Fermion masses and mixings

- Quark masses and mixings
- CP violation
- Neutrino masses and mixings

Quark masses

Quarks are not free: different definitions

u,d,s **quark masses** determined indirectly from χPT. The s quark mass also determined from its effects in hadronic tau decays. Presented in terms of $\bar{m}(\mu = 2\,\mathrm{GeV})$.

$$\boxed{m_u \approx 3 \pm 1\,\mathrm{MeV}, \quad m_d \approx 6 \pm 2\,\mathrm{MeV}, \quad m_s \approx 110 \pm 30\,\mathrm{MeV}}$$

c,b, **quark masses** determined from **heavy quark bound-states**. b quark mass also determined from its effects in jet production at the Z peak. $\bar{m}(\bar{m})$.

$$\boxed{m_c \approx 1.3 \pm 0.1\,\mathrm{GeV}, \quad m_b \approx 4.25 \pm 0.15\,\mathrm{GeV}}$$

t **quark mass** determined from **direct production** at Fermilab and from radiative corrections. m_{pole}:

$$\boxed{m_t \approx 175 \pm 5\ \mathrm{GeV}}$$

Quark mixings

We saw that the only flavor non-diagonal interacctions in the SM are in the quark charged current interactions

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} \left\{ W^+_\mu \bar{u}_L \gamma^\mu V d_L + \textbf{h.c.} \right\}$$

For N_g generations, V general $N_g \times N_g$ unitary matrix $\Rightarrow$ ($N_g(N_g-1)/2$ moduli and $N_g(N_g+1)/2$ phases).
The rest of the Lagrangian is invariant under
$u_a \to e^{i\alpha_a} u_a \,, \quad d_a \to e^{i\beta_a} d_a \Rightarrow$
$2N_g - 1$ of those phases (baryon number is conserved) can be removed by field redefinition leading to a mixing matrix V with $(N_g-1)^2$ physical parameters

$$\boxed{N_g(N_g-1)/2 \text{ angles and } (N_g-1)(N_g-2)/2 \text{ phases}}$$

For $N_g = 2$, V contains 1 angle and 0 phases $\Rightarrow$
no CP violation.

For $N_g = 4$, V contains 6 angles and 3 phases

For $N_g = 3$, the CKM matrix is

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

it contains 3 angles and 1 phase: One needs at least 3 generations to have CP-violation. Conventionally we write

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

The mixings are obtained from **semileptonic decays** of hadrons $H \to H' l \bar{\nu}_l$ (associated with $d_j \to u_i l^- \bar{\nu}_l$) together with data from **hadronic decays of the W** and data from **top decays**. Thus one determines the magnitude of most entries of V. The results can be summarized as

$$|V_{ij}| = \begin{bmatrix} 0.9739 \,\mathrm{to}\, 0.9751 & 0.221 \,\mathrm{to}\, 0.227 & 0.0029 \,\mathrm{to}\, 0.0045 \\ 0.221 \,\mathrm{to}\, 0.227 & 0.9730 \,\mathrm{to}\, 0.9744 & 0.039 \,\mathrm{to}\, 0.044 \\ 0.0048 \,\mathrm{to}\, 0.014 & 0.037 \,\mathrm{to}\, 0.043 & 0.9990 \,\mathrm{to}\, 0.9992 \end{bmatrix}$$

which, in the standard parametrization, corresponds to

$$\begin{aligned} s_{12} &= 0.226 \pm 0.001 \\ s_{23} &= 0.042 \pm 0.001 \\ s_{13} &= 0.0039 \pm 0.0001 \end{aligned}$$

The resulting CKM matrix shows clearly a **hierarchical pattern**. One can use the approximate parametrization by Wolfenstein

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

with

$$\lambda = |V_{us}| \approx 0.22\,, \qquad A \approx 0.8\,, \qquad \sqrt{\rho^2+\eta^2} \approx 0.4$$

This parametrization is particularly useful to estamate the size of the different decays and in analyzing CP violation.

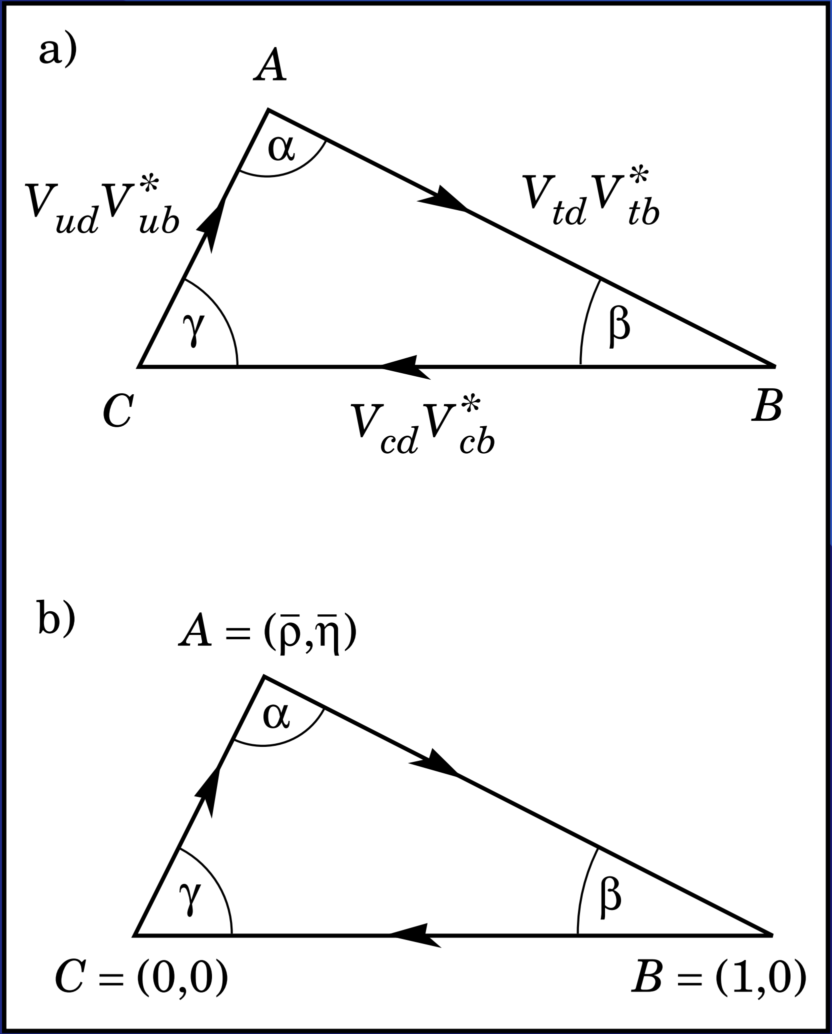
CP Violation

To disentangle CP violation and determine δ_{13}, the **only source of CP-violation** in the SM, it is important to use the unitarity of the CKM matrix $\sum_{k=u,c,t} V_{ki} V_{kj}^* = \delta_{ij}$ with $i, j = d, s, b$. For instance, for $i = d$ and $j = b$, we have

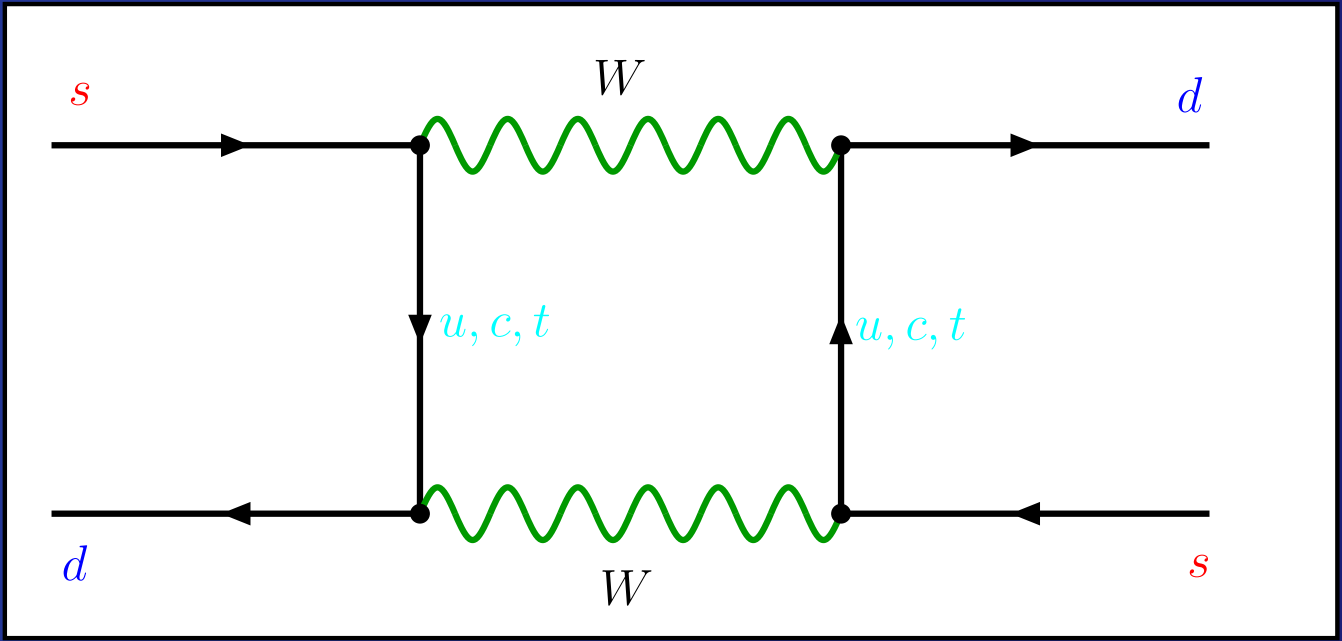
$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

This is the **Unitarity triangle**

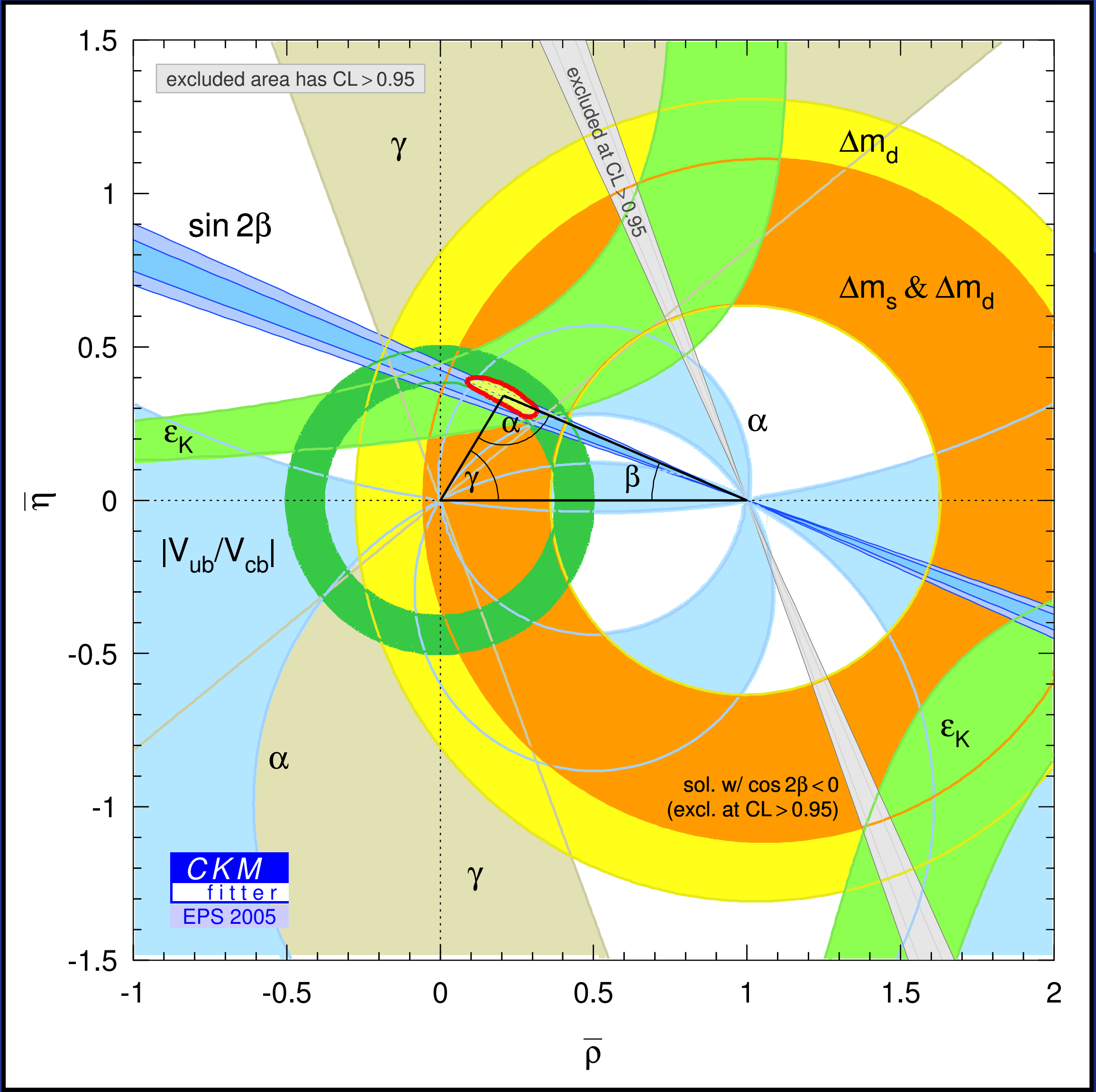
An area $\neq 0$ means CP violation



To constrain the sides and angles from the triangle one uses both, flavour changing processes which do not violate CP (basically the value of $|V_{ub}|$ and data on B-$\overline{B}$ mixing) and data on CP violating processes (ϵ_K and asymmetries in B_d decays which provide $\sin 2\beta$). All these contributions are generated in the SM by diagrams like



The unitarity triangle



Charged lepton masses

Charged Lepton masses are all **well known**

$$\begin{aligned} m_e &= 0.51099892 \pm 0.00000004\,\mathrm{MeV}\,, \\ m_\mu &= 105.658369 \pm 0.000009\,\mathrm{MeV}\,, \\ m_\tau &= 1777.0 \pm 0.3\,\mathrm{GeV} \end{aligned}$$

In the SM we studied there are no **righthanded neutrinos** and there is just **one Higgs doublet**. Then, we can choose M_e diagonal. As a consequence the theory is diagonal in lepton flavour (no CKM in the lepton sector): Individual **lepton numbers are conserved.**

$$\mu \not\to e\gamma\,, \quad \tau \not\to \mu\gamma\,, \quad \mu \not\to e e \bar{e}, \quad \tau \not\to e\bar{e}\mu$$

Its non-observation suggested that neutrinos are massless, however...

Evidence for lepton flavour change

- The Sun only produces ν_e in nuclear reactions. However the measured flux of ν_e coming from the Sun is much lower than expected. This could be explained if ν_e can transform into neutrinos of other flavours (neutrino mixing) and we miss the other flavours.
  In **agreement with accelerator experiments** with
  $L = 250\,\mathrm{Km}$ (K2K)
- The measured ratio of atmospheric fluxes ν_e and ν_μ (coming as secondaries of cosmic rays) is not in agreement with calculations, and, again this could be explained by neutrino mixing.
  Confirmed by reactor experiments with
  $L = 180\,\mathrm{Km}$ (KamLAND)

Neutrino oscillations

If ν's are massive, mass eigenstates are no flavour eigenstates ($W^+ \to \ell^+_\alpha \nu_\alpha$, $\alpha = e, \mu, \tau$)

$$|\nu_\alpha\rangle = \sum_i V^*_{\alpha i} |\nu_i\rangle$$

Where V is a matrix that can be parametrized as the CKM matrix.
After traveling some distance, L, time evolution gives ($p \gg m_i$)

$$|\nu_\alpha(L)\rangle = \sum_i V^*_{\alpha i} e^{-im_i^2 L/2E} |\nu_i\rangle$$

Then,

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta |\nu_\alpha(L)\rangle|^2$$

for only 2 flavours

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \, \sin^2 1.27\frac{\Delta m^2 L}{E}$$

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \, \sin^2 1.27\frac{\Delta m^2 L}{E}$$

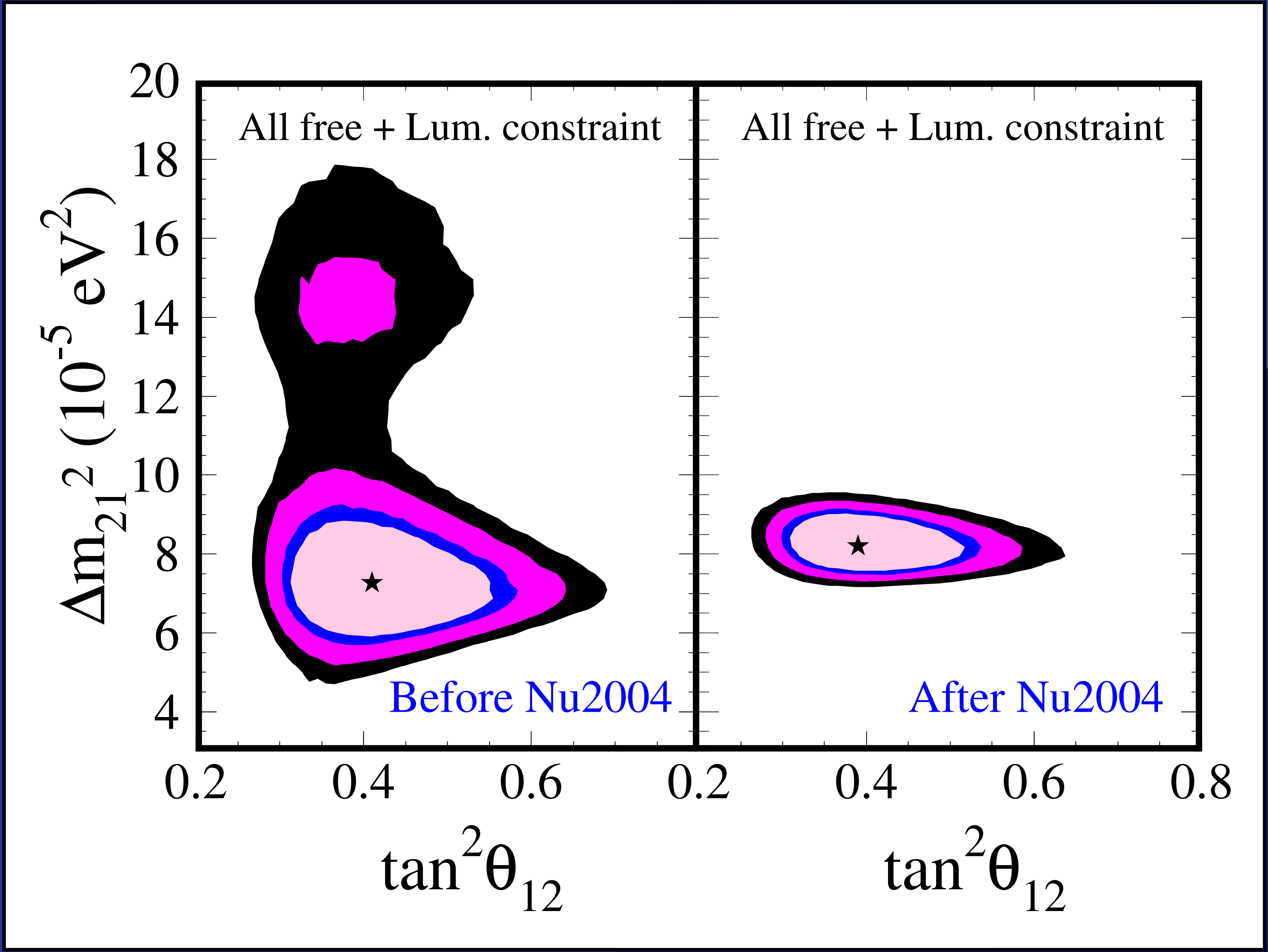
Δm^2 is given in eV, L in km, and E in GeV.

Even if masses are very small we can adjust L/E to have a sizesable transintion probability.

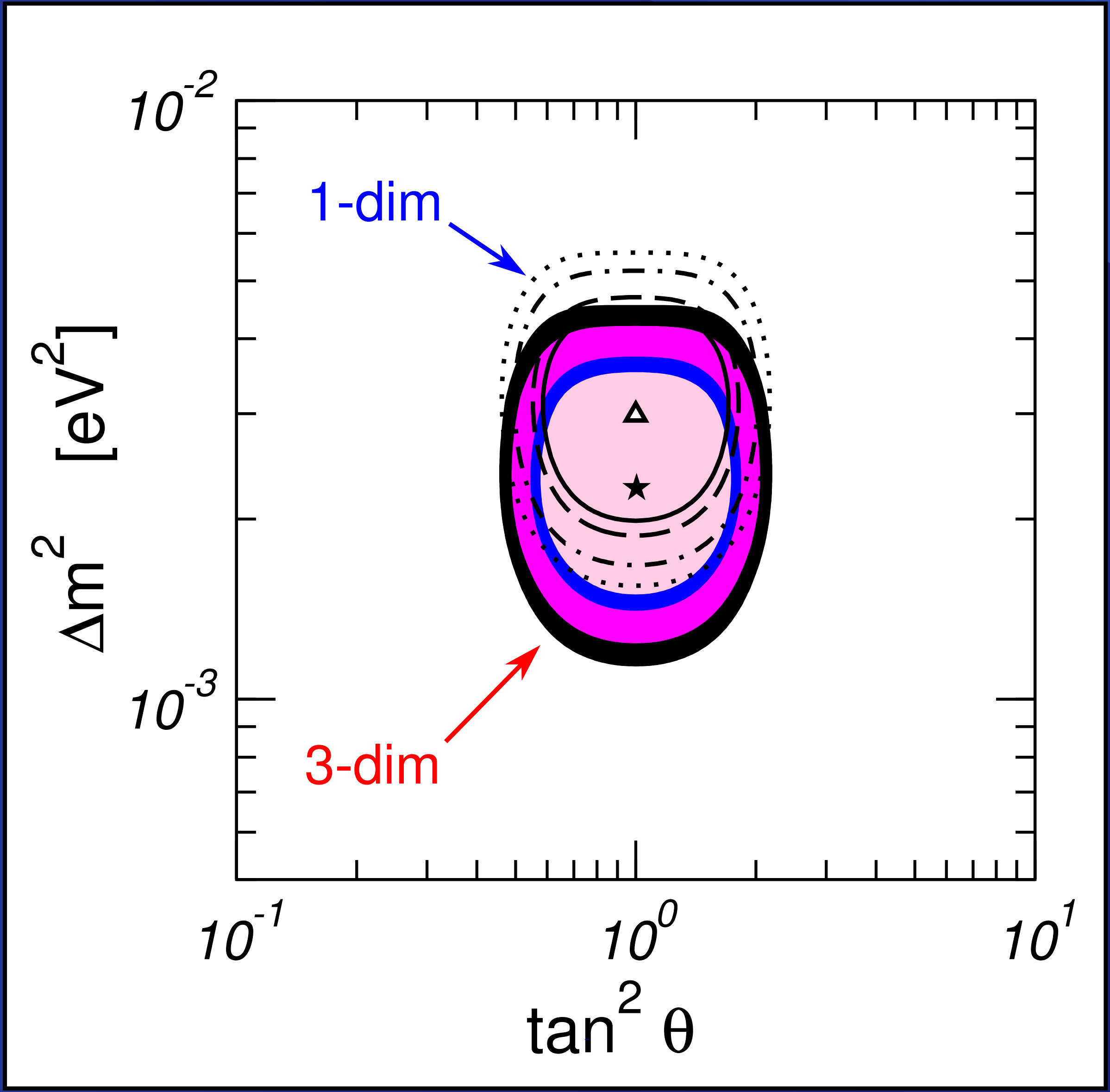
This could explain why we see flavour transitions in neutrinos but not in charged leptons.

No better explanation known.

Solar neutrino fit



Atmospheric neutrino fit



Results of the global fit

All solar neutrino data, atmospheric neutrino data and reactor experiment data can be fitted by neutrino oscillations. A global fit gives

$$7.3 \cdot 10^{-5} < \Delta m^2_{21}/\mathrm{eV}^2 < 9.3 \cdot 10^{-5}\,,$$

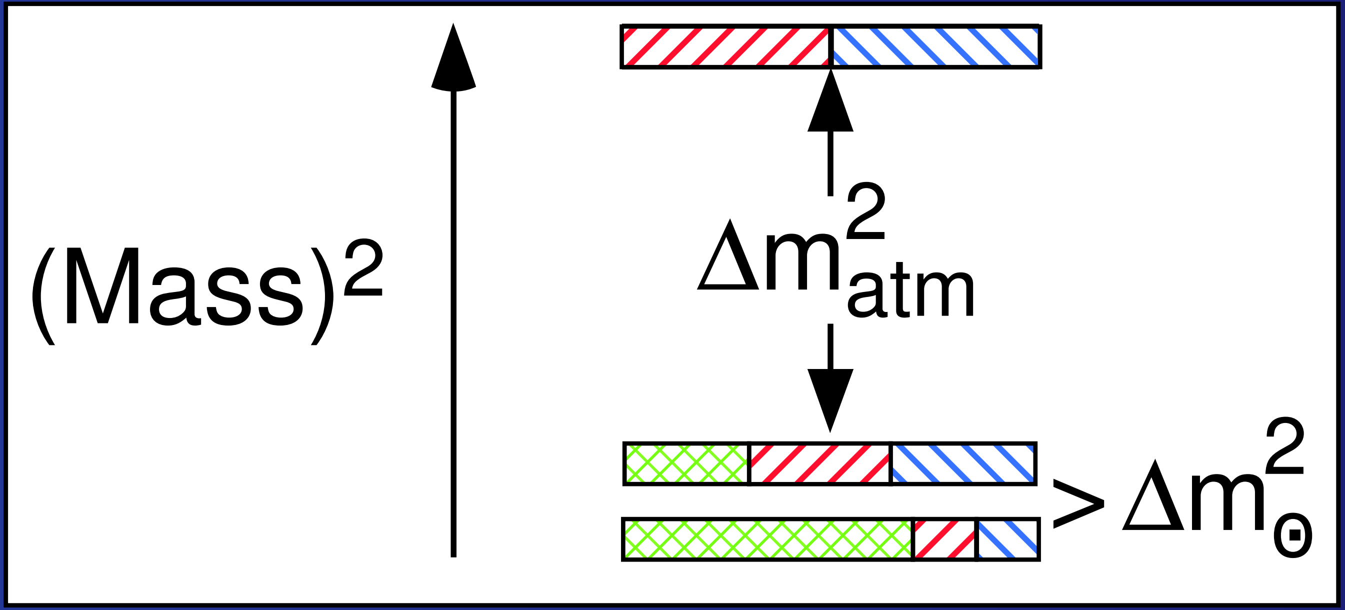
$$1.6 \cdot 10^{-3} < \Delta m^2_{32}/\mathrm{eV}^2 < 3.6 \cdot 10^{-5}$$

$$0.28 < \tan^2\theta_{12} < 0.6\,, \quad 0.5 < \tan^2\theta_{23} < 2.1\,, \quad \sin^2\theta_{13} < 0.041$$

only differences $\Delta m^2_{ij} = m^2_i - m^2_j$; overal scale unknown (from cosmology $m_\nu \lesssim 1\,\mathrm{eV}$). Since θ_{13} small solar and atmospheric mixing decouple (12 is solar and 23 atmospheric). Thus we have large mixing in solar oscillations and maximal mixing in atmospheric neutrino oscillations.

$$V \approx \begin{bmatrix} \frac{1}{\sqrt{2}}(1+\lambda) & \frac{1}{\sqrt{2}}(1-\lambda) & \epsilon \\ -\frac{1}{2}(1-\lambda+\epsilon) & \frac{1}{2}(1+\lambda-\epsilon) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(1-\lambda-\epsilon) & -\frac{1}{2}(1+\lambda+\epsilon) & \frac{1}{\sqrt{2}} \end{bmatrix}, \qquad \begin{array}{l} \lambda \approx 0.2 \\ \epsilon < 0.2 \end{array}$$

A possibility is



Neutrino masses

If there are no right-handed neutrinos and with the minimal scalar content neutrinos are exactly massless. In addition global-lepton number appears as an accidental symmetry.

With additional scalar multiplets the situation changes: with a complex triplet, χ, one can get Majorana neutrino masses for the left-handed neutrinos when χ develops a VEV.

With right-handed neutrinos one can write

$$\mathcal{L}_{YL} = -\bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi} \nu_R + \mathrm{h.c.}$$

which leads to a situation similar to the one in the quark sector. Neutrino masses would naturally be of the same order of magnitude than quark masses.

The see-saw mechanism

There is strong evidence for very small ($< 1\,\mathrm{eV}$) neutrino masses. Why? **Right-handed neutrinos** are special, they are completely **neutral** with respect to the **gauge group**. Nothing forbids a **Majorana mass** term like $\overline{\nu_R^c} M \nu_R$, then

$$\mathcal{L}_{YL} \to \mathcal{L}_{YL} = -\bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi} \nu_R - \frac{1}{2} \overline{\nu_R^c} M \nu_R + \mathrm{h.c.}$$

which after SSB leads to a neutrino mass term

$$\mathcal{L}_{\nu M} = -\frac{1}{2} \left(\bar{\nu}_L , \overline{\nu_R^c} \right) \begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \mathrm{h.c.}$$

diagonalized easily if $M \gg M_D$:
N_g **heavy** Majorana neutrinos $\sim \nu_R$ with masses $\sim M$
N_g **light** Majorana neutrinos $\sim \nu_L$ with masses $\sim M_D^2/M$

Not the only possibility. If there are additional scalars, fermions or gauge bosons lefthanded, neutrinos could get a Majorana mass even in the absence of righthanded neutrinos.
Independently of the mechanism there are two types of ν-mass terms

$$\mathcal{L}_{\text{Dirac}} = \overline{\nu_R} M_\nu \nu_L + \text{h.c.}$$

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2}\overline{\nu_L^c} M_\nu \nu_L + \text{h.c.}$$

Both explain equally well ν oscillations but

- Majorana ν's violate total lepton number (ν-less 2β decay). Additional CP phases.
- Dirac ν's conserve total lepton number. Less natural.

What else?

- Determine better θ_{12} and θ_{23}
- Determine θ_{13}
- Determine the individual masses of the eigenstates ν_i
- Is there CP violation in the lepton sector?
- Is total lepton number conserved? (ν-less 2β decay)
- Are neutrinos Majorana or Dirac particles?
- Can leptogenesis provide enough baryon number in the universe?
- Build and **TEST** a model of massive neutrinos!

Is the SM complete?

- Neutrinos
- Find the Higgs. What if it is not there?
- Understand CP violation
- Understand hierarchies of masses and mixings
- Why 3 generations?
- Unification of couplings
- Solve the Hierarchy problem
- Unify with gravity