QUANTUM FIELD THEORY

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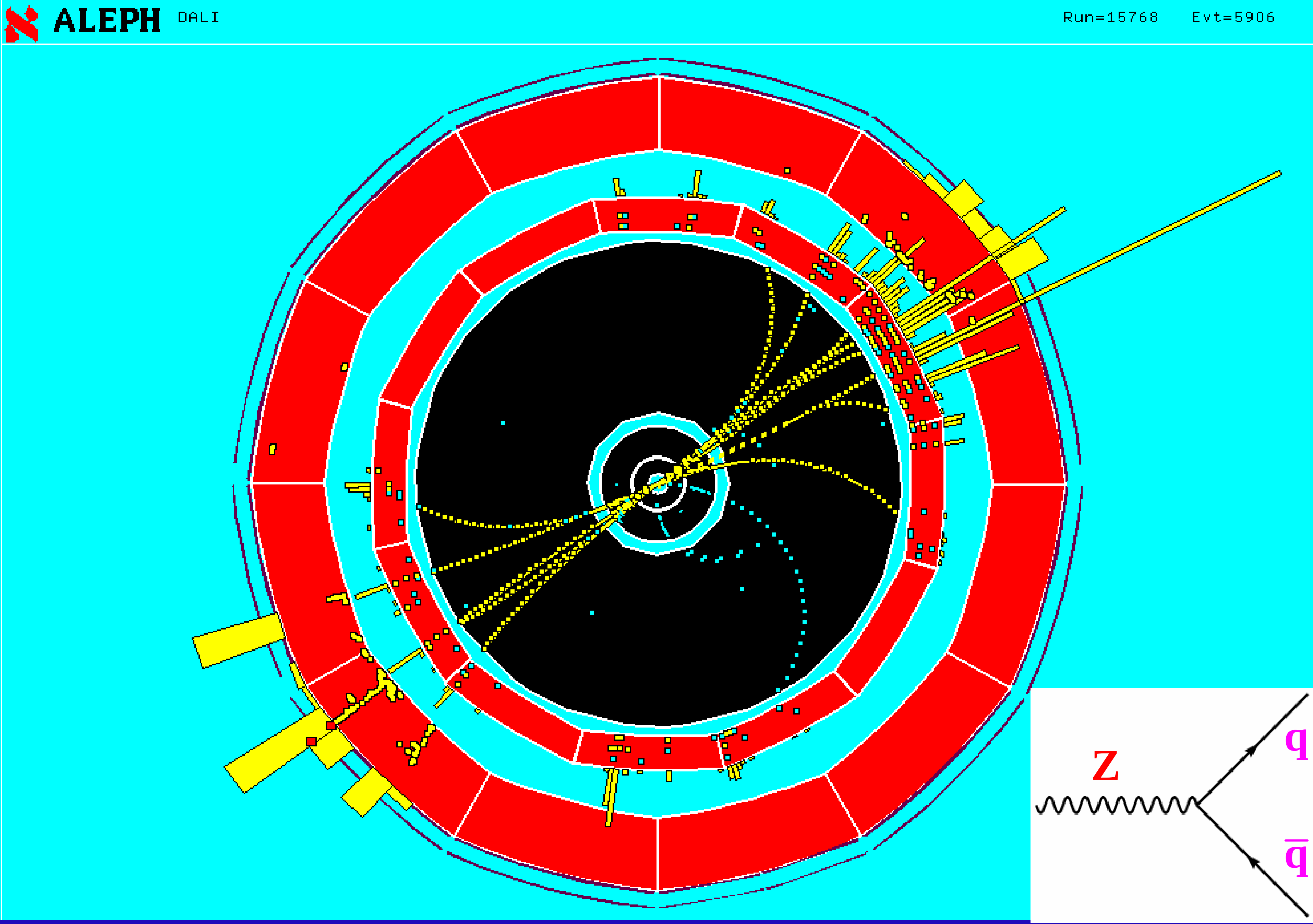
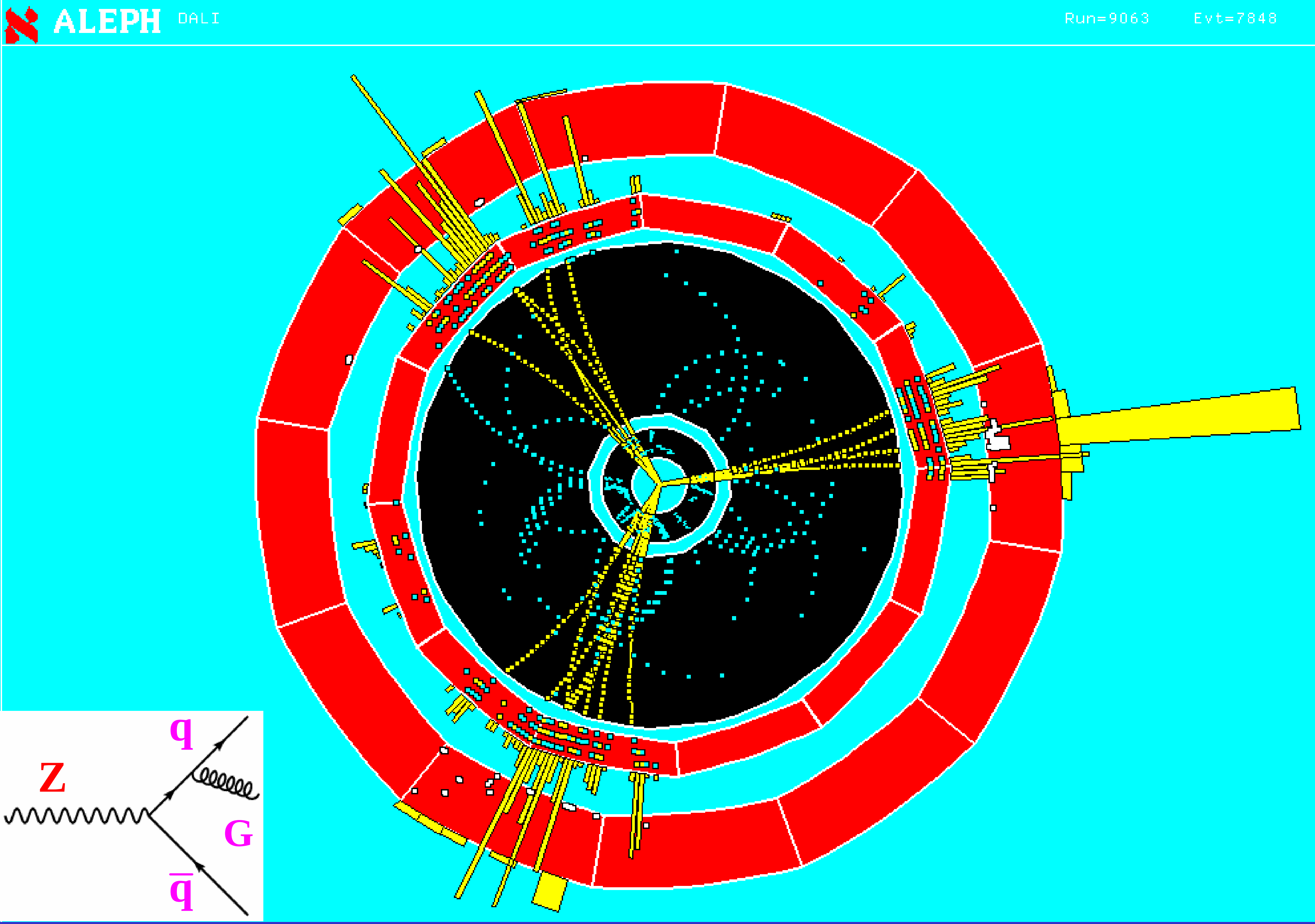
4. QCD Phenomenology

- Jets
- $e^+e^- \rightarrow$ Hadrons
- $\tau^- \rightarrow \nu_\tau$ + Hadrons
- α_s Determinations
- **m_s & V_{us}** from τ decay


ALEPH
DALI
Run=15768 Evt=5906
Z
q
$\bar{q}$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DALI
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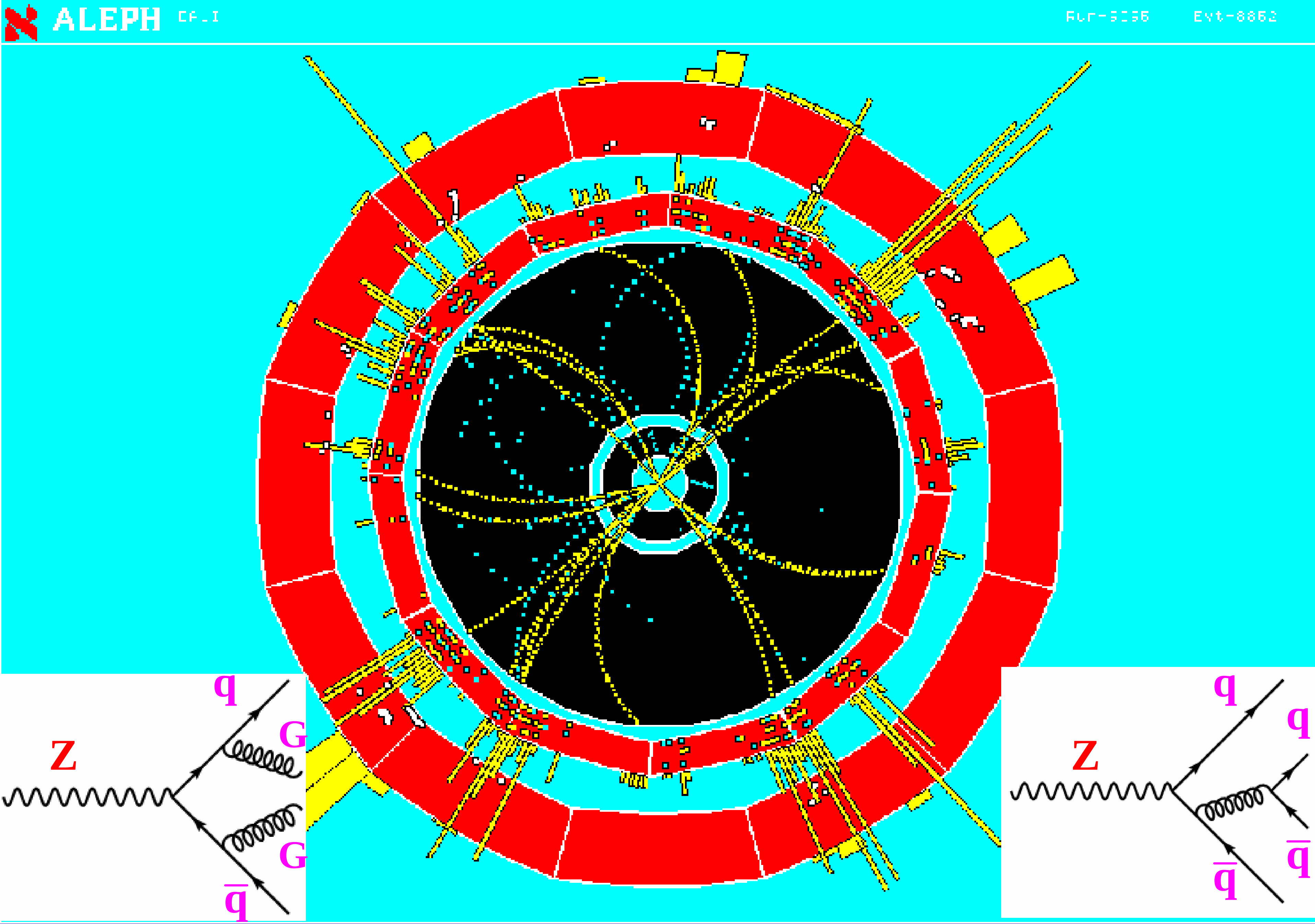
Evt=7848
q
Z
G
q̄


ALEPH
q
G
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Z
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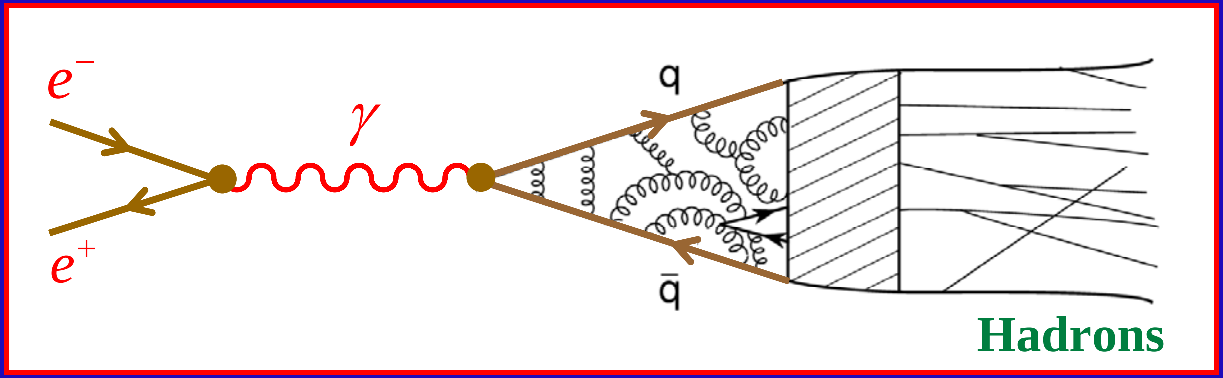
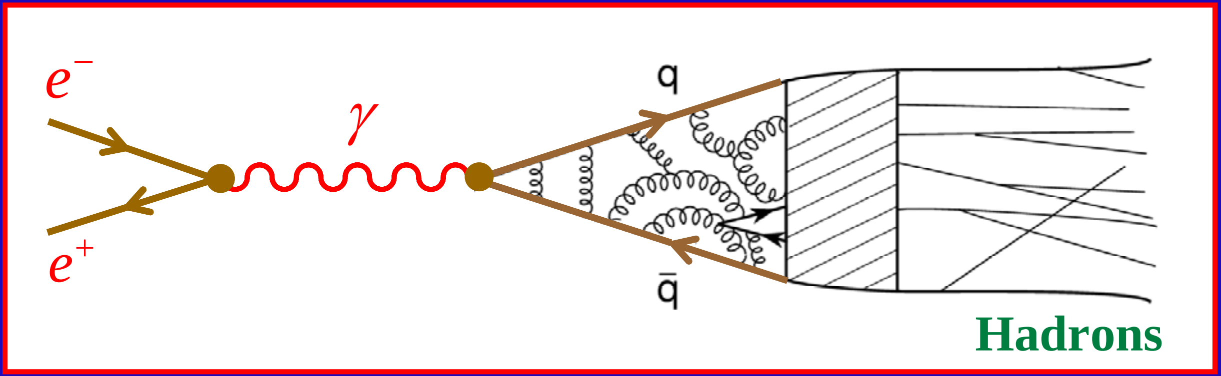
q̄

$e^+e^- \to H^0$ probes the hadronic electromagnetic current



$$\left\langle H^0 \left| \sum_q Q_q \; \bar{q}\gamma^\mu q \right| 0 \right\rangle$$

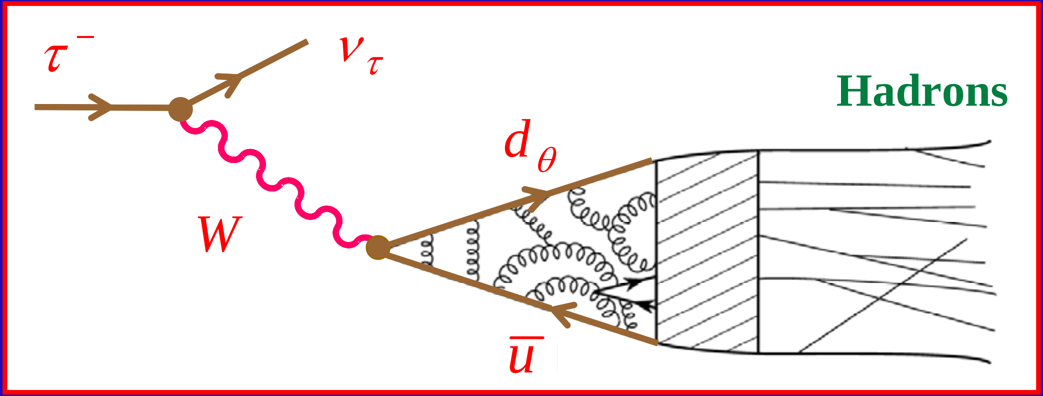
$e^+e^- \to H^0$ probes the hadronic electromagnetic current



$$\langle H^0 | \sum_q Q_q \, \bar{q} \gamma^\mu q | 0 \rangle$$

$\tau^- \to \nu_\tau H^-$ probes the hadronic V–A current

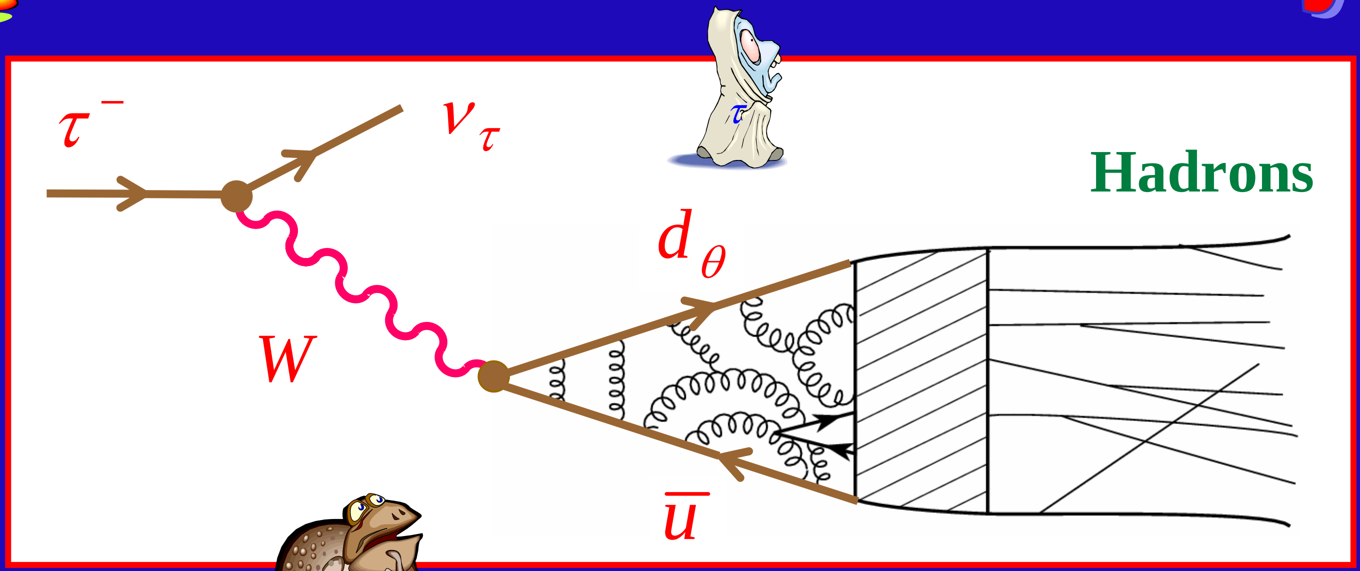
$$\langle H^- | \bar{d}_\theta \gamma^\mu (1-\gamma_5) u | 0 \rangle$$

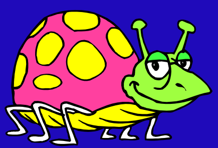


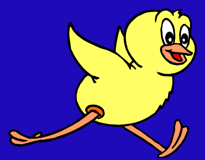
Isospin :

$$\frac{\Gamma(\tau^- \to \nu_\tau V^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = \frac{3\cos^2\theta_C}{2\pi\alpha^2} S_{\mathrm{EW}} \int_0^1 dx\, (1-x)^2 (1+2x)\, x\;\; \sigma^{I=1}_{e^+e^- \to V^0}(x m_\tau^2)$$

Hadronic Tau Decay





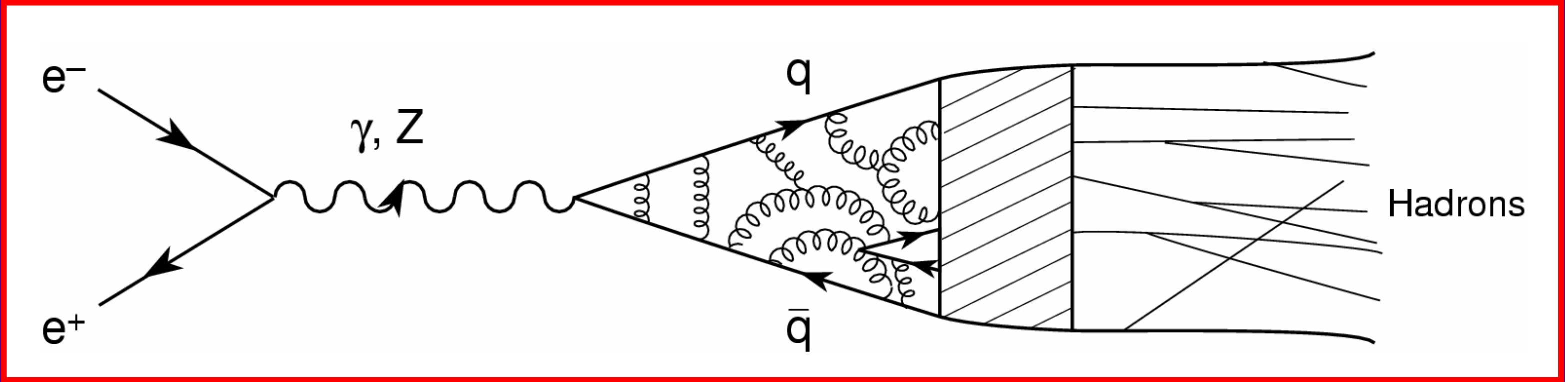


$$d_\theta = \cos\theta_C\, d + \sin\theta_C\, s$$

Only lepton massive enough to decay into hadrons

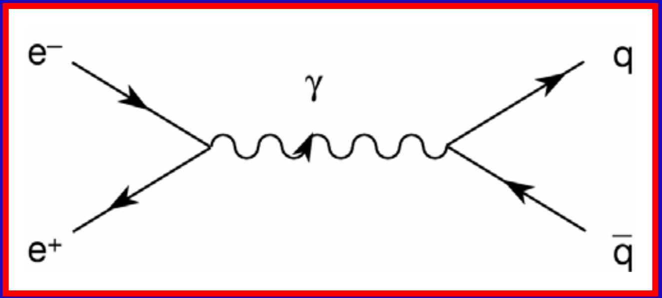
$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau\, e^-\, \bar{\nu}_e\right)} \approx N_C \qquad ; \qquad R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.642 \pm 0.013$$

Confinement ⟷ Probability Hadronization = 1



$$\sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to q\bar{q} + q\bar{q}G + q\bar{q}GG + q\bar{q}q\bar{q} + \cdots)$$

$$\mathbf{T}\left(e^{+}e^{-} \to q\,\bar{q}\right) \;=$$



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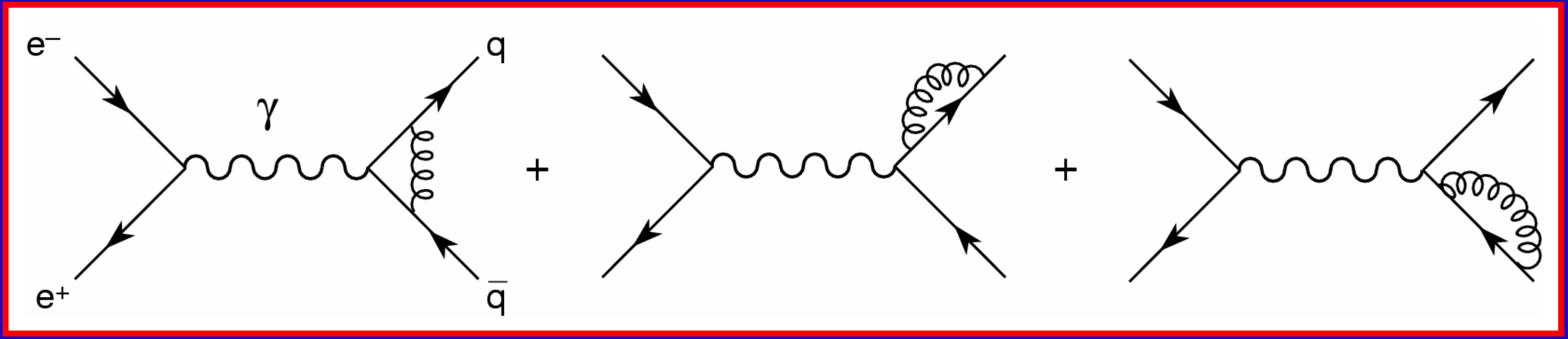
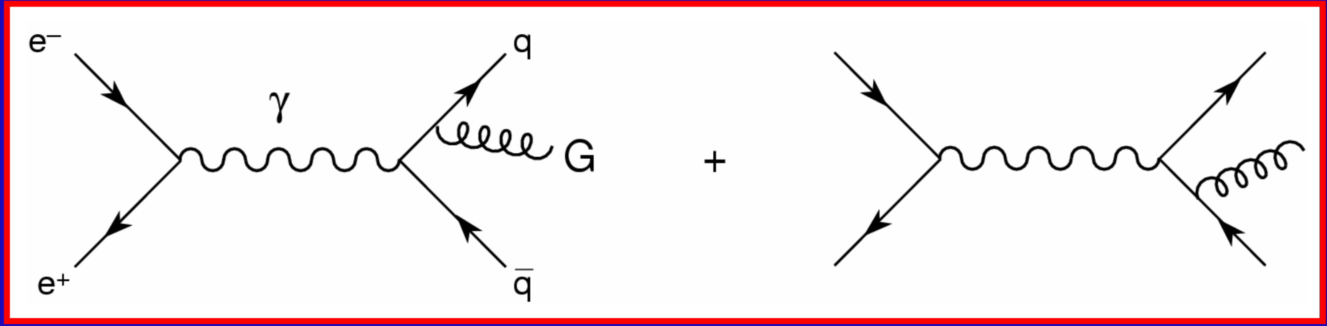


$$\mathbf{T}\left(e^{+}e^{-} \to q\,\bar{q}\,G\right) \;=$$



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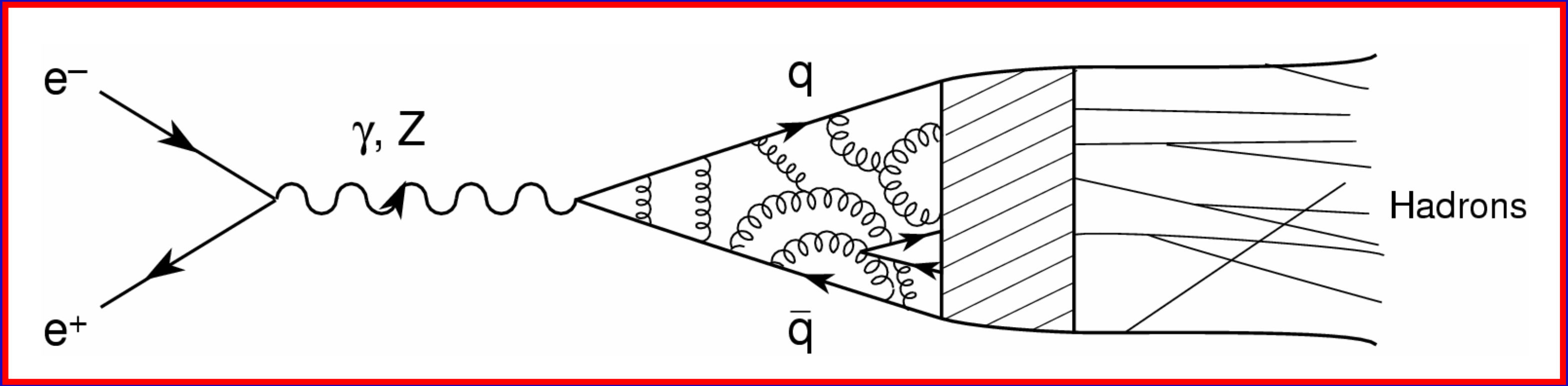
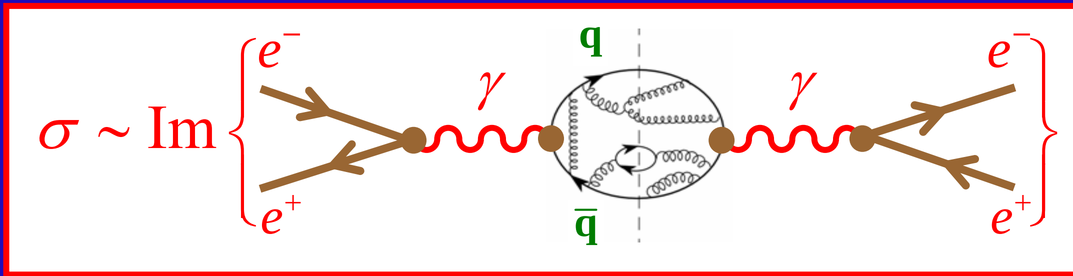


$$\sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to q\bar{q} + q\bar{q}G + q\bar{q}GG + q\bar{q}q\bar{q} + \cdots)$$

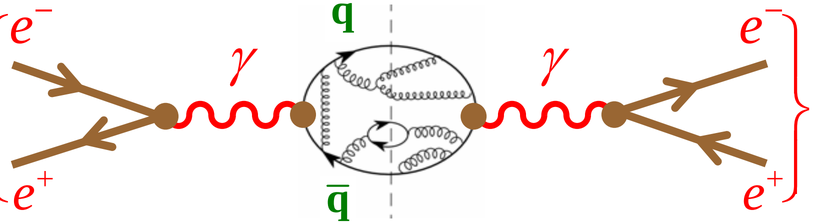
$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q Q_q^2 \; N_C \; \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\}$$

$$R_Z \equiv \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to e^+e^-)} = R_Z^{EW} \; N_C \; \left\{1 + \frac{\alpha_s(M_Z^2)}{\pi} + \cdots\right\}$$



$$\frac{\sigma(e^+e^-\to \text{had})}{\sigma(e^+e^-\to \mu^+\mu^-)} = 12\pi\, \text{Im}\,\Pi_{\text{em}}(s)$$

$$\Pi^{\mu\nu}_{\text{em}}(q) \equiv i\int d^4x\; e^{iqx}\,\langle 0|T[J^{\mu}_{\text{em}}(x)\,J^{\nu}_{\text{em}}(0)]|0\rangle \;=\; \left(-g^{\mu\nu}q^2 + q^{\mu}q^{\nu}\right)\Pi_{\text{em}}(q^2)$$

$\sigma \sim \mathrm{Im}\Big\{$  $\Big\}$

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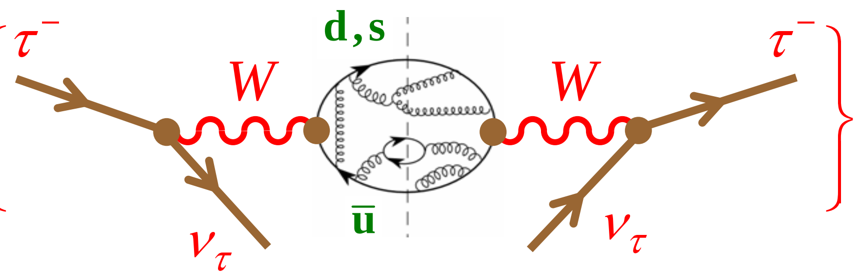
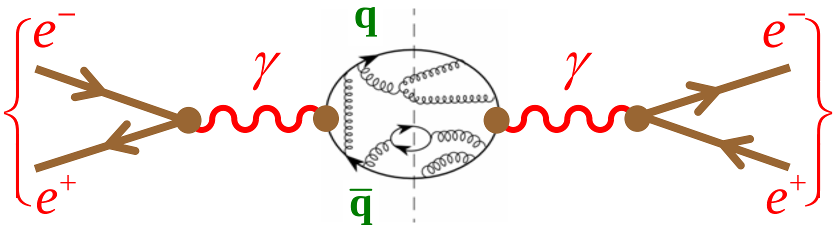
$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}\Big\{$  $\Big\}$

$$\sigma \sim \mathrm{Im}\left\{ \right.$$



$$\left. \right\}$$

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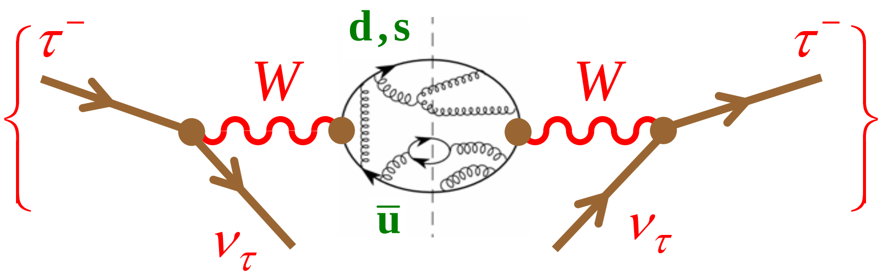


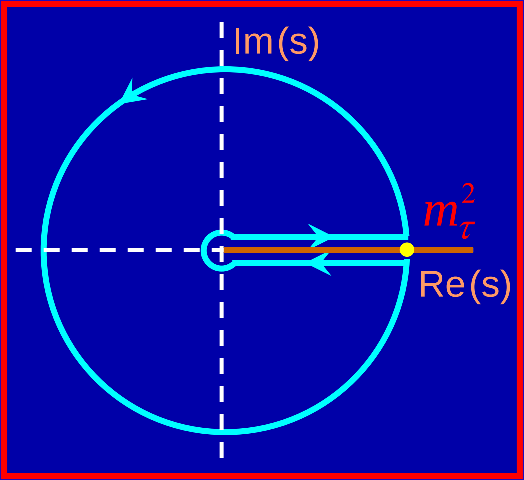
$$\left. \right\}$$

$$R_\tau \equiv \frac{\Gamma(\tau^-\to\nu_\tau+\mathrm{had})}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)} = 12\pi\int_0^{m_\tau^2} dx\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s)+\mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2\left[\Pi^{(J)}_{ud,V}(s)+\Pi^{(J)}_{ud,A}(s)\right] + |V_{us}|^2\left[\Pi^{(J)}_{us,V}(s)+\Pi^{(J)}_{us,A}(s)\right]$$

$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i\int d^4x\; e^{iqx}\,\langle 0|T[J^{\mu}_{ij}(x)\,J^{\nu}_{ij}(0)^{\dagger}]|0\rangle = \left(-g^{\mu\nu}q^2+q^{\mu}q^{\nu}\right)\Pi^{(1)}_{ij,J}(q^2)+q^{\mu}q^{\nu}\;\Pi^{(0)}_{ij,J}(q^2)$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\text{Im}\,\Pi^{(1)}(x m_\tau^2) + \text{Im}\,\Pi^{(0)}(x m_\tau^2) \right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx\,(1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2) \right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\,\langle O_D(\mu) \rangle}{(-s)^{D/2}}$$

OPE

$$R_\tau = N_C\, S_{\text{EW}} \left(1 + \delta'_{\text{EW}} + \delta_{\text{P}} + \delta_{\text{NP}}\right) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$S_{\text{EW}} = 1.0194$; $\delta'_{\text{EW}} = 0.0010$; $\delta_{\text{NP}} = -0.004 \pm 0.002$

(fitted from data)

$\delta_{\text{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + \ldots \approx 20\%$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

Perturbative: $(m_q=0)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n \quad ; \quad K_0 = K_1 = 1 \;,\; K_2 = 1.63982 \;,\; K_3 = 6.37101$$

$$\Rightarrow \quad \delta_{\mathrm{P}} = \sum_{n=1} K_n\, A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + \cdots$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Power Corrections:

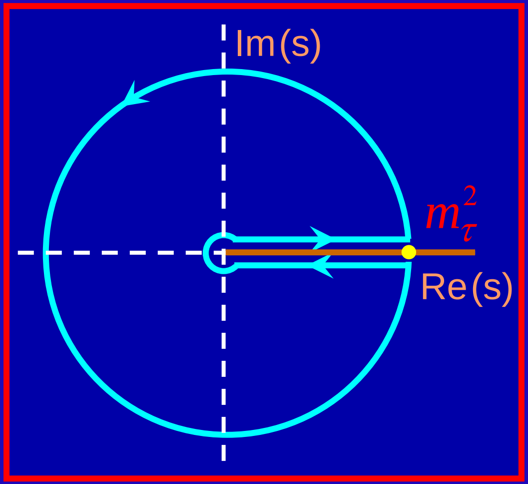
$$\Pi^{(0+1)}_{\mathrm{OPE}}(s) \approx \frac{1}{4\pi^2}\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{\mathrm{NP}} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;(1-3x^2+2x^3)\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

Suppressed by m_τ^6 [additional chiral suppression in $C_6\langle O_6\rangle^{V+A}$]

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\text{Im}\,\Pi^{(1)}(x m_\tau^2) + \text{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx\,(1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2)\right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

OPE

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$S_{\text{EW}} = 1.0194$; $\delta'_{\text{EW}} = 0.0010$; $\delta_{\text{NP}} = -0.004 \pm 0.002$

(fitted from data)

$\delta_{\text{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + \ldots \approx 20\%$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

Similar predictions for $R_{\tau,V}$, $R_{\tau,A}$, $R_{\tau,S}$ and the moments

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$

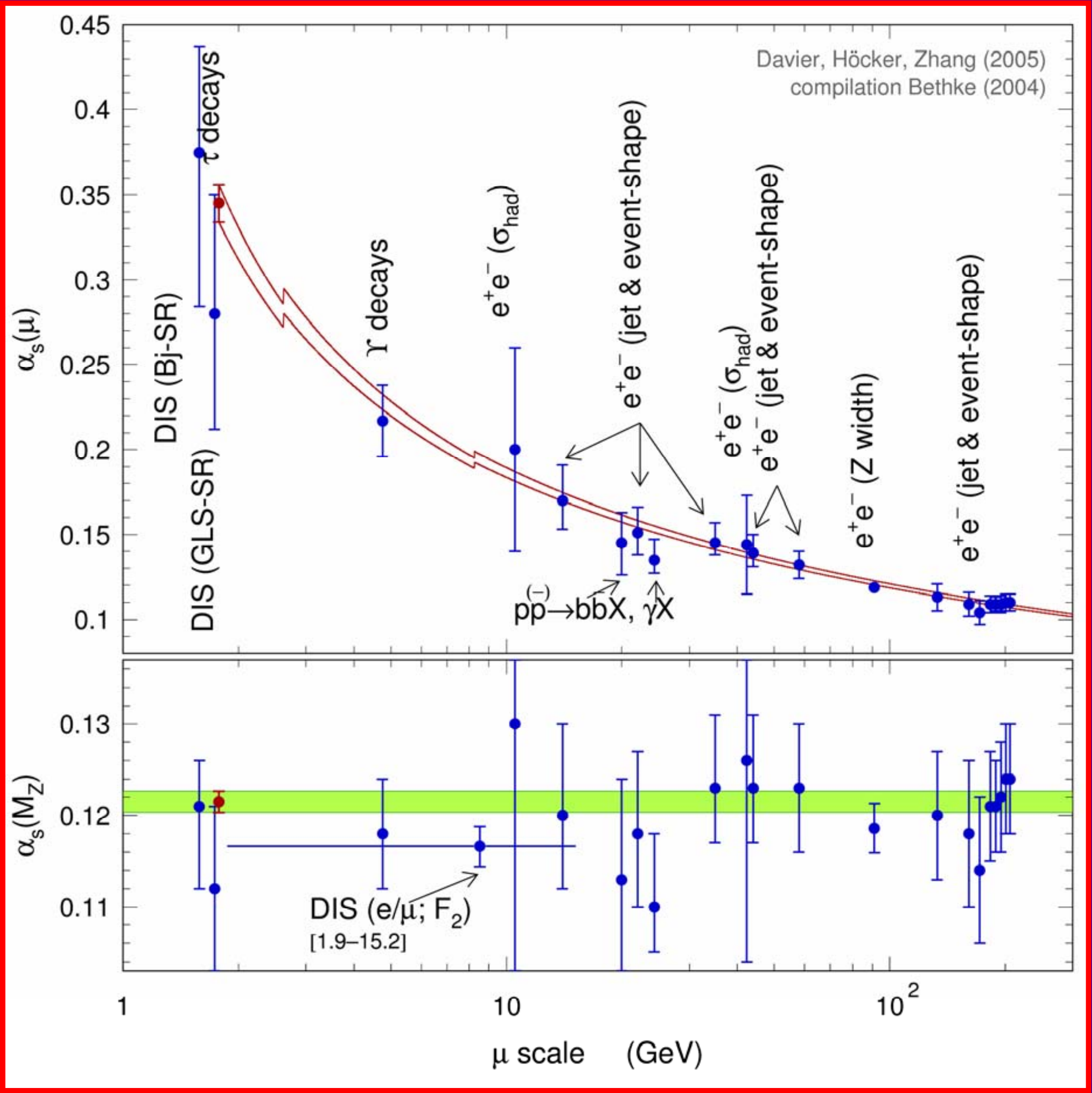
Different sensitivity to power corrections through k , l

The non-perturbative contribution to R_τ can be obtained from the invariant-mass distribution of the final hadrons:

$$\delta_{\mathrm{NP}} = -0.004 \pm 0.002$$

ALEPH, CLEO, OPAL

$R_{\tau,V} = 1.787 \pm 0.013 \quad ; \quad R_{\tau,A} = 1.695 \pm 0.013 \quad ; \quad R_{\tau,V+A} = 3.482 \pm 0.014$ (ALEPH 2005)



$\alpha_s(m_\tau^2) = 0.345 \pm 0.010$

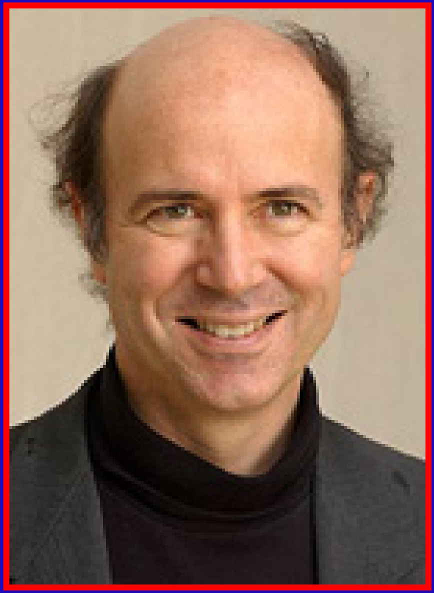
$\alpha_s(M_Z^2) = 0.1215 \pm 0.0012$

$\alpha_s(M_Z^2)_{Z\,\mathrm{width}} = 0.1186 \pm 0.0027$

The most precise test of Asymptotic Freedom

$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0029 \pm 0.0010_\tau \pm 0.0027_Z$

NOBEL PRIZE IN PHYSICS 2004



Frank Wilczek
Massachusetts
Institute of Technology

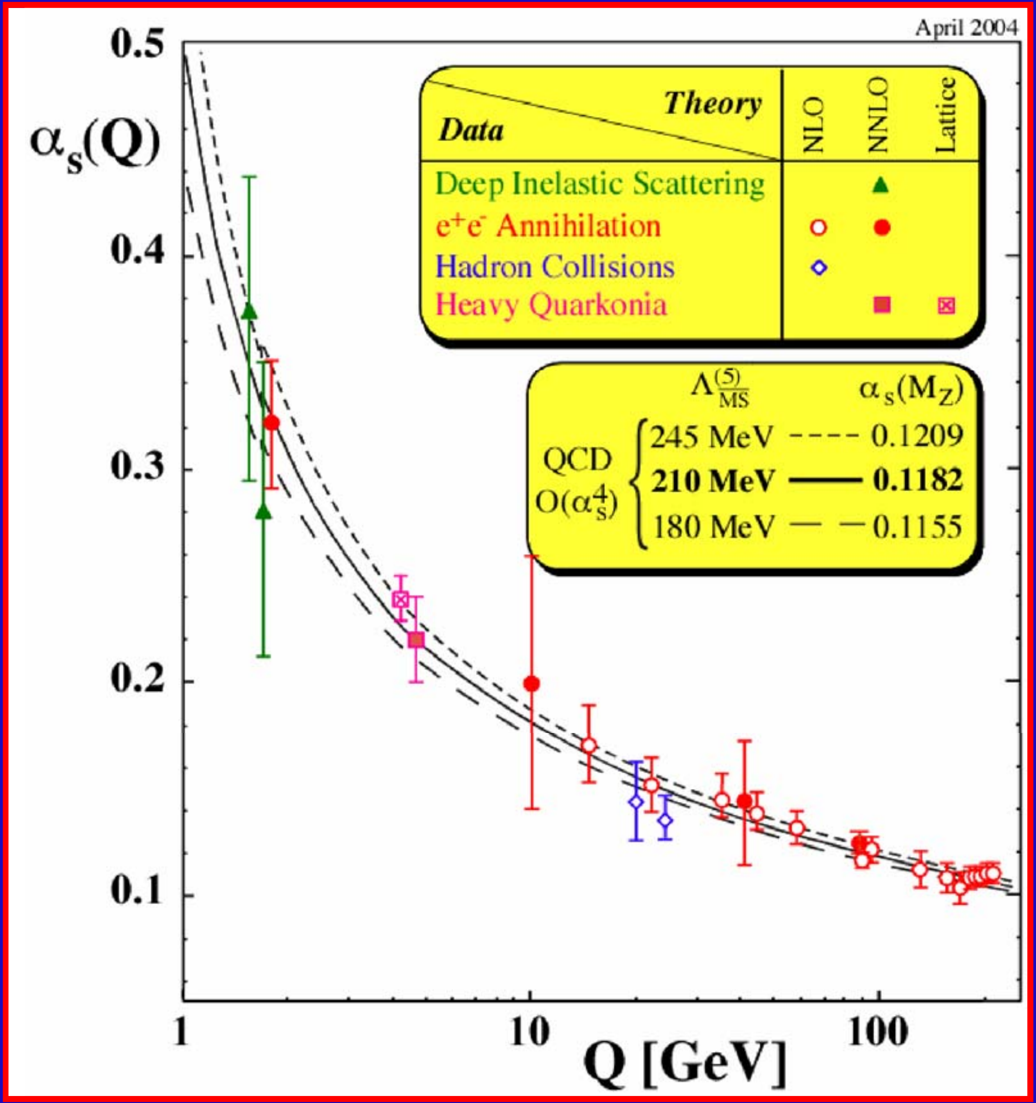
David Gross
Kavli Institute
University of California

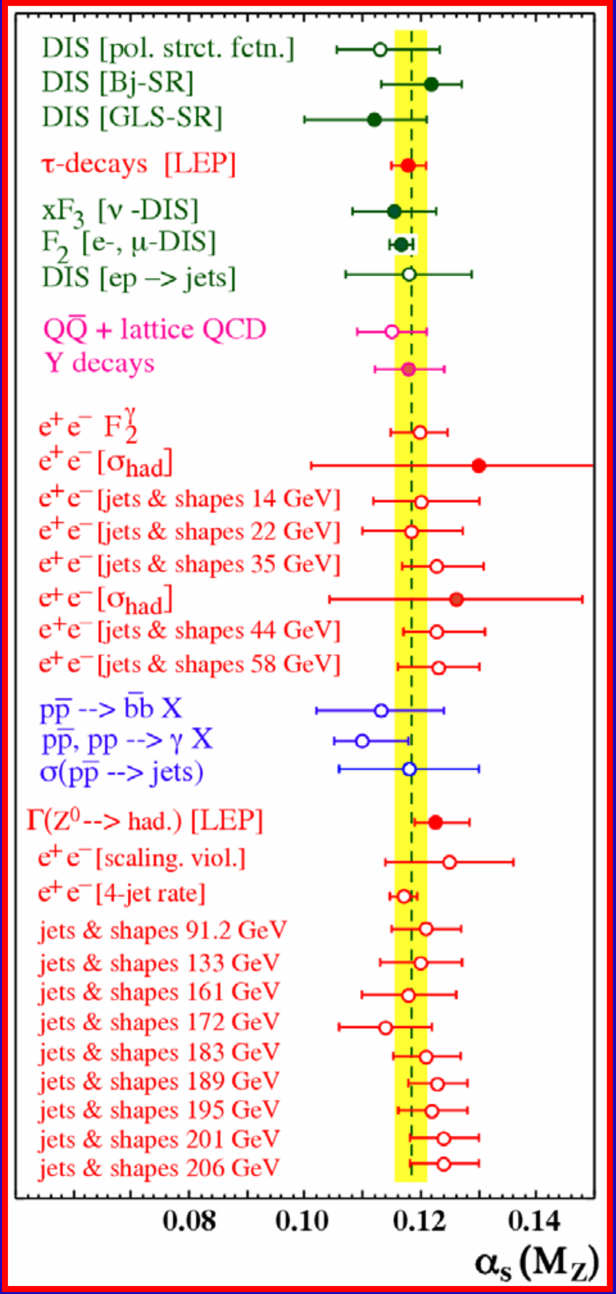
David Politzer
California
Institute of Technology

S. Bethke

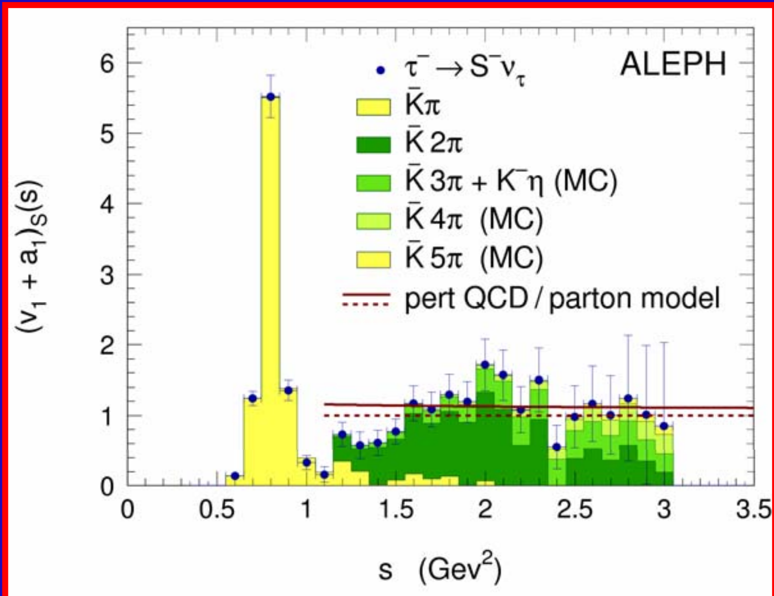
MEASUREMENTS OF α_s

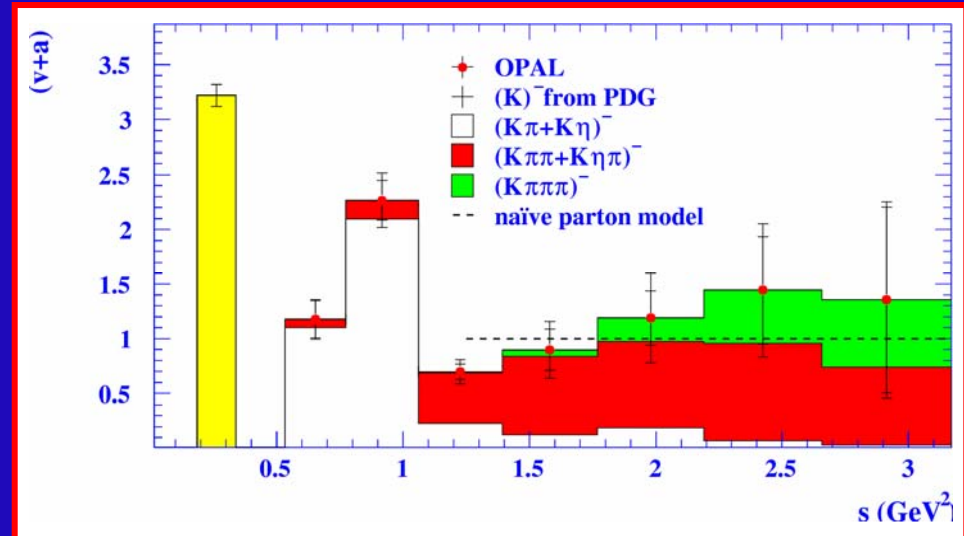
$$\alpha_s(M_Z^2) = 0.1182 \pm 0.0027$$





Strange Spectral Function: SU(3) Breaking





(k,l)	ALEPH	OPAL
(0,0)	0.39 ± 0.14	0.26 ± 0.12
(1,0)	0.38 ± 0.08	0.28 ± 0.09
(2,0)	0.37 ± 0.05	0.30 ± 0.07
(3,0)	0.40 ± 0.04	0.33 ± 0.05
(4,0)	0.40 ± 0.04	0.34 ± 0.04

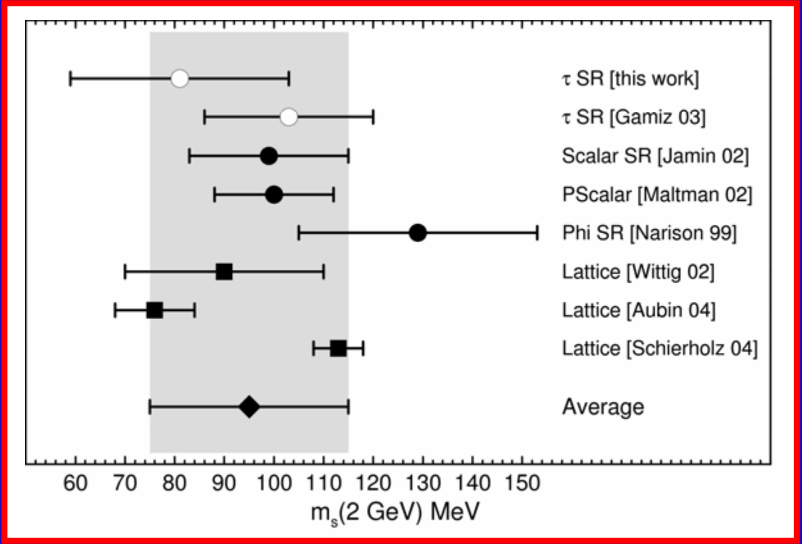
$$\delta R_\tau^{kl} \equiv \frac{R^{kl}_{\tau,V+A}}{|V_{ud}|^2} - \frac{R^{kl}_{\tau,S}}{|V_{us}|^2} \approx 24\, \frac{m_s^2(m_\tau^2)}{m_\tau^2}\, \Delta_{kl}(\alpha_s)$$

$$m_s(m_\tau) = 84 \pm 23 \;\text{MeV}$$

$$m_s(2\,\text{GeV}) = 81 \pm 22 \;\text{MeV}$$

Gámiz-Jamin-Pich-Prades-Schwab

Gámiz-Jamin-Pich-Prades-Schwab



τ OPAL data

τ ALEPH data

Strong sensitivity to V_{us}

Taking as input (from non τ sources) $m_s(2\,\text{GeV}) = 95 \pm 20\ \text{MeV}$:

(k=0,l=0)

$$|V_{us}| = 0.2208 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}}$$

$$\left(|V_{us}|_{K_{l3}} = 0.2233 \pm 0.0028\right)$$

(k≠0,l=0)

$$m_s(2\,\text{GeV}) = 81 \pm 22\ \text{MeV}$$

Simultaneous m_s & V_{us} fit possible with better data

The τ could give the most precise V_{us} determination