
Testing the Standard Model

- Gauge boson masses and couplings
- The Higgs boson

The line-shape of the Z

Close to the Z **peak** the cross section for $e^+e^- \rightarrow f\bar{f}$ is completely dominated by the resonance, **photon exchange** diagrams and **box diagrams** can be **neglected**.

$$\sigma^0(e^+e^- \rightarrow f\bar{f}) \approx \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2} \frac{s}{(s - m_Z^2)^2 + s^2 \Gamma_Z^2/m_Z^2}$$

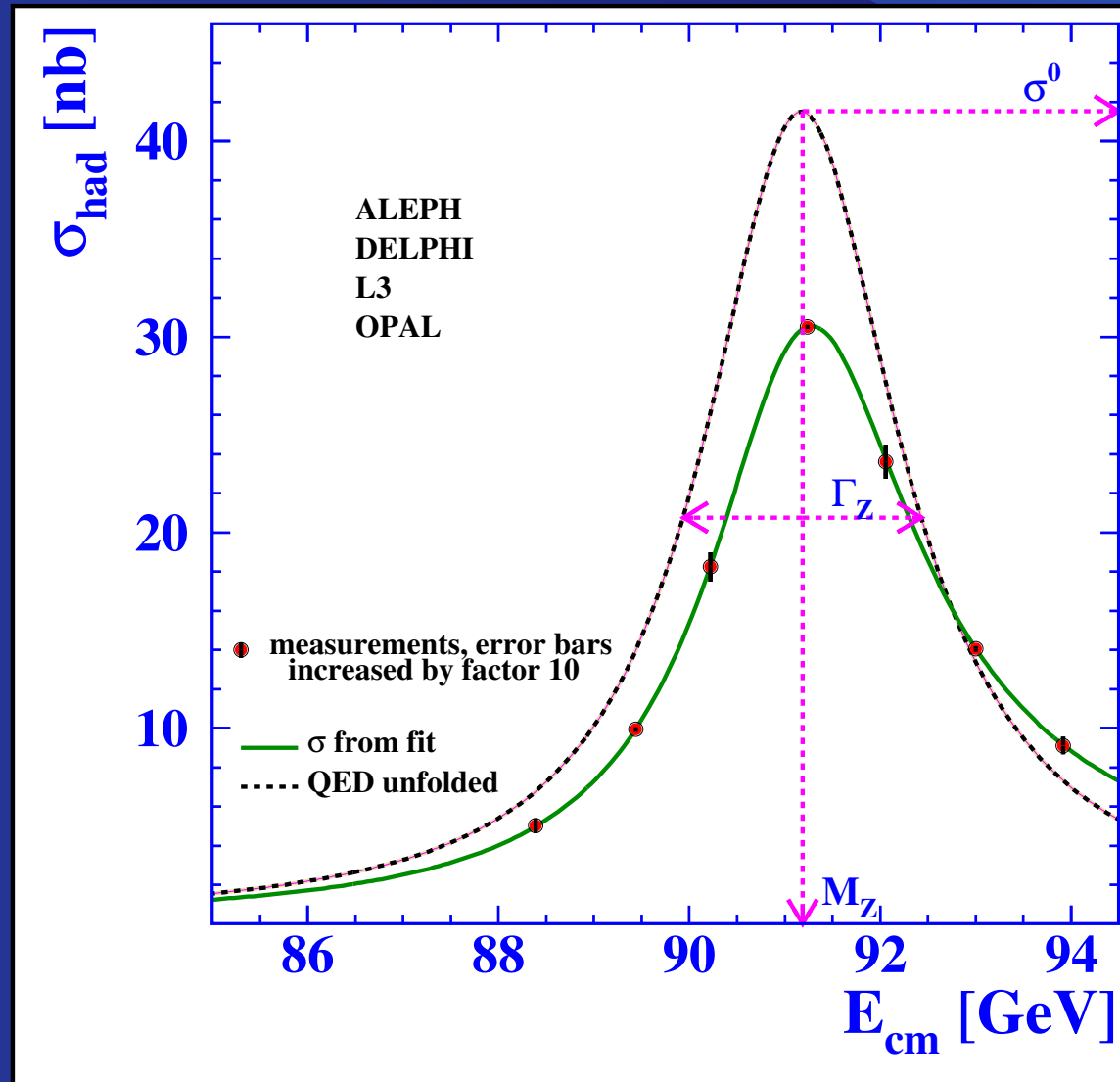
where $\Gamma_f, \Gamma_e, \Gamma_Z$ include the appropriate radiative corrections.

Including ISR as commented before one obtains

$$\sigma_{ISR}(s) \approx \left(1 + \frac{3}{4}\beta\right) \left(\frac{(s - m_Z^2)^2 + s^2 \Gamma_Z^2/m_Z^2}{s^2}\right)^{\beta/2} \sigma^0(s)$$

with $\beta = \frac{4\alpha}{\pi} \ln \frac{m_Z}{m_e}$. This amounts to **26% on the peak**.

LEP gives:



Decay widths of gauge bosons

The decay widths of the weak gauge bosons can be easily computed:

$$\Gamma(Z \rightarrow \bar{f}f) = \frac{\hat{\alpha}}{12s_Z^2c_Z^2} C_f (|v_f|^2 + |a_f|^2)$$

C_f takes into account the color of quarks, QCD corrections and final state QED corrections

$$C_f = \begin{cases} \delta_{f\text{QED}} & \text{leptons} \\ 3(1 + \alpha_s(m_Z)/\pi + \dots) \delta_{f\text{QED}} & \text{quarks} \end{cases}$$

$\delta_{f\text{QED}} = 1 + Q_f^2 3\alpha/(4\pi)$ and v_f and a_f are the tree-level neutral-current couplings written in terms of s_Z . For the b -quark additional corrections needed.

Similar expressions obtained for the **W decay widths**

Since **parity violation** comes from the **axial-vector couplings** it is customary to define the combination of the vector and axial couplings of the fermions as

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2}$$

In $e^+e^- \rightarrow f^+f^-$ collisions one can define the **forward-backward asymmetry**

$$A_{FB} \equiv \frac{N_F - N_B}{N_F + N_B}$$

with N_F (N_B) denote the number of f emerging in the **forward** (**backward**) directions.

At the Z pole, it is given by

$$\mathcal{A}_{FB}^{0,f} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

The measurement of $\mathcal{A}_{FB}^{0,f}$ for charged leptons, and c and b quarks give us information only on the **product of A_e and A_f** .

On the other hand, the measurement of the τ lepton polarization is able to determine the values of A_e and A_τ **separately**. The **longitudinal τ polarization** is defined as

$$\mathcal{P}_\tau \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

where $\sigma_{R(L)}$ is the cross section for tau-lepton pair production of a right (left) handed τ^- . At the Z pole, \mathcal{P}_τ can be written in terms of scattering (e^-, τ^-) angle θ as,

$$\mathcal{P}_\tau = -\frac{\mathcal{A}_\tau(1 + \cos^2 \theta) + 2\mathcal{A}_e \cos \theta}{1 + \cos^2 \theta + 2\mathcal{A}_e \mathcal{A}_\tau \cos \theta}$$

Another interesting asymmetry that can be measured by using **polarized beams** (in SLD) is the **left-right cross section asymmetry**,

$$\mathcal{A}_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\mathcal{P}_e$$

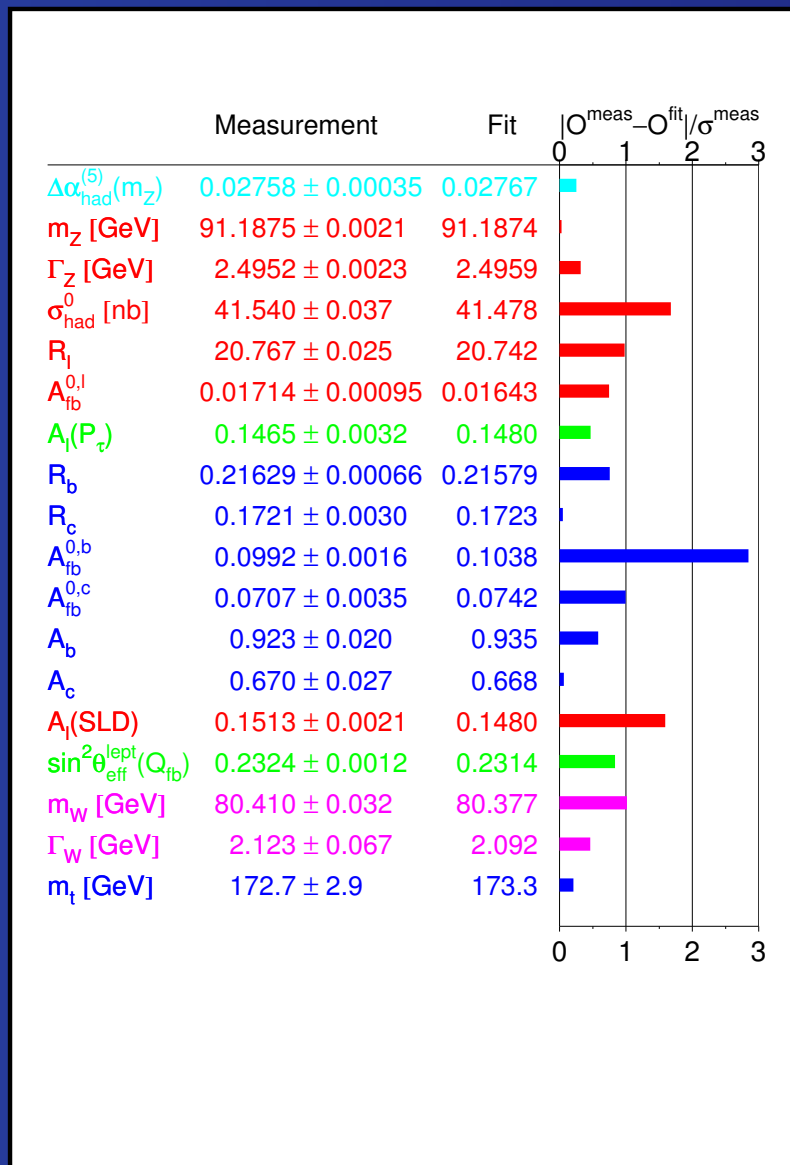
where $\sigma_{L(R)}$ is the cross section for (left-) right-handed incident electron with the positron kept unpolarized.

Observables can be expressed in terms of a few parameters $G_F, \hat{\alpha}(m_Z), m_Z, m_t, m_H, \alpha_s(m_Z)$. G_F **well known** from muon decay. The hadronic contributions to $\hat{\alpha}(m_Z)$ are not so well known and one leaves them also free in the global fit. Thus

$$\chi^2(\text{parameters}) = \sum_i \left(\frac{\mathcal{O}_{\text{th}}^i(\text{parameters}) - \mathcal{O}_{\text{exp}}^i}{\Delta \mathcal{O}^i} \right)^2$$

by **minimizing** χ^2 one **determines the parameters** and gives **predictions** for the rest of the observables which can be compared back with measured values using the "Pull"

$$\text{Pull}_i = \frac{\mathcal{O}_{\text{th}}^i(\text{fitted parameters}) - \mathcal{O}_{\text{exp}}^i}{\Delta \mathcal{O}^i}$$

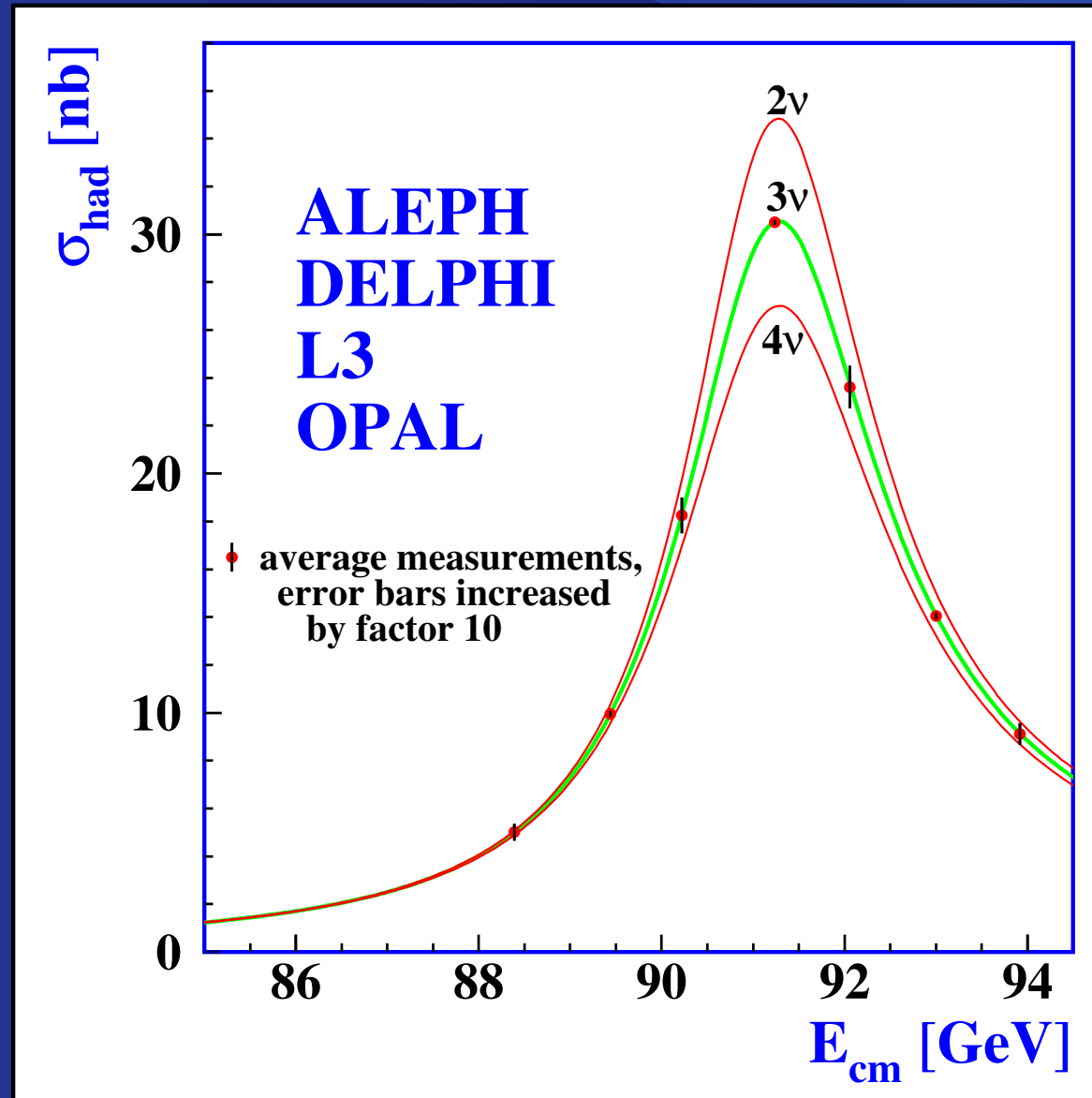


Z-pole fit	
m_Z	91.1874 ± 0.0021 GeV
m_H	$111 \pm_{60}^{190}$ GeV
m_t	$173 \pm_{10}^{13}$ GeV
$\alpha_s(m_Z)$	0.1190 ± 0.0028
$1/\hat{\alpha}(m_Z)$	127.918 ± 0.018

To be compared with the recent measurement of m_t at Fermilab

$$172.7 \pm 2.9 \text{ GeV}$$

Number of Neutrino Species



We can extract information on the number of light neutrino species by assuming that they are the **only** particles **responsible for the invisible width**, i.e. $\Gamma_{inv} = N_\nu \Gamma_\nu$. The LEP data gives the ratio of the invisible and leptonic Z partial widths, $\Gamma_{inv}/\Gamma_\ell = 5.941 \pm 0.016$ and the SM predicts $(\Gamma_\nu/\Gamma_\ell)_{SM} = 1.9912 \pm 0.0008$. Γ_ℓ cancels out and then

$$N_\nu = 2.984 \pm 0.008$$

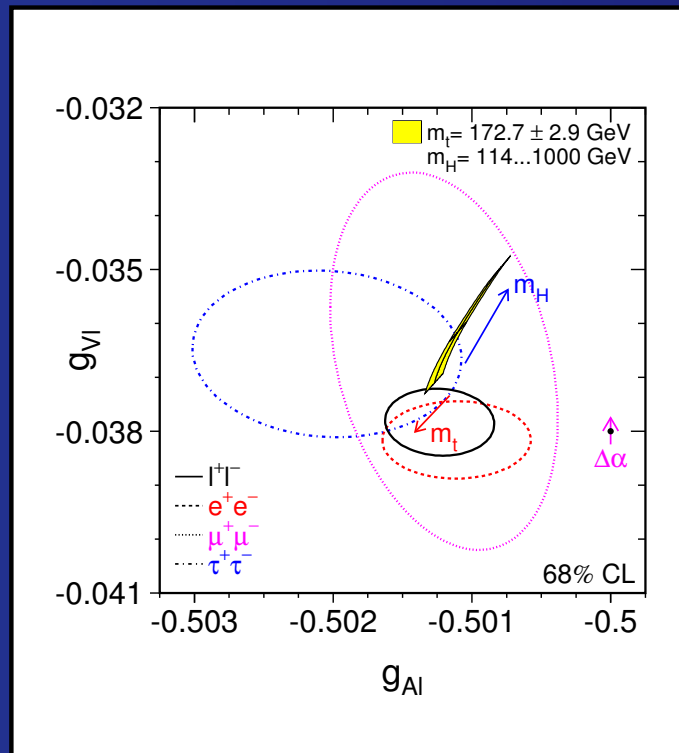
N_ν is the number of neutrino flavors that are **accessible kinematically to the Z** . This result indicates that there exist **only three families of fermions**.

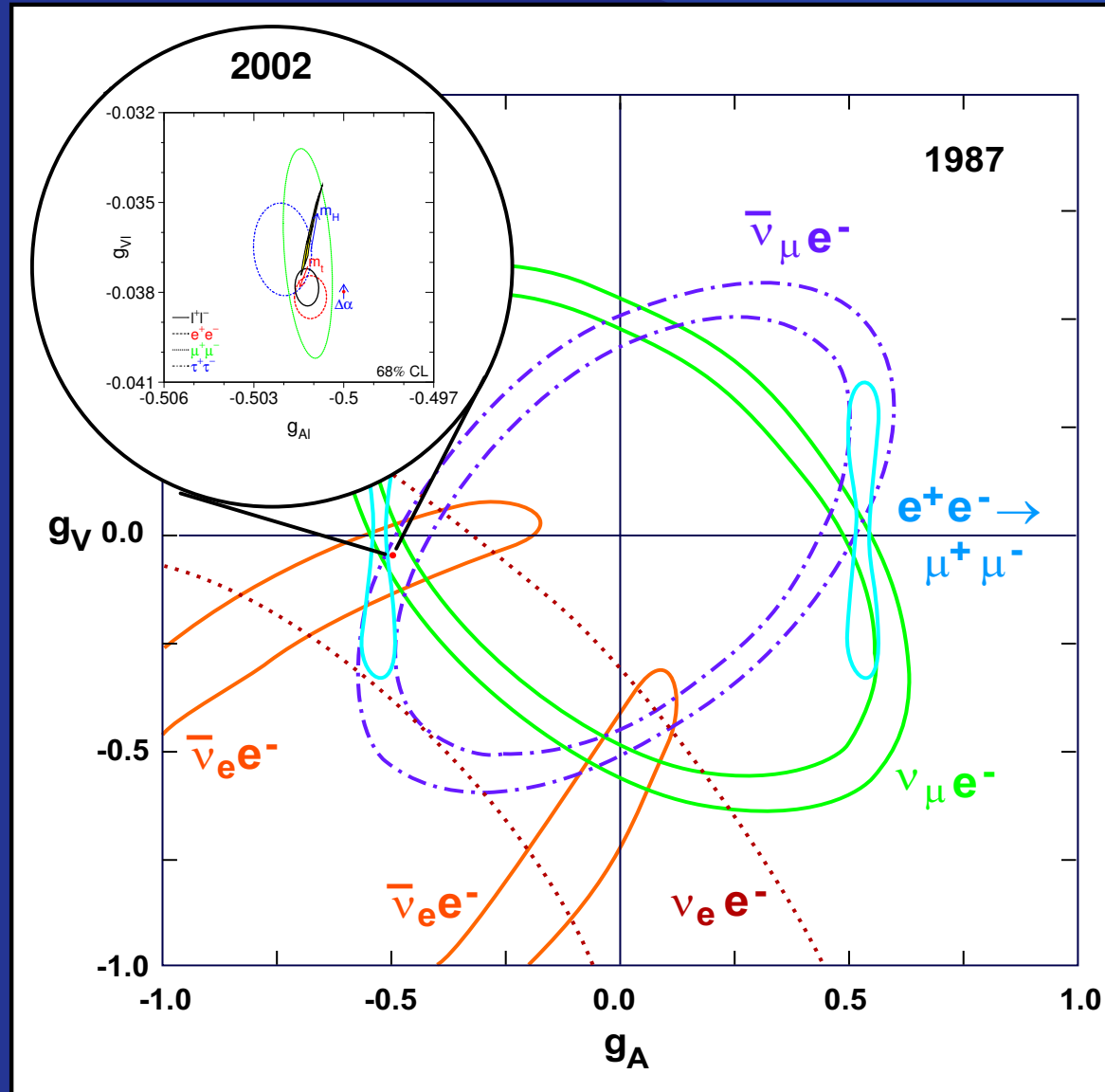
If we assume $N_\nu = 3$ we can put bounds on additional contributions to Γ_{inv} .

$$\Delta\Gamma_{inv} = -2.7 \pm 1.7 \text{ MeV} \rightarrow \Delta\Gamma_{inv} < 2 \text{ MeV} \quad 95\% \text{ CL}$$

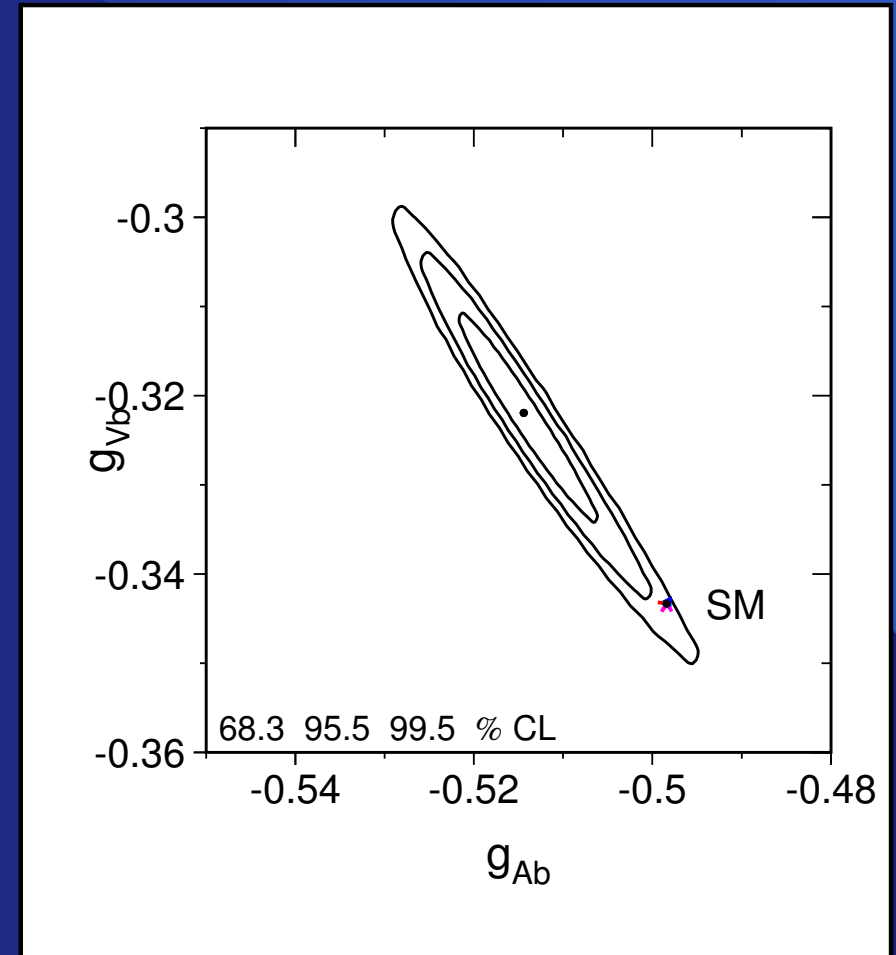
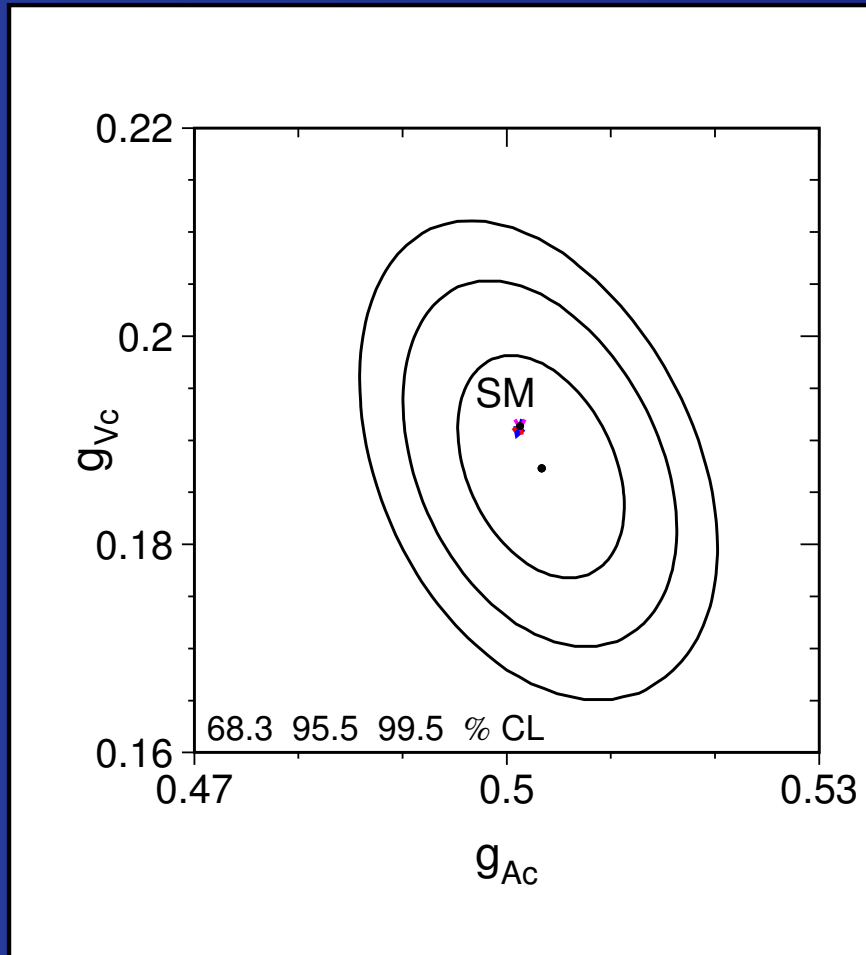
The couplings of leptons and universality

The **partial Z widths** in the different lepton flavors together with the **asymmetries** allows for a determination of all **lepton neutral-current couplings**, $v_\ell \equiv g_{V\ell}$ and $a_\ell \equiv g_{A\ell}$. The values of $g_{V\ell}$ and $g_{A\ell}$ can be plotted for $\ell = e, \mu, \tau$.

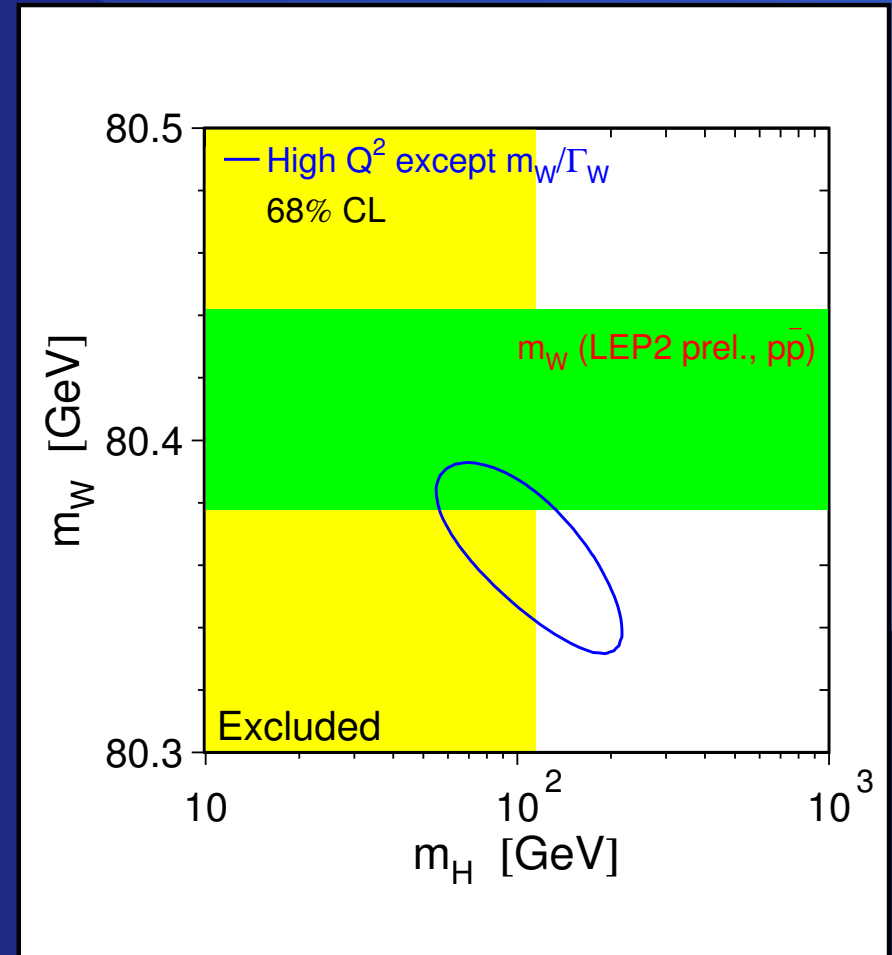
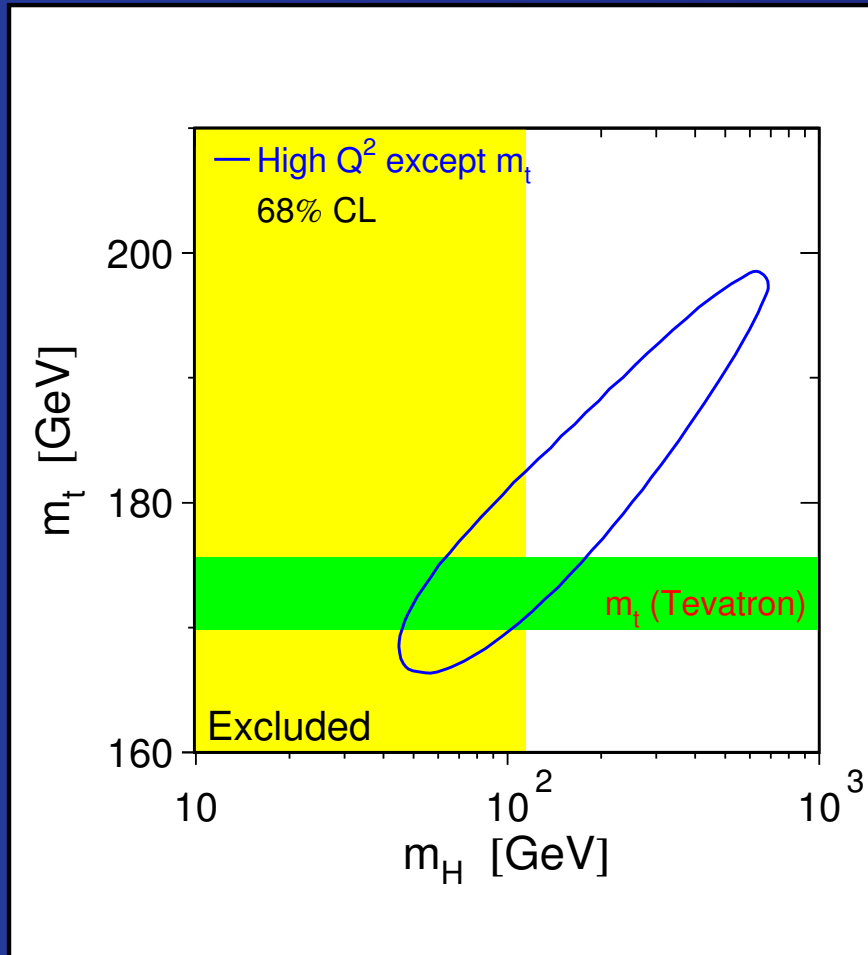




The couplings of heavy quarks

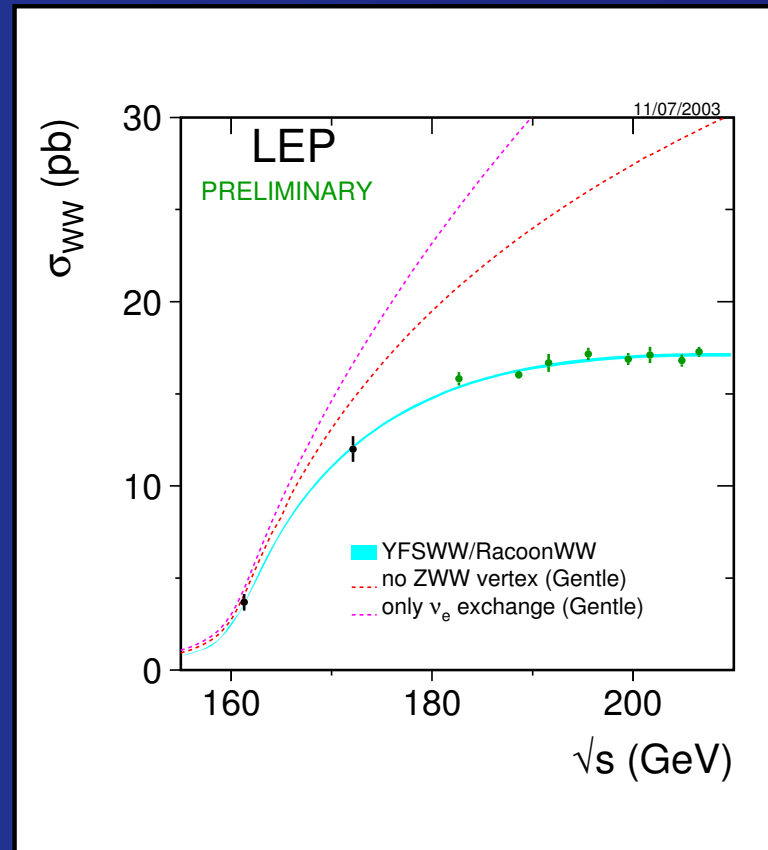


top-quark, W , and Higgs masses



LEP2 and the non-Abelian couplings

The **unitarity problems of the IVB** and the need for non-Abelian couplings were one of the main points that triggered the development of the SM. These have been **tested at LEP2**



The Higgs Couplings

Coupling	Intensity
$H f \bar{f}$	M_f / v
HW^+W^-	$2M_W^2 / v$
HZ^0Z^0	M_Z^2 / v
HHW^+W^-	M_W^2 / v^2
HHZ^0Z^0	$M_Z^2 / 2v^2$
HHH	$M_H^2 / 2v$
$HHHH$	$M_H^2 / 8v^2$

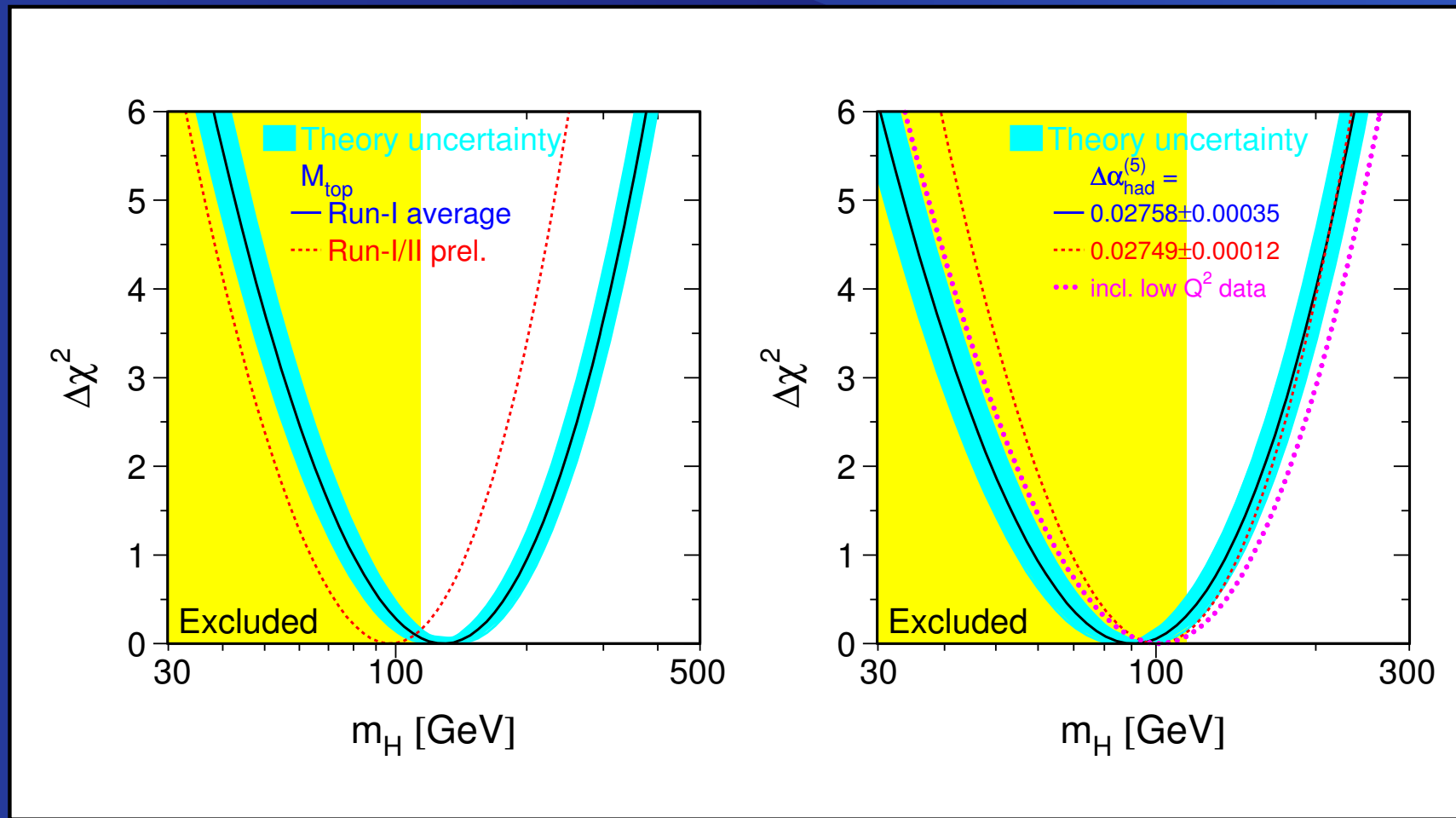
The Higgs also couples at **higher orders** with other gauge bosons

$$H\gamma\gamma, HZ\gamma, Hgg$$

Higgs coupling proportional to particle masses:

- Produced in association with heavy particles
- Decay into the heaviest accessible particles

Direct searches and global fit



$$114 < M_H < 285 \text{ GeV} \quad 95\% \text{ CL}$$

Unitarity and perturbativity bounds

Decay widths of the Higgs into gauge bosons grow like the Higgs mass

$$\Gamma(H \rightarrow W^+W^-) = \frac{G_F m_H^3}{8\pi\sqrt{2}}, \quad \Gamma(H \rightarrow ZZ) = \frac{G_F m_H^3}{16\pi\sqrt{2}}$$

Requiring $\Gamma_{\text{tot}}(H) \leq m_H$ gives

$$m_H \leq 1.6 \text{ TeV}$$

Requiring that tree-level unitarity is not violated in $W^+W^- \rightarrow W^+W^-$ leads to a slightly better bound

$$m_H \leq 1.2 \text{ TeV}$$

These are **not strict bounds**, just say that for larger m_H one should **not trust perturbation theory**.

The λ coupling in the scalar potential grows with energy

$$\frac{d\lambda}{d \ln q^2} = \frac{3\lambda^2}{4\pi^2} + \dots$$

then, λ diverges at some scale Λ , unless it is strictly zero.
Taking $\lambda(\Lambda) = \infty$ (the theory only makes sense up to $q^2 \sim \Lambda^2$) one finds

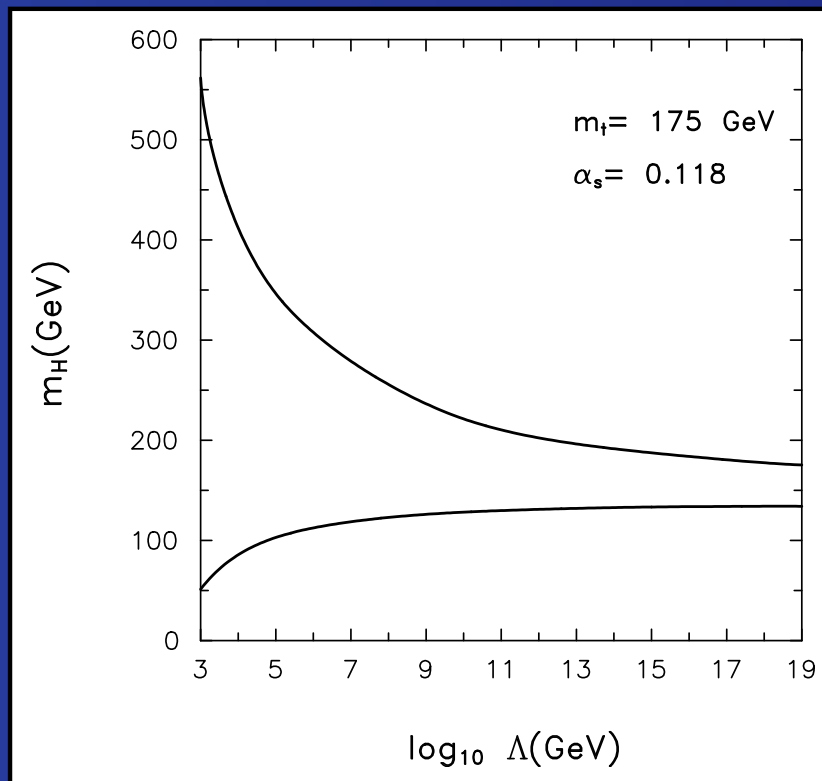
$$\lambda(q^2) = \frac{4\pi^2}{3 \log(\Lambda^2/q^2)} \quad m_H^2 = 2\lambda(v^2)v^2 \approx \frac{4\pi^2}{3 \log(\Lambda^2/v^2)}$$

Since Λ should be larger than m_H one finds

$$m_H \leq \frac{4\pi^2}{3\sqrt{2}G_F \log(m_H^2/v^2)} \approx 850 \text{ GeV}$$

Stability of the Higgs Potential

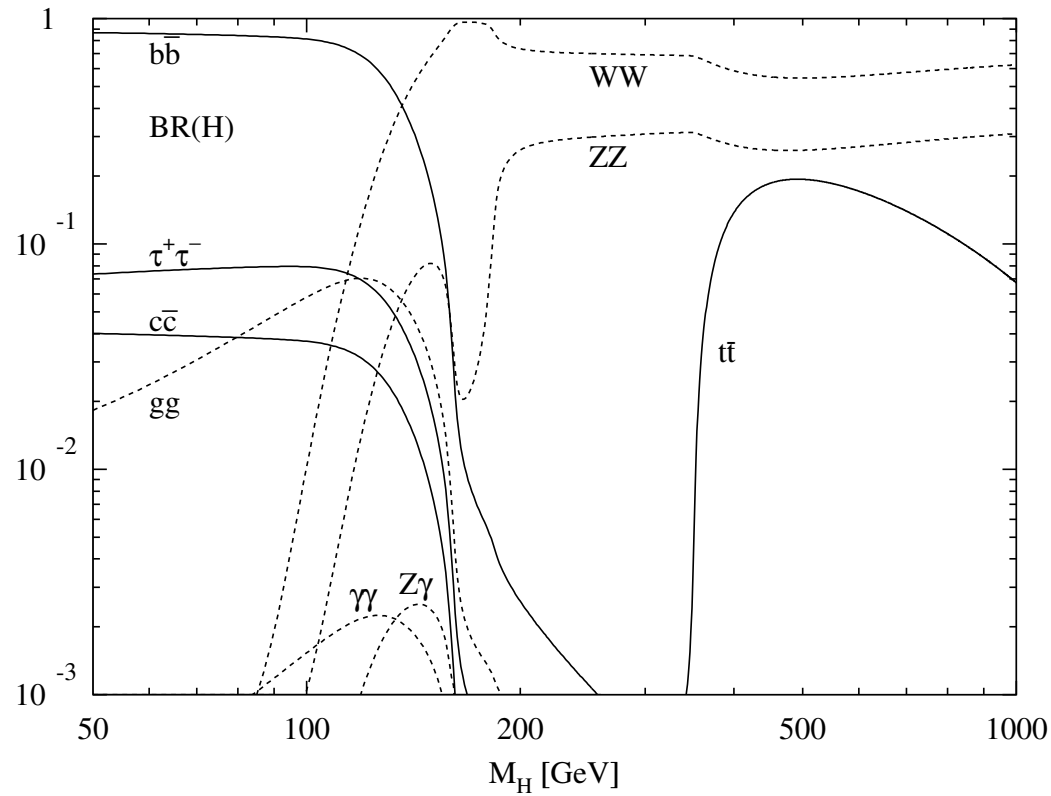
Radiative corrections modify the shape of the Higgs potential and could destabilize it. Requiring this does not happen gives a lower bounds on the Higgs mass (at one loop).



$$m_H > 100 \text{ GeV} \quad (\text{Stability})$$

$$m_H < 850 \text{ GeV} \quad (\text{Triviality})$$

The Decay Modes of the Higgs Boson



$95 \text{ GeV} < m_H < 130 \text{ GeV}, \Gamma_H < 10 \text{ MeV}$

$$BR(H \rightarrow b\bar{b}) \sim 90\% ,$$

$$BR(H \rightarrow c\bar{c}) \simeq BR(H \rightarrow \tau^+\tau^-) \sim 5\%$$

$$BR(H \rightarrow gg) \sim 5\% \quad \text{for } m_H \sim 120 \text{ GeV}$$

$m_H > 130 \text{ GeV}$

$$BR(H \rightarrow W^+W^-) \sim 65\% , \quad BR(H \rightarrow Z^0Z^0) \sim 35\%$$

$$m_H \simeq 500 \text{ GeV} \quad BR(H \rightarrow t\bar{t}) \sim 20\%$$

- **Bjorken:** $e^+e^- \rightarrow Z \rightarrow ZH$
- **WW fusion:** $e^+e^- \rightarrow \nu\bar{\nu}(WW) \rightarrow \nu\bar{\nu}H$
- **ZZ fusion:** $e^+e^- \rightarrow e^+e^-(ZZ) \rightarrow e^+e^-H$

At **LEP1 and 2**, where $\sqrt{s} \simeq M_Z$ or $2M_W$ the Higgs production is dominated by the **Bjorken mechanism**. Present bounds come from the analysis of LEP2 results. At the future e^+e^- accelerators, like the **Next Linear Collider**, where $\sqrt{s} = 500 \text{ GeV}$, the production of a Higgs with $100 < M_H < 200 \text{ GeV}$ will be dominated by the **WW fusion**. One expects $M_H \sim 350 \text{ GeV}$.

At proton-(anti)proton collisions

- **Gluon fusion:** $pp \rightarrow gg \rightarrow H$
- **VV fusion:** $pp \rightarrow VV \rightarrow H$
- **Association with V :** $pp \rightarrow qq' \rightarrow VH$

Fermilab Tevatron, with $\sqrt{s} = 1.8$ (2) TeV: better produced in **association with vector bosons**, look for the $VH(\rightarrow b\bar{b})$ signature. Will be able to explore to explore up to $M_H \sim 100$ GeV.

CERN Large Hadron Collider (LHC), with $\sqrt{s} = 14$ TeV: the dominant mechanism is **gluon fusion** and the best signature $H \rightarrow ZZ \rightarrow 4\ell^\pm$ for $M_H > 130$ GeV. For $M_H < 130$ GeV rely on the small $BR(H \rightarrow \gamma\gamma) \sim 10^{-3}$. Will explore up to $M_H \sim 700$ GeV.