The SM Lagrangian

- Charged current interactions
- Neutral current interactions
- Gauge boson selfinteractions
- Higgs couplings
- Radiative corrections

Charged current interactions

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$\mathcal{L}_\psi$ contains interactions of fermions with gauge bosons,

$$\mathcal{L} \longrightarrow \frac{g}{2}\,\overline{\Psi}_1\gamma^\mu(\vec{\tau}\cdot\vec{W}_\mu)\Psi_1 + g'\,B_\mu\sum_j y_j\,\overline{\Psi}_j\gamma^\mu\Psi_j\,.$$

with

$$\vec{\tau}\cdot\vec{W}_\mu = \begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix}$$

For a single family of quarks and leptons,

$$\mathcal{L}_{\mathrm{CC}} = \frac{g}{\sqrt{2}}\left\{W_\mu^+\,[\bar{u}_L\gamma^\mu d_L + \bar{\nu}_L\gamma^\mu e_L] + \textbf{h.c.}\right\}$$

This is precisely the IVB interaction. **Universality** of the quark and lepton interactions is just a direct **consequence of the gauge symmetry**.

Neutral current interactions

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$\mathcal{L}_\psi$ also contains interactions with the neutral gauge fields W^3_μ and B_μ with can be expressed in terms of the physical fields Z_μ and A_μ

$$\mathcal{L}_{\rm NC} = \sum_j \overline{\Psi}_j \gamma^\mu \left\{ A_\mu \left[\frac{g}{2}\tau_3 \sin\theta_W + \frac{g'}{2} Y_j \cos\theta_W \right] \right\} \Psi_j + \sum_j \overline{\Psi}_j \gamma^\mu \left\{ Z_\mu \left[\frac{g}{2}\tau_3 \cos\theta_W - \frac{g'}{2} Y_j \sin\theta_W \right] \right\} \Psi_j$$

Using $\tan\theta_W = g'/g$ and the relationship between charge and hypercharge $Q = T_3 + Y/2$ ($T_3 = \tau_3/2$) one immediately finds

$$\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm QED} + \mathcal{L}^Z_{\rm NC}\,,$$

where

$$\mathcal{L}_{\rm QED} = e\,A_\mu \sum_j \overline{\Psi}_j \gamma^\mu Q_j \Psi_j \equiv e\,A_\mu\,J^\mu_{\rm em}$$

is the usual QED Lagrangian if we identify the QED coupling

$$e = g\,\sin\theta_W = g'\,\cos\theta_W$$

while

$$\mathcal{L}^Z_{\rm NC} = \frac{e}{2\sin\theta_W\cos\theta_W}\,J^\mu_Z\,Z_\mu$$

$$J^\mu_Z \equiv \sum_j \overline{\Psi}_j \gamma^\mu \left(\tau_3 - 2\sin^2\theta_W Q_j\right)\Psi_j$$

contains the Z-boson interactions.

In terms of the more usual fermion fields, $\mathcal{L}_{\rm NC}^{Z}$ has the form

$$\mathcal{L}_{\rm NC}^{Z} = \frac{e}{2\sin\theta_W\cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu(v_f - a_f\gamma_5)\, f \,,$$

where $a_f = T_3^f$ and $v_f = T_3^f\left(1 - 4|Q_f|\sin^2\theta_W\right)$.

	v_f	a_f
u	$(1-\frac{8}{3}\sin^2\theta_W)/2$	$1/2$
d	$(-1+\frac{4}{3}\sin^2\theta_W)/2$	$-1/2$
ν	$1/2$	$1/2$
e	$(-1+4\sin^2\theta_W)/2$	$-1/2$

Gauge self interactions

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In addition to the usual kinetic terms, the Lagrangian generates cubic and quartic self-interactions among the gauge bosons:

$$\mathcal{L}_3 = -ie\cot\theta_W \left\{ \left(\partial^\mu W^\nu - \partial^\nu W^\mu\right) W^\dagger_\mu Z_\nu \right.$$

$$\left. - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right) W_\mu Z_\nu + W_\mu W^\dagger_\nu \left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right) \right\}$$

$$-ie \left\{ \left(\partial^\mu W^\nu - \partial^\nu W^\mu\right) W^\dagger_\mu A_\nu \right.$$

$$\left. - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right) W_\mu A_\nu + W_\mu W^\dagger_\nu \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) \right\}$$

$$\mathcal{L}_4 = -\frac{e^2}{2\sin^2\theta_W}\left\{\left(W^\dagger_\mu W^\mu\right)^2 - W^\dagger_\mu W^{\mu\dagger} W_\nu W^\nu\right\}$$

$$-e^2\cot^2\theta_W\,\left\{W^\dagger_\mu W^\mu Z_\nu Z^\nu - W^\dagger_\mu Z^\mu W_\nu Z^\nu\right\}$$

$$-e^2\cot\theta_W\left\{2W^\dagger_\mu W^\mu Z_\nu A^\nu - W^\dagger_\mu Z^\mu W_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu Z^\nu\right\}$$

$$-e^2\,\left\{W^\dagger_\mu W^\mu A_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu A^\nu\right\}$$

Notice that $\mathcal{L}_3$ has only terms with **two charged W's** and **one neutral (Z or γ)** boson.

Higgs couplings

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Higgs boson interactions can separated in three parts

$$\mathcal{L}_H = \mathcal{L}_{HH} + \mathcal{L}_{HG} + \mathcal{L}_Y$$

where

$$\begin{aligned}
\mathcal{L}_{HH} &= -\frac{1}{2} m_H^2 H^2 \left(1 + \frac{H}{v} + \frac{H^2}{4v^2}\right) \\
\mathcal{L}_{HG} &= \left(m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right) \left(1 + \frac{H}{v}\right)^2 \\
\mathcal{L}_Y &= -\left(m_e \bar{e} e + m_d \bar{d} d + m_u \bar{u} u\right) \left(1 + \frac{H}{v}\right)
\end{aligned}$$

Higgs couplings to particles are always **proportional to the masses** of the particles, **linearly for fermions** and **quadratically for bosons**.

(*) Quantization and gauge fixing

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The Lagrangian we just studied, written in the Unitary gauge, is appropriate for all tree-level calculations. However, at the loop level other subtleties enter. Quantization of **non-Abelian gauge theories** requires the introduction of additional unphysical particles, the so-called **Fadeev-Popov ghosts**. In addition the **unitary gauge**, some times, is **not appropriate** for **loop calculations**:

In this gauge the vector boson (V) propagator is the standard **Proca propagator**

$$P^U_{\mu\nu} = \frac{-i}{q^2 - m_V^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_V^2}\right)$$

$P^U_{\mu\nu}$ does **not go like** $\sim 1/q^2$ as $q \to \infty$ due to the term proportional to $q_\mu q_\nu$.

Renormalizability is **not obvious** in the unitary gauge.
Although S **matrix elements** should be **the same** in any gauge, **individual Green functions** could show a rather **wild behavior** in the **Unitary gauge**.
If one comes back to the original Lagrangian and writes $\phi^0 \equiv \frac{1}{\sqrt{2}}(v + H + i\omega^0)$ and $\phi^\pm \equiv \omega^\pm$, instead the exponential parametrization we used, one immediately finds terms that mix the gauge bosons with the scalars like $B^\mu \partial_\mu \omega^0$, $W^\pm_\mu \partial^\mu \omega^\pm$ which complicate the analysis of the spectrum and are at the origin of the $q_\mu q_\nu / m_V^2$ in the propagator.

The R_ξ gauge

A way out to this problem is to **use the gauge freedom** to **fix the gauge** in such a way that those terms are canceled. In the same spirit that covariant Lorentz gauges one adds a term like

$$\begin{aligned}\mathcal{L}_{\rm gf} &= -\frac{1}{2\xi}\left(\partial_\mu \vec{W}^\mu + i\frac{g}{2}\xi\left(\Phi'^\dagger\vec{\tau}\left\langle\Phi\right\rangle - \left\langle\Phi^\dagger\right\rangle\vec{\tau}\Phi'\right)\right)^2 \\ &\quad -\frac{1}{2\xi}\left(\partial_\mu B^\mu + i\frac{g'}{2}\xi\left(\Phi'^\dagger\left\langle\Phi\right\rangle - \left\langle\Phi^\dagger\right\rangle\Phi'\right)\right)^2\end{aligned}$$

with $\Phi' \equiv \Phi - \langle\Phi\rangle$. The addition of those terms removes the unwanted terms. This is called the **R_ξ gauge**.

In the R_ξ gauge the **would be Goldstone bosons remain in the Lagrangian**, although cannot appear in external legs. The **vector boson** propagator is

$$P^{\xi}_{\mu\nu} = \frac{-i}{q^2 - m_V^2}\left[g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2 - \xi m_V^2}\right]$$

while the would be **Goldstone bosons** have the propagator

$$P^{\xi} = \frac{i}{q^2 - \xi m_V^2}$$

The physical Higgs propagator remains the same. For $\xi \to \infty$ the **unitary gauge** is recovered. $\xi \to 1$. gives the **Feynman** gauge. Although, individual diagrams can depend on ξ **physical amplitudes** should be **independent** of if.

Fixing the SM parameters

The gauge and scalar sector of the SM Lagrangian contains **only 4 for parameters:** g, g', μ^2 and λ.
At tree level we found simple expressions that relate those parameters to more physical parameters like α, $\sin^2\theta_W$, m_Z and m_H or α, G_F, m_Z and m_H.
The rest of the parameters or **observables**, like m_W decay widths, cross sections, etc. can be expressed in terms of **only 4 parameters** which must be **fixed from experiment**.
The last set of parameters (apart from the **Higgs mass**) is the best known set. We will use as input values

$$\begin{aligned} \alpha^{-1} &= 137.03599976 \pm 0.00000050 \\ G_F &= (1.16639 \pm 0.00001) \times 10^{-5}\,\mathrm{GeV}^{-2} \\ m_Z &= (91.1874 \pm 0.0021)\,\mathrm{GeV} \end{aligned}$$

Can be written in terms of g, g' and $v = \sqrt{-2\mu^2}$

$$\alpha = \frac{g^2 g'^2}{4\pi(g^2 + g'^2)}\,, \quad G_F = \frac{1}{\sqrt{2}v^2}\,, \quad m_Z^2 = \frac{v^2}{4}(g^2 + g'^2)$$

other quantities written in terms of g, g' and $v \Rightarrow$ in terms of α, G_F and m_Z. At **tree level**

$$m_W s_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}} \equiv A_0\,, \qquad c_W = \frac{m_W}{m_Z}$$

we use $s_W \equiv \sin\theta_W$, $c_W = \cos\theta_W$
and
$A_0 \equiv (37.2802 \pm 0.00012)\,\mathrm{GeV}^2$

From where we can determine s_W and m_W:

$$m_W = \frac{m_Z}{\sqrt{2}} \left\{ 1 + \sqrt{1 - \frac{4A_0^2}{m_Z^2}} \right\}^{1/2} = 80.94 \, \textbf{GeV}$$

$$s_W^2 = \frac{1}{2} \left\{ 1 - \sqrt{1 - \frac{4A_0^2}{m_Z^2}} \right\} = 0.21215$$

Good agreement with experiment,

$$m_W = 80.410 \pm 0.032 \, \mathrm{GeV}$$

We could also use the **tree-level** Lagrangian to compute the most interesting observables: gauge boson **decay widths**, **cross sections** at the Z peak and WW **production** with also very **good agreement**.
However, the experimental **precision** is **good** enough to go beyond the tree level approximation and **check** also the **SM at the quantum level**.
On the other hand, **radiative corrections** are interesting because the corrections to the tree-level relationships **depend** on the **top-quark mass** and on the **Higgs mass**. So studying them and comparing them with experiment one can get some **information** on the **Higgs mass** (once the top quark mass is known).

Radiative corrections

The most precise tests of the SM have been performed at LEP where a very good precision has been reached. Therefore we will mainly study corrections relevant at the Z peak. As we will see, if **Z-peak data** is expressed in terms of **Z-peak parameters**, the bulk of the radiative corrections come when we connect those parameters with the low-energy input parameters, namely α and G_F. Then, the remaining corrections are pushed **below 1%**.

Usually radiative corrections are classified according the the following schema.

- Photonic corrections
- QCD corrections
- Pure electro-weak corrections

Photonic corrections

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QED diagrams involving the emission of **real photons** or the exchange of **virtual photons** in loops, but **not including vacuum polarization** diagrams. These are very important numerically. Often lead to infrared problems and therefore depend on the details of the experiment (cuts, etc). Usually experimental data are presented after subtracting these corrections. Notice that these type of corrections are already subtracted in the definition of G_F from μ-decay. **One defines**:

$$\frac{1}{\tau_\mu} \equiv \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1+\frac{3}{5}\frac{m_\mu^2}{m_W^2}\right) \times \left[1+\frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right]$$

Other important pure QED corrections are the initial and final state photon radiation. The **initial state radiation** (ISR) is taken into account by convoluting the cross section with the radiator function $H(x)$,

$$\sigma(s) \approx \int_0^1 dk\ H(x)\sigma_0[x\ s]$$

The radiator function takes into account virtual and real photon emissions and includes soft photon re-summation. The **final state radiation** is included by multiplying the bare cross sections and widths by the QED correction factor,

$$\delta_{\text{fQED}} = \left(1 + \frac{3\alpha Q_f^2}{4\pi}\right) \simeq (1 + 0.002\ Q_f^2)$$

QCD corrections

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Very important in processes involving quarks.
Often they can be incorporated (Z decays, W decays, e^+e^+ cross section, etc) by using an **effective number of colors** ($s = m_Z$)

$$N_C \Longrightarrow N_C \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\} \approx 3.115$$

Electro-weak corrections

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- **Oblique:**
  Self-energies of gauge bosons. Dominated by the **running of α**
- **Vertex corrections:**
  at LEP small except for the $Z \to b\bar{b}$ vertex
- **Box:**
  negligible at LEP (but very important in some low energy processes)

Oblique corrections

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The net effect of including radiative corrections in the photon propagator amounts to the replacement:

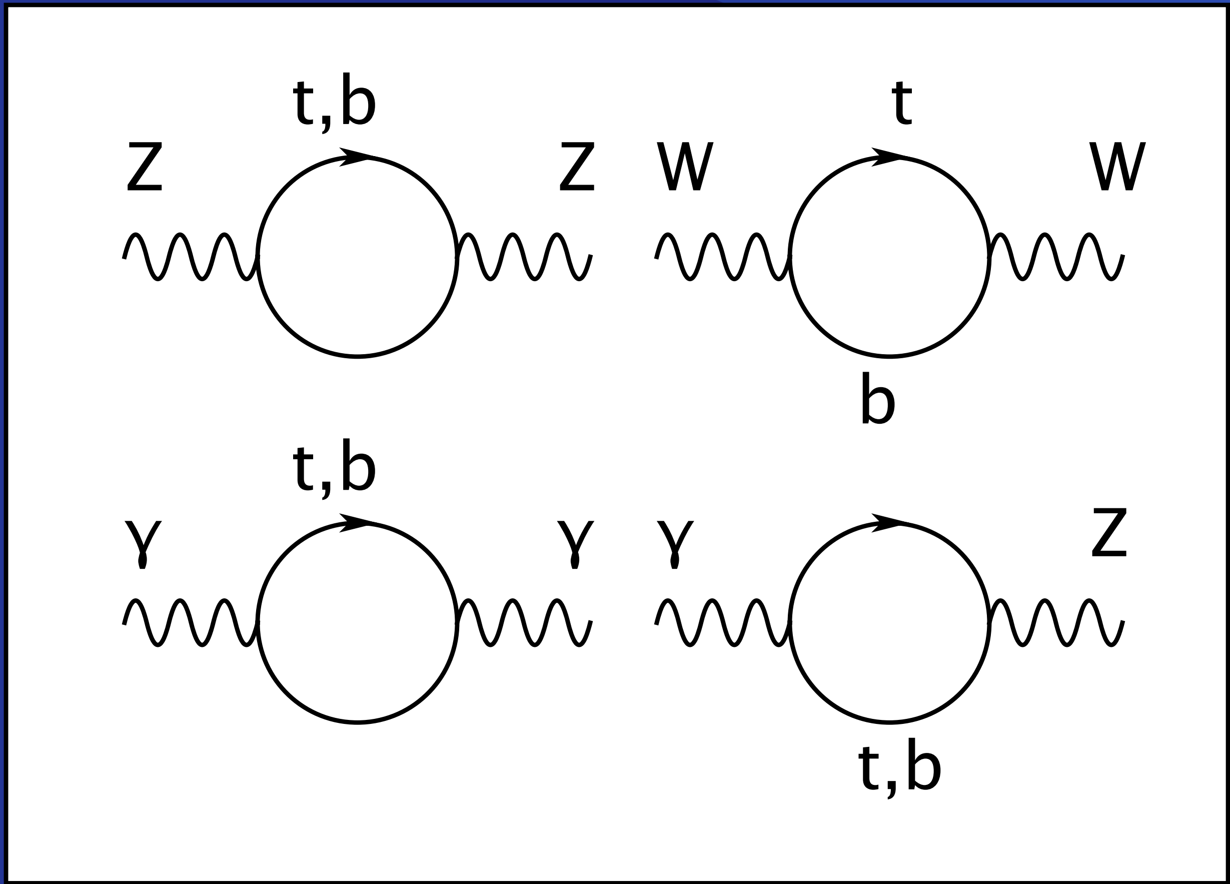
$$\frac{e_0^2}{s} \longrightarrow \frac{e^2}{s\,[1+\Pi_\gamma(s)]} \equiv \frac{e^2(s)}{s}$$

with the running coupling constant

$$e^2(s) = \frac{e^2}{1+\Pi_\gamma(s)}\,, \qquad \Pi_\gamma(s) \sim -\frac{\alpha}{3\pi} Q_f^2 \log\frac{s}{m_f^2}\,, \qquad s \gg m_f$$

at $s = m_Z^2$ very important,

$$\Delta\alpha \equiv 1 - \frac{\alpha}{\hat{\alpha}(m_Z)} = 0.0664$$



The vacuum polarization contributions also modify the $W^{\pm}$ and Z propagators

$$\frac{g_{V0}^2}{s - m_V^2} \Longrightarrow \frac{g_V^2}{s - m_V^2 + \mathrm{Re}\,\Sigma_V(s) + i\mathrm{Im}\,\Sigma_V(s)}$$
$$= \frac{g_V^2(s)}{s - m_V^2 + i m_V \Gamma_V(s)}$$

with $\mathrm{Re}\,\Sigma_V(s) = (s - m_V^2)\Pi_V(s)$ and

$$g_V^2(s) \equiv \frac{g_V}{1 + \Pi_V(s)}, \quad m_V \Gamma_V(s) \equiv \frac{\mathrm{Im}\,\Sigma_V(s)}{1 + \Pi_V(s)} \approx \frac{\Gamma_V}{m_Z} s$$

$\Gamma_V(s)$, dominates the propagator near the peak, $s \sim m_V^2$

The weak mixing angle

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At tree level one can define the weak mixing in a number of equivalent ways:

1) Via the **unification condition:**

$$e_0 = g_0 s_{0W} = g'_0 c_{0W}$$

2) Via the **relation between** the **masses** of the gauge bosons, valid for scalar doublets

$$s_{0W}^2 = 1 - \frac{m_{0W}^2}{m_{0Z}^2}$$

3) From low energy charged current interactions

$$\frac{G_F^0}{\sqrt{2}} = \frac{e_0^2}{8 s_{0W} m_{0W}^2}$$

4) From the vector part of neutral current couplings

$$v_f = T_3^f - 2 Q_f s_{0W}^2$$

Quantum loops generate different corrections to these relationships. At the loop level, one can use one of them to define the weak mixing angle then the other three will receive finite and calculable corrections. Among the different choices, there are two particularly interesting.

The "on-shell" scheme

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In the **"on-shell" scheme** one chooses to **keep 2**:

$$s_W^2 \equiv 1 - M_W^2/M_Z^2 \,.$$

Then the other two get corrections which are usually parametrized as

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2 s_W^2 m_W^2} \frac{1}{1-\Delta r}$$

$$v_f = T_3^f - 2 Q_f \kappa_f s_W^2$$

the correction to the unification relation is expressed in terms of the neutral current to charged current ratio

$$\rho \equiv \frac{g_Z^2(0)/m_Z^2}{g_W^2(0)/m_W^2} \approx 1 + \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2}$$

In this scheme large top-quark effects are everywhere

$$\Delta r = \frac{\Sigma_W(0)}{m_W^2} \sim -\frac{c_W^2}{s_W^2}\Delta\rho_t + \cdots$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2} \sim \Delta\rho_t + \cdots$$

$$\Delta\kappa_{\text{univ}} \approx \frac{c_W^2}{s_W^2}\Delta\rho_t$$

which increase quadratically with the top-quark mass,

$$\Delta\rho_t \equiv \frac{\alpha N_C}{16\pi s_W^2 c_W^2}\,\frac{m_t^2}{m_Z^2}$$

Owing to an accidental $SU(2)_R$ symmetry of the scalar sector (the so-called **custodial symmetry**), the virtual production of Higgs particles does not generate any m_H^2 dependence at one loop (Veltman screening). The **dependence on the Higgs mass** is only **logarithmic**:

$$\Delta r_H \sim -\frac{c_W^2}{s_W^2}\Delta\rho \sim \frac{\alpha}{16\pi s_W^2}\frac{11}{3}\left[\ln\left(\frac{M_H^2}{M_W^2}\right)-\frac{5}{6}\right]$$

This correction is small -0.0041 (0.0098) for $M_H = 50$ (1000) GeV however it is very important because it provides a **unique information** on the **Higgs mass**.

The $\overline{\rm MS}$ scheme

Alternatively one can choose to **keep 1: the unification relation**.
The $\overline{\rm MS}$ **scheme** just takes this option and one defines

$$s_Z^2 = \frac{g'^2(m_Z)}{g'^2(m_Z) + g^2(m_Z)}$$

Corrections to the other relations are expressed as follows

$$\begin{aligned}
\hat{\rho} &= \frac{m_W^2}{\hat{c}_Z^2 m_Z^2} \\
\frac{G_F}{\sqrt{2}} &= \frac{\pi \hat{\alpha}(m_Z)}{2 s_Z^2 m_W^2} \frac{1}{1 - \Delta \hat{r}} \\
v_f &= T_3^f - 2 Q_f \hat{k}_f s_Z^2
\end{aligned}$$

In this scheme the dominant **top-quark mass** corrections are **only in** $\hat{\rho}$ and the **rest** of the corrections are **very small**

$$\hat{\rho} \approx 1 + \rho_t \approx 1.009\,, \quad \Delta\hat{r} \approx 0.003\,, \quad \Delta k_\ell \approx 0.001$$

Therefore, **at LEP** energies one can reach precisions **better than** 1% by computing observables at **tree level** but written **in terms of** s_Z, $\hat{\alpha}(m_Z)$ **and** m_Z.

All the important **corrections** are **encapsulated in the relationship** between high energy parameters and low energy parameters: $\hat{\alpha}(m_Z)$ **in terms of** α and s_Z **in terms of** G_F **and** α.

There is one **exception** to this rule, the $Zb\bar{b}$ **vertex**.

The $Z \to \bar{b}b$ vertex

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The $Z\bar{f}f$ vertex gets 1–loop corrections where a virtual $W^\pm$ is exchanged between the two fermionic legs.

Hard m_t^2 corrections to the $Z \to \bar{b}b$ vertex arise from the **Goldstone boson vertices** which are proportional to quark masses.

The induced correction is the **same** for the **vector** and **axial-vector** couplings ($\delta v_b = \delta a_b$):

$$\delta v_b = \delta a_b = -\frac{\alpha}{8\pi s_W^2}\left\{\frac{m_t^2}{m_W^2} + \cdots\right\}$$

$$\delta_b \equiv \frac{\delta\Gamma(Z \to \bar{b}b)}{\Gamma(Z \to \bar{b}b)} \approx -0.016$$

Improved Born Approximation

The above discussion allows us to formulate a **simple approximation**:

- Compute all LEP observables (except $Z \to b\bar{b}$ processes) with **tree-level formulae** but written in terms of s_Z, $\hat{\alpha}(m_Z)$ and m_Z (also $\alpha_s(m_Z)$)
- In the $Zb\bar{b}$ **vertex** change

$$a_b \longrightarrow a_b + \delta a_b\,, \qquad v_b \longrightarrow v_b + \delta v_b$$

All **large corrections** are encapsulated in **high-energy-low-energy connection**. $1/\hat{\alpha}(m_Z) = 127.918$ is obtained from α by using the RG plus data on $e^+e^- \longrightarrow \text{hadrons}$.

And

$$s_Z^2 = \frac{1}{2}\left\{1 - \sqrt{1 - \frac{4\pi\hat{\alpha}(m_Z)}{\sqrt{2}G_F m_Z^2 \hat{\rho}}}\right\}$$

with $\hat{\rho} \approx 1 + \rho_t + \mathrm{Higgs} + \cdots \approx 1.009$. m_W determined using

$$m_W^2 = \hat{\rho}\, c_Z^2\, m_Z^2$$

These expressions give $s_Z^2 = 0.231$ and $m_W = 80.34\,\mathrm{GeV}$ to be compared with the global fit $s_Z^2 = 0.23120 \pm 0.00015$, $m_W = 80.377 \pm 0.020$, or with the measured value $m_W = 80.410 \pm 0.032$.
Of course one also includes, when necessary, **initial state radiation** and **QCD corrections**, which will bring in a dependence in the strong coupling constant α_s.