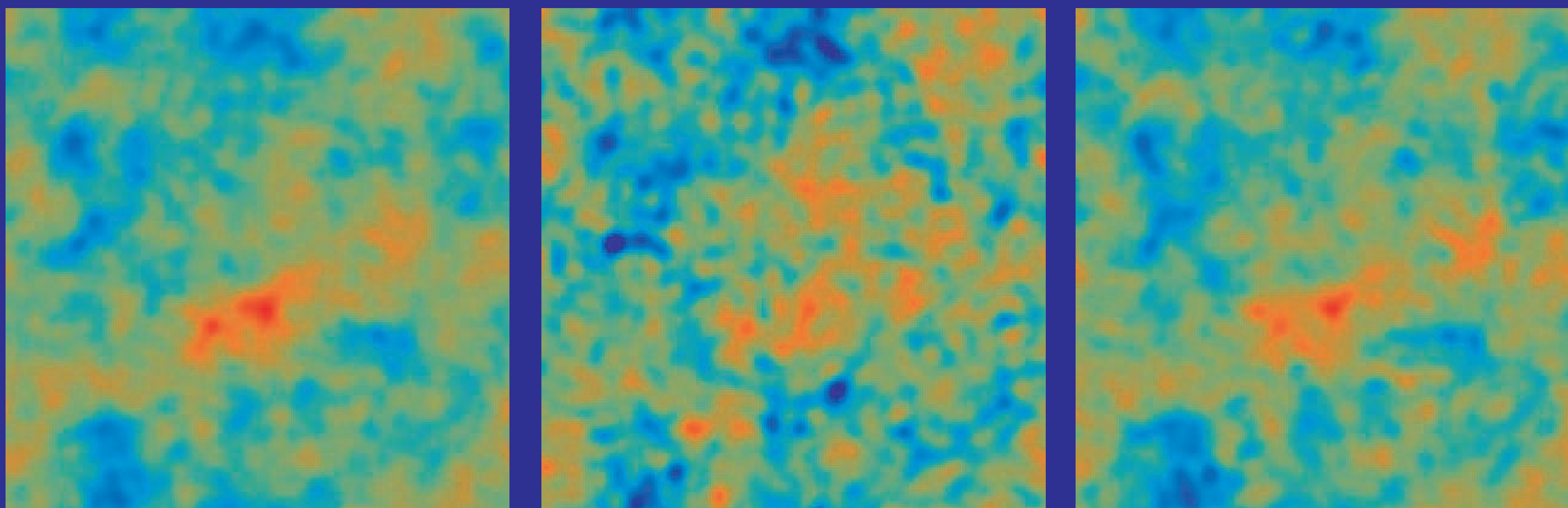
$$\langle x(\mathbf{l}) x'(\mathbf{l}')\rangle_{\rm CMB} = f_\alpha(\mathbf{l},\mathbf{l}')\phi(\mathbf{l}+\mathbf{l}') \,,$$

  where $x \in$ temperature, polarization fields and f_α is a fixed weight that reflects geometry

- Each pair forms a noisy estimate of the potential or projected mass - just like a pair of galaxy shears
- Minimum variance weight all pairs to form an estimator of the lensing mass
- Generalize to inhomogeneous noise, cut sky and maximum likelihood by iterating the quadratic estimator (Seljak & Hirata 2002)

High Signal-to-Noise B-modes

- Cosmic variance of CMB fields sets ultimate limit for T,E
- B-polarization allows mapping to finer scales and in principle is not limited by cosmic variance of E (Hirata & Seljak 2003)



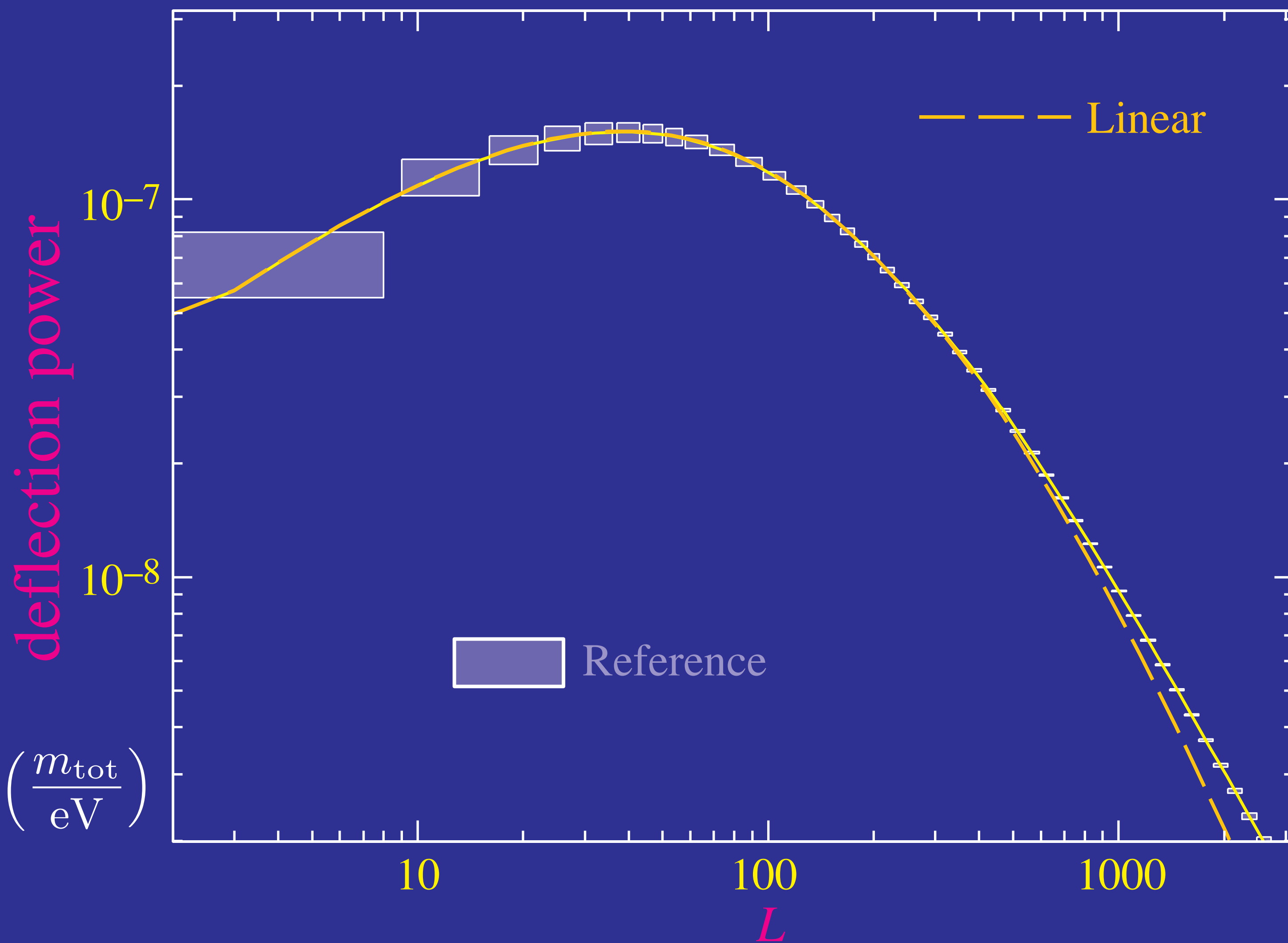
mass | temp. reconstruction | EB pol. reconstruction

100 sq. deg; 4' beam; 1μK-arcmin

Hu & Okamoto (2001)

Matter Power Spectrum

- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime $0.002 < k < 0.2\ h$/Mpc

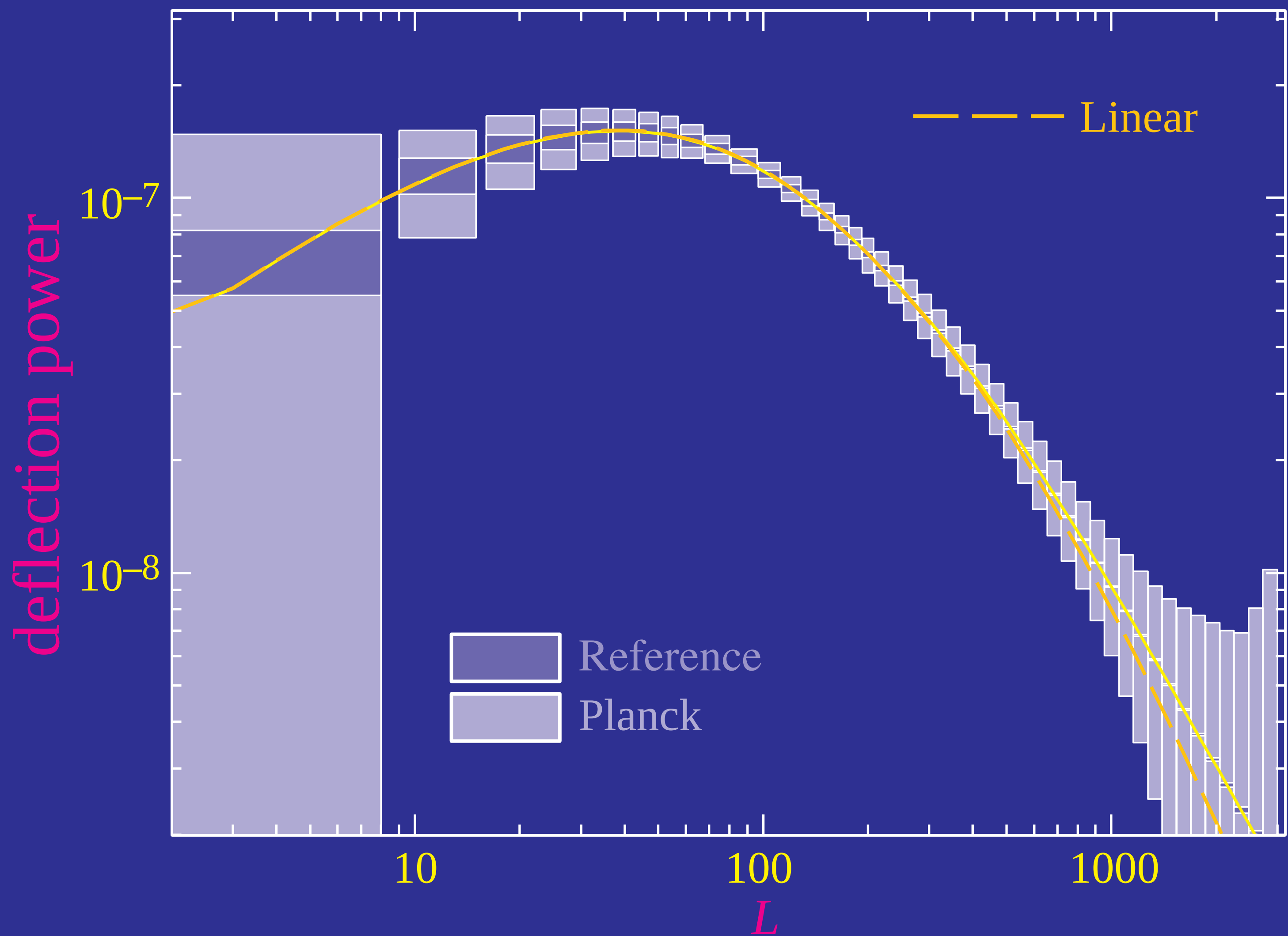


$$\frac{\Delta P}{P} \approx -0.6 \left(\frac{m_{\rm tot}}{\rm eV} \right)$$

Hu & Okamoto (2001)

Matter Power Spectrum

- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime 0.002< k < 0.2 *h*/Mpc



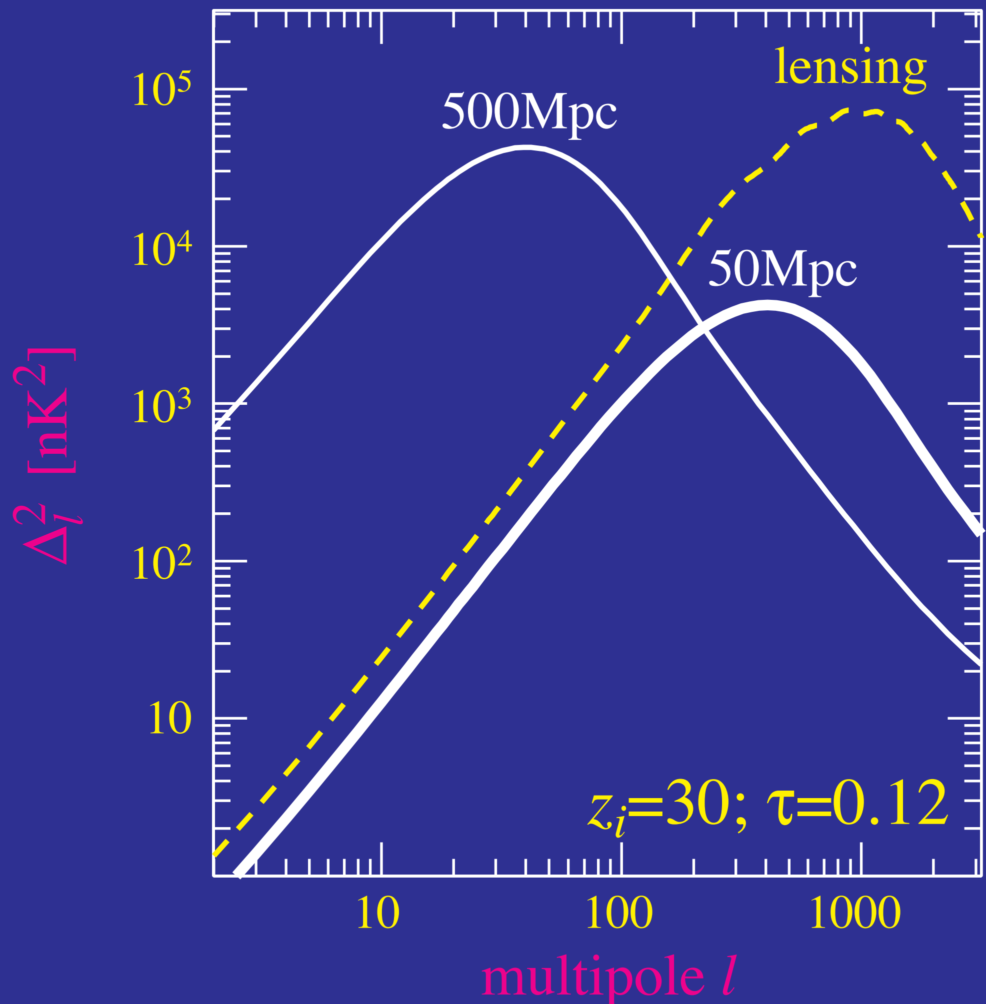
Hu & Okamoto (2001)

Breaking Degeneracies

- Reconstructed power spectrum comes from the non-Gaussian part of the CMB: 4pt and higher correlations
- Contains more information than the two lensing observables which probe the amplitude of the power spectrum at around only two multipoles
- Degeneracies between neutrinos, curvature and the dark energy can potentially be broken (Hu 2002, Kaplinghat, Knox & Song 2003)
- Small biases must be removed from higher order Taylor terms and other non-Gaussian secondaries and foregrounds
- Further study of techniques needed to insure accuracy of measurements (Kuo et al, ACBAR, in prep)

B-mode Contamination from Reionization

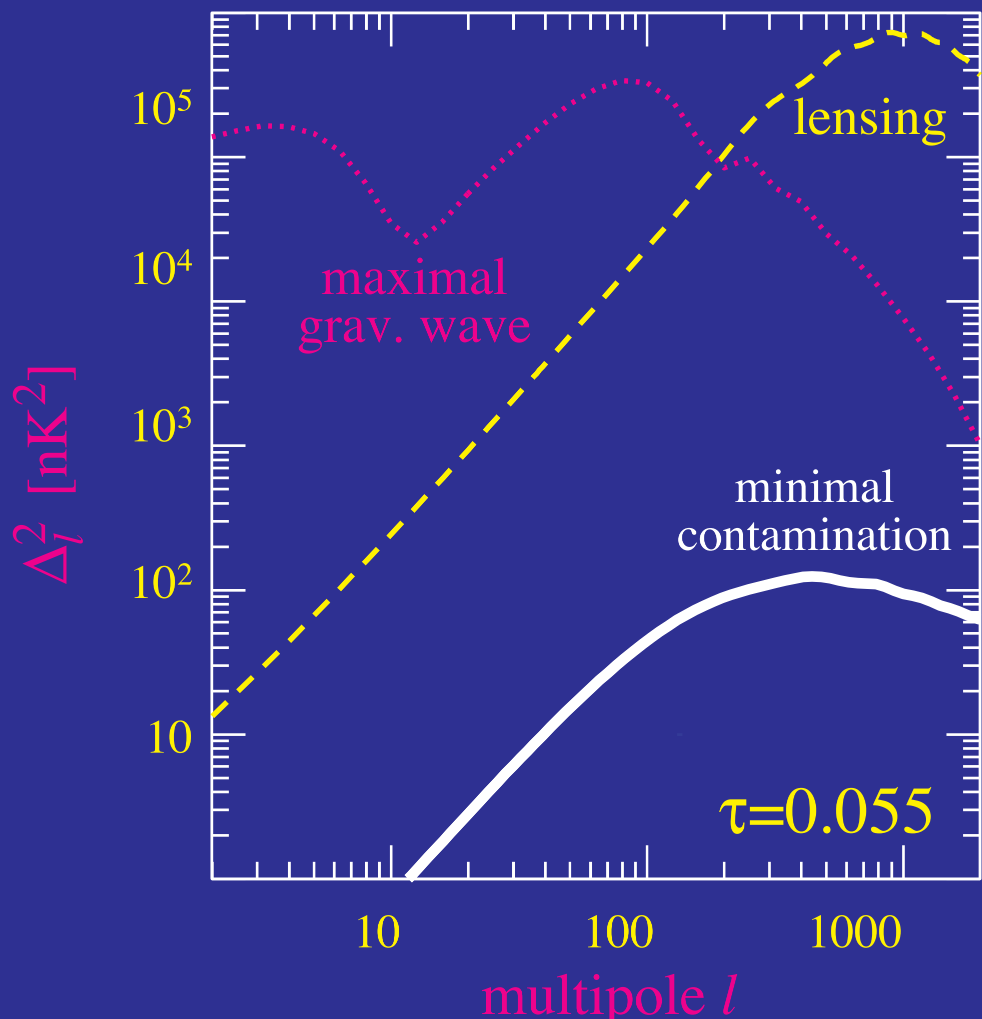
- Inhomogeneous reionization modulates polarization into B-modes (Hu 2000)
- Large signals if ionization bubbles >100Mpc at z~20-30



Mortonson & Hu (2006)

B-mode Contamination from Reionization

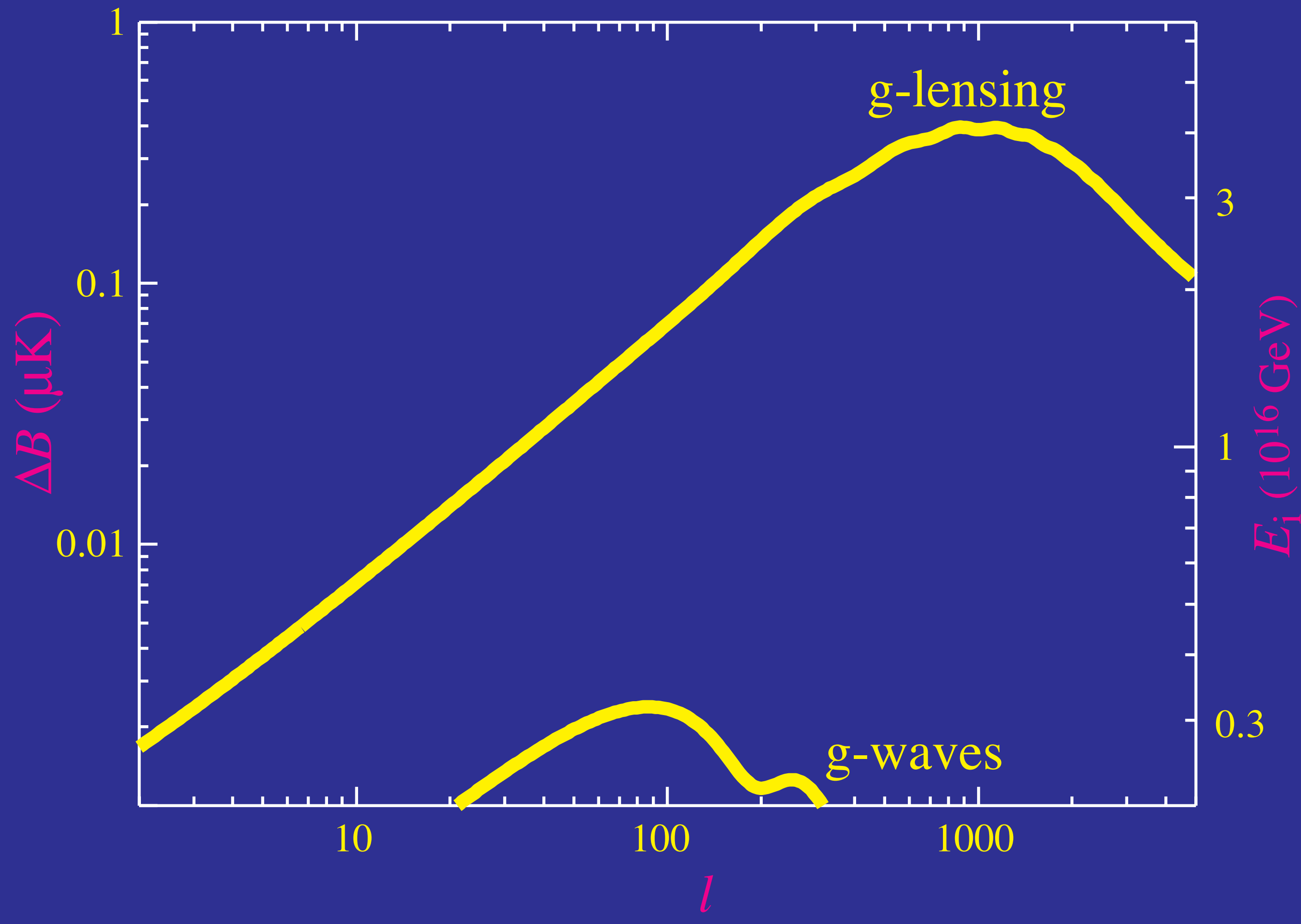
- Inhomogeneous reionization modulates polarization into B-modes
  (Hu 2000)
- Current expectation: grow to 10-100Mpc only at $z<10$
  (Furlanetto et al 2004; Zahn et al 2006)



Mortonson & Hu (2006)

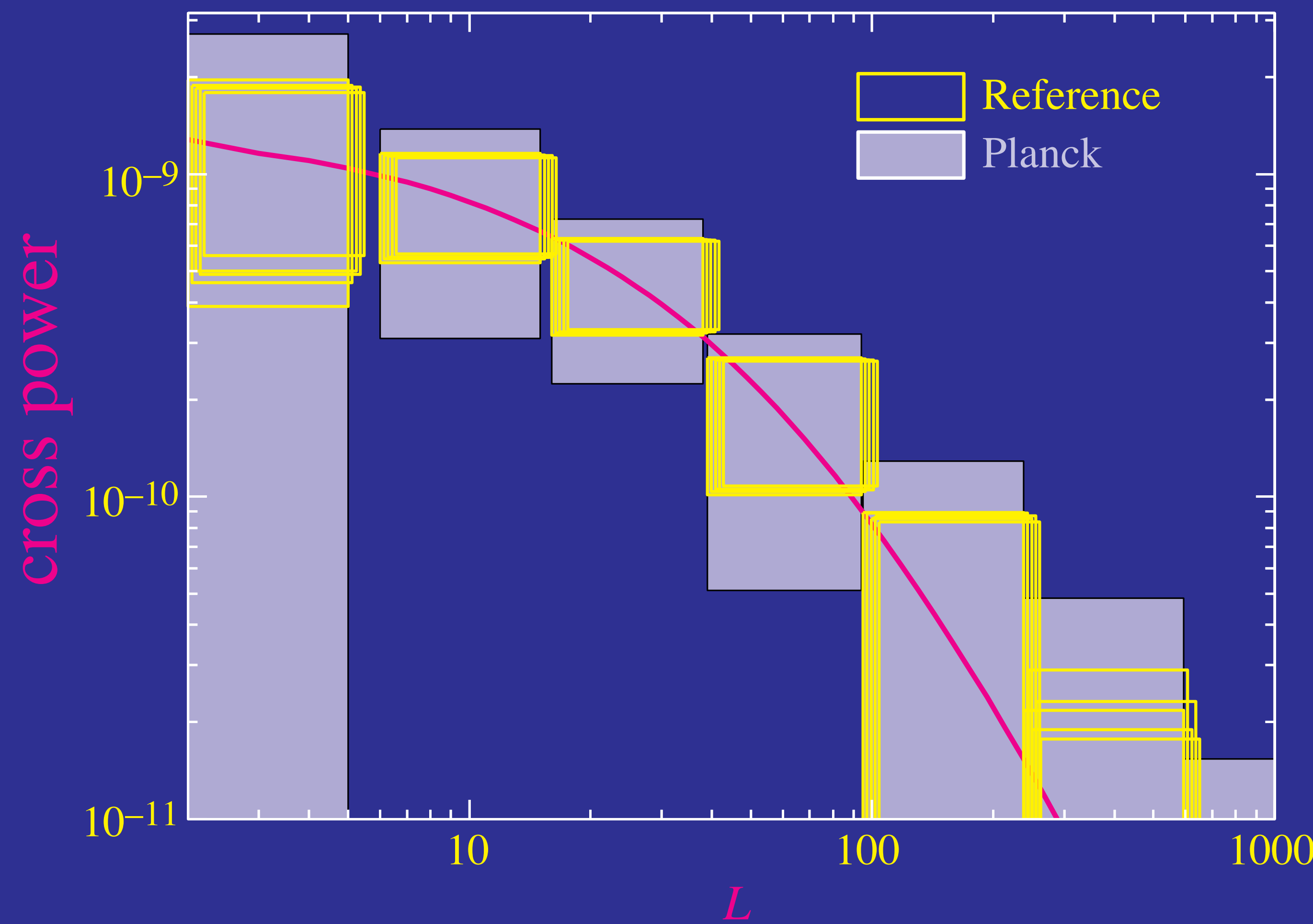
De-Lensing the Polarization

- Gravitational lensing contamination of B-modes from gravitational waves cleaned to E_i~0.3 x 10^{16} GeV
  Hu & Okamoto (2002); Knox & Song (2002); Cooray, Kedsen, Kamionkowski (2002)
- Potentially further with maximum likelihood Hirata & Seljak (2004)



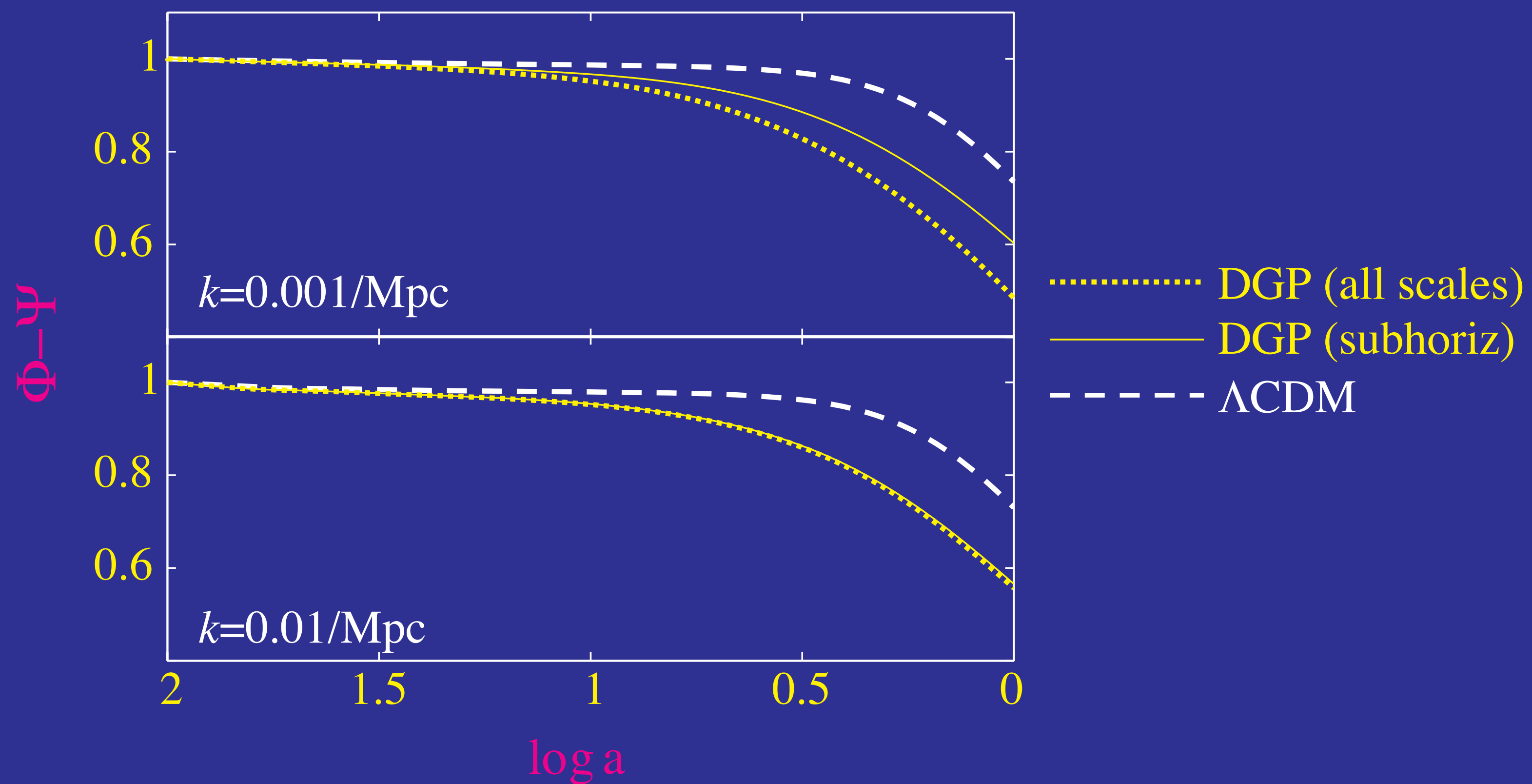
Cross Correlation with Temperature

- Correlation with ISW effect tests the nature of acceleration
- Tests smoothness of dark energy (scalar field) Hu & Okamoto (2002)
- Tests modified gravity (e.g. DGP braneworld) Zhang (2006)



DGP Example

- Difference in expansion history gives excess decay of grav. potential on subhorizon scales (Lue, Scoccimarro, Starkmann 2004; Koyama & Maartins 2005)
- Self-consistent iterative solution of master equation dynamics in the bulk enhances decay further on horizon scales and beyond



Sawicki, Song & Hu 2006

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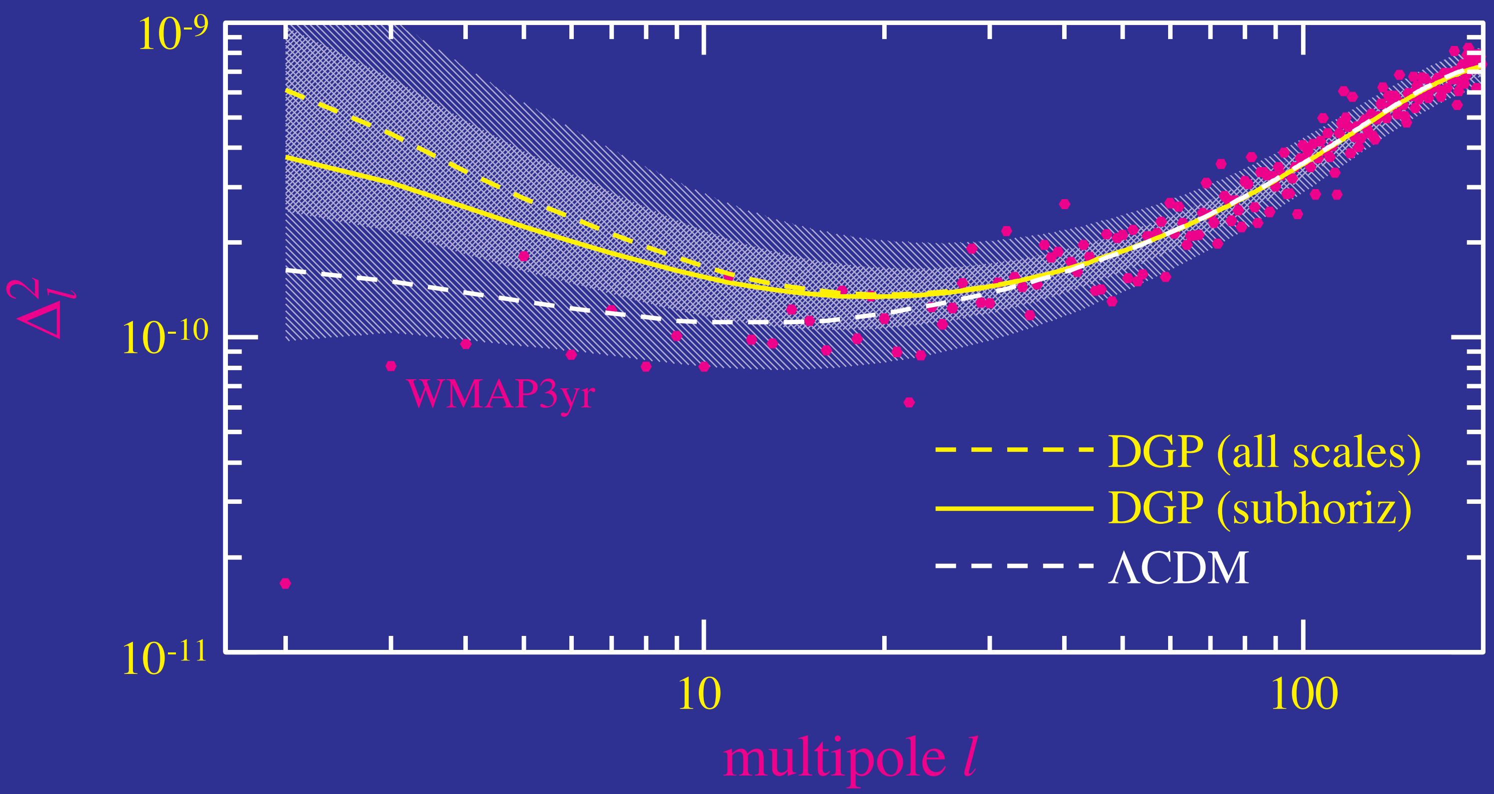
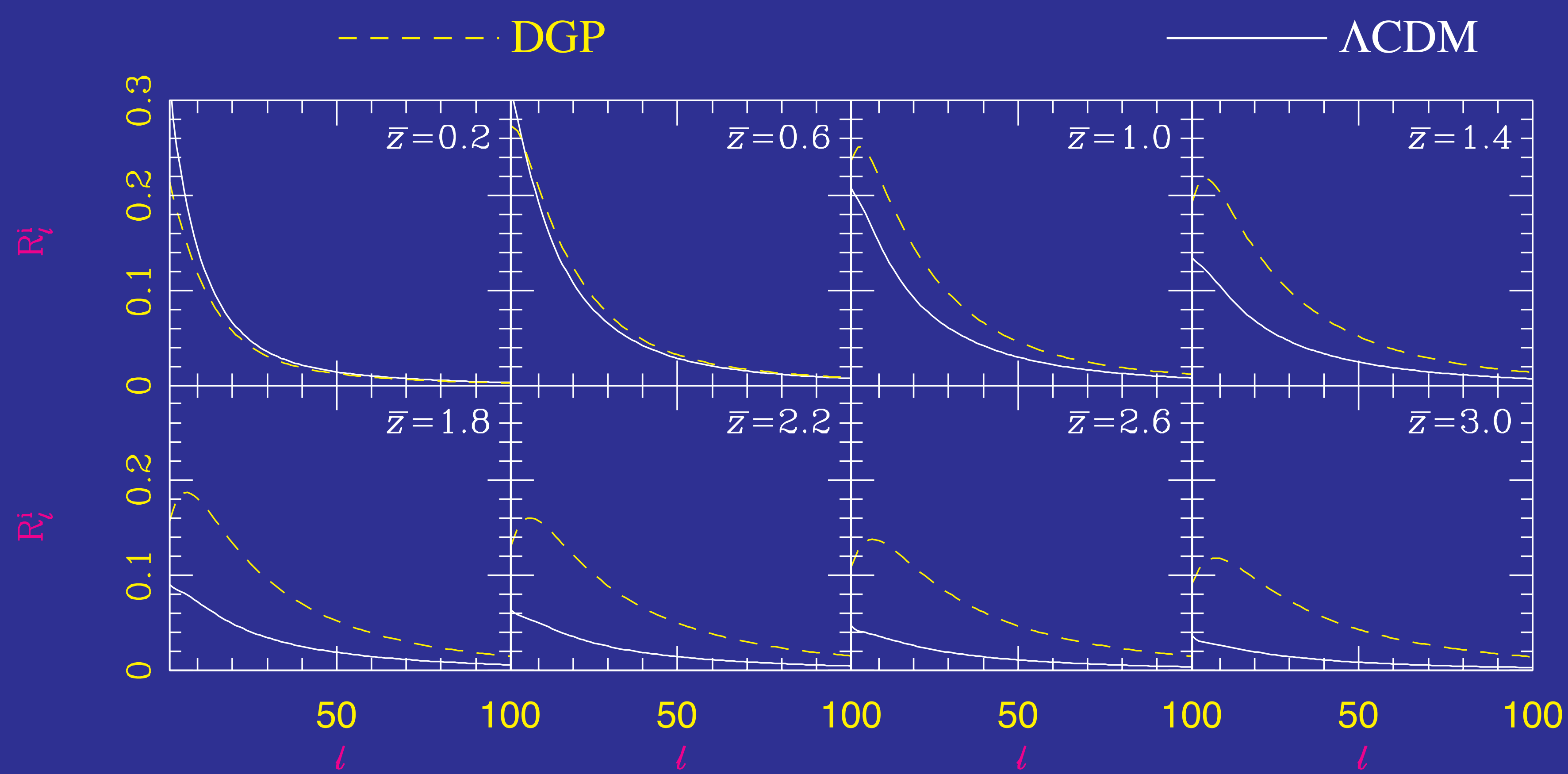


Song, Sawicki & Hu (2006)

Other Routes to Testing ISW

- Cosmic shear cross correlation (Zhang 2006)
- Galaxy/quasar cross correlation at high z (quasars: Giannantonio et al 2006)



Song, Sawicki, Hu (2006)

Summary

- Gravitational lensing of the CMB should be detectable in next generation temperature and polarization measurements
- CMB power spectra measure two lensing observables, associated with convergence at $\ell \sim 100, 500$ and $z \sim 1-4$
- Observable measurements limited by sample variance of lenses regardless of how well arcminute CMB anisotropy measured
- Lensing observables is a useful framework for studying parameter degeneracies and complementarity with other cosmological probes
- SPTpol and other experiments can fix spatial curvature well enough for SNAP and Planck dark energy measurements
- Mass reconstruction can help break degeneracies, test fundamental principles in acceleration, and clean the polarization field for gravitational wave studies