5 Chiral Perturbation Theory

- Sigma Model
- Goldstone Theorem
- Chiral Symmetry
- Effective Goldstone Theory
- Explicit Symmetry Breaking
- Higher Orders

SIGMA MODEL: $\boldsymbol{\Phi}^{\mathsf{T}} \equiv (\sigma, \vec{\pi})$

$$\mathcal{L}_\sigma = \frac{1}{2}\,\partial_\mu \boldsymbol{\Phi}^{\mathsf{T}} \partial^\mu \boldsymbol{\Phi} \,-\, \frac{\lambda}{4}\,\left(\boldsymbol{\Phi}^{\mathsf{T}}\boldsymbol{\Phi} - v^2\right)^2$$

Global Symmetry: $\mathbf{O(4)} \sim \mathbf{SU(2)} \otimes \mathbf{SU(2)}$

- $v^2 < 0$: $\quad m_\Phi^2 = -\lambda\, v^2$
- $v^2 > 0$: $\quad \langle 0|\sigma|0\rangle = v \quad , \quad \langle 0|\vec{\pi}|0\rangle = 0$

SSB: $\mathbf{O(4)} \to \mathbf{O(3)}$ $\quad$ [$\frac{4\times 3}{2} - \frac{3\times 2}{2} = 3$ broken generators]

$$\mathcal{L}_\sigma = \frac{1}{2}\left\{\partial_\mu\hat{\sigma}\,\partial^\mu\hat{\sigma} + \partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} - M^2\hat{\sigma}^2\right\} - \frac{M^2}{2v}\,\hat{\sigma}\left(\hat{\sigma}^2 + \vec{\pi}^2\right) - \frac{M^2}{8v^2}\left(\hat{\sigma}^2 + \vec{\pi}^2\right)^2$$

$$\hat{\sigma} \equiv \sigma - v \qquad ; \qquad M^2 = 2\,\lambda\, v^2$$

3 Massless Goldstone Bosons

1) $\mathbf{\Sigma}(x) \equiv \sigma(x)\, \mathbf{I}_2 + i\, \vec{\tau}\, \vec{\pi}(x)$; $\langle \mathbf{A} \rangle \equiv \mathrm{Tr}\,(\mathbf{A})$

$$\mathcal{L}_\sigma = \frac{1}{4} \langle \partial_\mu \mathbf{\Sigma}^\dagger \partial^\mu \mathbf{\Sigma} \rangle - \frac{\lambda}{16} \left(\langle \mathbf{\Sigma}^\dagger \mathbf{\Sigma} \rangle - 2\, v^2 \right)^2$$

$\mathbf{O(4)} \sim \mathbf{SU(2)_L} \otimes \mathbf{SU(2)_R}$ **Symmetry:** $\mathbf{\Sigma} \longrightarrow g_R\, \mathbf{\Sigma}\, g_L^\dagger$; $g_{L,R} \in SU(2)_{L,R}$

2) $\mathbf{\Sigma}(x) \equiv [v + S(x)]\ \mathbf{U}(x)$; $\mathbf{U} \equiv \exp\left\{ i\, \frac{\vec{\tau}\, \vec{\phi}}{v} \right\} \longrightarrow g_R\, \mathbf{U}\, g_L^\dagger$

$$\mathcal{L}_\sigma = \frac{v^2}{4} \left(1 + \frac{S}{v}\right)^2 \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle + \frac{1}{2} \left(\partial_\mu S\, \partial^\mu S - M^2 S^2\right) - \frac{M^2}{2v} S^3 - \frac{M^2}{8v^2} S^4$$

Derivative Golstone Couplings

3) $E \ll M \sim v$:

$$\boxed{\mathcal{L}_\sigma \approx \frac{v^2}{4} \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle}$$

SYMMETRY REALIZATIONS

Symmetry **G** $\{T_a\}$ ⟷ **Conserved charges** $\mathcal{Q}_a$

Noether Theorem: $\partial_\mu j_a^\mu = 0 \quad ; \quad \mathcal{Q}_a = \int d^3x \, j_a^0(x) \quad ; \quad \frac{d}{dt}\mathcal{Q}_a = 0$

Wigner–Weyl	**Nambu–Goldstone**
$\mathcal{Q}_a \,\vert\, 0 \,\rangle = 0$	$\mathcal{Q}_a \,\vert\, 0 \,\rangle \neq 0$
• Exact Symmetry	• Spontaneously Broken Symmetry
• Degenerate Multiplets	• Massless Goldstone Bosons
• Linear Representation	• Non-Linear Representation

GOLDSTONE THEOREM

$$\mathcal{Q} = \int d^3x\, j^0(x) \quad ; \quad \partial_\mu j_a^\mu = 0 \quad ; \quad \exists\, \mathcal{O} :\; v(t) \equiv \langle 0|\, [\mathcal{Q}(t), \mathcal{O}]\, |0\rangle \neq 0$$

$$\Downarrow$$

$$\exists\, |n\rangle :\; \langle 0|\mathcal{O}|n\rangle\, \langle n|j^0|0\rangle \neq 0 \quad ; \quad E_n\, \delta^{(3)}(\vec{p}_n) = 0 \quad ; \quad M_n = 0$$

Proof: $\quad j^0(x) \,=\, \mathrm{e}^{iP\cdot x}\, j^0(0)\, \mathrm{e}^{-iP\cdot x} \quad ; \quad \sum_n |n\rangle\langle n| \,=\, 1$

$$v(t) \,=\, \sum_n \int d^3x\; \left\{\langle 0|j^0(x)|n\rangle\langle n|\mathcal{O}|0\rangle \,-\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(x)|0\rangle\right\}$$

$$= \sum_n \int d^3x\; \left\{\mathrm{e}^{-ip_n\cdot x}\, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle \,-\, \mathrm{e}^{ip_n\cdot x}\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle\right\}$$

$$= \,(2\pi)^3 \sum_n \delta^{(3)}(\vec{p}_n)\; \left\{\mathrm{e}^{-iE_n t}\, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle \,-\, \mathrm{e}^{iE_n t}\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle\right\} \neq 0$$

$$\frac{d}{dt}\, v(t) \,=\, 0 \,=\, -i\,(2\pi)^3 \sum_n \delta^{(3)}(\vec{p}_n)\, E_n\; \Big\{\, \mathrm{e}^{-iE_n t}\, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle$$

$$+\, \mathrm{e}^{iE_n t}\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle \Big\} \qquad \square$$

CHIRAL SYMMETRY

$\mathbf{q} \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}$; $\mathbf{m_q = 0}$ (Chiral Limit)

$$\mathcal{L}^0_{QCD} = -\frac{1}{4} G^{\mu\nu}_a G^a_{\mu\nu} + \bar{\mathbf{q}}_L \, i \gamma^\mu D_\mu \mathbf{q}_L + \bar{\mathbf{q}}_R \, i \gamma^\mu D_\mu \mathbf{q}_R$$

$$q = \left(\frac{1-\gamma_5}{2}\right) q + \left(\frac{1+\gamma_5}{2}\right) q \equiv q_L + q_R$$

- $\mathcal{L}^0_{QCD}$ **invariant under** $\mathbf{G \equiv SU(3)_L \otimes SU(3)_R}$**:**

$$\bar{\mathbf{q}}_L \to g_L \, \bar{\mathbf{q}}_L \quad ; \quad \bar{\mathbf{q}}_R \to g_R \, \bar{\mathbf{q}}_R \quad ; \quad (g_L, g_R) \in \mathbf{G}$$

- **Only** $\mathbf{SU(3)_V}$ **in the hadronic spectrum:** $(\pi, K, \eta)_{0^-}$; $(\rho, K^*, \omega)_{1^-}$; $\cdots$

$$M_{0^-} < M_{0^+} \quad ; \quad M_{1^-} < M_{1^+}$$

- **The** 0^- **octet is nearly massless:** $\mathbf{m_\pi \approx 0}$

- **The vacuum is not invariant (SSB):** $\langle 0 | \left(\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L \right) | 0 \rangle \neq 0$

8 Massless 0^- Goldstone Bosons

Noether QCD Currents: $G \equiv SU(3)_L \otimes SU(3)_R$

$$J_X^{a\mu} = \bar{\mathbf{q}}_X \gamma^\mu \frac{\lambda^a}{2} \mathbf{q}_X \quad ; \quad \mathcal{Q}_X^a = \int d^3x \, J_X^{a0}(x) \qquad (a = 1, \cdots, 8 \; ; \; X = L, R)$$

Current Algebra ('60) : $\left[\mathcal{Q}_X^a , \mathcal{Q}_Y^b\right] = i\, \delta_{XY}\, f^{abc}\, \mathcal{Q}_X^c$

Dynamical Symmetry Breaking:

- 8 Pseudoscalar Goldstones $\quad \pi^a = (\pi, K, \eta)$
- $\mathcal{Q}_A^a = \mathcal{Q}_R - \mathcal{Q}_L \quad ; \quad \mathcal{O}^b = \bar{\mathbf{q}}\, \gamma_5\, \lambda^b\, \mathbf{q}$

$$\langle 0| \left[\mathcal{Q}_A^a , \mathcal{O}^b\right] |0\rangle = -\frac{1}{2} \langle 0| \bar{\mathbf{q}} \left\{\lambda^a , \lambda^b\right\} \mathbf{q} |0\rangle = -\frac{2}{3} \langle 0| \bar{\mathbf{q}}\, \mathbf{q} |0\rangle$$

$$\Longrightarrow \quad \langle 0| \bar{u}\, u |0\rangle = \langle 0| \bar{d}\, d |0\rangle = \langle 0| \bar{s}\, s |0\rangle \neq 0$$

- $\langle 0| J_A^{a\mu} |\pi^b(p)\rangle = i\, \delta^{ab} \sqrt{2}\, f_\pi\, p^\mu$

EFFECTIVE GOLDSTONE THEORY

- **Mass Gap:** $m_\pi \approx 0 \ll M_\rho$

- **Low-Energy Goldstone Theory:** $E \ll M_\rho$

$$\langle 0| \bar{\mathbf{q}}^j_L \, \mathbf{q}^i_R |0\rangle \quad \Longrightarrow \quad \mathbf{U}_{ij}(\phi) = \left\{ \exp\left(i\sqrt{2}\,\mathbf{\Phi}/f \right) \right\}_{ij}$$

$$\mathbf{\Phi} \equiv \frac{\vec{\lambda}}{\sqrt{2}}\,\vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\,\pi^0 + \frac{1}{\sqrt{6}}\,\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\,\pi^0 + \frac{1}{\sqrt{6}}\,\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix}$$

$$\mathbf{U} \longrightarrow g_R \; \mathbf{U} \; g_L^\dagger \qquad ; \qquad g_{L,R} \in SU(3)_{L,R}$$

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	↓ OPE	
$\lesssim m_c$	γ, g ; μ, e, ν_i s, d, u	$\mathcal{L}_{\rm QCD}^{(n_f=3)}, \mathcal{L}_{\rm eff}^{\Delta S=1,2}$
	↓ $N_C \to \infty$	
M_K	γ ; μ, e, ν_i π, K, η	χPT

EFFECTIVE LAGRANGIAN: $\mathcal{L}(\mathbf{U}) = \sum_n \mathcal{L}_{2n}$

- Goldstone Lagrangian

$$\langle 0|\, \bar{\mathbf{q}}^j_L\, \mathbf{q}^i_R\, |0\rangle \;\longrightarrow\; \mathbf{U}_{ij}(\phi) = \left\{ \exp\left(i\sqrt{2}\,\mathbf{\Phi}/f \right) \right\}_{ij}$$

- Expansion in powers of momenta ⟷ derivatives

Parity ⟶ even dimension ; $\mathbf{U}\,\mathbf{U}^\dagger = 1$ ⟶ $2n \geq 2$

- $SU(3)_L \otimes SU(3)_R$ Invariant

$\mathbf{U}$ ⟶ $g_R\, \mathbf{U}\, g_L^\dagger$; $g_{L,R} \in SU(3)_{L,R}$

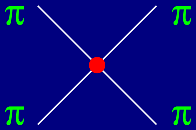
$$\Longrightarrow \quad \boxed{\mathcal{L}_2 = \frac{f^2}{4}\, \langle \partial_\mu \mathbf{U}^\dagger\, \partial^\mu \mathbf{U} \rangle}$$

$$\mathcal{L}_2 = \frac{f^2}{4} \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle = \partial_\mu \pi^- \partial^\mu \pi^+ + \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 + \cdots$$

$$+ \frac{1}{6f^2} \left\{ \left(\pi^+ \overleftrightarrow{\partial}_\mu \pi^- \right) \left(\pi^+ \overleftrightarrow{\partial}^\mu \pi^- \right) + 2 \left(\pi^0 \overleftrightarrow{\partial}_\mu \pi^+ \right) \left(\pi^- \overleftrightarrow{\partial}^\mu \pi^0 \right) + \cdots \right\}$$

$$+ O\left(\pi^6 / f^4 \right)$$

Chiral Symmetry Determines the Interaction:



$$T\left(\pi^+ \pi^0 \to \pi^+ \pi^0 \right) = \frac{t}{f^2}$$

$t \equiv (p'_+ - p_+)^2$

Weinberg

Non-Linear Lagrangian: $2\pi \to 2\pi, 4\pi, \cdots$ related

EXPLICIT SYMMETRY BREAKING

$$\mathcal{L}_{QCD} \equiv \mathcal{L}^0_{QCD} + \bar{\mathbf{q}}\,(\not{\mathbf{v}} + \gamma_5\,\not{\mathbf{a}})\,\mathbf{q} - \bar{\mathbf{q}}\,(\mathbf{s} - i\,\gamma_5\,\mathbf{p})\,\mathbf{q}$$
$$= \mathcal{L}^0_{QCD} + \bar{\mathbf{q}}_L\,\not{\mathbf{l}}\,\mathbf{q}_L + \bar{\mathbf{q}}_R\,\not{\mathbf{r}}\,\mathbf{q}_R - \bar{\mathbf{q}}_R\,(\mathbf{s} + i\,\mathbf{p})\,\mathbf{q}_L - \bar{\mathbf{q}}_L\,(\mathbf{s} - i\,\mathbf{p})\,\mathbf{q}_R$$

$$\mathbf{l}_\mu \equiv \mathbf{v}_\mu - \mathbf{a}_\mu = e\,\mathcal{Q}\,A_\mu + \cdots$$
$$\mathbf{r}_\mu \equiv \mathbf{v}_\mu + \mathbf{a}_\mu = e\,\mathcal{Q}\,A_\mu + \cdots \qquad \mathcal{Q} \equiv \tfrac{1}{3}\,\mathrm{diag}(2,-1,-1)$$
$$\mathbf{s} = \mathcal{M} + \cdots \qquad ; \qquad \mathcal{M} \equiv \mathrm{diag}(m_u, m_d, m_s)$$

Local $\mathrm{SU(3)_L} \otimes \mathrm{SU(3)_R}$ Symmetry:

$$\mathbf{q}_L \longrightarrow g_L\,\mathbf{q}_L \qquad \mathbf{q}_R \longrightarrow g_R\,\mathbf{q}_R$$

$$\mathbf{l}_\mu \longrightarrow g_L\,\mathbf{l}_\mu\,g_L^\dagger + i\,g_L\,\partial_\mu g_L^\dagger$$
$$\mathbf{r}_\mu \longrightarrow g_R\,\mathbf{r}_\mu\,g_R^\dagger + i\,g_R\,\partial_\mu g_R^\dagger$$
$$(\mathbf{s} + i\,\mathbf{p}) \longrightarrow g_R\,(\mathbf{s} + i\,\mathbf{p})\,g_L^\dagger$$

Lowest–Order Effective Lagrangian

$$\mathcal{L} = \frac{f^2}{4} \langle D_\mu \mathbf{U}\, D^\mu \mathbf{U}^\dagger + \chi\, \mathbf{U}^\dagger + \mathbf{U}\, \chi^\dagger \rangle$$

$$D_\mu \mathbf{U} = \partial_\mu \mathbf{U} - i\, \mathbf{r}_\mu\, \mathbf{U} + i\, \mathbf{U}\, \mathbf{l}_\mu$$

$$\chi \equiv 2\, B_0\, (\mathbf{s} + i\, \mathbf{p})$$

Currents:

$$\mathbf{J}^\mu_{L} = \frac{\partial}{\partial \mathbf{l}_\mu} \mathcal{L}_2 = \frac{i}{2} f^2\, D^\mu \mathbf{U}^\dagger\, \mathbf{U} = \frac{f}{\sqrt{2}}\, D^\mu \mathbf{\Phi} + \cdots$$

$$\mathbf{J}^\mu_{R} = \frac{\partial}{\partial \mathbf{r}_\mu} \mathcal{L}_2 = \frac{i}{2} f^2\, D^\mu \mathbf{U}\, \mathbf{U}^\dagger = -\frac{f}{\sqrt{2}}\, D^\mu \mathbf{\Phi} + \cdots$$

$$\langle 0 |\, (J^\mu_A)_{12}\, | \pi^+(p) \rangle = i\, \sqrt{2}\, f\, p^\mu \quad \longrightarrow \quad f = f_\pi \approx 92.4\ \mathrm{MeV}$$

$(\pi^+ \to \mu^+ \nu_\mu)$

$$\bar{\mathbf{q}}^j_{L}\, \mathbf{q}^i_{R} = -\frac{\partial\, \mathcal{L}_2}{\partial (\mathbf{s} - i\, \mathbf{p})^{ji}} = -\frac{f^2}{2}\, B_0\, \mathbf{U}^{ij} \quad \longrightarrow \quad \langle 0 | \bar{\mathbf{q}}^j \mathbf{q}^i | 0 \rangle = -f^2\, B_0\, \delta_{ij}$$

QUARK MASSES: $\frac{f^2}{4}\langle\chi\,\mathbf{U}^\dagger+\mathbf{U}\,\chi^\dagger\rangle \quad\longrightarrow\quad \mathcal{L}_m = -B_0\,\langle\mathcal{M}\,\Phi^2\rangle$

$$\mathcal{L}_m = -B_0\left\{(m_u+m_d)\left[\pi^+\pi^- + \frac{1}{2}\,\pi^0\pi^0\right] + (m_u+m_s)\,K^+K^- \right.$$
$$\left. + (m_d+m_s)\,K^0\bar{K}^0 + \frac{1}{6}\,(m_u+m_d+4\,m_s)\,\eta^2 + \frac{1}{\sqrt{3}}\,(m_u-m_d)\,\pi^0\eta\right\}$$

Isospin limit: $\mathbf{m_u = m_d = \hat{m}}$

$$\frac{M_\pi^2}{2\,\hat{m}} = \frac{M_K^2}{\hat{m}+m_s} = \frac{3\,M_\eta^2}{2\,\hat{m}+4\,m_s} = B_0$$

- **Gell-Mann–Okubo:** $4\,M_K^2 = M_\pi^2 + 3\,M_\eta^2$
- **Gell-Mann–Oakes–Renner:** $f^2\,M_\pi^2 = -\hat{m}\,\langle 0|\bar{u}\,u + \bar{d}\,d|0\rangle$

QUARK MASS RATIOS

Dashen Theorem:

$$\left(M^2_{K^0} - M^2_{K^\pm}\right)_{\rm em} = \left(M^2_{\pi^0} - M^2_{\pi^\pm}\right)_{\rm em} + \mathcal{O}(e^2 p^2)$$

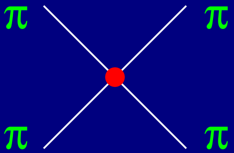
Proof: $e^2 \langle \mathcal{Q}_{\rm R} \mathrm{U} \mathcal{Q}_{\rm L} \mathrm{U}^\dagger \rangle = -\frac{2e^2}{f^2} \left(\pi^+\pi^- + K^+K^-\right) + \mathcal{O}(\phi^2)$; $\mathcal{Q}_X \to g_X\, \mathcal{Q}_X\, g_X^\dagger$ $\square$

$$\frac{m_d - m_u}{m_d + m_u} = \frac{\left(M^2_{K^0} - M^2_{K^\pm}\right) - \left(M^2_{\pi^0} - M^2_{\pi^\pm}\right)}{M^2_{\pi^0}} \approx 0.29$$

$$\frac{m_s - m_u}{m_u + m_d} = \frac{M^2_{K^0} - M^2_{\pi^0}}{M^2_{\pi^0}} \approx 12.6$$

$\Rightarrow$ $\mathbf{m_u : m_d : m_s = 0.55 : 1 : 20.3}$ Weinberg

$$\frac{f^2}{4}\,\langle \chi\, \mathbf{U}^\dagger + \mathbf{U}\, \chi^\dagger \rangle \;=\; -B_0\,\langle \mathcal{M}\, \Phi^2 \rangle \;+\; \frac{B_0}{6\, f^2}\,\langle \mathcal{M}\, \Phi^4 \rangle \;+\; \cdots$$



$$T\left(\pi^+\pi^0 \to \pi^+\pi^0\right) \;=\; \frac{t - M_\pi^2}{f_\pi^2}$$

$t \equiv (p'_+ - p_+)^2$

Weinberg

$\mathcal{L}_2$ ⟷ **Current Algebra** **60's**

Chiral Power Counting

$\mathbf{U}$	$\mathcal{O}(p^0)$
$D_\mu \mathbf{U}$, $\mathbf{l}_\mu$, $\mathbf{r}_\mu$	$\mathcal{O}(p^1)$
χ , $\mathbf{F}^{\mu\nu}_{L,R}$	$\mathcal{O}(p^2)$

$$\mathbf{F}_L^{\mu\nu} \equiv \partial^\mu \mathbf{l}^\nu - \partial^\nu \mathbf{l}^\mu - i\, [\mathbf{l}^\mu, \mathbf{l}^\nu]$$

$$\mathbf{F}_R^{\mu\nu} \equiv \partial^\mu \mathbf{r}^\nu - \partial^\nu \mathbf{r}^\mu - i\, [\mathbf{r}^\mu, \mathbf{r}^\nu]$$

General connected diagram with N_d vertices of $\mathcal{O}(p^d)$ and L loops:

$$D \;=\; 2\,L + 2 + \sum_d N_d\,(d-2)$$

Weinberg

- $D = 2$: $\quad L = 0$, $d = 2$
- $D = 4$: $\quad L = 0$, $d = 4$, $N_4 = 1$
  $L = 1$, $d = 2$

$\mathcal{O}(\mathbf{p}^4)$ χPT

i) $\mathcal{L}_4$ **at tree level** (Gasser–Leutwyler)

$$\begin{aligned} \mathcal{L}_4 \; = \; & L_1 \, \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \, \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\ & + L_3 \, \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle + L_4 \, \langle D_\mu U^\dagger D^\mu U \rangle \langle U^\dagger \chi + \chi \dagger U \rangle \\ & + L_5 \, \langle D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U \right) \rangle + L_6 \, \langle U^\dagger \chi + \chi^\dagger U \rangle^2 \\ & + L_7 \, \langle U^\dagger \chi - \chi^\dagger U \rangle^2 + L_8 \, \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi \rangle \\ & - i \, L_9 \, \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle + L_{10} \, \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle \end{aligned}$$

ii) $\mathcal{L}_2$ **at one loop** **(unitarity):** $T_4 \sim p^4 \left\{ a \log(p^2/\mu^2) + b(\mu) \right\}$

- Chiral Logarithms unambiguously predicted
- L_i's fixed by QCD dynamics. 1–loop divergences ⟶ $L_i^r(\mu)$

iii) **Wess–Zumino–Witten term** **(chiral anomaly):** $\pi^0, \eta \to \gamma\gamma$

$O(p^4)$ χPT COUPLINGS

i	$L_i^r(M_\rho) \times 10^3$	Source	Γ_i
1	0.4 ± 0.3	K_{e4}, $\pi\pi \to \pi\pi$	$3/32$
2	1.4 ± 0.3	K_{e4}, $\pi\pi \to \pi\pi$	$3/16$
3	-3.5 ± 1.1	K_{e4}, $\pi\pi \to \pi\pi$	0
4	-0.3 ± 0.5	Zweig rule	$1/8$
5	1.4 ± 0.5	F_K/F_π	$3/8$
6	-0.2 ± 0.3	Zweig rule	$11/144$
7	-0.4 ± 0.2	GMO, $L_{5,8}$	0
8	0.9 ± 0.3	$M_{K^0} - M_{K^+}$, L_5, $(m_s - \hat{m})/(m_d - m_u)$	$5/48$
9	6.9 ± 0.7	$\langle r^2 \rangle_V^\pi$	$1/4$
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$	$-1/4$

- $L_i = L_i^r(\mu) + \Gamma_i \, \frac{\mu^{D-4}}{32\pi^2} \left\{ \frac{2}{D-4} + \gamma_E - \log(4\pi) - 1 \right\}$
- $\Lambda_\chi \sim 1\,\mathrm{GeV}$ $\longrightarrow$ $L_i \sim \frac{f_\pi^2/4}{\Lambda_\chi^2} \sim 2 \times 10^{-3}$
- χPT Loops $\sim$ $1/(4\pi f_\pi)^2$