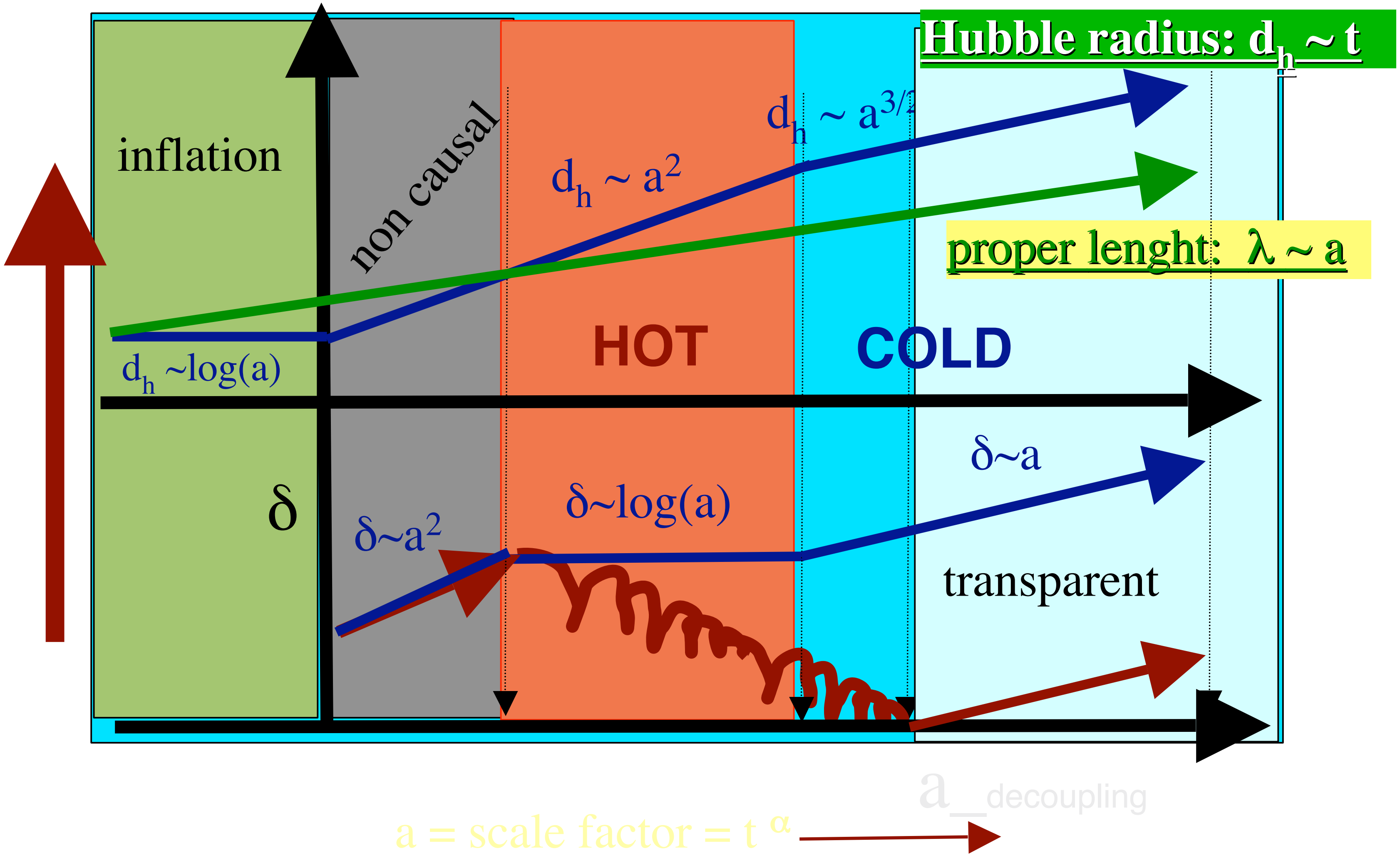
growth of fluctuations



$$\delta'' + \alpha H \delta' - (\mu H^2 - k^2 v^2/a^2)\ \delta = 0$$

harmonic osc. for $\delta(k)$, $k = 2\pi/\lambda$

HOT Damped oscilations: $a^2 \mu H^2 < k^2 v^2$

COLD Growing fluctuations $a^2 \mu H^2 > k^2 v^2$

"Large-scale" modes enter during MAT, potential stays constant:

$$-k^2\Phi \propto \mathcal{H}^2\delta \propto H^2 a^3 \propto \frac{1}{t^2}(t^{2/3})^3 = \mathrm{const.}$$

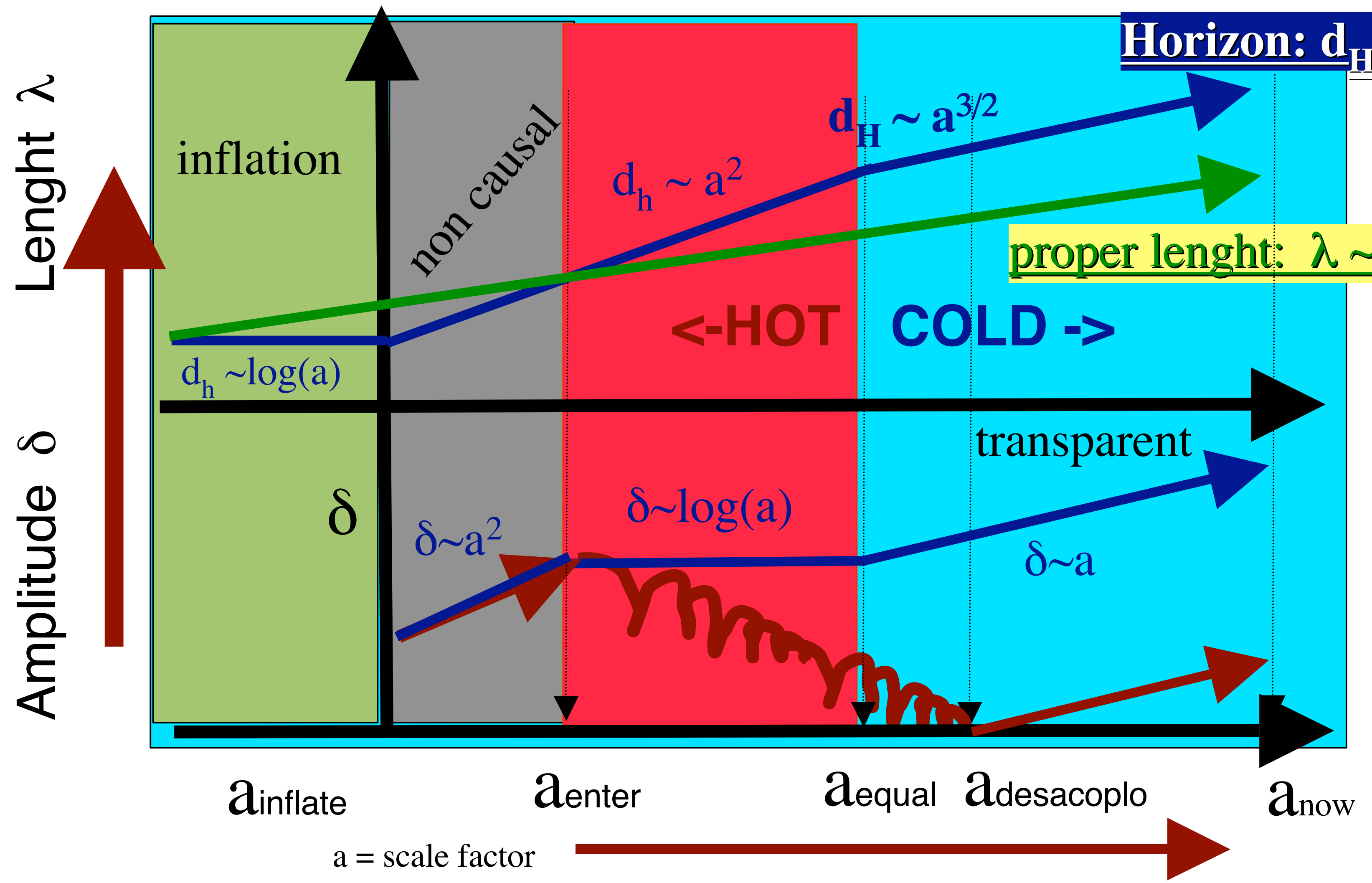
"Small-scale" modes enter during RAD, potential decays:

$$-k^2\Phi \propto \mathcal{H}^2\delta \propto H^2 a^2 \ln(a) \propto \frac{1}{t^2} a^2 \ln(a) \propto \frac{\ln(a)}{a^2}$$

and thus they are suppressed relative to large scales by

$$\ln\left[\frac{a_{\mathrm{eq}}}{a_e(k)}\right]\left(\frac{a_e(k)}{a_{\mathrm{eq}}}\right)^2 = \ln\left(\frac{k}{k_{\mathrm{eq}}}\right)\left(\frac{k_{eq}}{k}\right)^2, \quad k_{\mathrm{eq}} \sim 0.07\Omega_m h^2/\mathrm{Mpc}$$

Gravitational Instability: growth of fluctuations $\delta \equiv \rho / \langle\rho\rangle - 1$



$\delta'' + \alpha H \delta' - (\mu H^2 - k^2 v^2/a^2)\ \delta = 0$ harmonic osc. for $\delta(k)$, $k=2\pi/\lambda$

HOT Damped oscillations: $a^2 \mu H^2 < k^2 v^2$

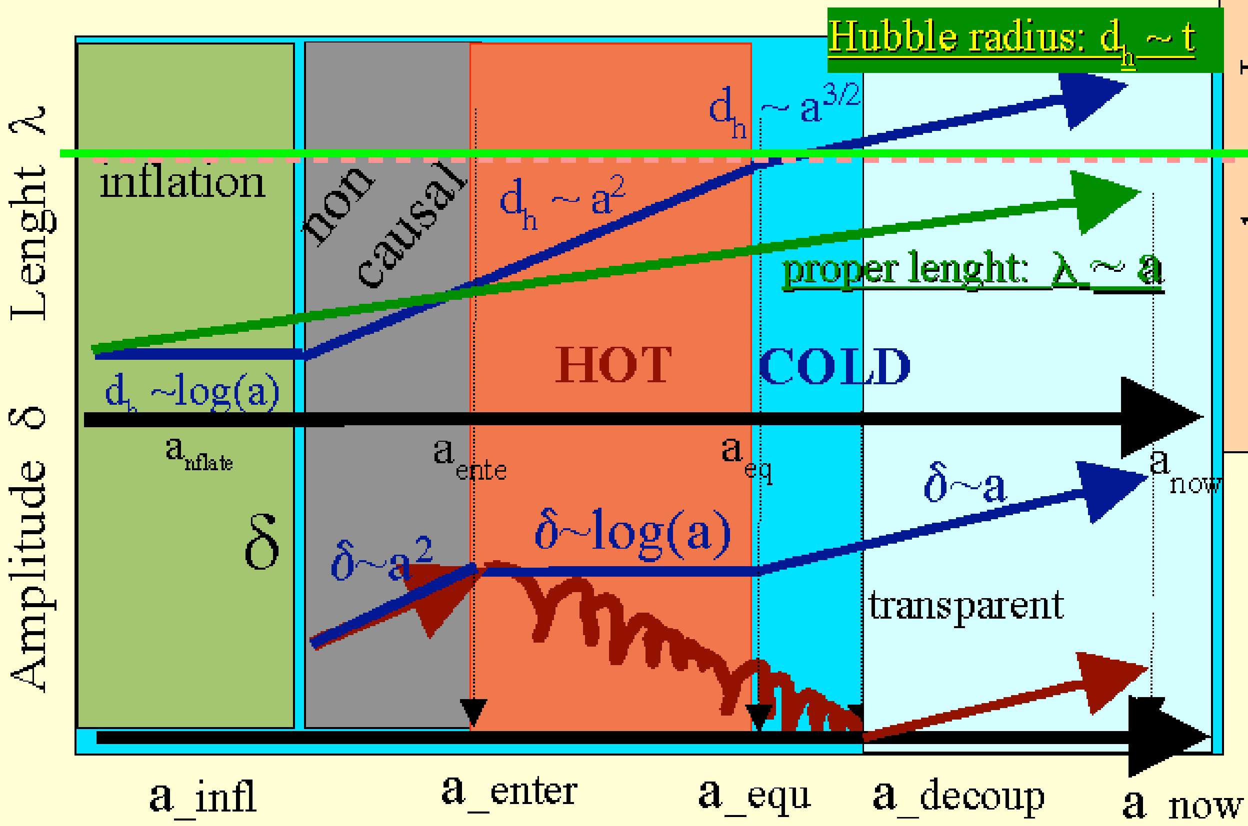
COLD Growing fluctuations $a^2 \mu H^2 > k^2 v^2$

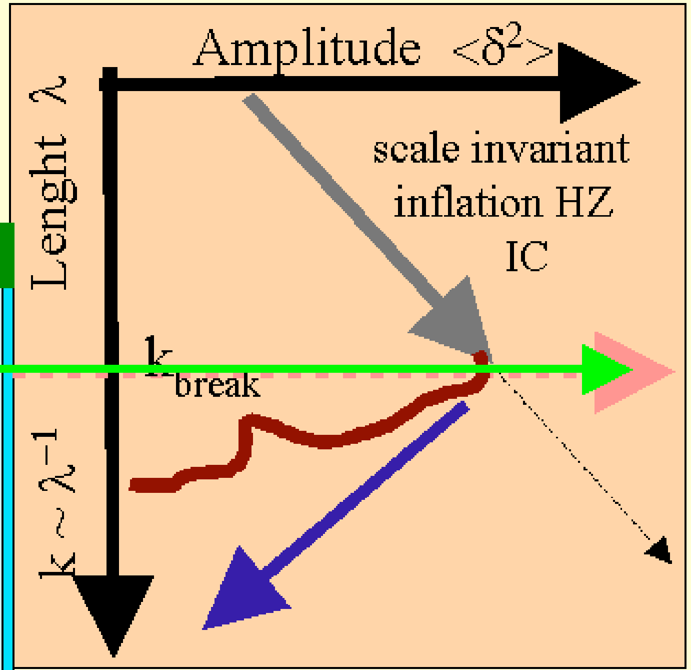
Spectrum of fluctuations

Given IC spectrum: $P_0(k) = \langle\delta^2\rangle = k$ (HZ)

Transfer function: $P(k) = P_0(k)\, T^2(k)$

$T^2(k) = F[k,\Gamma]$; $\Gamma = \Omega_0 h\, e^{-[\Omega b(1+2h/\Omega o)0.06]}$





k_{break} = Hubble radius at MD
(CDM) ~ 0.1 (Ω_0 h) h/Mpc
$\Rightarrow$ 30 Mpc/h /(Ω_0 h) \Γ

a = scale factor = t $^{\alpha}$

Observations require an statistical approach:

Evolution of (rms) variance $\xi_2 = <\delta^2>$ instead of δ

Or power spectrum $P(k) = <\delta^2(k)> \Rightarrow \xi_2 = \int dk\, P(k)\, k^2\, W(k)\, dk$

IC problem: Linear Theory $\delta = a\, \delta_0$

$$\xi_2 = <\delta^2> = D^2 <\delta_0^2>$$

Normalization $\sigma_8^2 \equiv <\delta^2(R=8)>$

To find D(z) -> Compare rms at two times or find evolution invariants

Initial Gaussian distribution of density fluctuations:

$\xi_p(V) = <\delta^P> = 0$ for all $p \neq 2$

Perturbations due to gravity generate non-zero ξ_p

-> $\xi_3 = S_3\, \xi_2^2$ with $S_3(m) = 34/7$ (time invariant)

Simplest form of random fields, just characterized by their second moment,

$$\xi(r) = \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle$$

The physical interpretation of this two-point function has to do with probability of finding pairs of objects at some distance from each other

$$\mathrm{d}P_{12} = n^2[1+\xi(x_{12})]\mathrm{d}V_1\mathrm{d}V_2$$

Gaussian Fields are easiest to describe in Fourier space,

$$\delta(\mathbf{x}) = \int \mathrm{d}^3\mathbf{k}\,\delta(\mathbf{k})\,\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{x})$$

$$\delta(\mathbf{k}) = \delta^*(-\mathbf{k})$$

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}')\rangle = \delta_D(\mathbf{k}+\mathbf{k}')\,P(k)$$

Translation Invariance Rotational Invariance where the power spectrum P(k) is the Fourier transform of the 2-pt function,

$$\xi(r) = \int \mathrm{d}^3\mathbf{k}\, P(k) \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})$$

In a Gaussian field Fourier modes are uncorrelated, by this we mean

$$\begin{aligned}\langle \delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_{2p+1})\rangle &= 0\\ \langle \delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_{2p})\rangle &= \sum_{\text{all pair associations}} \prod_{p \text{ pairs (i,j)}} \langle \delta(\mathbf{k}_i)\delta(\mathbf{k}_j)\rangle\end{aligned}$$

remember that $\langle \delta(\mathrm{k})\delta(\mathrm{k}')\rangle = \delta_D(\mathrm{k}+\mathrm{k}')\, P(k)$

These properties are usually summarized by saying that connected moments of order larger than 2 are zero,

$$\langle \delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_N)\rangle_c = 0, \quad N > 2$$

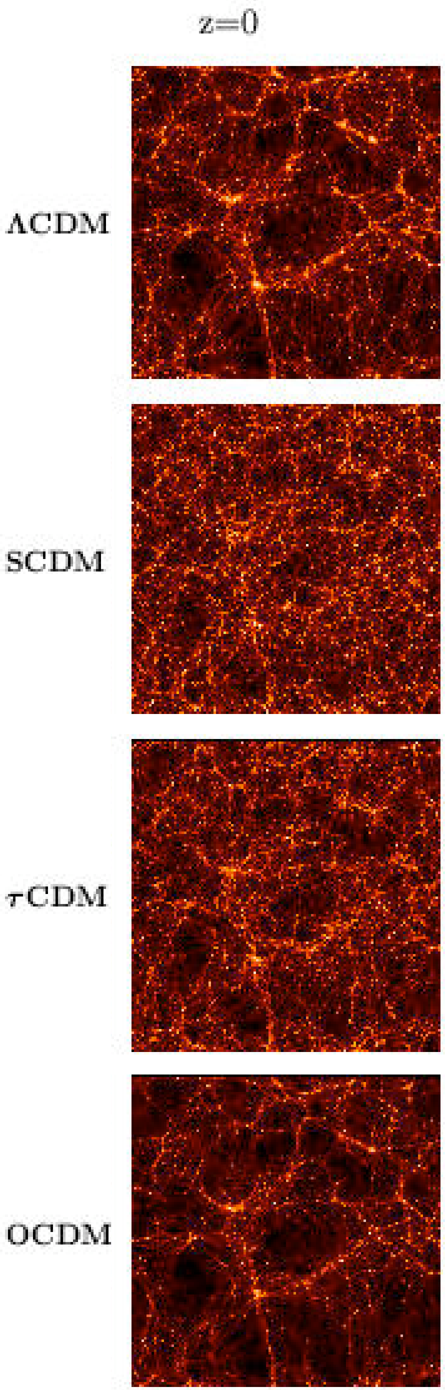
Thus, to generate a Gaussian field, just draw 2 random numbers per mode...

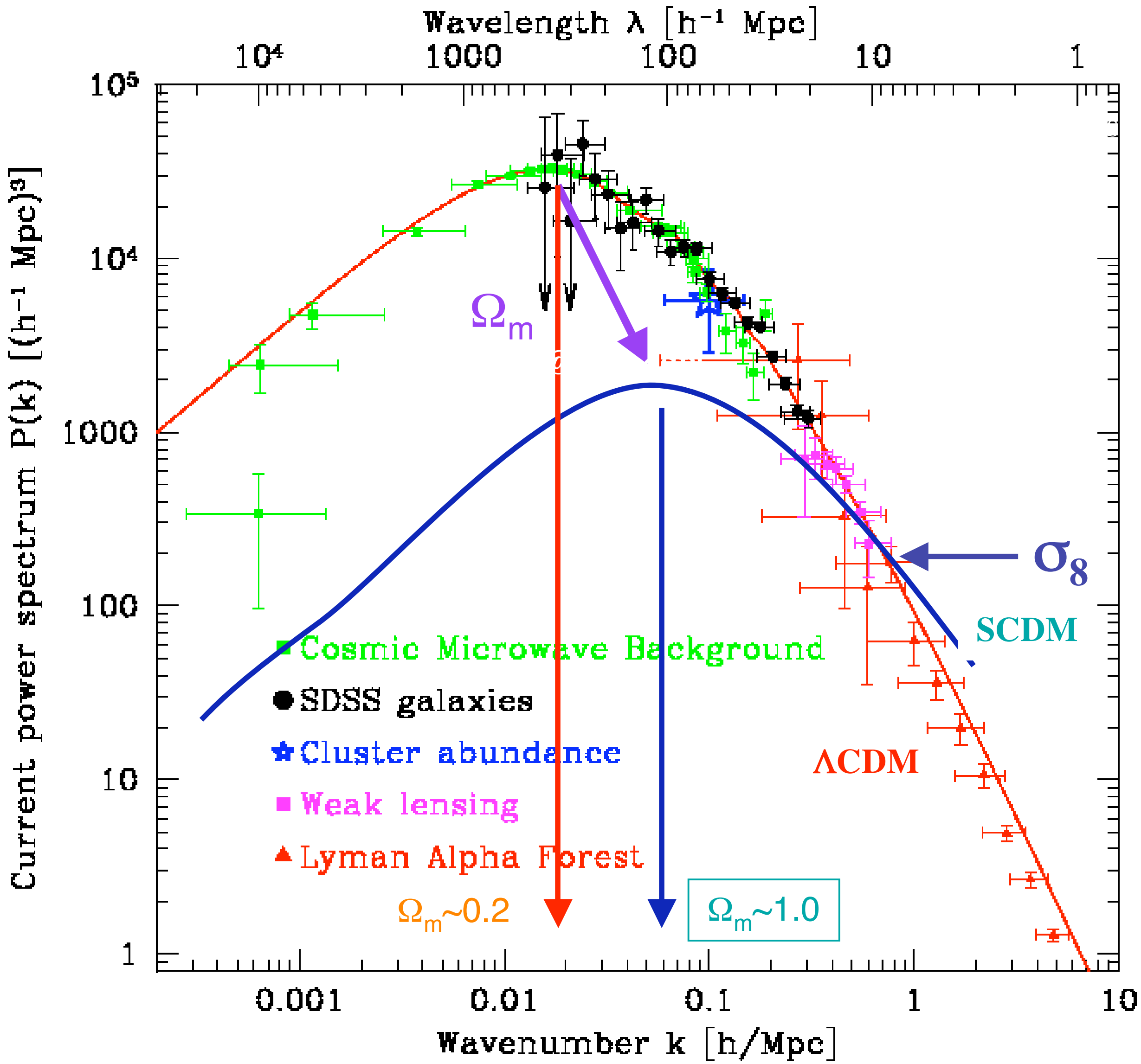
Local spectral index $P(k) \sim k^n$ (initial spectrum + transfer function)

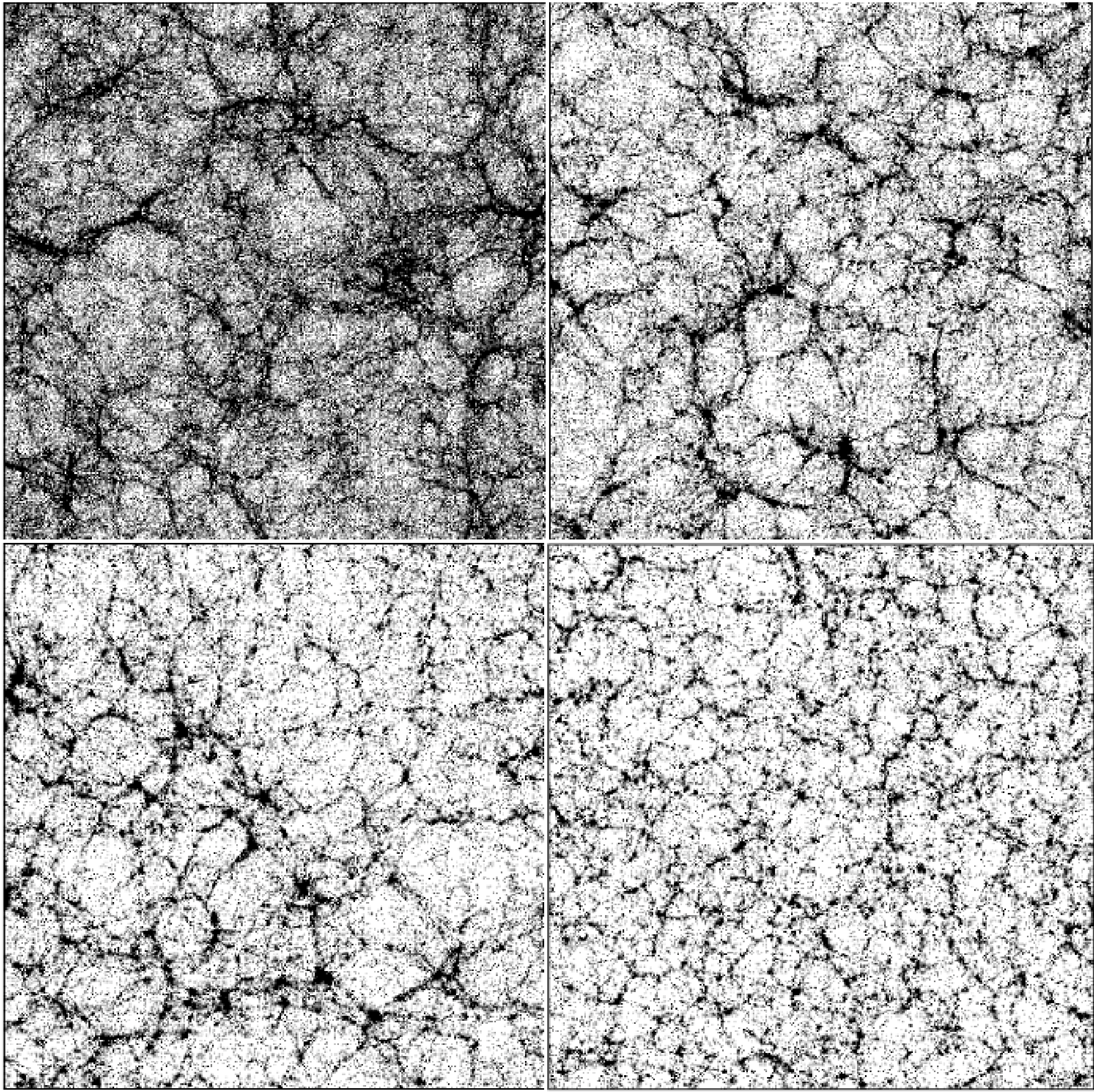
$$\xi_2[r] = \int dk\, P(k)\, k^2\, W(k)\, dk \sim r^{-(n+3)}$$

$n \sim -2 \Rightarrow \xi_2[r] \sim r^{-1}$ (1D fractal) equal power on all scales ($\Omega_m \sim 0.2$)

$n \sim -1 \Rightarrow \xi_2[r] \sim r^{-2}$ (2D fractal) less power on large scales ($\Omega_m \sim 1.0$)





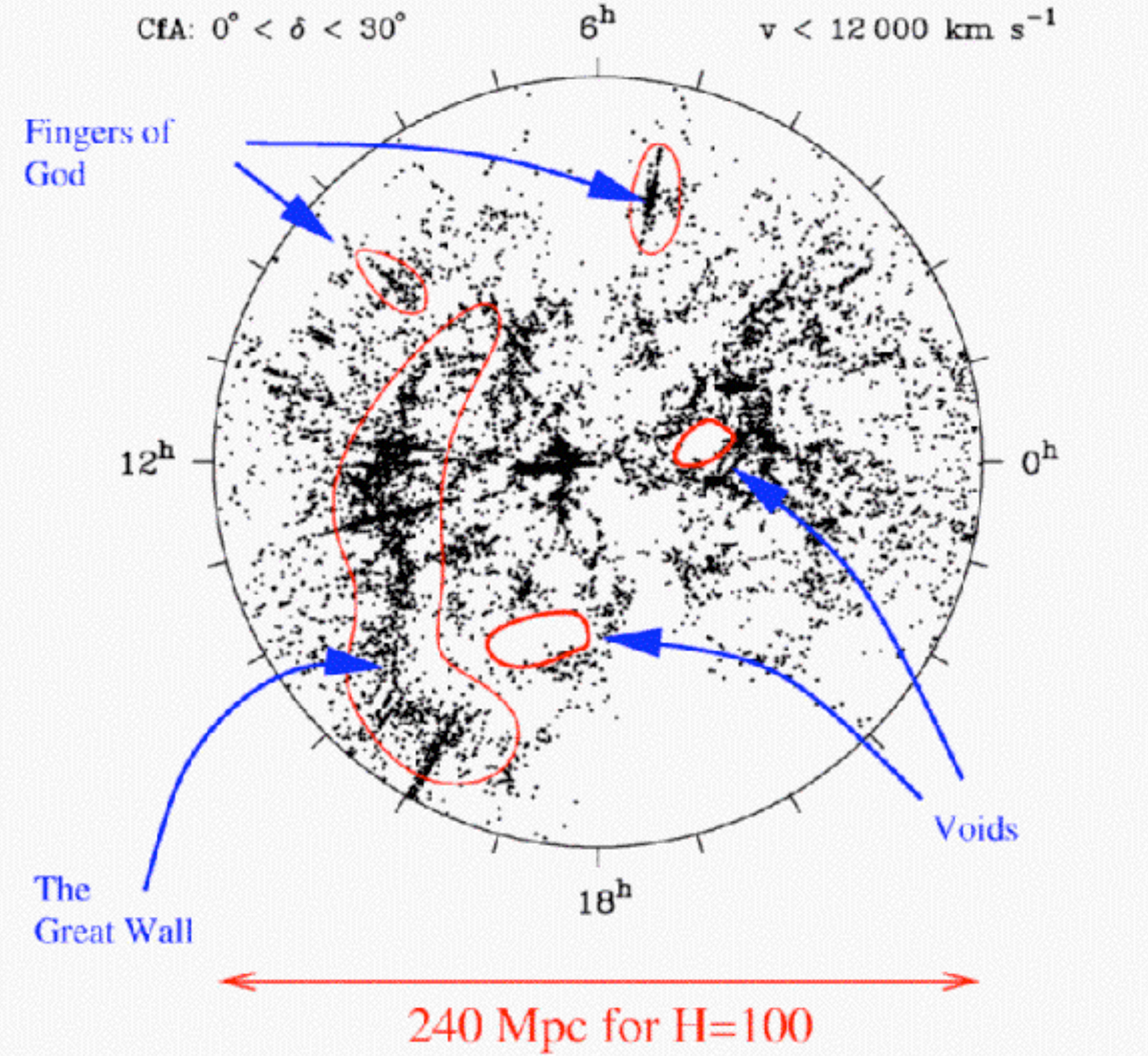


$n = -2$

$n = -1.5$

$n = -1$

$n = 0$

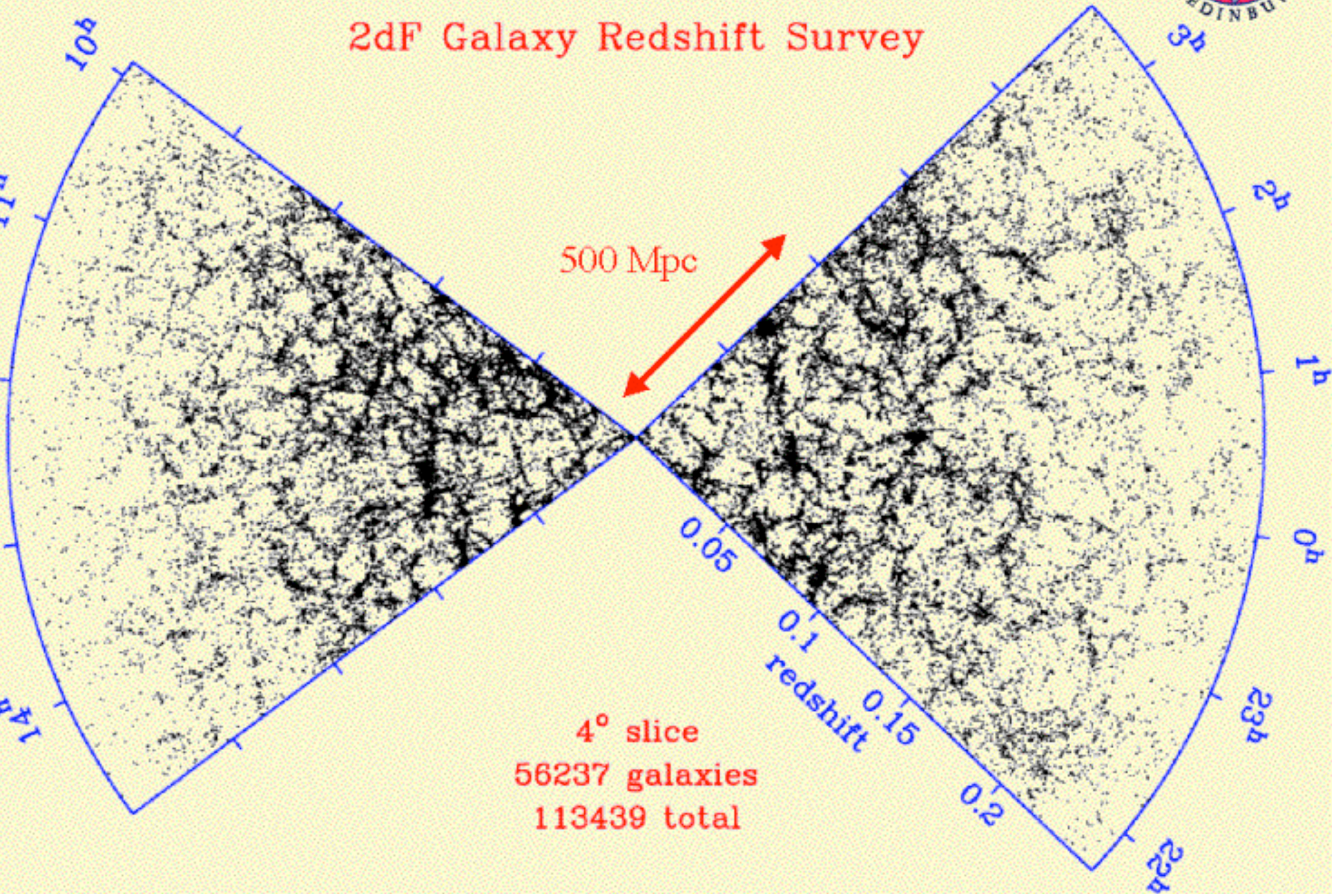


Redshift surveys (mid-1980s)

Inverting v = cz = Hd gives an approximate distance.

Applied to galaxies on a strip on the sky, gives a 'slice of the universe'

The state of the art in galaxy clustering



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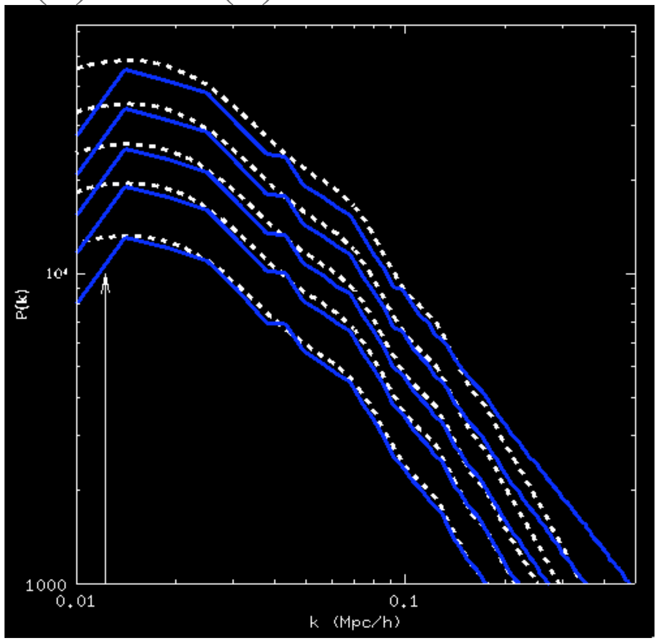
Or power spectrum $P(k) = <\delta^2(k)> \Rightarrow \xi_2 = \int dk\, P(k)\, k^2\, W(k)\, dk$



IC problem: Linear Theory $\delta = a\, \delta_0$

$$\xi_2 = <\delta^2> = D^2 <\delta_0^2>$$

Normalization $\sigma_8^{\;2} \equiv <\delta^2(R=8)>$

To find D(z) -> Compare rms at two times
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Initial Gaussian distribution of density fluctuations:

$$\xi_p(V) = <\delta^P> = 0 \quad \text{for all} \quad p \neq 2$$

Perturbations due to gravity generate non-Gaussian statistics ξ_p

-> $\xi_3 = S_3\, \xi_2^{\,2}$ with $S_3(m) = 34/7$ (time & Cosmo invariant)

Predictions of Inflation

- Flat universe
- scale invariance IC: $n \sim 1$

+ CDM transfer funcion: $P(k) = k^n \; T(k)$

=> Gaussian IC

$\Delta\phi$ is time (and scale!) invariant

even when density fluctuations grow!

Perturbation theory:

$$\rho = \rho_b\,(1 + \delta) \quad \Rightarrow \quad \Delta\rho = (\rho - \rho_b) = \rho_b\,\delta$$

$$\rho_b = M / V \quad \Rightarrow \quad \Delta M / M = \delta$$

With : $\delta'' + H\,\delta' - 3/2\,\Omega_m H^2\,\delta = 0$ in EdS linear theory: $\delta = a\,\delta_0$

Gravitation potential:

$$\Phi = -\,G M / R \quad \Rightarrow \Delta\Phi = G\,\Delta M / R = GM/R\ \ \delta$$

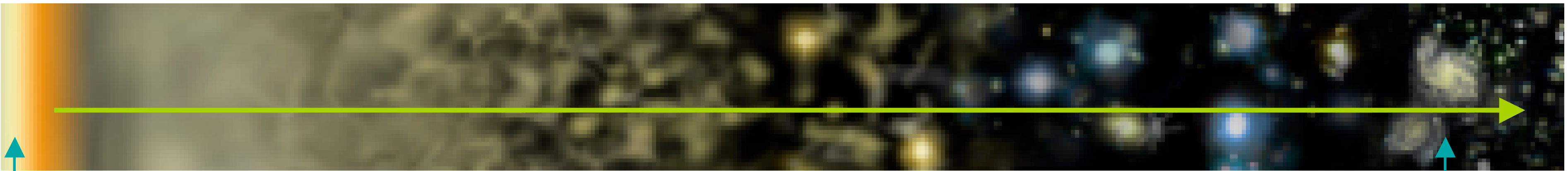
in EdS linear theory: $\delta = a\,\delta_0 \Rightarrow \Delta\Phi = GM\,(\delta / R) = GM\,(\delta_0 / R_0)$!!

PRIMARY CMB ANISOTROPIES

Sachs-Wolfe (ApJ, 1967)

$$\Delta T/T(n) = [\Phi\ (n)\ ]_i^f$$

Temp. F. = diff in N.Potential (SW)



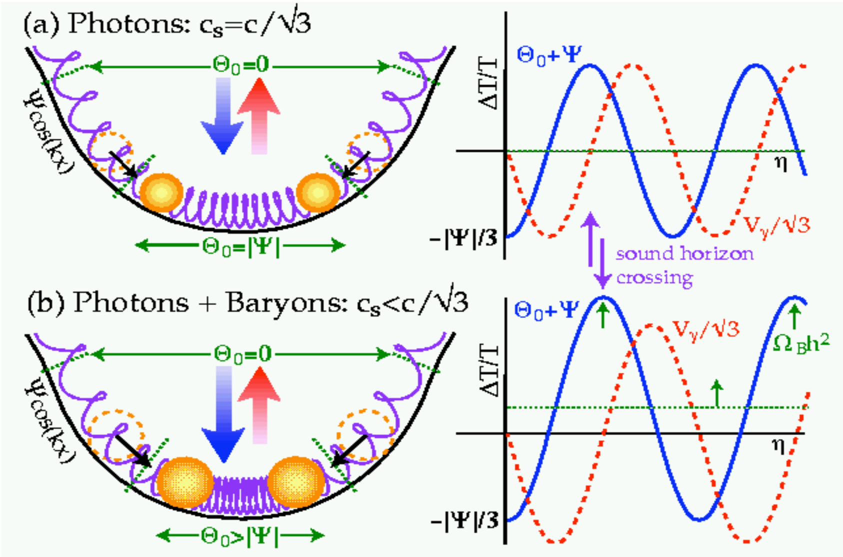
Φ_i Φ_f

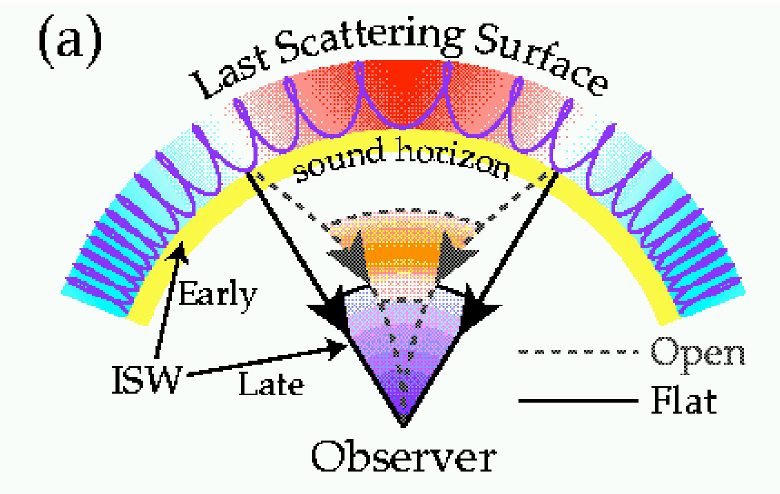
$$\left.\begin{array}{l}\Delta T/T=(SW)= \Delta\Phi\ /c^2 \\ \Delta\Phi\ = GM\ (\delta/\ R)\ /c^2\end{array}\right\}\ \Delta T/T= G\ \rho_m\ 4/3\ \pi\ (R/c)^2\ \delta$$

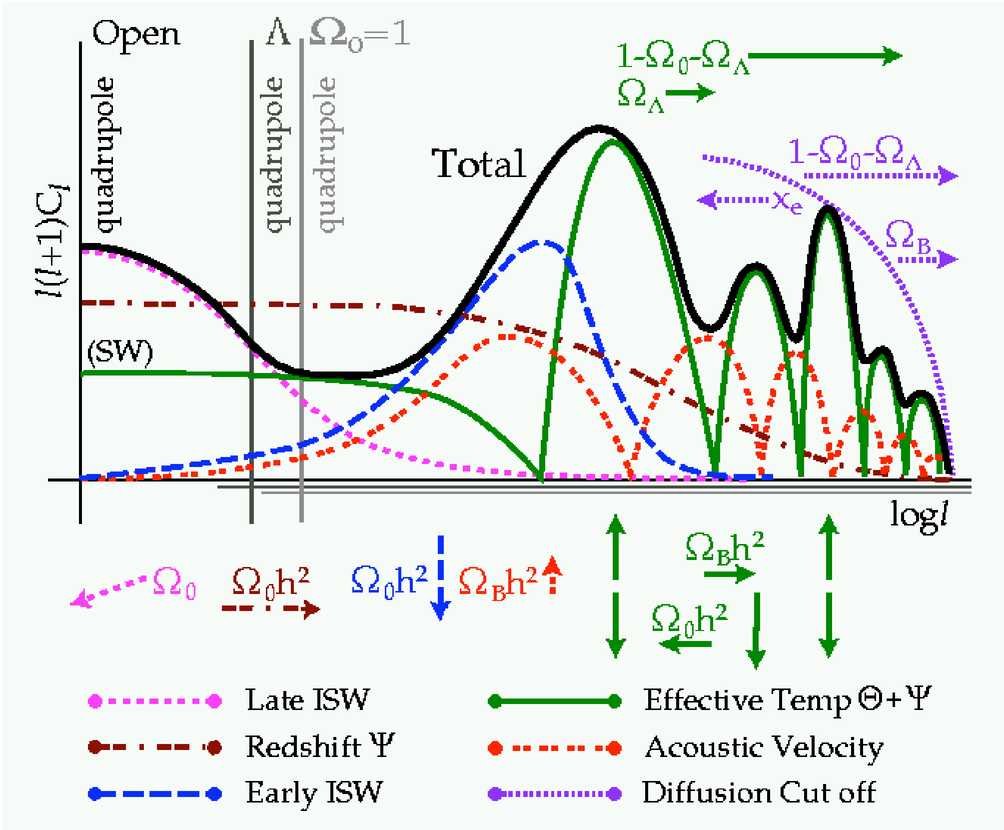
$$\boxed{\Delta T/T = \Omega_m\ /2\ \ (H_0R/c)^2\ \delta\ \ \sim\ \ \Omega_m/2\ \ (R/3000Mpc)^2\ \delta}$$

$$<\Delta T/T>_{rms} \sim 10^{-5}\ \sigma_8 \quad \textit{for} \quad (R\sim 8\ Mpc\ ,\ <\delta>\sim 1)$$

$$\frac{d^2\delta_k}{d\tau^2} + \mathcal{H}\,\frac{d\delta_k}{d\tau} - \left(\frac{3}{2}\;\mathcal{H}^2\,\Omega_m - k^2\;v_s^2\right)\;\delta_k = 0$$

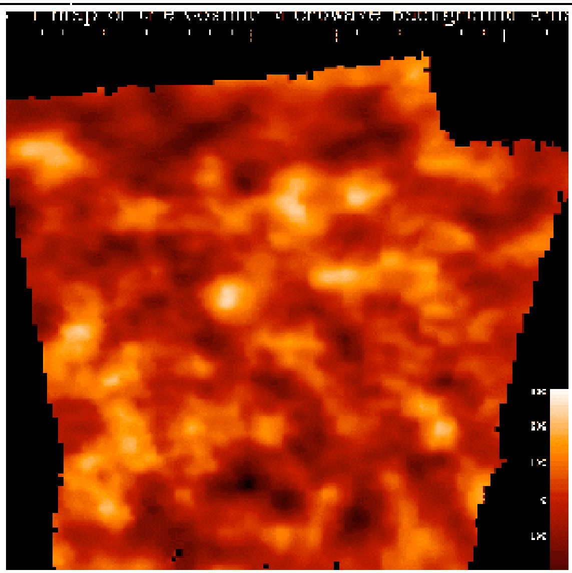


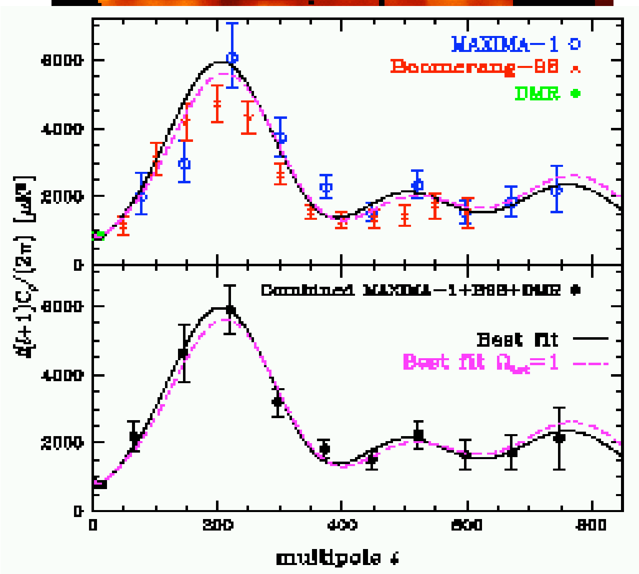


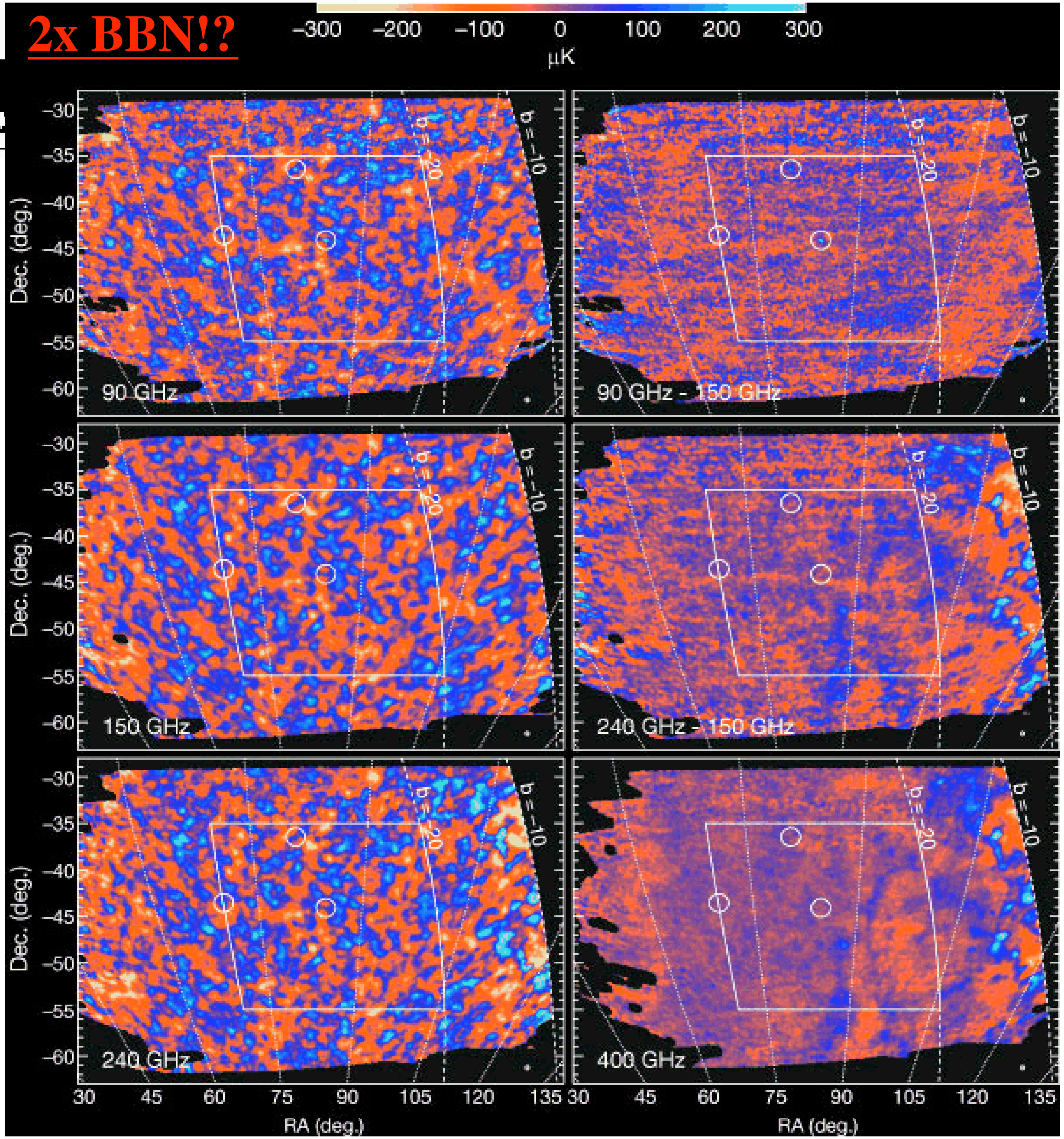


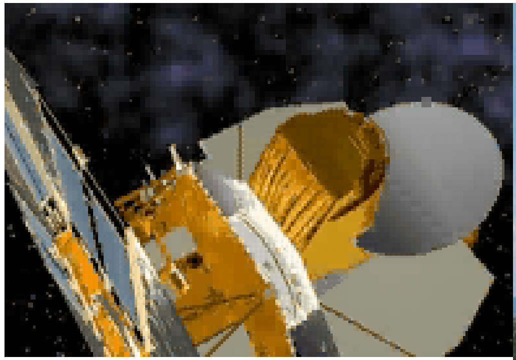
A flat Universe from high-resolution maps of the cosmic microwave background radiation

MAXIMA-1

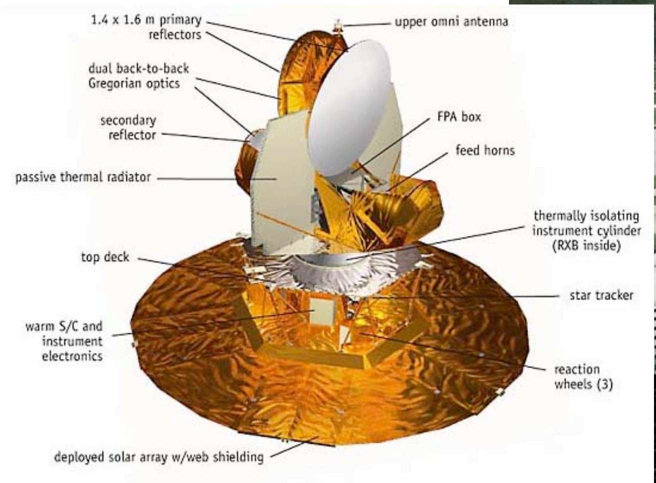


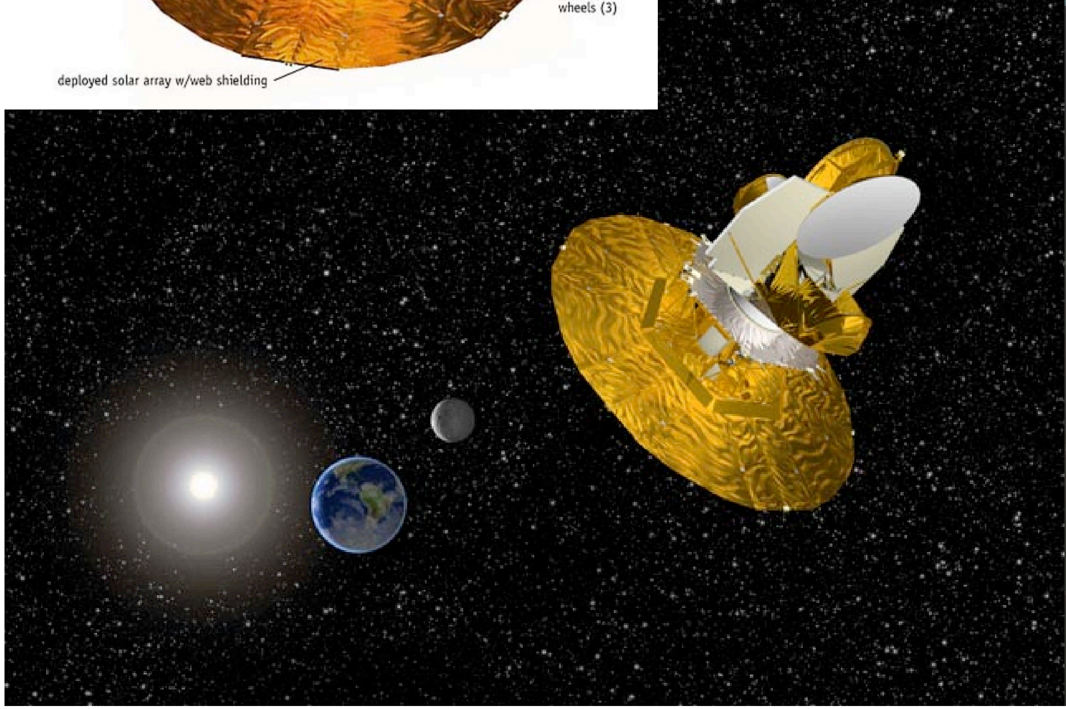


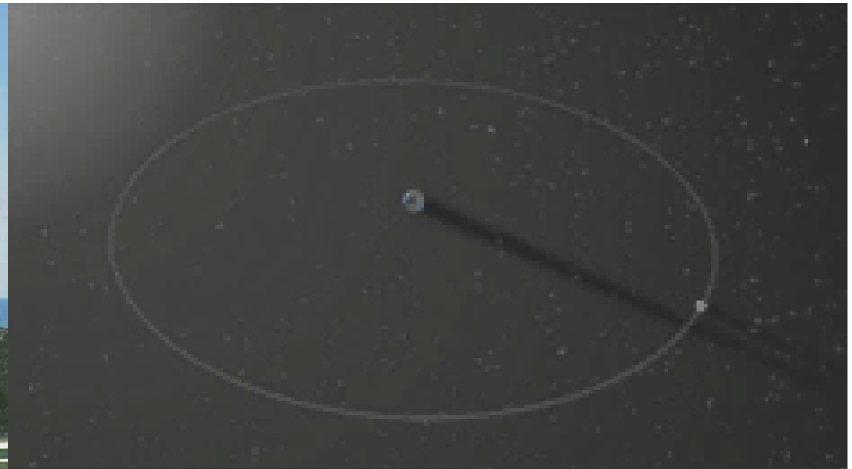


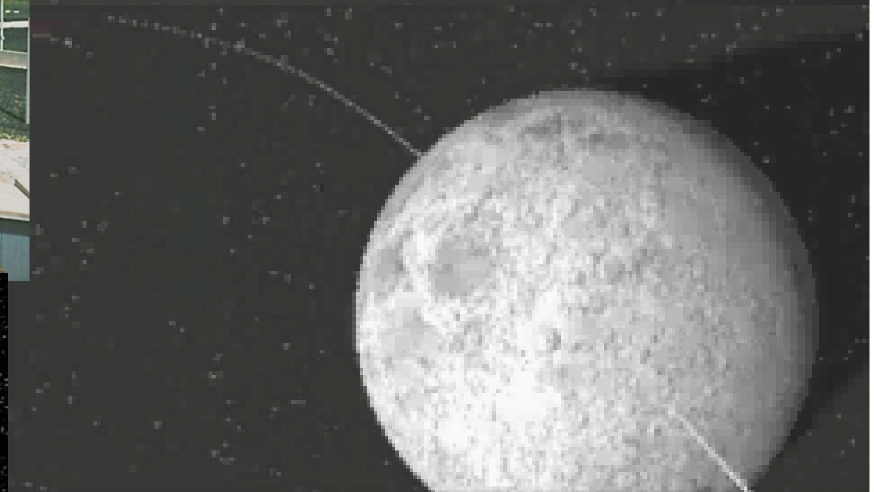


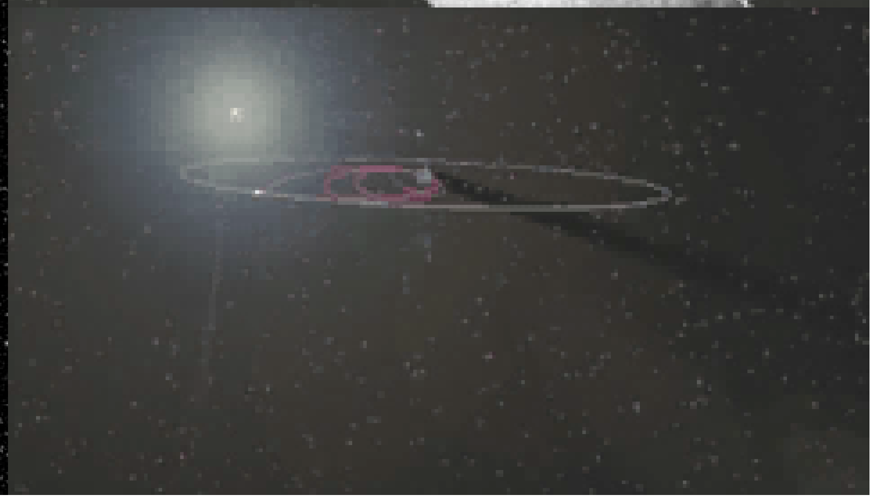

USAF
DELTA
MAP

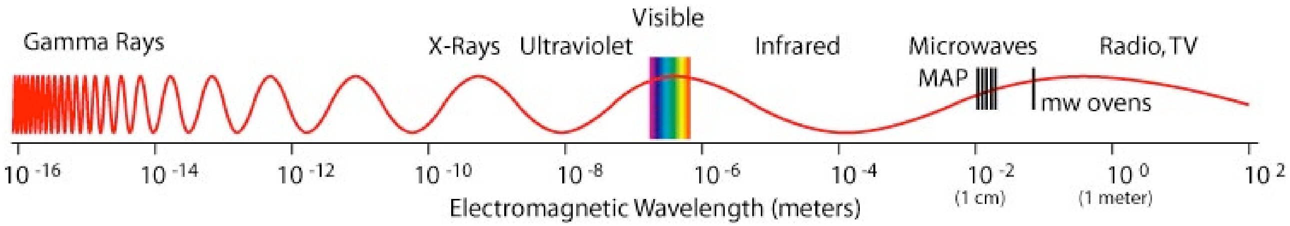

1.4 x 1.6 m primary reflectors
upper omni antenna
dual back-to-back Gregorian optics
secondary reflector
FPA box
feed horns
passive thermal radiator
thermally isolating instrument cylinder (RXB inside)
top deck
star tracker
warm S/C and instrument electronics
reaction wheels (3)
deployed solar array w/web shielding



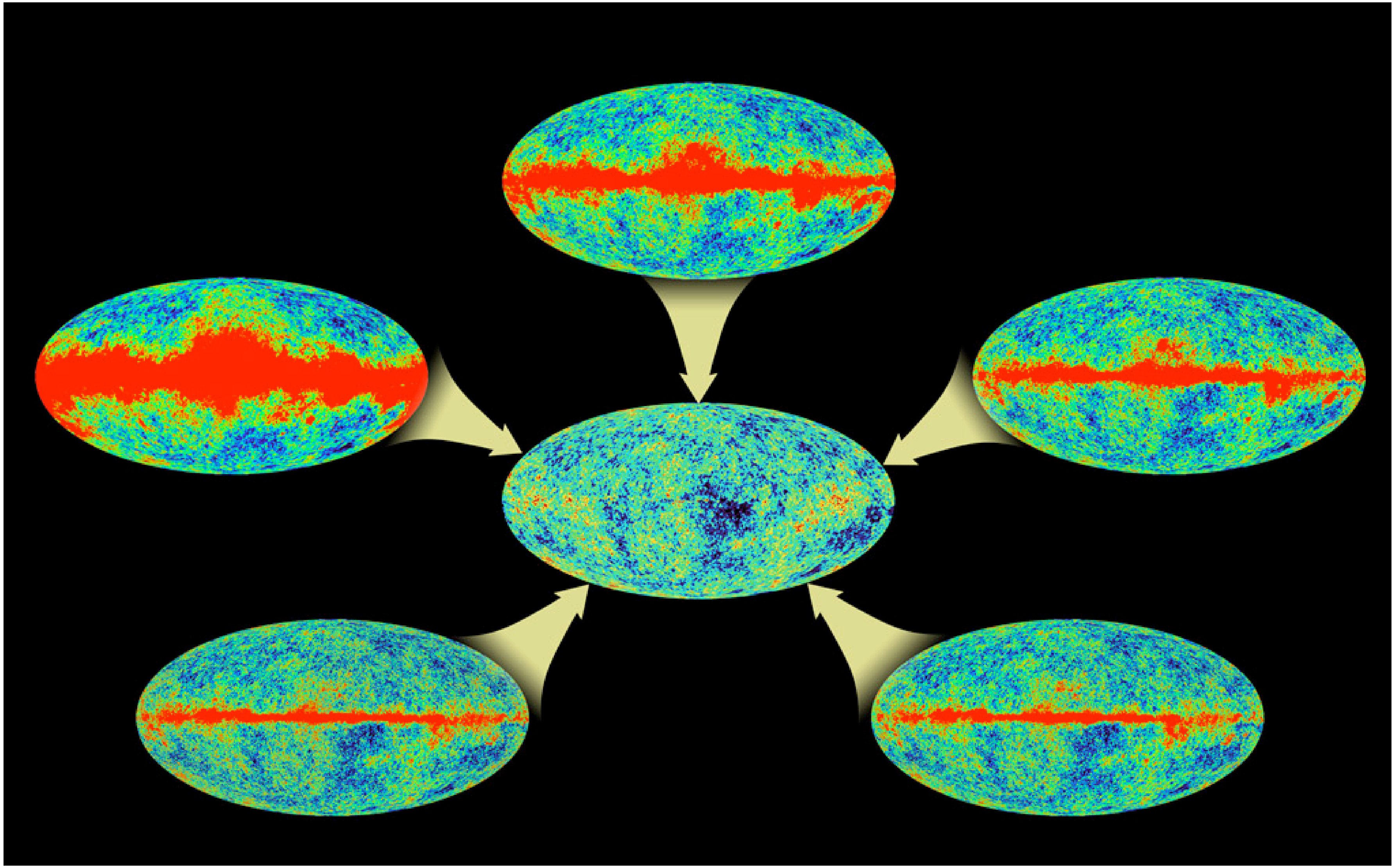




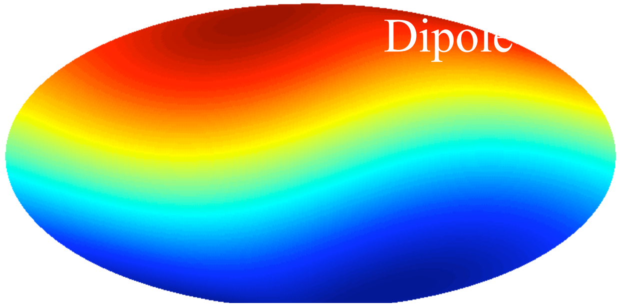




Visible
Gamma Rays
X-Rays
Ultraviolet
Infrared
Microwaves
Radio, TV
MAP
mw ovens
10 -16
10 -14
10 -12
10 -10
10 -8
10 -6
10 -4
10 -2
(1 cm)
10 0
(1 meter)
10 2
Electromagnetic Wavelength (meters)



$$Y_\ell^m(\theta,\phi) = \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}P_\ell^m(\cos\theta)e^{im\phi}.$$

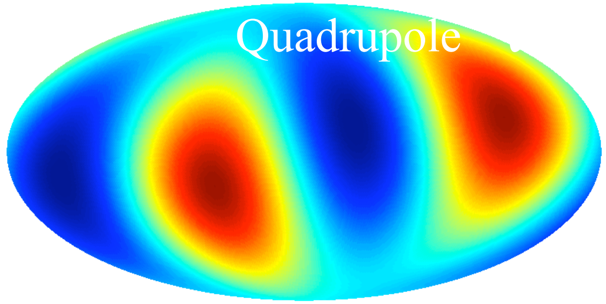


CMB temperature fluctuations can then be expanded as

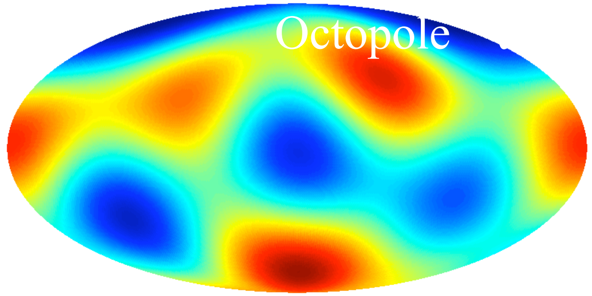
Spherical-Harmonics

$$\Delta T(\hat{q}) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\hat{q}),$$

$$a_{\ell m} = \int d\Omega Y_\ell^m(\hat{q})\Delta T(\hat{q}),$$

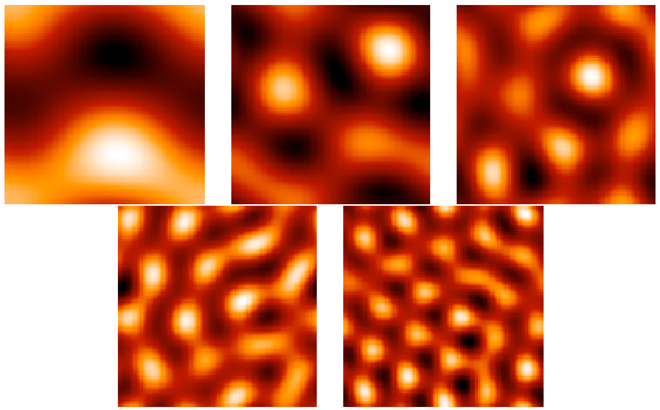


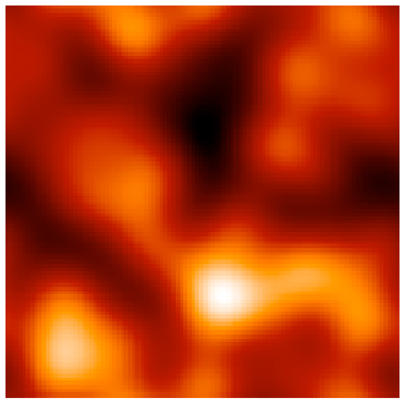
Gaussian fields

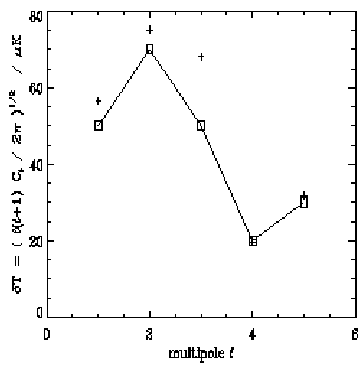


$$\langle a_{\ell_q m_q} a_{\ell_p m_p}\rangle = \delta_{\ell_q \ell_p}\delta_{m_q m_p} C_\ell \quad \rightarrow \quad C_\ell \simeq \sum_l a_{\ell m}^2$$

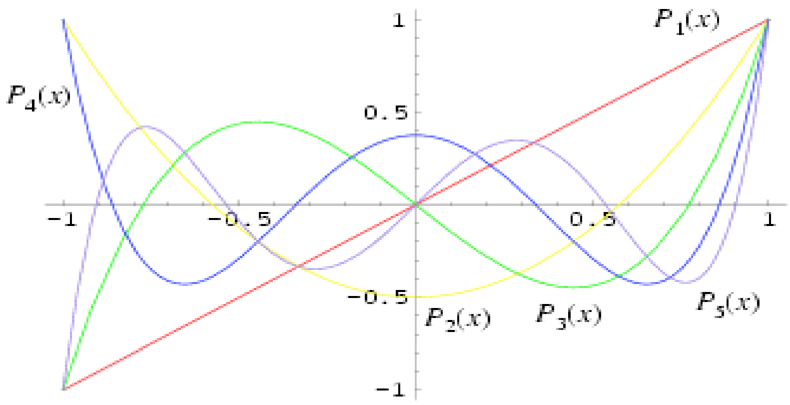
$$w_2(\theta) = \langle \Delta T(\hat{q})\Delta T(\hat{p})\rangle = \sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell} \widehat{C}_\ell Y_{\ell_q m_q}^\dagger(\hat{q}) Y_{\ell_p m_p}(\hat{p}) = \frac{1}{4\pi}\sum_\ell (2\ell+1)\,\widehat{C}_\ell\, P_\ell(\cos\theta)$$

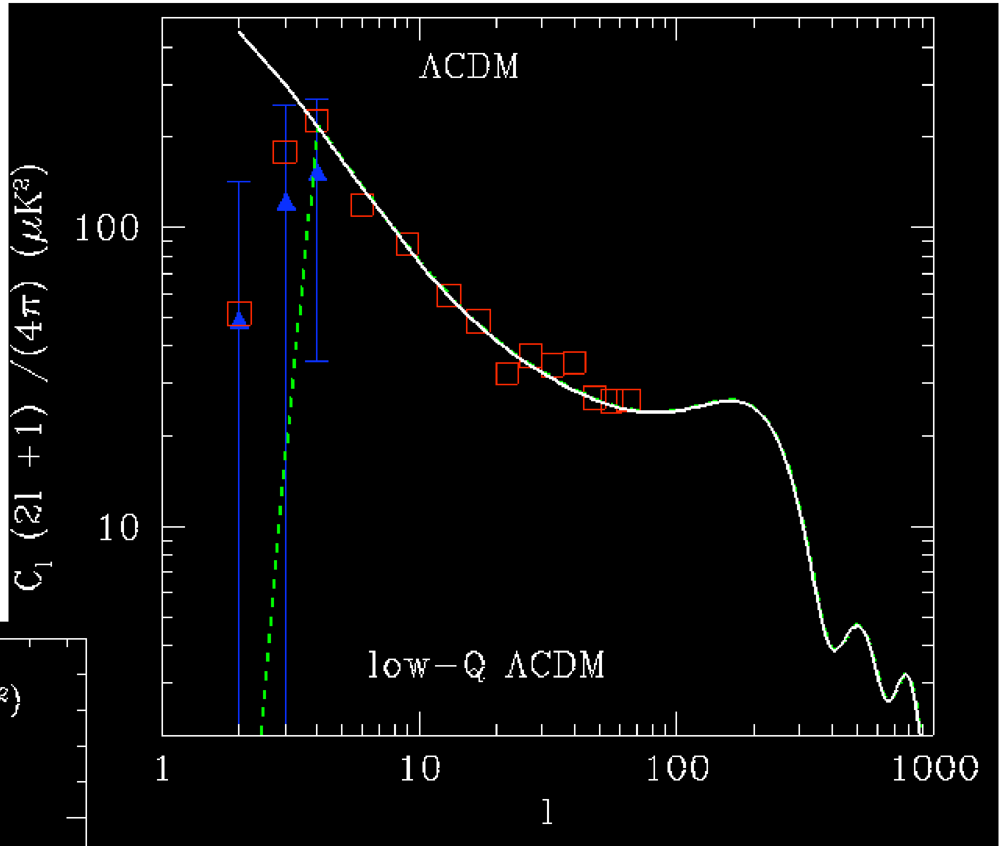


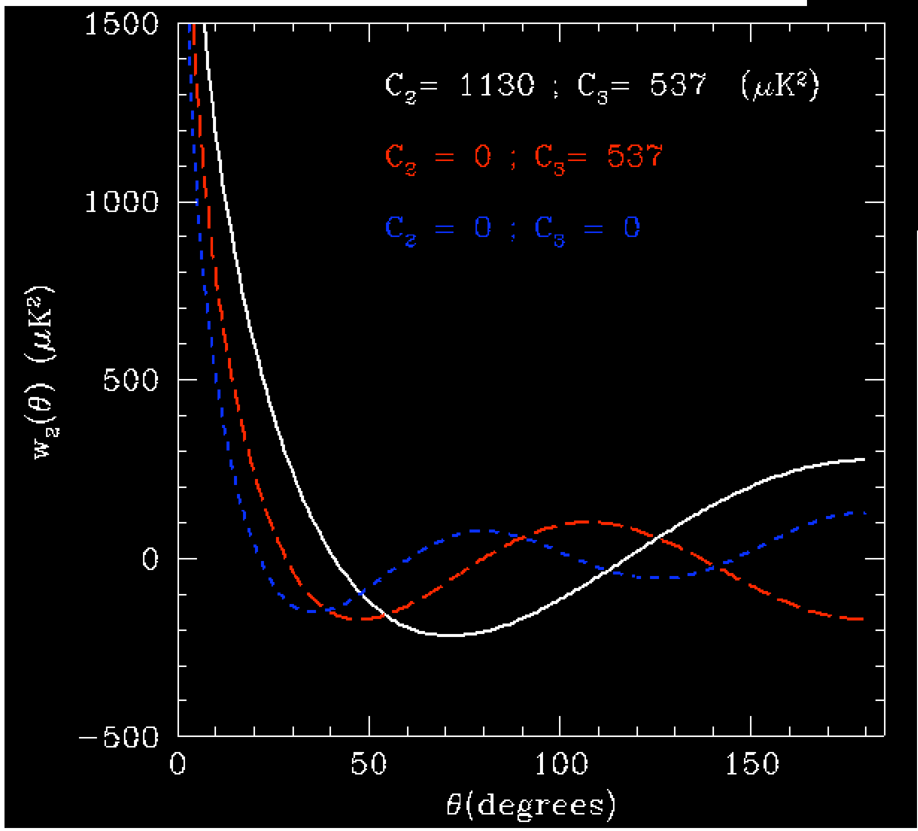


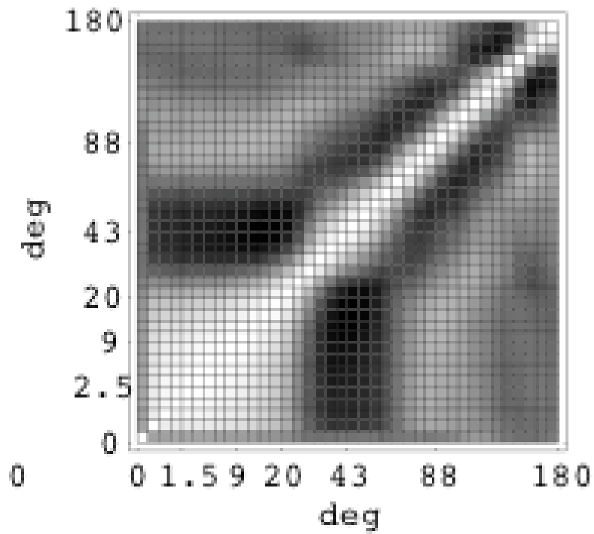


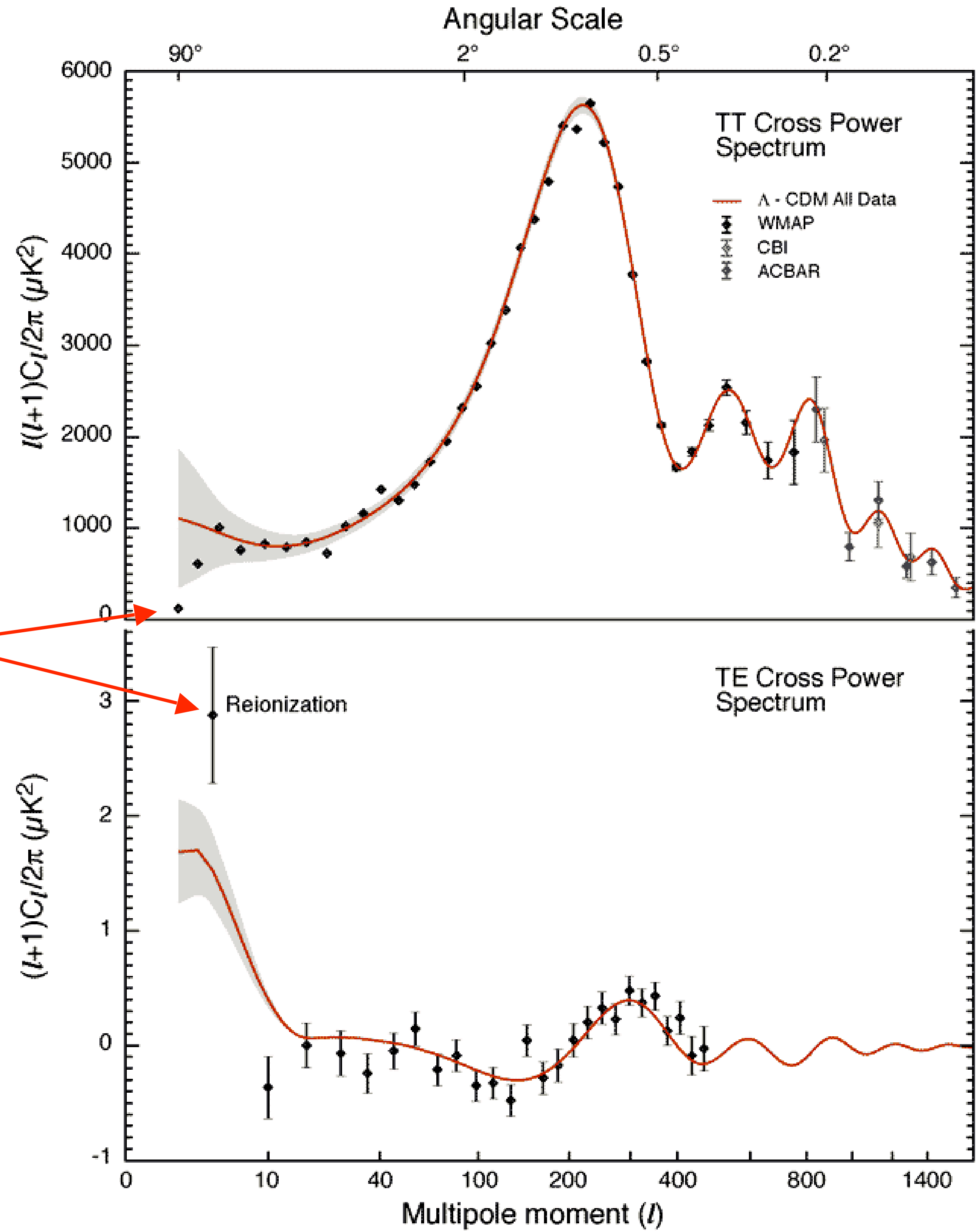
$$w_2(\theta) = \frac{1}{4\pi} \sum_{l=2} (2l+1)\ C_l\ P_l(\cos\theta),$$










Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
$l(l+1)C_l/2\pi\ (\mu K^2)$
Reionization
TE Cross Power Spectrum
3
2
1
0
-1
$(l+1)C_l/2\pi\ (\mu K^2)$
0
10
40
100
200
400
800
1400
Multipole moment (l)

Quadrupole

Non-Gaussian Conclusions

EG, & J.Wagg Phys.Rev.D68p021302 (2003)

- WMAP team (Komatsu astro-ph/0305467) measured bispectrum and find: $-58 < f_{NL} < 134$ (95%)

Where: $\Phi = \Phi_L + f_{NL}\ \Phi_L^{\ 2}$ (curvature)

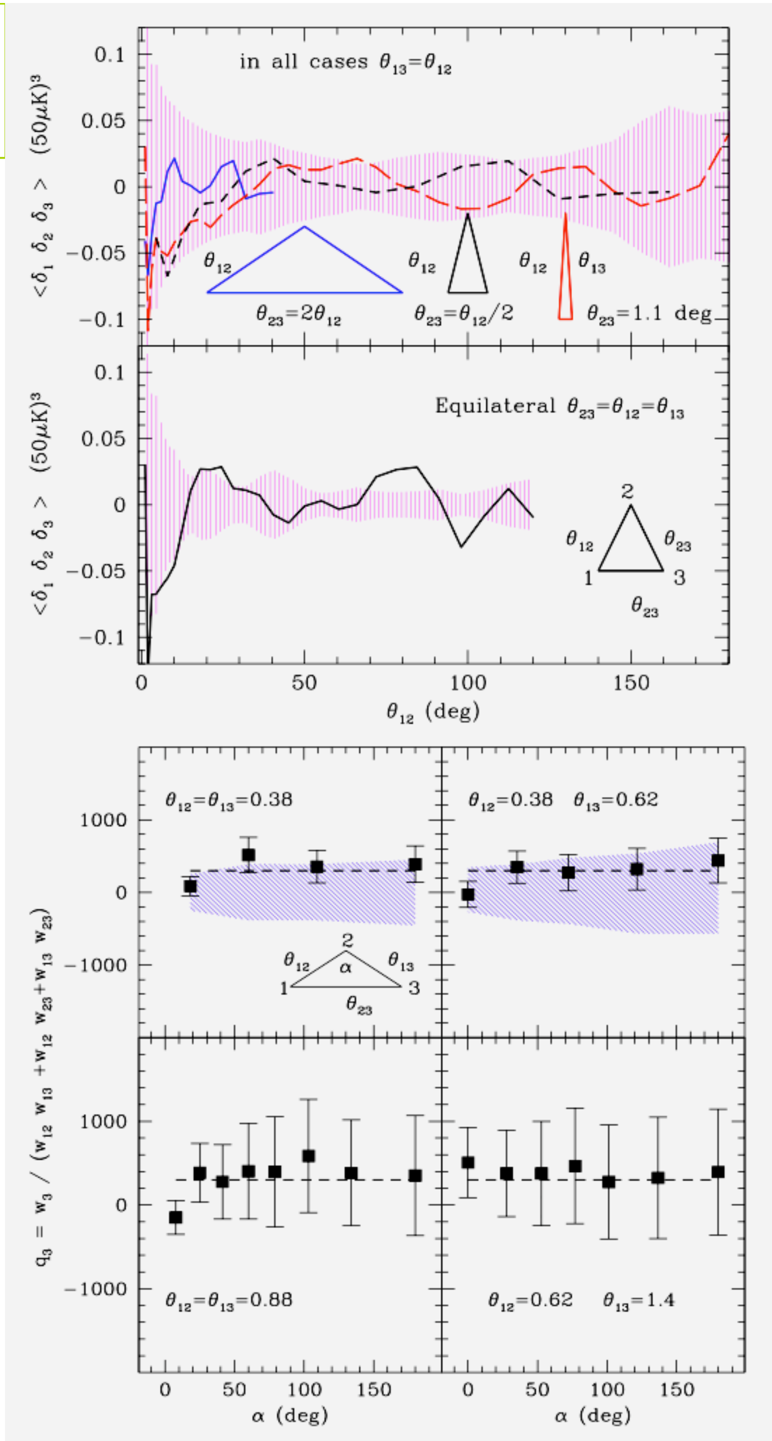
- We find Cosmic variance domination: $\Delta d_3 \sim 1$ where $d_3 \sim w_3/\ w_2^{3/2}$ (dimensionless scaling)

- Hierarchical scaling: $q_3 \sim w_3/\ w_2^{2}$ comes from: $\Delta T = \Delta T_L + f_{NLT}\ \Delta T_L^{\ 2}$ with $f_{NLT} = q_3/2$ (non-linear effects, including lensing, ISW or point-source).

- $q_3 = 19 \pm 141$ @ 0.5-1 deg collapse

- $q_3 = 336 \pm 218$ @ 0.5-1 deg for non-collapsed

- SW: $f_{NL} \sim q_3/6$ Acoustic: $f_{NL} \sim q_3/30$



Non-linear Evolution

- Spherical Collapse model
- Non-linear Perturbation Theory
- N-body simulations

Most of the volume of the universe is filled with small fluctuations that slowly evolve in the linear regime (growth ~1000). Once these fluctuations reach a critical point ($\delta \sim 1$) they start a very rapid non-linear evolution ($\sim 10^{30}$)

Weakly non-linear Perturbation Theory (Spherical average)

$$\delta = \delta_L + \nu_2 \, \delta_L^{\,2} + \ldots$$

$$\delta^3 = \delta_L^{\,3} + 3\,\nu_2 \, \delta_L^{\,4} + \ldots$$

$$< \delta^3 > = < \delta_L^{\,3} > + 3\,\nu_2 < \delta_L^{\,4} > + \ldots$$

$$< \delta^3 > = 3\,\nu_2 < \delta_L^{\,2} >^2 + \ldots$$

Gaussian Initial conditions

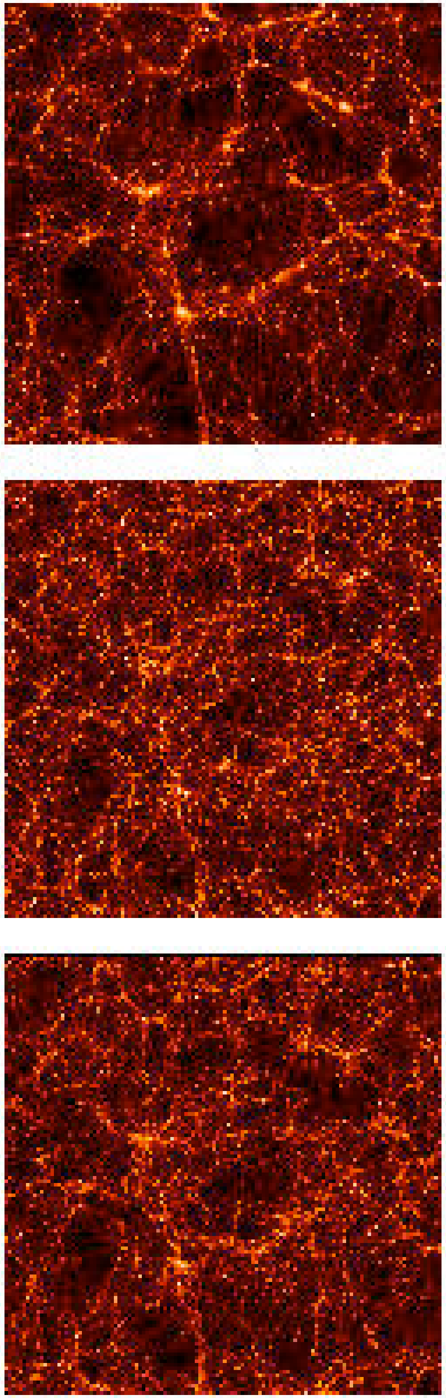
$$< \delta_L^{\,3} > = a^3 <\delta_0^{\,3}> = 0$$

$$< \delta_L^{\,4} > = < \delta_L^{\,2} >^2$$

$$S_3 \equiv < \delta^3 > / < \delta^2 >^2 = 3\,\nu_2 = 34/7$$

gravity?

High order statistics -> vertices of non-linear growth!



Depends on local spectral index $P(k) \sim k^n$ (not on Ω_m)

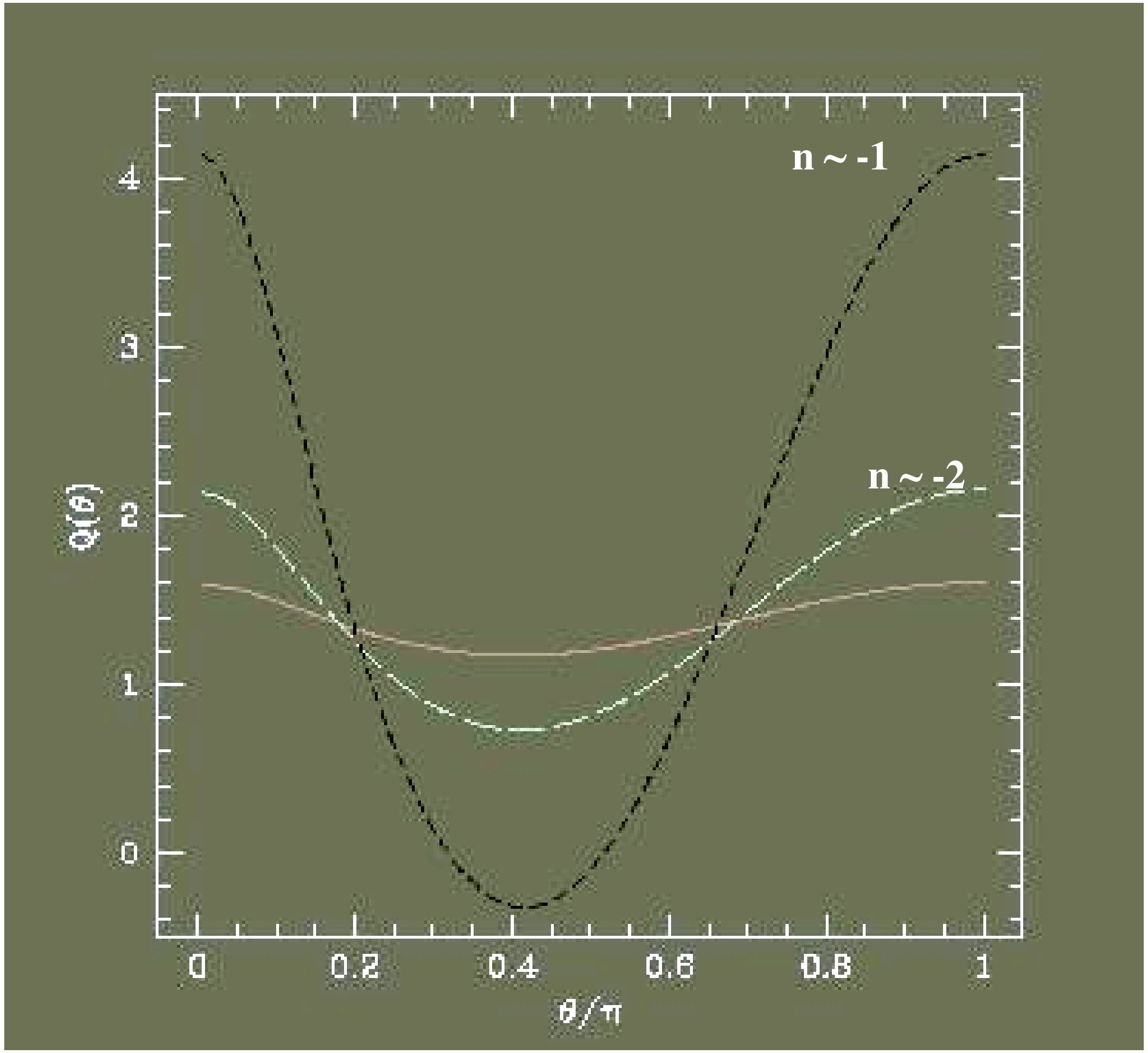
$$\xi_2[r] = \int dk\, P(k)\, k^2\, W(k)\, dk \sim r^{-(n+3)}$$

$n \sim -2 \Rightarrow \xi_2[r] \sim r^{-1}$ (1D fractal) equal power on all scales ($\Omega_m \sim 0.2$)

$n \sim -1 \Rightarrow \xi_2[r] \sim r^{-2}$ (2D fractal) less power on large scales ($\Omega_m \sim 1.0$)

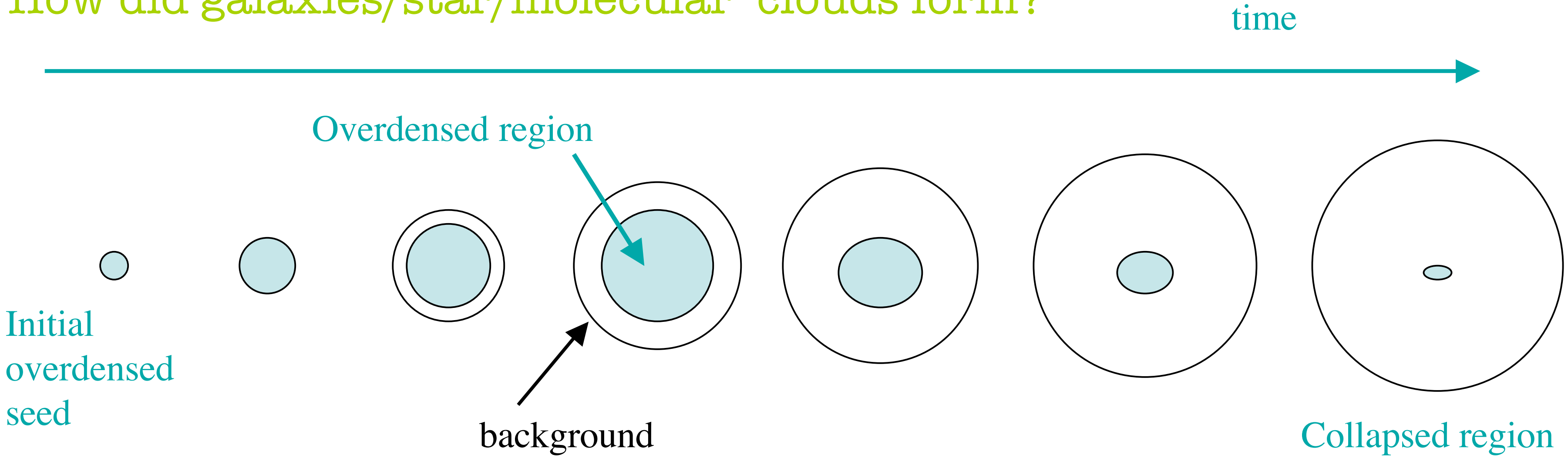
n ~ -2

n ~ -1

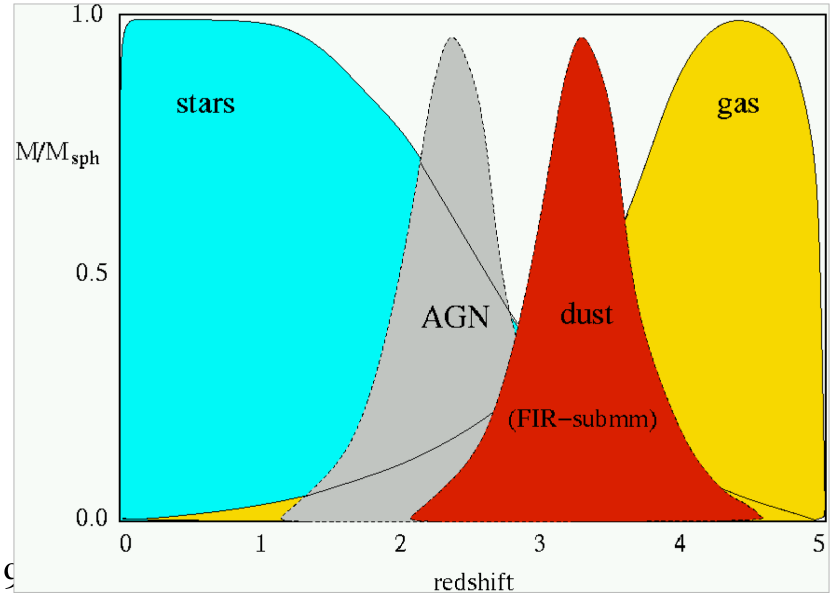


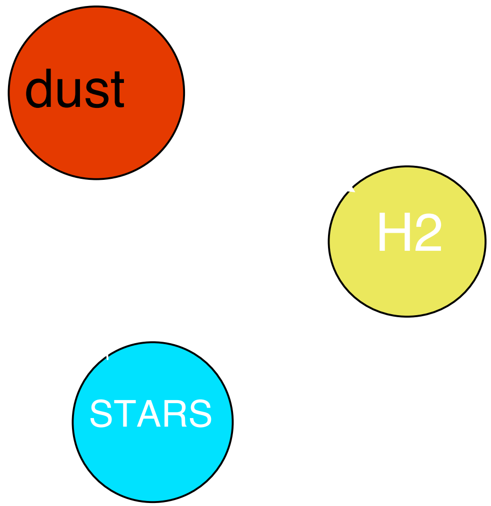
Where does Structure in the Universe come From?

How did galaxies/star/molecular clouds form?



IC + **Gravity** + Chemistry = Star/Galaxy (tracer of mass?)





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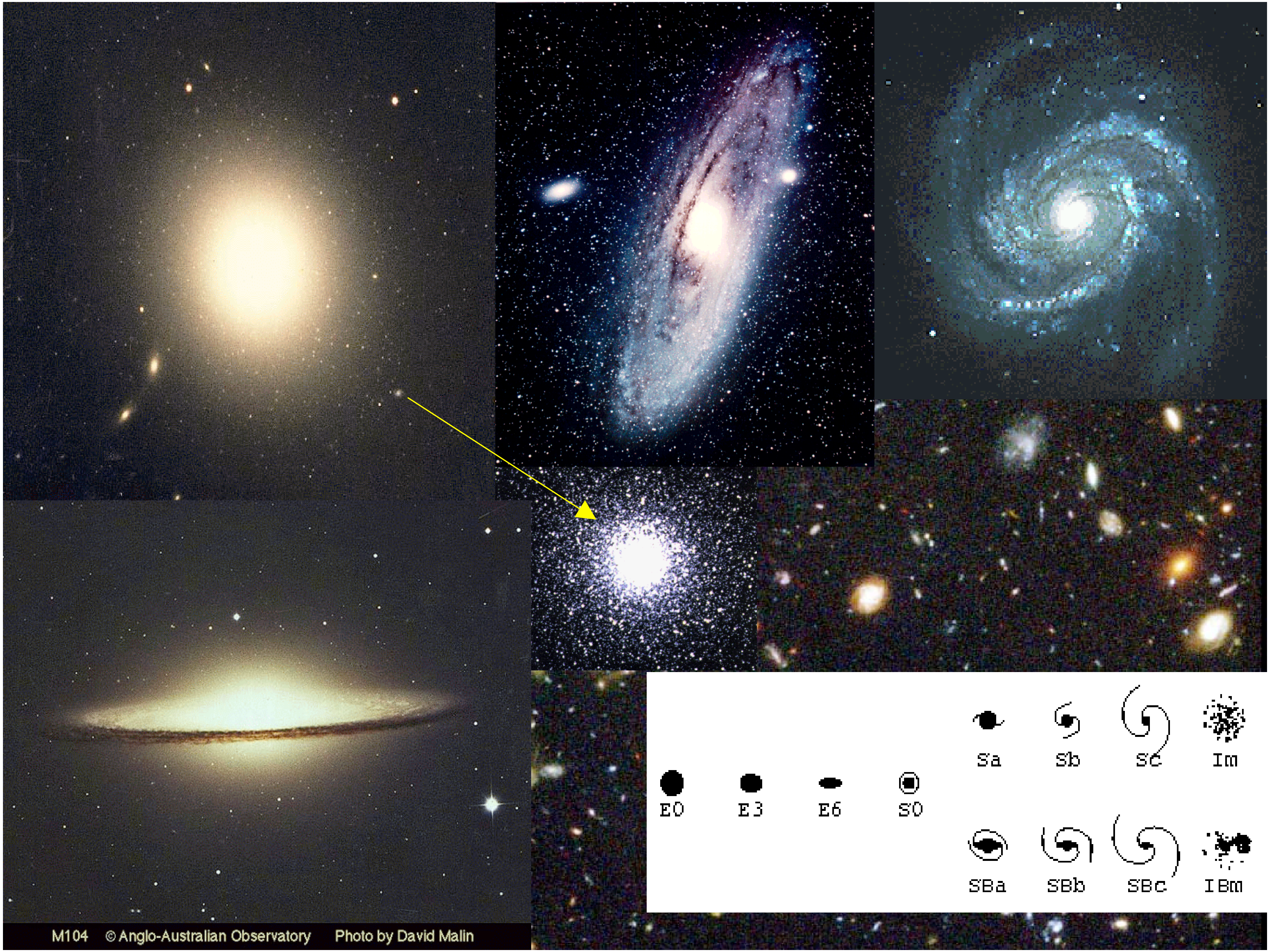
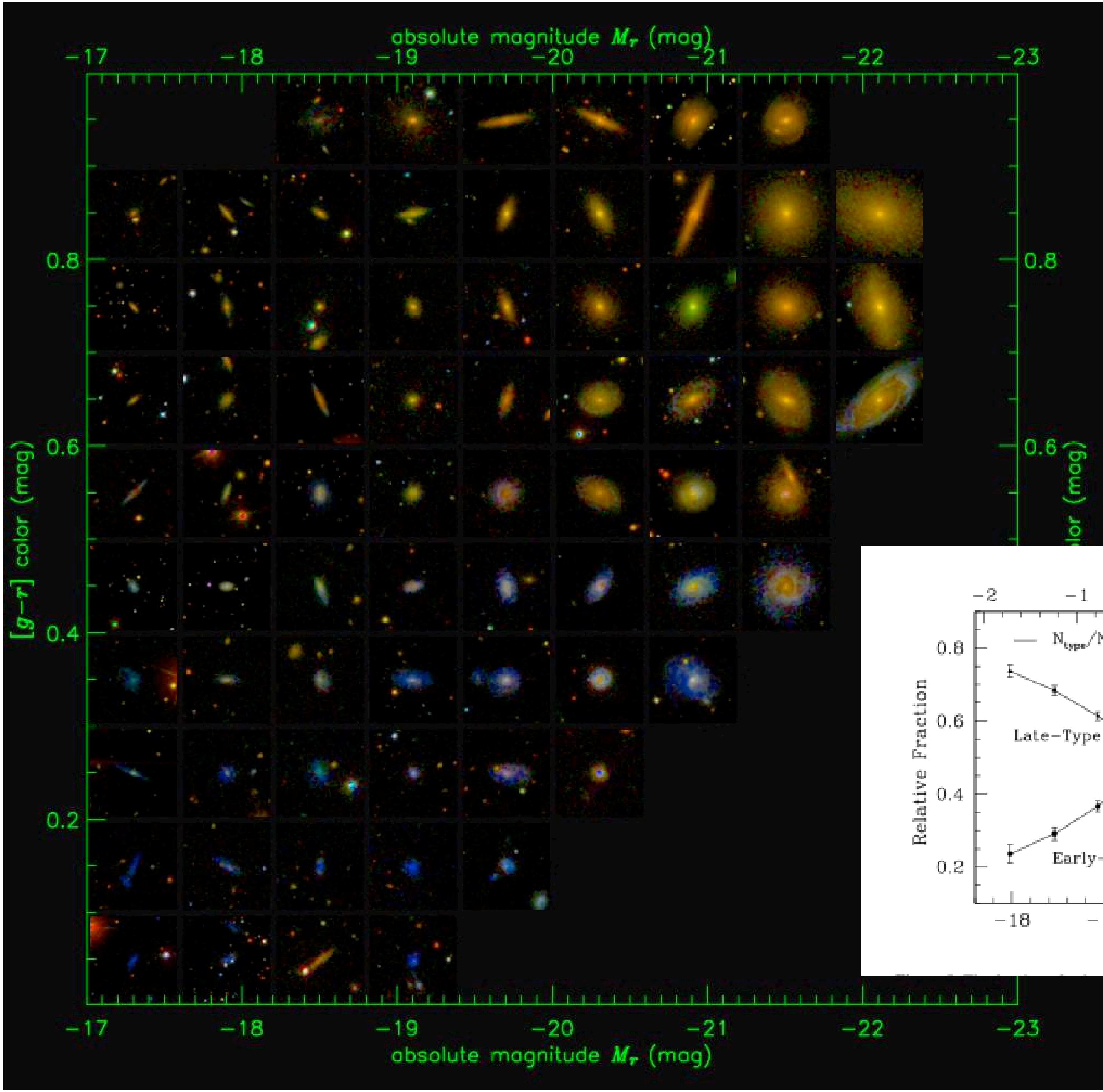
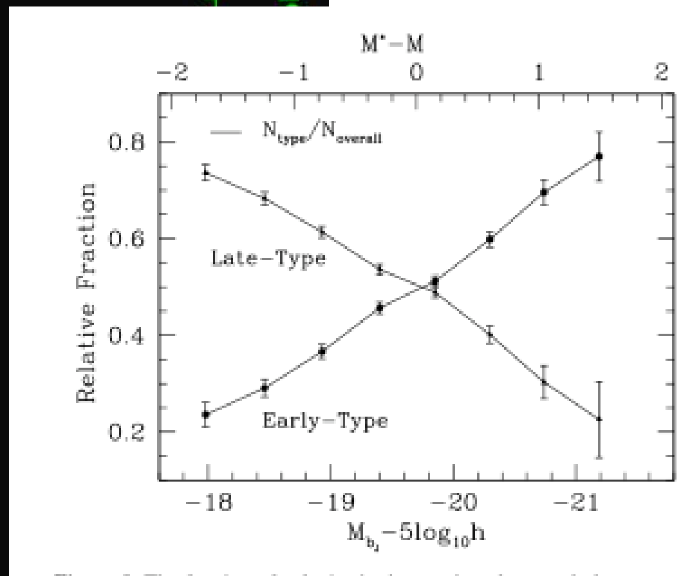
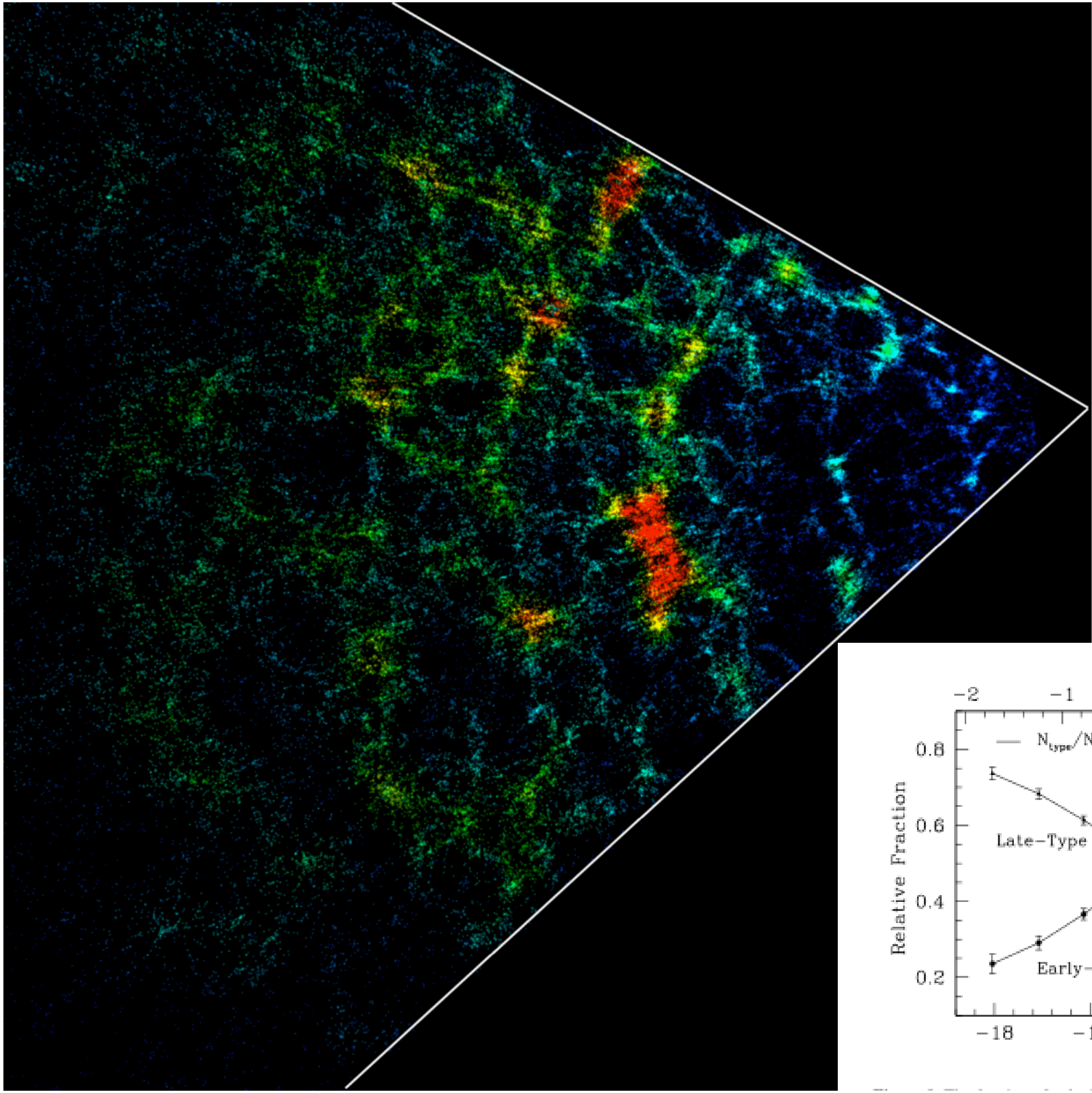
D.Hughes

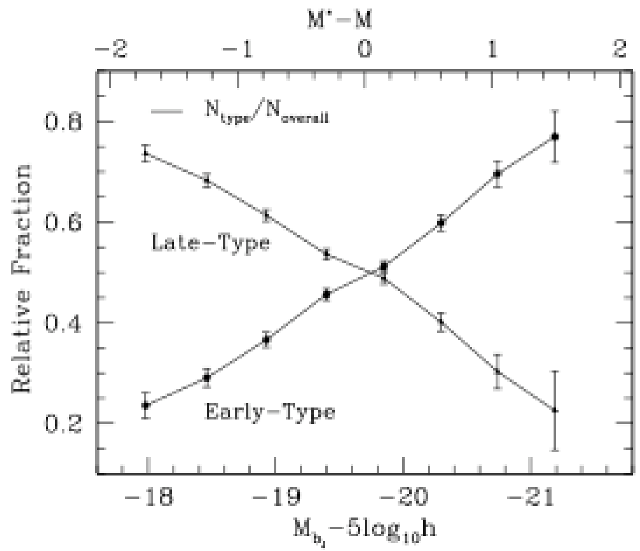

M104 © Anglo-Australian Observatory Photo by David Malin
Sa
Sb
Sc
Im
E0
E3
E6
S0
SBa
SBb
SBc
IBm

Hogg & Blanton









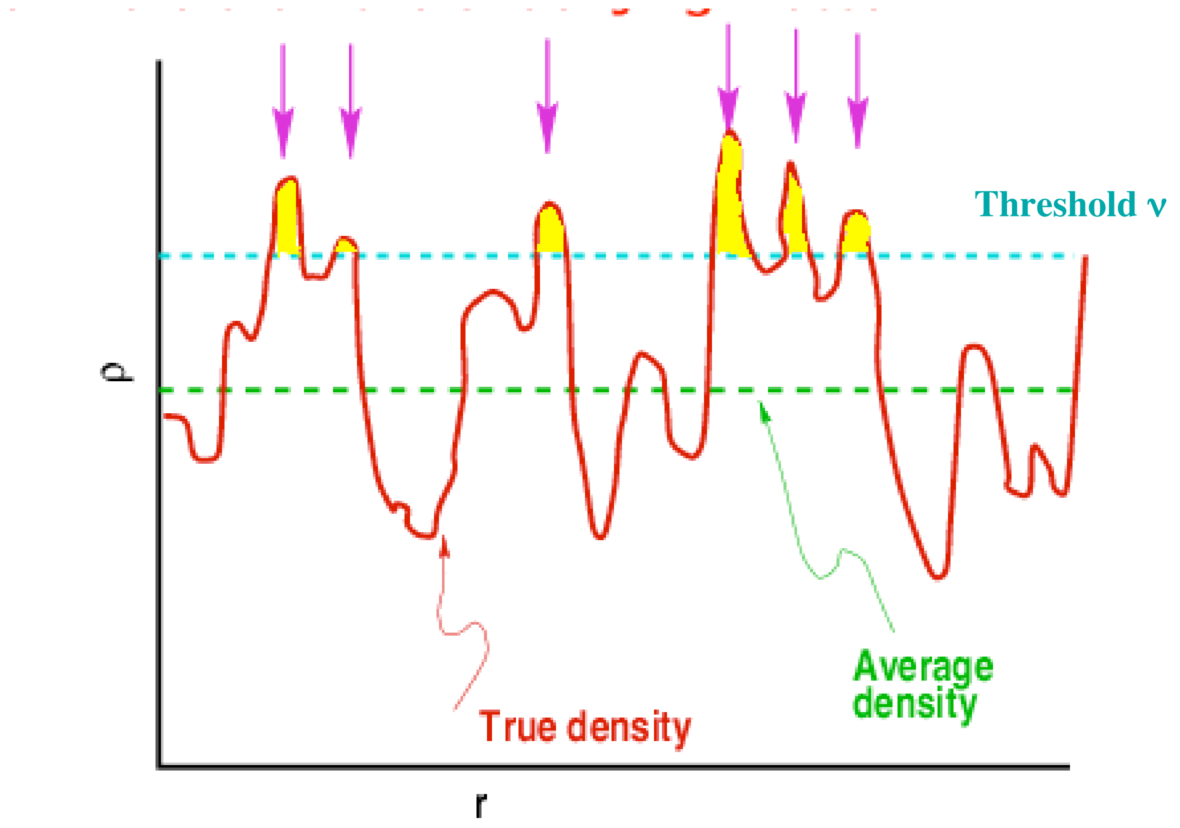
M*−M
−2
−1
0
1
2
Ntype/Noverall
0.8
0.6
0.4
0.2
Relative Fraction
Late−Type
Early−Type
−18
−19
−20
−21
Mbj−5log10h

Bias: lets take a very simple model.

rare peaks in a Gaussian field (Kaiser 1984, BBKS)

Linear bias "b": $\delta\,(\text{peak}) = b\;\;\delta(\text{mass})$ with $b = \nu/\sigma$ (SC: $\nu = \delta_c/\sigma$

$$\to \xi_2\,(\text{peak}) = b^2\,\xi_2\,(m)$$

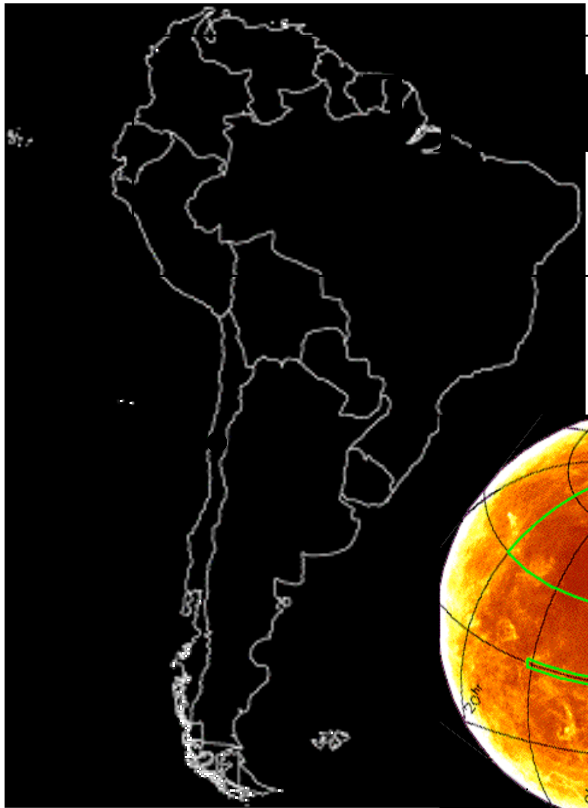


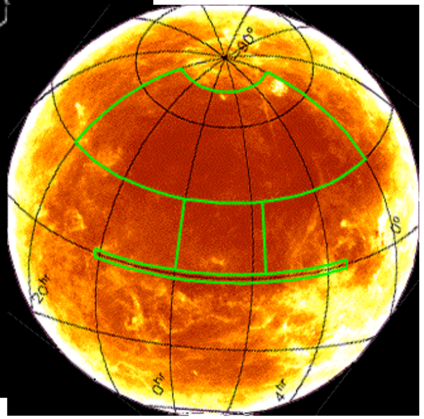
The Science Case for the Dark Energy Survey

Institut de Física d'Altes Energies

E.Gaztanaga

Dark Energy Survey





- We propose to make precision measurements of Dark Energy
  - Cluster counting, weak lensing, galaxy clustering and supernovae
  - Independent measurements
- by mapping the cosmological density field to z=1
  - Measuring 300 million galaxies
  - Spread over 5000 sq-degrees
- using new instrumentation of our own design.
  - 500 Megapixel camera
  - 2.1 degree field of view corrector
  - Install on the existing CTIO 4m