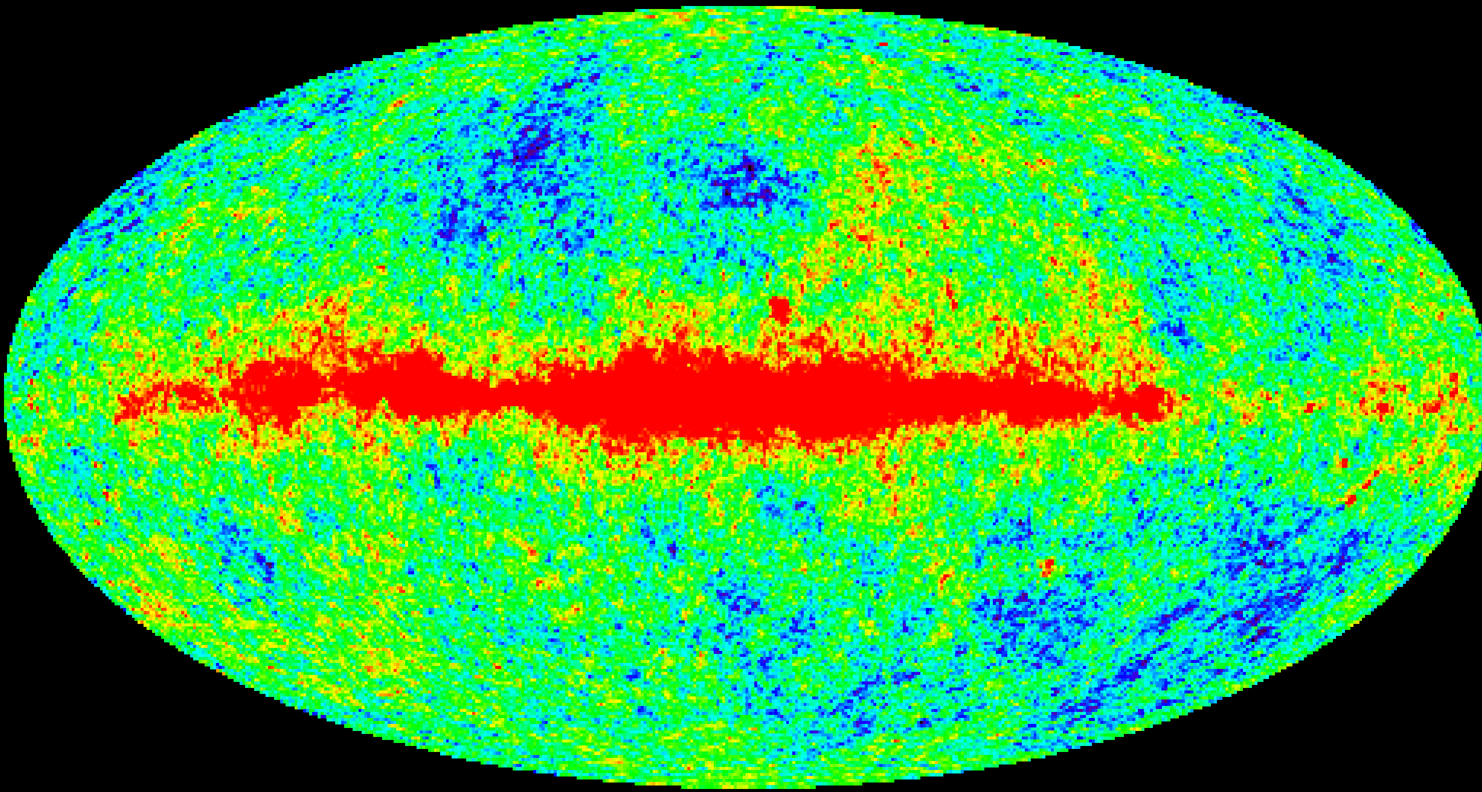
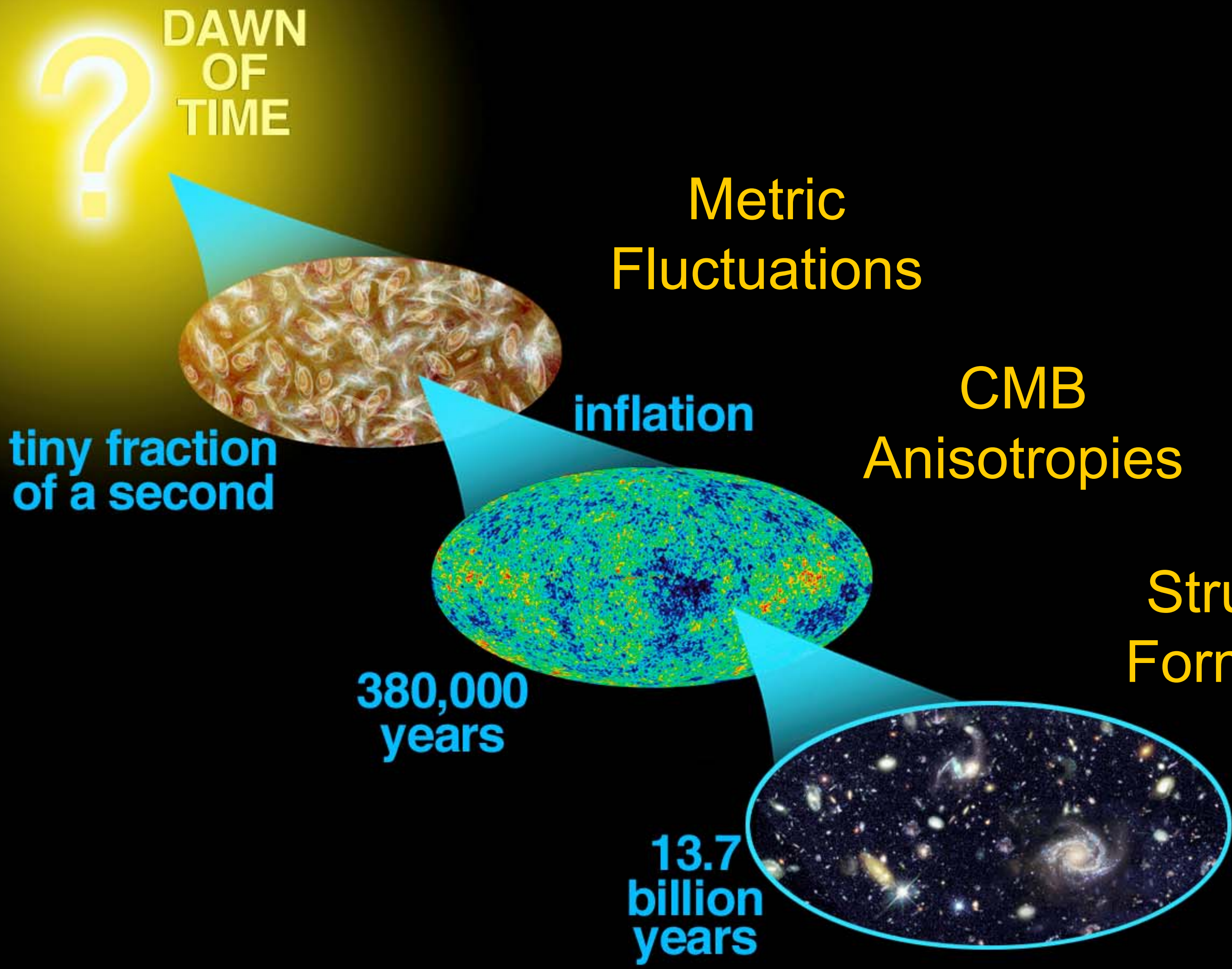
Cosmology (III)



Juan García-Bellido
Física Teórica UAM
9th March 2005

Winter Meeting 2005
(XXXIII I.M.F.P.)



Metric
Fluctuations

CMB
Anisotropies

Structure
Formation

Inflationary Paradigm

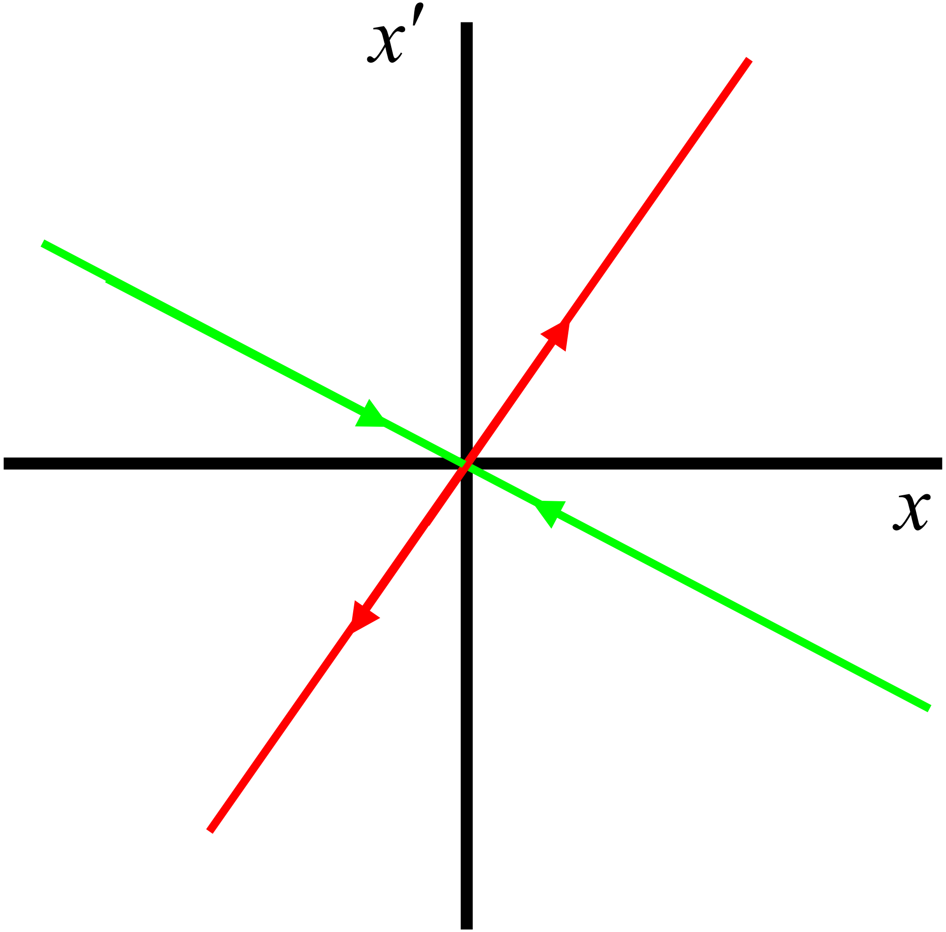
- Why is the Universe spatially flat?

- Why is the Universe homogeneous on large scales?

- What is the origin of all matter and radiation?

- What is the origin of the fluctuations that gave rise to galaxies and other large structures?

Flatness



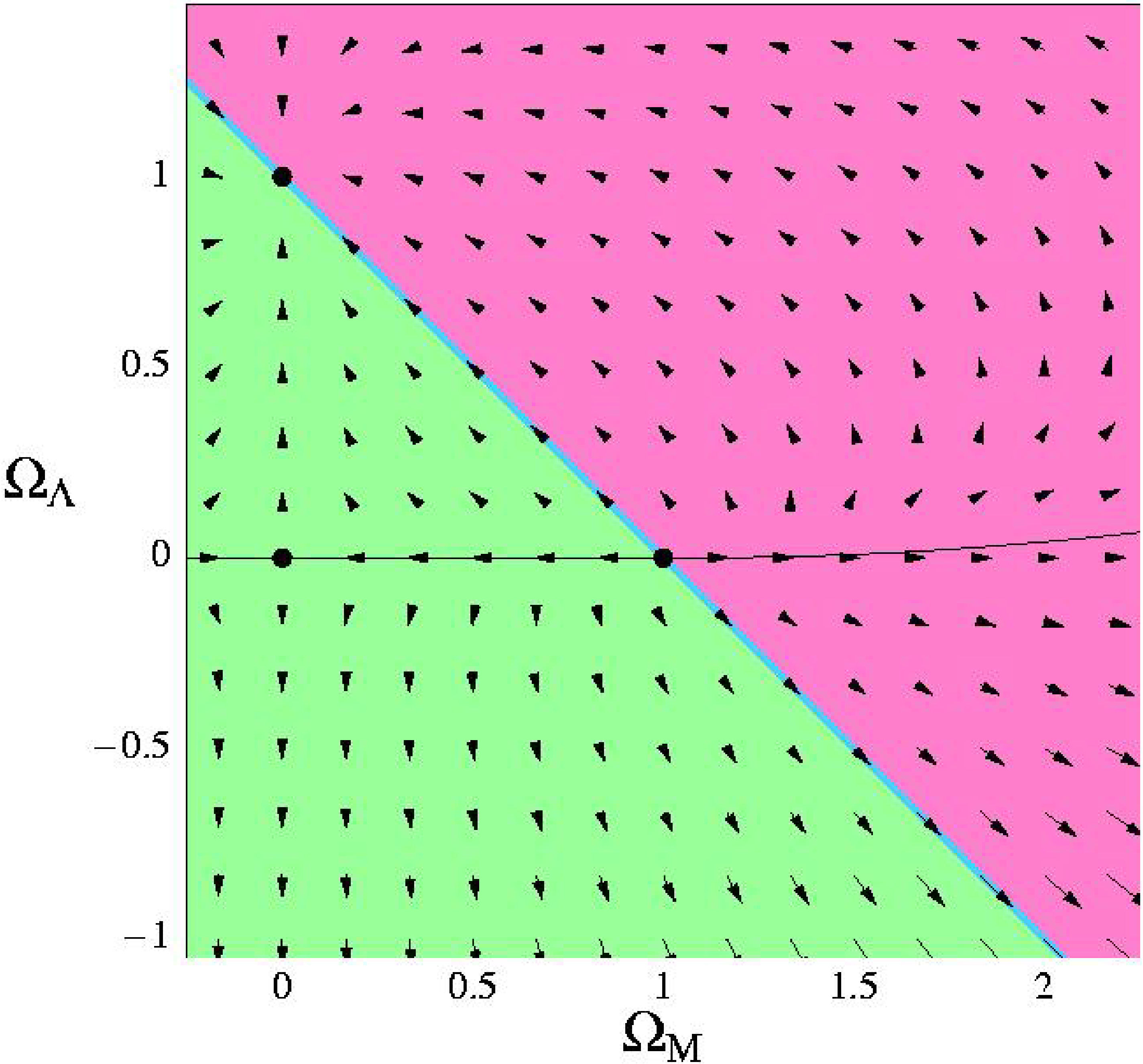
$$x \equiv \frac{\Omega - 1}{\Omega} = \frac{const.}{\rho a^2}$$

$$x' = (1 + 3w)x$$

Matter & Radiation — $x = 0$ unstable

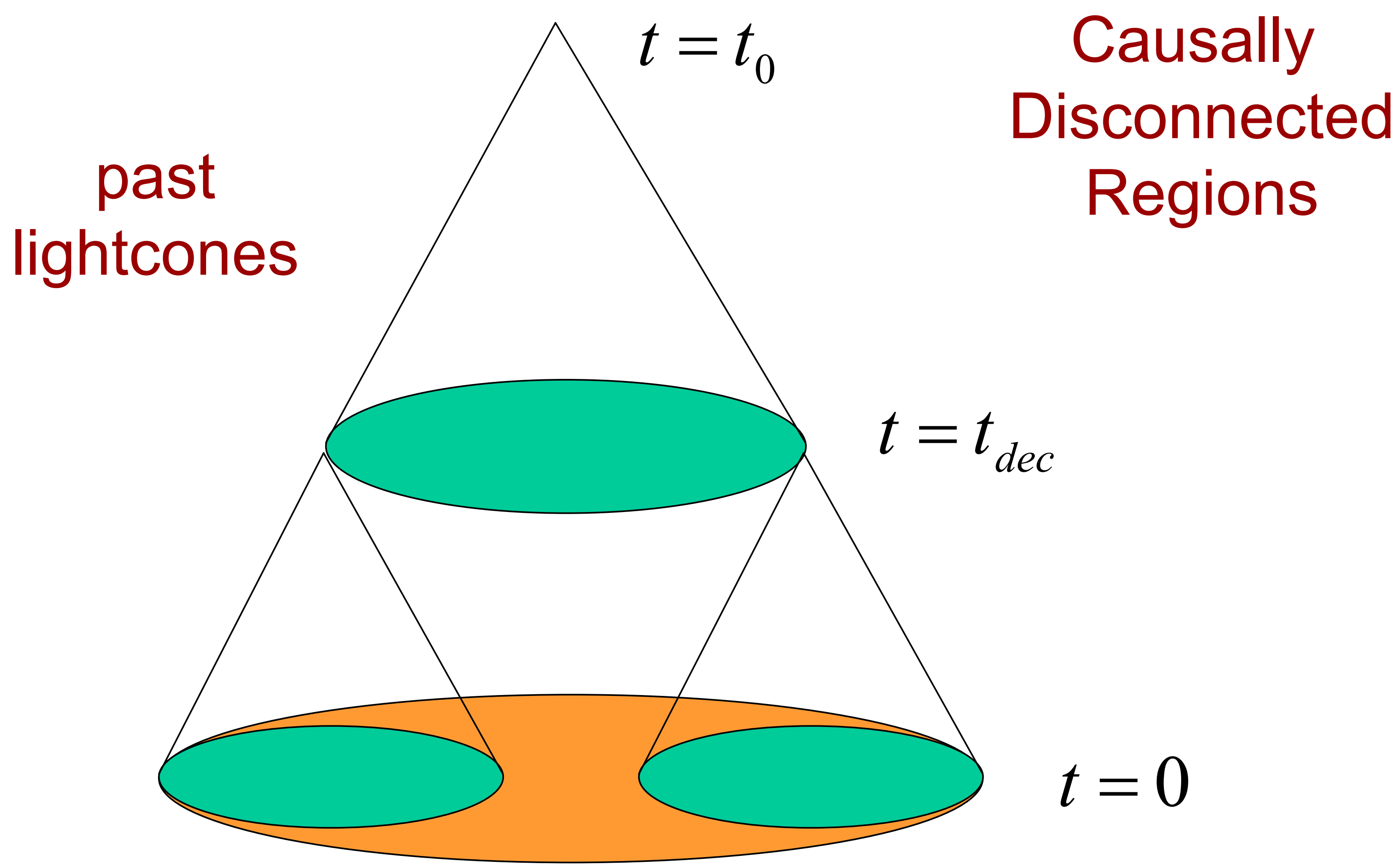
Vacuum — $x = 0$ stable

$$x_0 = x(t_{BBN}) \left(\frac{T_{BBN}}{T_{eq}} \right)^2 (1 + z_{eq}) = 0.01 \Rightarrow \Omega(t_{BBN}) = 1 \pm 10^{-17}$$

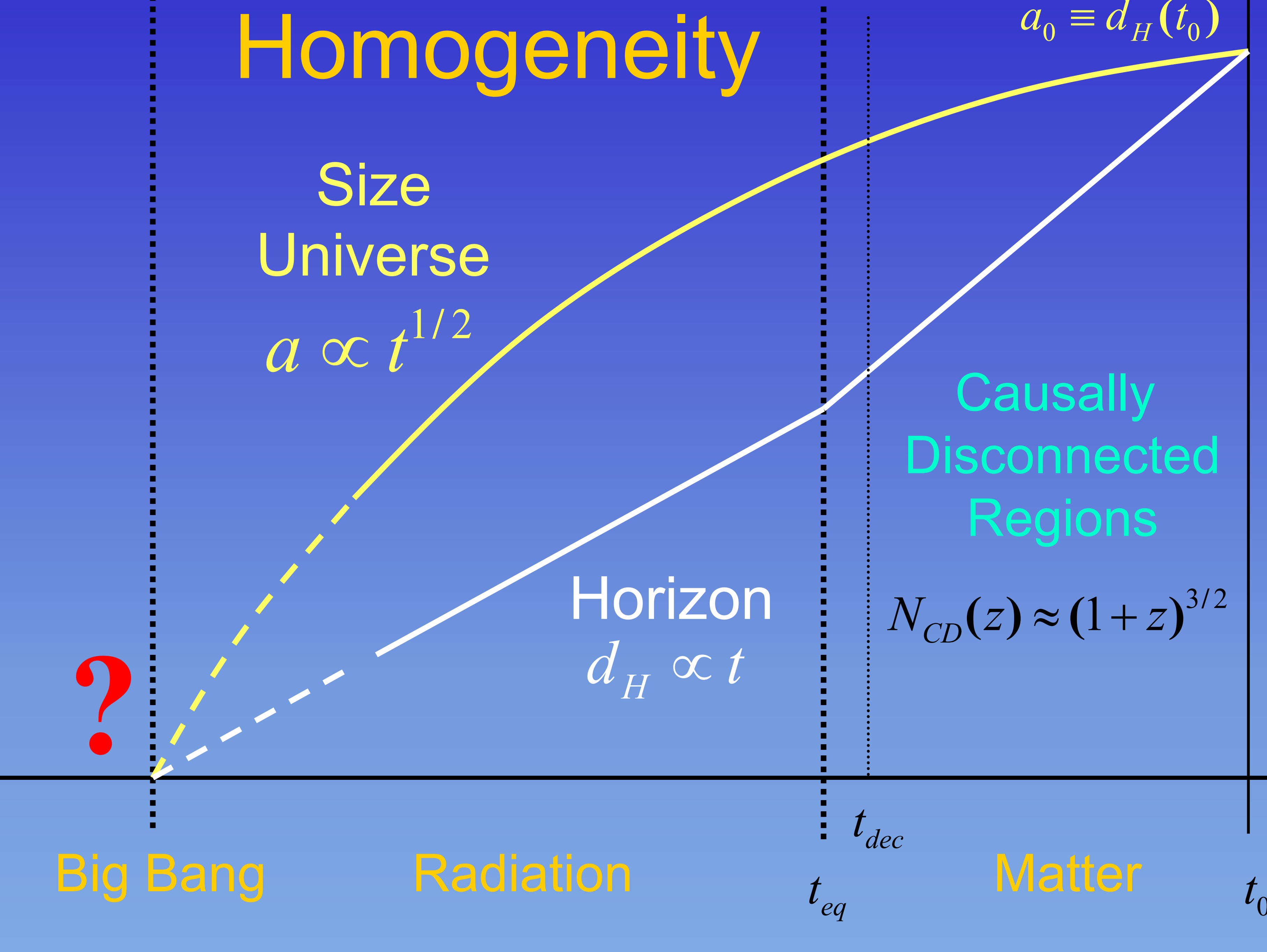


1
0.5
0
−0.5
−1
Ω_Λ
0
0.5
1
1.5
2
Ω_M

Homogeneity

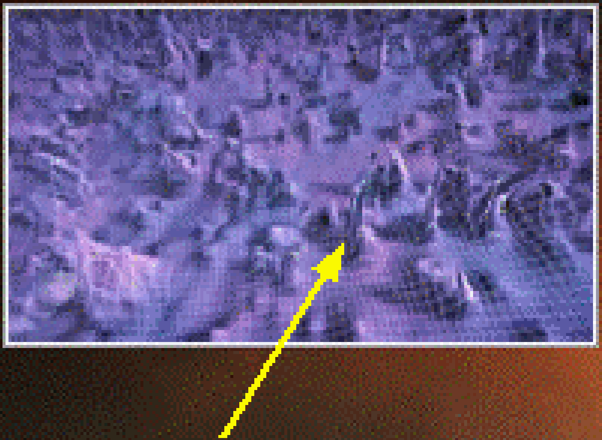


Homogeneity



Inflation

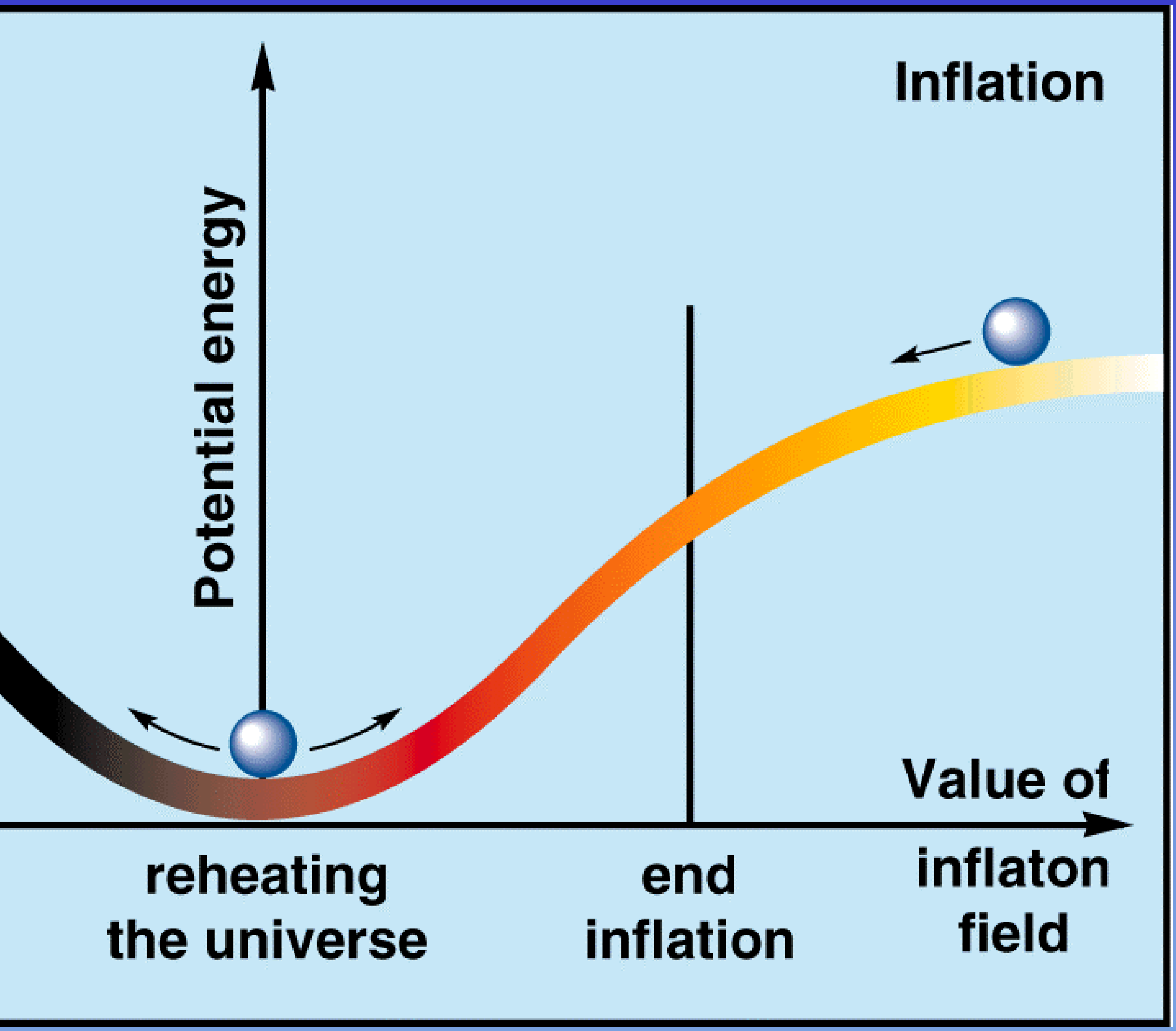
The universe itself could be a product of quantum uncertainty.



Inflation: a small bubble of quantum vacuum (*i.e.*, *nothing*) expands exponentially and becomes the entire universe.

Alan Guth

Andrei Linde

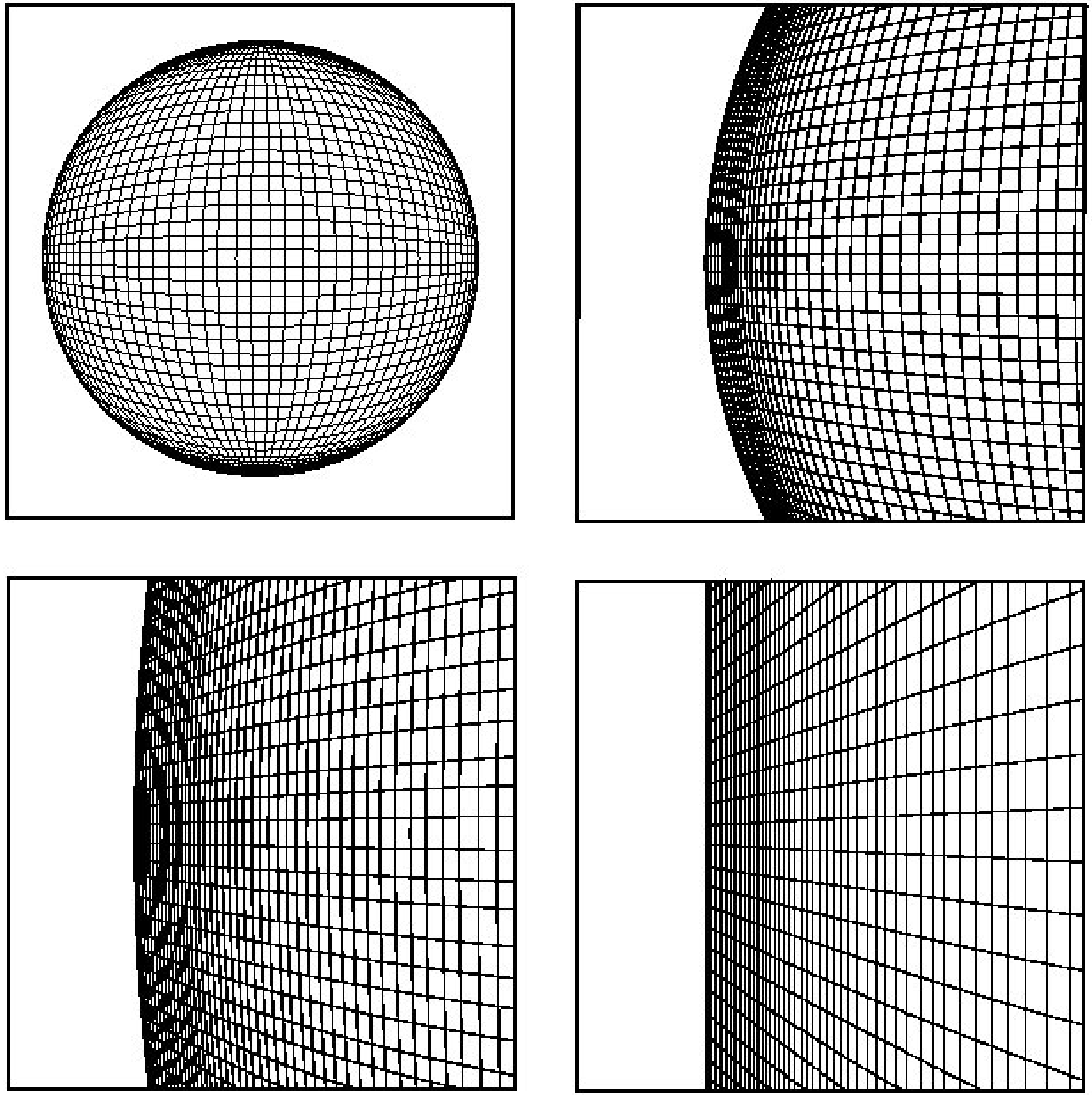


Constant Density

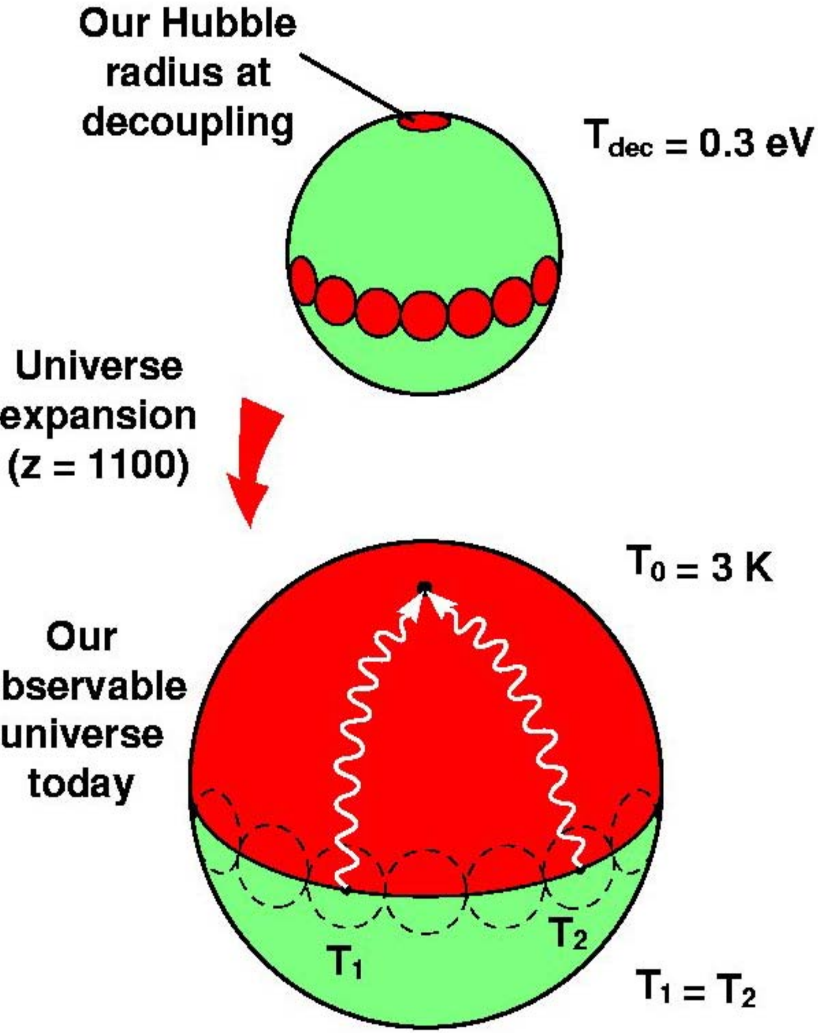
GR

Exponential growth

Flatness



Homogeneity



Scale Factor

$$a(t) \propto t^p \qquad p < 1$$

Particle Horizon

$$d_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} \propto t$$

Causally Disconnected

$$N_{CD}(z) \approx \left(\frac{a}{d_H}\right)^3 \approx (1+z)^{3/2}$$

Homogeneous scalar field

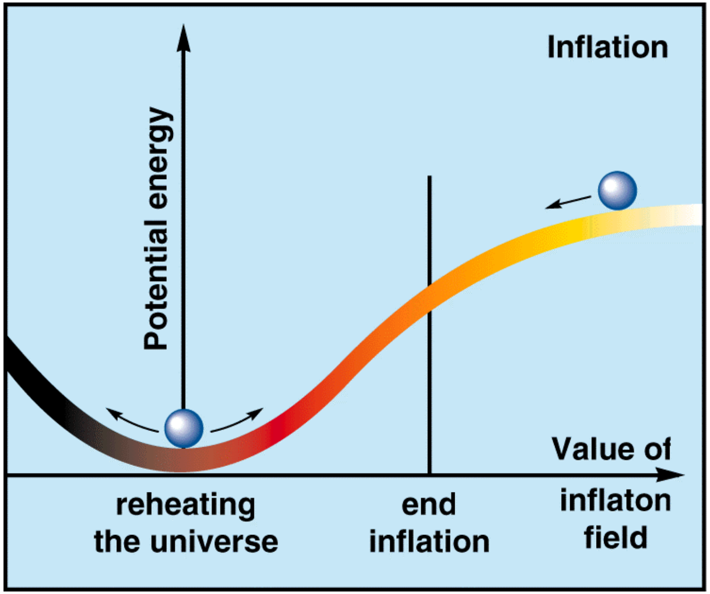
(effective description)

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

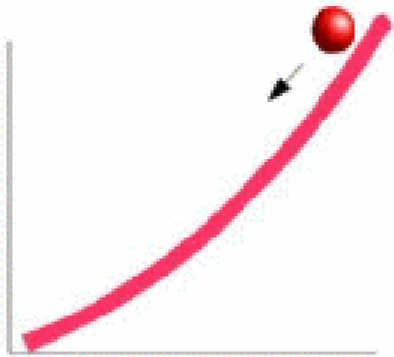
$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\dot{H} = \frac{\kappa^2}{2}\dot{\phi}^2$$

$$\kappa^2 \equiv 8\pi G = \frac{8\pi}{M_P^2}$$



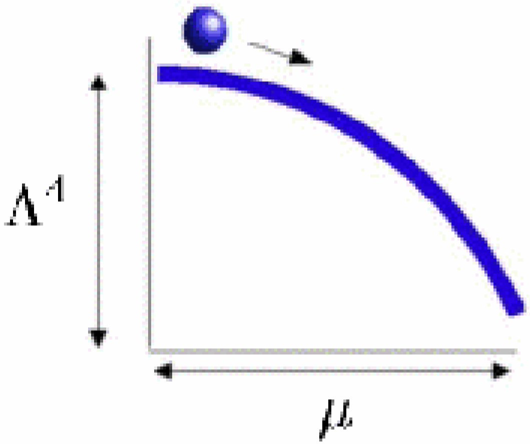
single field inflation



Large field

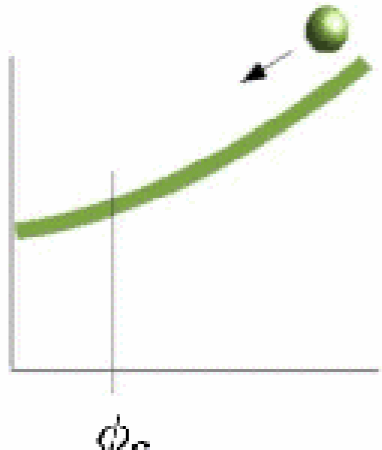
$$V(\phi) = \Lambda^4 (\phi/\mu)^p$$

$$V(\phi) = \Lambda^4 e^{\phi/\mu}$$



Small field

$$V(\phi) = \Lambda^4 \left[1 - (\phi/\mu)^p\right]$$



Hybrid

$$V(\phi) = \Lambda^4 \left[1 + (\phi/\mu)^p\right]$$

Slow-roll approximation

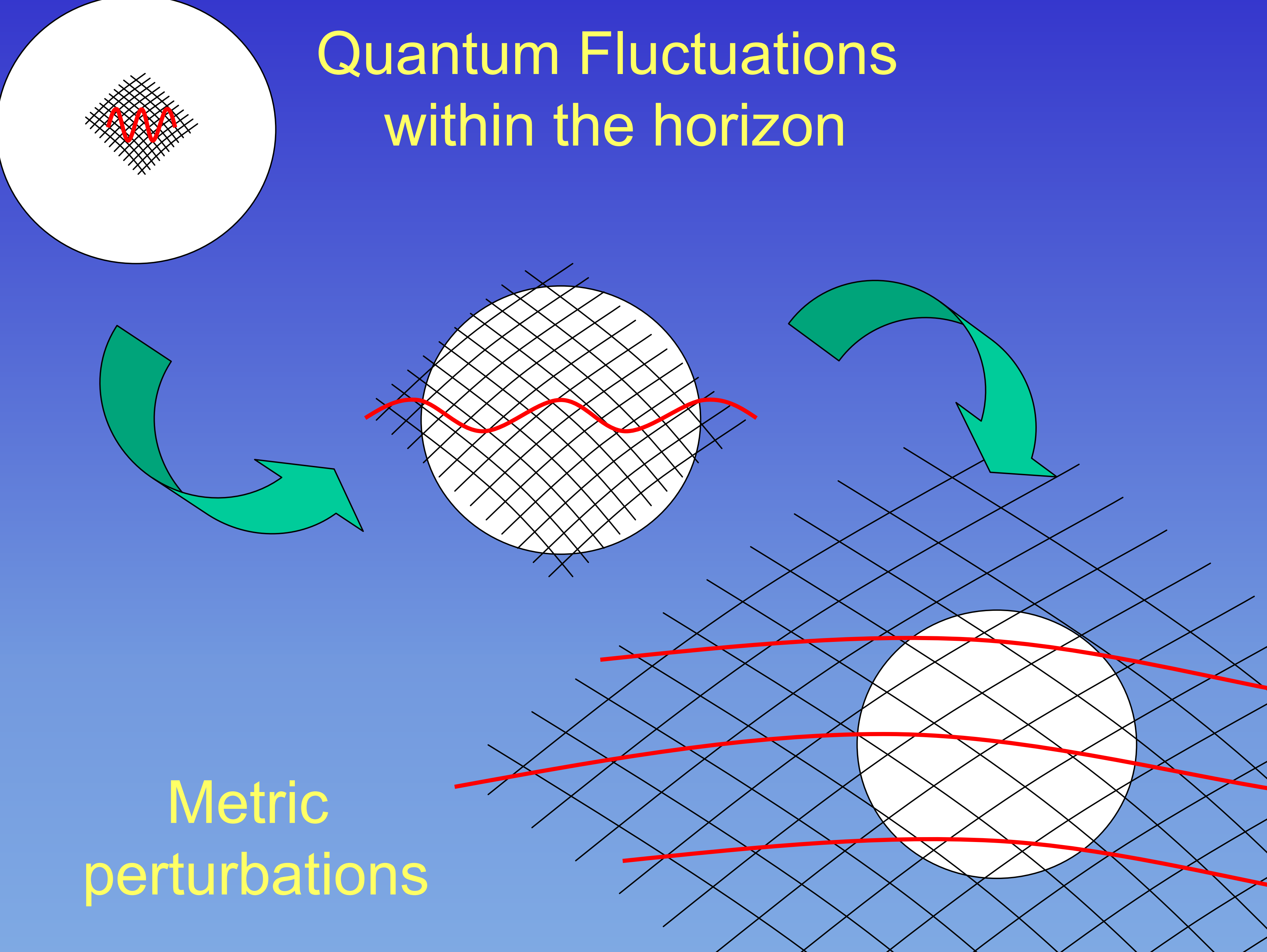
$$\varepsilon = \frac{1}{2\kappa^2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 << 1 \quad \Rightarrow \quad H^2(\phi) \approx \frac{\kappa^2}{3} V(\phi)$$

$$\eta = \frac{1}{\kappa^2}\frac{V''(\phi)}{V(\phi)} << 1 \quad \Rightarrow \quad 3H(\phi)\,\dot{\phi} \approx -V'(\phi)$$

$$dN = H dt = \frac{\kappa d\phi}{\sqrt{2\varepsilon(\phi)}} \quad \Rightarrow \quad N \approx \kappa^2 \int \frac{V(\phi)\, d\phi}{V'(\phi)} \approx 60$$

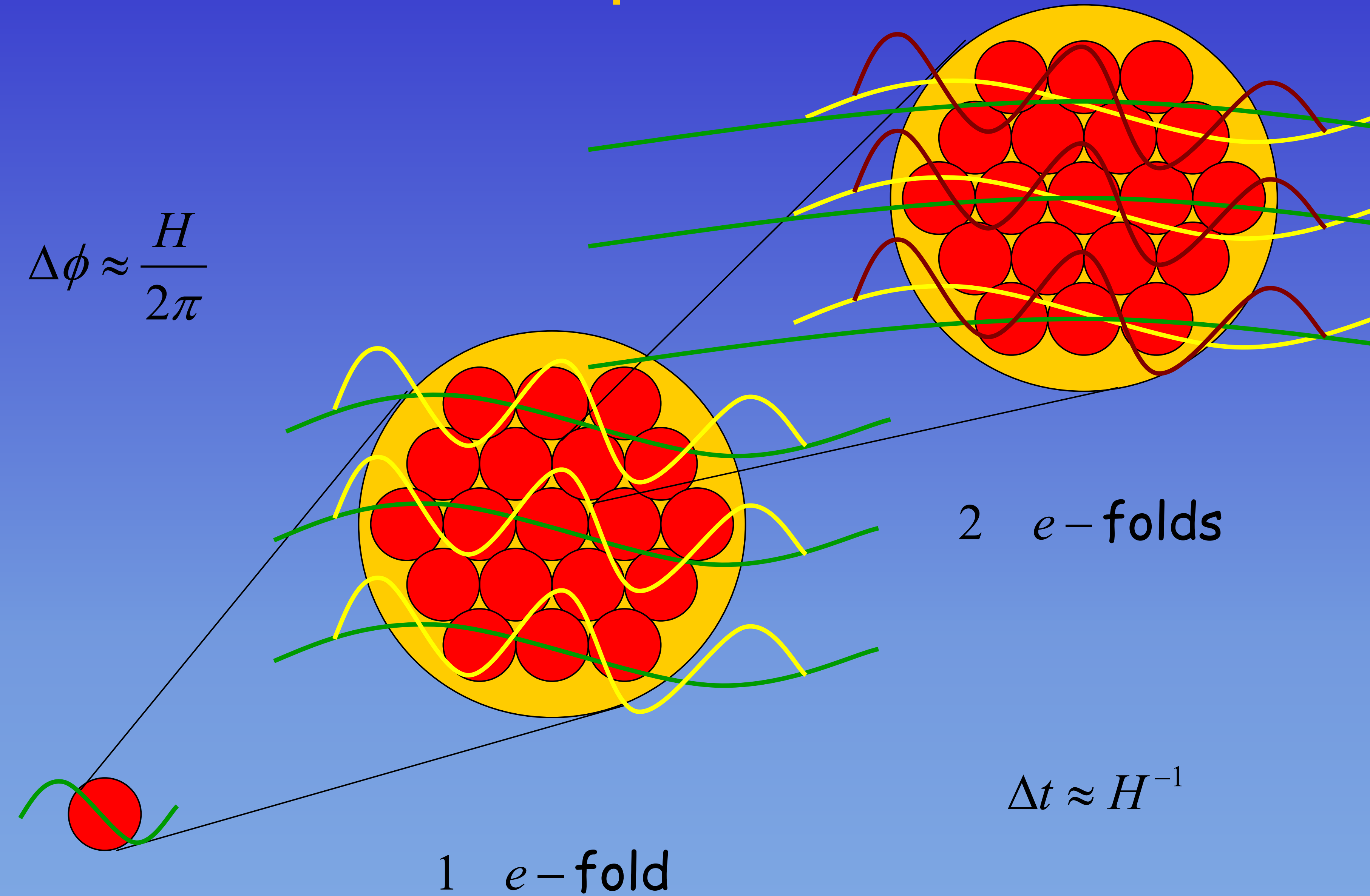
$N \equiv \ln a$ number of e – folds

Quantum Fluctuations within the horizon



Metric perturbations

Scale Invariant Spectrum



Power spectra

Scalar

$$P_S(k) = \frac{\kappa^2}{2\varepsilon}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{k_*}\right)^{n_S - 1}$$

$$n_S - 1 = 2\eta - 6\varepsilon \quad = O(N^{-1})$$

approx. scale invariant

($k_* = aH$ horizon crossing)

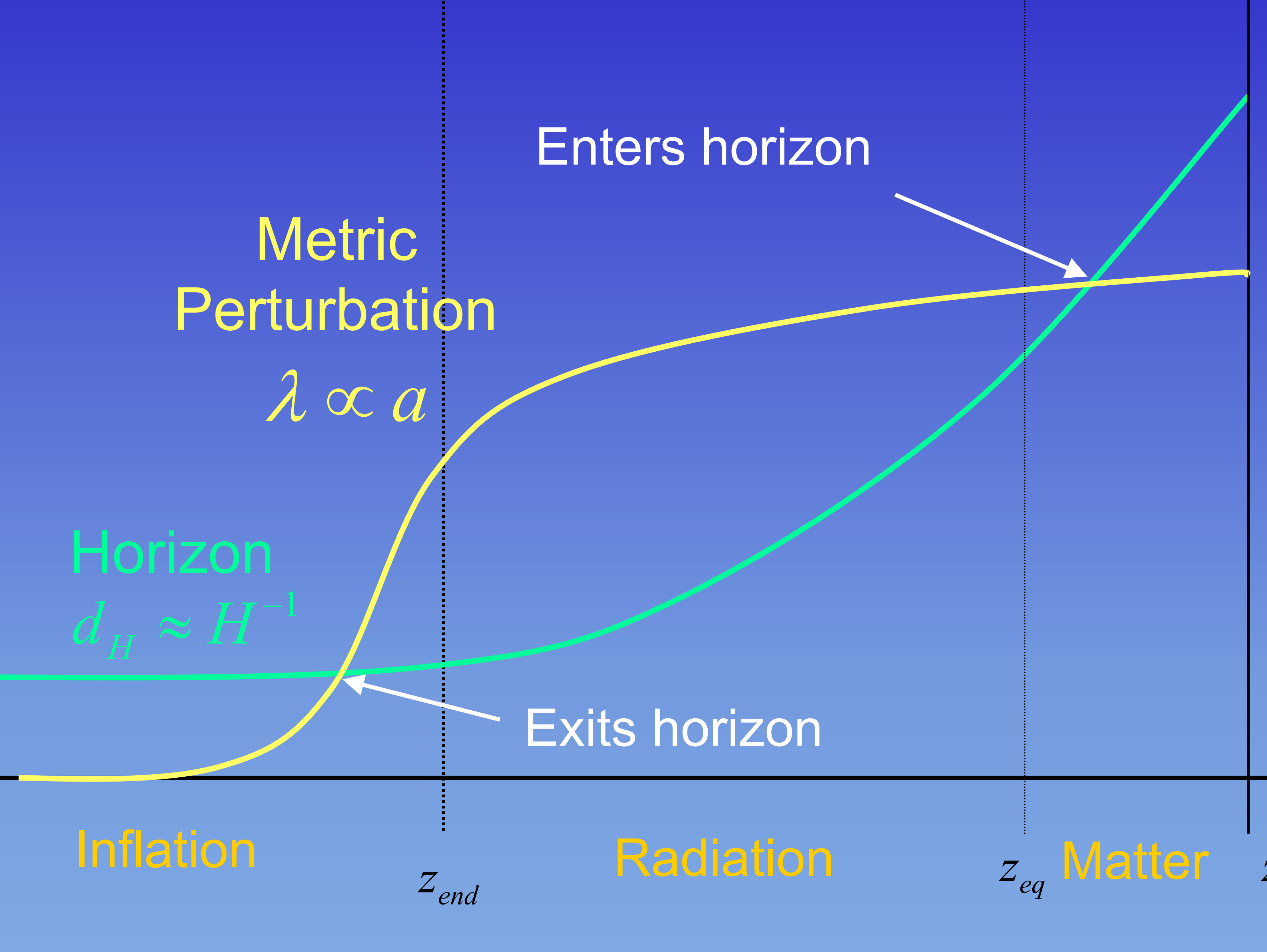
Tensor

$$P_T(k) = 8\kappa^2\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{k_*}\right)^{n_T}$$

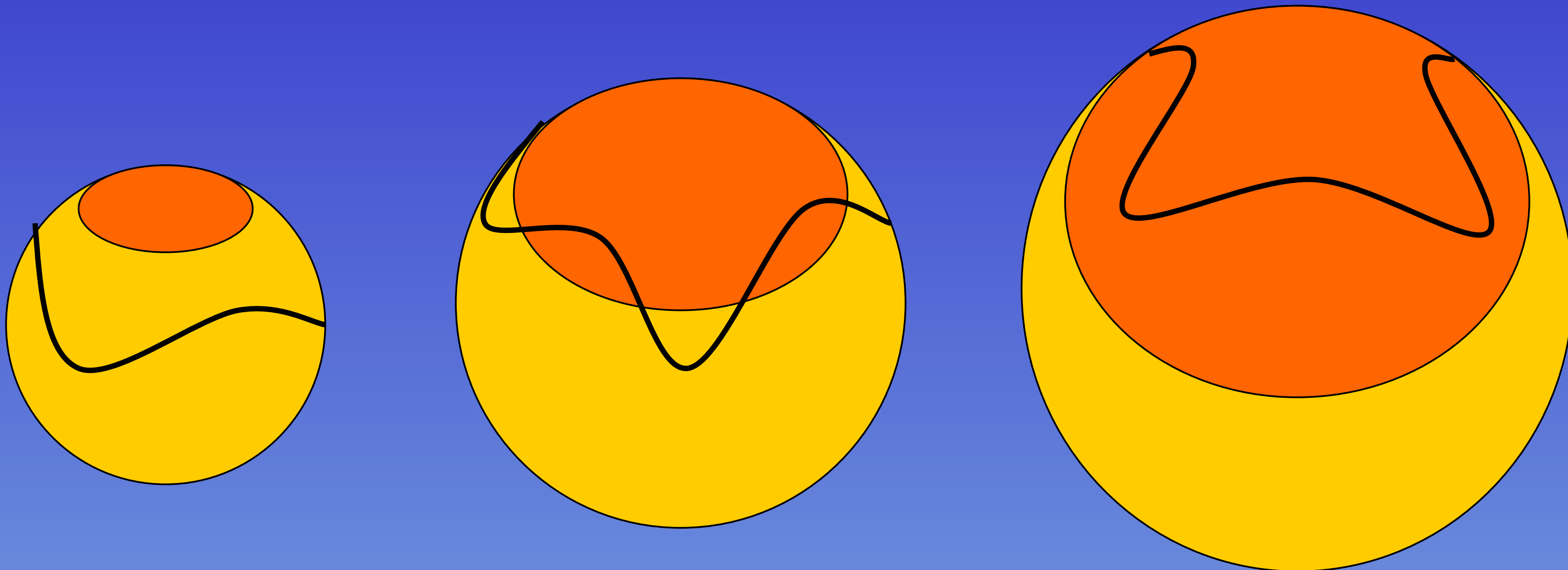
$$n_T = -2\varepsilon \quad = O(N^{-1})$$

Consistency relation

$$r = \frac{A_T^2}{A_S^2} = 16\varepsilon = -8n_T$$


Enters horizon
Metric
Perturbation
$\lambda \propto a$
Horizon
$d_H \approx H^{-1}$
Exits horizon
Inflation
z_{end}
Radiation
z_{eq}
Matter

After Inflation

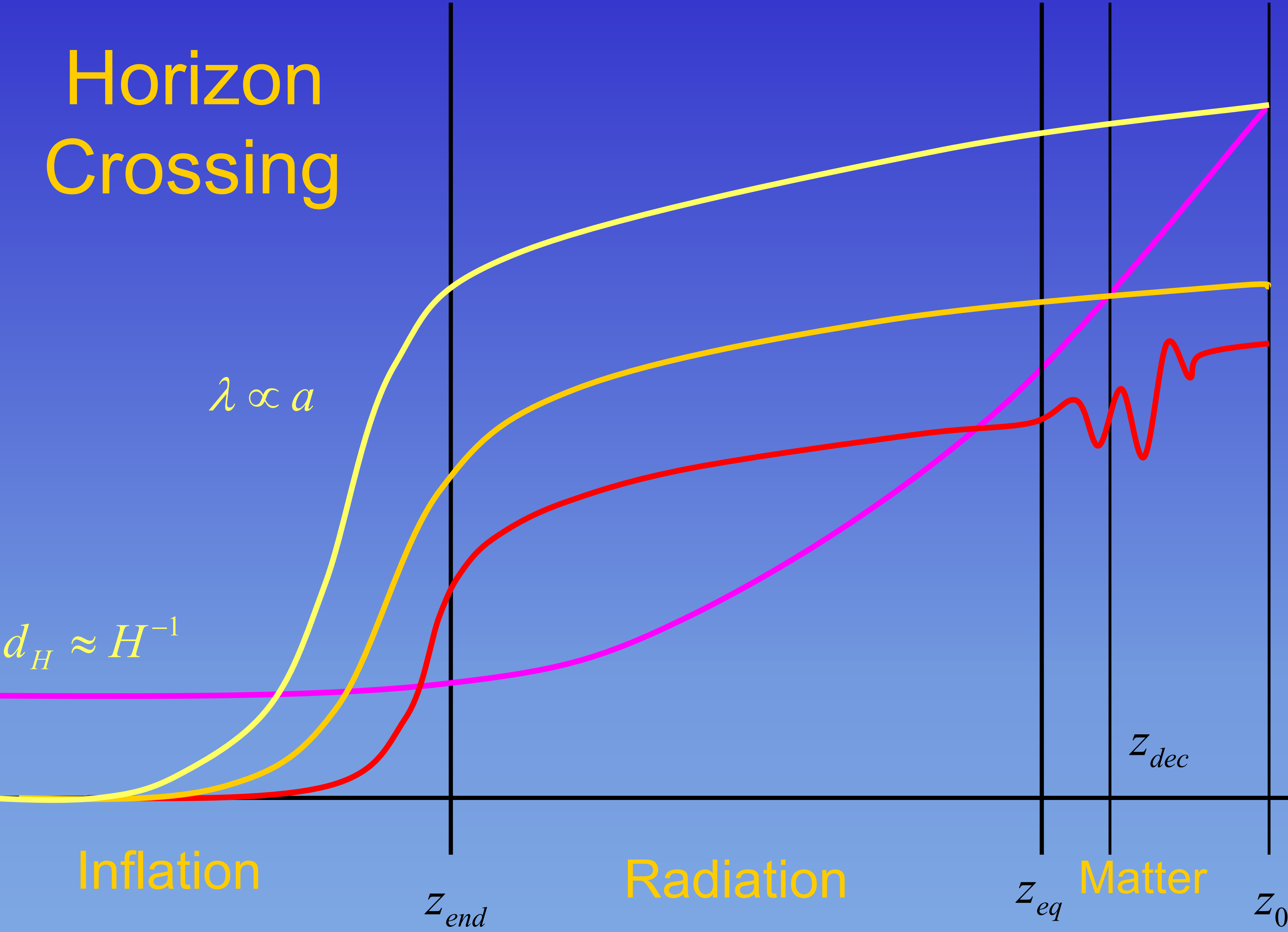


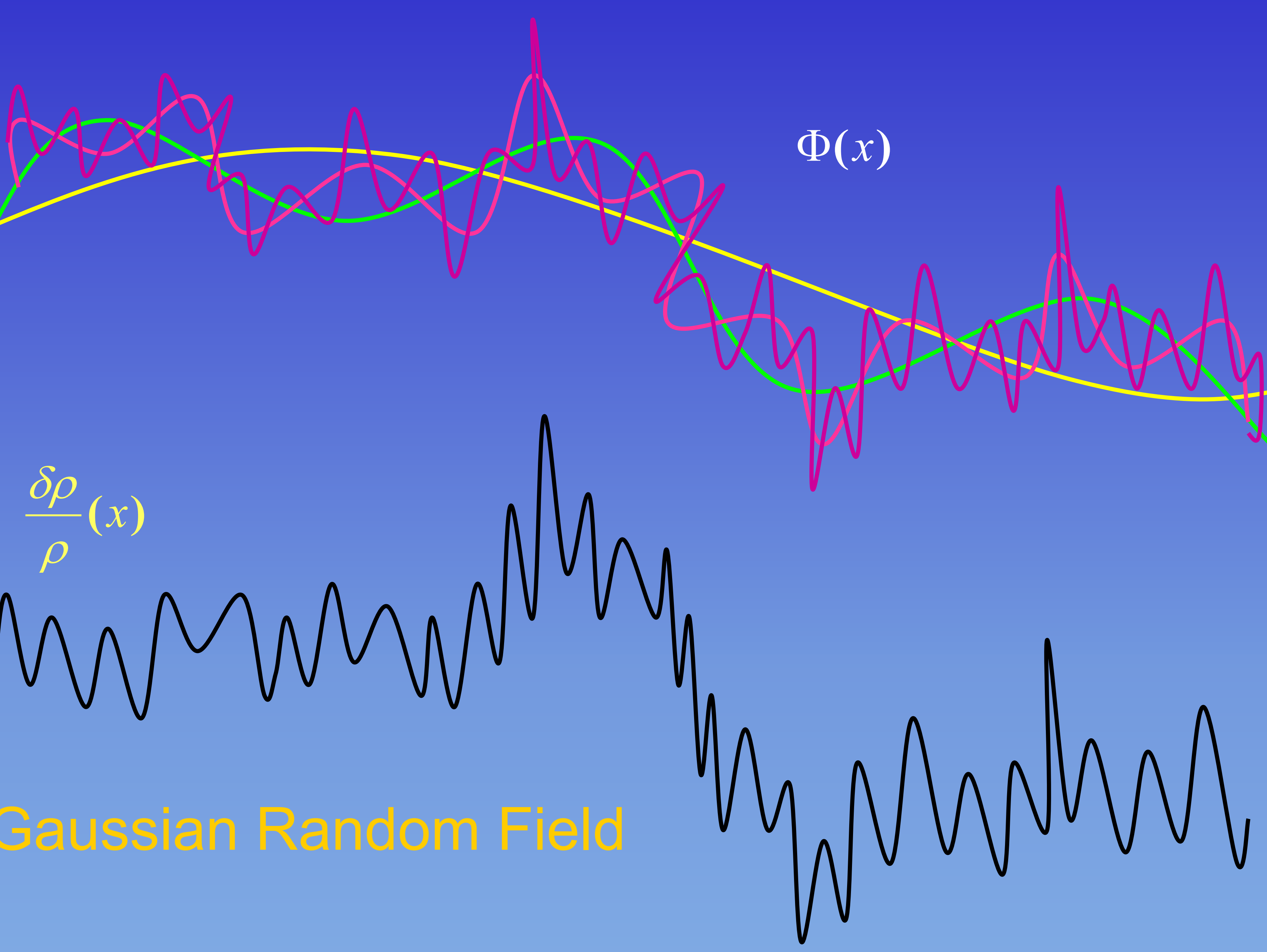
$\lambda > d_H$ — Outside Horizon

$\lambda \approx d_H$ — Enters the Horizon

$\lambda < d_H$ — Inside Horizon

Horizon Crossing




$\Phi(x)$
$\frac{\delta\rho}{\rho}(x)$
Gaussian Random Field

Basic Inflationary Predictions

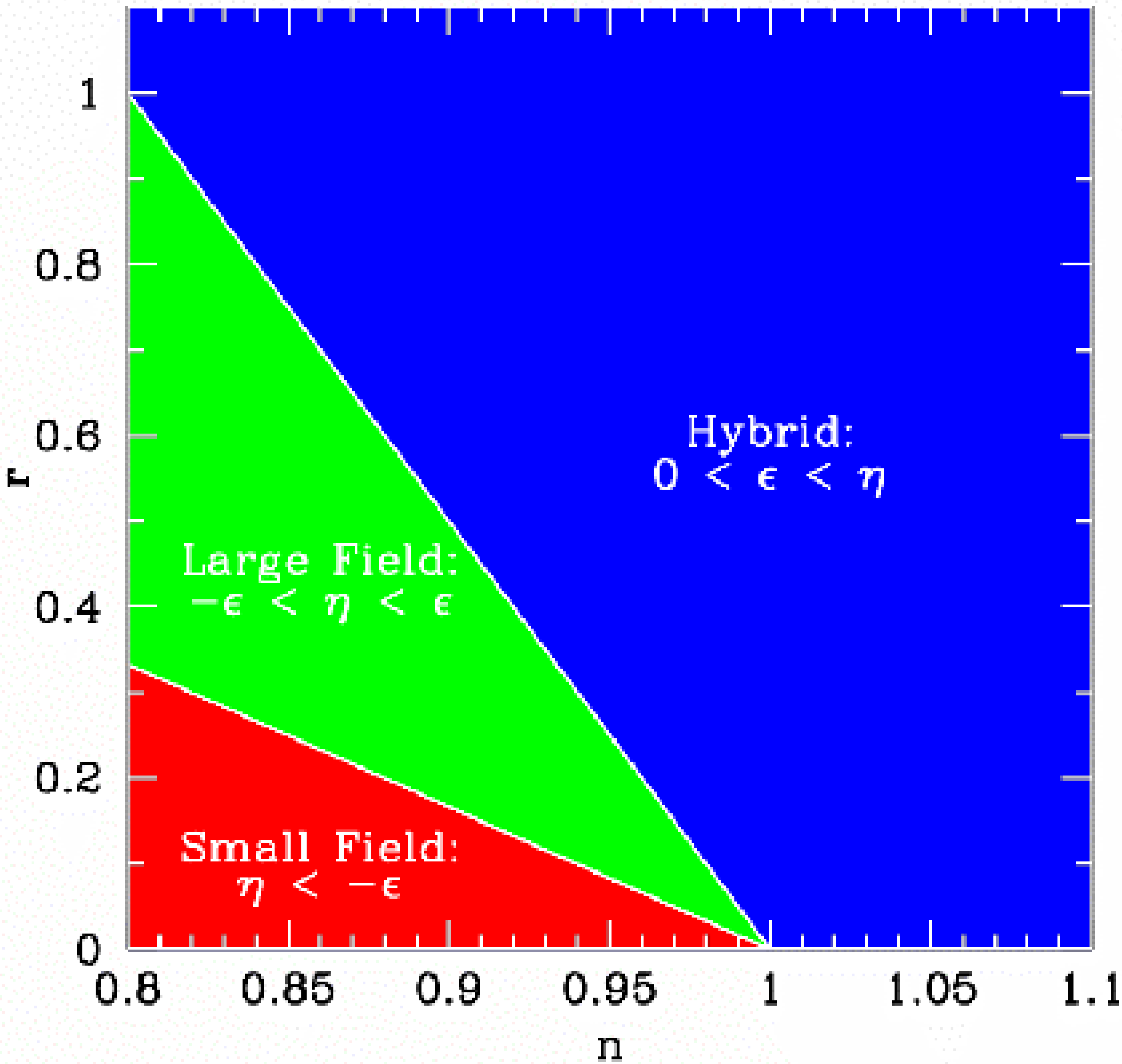
Geometry and matter:

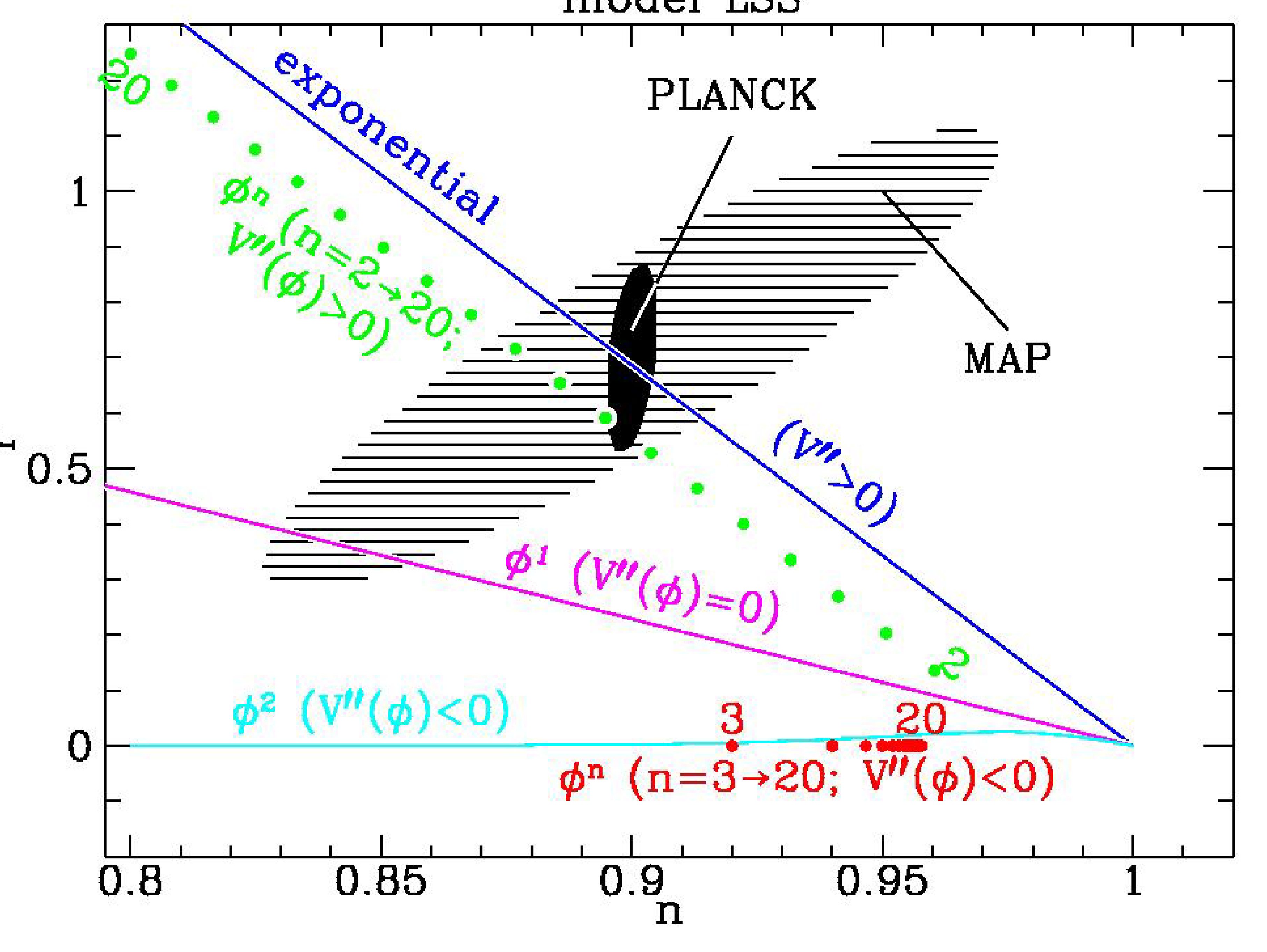
- Homogeneity and Isotropy
- Flat Spatial Sections
- Origin matter & radiation (reheating)

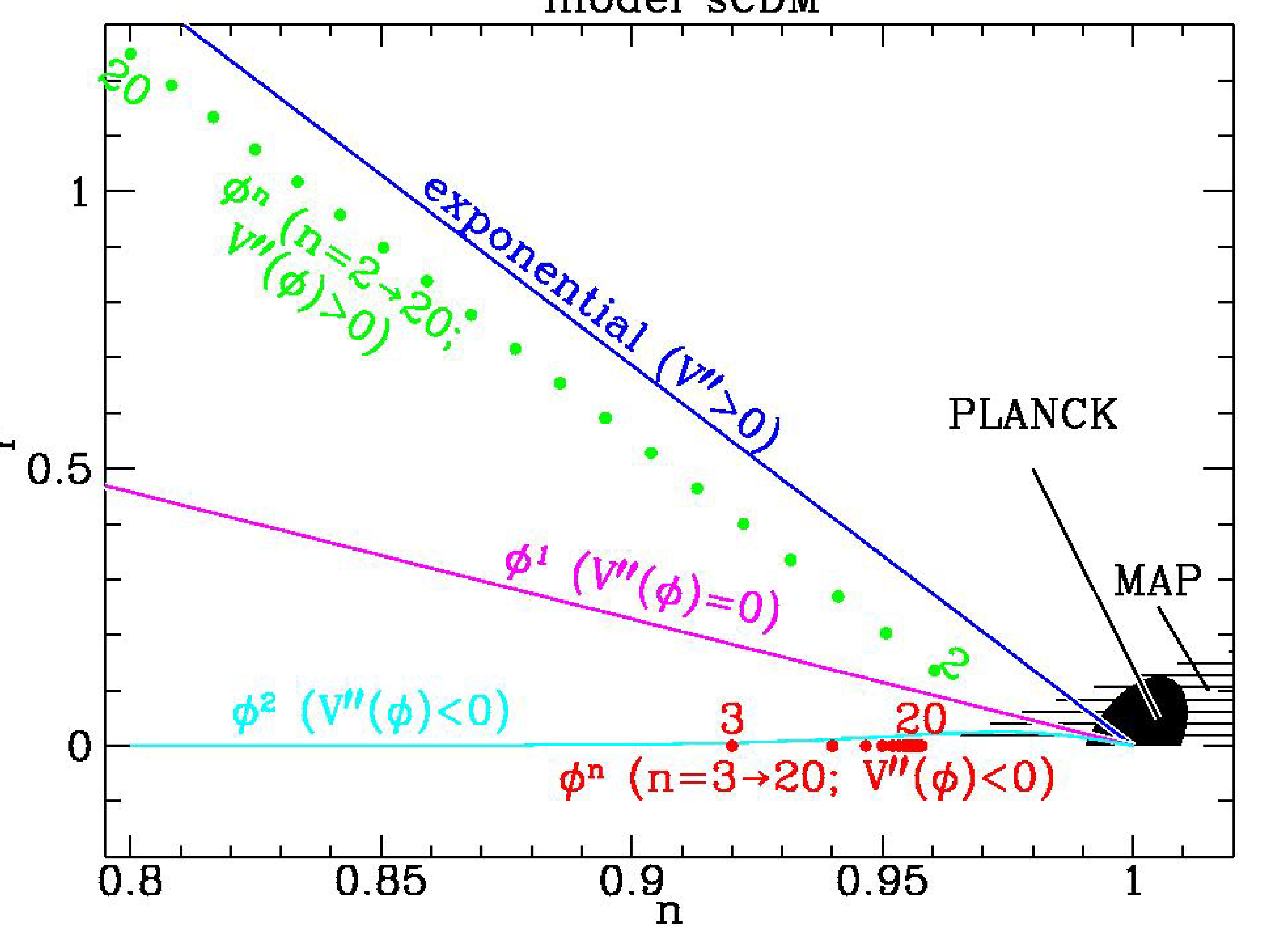
Metric Perturbations:

- Gaussian spectrum
- Aprox. scale invariant
- Adiabatic density fluctuations
- Gravitational waves
- No vector perturbations

single field inflation




model LSS
exponential
PLANCK
MAP
ϕⁿ (n=2→20; V''(ϕ)>0)
20
2
(V''>0)
ϕ¹ (V''(ϕ)=0)
ϕ² (V''(ϕ)<0)
3
20
ϕⁿ (n=3→20; V''(ϕ)<0)
0.5
0
1
r
0.8
0.85
0.9
0.95
1
n


model sCDM
r
n
0.8
0.85
0.9
0.95
1
0
0.5
1
20
ϕ^n (n=2→20; V''(φ)>0)
2
exponential (V''>0)
ϕ^1 (V''(φ)=0)
ϕ^2 (V''(φ)<0)
3
20
ϕ^n (n=3→20; V''(φ)<0)
PLANCK
MAP

Cosmological Observations

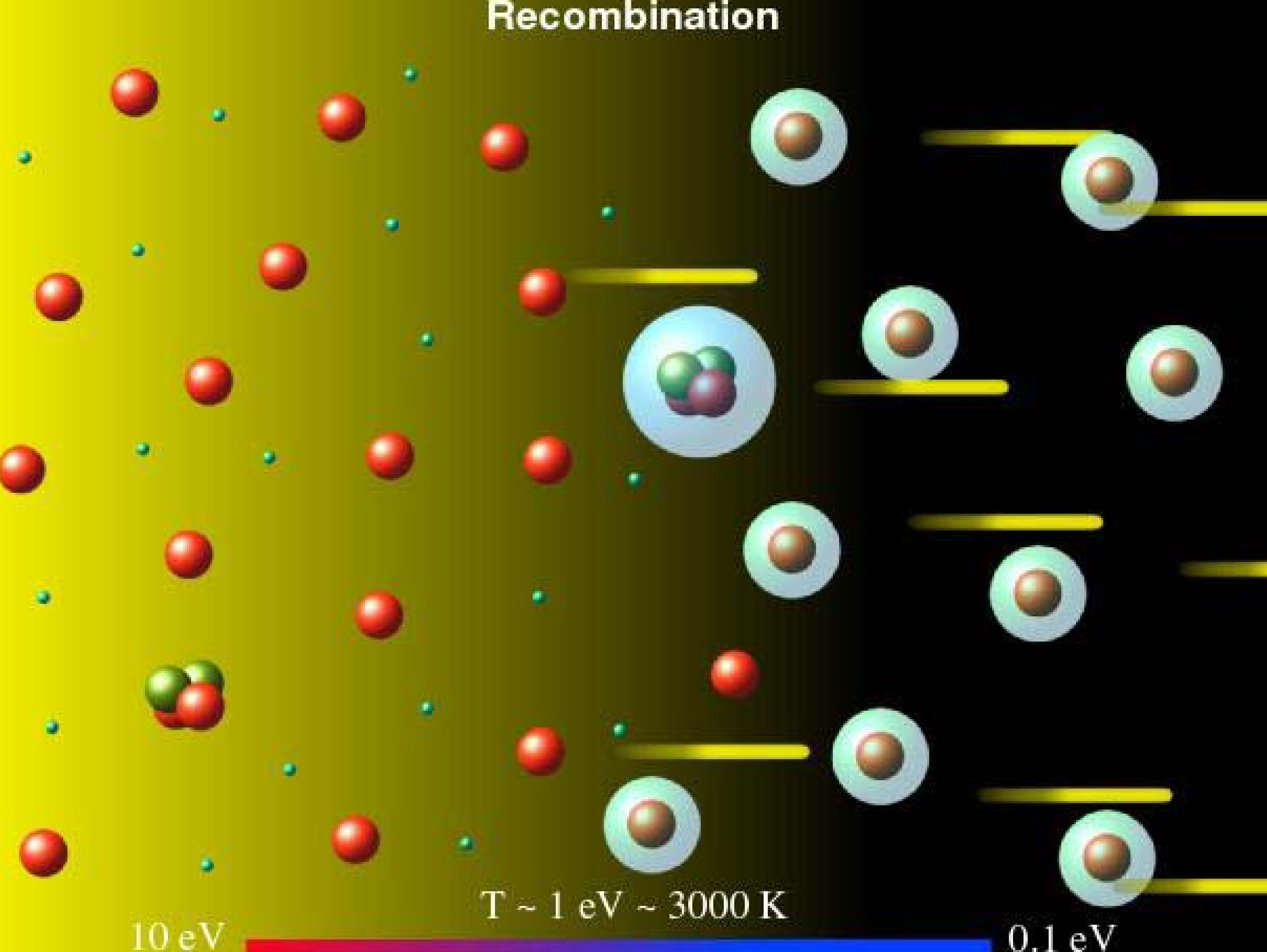
Cosmic Microwave Background:

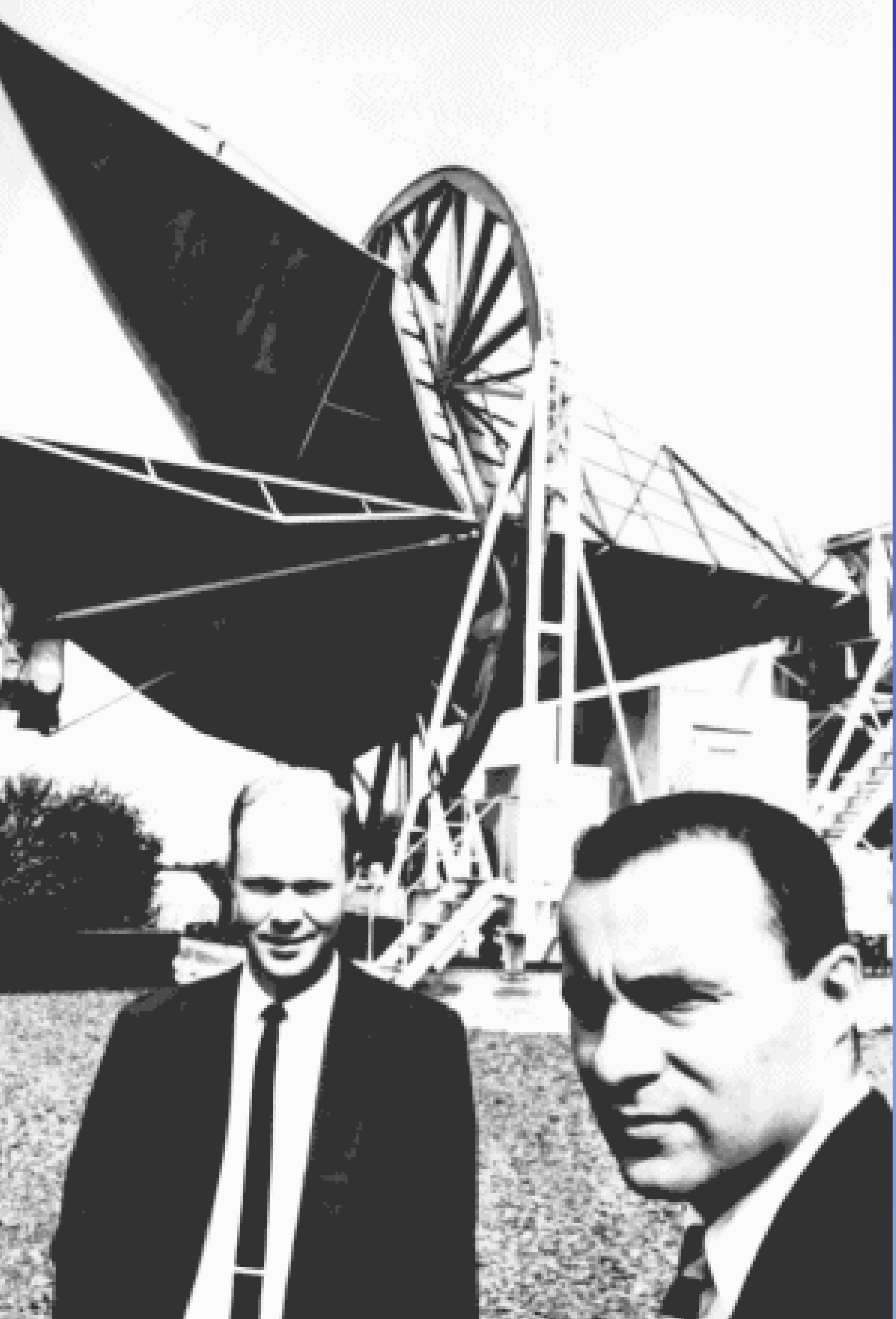
- Temperature Anisotropies (WMAP et al.)
- Polarization Anisotropies (WMAP-TE+DASI)

Large Scale Structure:

- 2dF Galaxy Redshift Survey
- Sloan Digital Sky Survey

Cosmic Microwave Background

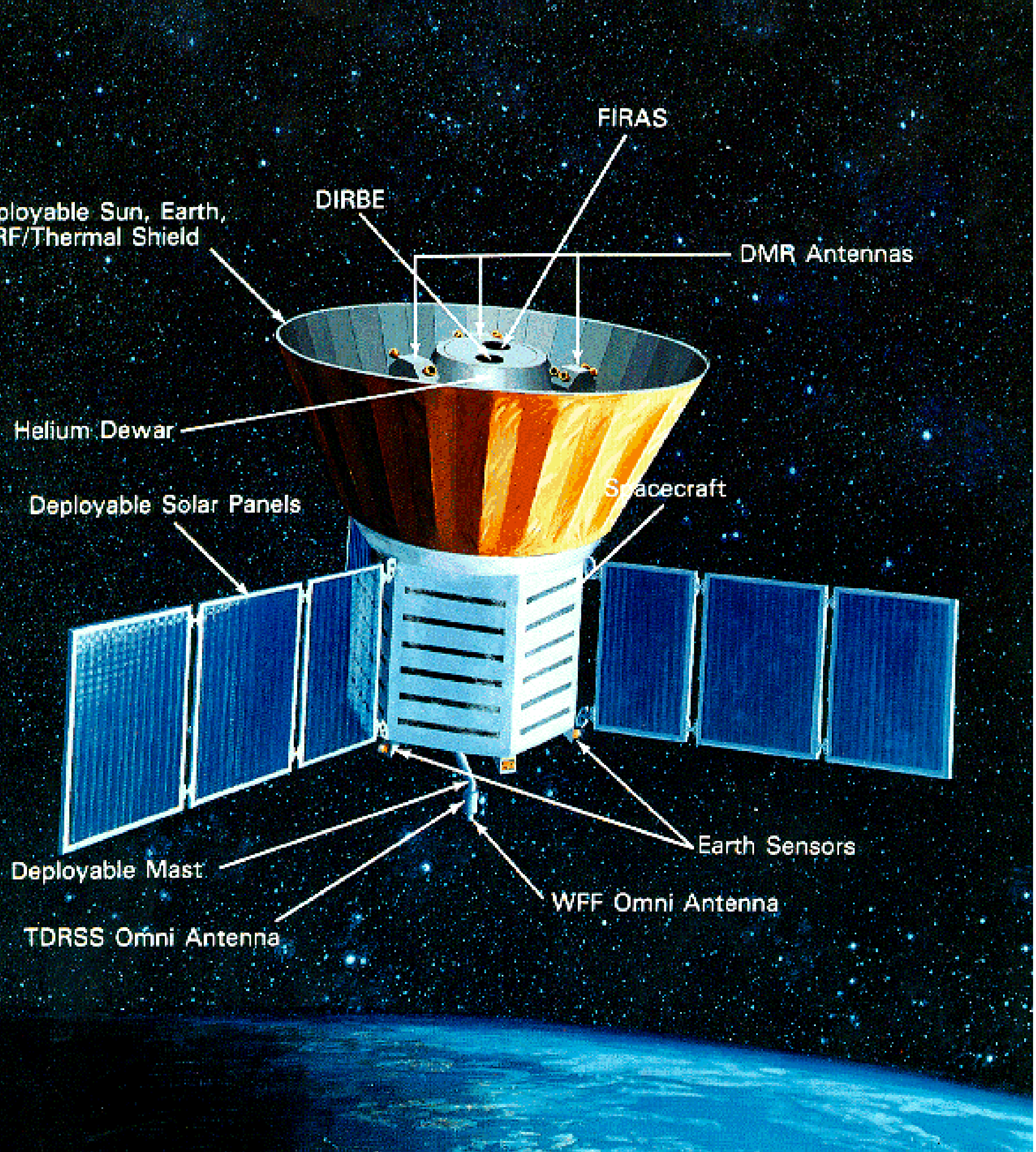

Recombination
T ~ 1 eV ~ 3000 K
10 eV
0.1 eV

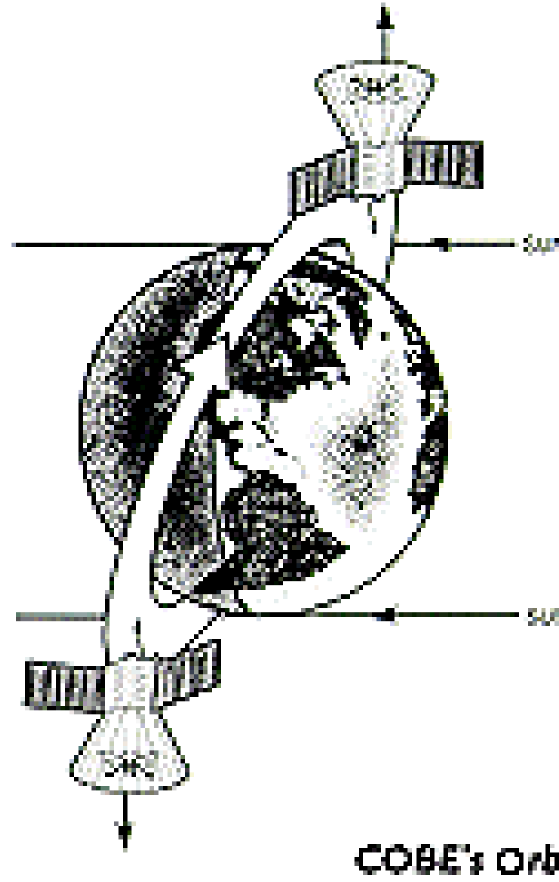


Discovery of CMB

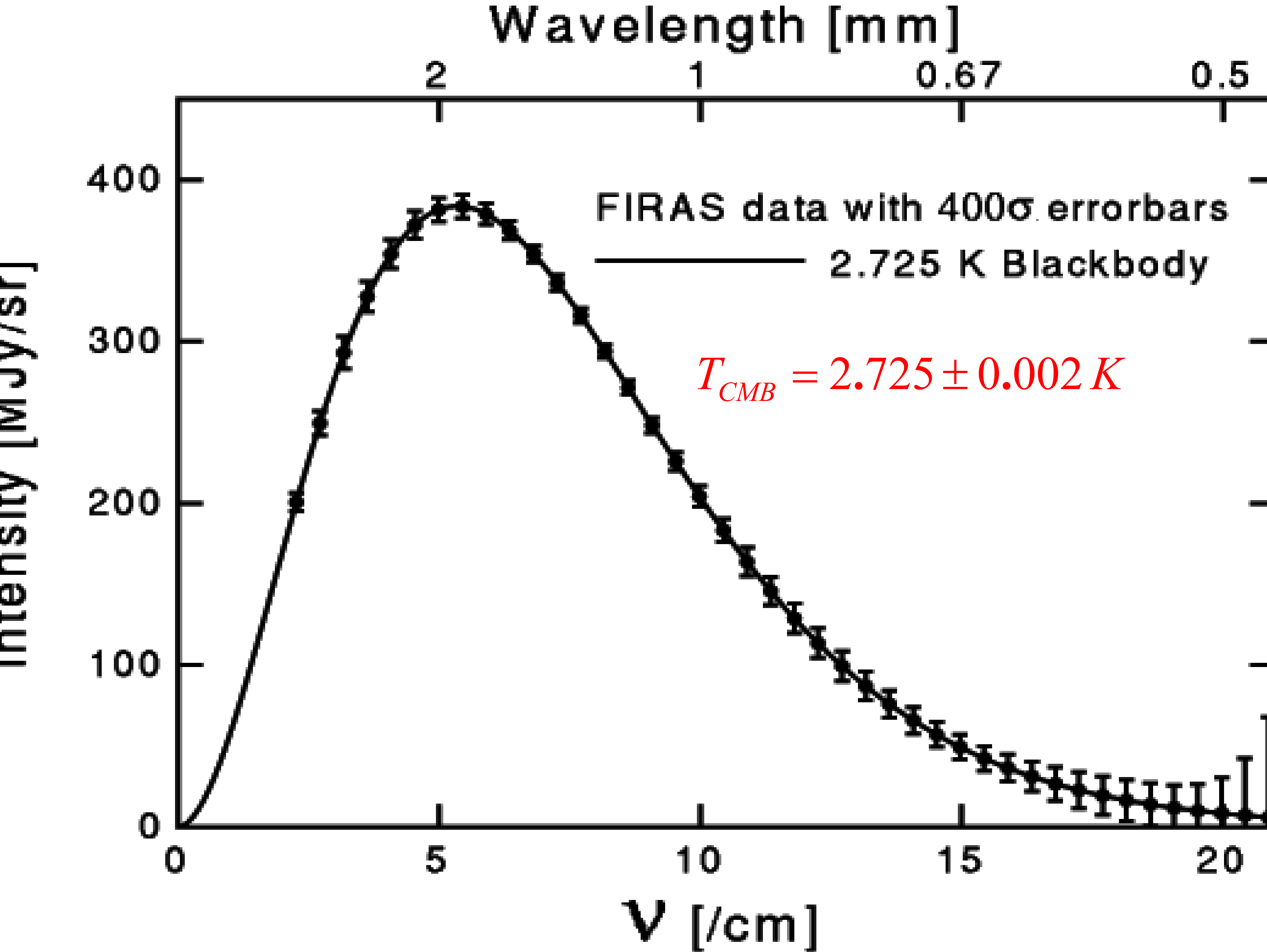
Arno Penzias
Robert Wilson
(1965)

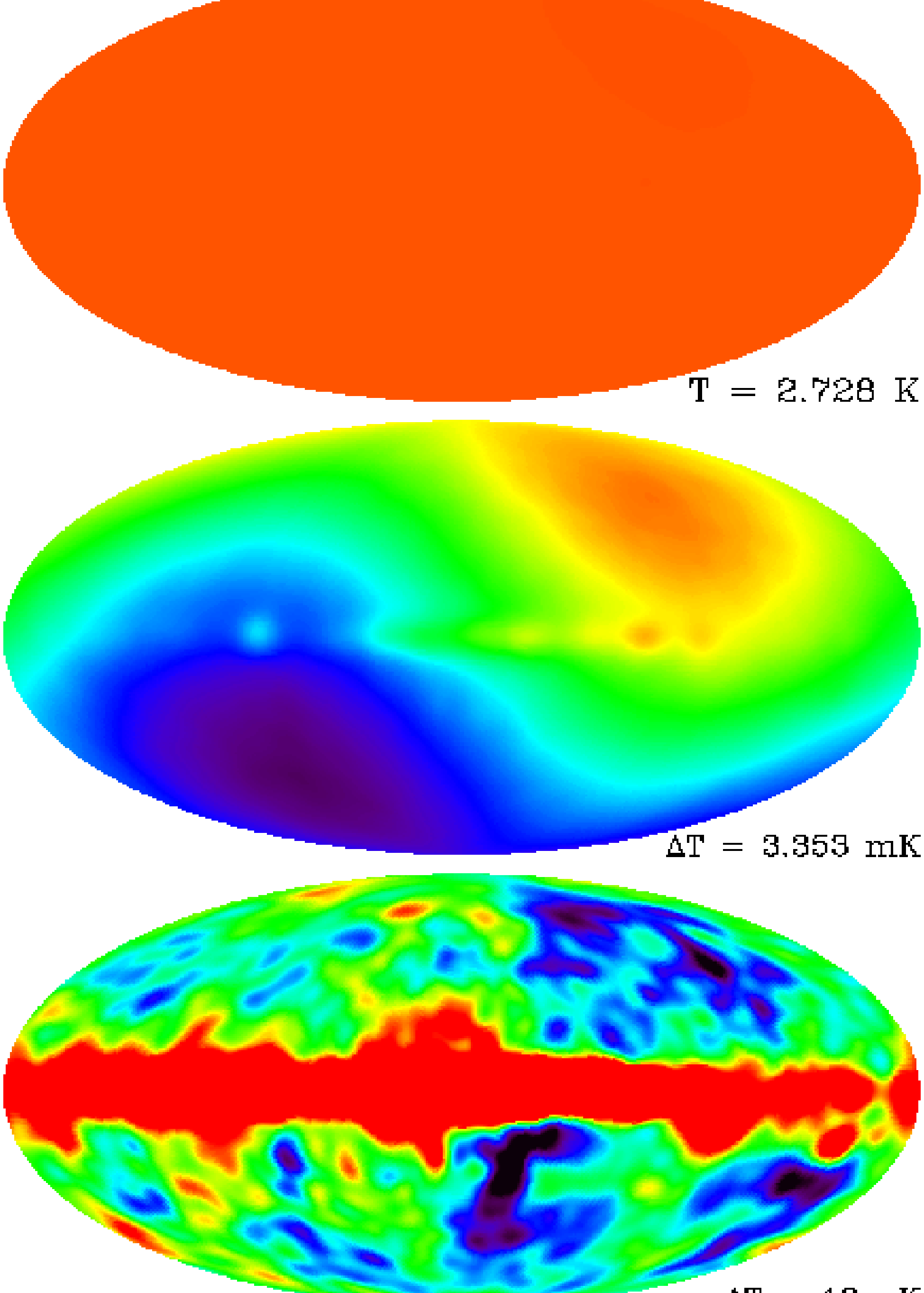
Blackbody Spectrum
T=3K
isotropic





COBE
(1989-1992)


Wavelength [mm]
2
1
0.67
0.5
400
300
200
100
0
Intensity [MJy/sr]
FIRAS data with 400σ errorbars
2.725 K Blackbody
$T_{CMB} = 2.725 \pm 0.002\, K$
0
5
10
15
20
ν [/cm]



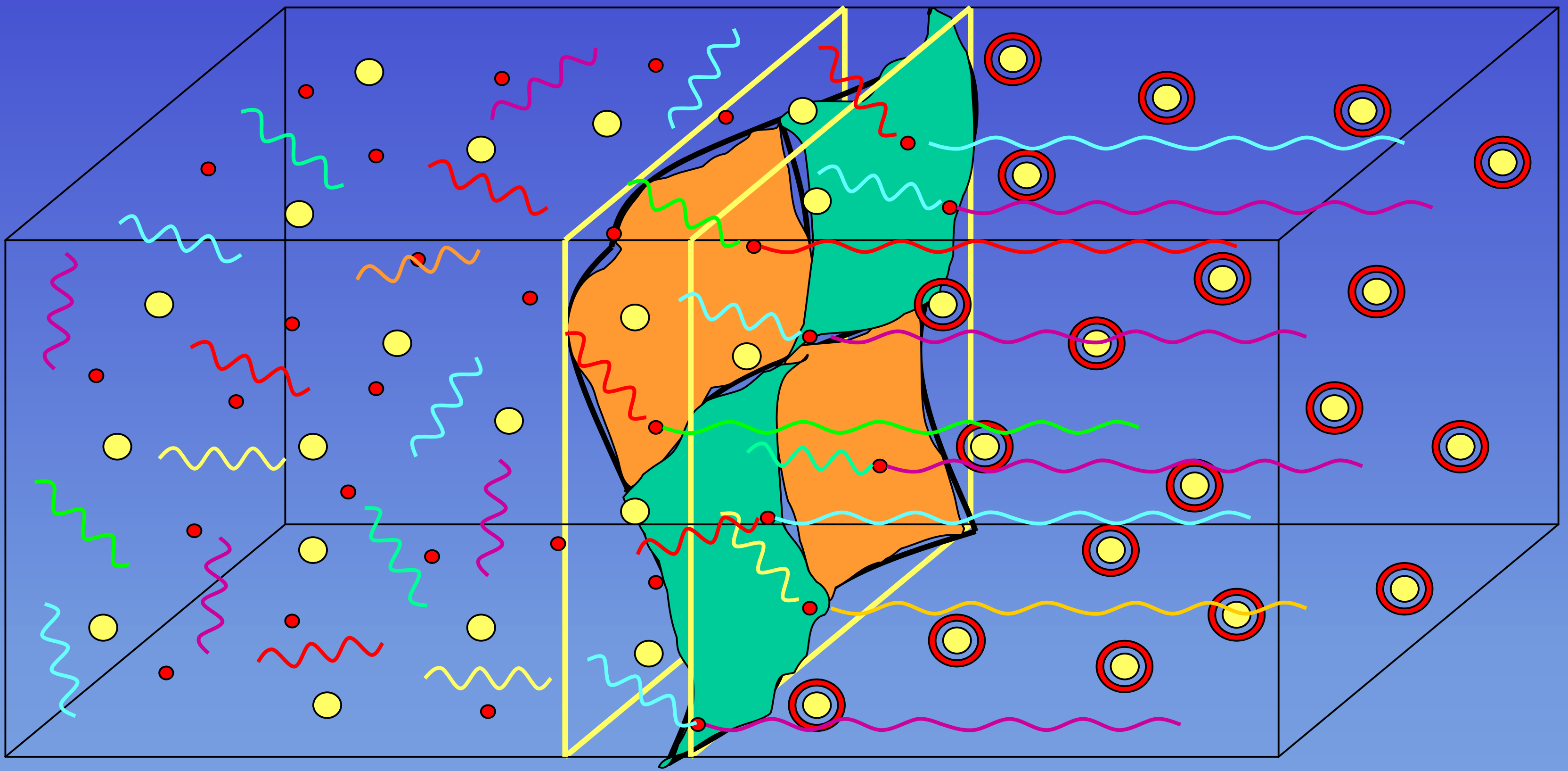
COBE 4-year Measurements (1992-1996)

First Measurements Temperature Anisotropies (1992)

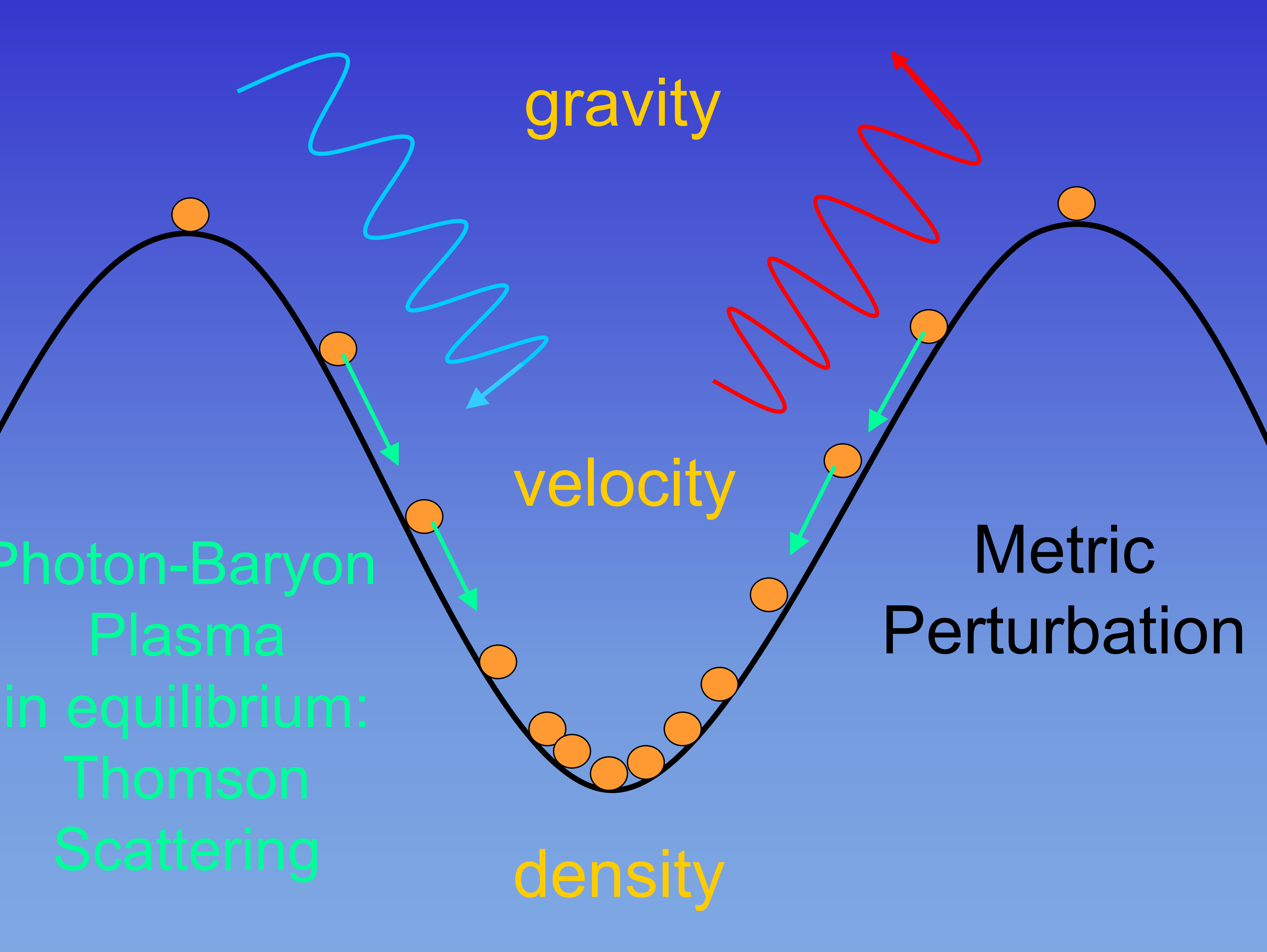
$$\frac{\Delta T}{T_0} \approx 10^{-5}$$

Temperature Anisotropies

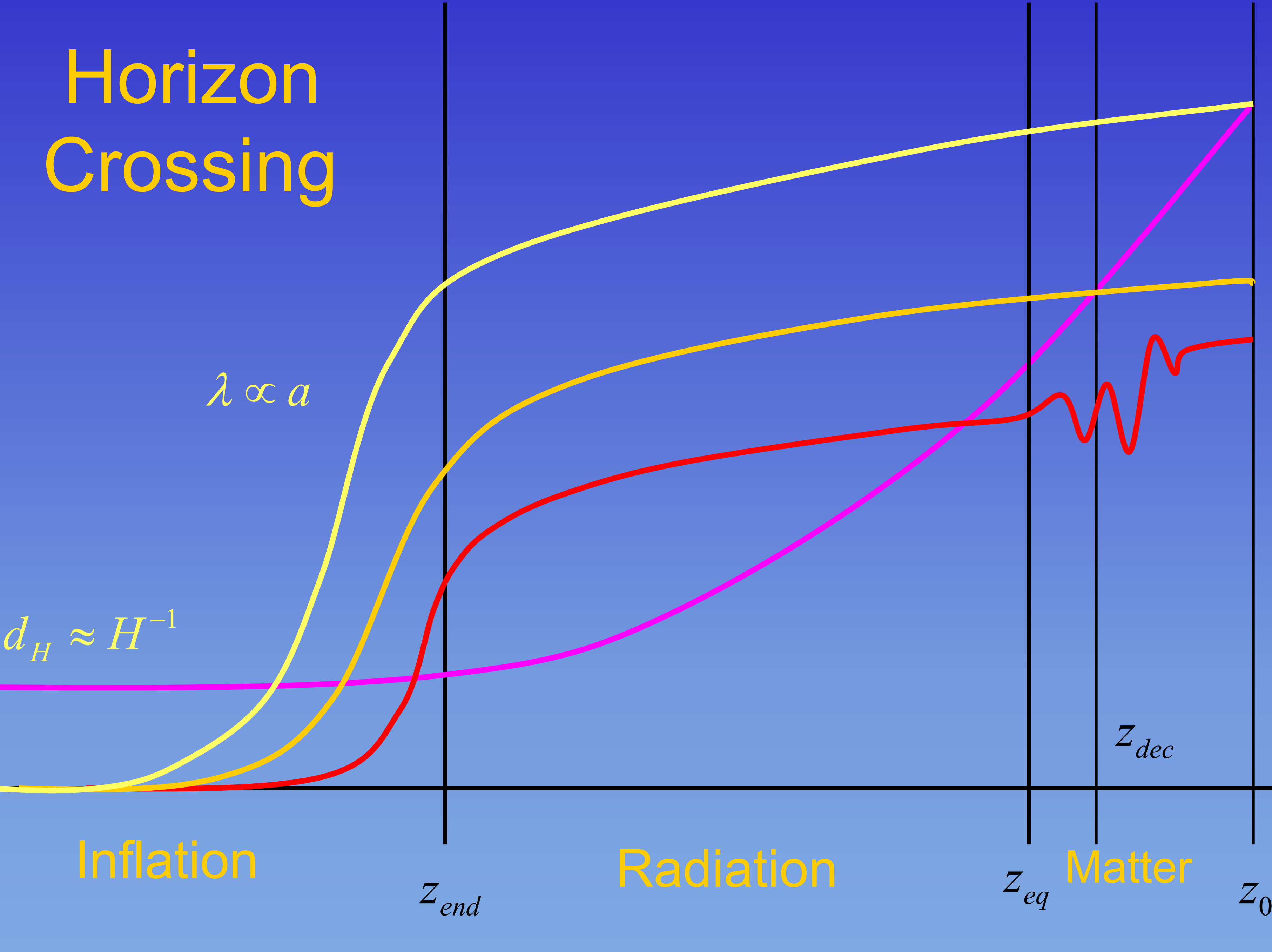
The microwave background is a snapshot of the last scattering surface

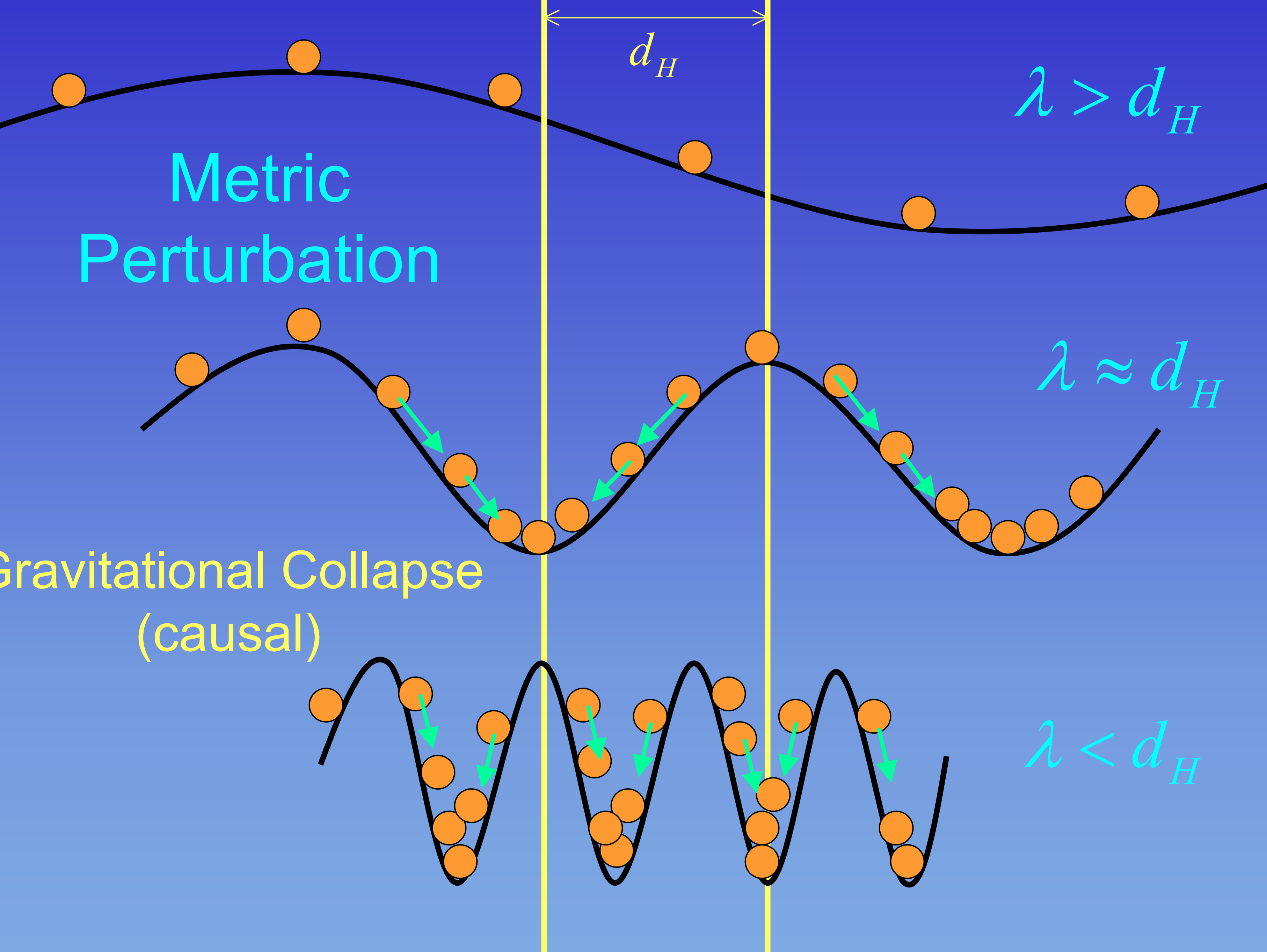


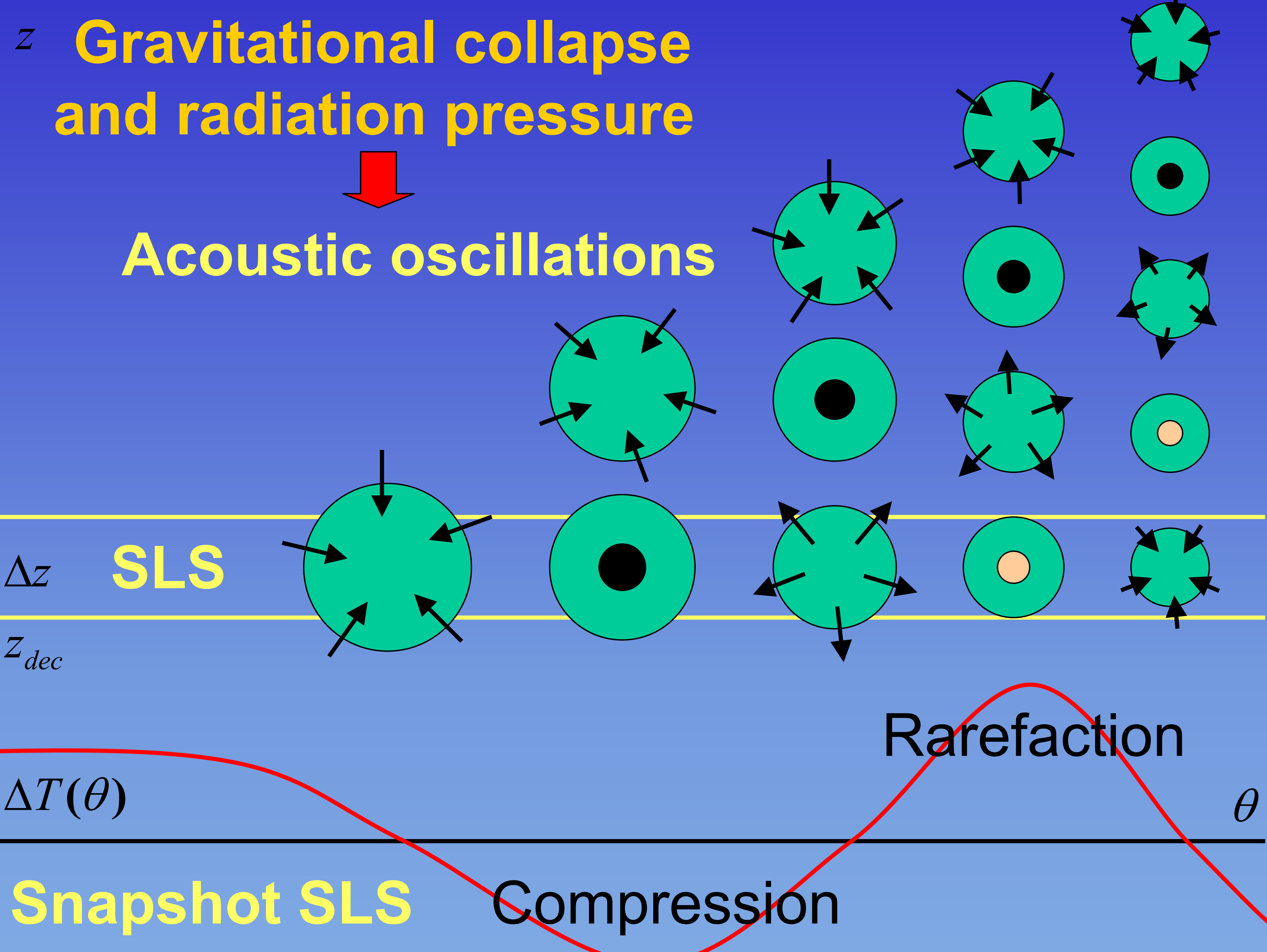
The anisotropies reflect the perturbations in the surface of last scattering


gravity
velocity
density
Photon-Baryon
Plasma
in equilibrium:
Thomson
Scattering
Metric
Perturbation

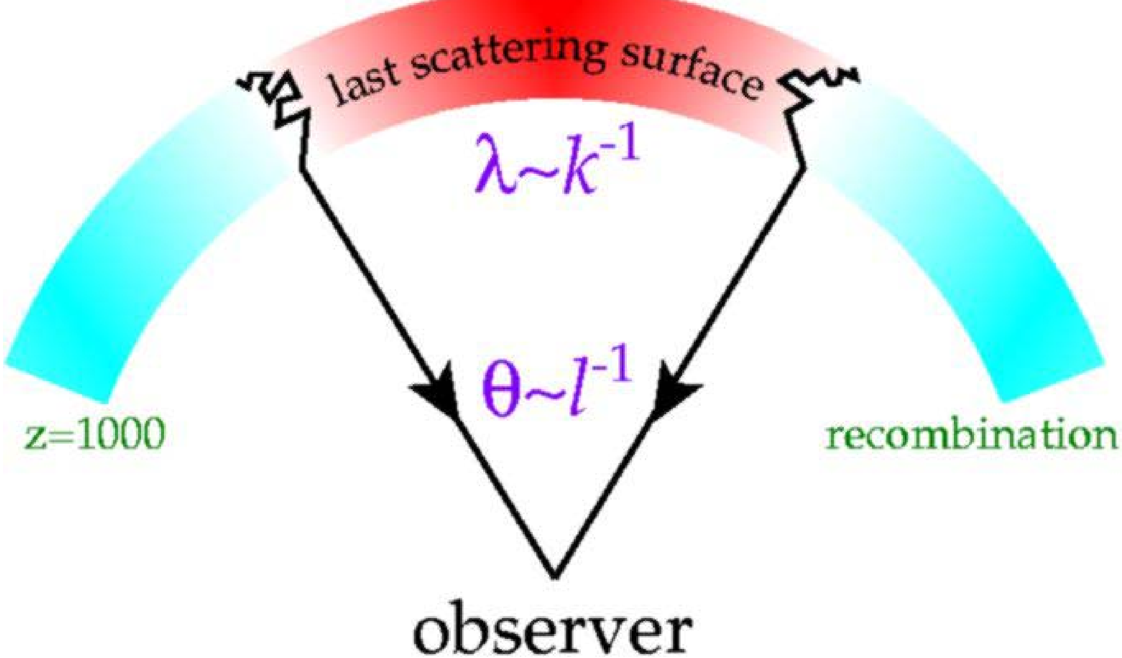
Horizon Crossing

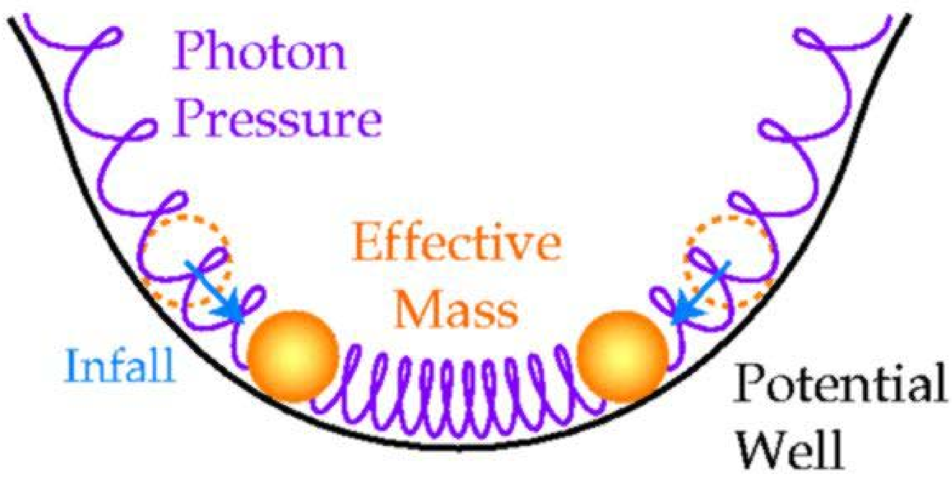


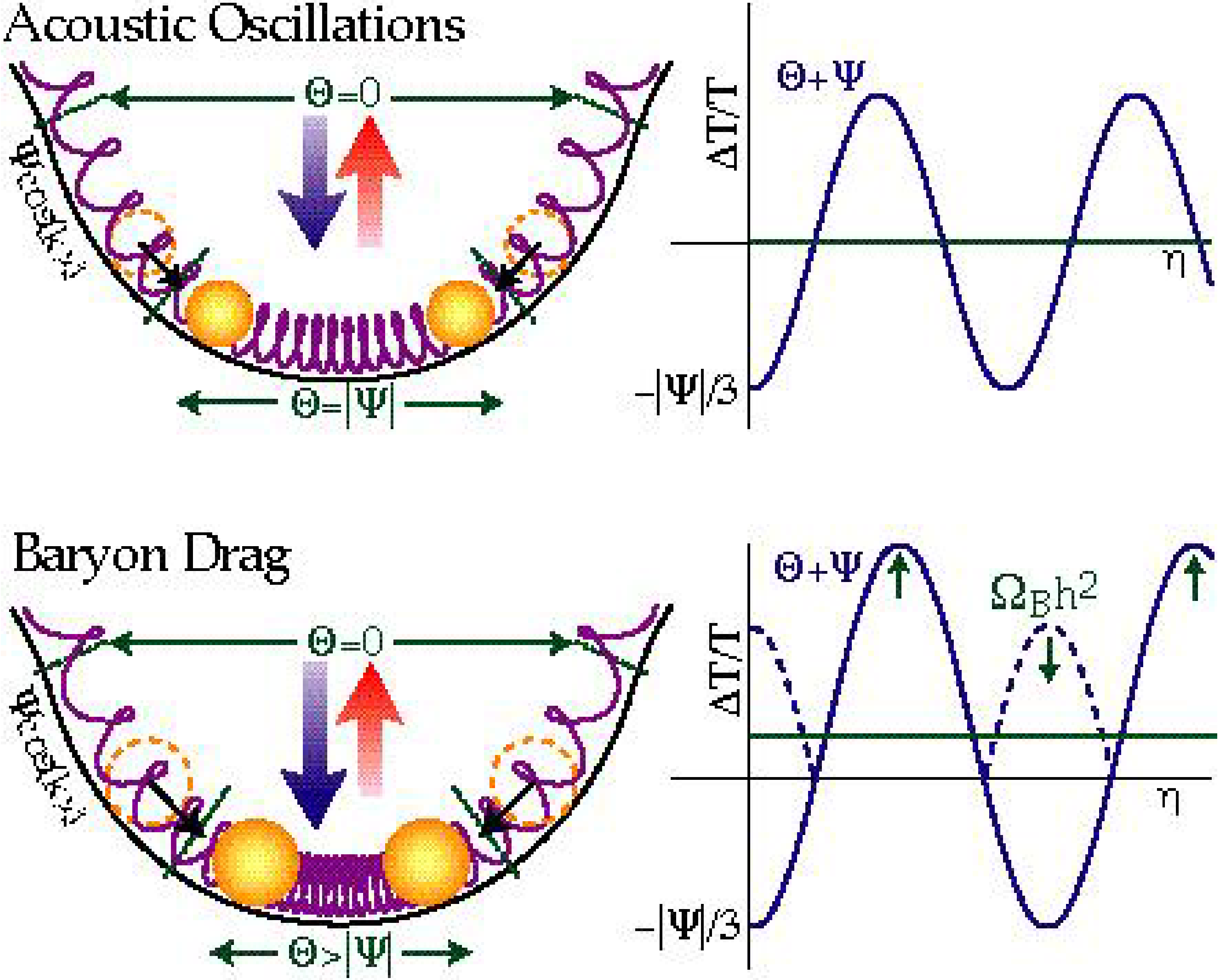

d_H
$\lambda > d_H$
Metric Perturbation
$\lambda \approx d_H$
Gravitational Collapse (causal)
$\lambda < d_H$

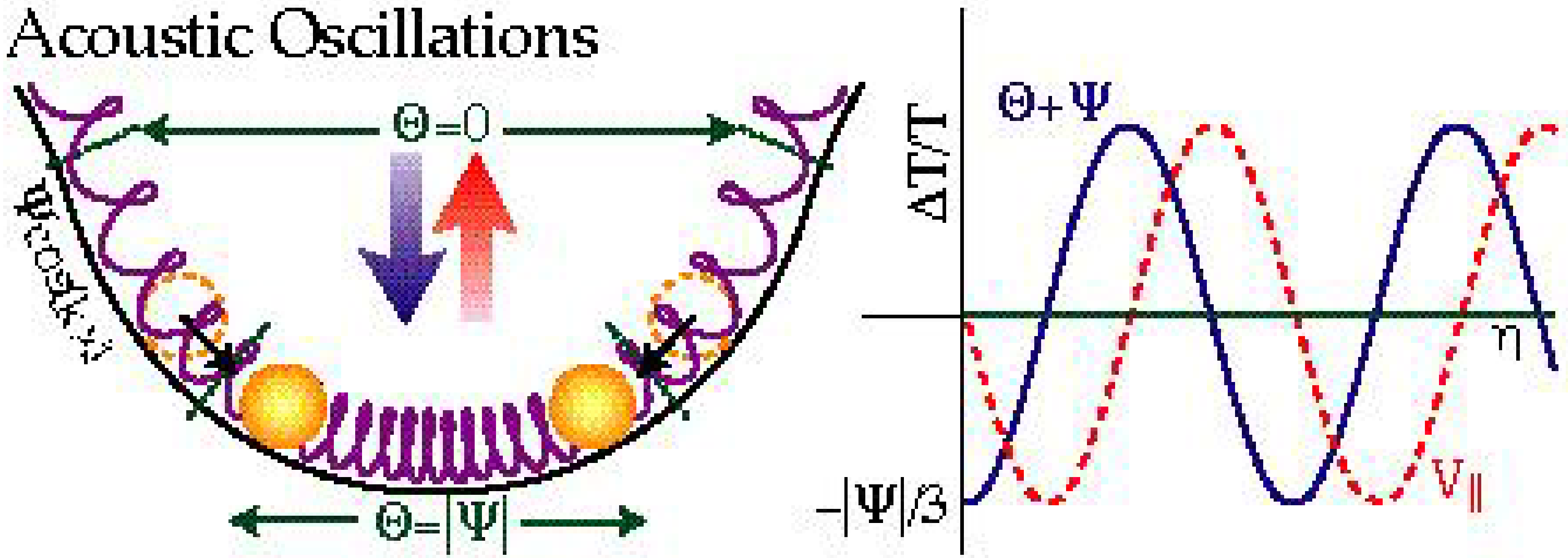


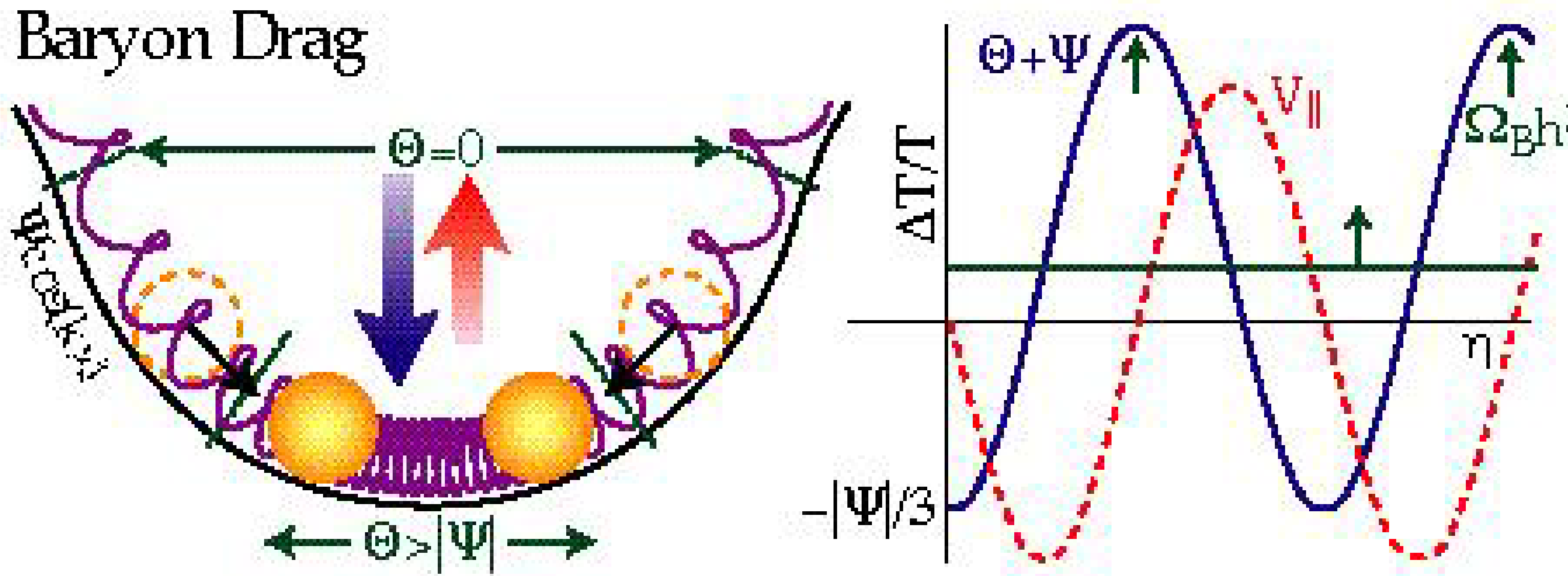
z
Gravitational collapse
and radiation pressure
Acoustic oscillations
Δz
SLS
z_{dec}
Rarefaction
$\Delta T(\theta)$
θ
Snapshot SLS
Compression

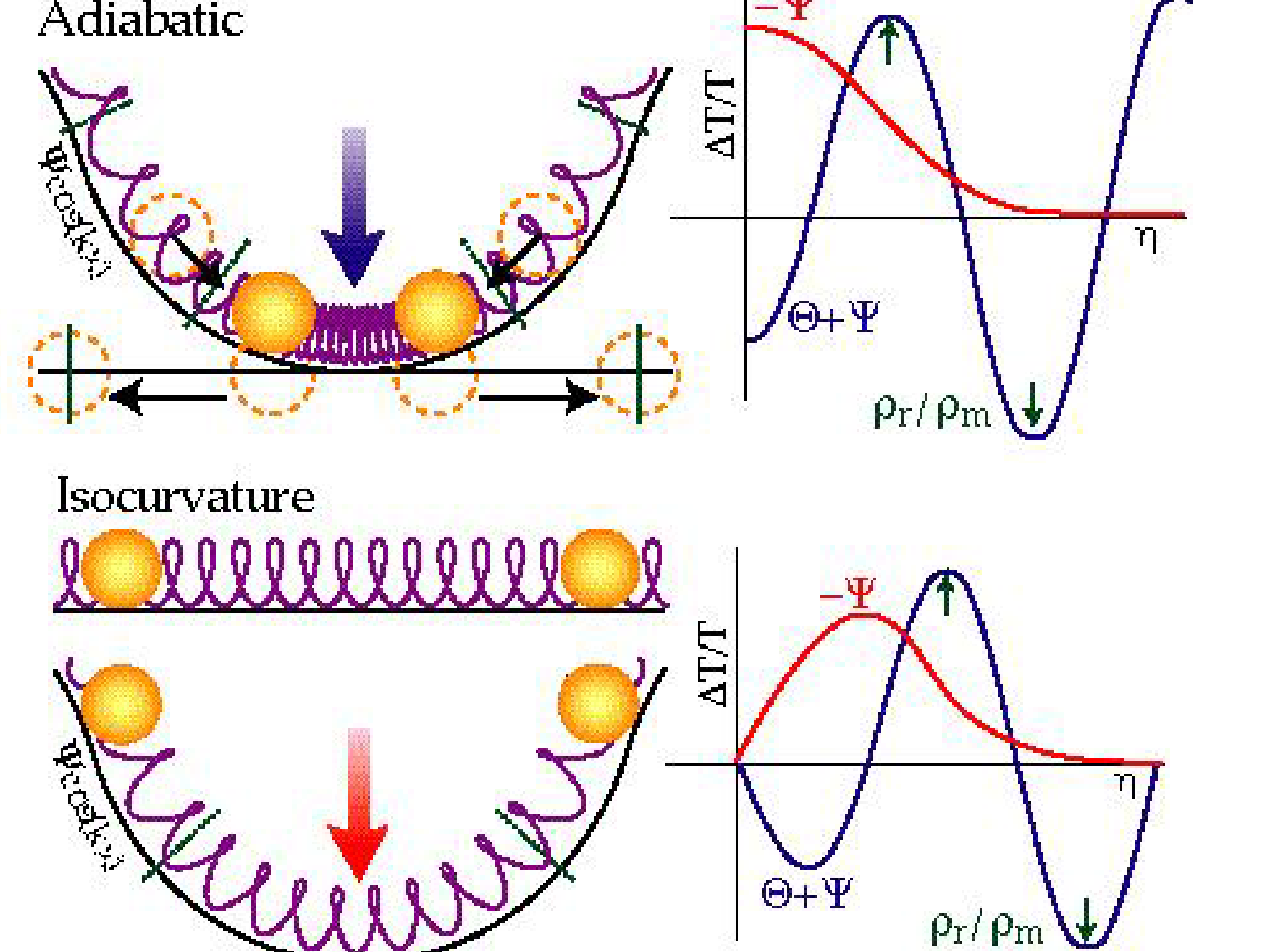

last scattering surface
λ~k^-1
θ~l^-1
z=1000
recombination
observer


Photon
Pressure
Effective
Mass
Infall
Potential
Well

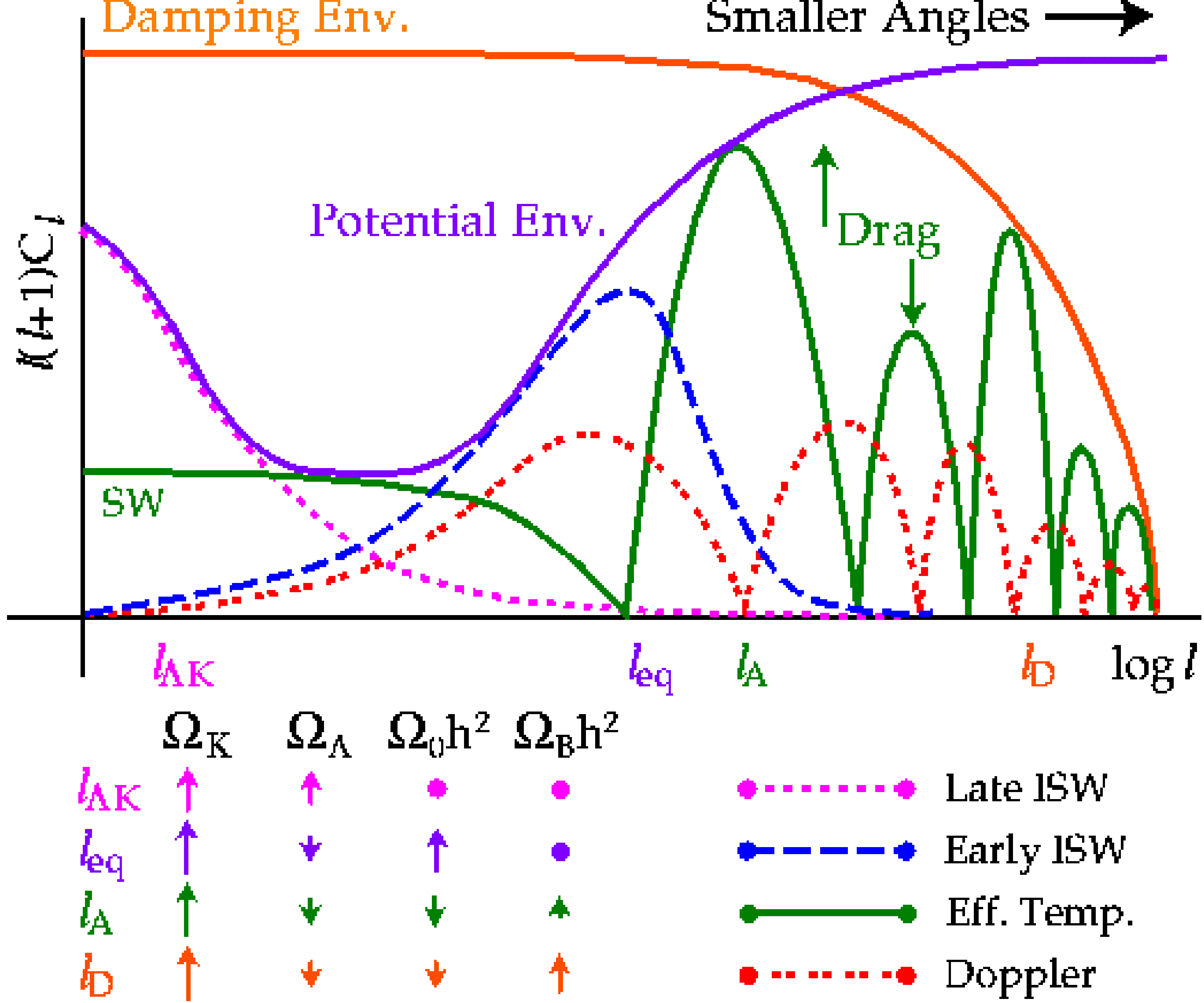

Acoustic Oscillations
Θ=0
Θ=|Ψ|
Θ+Ψ
ΔT/T
−|Ψ|/3
η
Baryon Drag
Θ=0
Θ>|Ψ|
Θ+Ψ
$\Omega_B h^2$
ΔT/T
−|Ψ|/3
η


Acoustic Oscillations
Θ=0
Θ=|Ψ|
Θ+Ψ
ΔT/T
η
V∥
−|Ψ|/3


Baryon Drag
Θ=0
Θ>|Ψ|
Θ+Ψ
ΔT/T
V∥
η
−|Ψ|/3



Adiabatic
−Ψ
ΔT/T
Θ+Ψ
η
ρr / ρm
Ψcos(kx)
Isocurvature
−Ψ
ΔT/T
Θ+Ψ
η
ρr / ρm
Ψcos(kx)

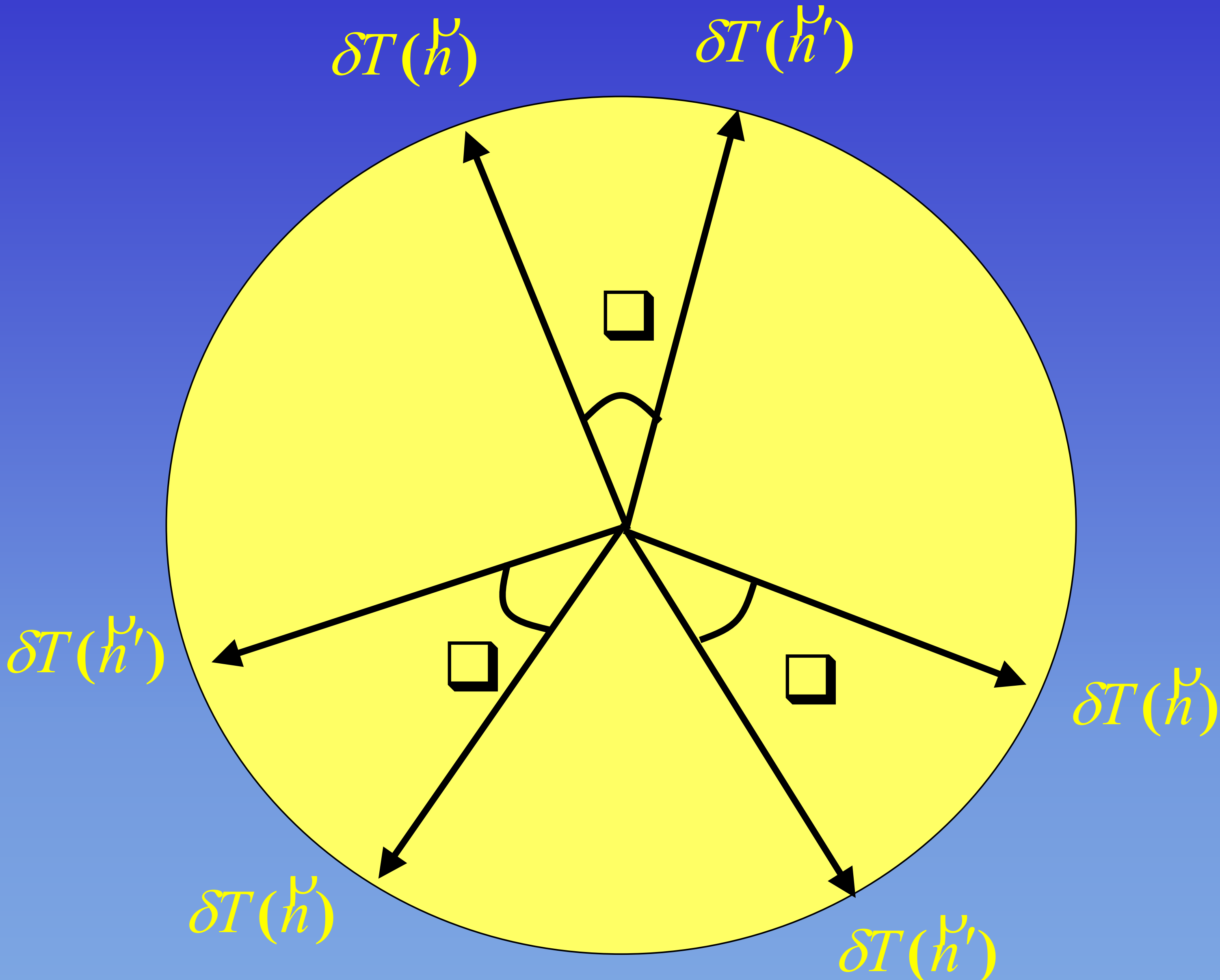


Damping Env.
Smaller Angles
Potential Env.
Drag
l(l+1)C_l
SW
l_ΛK
l_eq
l_A
l_D
log l
Ω_K Ω_Λ Ω_0h^2 Ω_Bh^2
l_ΛK
l_eq
l_A
l_D
Late ISW
Early ISW
Eff. Temp.
Doppler

Angular power spectrum

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{lm} a_{lm}\, Y_{lm}(\theta,\phi)$$

$$C(\theta) = \left\langle \delta T^*(\vec{n})\, \delta T(\vec{n}') \right\rangle_{n\cdot n' = \cos\theta} = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l\, P_l(\cos\theta)$$



$\delta T(\vec{n})$
$\delta T(\vec{n}')$
$\delta T(\vec{n}')$
$\delta T(\vec{n})$
$\delta T(\vec{n})$
$\delta T(\vec{n}')$

Angular power spectrum

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{lm} a_{lm}\, Y_{lm}(\theta,\phi)$$

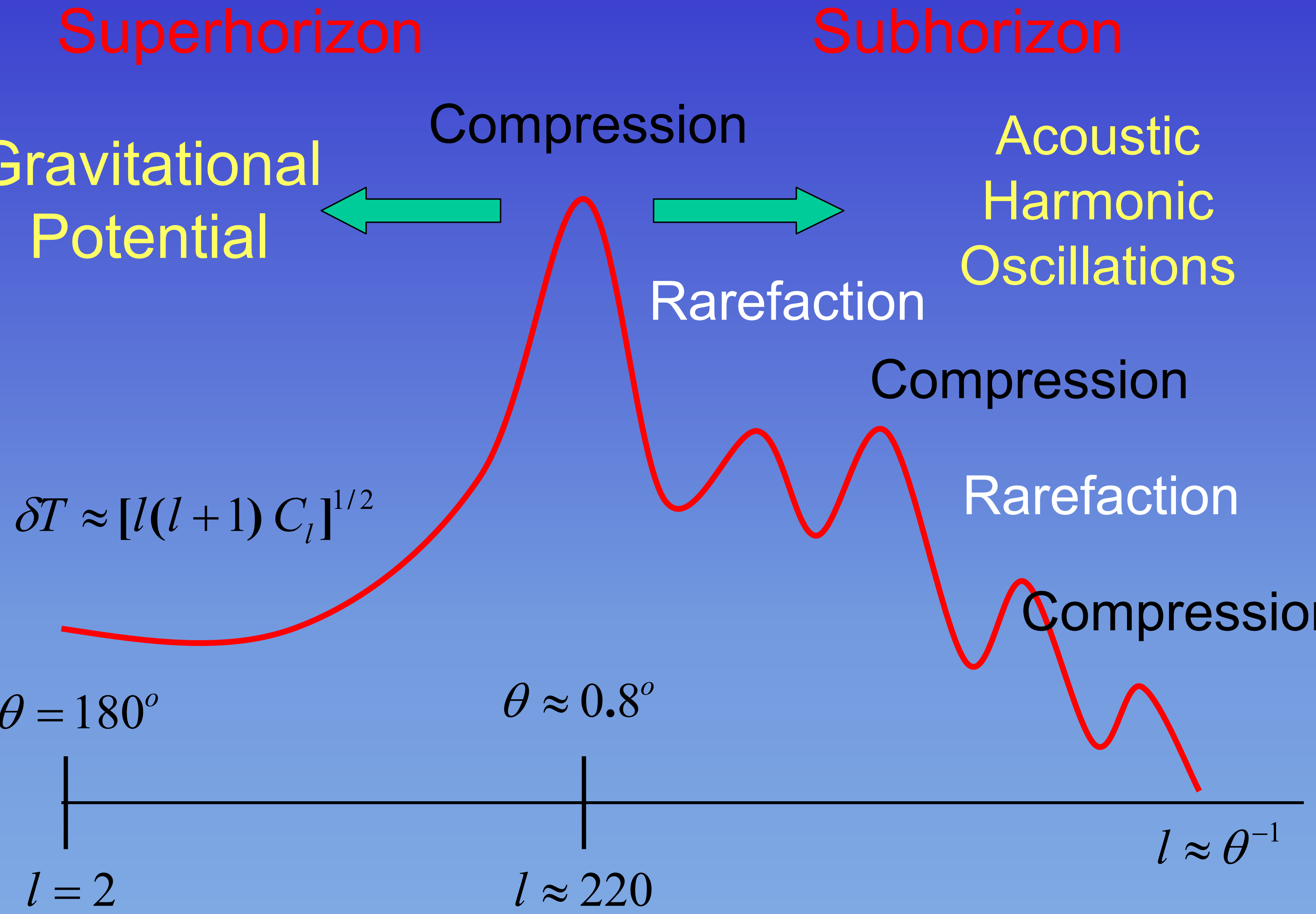
$$C(\theta) = \left\langle \delta T^*(\vec{n})\, \delta T(\vec{n}') \right\rangle_{n \cdot n' = \cos\theta} = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l\, P_l(\cos\theta)$$

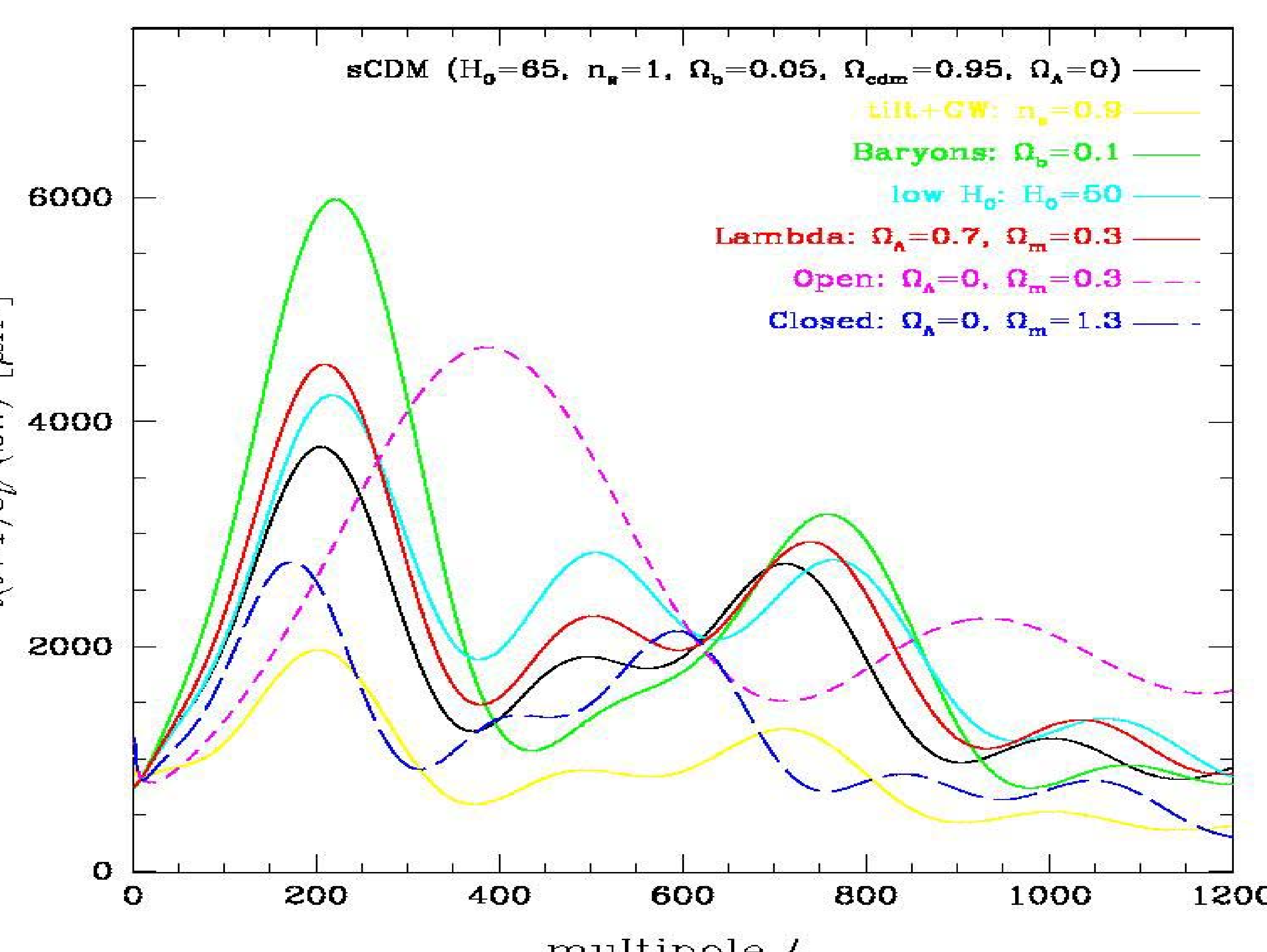
$$C_l^S = \frac{4\pi}{25} \int_0^\infty \frac{dk}{k} P_S(k)\, j_l^2(kr) \qquad P_S(k) = A_S^2 \left(\frac{k}{k_*}\right)^{n-1}$$

Sachs-Wolfe plateau

$$\frac{l(l+1)C_l^S}{2\pi} = \frac{A_S^2}{25} = const$$

Angular Power Spectrum



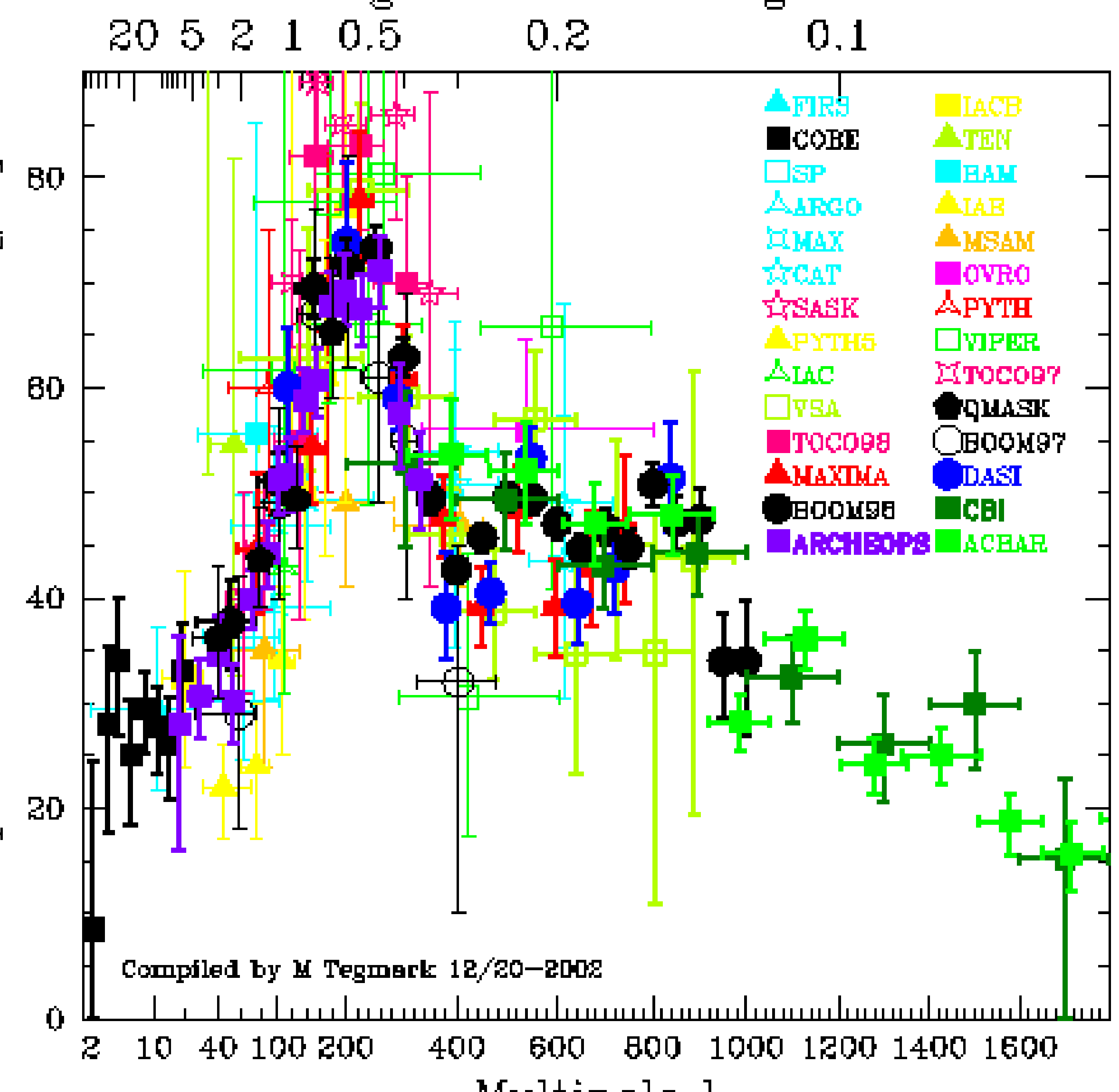


sCDM (H0=65, ns=1, Ωb=0.05, Ωcdm=0.95, ΩΛ=0)
tilt+GW: ns=0.9
Baryons: Ωb=0.1
low H0: H0=50
Lambda: ΩΛ=0.7, Ωm=0.3
Open: ΩΛ=0, Ωm=0.3
Closed: ΩΛ=0, Ωm=1.3
6000
4000
2000
0
0
200
400
600
800
1000
1200

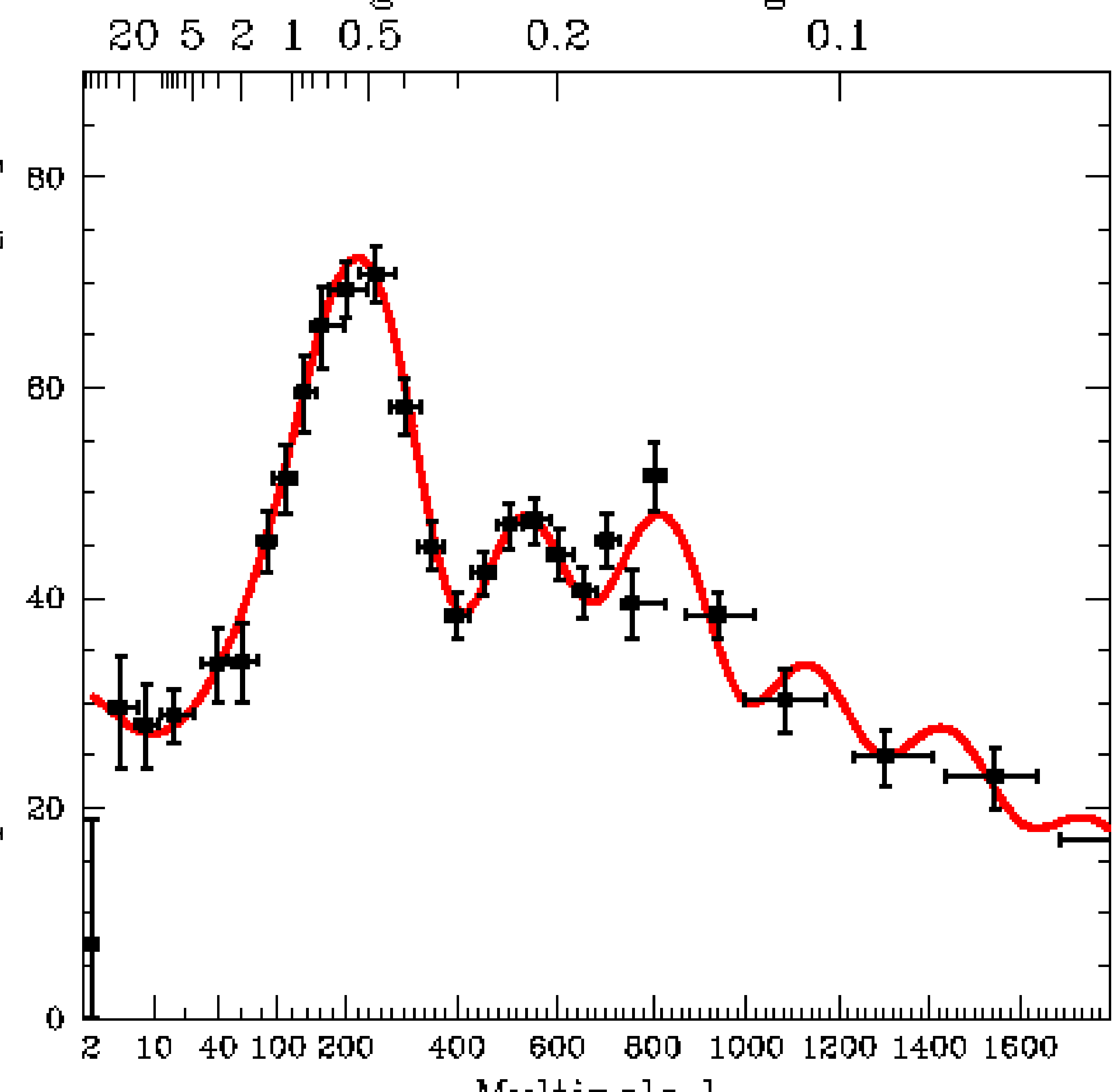
Degeneracies in the determination of parameters

↓

It is necessary to make a multiparameter fit with the largest possible data set

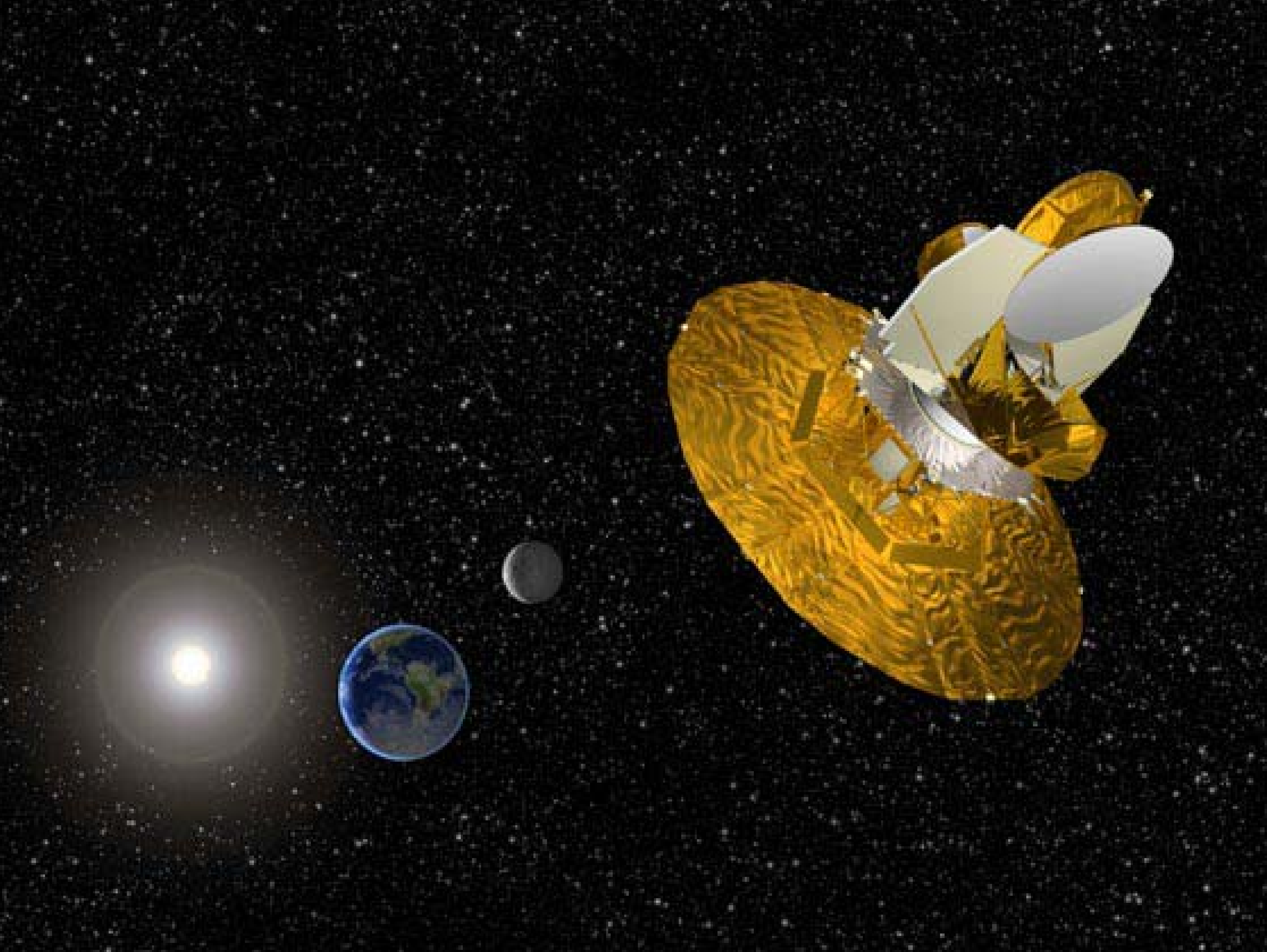


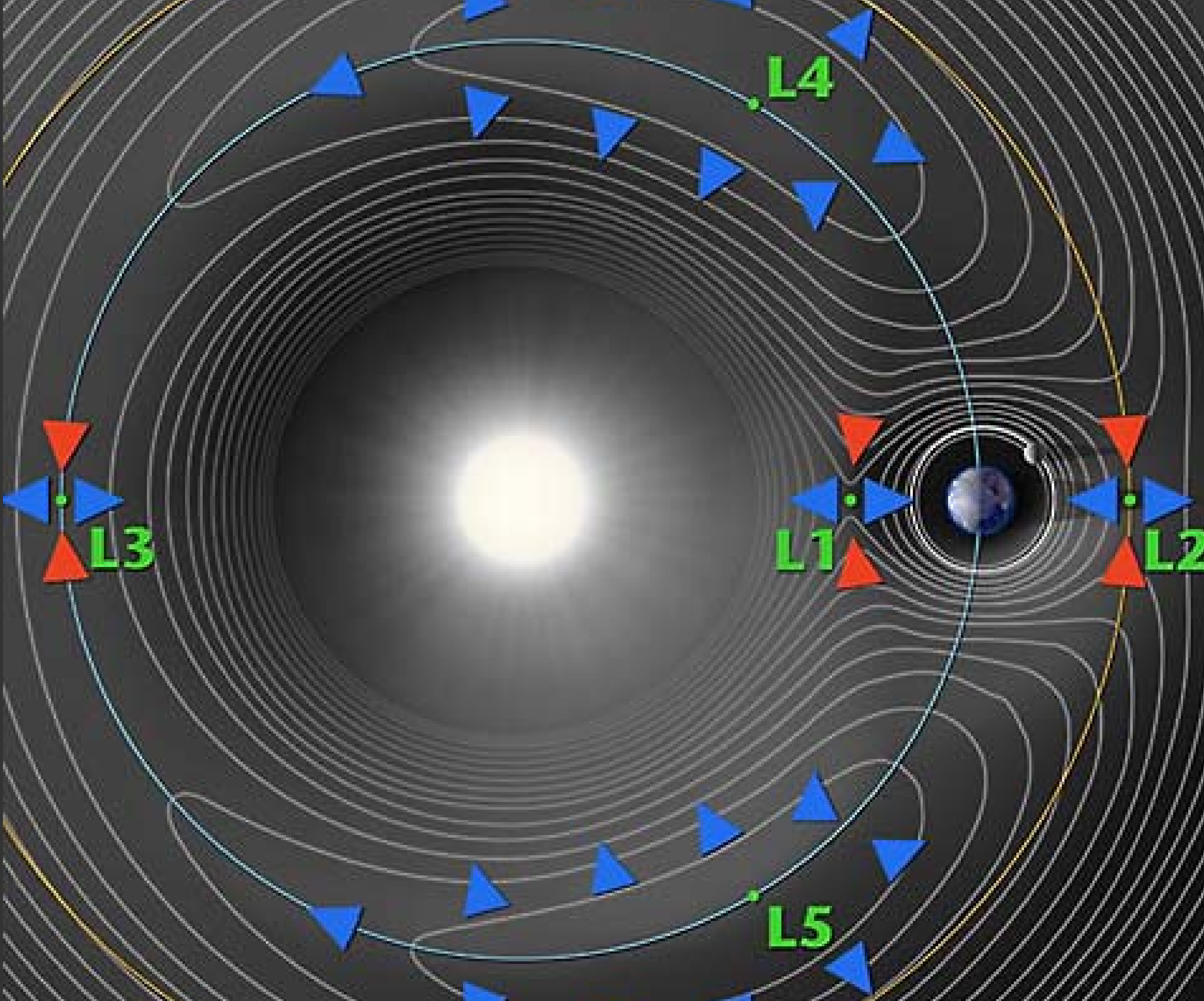
All CMB Exp. (2002)



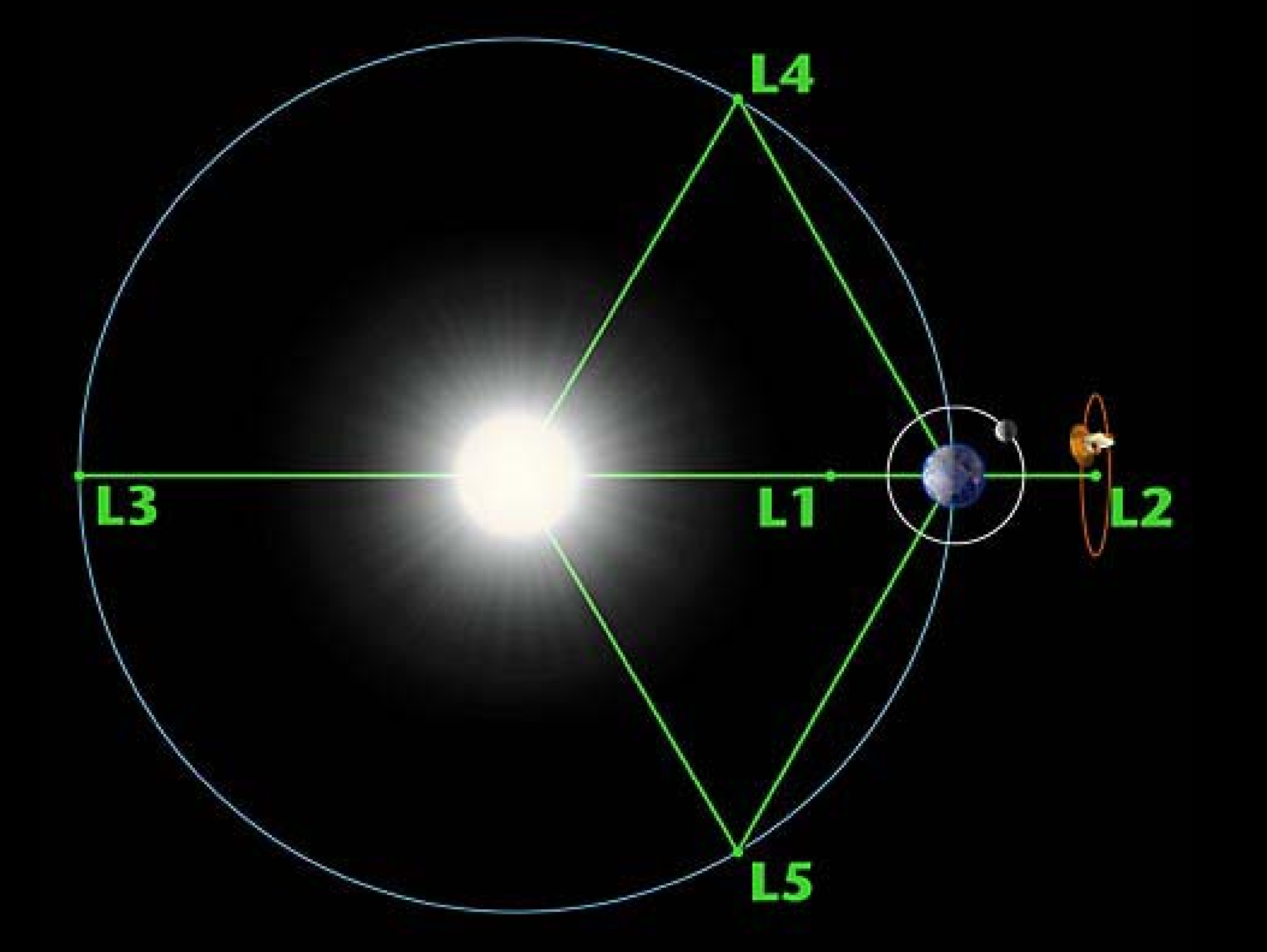
Best fit to all CMB data (2002)

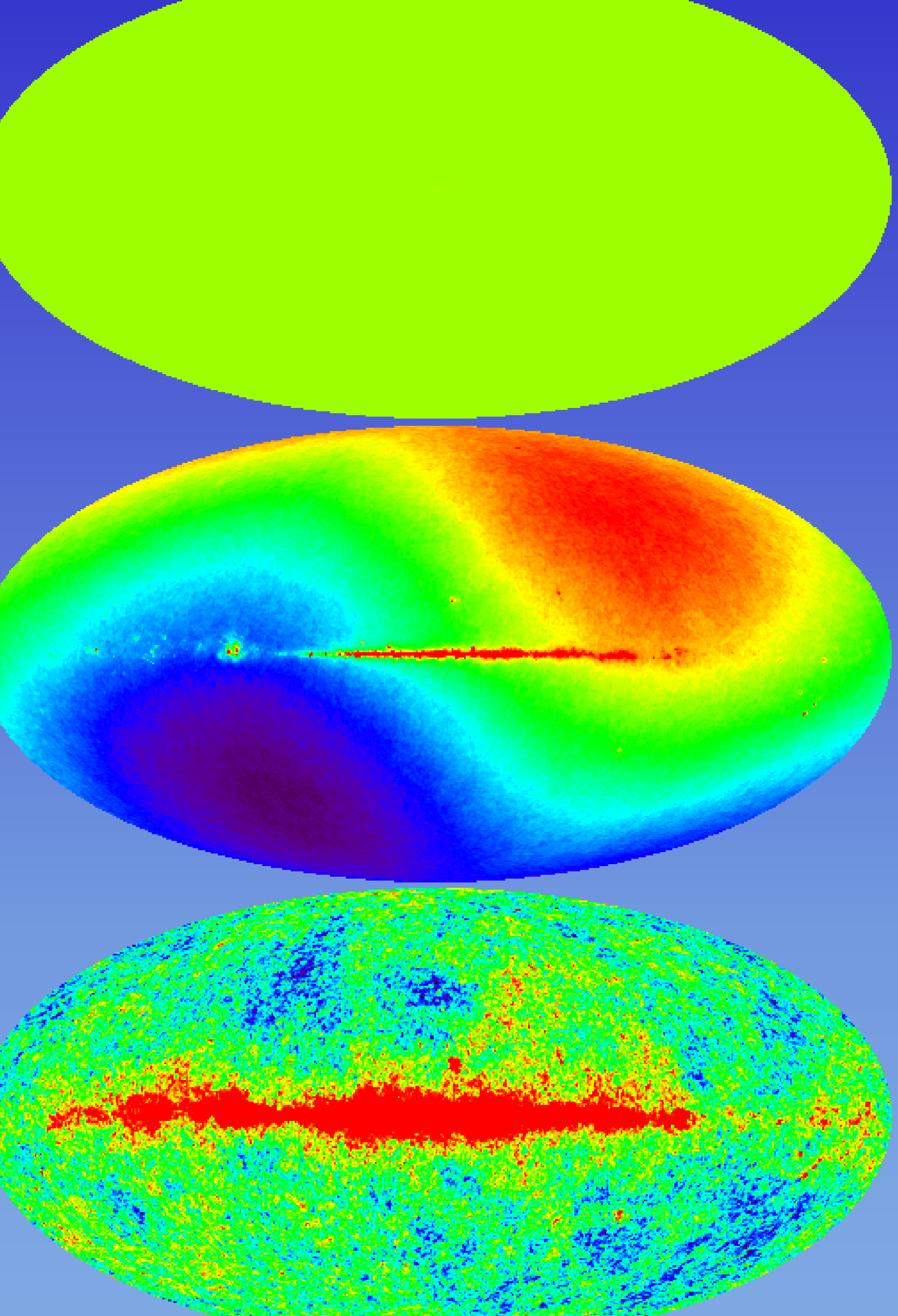
Wilkinson Microwave Anisotropy Probe





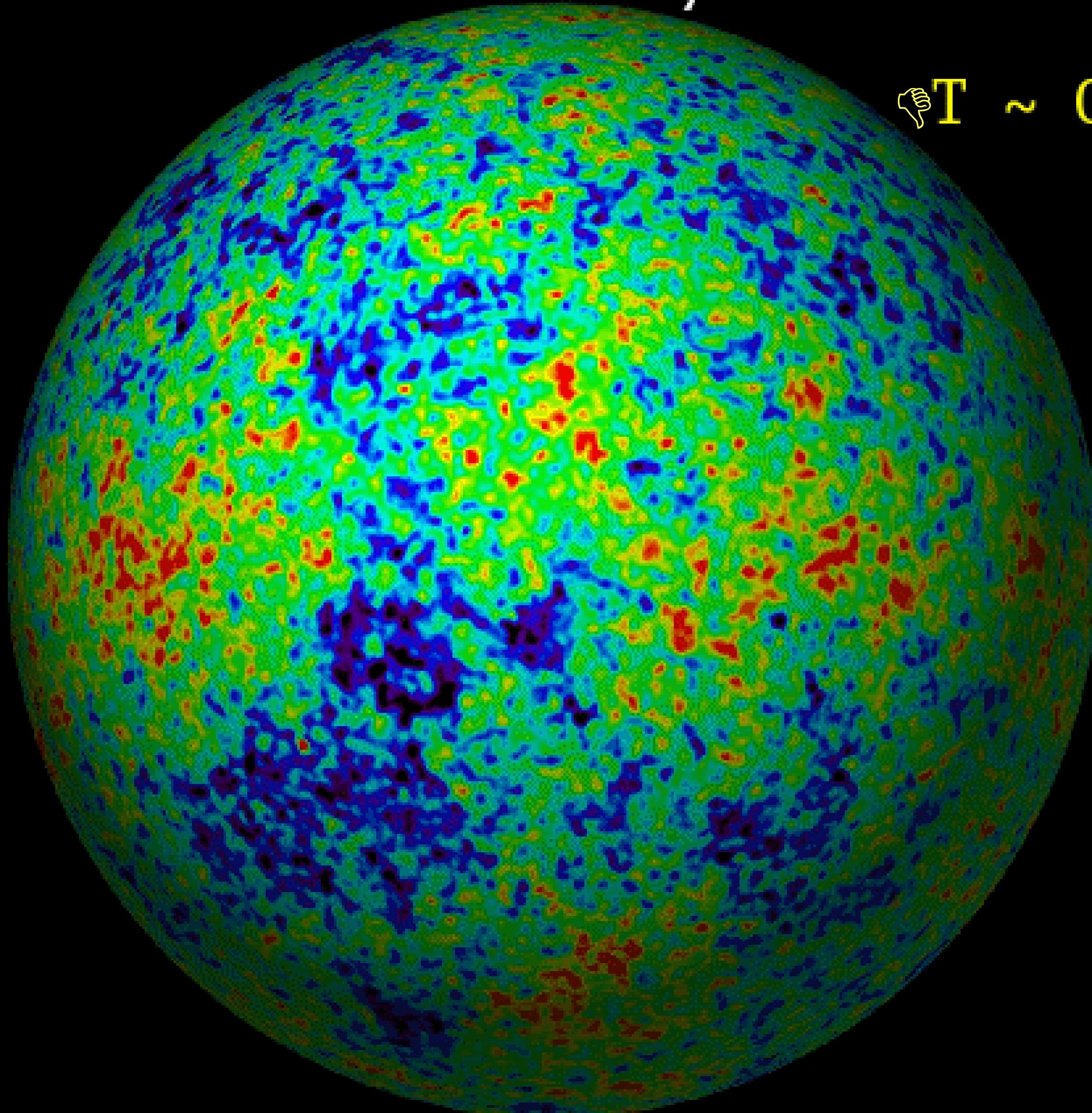
L4
L3
L1
L2
L5


L4
L3
L1
L2
L5



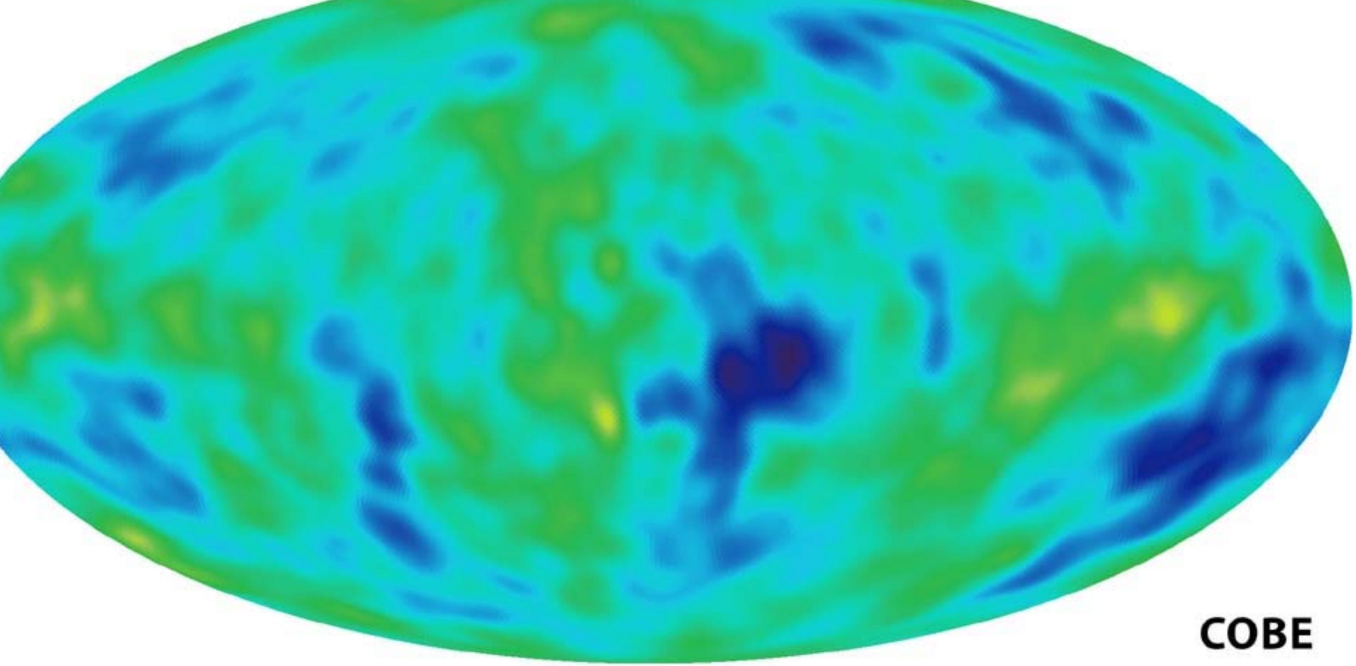
Wilkinson Microwave Anisotropy Probe (2003)

The CMB as seen by WMAP

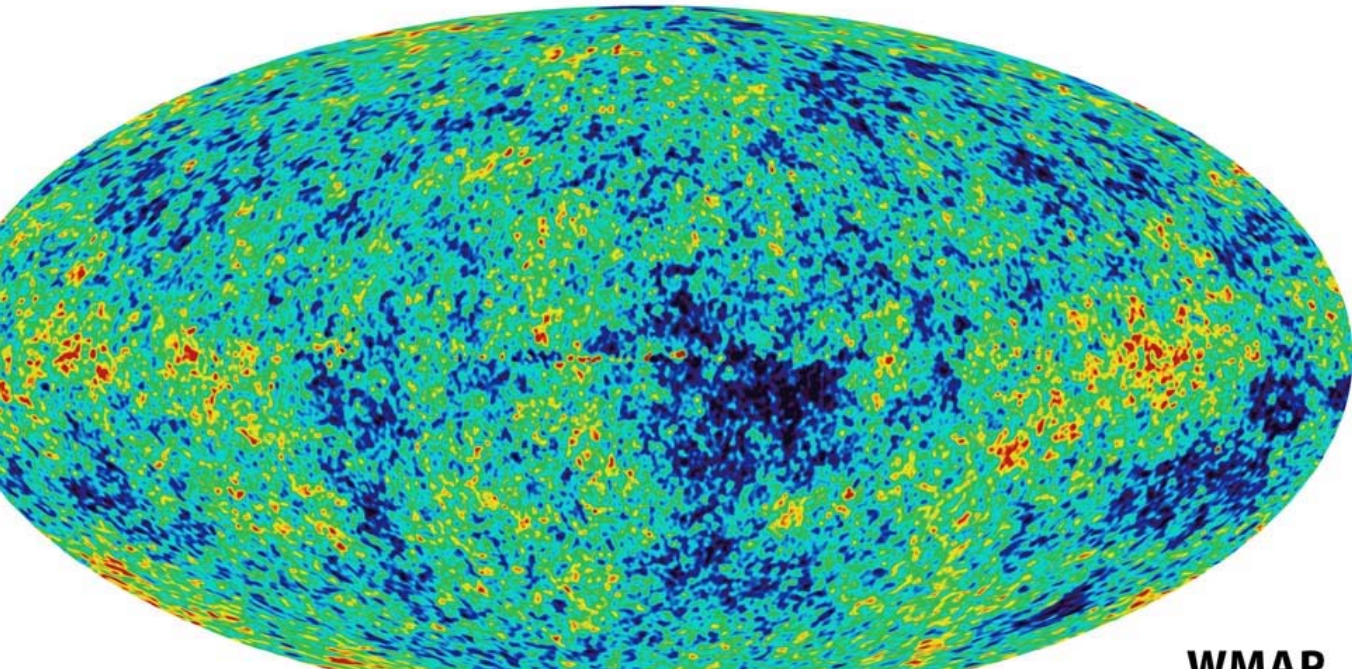


$\delta T \sim 0.00001$

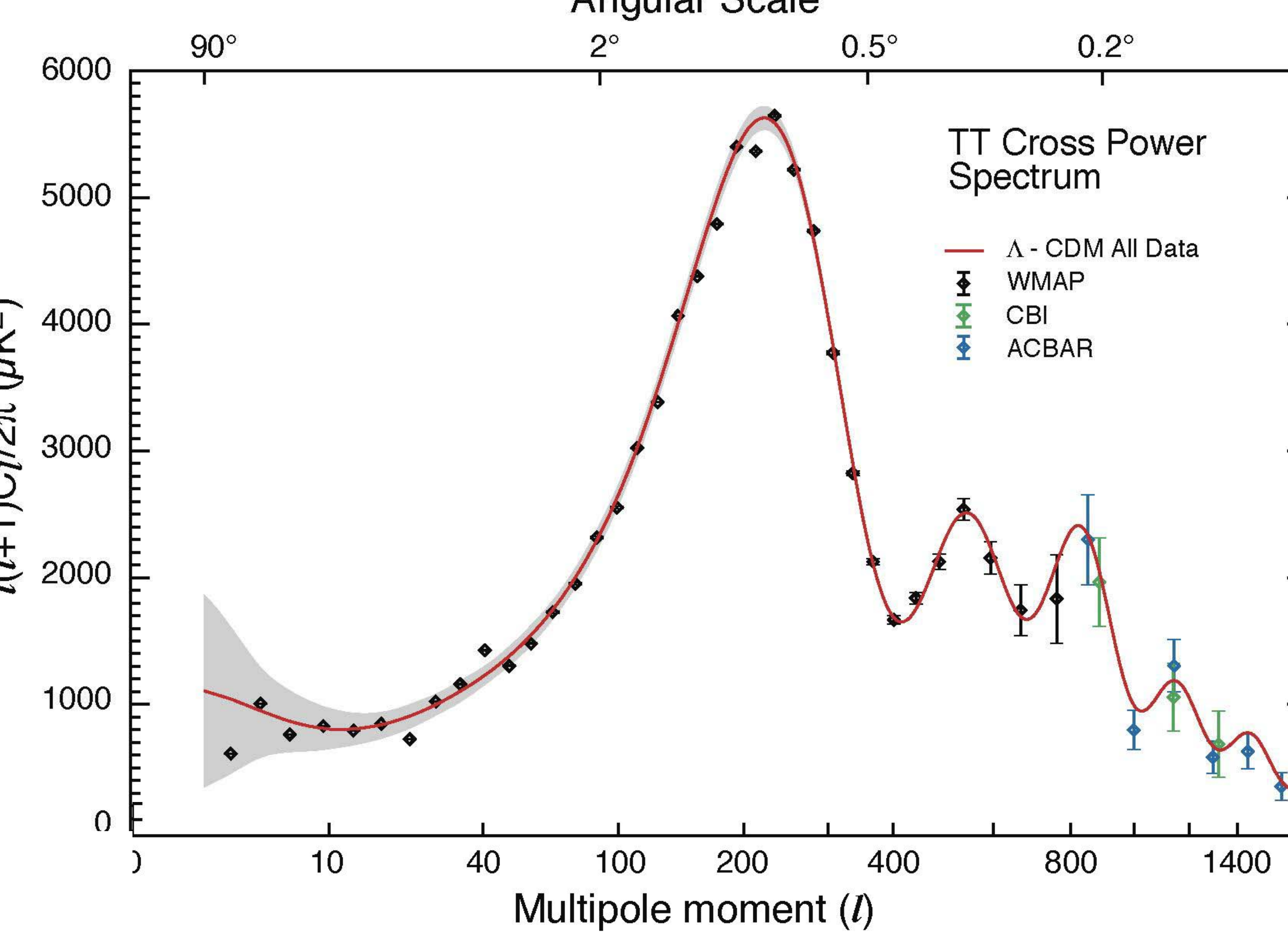
Figure courtesy of the WMAP science team

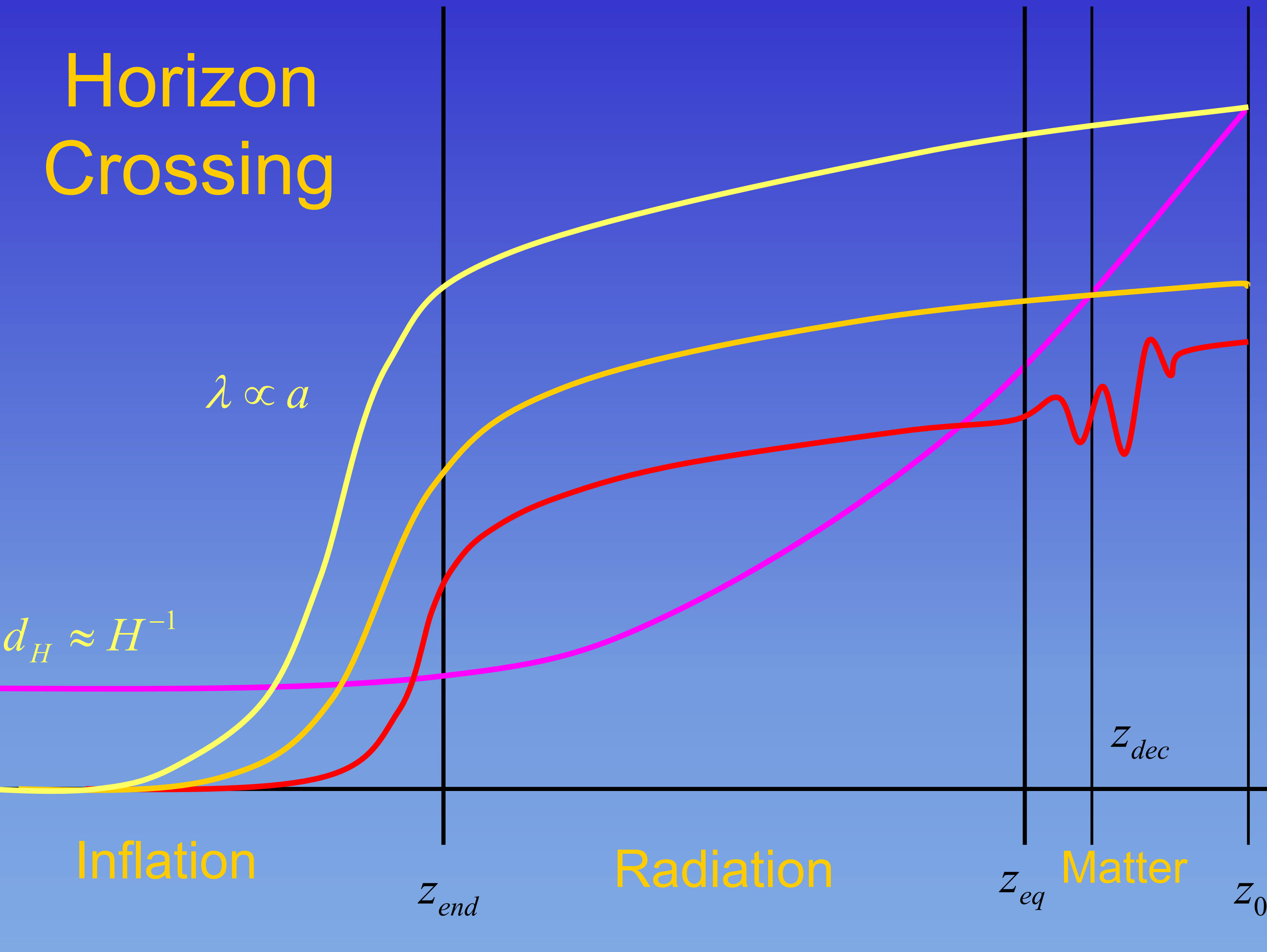


COBE
(1992)
7°

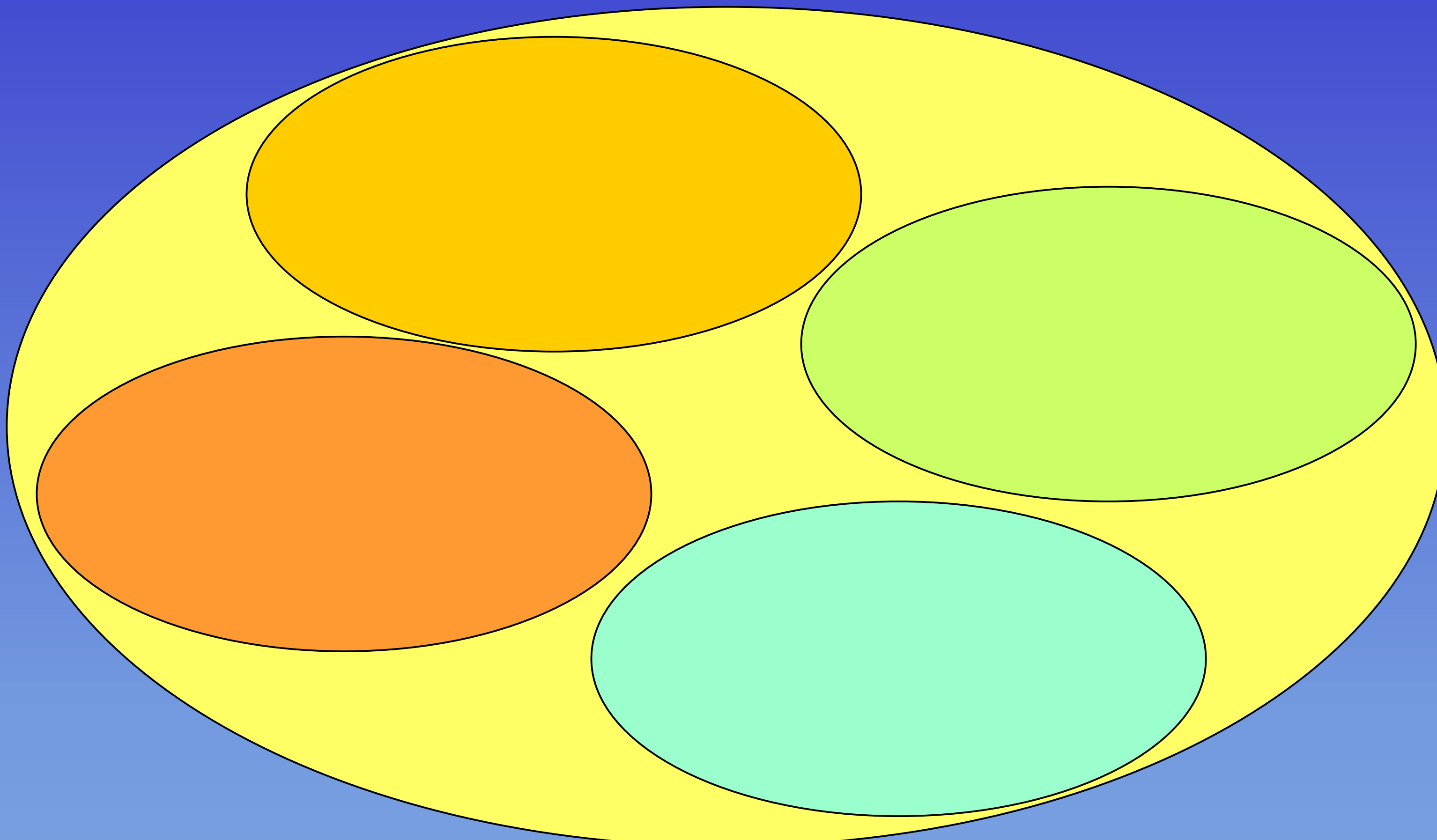


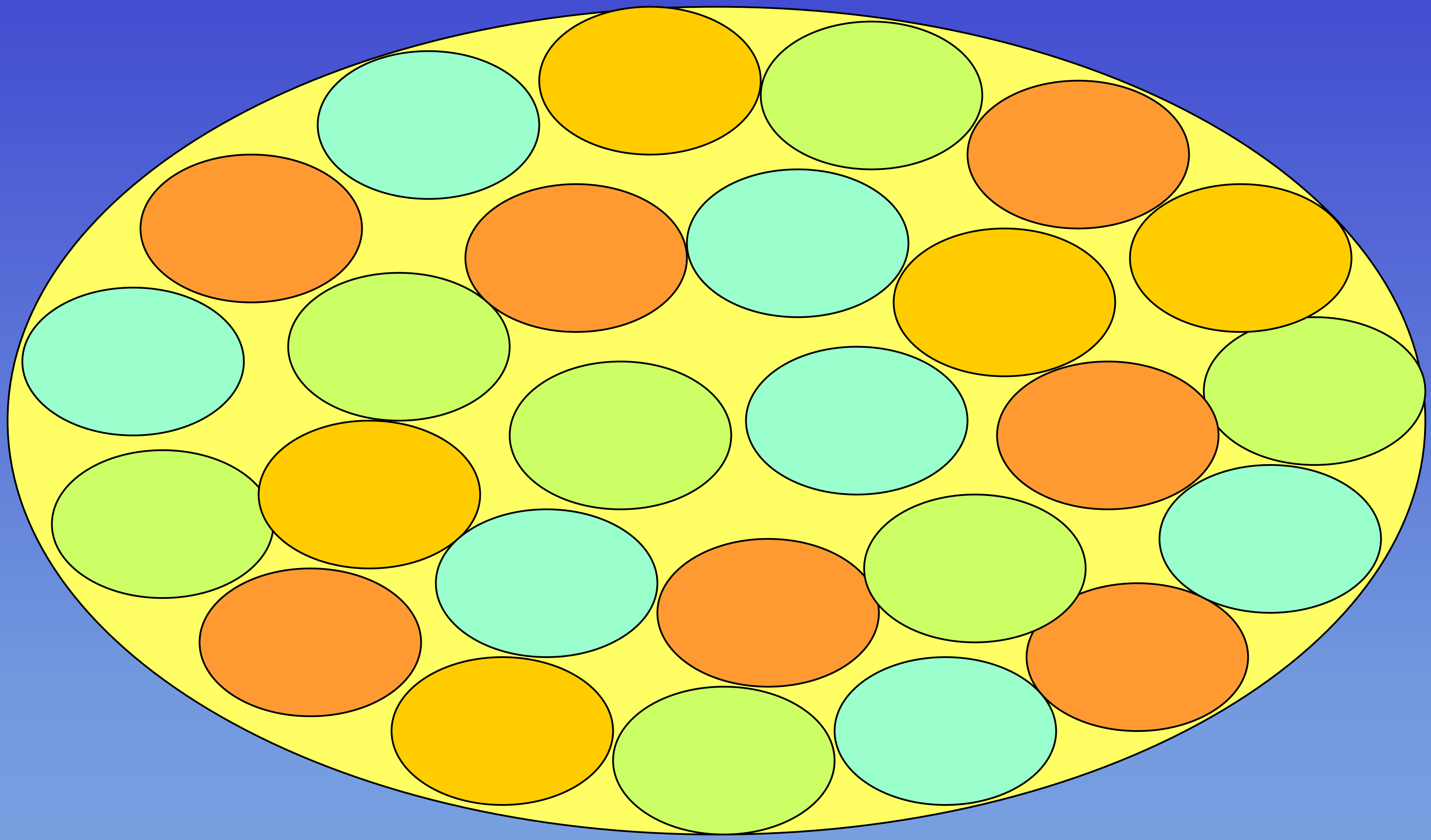
WMAP
(2003)
10’

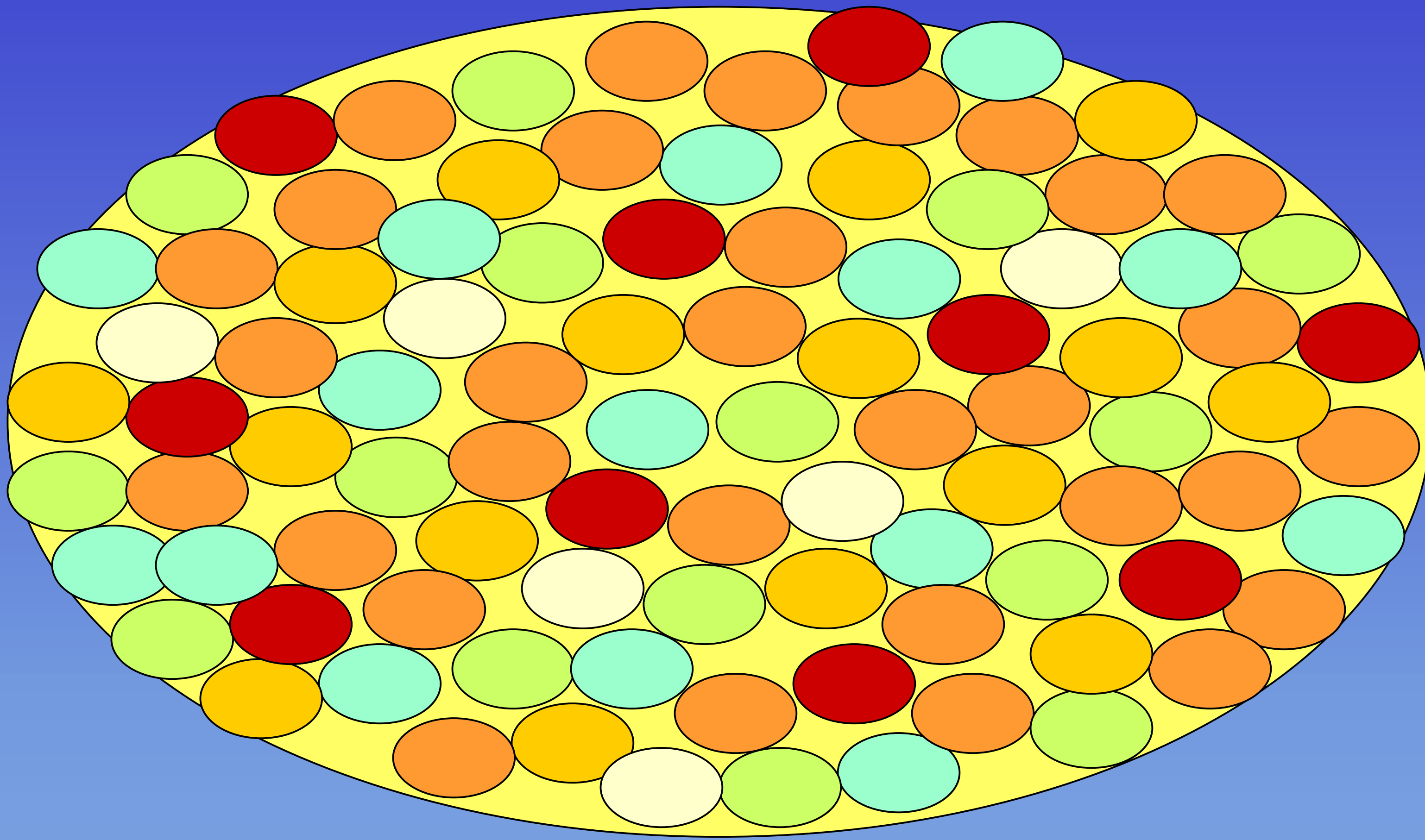

Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
TT Cross Power
Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
10
40
100
200
400
800
1400
Multipole moment (l)

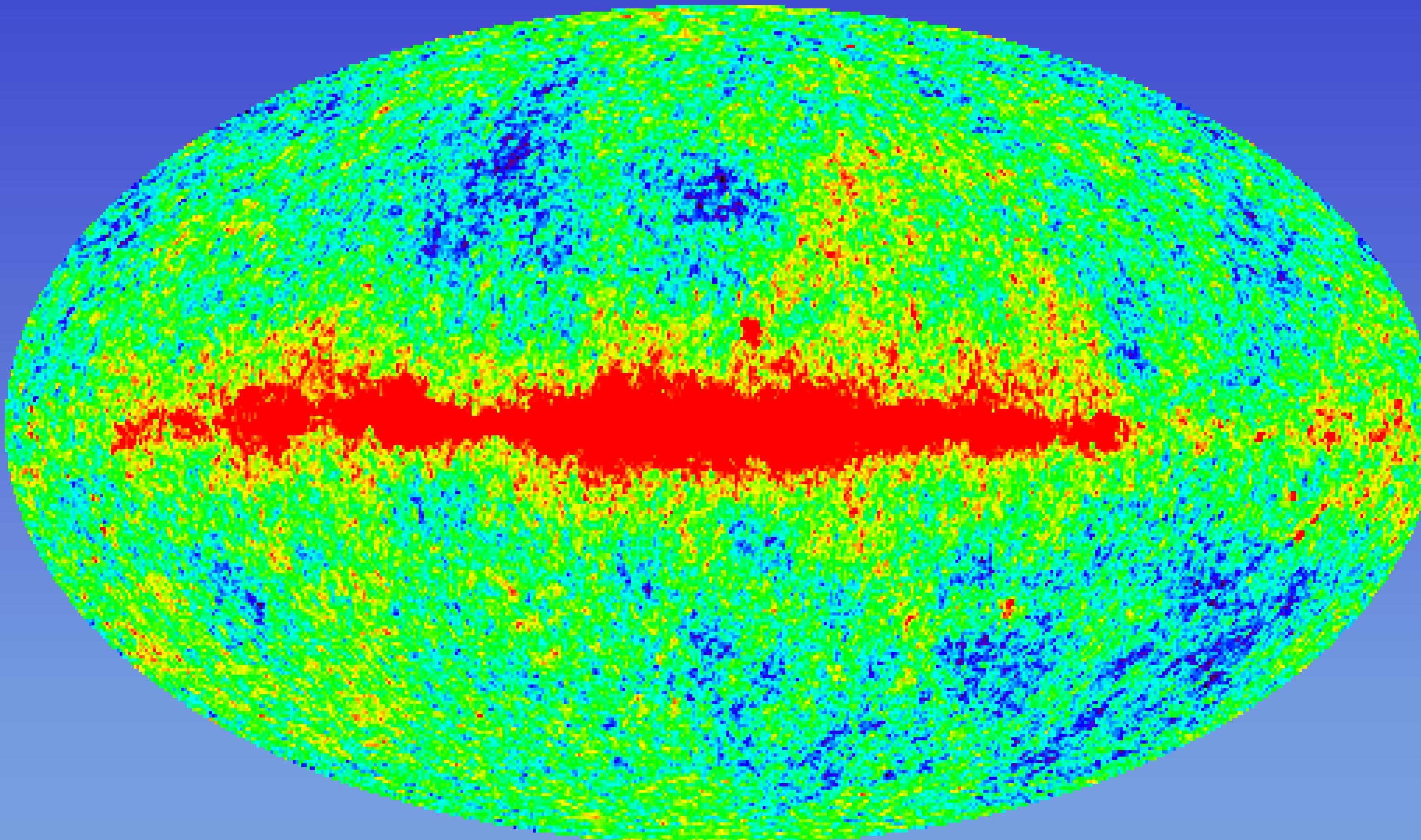


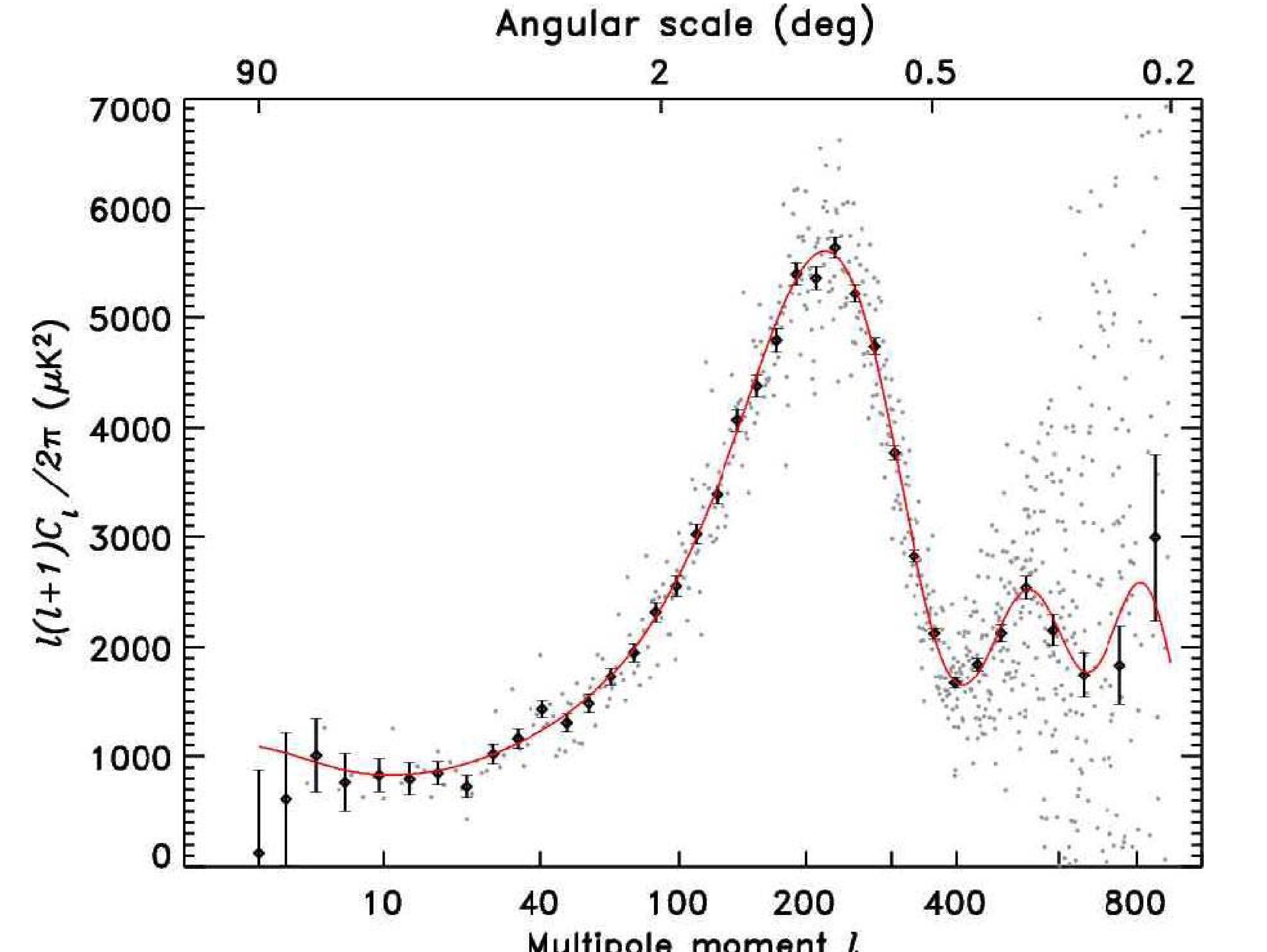
Horizon Crossing
$\lambda \propto a$
$d_H \approx H^{-1}$
z_{dec}
Inflation
z_{end}
Radiation
z_{eq}
Matter
z_0



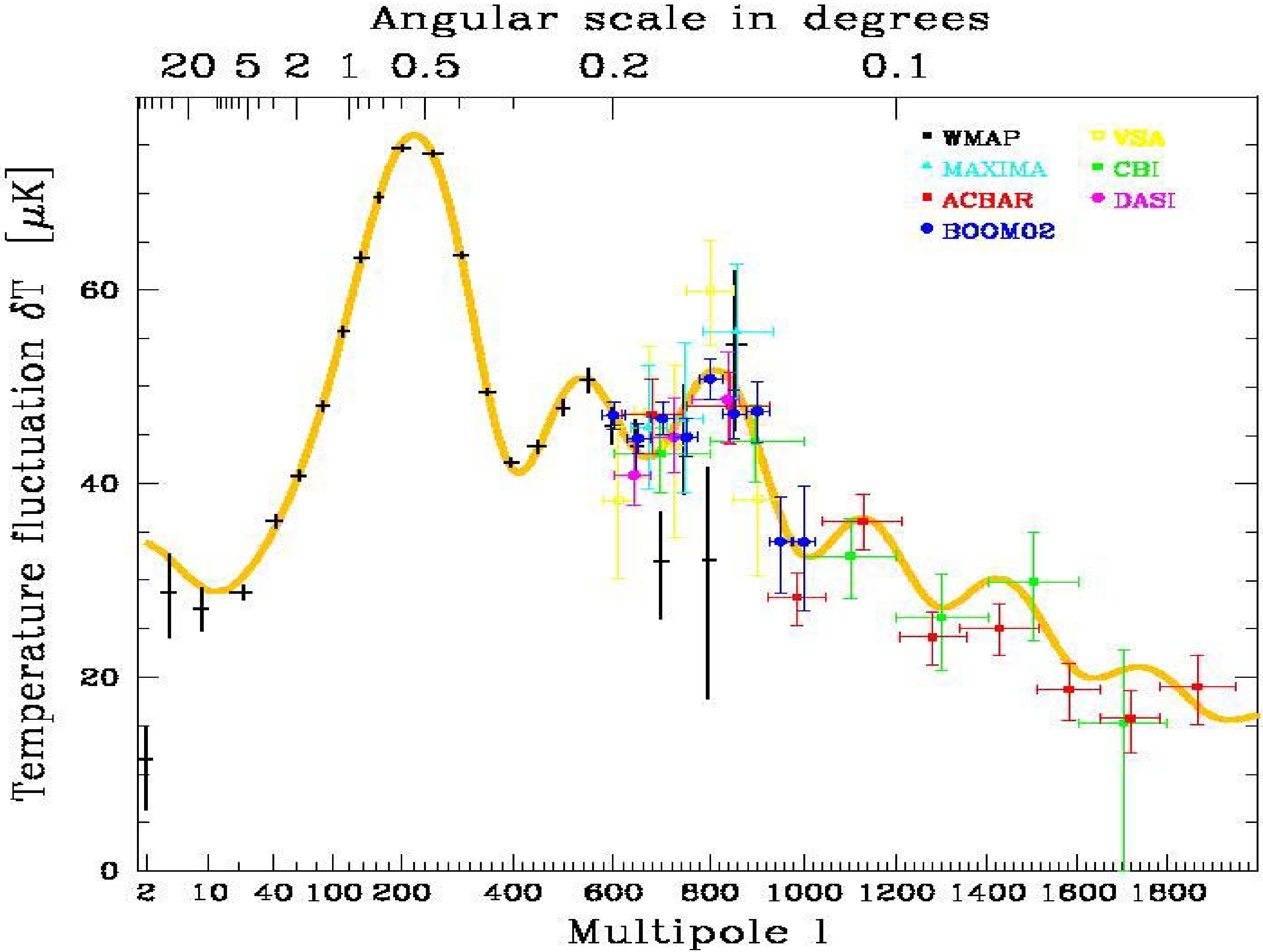








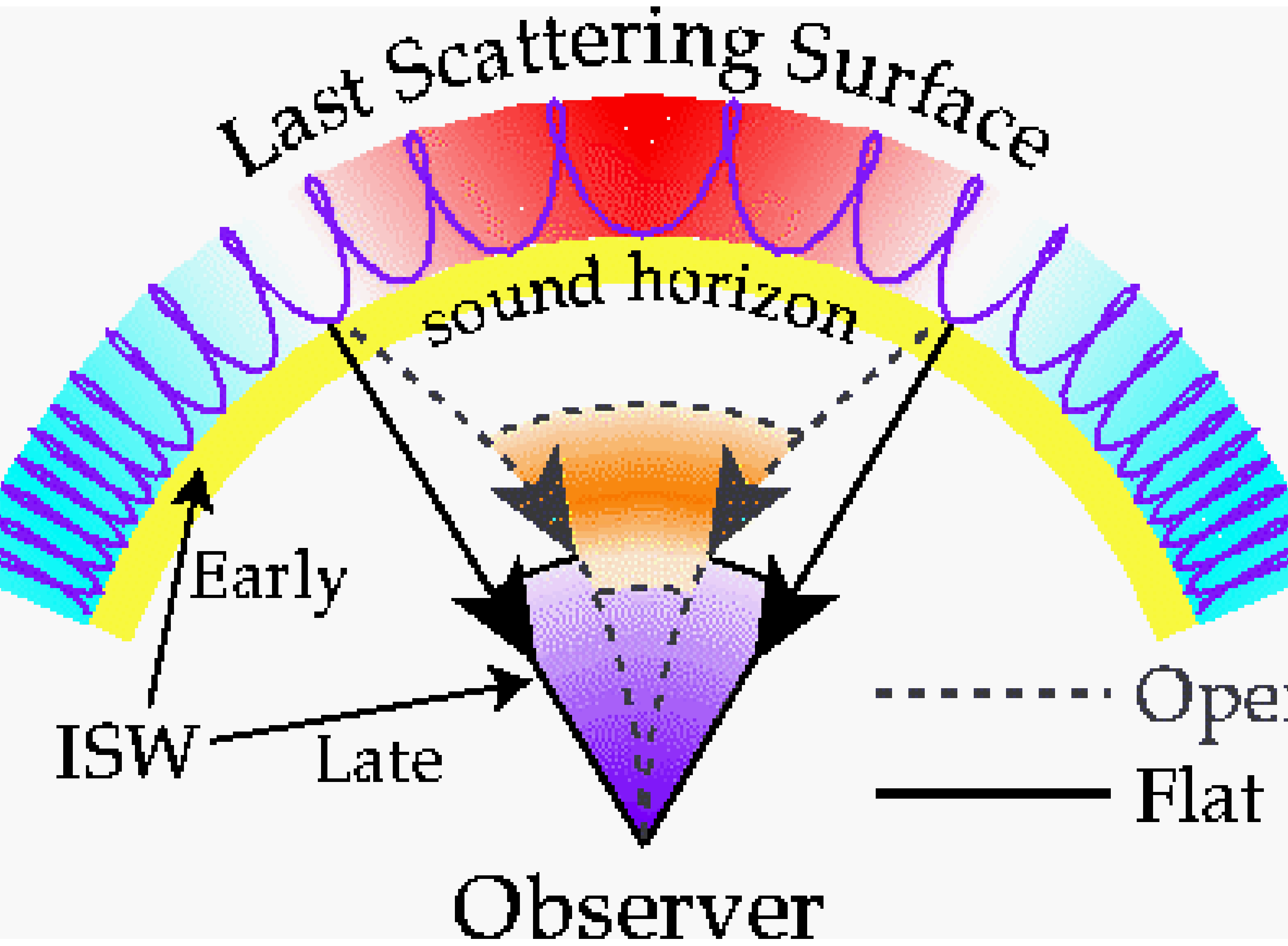
Angular scale (deg)
90
2
0.5
0.2
7000
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ (μK^2)
10
40
100
200
400
800
Multipole moment l


Angular scale in degrees
20 5 2 1 0.5 0.2 0.1
Temperature fluctuation δT [μK]
0
20
40
60
2 10 40 100 200 400 600 800 1000 1200 1400 1600 1800
Multipole l
WMAP
MAXIMA
ACBAR
BOOM02
VSA
CBI
DASI

Cosmological Parameters: WMAPext

Rate of expansion	$H_0 = 71 \pm 3$ km/s/Mpc
Age of the Universe	$t_0 = 13.7 \pm 0.2$ Gyr
Spatial Curvature	$\Omega_K < 0.02 \quad (95\%\ c.l.)$
Cosmological Constant	$\Omega_\Lambda = 0.73 \pm 0.04$
Dark Matter	$\Omega_M = 0.23 \pm 0.04$
Baryon Density	$\Omega_B = 0.044 \pm 0.004$
Neutrino Density	$\Omega_\nu < 0.0076 \quad (95\%\ c.l.)$
Spectral Amplitude	$A_s = 0.833 \pm 0.085$
Spectral tilt	$n_s = 0.93 \pm 0.03$
Tensor-scalar ratio	$r < 0.71 \ (95\%\ c.l.)$

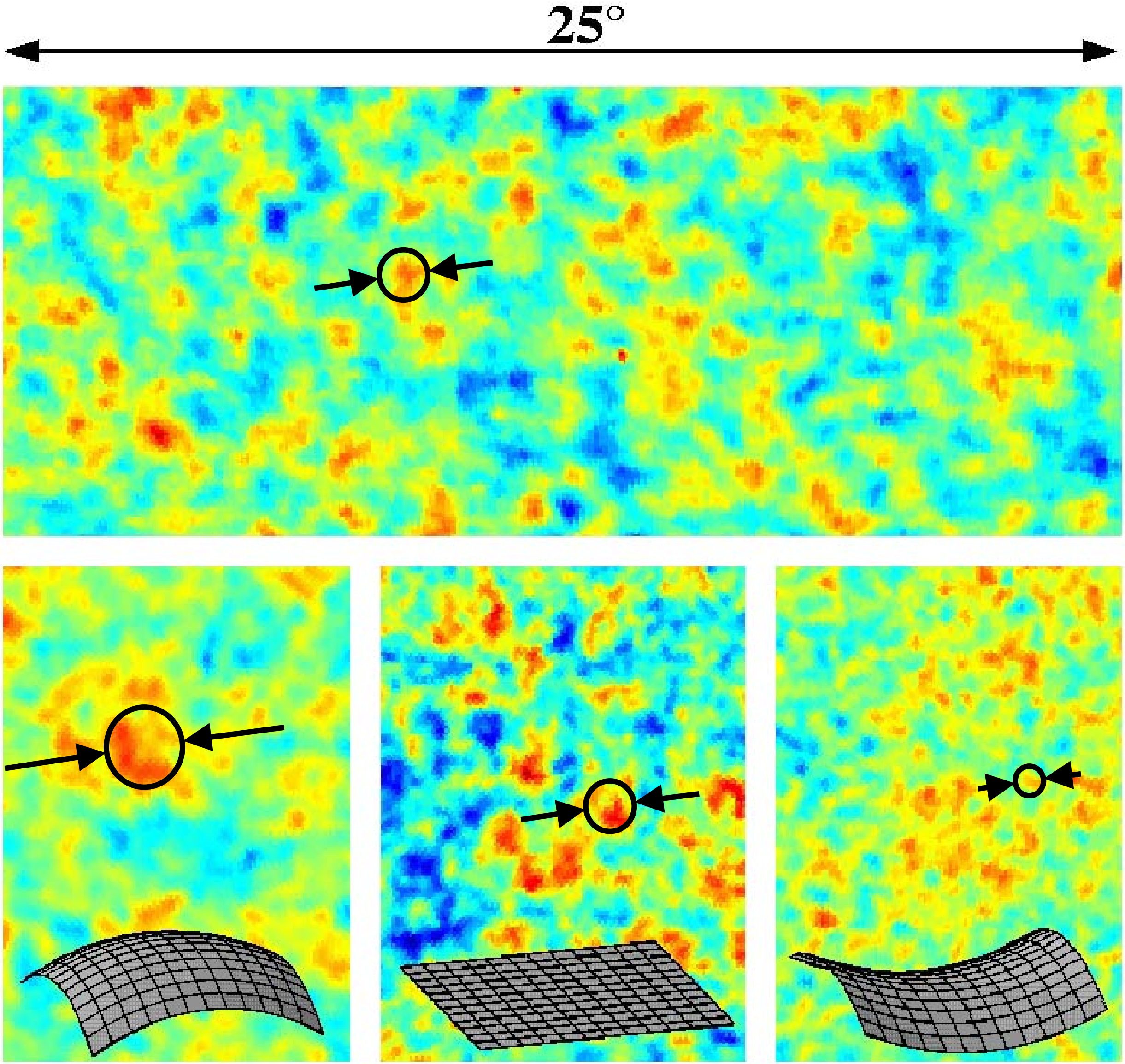
Spatial Curvature

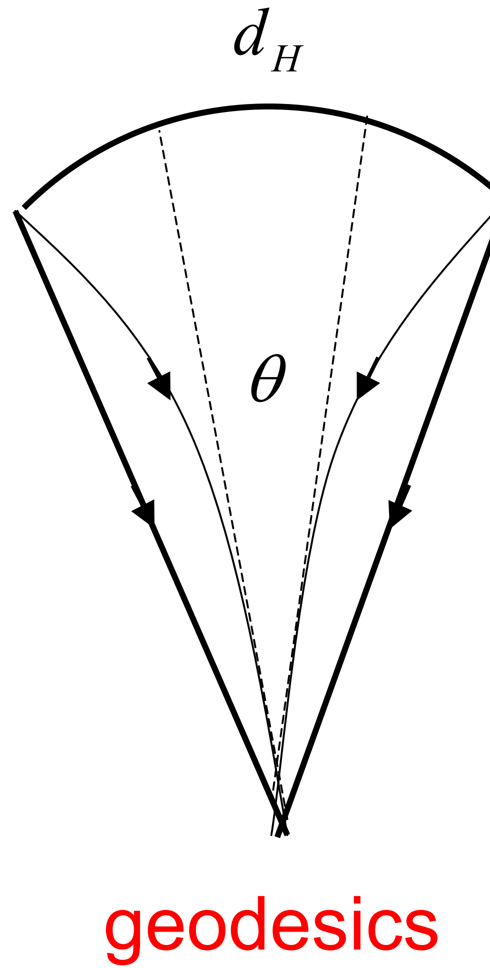


Last Scattering Surface
sound horizon
Early
ISW
Late
Open
Flat
Observer

$$\Omega_K = |\Omega_0 - 1| < 0.02$$ WMAP

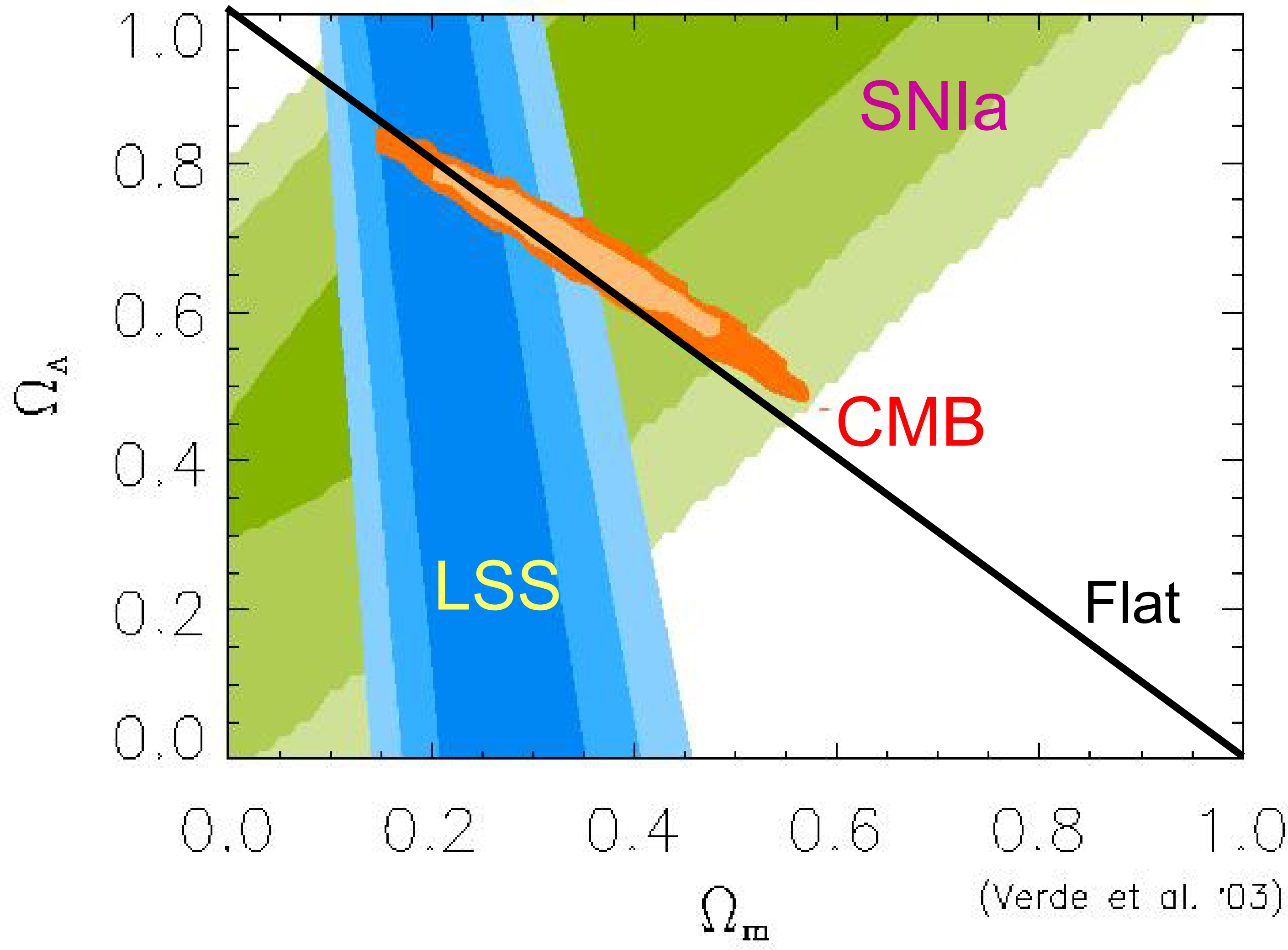
Spatial Curvature



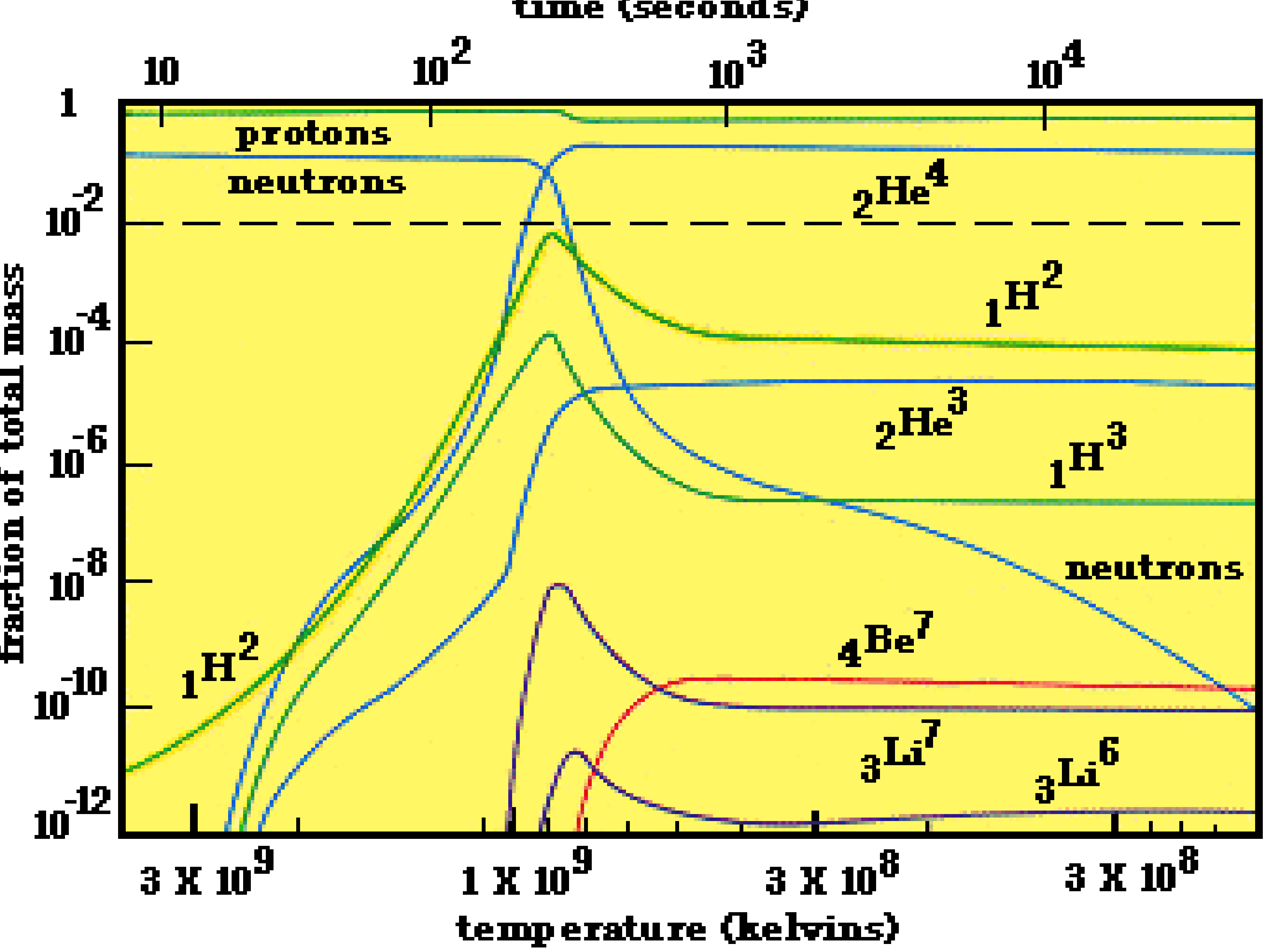


$$\Omega_\Lambda = 0.73 \pm 0.04$$ COSMOLOGICAL CONSTANT

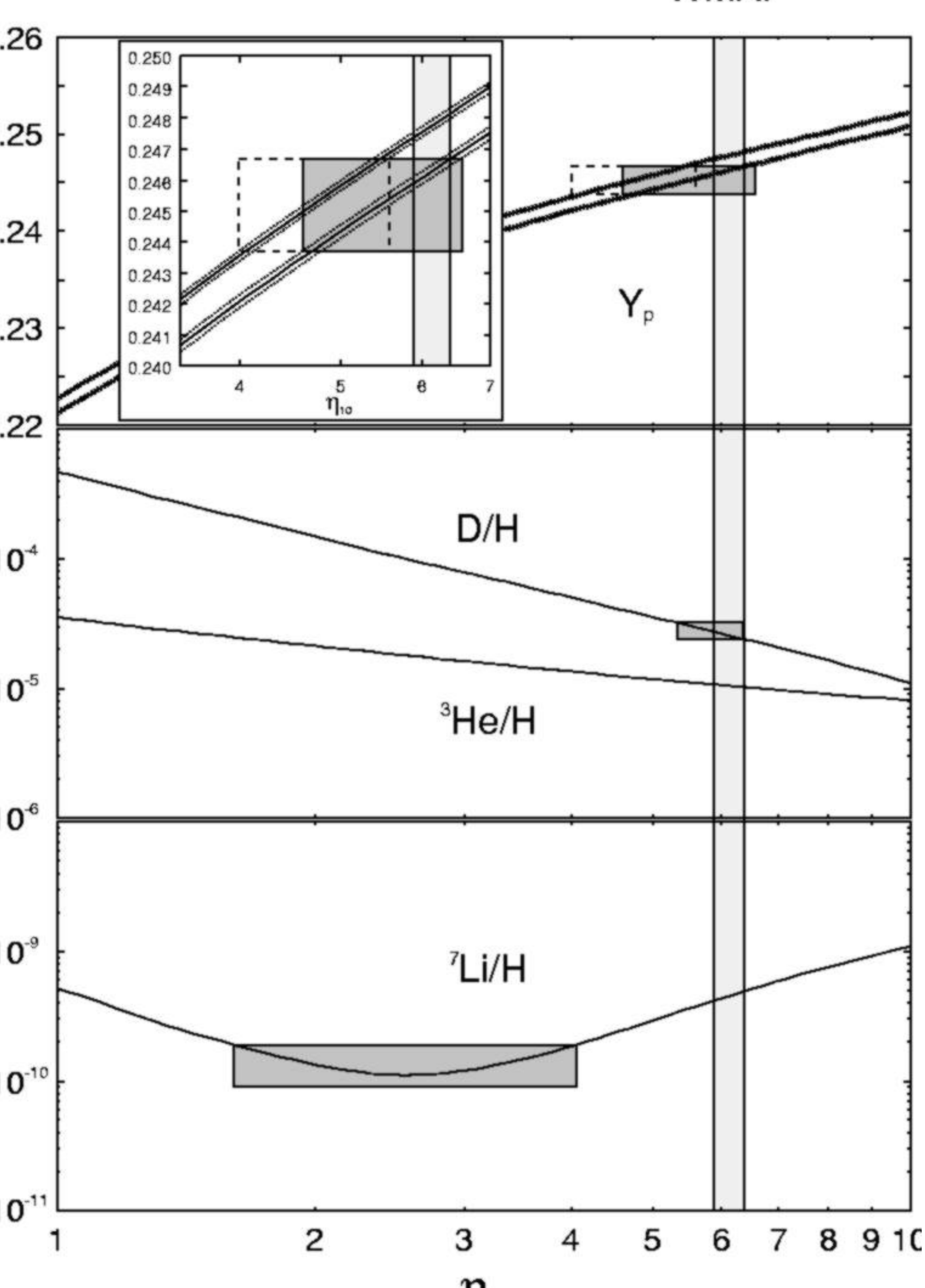
$$\Omega_M = 0.27 \pm 0.04$$ DARK MATTER DENSITY



Baryon Fraction



time (seconds)
10
10^2
10^3
10^4
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
fraction of total mass
protons
neutrons
$_2He^4$
$_1H^2$
$_2He^3$
$_1H^3$
neutrons
$_4Be^7$
$_3Li^7$
$_3Li^6$
$_1H^2$
3 X 10^9
1 X 10^9
3 X 10^8
3 X 10^8
temperature (kelvins)



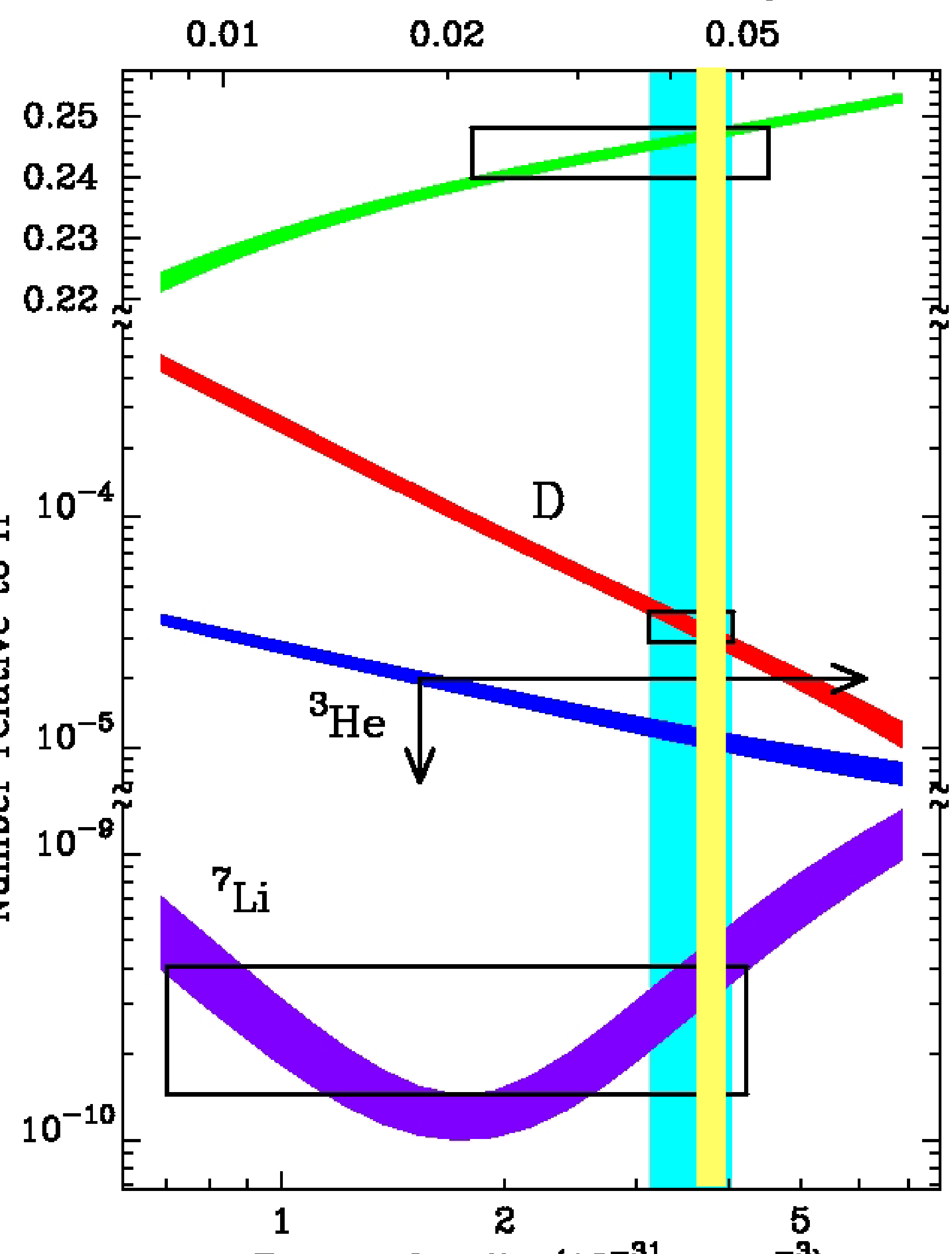
Neutron Lifetime

World average (PDG)

$$\tau_n = 885.7 \pm 0.8\ s$$

Mathews et al. (2005)

$$\tau_n = 878.5 \pm 0.8\ s$$



Primordial Nucleosynthesis

$$\Omega_B h^2 = 0.022 \pm 0.003$$

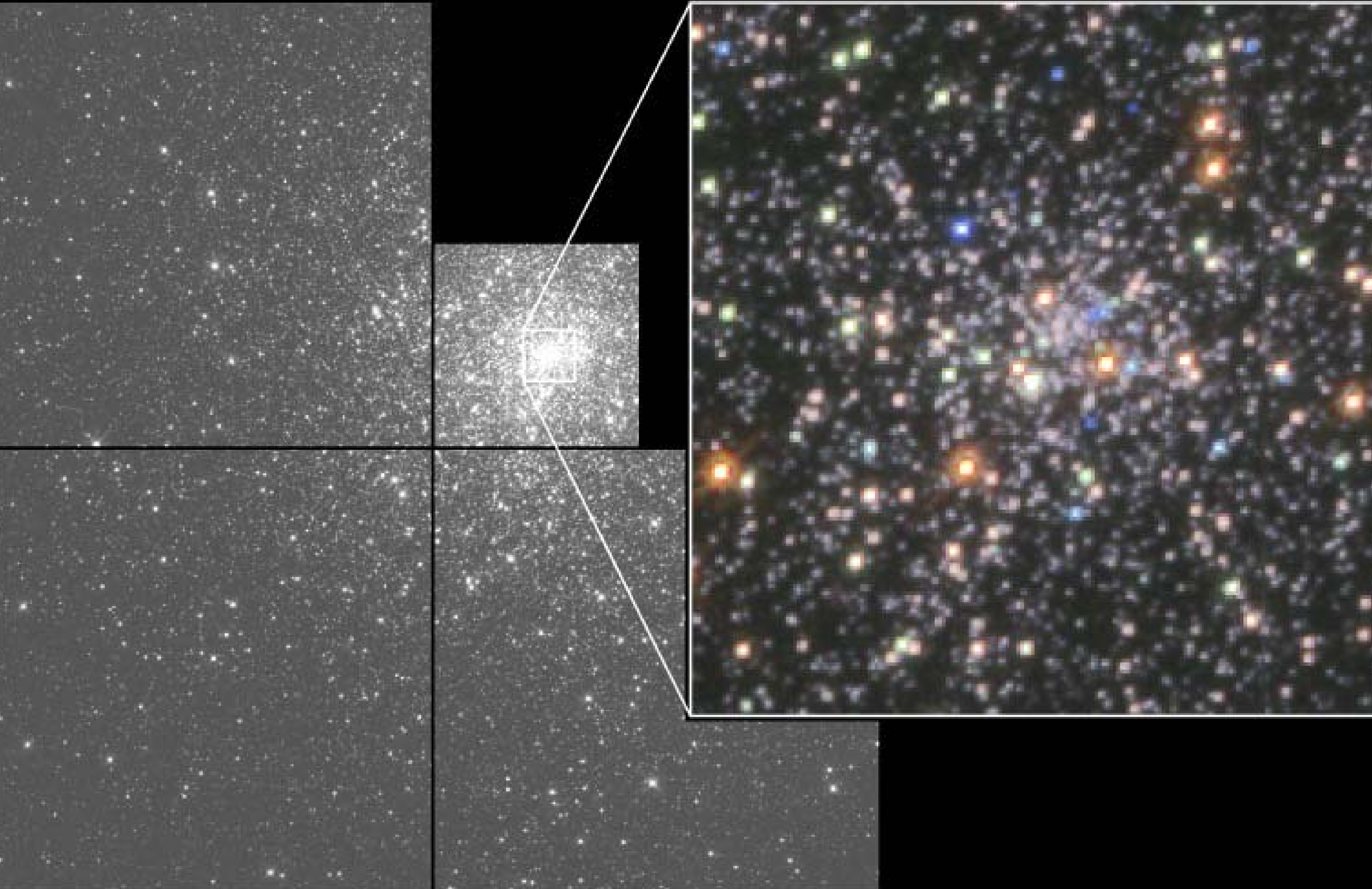
$$\Omega_B = 0.042 \pm 0.006$$

WMAP

$$\Omega_B h^2 = 0.0224 \pm 0.000$$

$$\Omega_B = 0.044 \pm 0.002$$

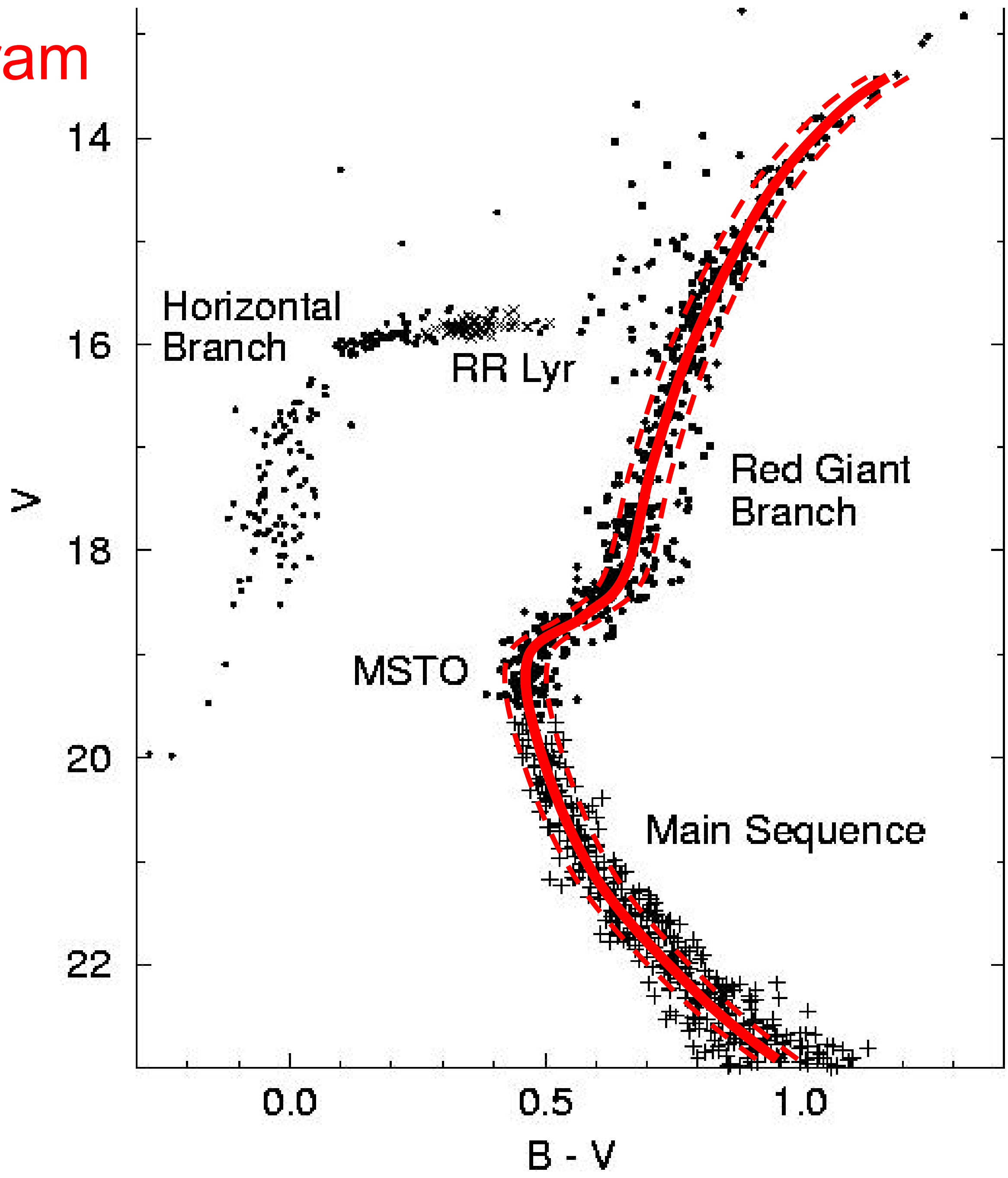
Age Universe



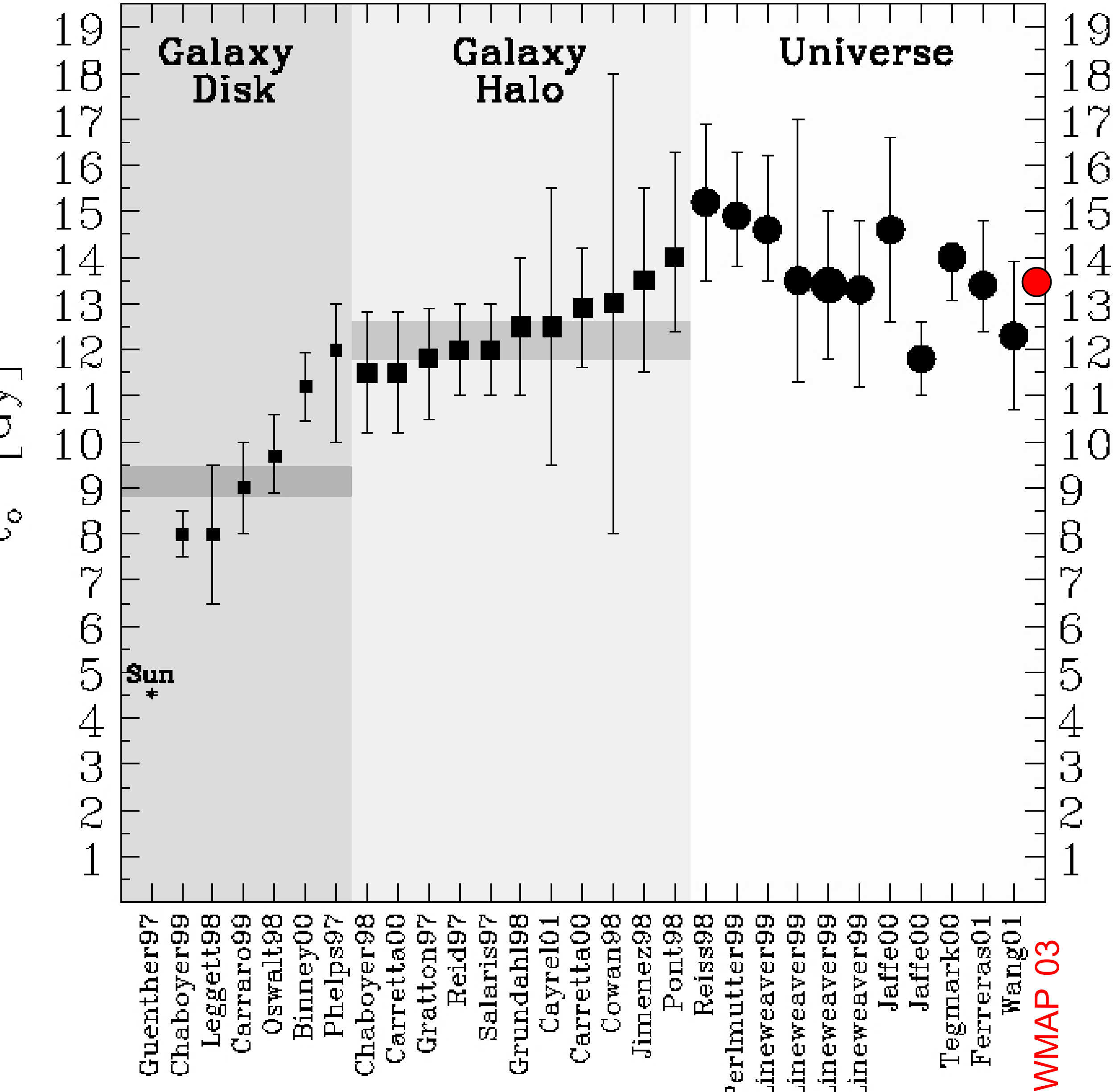
Globular Cluster M15 HST • WFPC

RC95-06 • ST ScI OPO • November 1995 • P. Guhathakurta (UC Santa Cruz), NASA

HR-diagram



14
16
18
20
22
V
0.0
0.5
1.0
B - V
Horizontal Branch
RR Lyr
Red Giant Branch
MSTO
Main Sequence



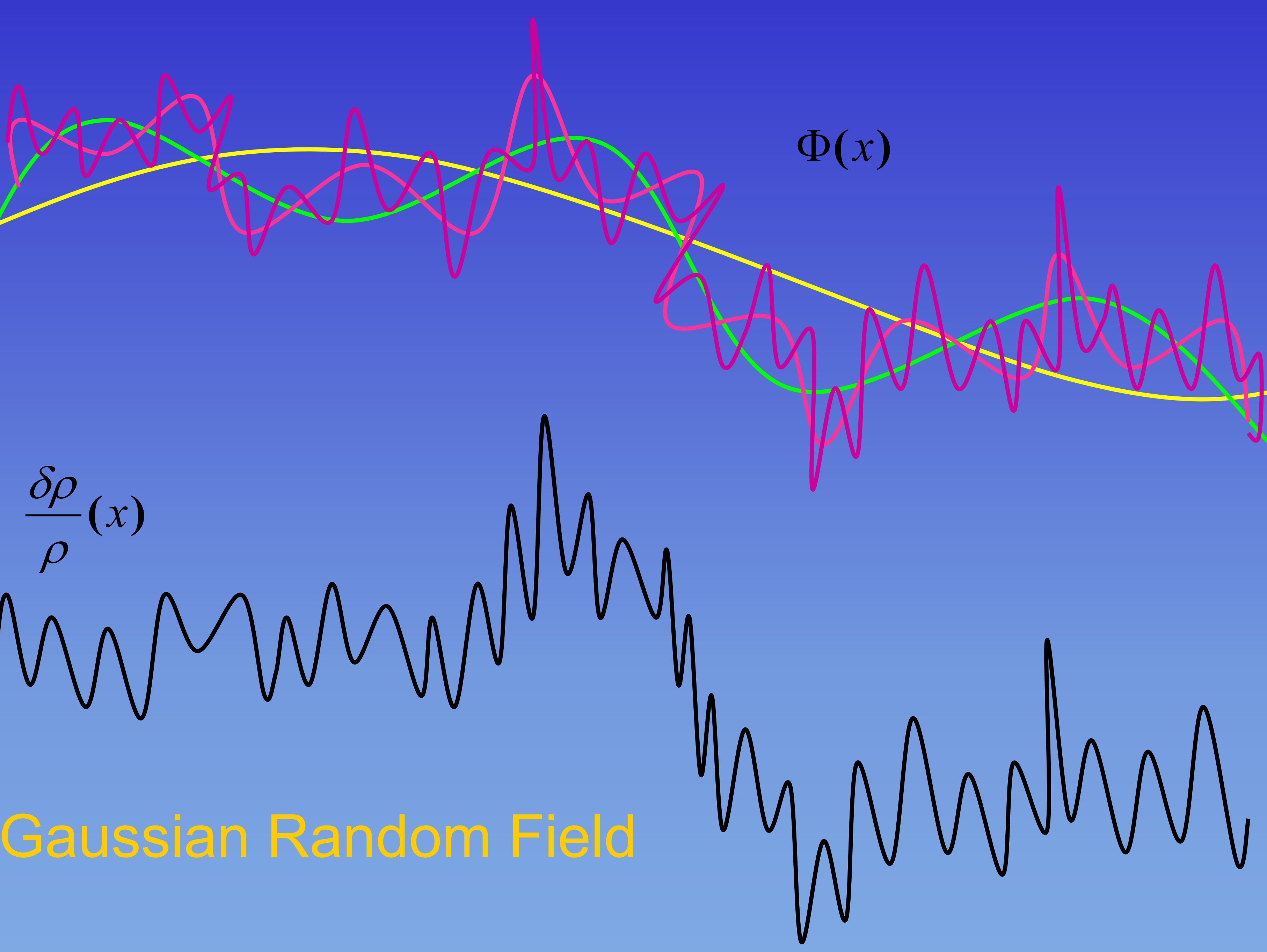
AGE OF UNIVERSE

WMAP (2003)

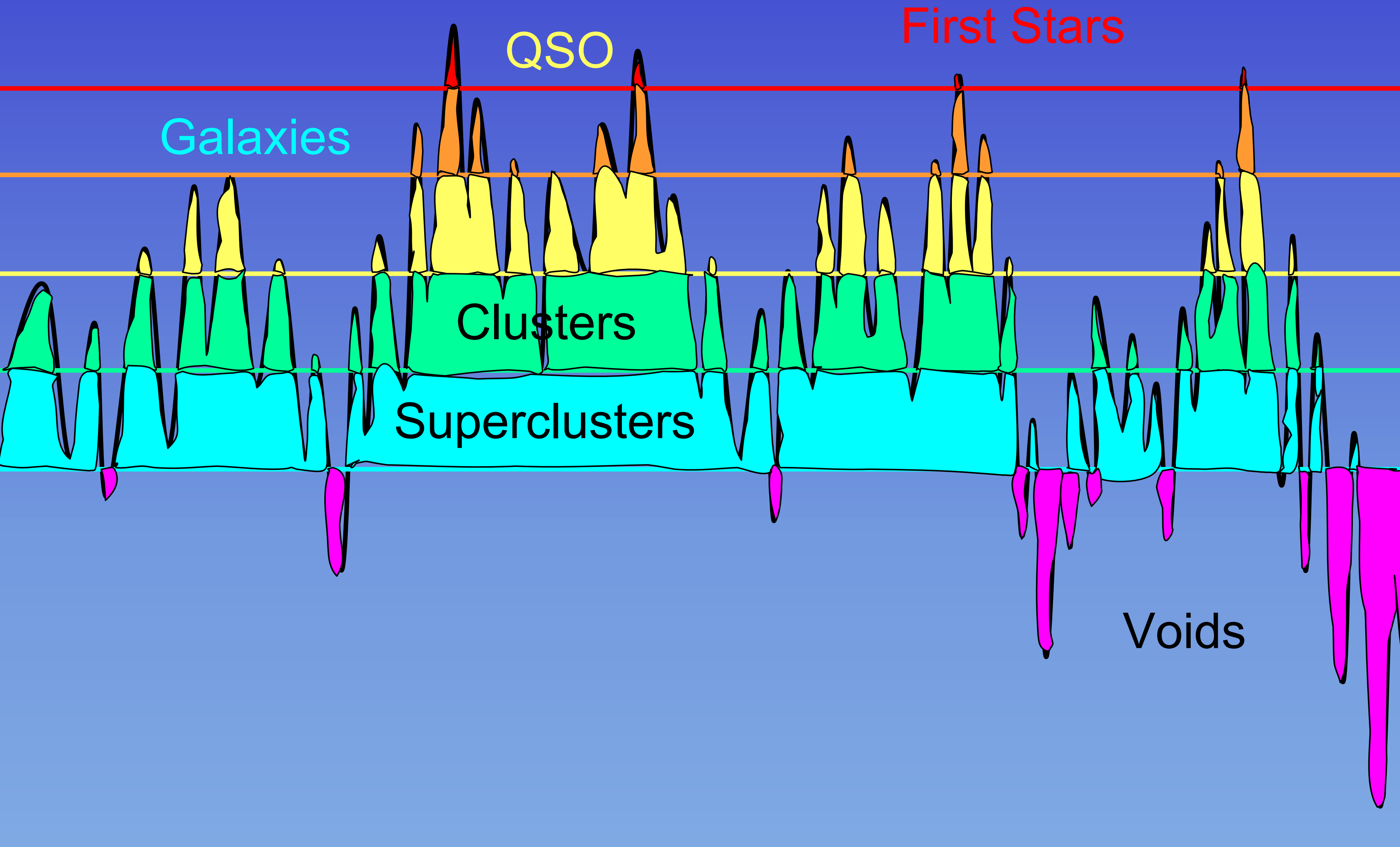
$$t_0 = 13.6 \pm 0.2$$

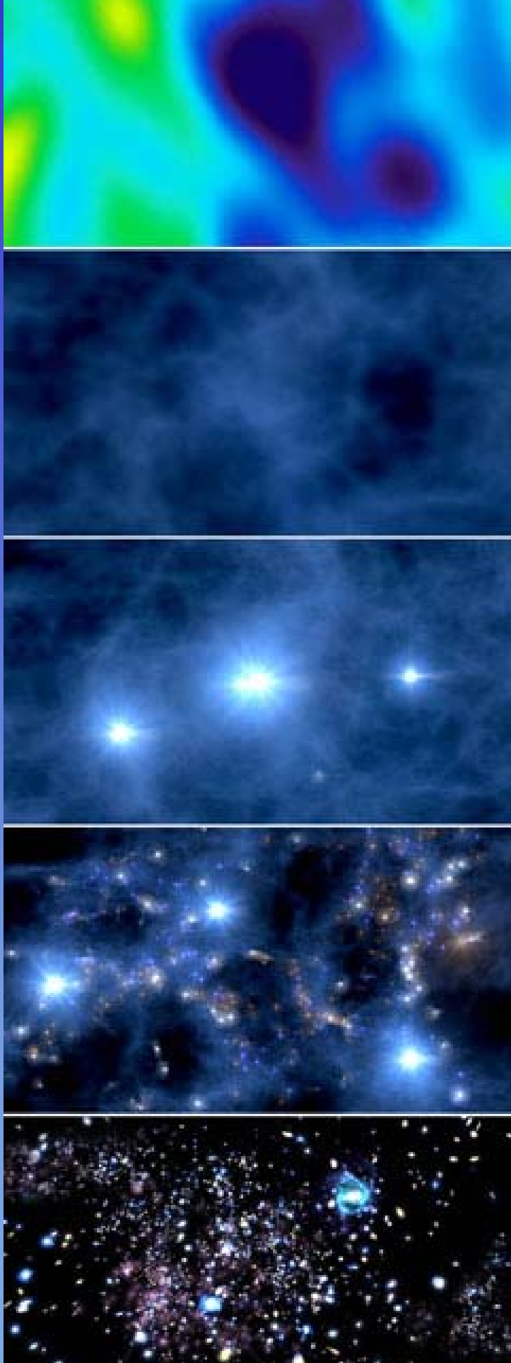
Gyr

Structure Formation


$\Phi(x)$
$\frac{\delta\rho}{\rho}(x)$
Gaussian Random Field

Density Contrast Thresholds





$z \approx 1100$ CMB Anisotropies

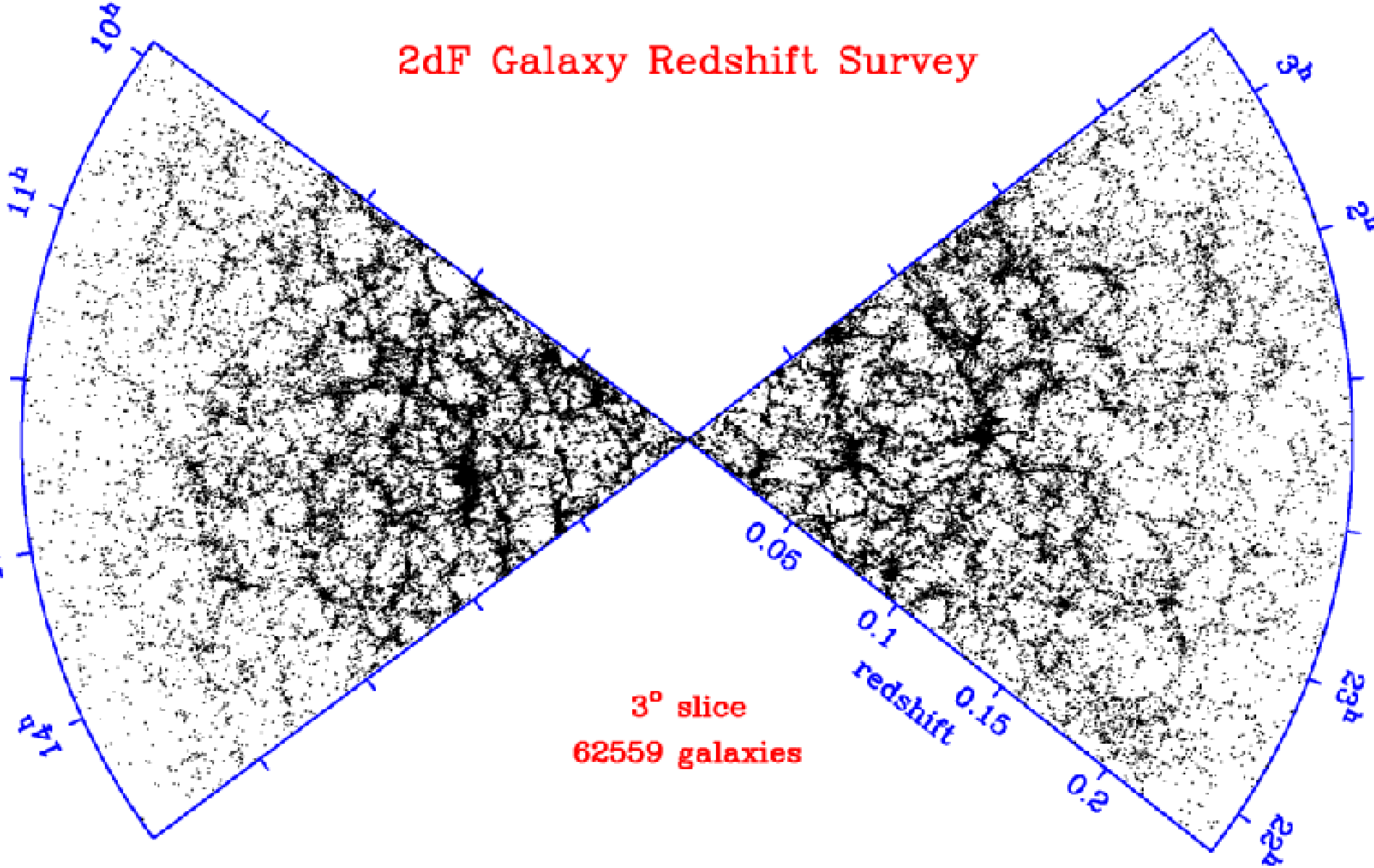
$z \approx 100$ Dark ages

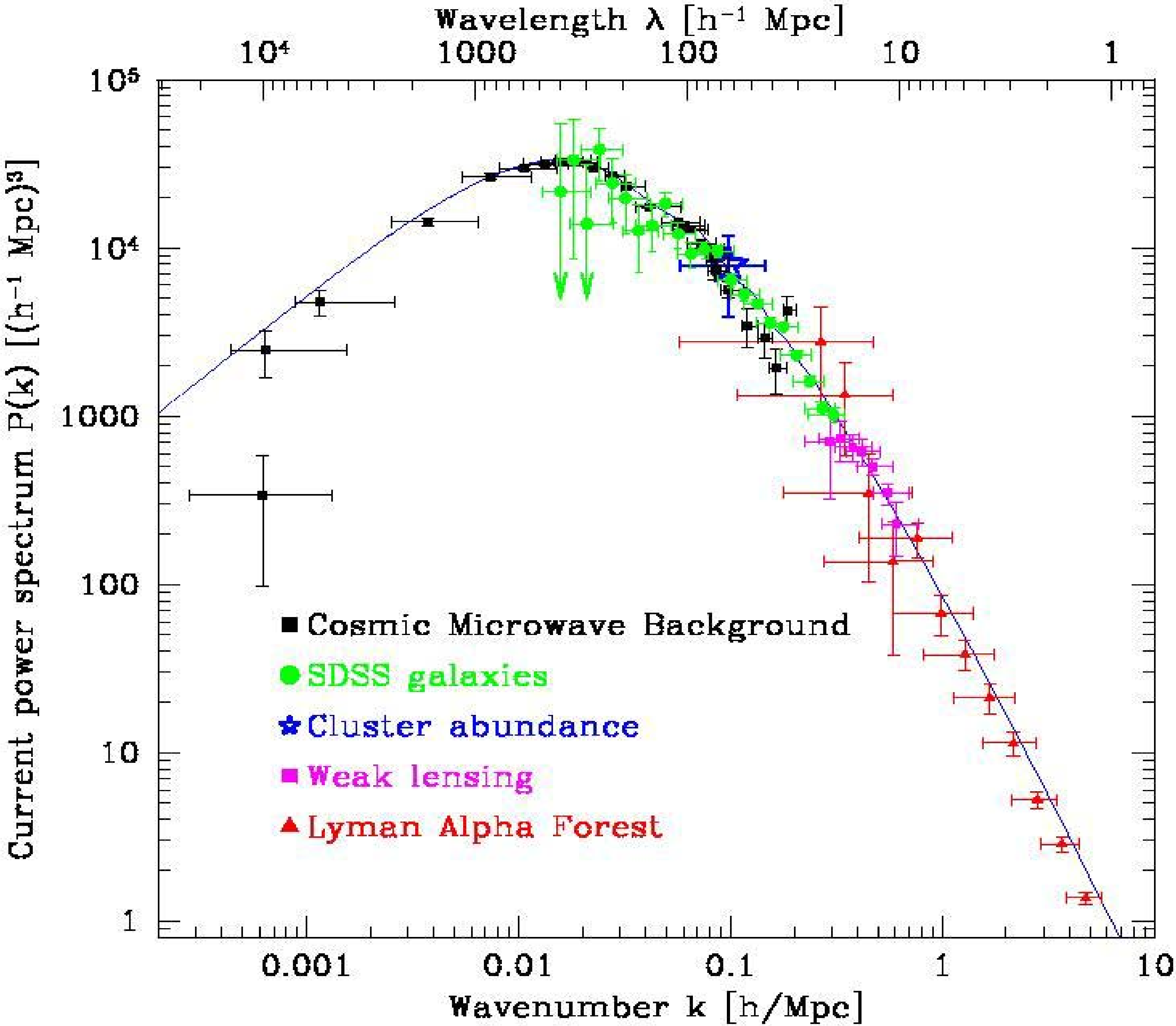
$z \approx 20$ First stars

$z \approx 10$ Galaxies & Quasars

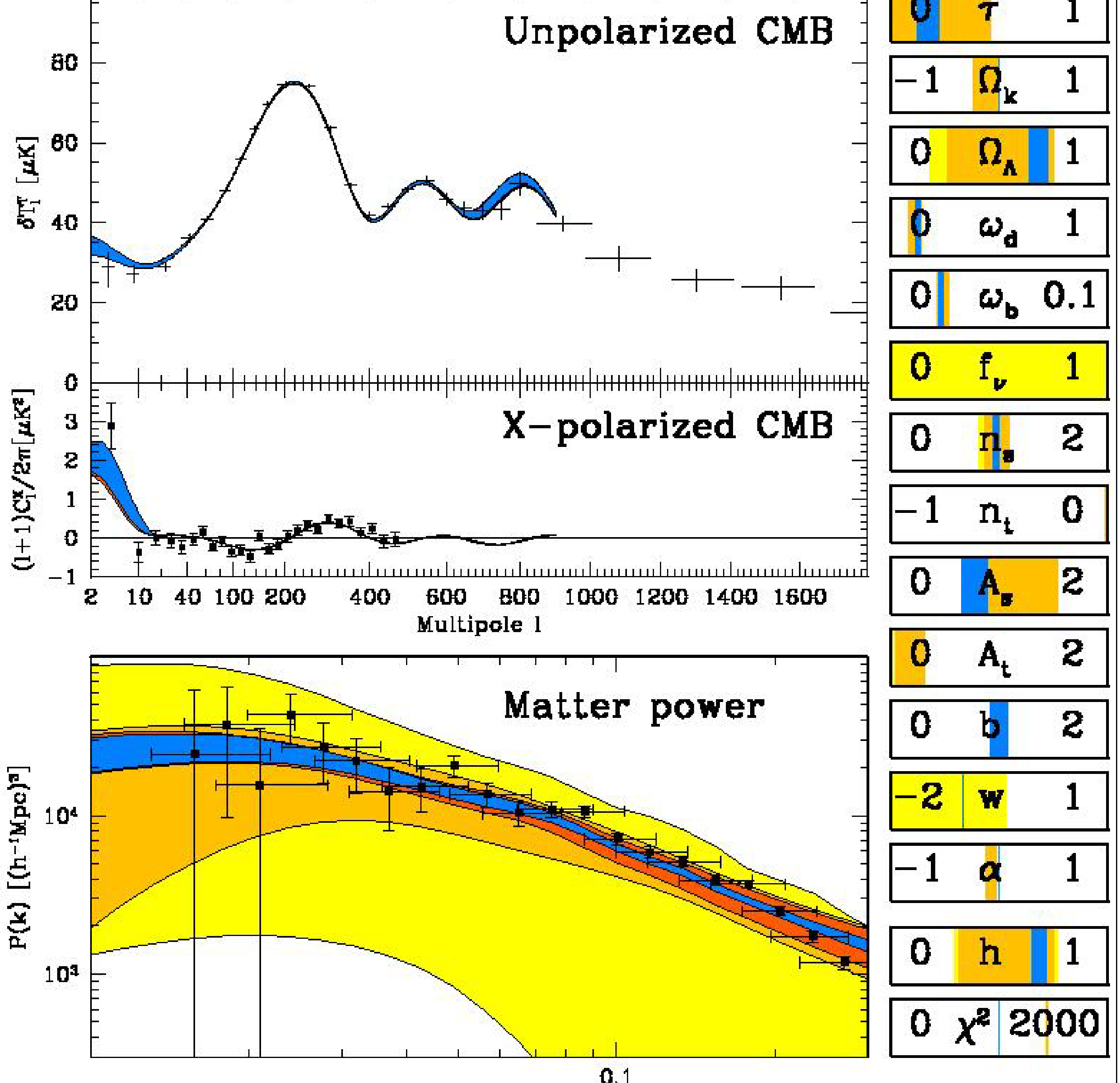
$z \approx 1$ Clusters & Superclusters

Cone diagram: 3-degree slice

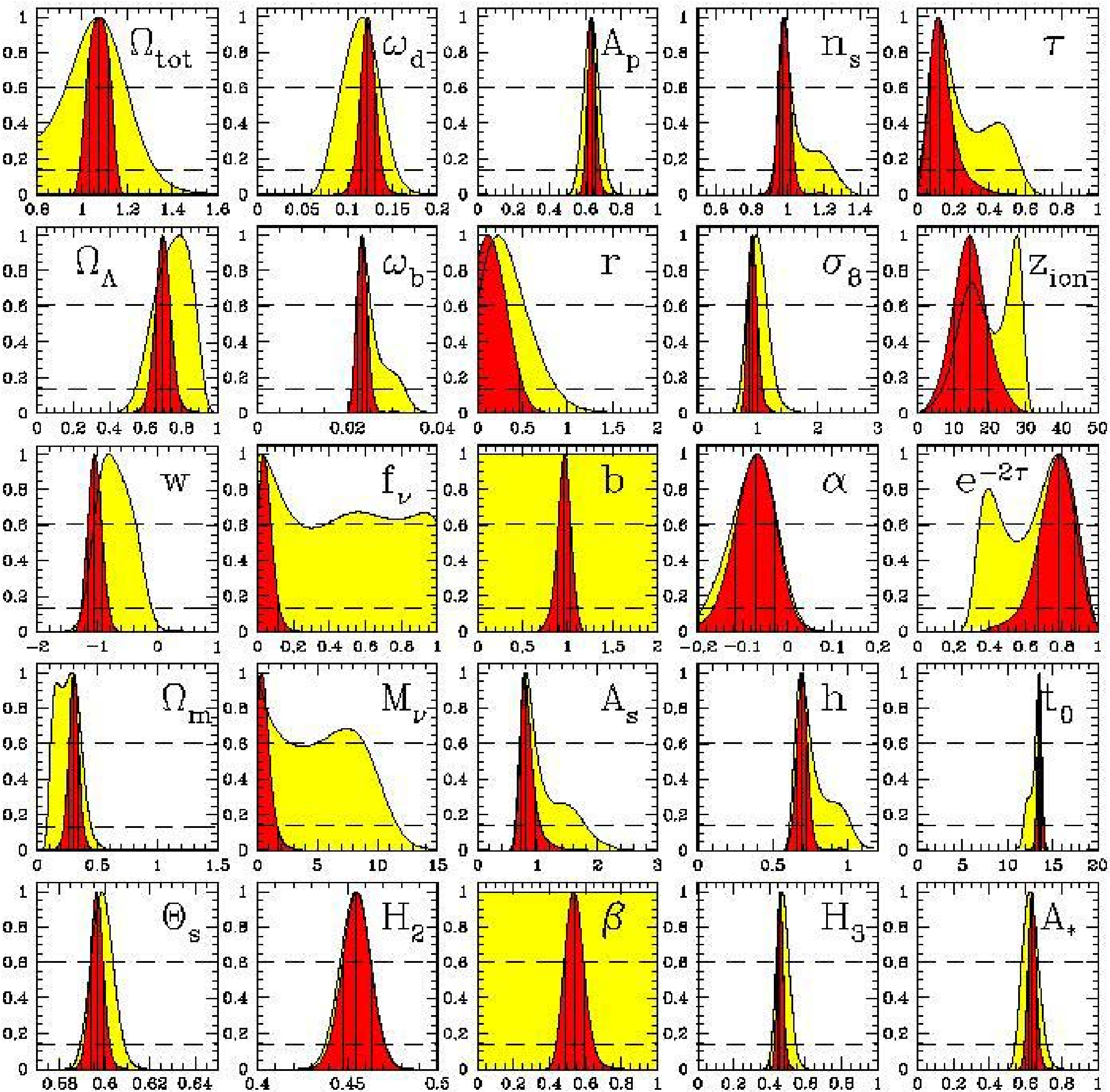




Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
10⁵
10⁴
1000
100
10
1
Current power spectrum P(k) [(h⁻¹ Mpc)³]
Cosmic Microwave Background
SDSS galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]



Unpolarized CMB
X-polarized CMB
Matter power
Multipole l
0.1
0 τ 1
−1 Ω_k 1
0 Ω_Λ 1
0 ω_d 1
0 ω_b 0.1
0 f_ν 1
0 n_s 2
−1 n_t 0
0 A_s 2
0 A_t 2
0 b 2
−2 w 1
−1 α 1
0 h 1
0 χ² 2000

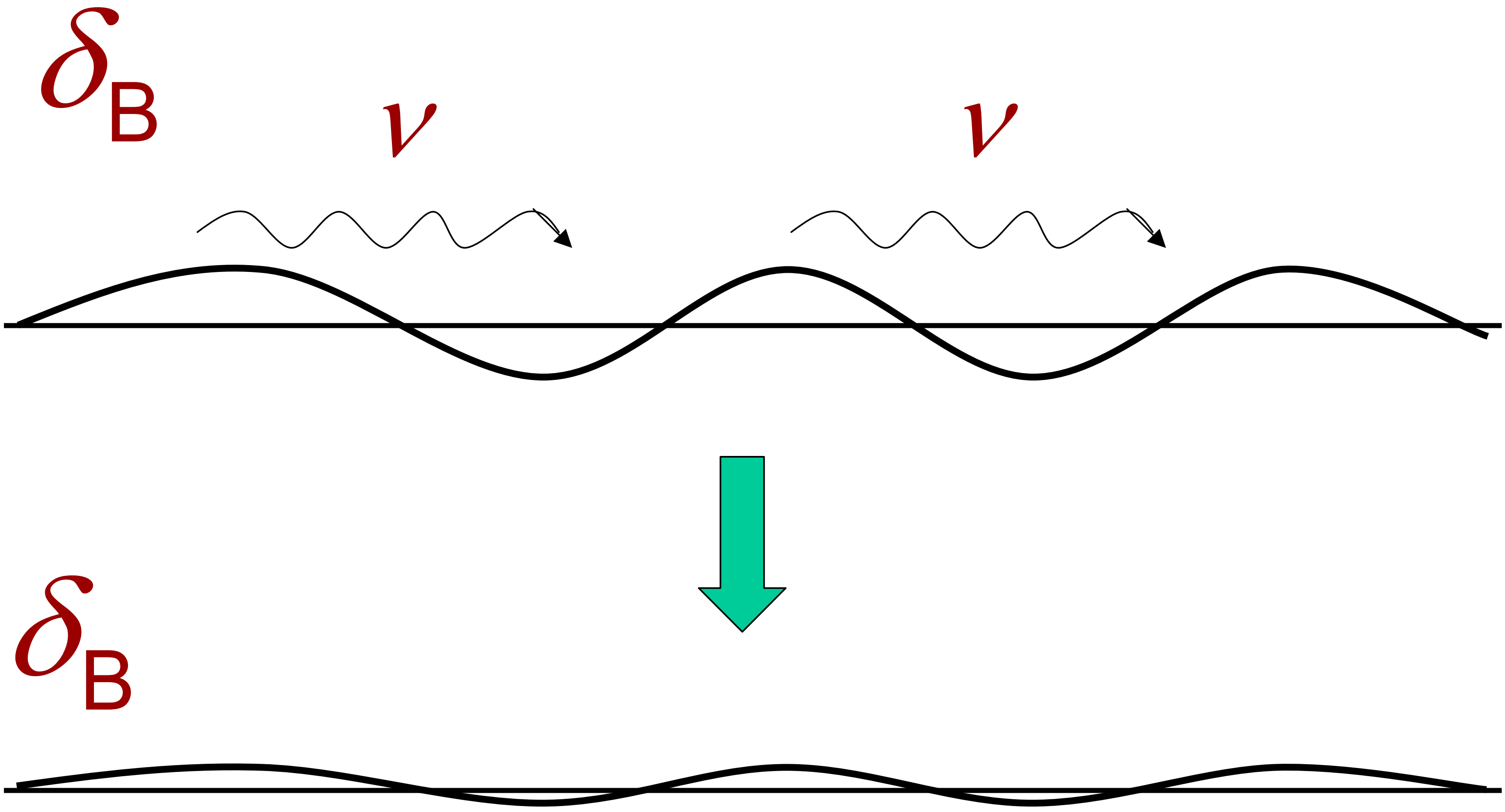


Tegmark et al. (2004)

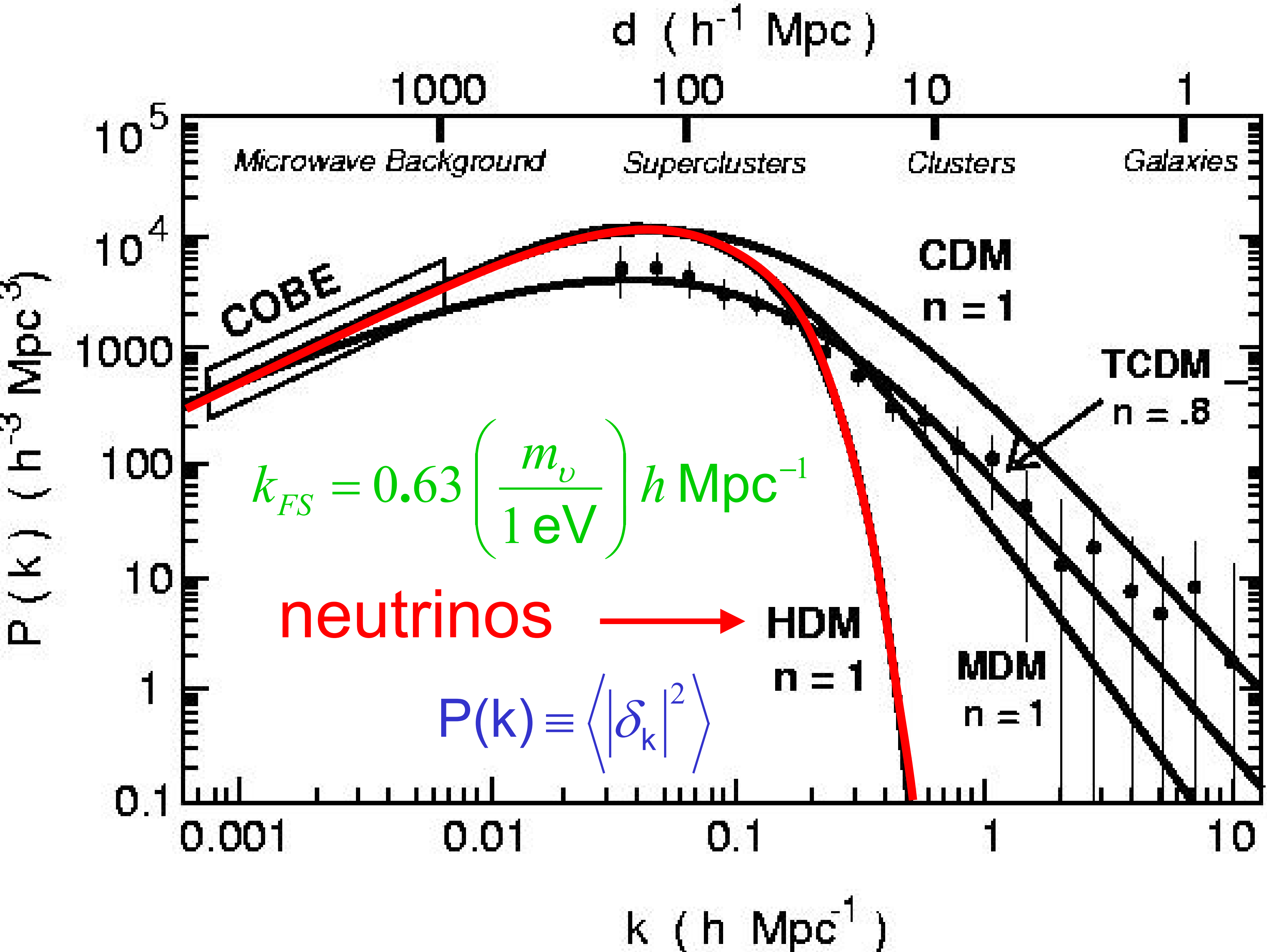
WMAP

SDSS

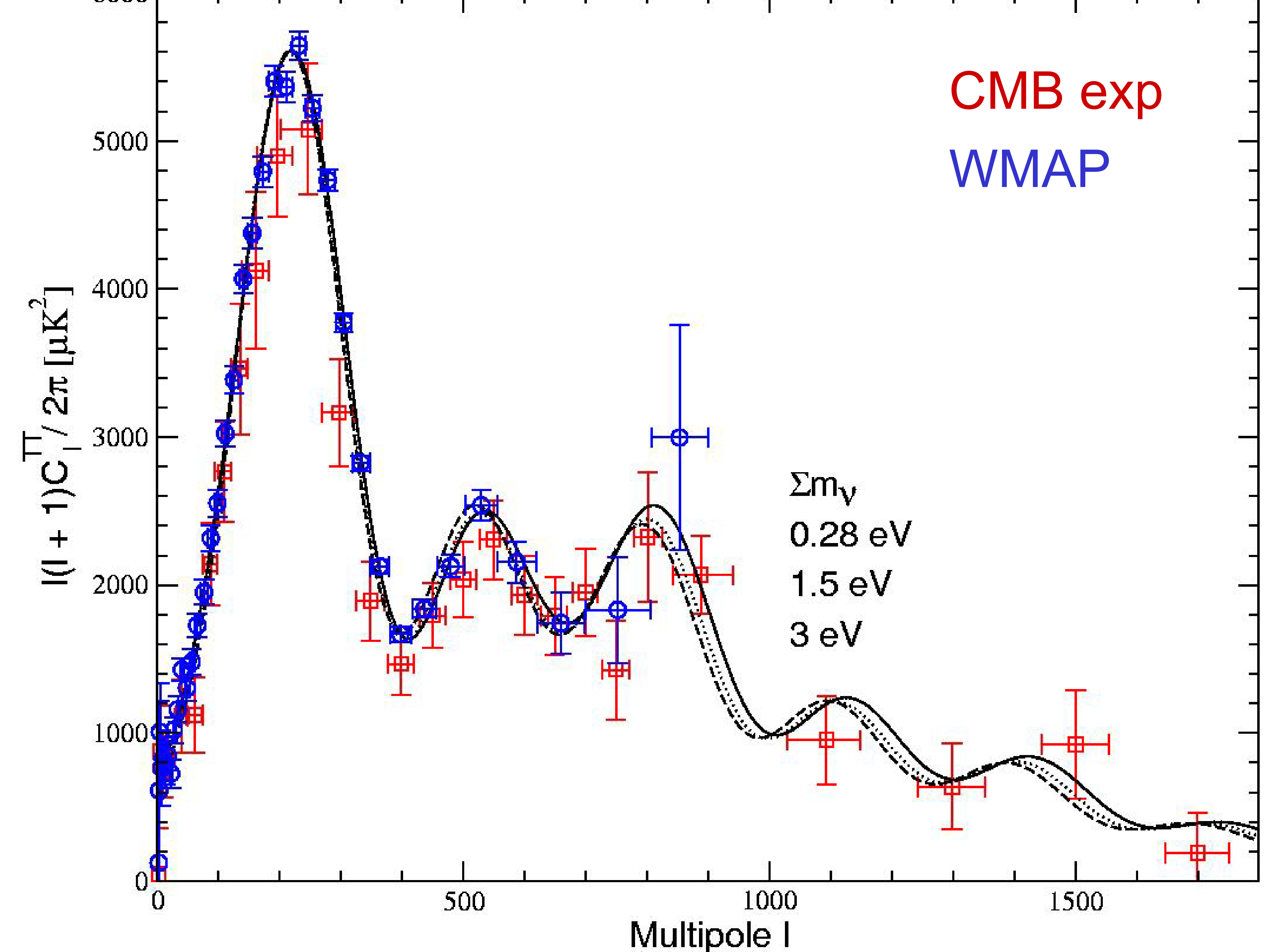
Are Neutrinos The Dark Matter?



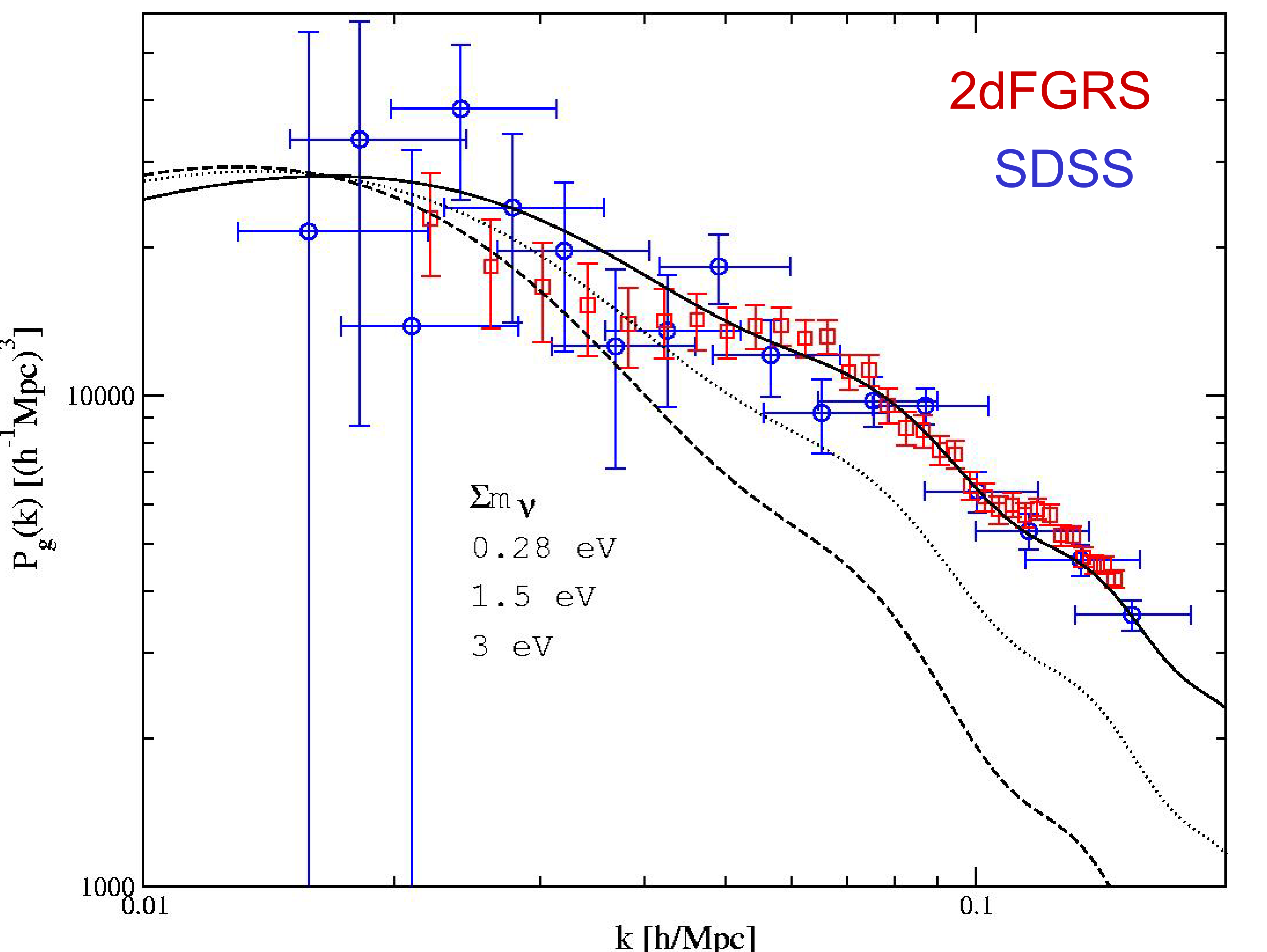
δ_B
ν
ν
δ_B



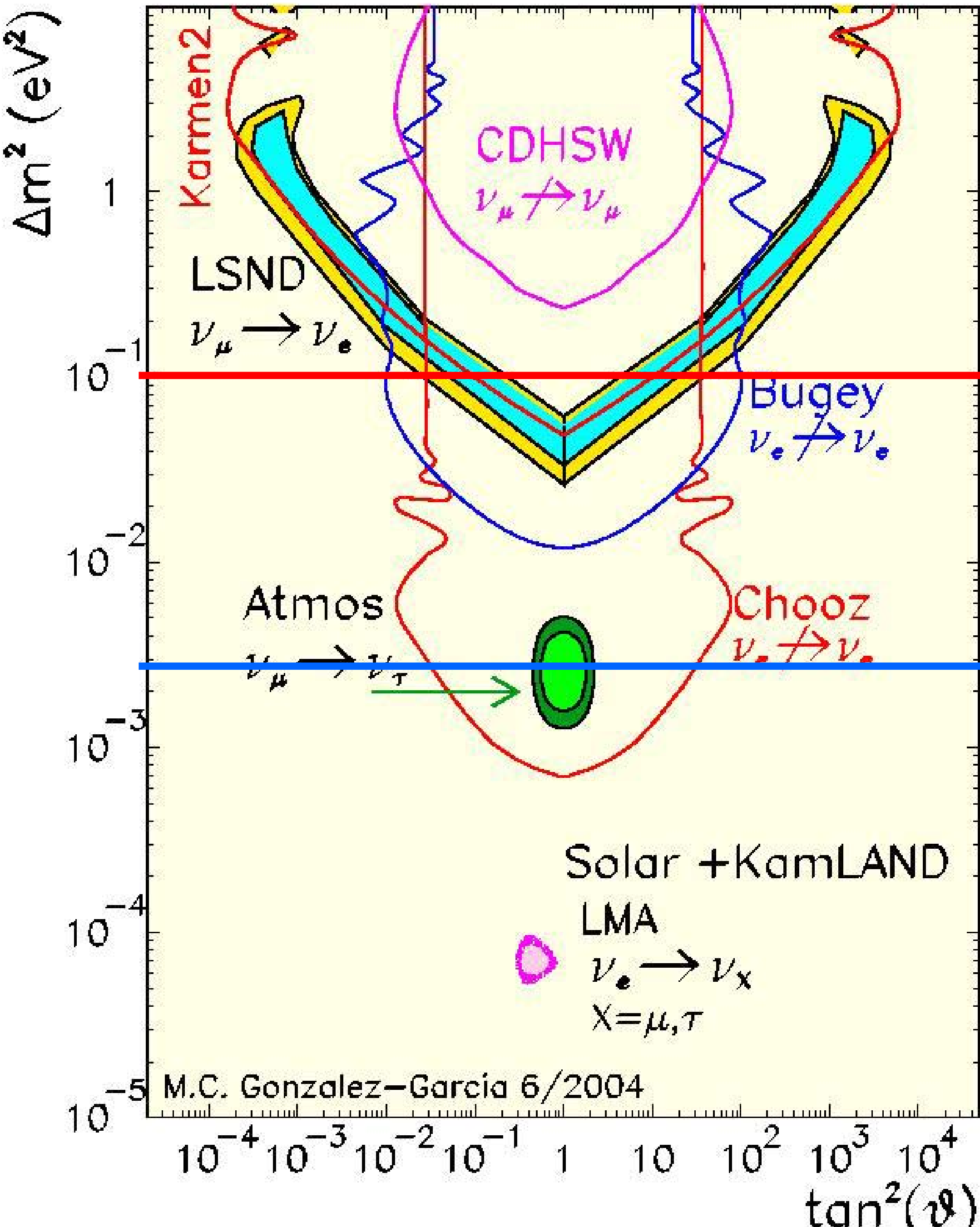
d (h⁻¹ Mpc)
1000
100
10
1
Microwave Background
Superclusters
Clusters
Galaxies
COBE
CDM
n = 1
TCDM
n = .8
$k_{FS} = 0.63\left(\frac{m_\upsilon}{1\,\mathrm{eV}}\right) h\,\mathrm{Mpc}^{-1}$
neutrinos
HDM
n = 1
MDM
n = 1
$P(k) \equiv \langle |\delta_k|^2 \rangle$
P (k) (h⁻³ Mpc³)
10⁵
10⁴
1000
100
10
1
0.1
0.001
0.01
0.1
1
10
k (h Mpc⁻¹)



CMB exp
WMAP
Σmν
0.28 eV
1.5 eV
3 eV
l(l + 1)C_l^TT / 2π [μK^2]
Multipole l
0
1000
2000
3000
4000
5000
0
500
1000
1500



2dFGRS
SDSS
Σm ν
0.28 eV
1.5 eV
3 eV
10000
1000
0.01
0.1
k [h/Mpc]
P_g(k) [(h^-1 Mpc)^3]

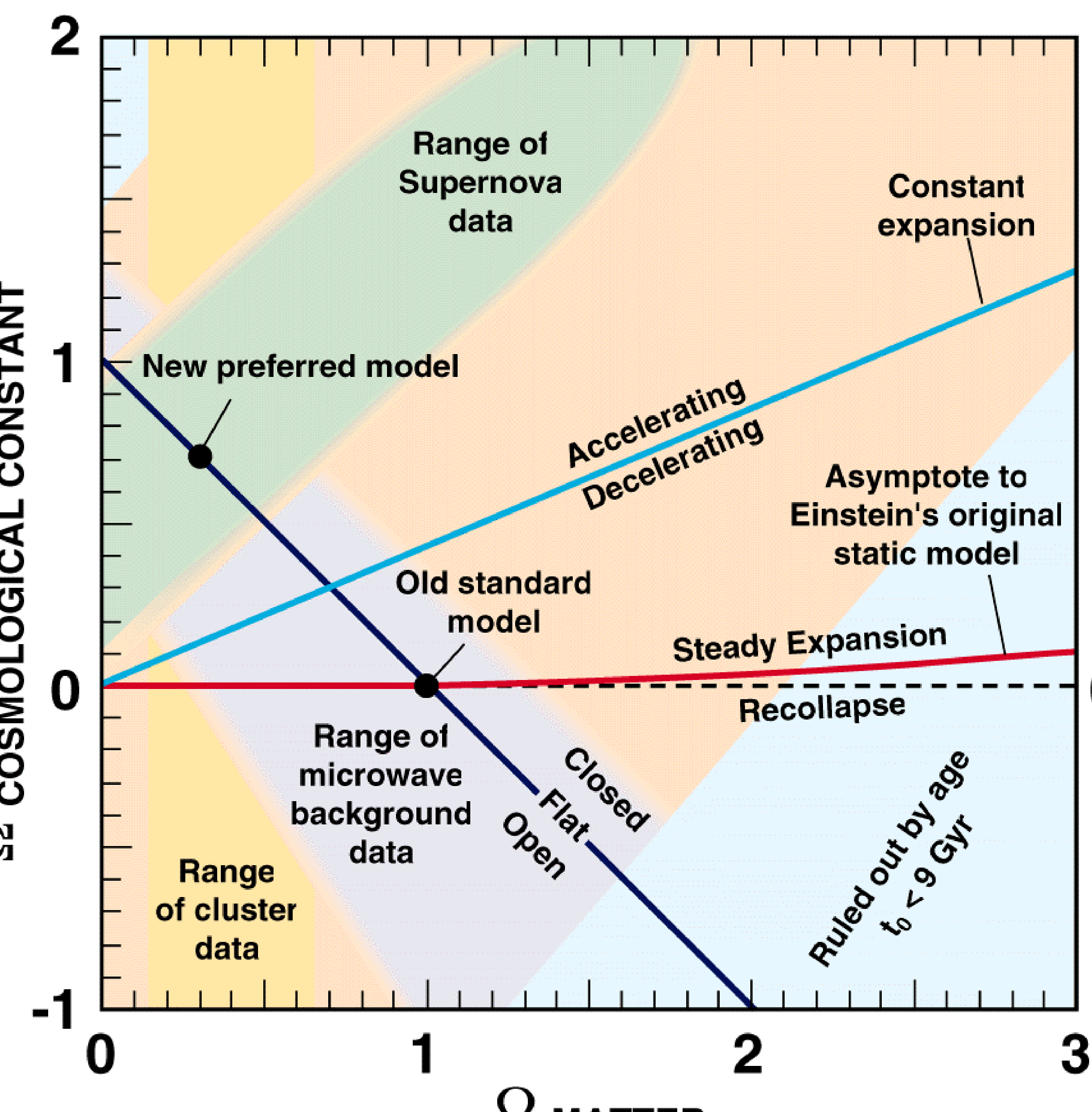


Cosmologically Excluded (WMAP/SDSS)

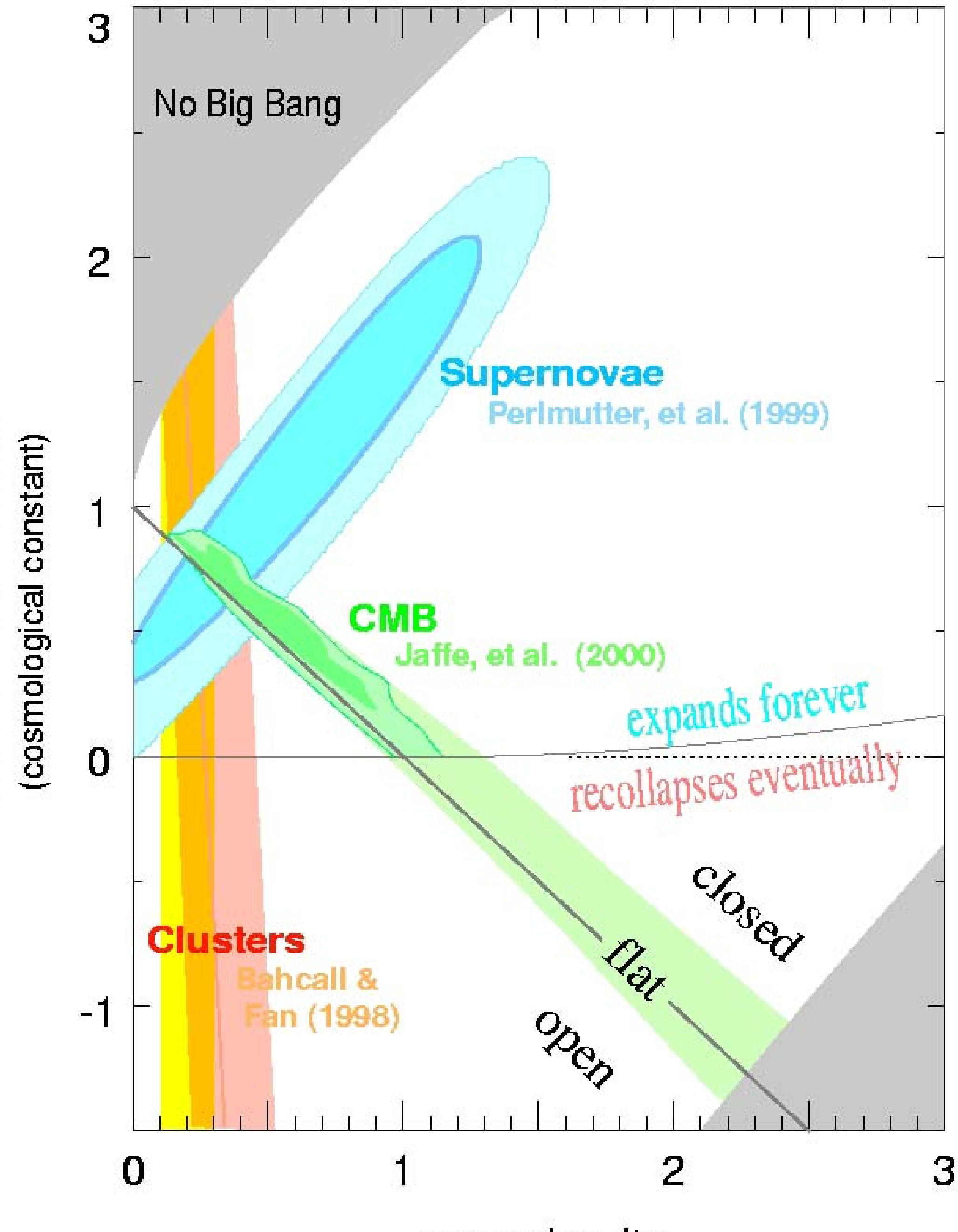
Cosmologically Detectable (Planck)

$$\Omega_\upsilon = \frac{\sum m_\upsilon\, h^{-2}}{93.2\ \text{eV}}$$

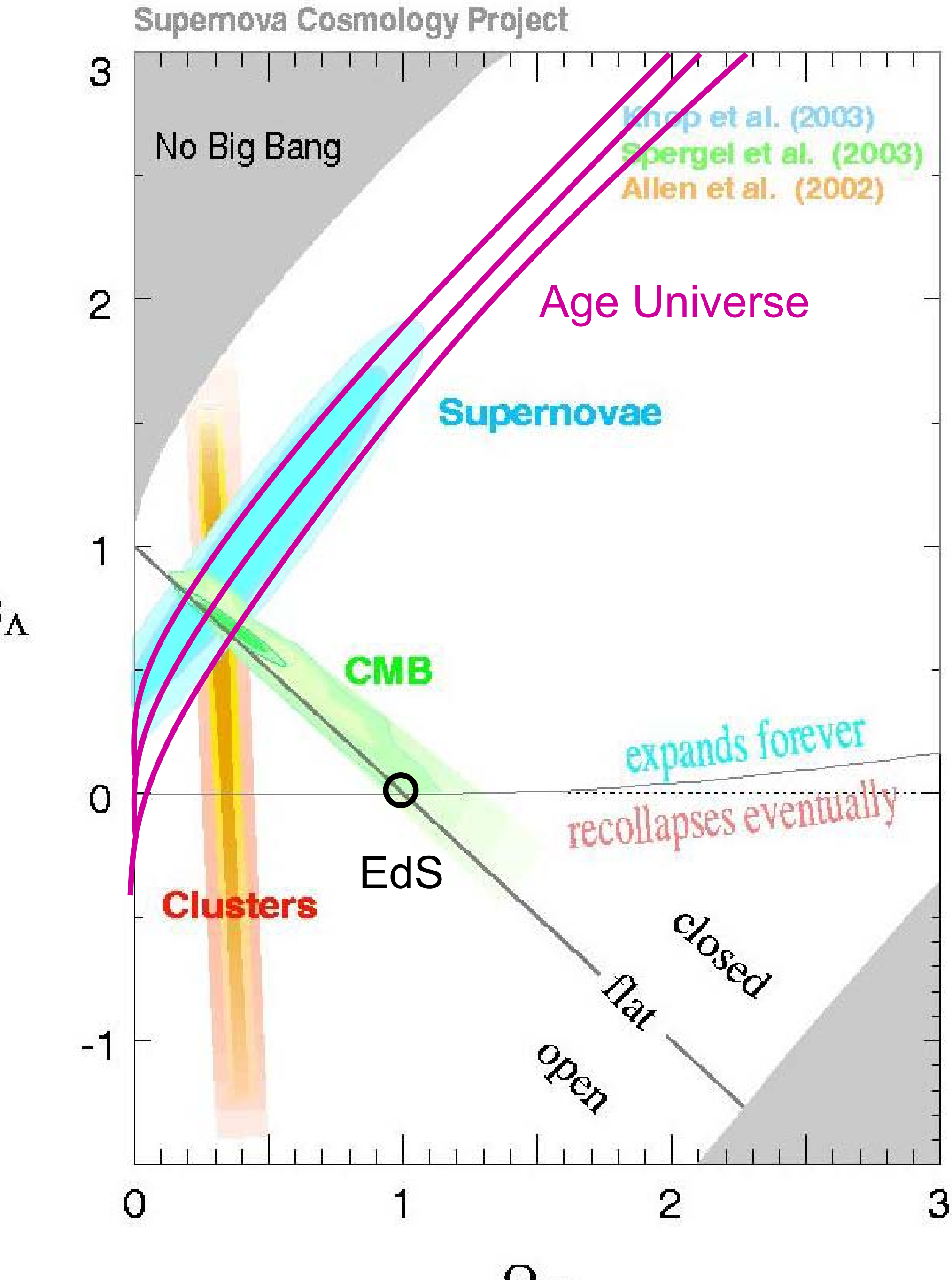
From Concordance to Standard Model



Cosmic Data (1999)



THE CONCORDANCE MODEL (2001)



STANDARD COSMOLOGICAL MODEL (2003)

$\Omega_M = 0.27 \pm 0.04$

$\Omega_\Lambda = 0.73 \pm 0.04$

$\Omega_0 = 1.02 \pm 0.02$

$\Omega_B = 0.044 \pm 0.004$

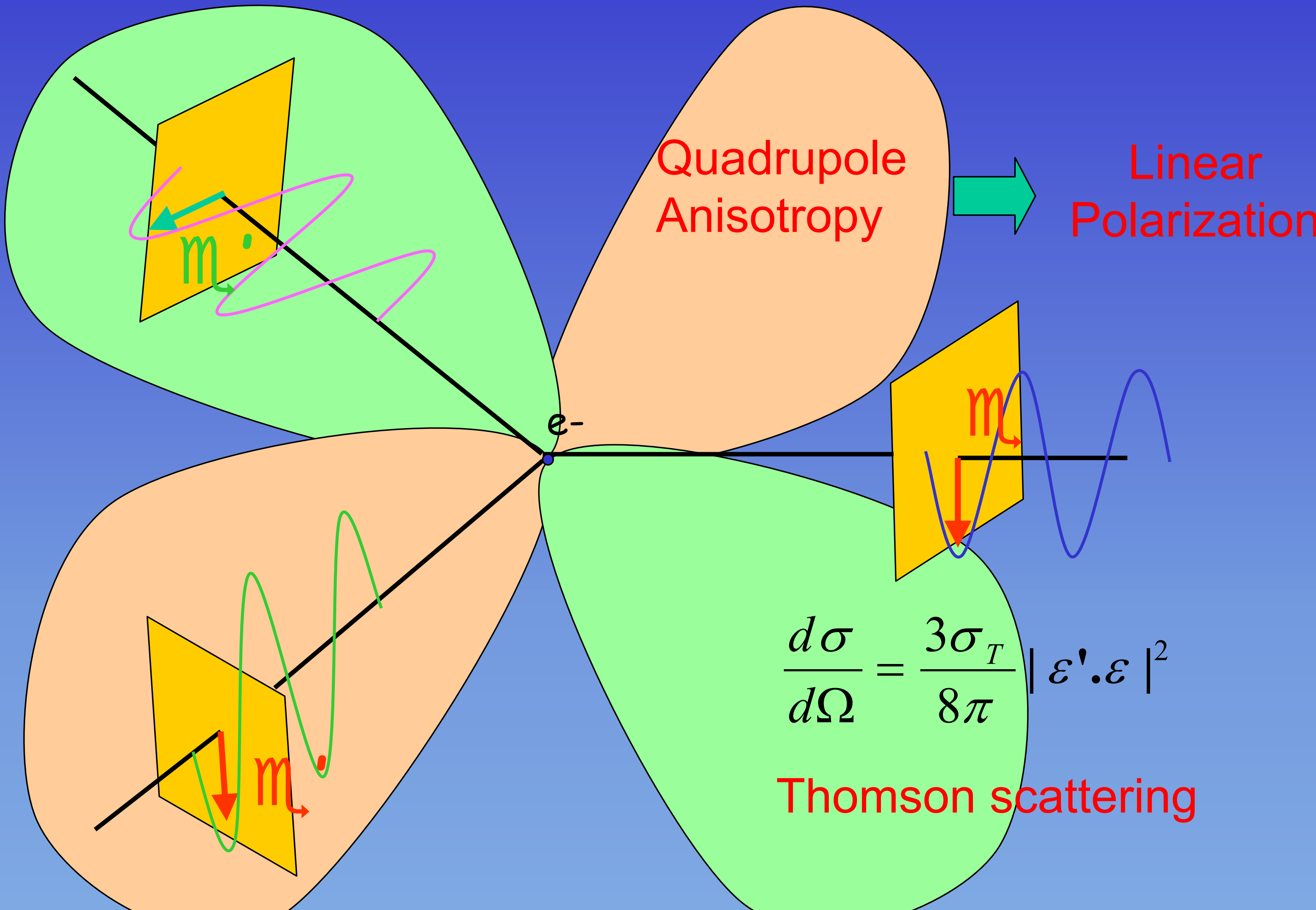
$H_0 = 71 \pm 3$ km/s/Mpc

$t_0 = 13.6 \pm 0.2$ Gyr

Precision Cosmology!
Errors < few%

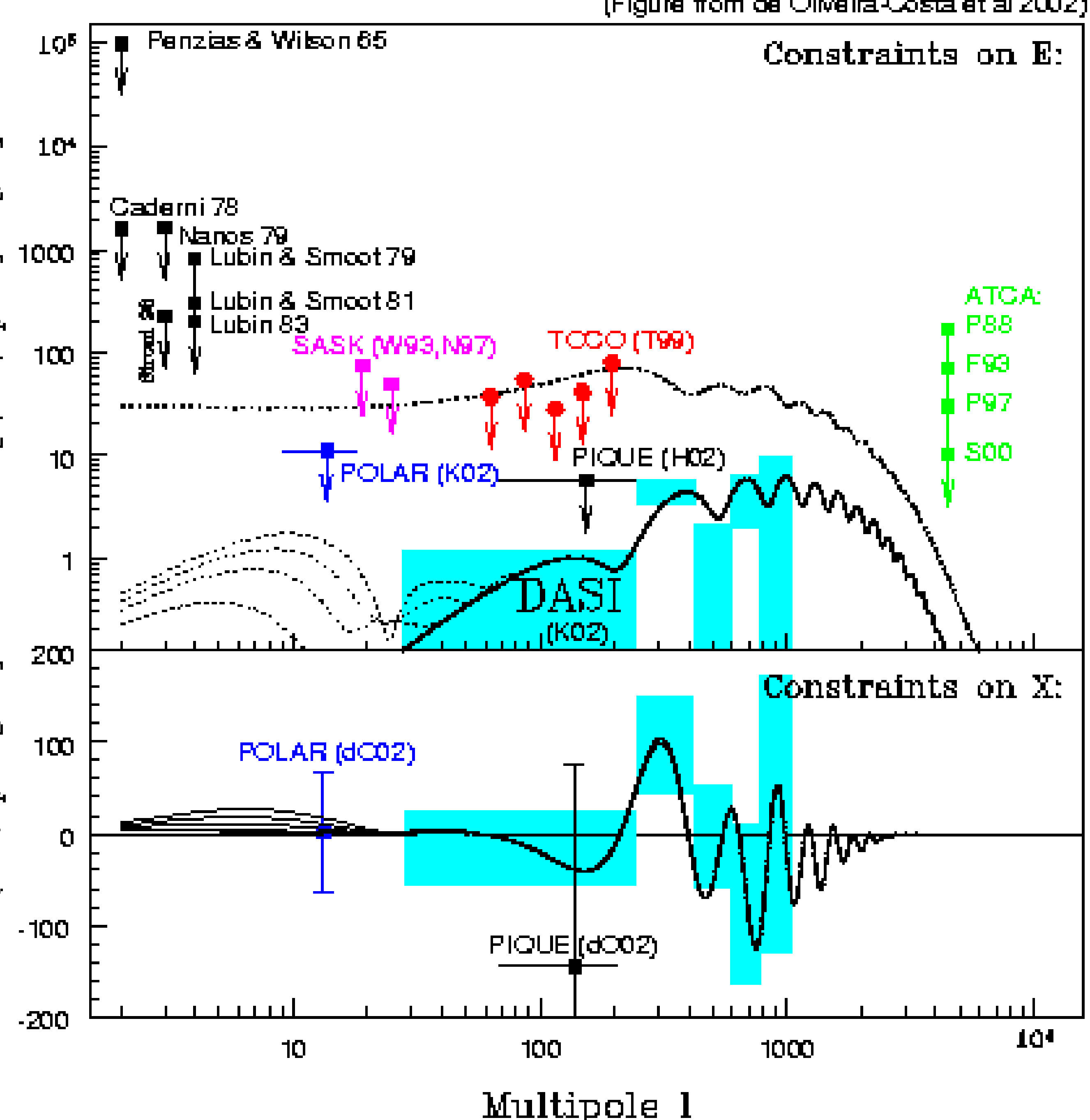
Polarization Anisotropies

Linear Polarization of CMB



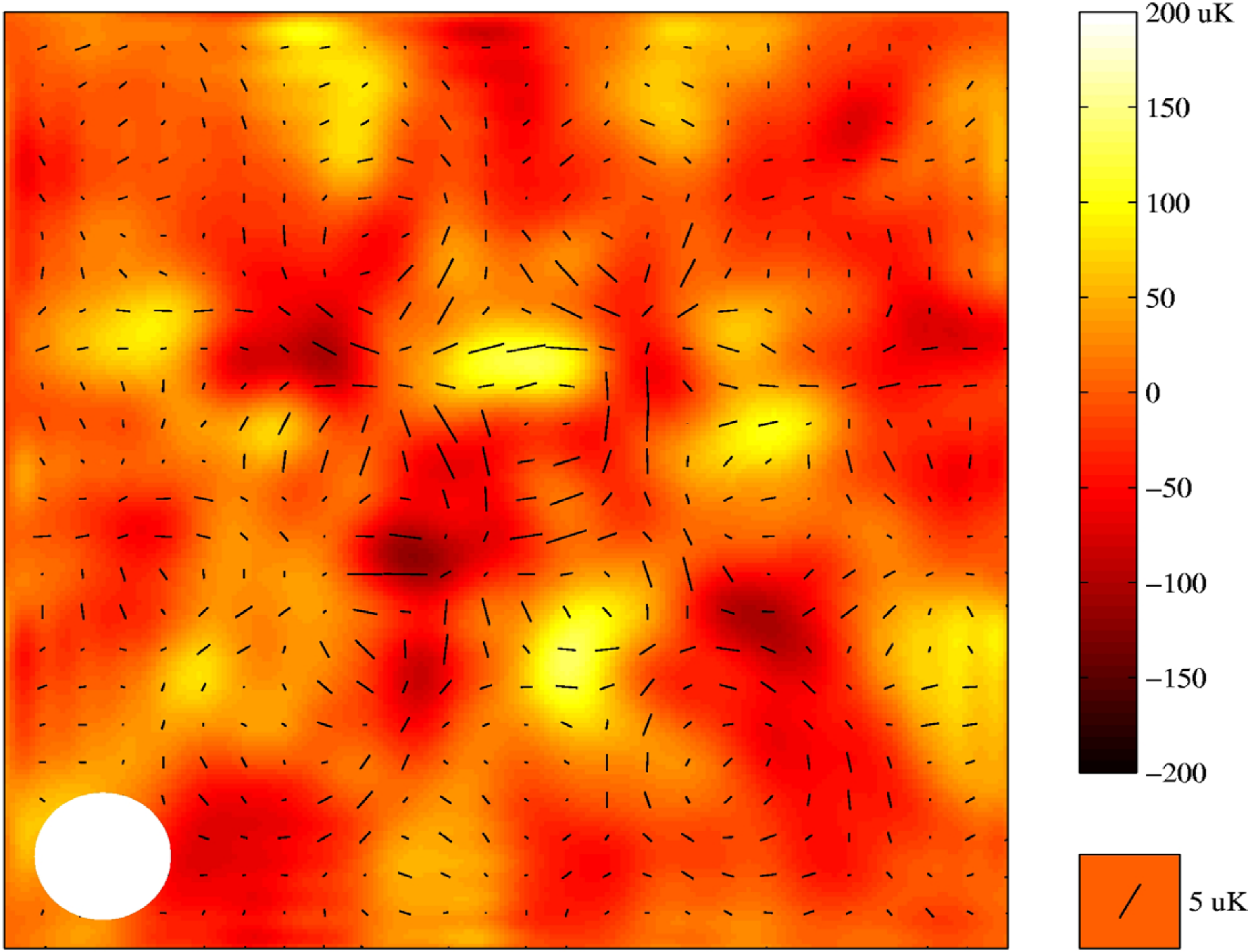
$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} |\varepsilon' . \varepsilon|^2$$

Thomson scattering



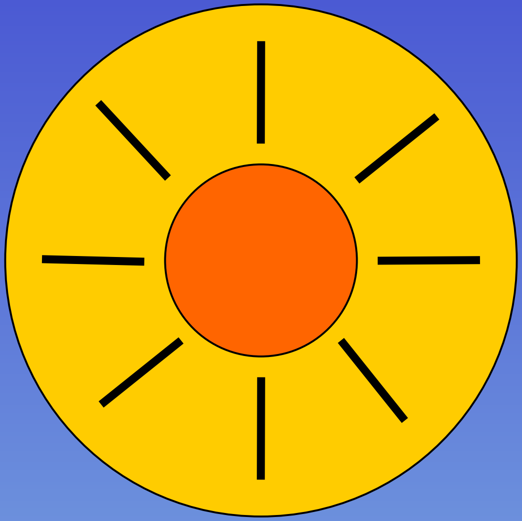
DASI: First detection of Polarization (2002)

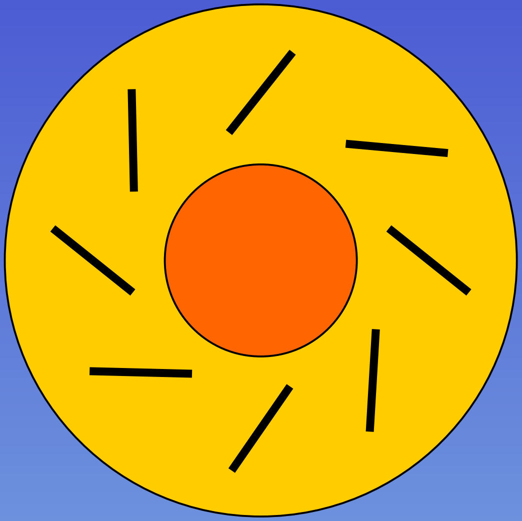
DASI: First measurement Polarization (2002)



Map is 5 degrees square

Polarization around Hot spots





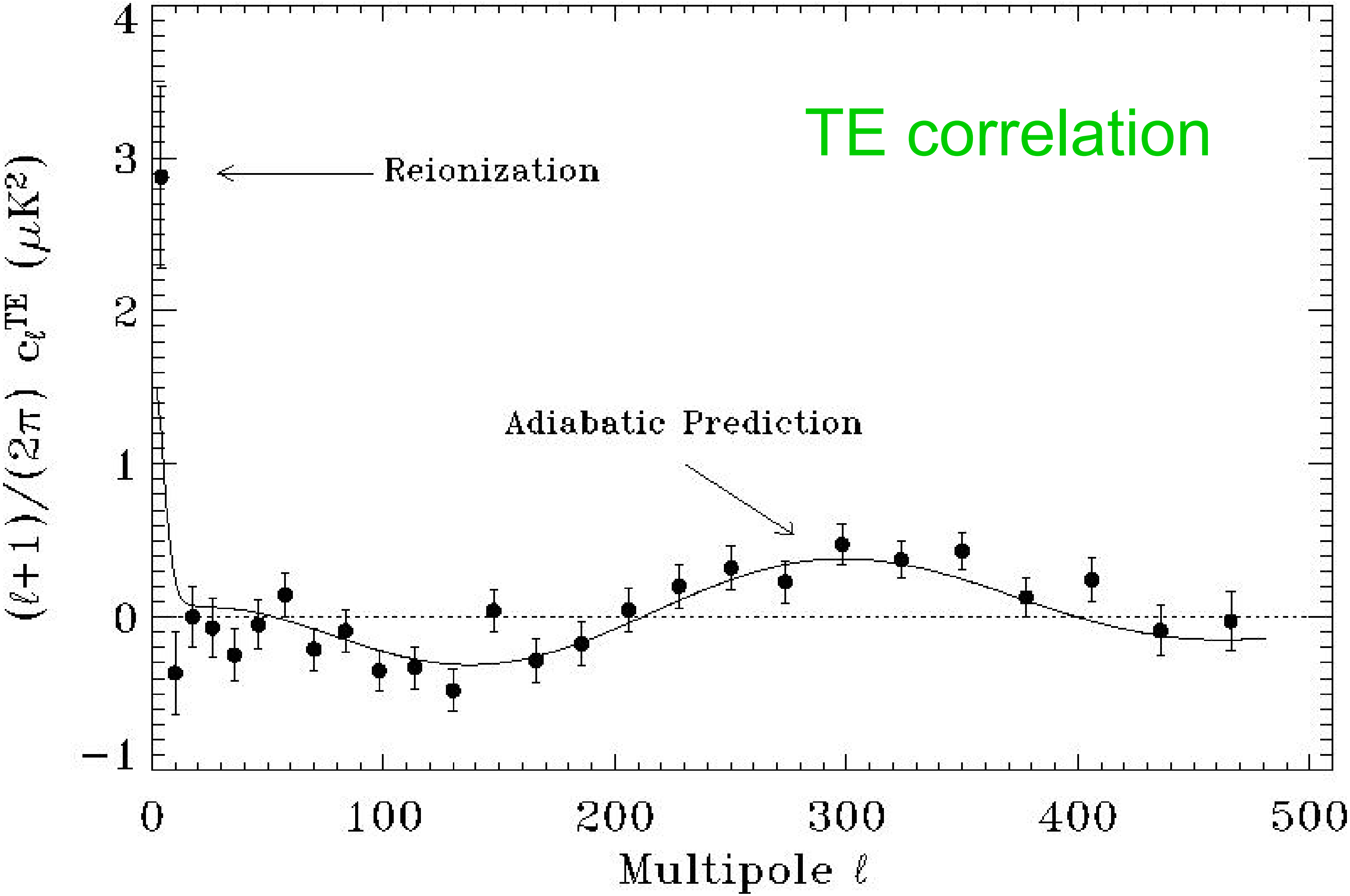
E Polarization

$$\nabla \times E = 0$$

B Polarization

$$\nabla \cdot B = 0$$

WMAP (2003)


TE correlation
Reionization
Adiabatic Prediction
$(\ell+1)/(2\pi)\ c_\ell^{TE}\ (\mu K^2)$
Multipole ℓ
4
3
2
1
0
−1
0
100
200
300
400
500

Confirmed predictions of inflation

The Universe is flat

Acoustic oscillations

E polarization anisotropies

Gaussian spectrum

(Approx.) scale invariant spectrum

No significant isocurvature component

Unconfirmed predictions of inflation

Tensor (gravitational wave) spectrum

B polarization spectrum

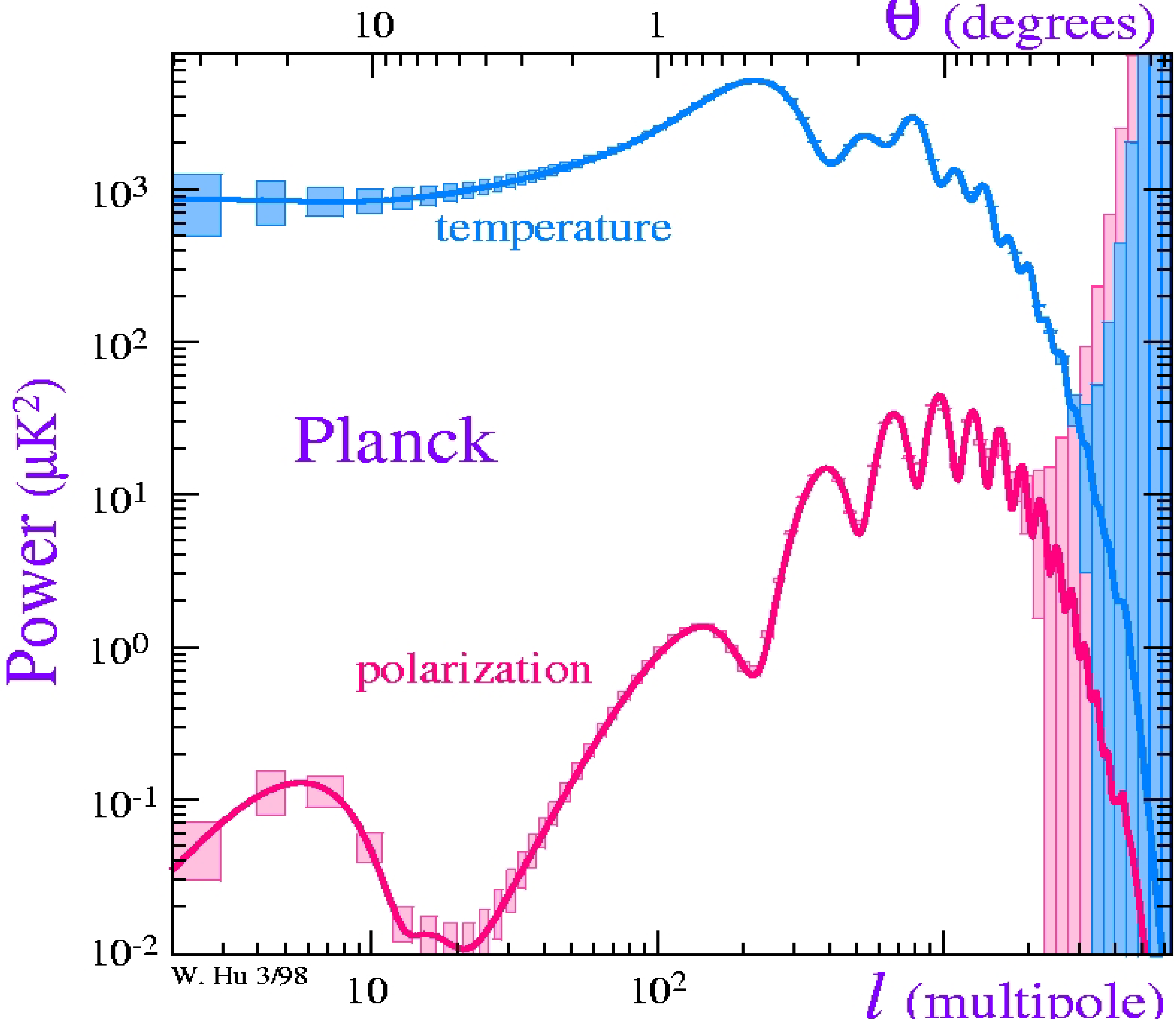
Consistency relation (tensor/scalar)

No vector (vorticity) spectrum

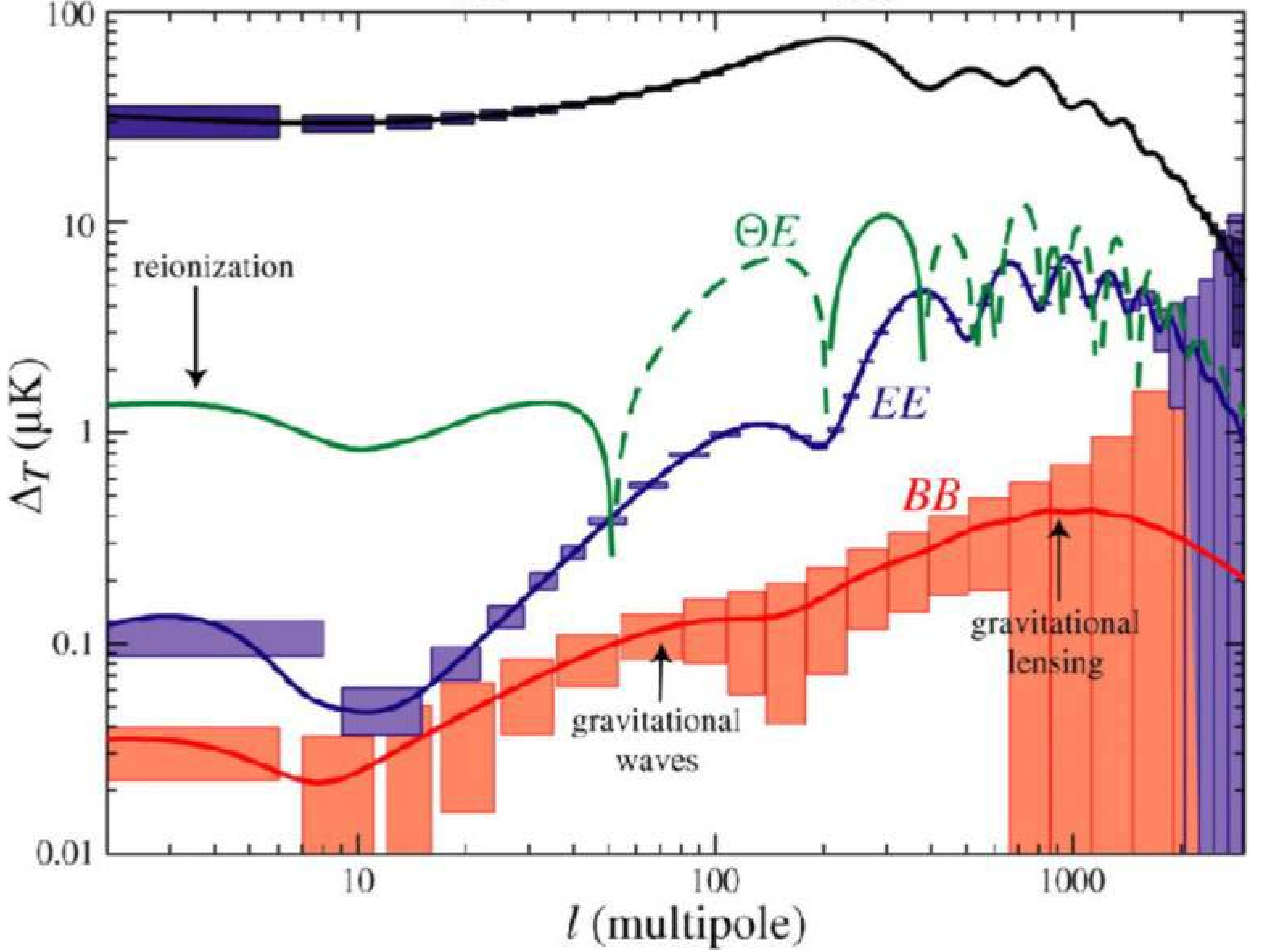
(Possibly) topological defects

Reheating after inflation

Planck Surveyor (2007)



θ (degrees)
10
1
10^3
10^2
10^1
10^0
10^-1
10^-2
Power (μK^2)
temperature
Planck
polarization
W. Hu 3/98
10
10^2
l (multipole)



100
10
1
0.1
0.01
10
100
1000
Δ_T (μK)
l (multipole)
reionization
ΘE
EE
BB
gravitational waves
gravitational lensing

Cosmological Parameters: MAP and Planck

Rate of expansion	$H_0 = 71 \pm 3$ km/s/Mpc	0.8%
Age of the Universe	$t_0 = 13.7 \pm 0.2$ Gyr	0.1%
Spatial Curvature	$\Omega_K < 0.02 \ \ (95\%\ c.l.)$	0.5%
Cosmological Constant	$\Omega_\Lambda = 0.73 \pm 0.04$	0.5%
Dark Matter	$\Omega_M = 0.23 \pm 0.04$	0.6%
Baryon Density	$\Omega_B = 0.044 \pm 0.004$	0.6%
Neutrino Density	$\Omega_\nu < 0.0076\,(95\%\ c.l.)$	1%
Spectral Amplitude	$A = 0.833 \pm 0.085$	0.1%
Spectral tilt	$n_s = 0.93 \pm 0.03$	0.2%
Tensor-scalar ratio	$r < 0.71 \ \ (95\%\ c.l.)$	5%

Conclusions

- Cosmology is becoming “Cosmonomy”, the science of measuring the Cosmos
- The stuff we are made of amounts to just a few percent of all the matter/energy
- Dark matter is here to stay. It could open the door to a new type of particle species (e.g. susy)
- Some kind of dark energy or “smooth tension” is responsible for the acceleration of the Universe. We have no idea of what it is
- We may measure our Local Universe but we ignore its origin and its fate

- The inflationary paradigm provides a general framework in which one can describe all cosmological observations

- The microwave background anisotropies contain a huge amount of information on the cosmological parameters, with small systematic errors

- The Standard Cosmological Model, with errors of few %, has two unsolved fundamental problems: the nature of dark matter and the dark energy

The future looks promising

- New observations:
  Planck & CMBPol (CMB)
  SZE & Grav. Lensing (CMB+LSS)
  SDSS Lyman ♋ (LSS)
  2dFGRS QSO (LSS)
  SNAP & SNConsortium (SNIa)
  MAGIC & GLAST (Dark Matter+Cosmic Rays)
- New theoretical ideas on the nature of vacuum energy (e.g. ghost condensate)
- New models of inflation based on fundamental physics (e.g. strings/branes)