QUANTUM FIELD THEORY

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1. **Gauge Theories**
2. **Renormalization**
3. **Renormalization Group**
4. **QCD Phenomenology**
5. **Chiral Perturbation Theory**

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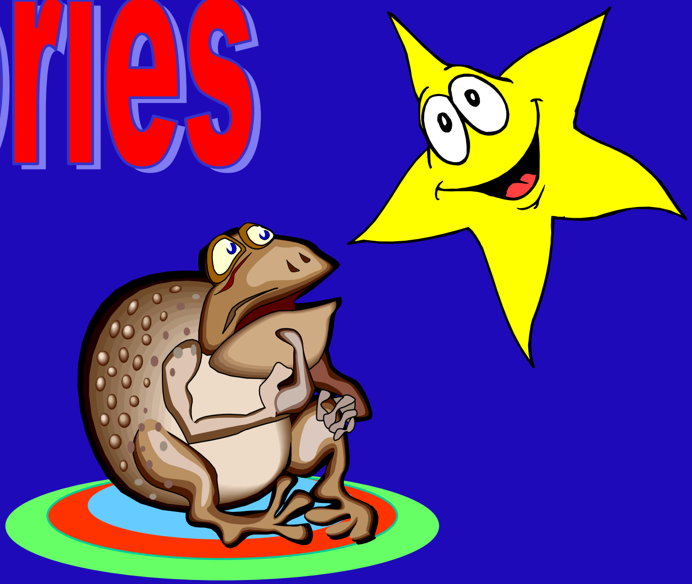
1. Gauge Theories



- Gauge Symmetry
- Quantum Electrodynamics (QED)
- SU(N) Algebra
- Non-Abelian Gauge Symmetry
- Quantum Chromodynamics (QCD)
- Gauge Fixing

FREE Dirac fermion:

$$\mathcal{L} = \bar{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi$$

Phase Invariance: $\psi \to \psi' = e^{iQ\theta}\,\psi \quad ; \quad \bar{\psi} \to \bar{\psi}' = e^{-iQ\theta}\,\bar{\psi}$

Absolute phases are not observable in Quantum Mechanics

GAUGE PRINCIPLE: $\theta = \theta(x)$

Phase Invariance should hold LOCALLY

BUT $\partial_{\mu}\psi \to e^{iQ\theta}\,(\partial_{\mu} + i\,Q\,\partial_{\mu}\theta)\,\psi$

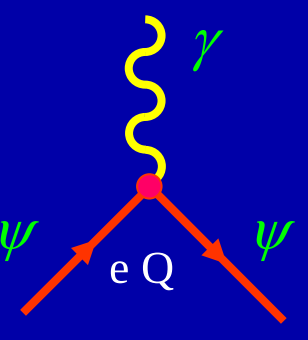
SOLUTION: Covariant Derivative

$$D_{\mu}\psi \equiv (\partial_{\mu} - i\,e\,Q\,A_{\mu})\,\psi \to e^{iQ\theta}\,D_{\mu}\psi$$

One needs a spin-1 field A_{μ} satisfying $A_{\mu} \to A_{\mu} + \frac{1}{e}\,\partial_{\mu}\theta$

QUANTUM ELECTRODYNAMICS

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi$$
$$= \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi + e\,Q\,A_\mu\;(\bar{\psi}\gamma^\mu\psi)$$



Kinetic term:

$$\mathcal{L}_K = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \Longrightarrow \quad \partial_\mu F^{\mu\nu} = -eQ\,(\bar{\psi}\gamma^\nu\psi)$$

Maxwell

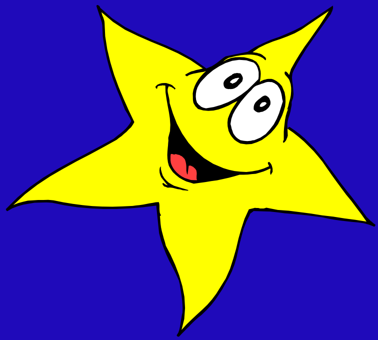
Mass term: [exp: $m_\gamma < 2\cdot 10^{-16}$ eV]

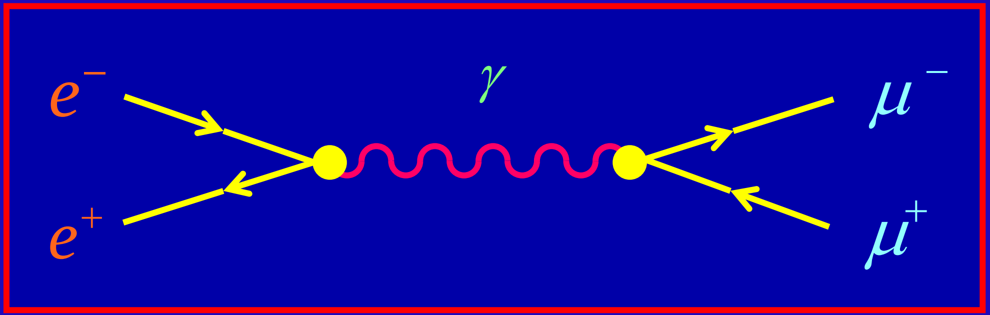
$$\mathcal{L}_M = \frac{1}{2} m_\gamma^2\, A^\mu A_\mu$$

Not Gauge Invariant ⟹ $m_\gamma = 0$

Gauge Symmetry ⟹ QED Dynamics

Successful Theory

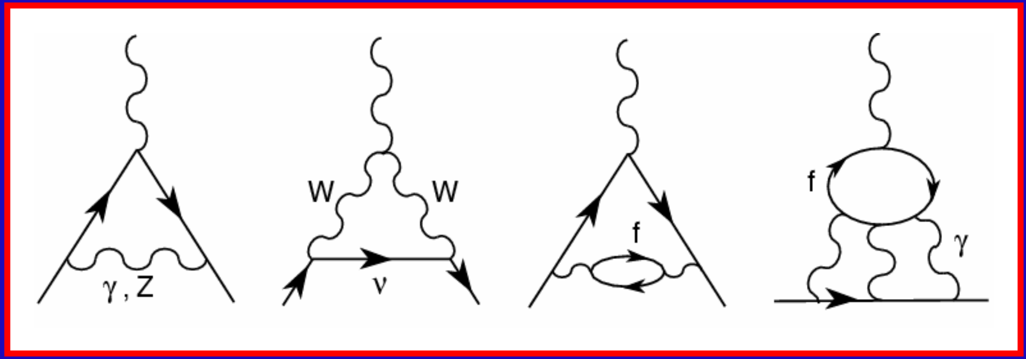




Anomalous Magnetic Moment

$$\mu_l \equiv g_l \, \frac{e}{2\, m_l}$$

$$a_l \equiv \frac{1}{2}\,(g_l - 2)$$



$$a_e = (115\,965\,218.59 \pm 0.38) \times 10^{-11} \quad \Longrightarrow \quad \alpha^{-1} = 137.035\,999\,11 \pm 0.000\,000\,46$$

$$\Longrightarrow \quad a_\mu^{\mathrm{th}} = (116\,591\,922 \pm 88) \times 10^{-11} \qquad [\,\mathrm{Exp:}\quad (116\,592\,080 \pm 60) \times 10^{-11}\,]$$

QUANTUM CHROMODYNAMICS

FREE QUARKS: $N_C = 3$

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

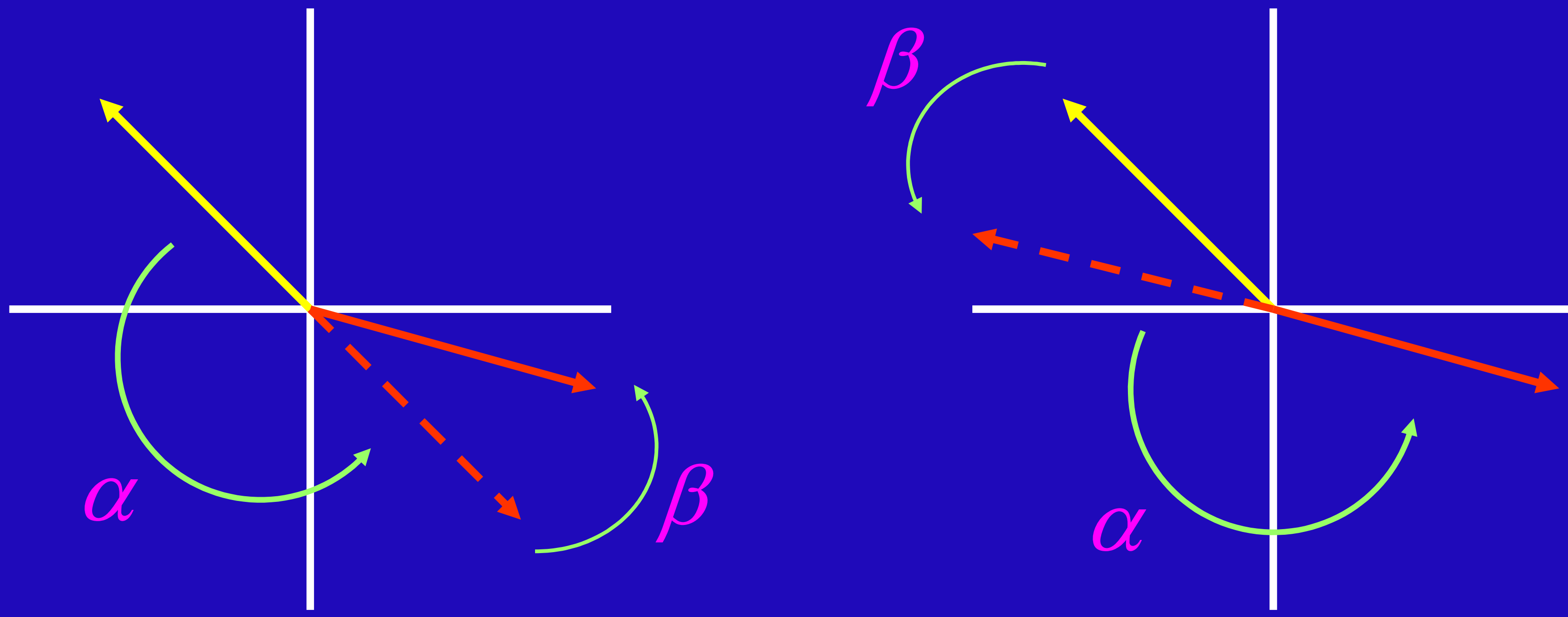
$$\mathcal{L} = \bar{\mathbf{q}} \, [\, i \gamma^\mu \partial_\mu - m \,] \, \mathbf{q}$$

SU(3) Colour Symmetry:

$$\mathbf{q} \to \mathbf{U}\,\mathbf{q} \qquad ; \qquad \bar{\mathbf{q}} \to \bar{\mathbf{q}}\,\mathbf{U}^\dagger$$

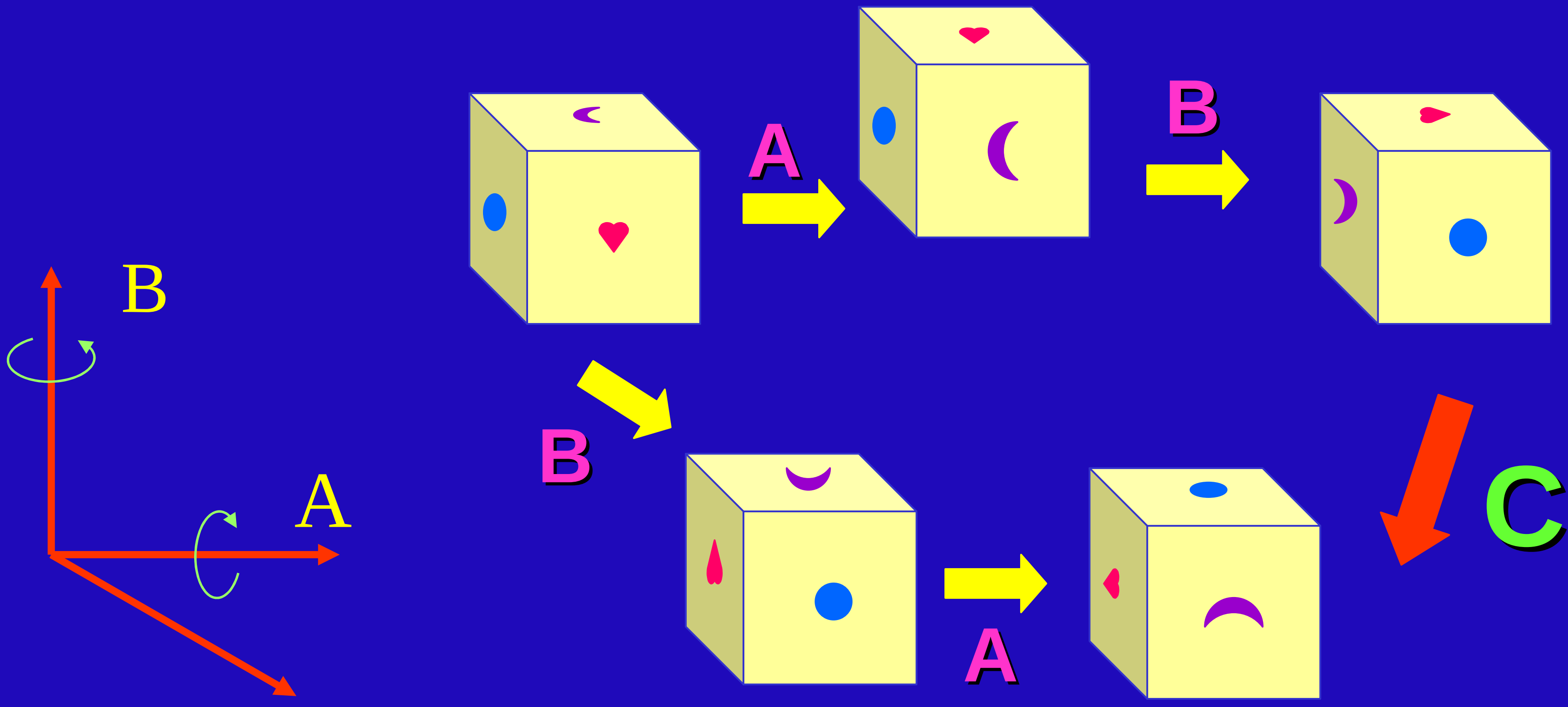
$$\mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\mathbf{U} = \mathbf{1} \qquad ; \qquad \det \mathbf{U} = 1$$

ABELIAN ROTATIONS



NON ABELIAN ROTATIONS

AB - BA = C



SU(N) ALGEBRA

$N \times N$ matrices: $\mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\,\mathbf{U} = \mathbf{1}$; $\det \mathbf{U} = 1$

$$\mathbf{U} = \exp\left\{ i\, \mathbf{T}^a \theta_a \right\} \quad ; \quad \mathbf{T}^a = \mathbf{T}^{a\dagger} \quad ; \quad \mathrm{Tr}\,(\mathbf{T}^a) = 0 \quad ; \quad \mathrm{a} = 1, \cdots, N^2 - 1$$

Commutation Relation: $\left[\mathbf{T}^a, \mathbf{T}^b \right] = i\, f^{abc}\, \mathbf{T}^c$

Structure Constants f^{abc} real , antisymmetric

Fundamental Representation: $\mathbf{T}_F^a = \frac{1}{2} \lambda^a$ $\quad N \times N$

Adjoint Representation: $(\mathbf{T}_A^a)_{bc} = -i\, f^{abc}$ $\quad (N^2 - 1) \times (N^2 - 1)$

$$\mathrm{Tr}\,(\mathbf{T}_F^a\, \mathbf{T}_F^b) = T_F\, \delta_{ab} \quad ; \quad \mathrm{Tr}\left(\mathbf{T}_A^a\, \mathbf{T}_A^b\right) = C_A\, \delta_{ab} \quad ; \quad \left(\mathbf{T}_F^a\, \mathbf{T}_F^a\right)_{\alpha\beta} = C_F\, \delta_{\alpha\beta}$$

$$T_F = \frac{1}{2} \quad ; \quad C_A = N \quad ; \quad C_F = \frac{N^2 - 1}{2N}$$

SU(2)

2×2 matrices: $\quad \mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\,\mathbf{U} = \mathbf{1} \quad ; \quad \det\mathbf{U} = 1$

$$\mathbf{U} = \exp\left\{ i\,\mathbf{T}^a \theta_a \right\} \quad ; \quad \mathbf{T}^a = \mathbf{T}^{a\dagger} \quad ; \quad \mathrm{Tr}\,(\mathbf{T}^a) = 0 \quad ; \quad \mathrm{a} = 1, \cdots, 3$$

Commutation Relation:

$$\left[\mathbf{T}^a, \mathbf{T}^b\right] = i\,\varepsilon^{abc}\,\mathbf{T}^c$$

Fundamental Representation:

$$\mathbf{T}^a_F = \frac{1}{2}\,\sigma^a$$

Pauli

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

SU(3)

$$[\mathbf{T}^a, \mathbf{T}^b] = i\, f^{abc}\, \mathbf{T}^c$$

Fundamental Representation:

$$\mathbf{T}_F^a = \frac{1}{2}\lambda^a$$

Gell-Mann

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;;\quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;;\quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;;\quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \;;\quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \;;\quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \;;\quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\frac{1}{2} f^{123} = f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{\sqrt{3}} f^{458} = \frac{1}{\sqrt{3}} f^{678} = \frac{1}{2}$$

QUANTUM CHROMODYNAMICS

FREE QUARKS: $N_C = 3$

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

$$\mathcal{L} = \bar{\mathbf{q}}\,[\,i\gamma^\mu \partial_\mu - m\,]\,\mathbf{q}$$

SU(3) Colour Symmetry:

$$\mathbf{q} \to \mathbf{U}\,\mathbf{q} = \exp\left\{ i\,\frac{\lambda^a}{2}\,\theta_a \right\}\,\mathbf{q}$$

Gauge Principle: Local Symmetry $\theta_a = \theta_a(x)$

$$\mathbf{D^\mu q} \equiv (\mathbf{I}_3\,\partial^\mu - i\,g_s\,\mathbf{G}^\mu)\,\mathbf{q} \quad \to \quad \mathbf{U}\;\mathbf{D^\mu q}$$

$$\mathbf{D}^\mu \to \mathbf{U}\;\mathbf{D}^\mu\;\mathbf{U}^\dagger \qquad ; \qquad \mathbf{G}^\mu \to \mathbf{U}\,\mathbf{G}^\mu\,\mathbf{U}^\dagger - \frac{i}{g_s}\,(\partial^\mu \mathbf{U})\;\mathbf{U}^\dagger$$

$$[\mathbf{G}^\mu]_{\alpha\beta} \equiv \frac{1}{2}\,(\lambda^a)_{\alpha\beta}\;G^\mu_a(x)$$

8 Gluon Fields

Infinitesimal SU(3) Transformation:

$$\delta \mathrm{q}^{\alpha} = i\, \delta\theta_a \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} \mathrm{q}^{\beta} \qquad ; \qquad \delta G_a^{\mu} = \frac{1}{g_s}\, \partial^{\mu}(\delta\theta_a) - f^{abc}\, \delta\theta_b\, G_c^{\mu}$$

Non Abelian Group:

$$f^{abc} \neq 0$$

- δG_a^{μ} depends on G_a^{μ}
- Universal g_s
- No Colour Charges

Kinetic Term:

$$\mathbf{G}^{\mu\nu} \equiv \frac{i}{g_s}\,[\mathbf{D}^\mu, \mathbf{D}^\nu] = \partial^\mu \mathbf{G}^\nu - \partial^\nu \mathbf{G}^\mu - i\, g_s\, [\mathbf{G}^\mu, \mathbf{G}^\nu] \quad \to \quad \mathbf{U}\;\mathbf{G}^{\mu\nu}\;\mathbf{U}^\dagger$$

$$\mathbf{G}^{\mu\nu} \equiv \frac{\lambda^a}{2} G_a^{\mu\nu} \quad ; \quad G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g_s\, f^{abc}\, G_b^\mu\, G_c^\nu$$

$$\mathcal{L}_K = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\mathbf{G}_{\mu\nu}) = -\frac{1}{4}\, G_a^{\mu\nu}\, G^a_{\mu\nu}$$

Mass Term:

$$\mathcal{L}_M = \frac{1}{2}\, m_G^2\; G_a^\mu\, G^a_\mu$$

Not Gauge Invariant ➡ $m_G = 0$

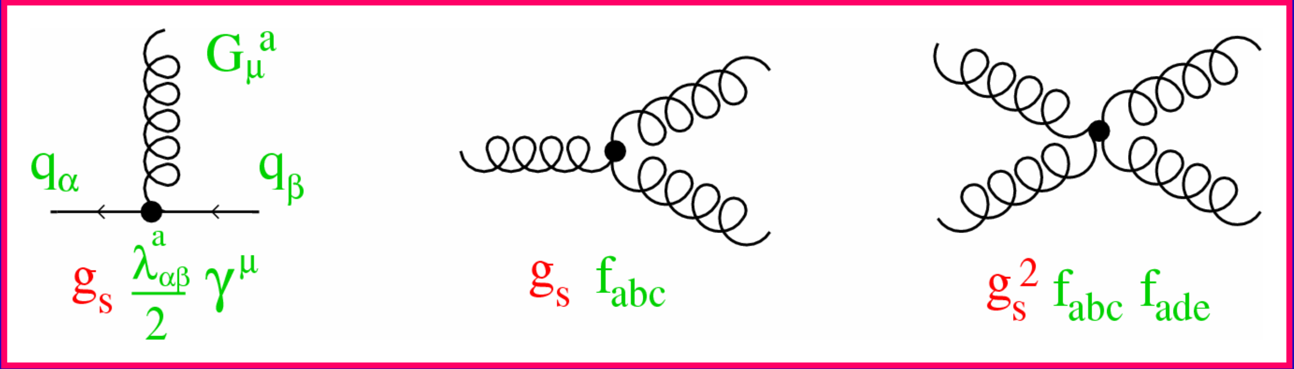
Massless Gluons

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\,\mathbf{G}_{\mu\nu}) + \bar{\mathbf{q}}\,[\,i\,\gamma^\mu\,\mathbf{D}_\mu - m_q\,]\,\mathbf{q}$$

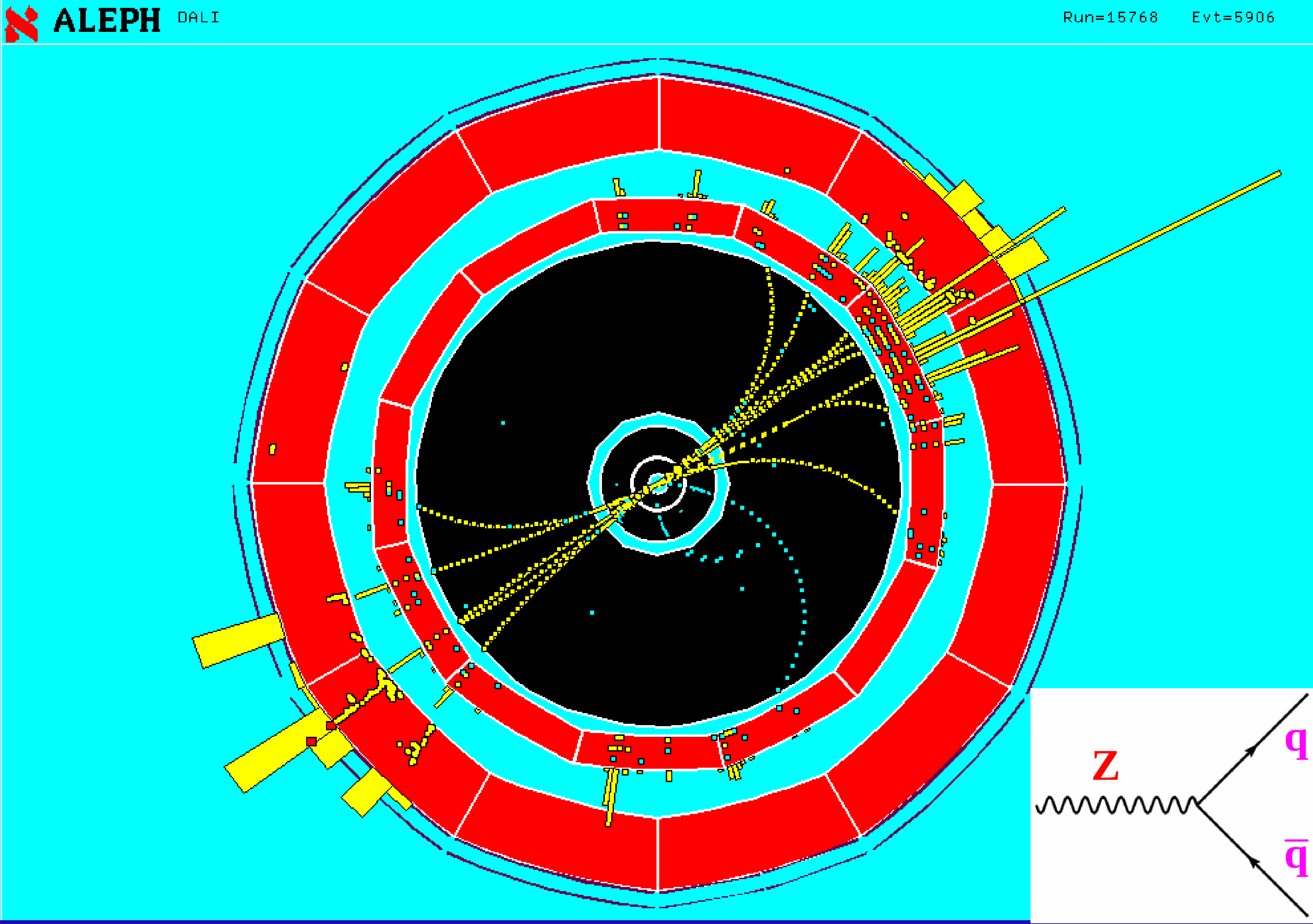
$$= -\frac{1}{4}\left(\partial^\mu G^\nu_a - \partial^\nu G^\mu_a\right)\left(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu\right) + \sum_q \bar{q}_\alpha\,[\,i\,\gamma^\mu\partial_\mu - m_q\,]\,q_\alpha$$

$$+ \frac{1}{2}\sum_q g_s\,[\bar{q}_\alpha\,(\lambda^a)_{\alpha\beta}\,\gamma^\mu q_\beta\,]\,G^a_\mu$$

$$- \frac{1}{2}\,g_s\,f_{abc}\,(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu)\,G^\mu_b\,G^\nu_c - \frac{1}{4}\,g_s^2\,f_{abc}\,f_{ade}\,G^\mu_b\,G^\nu_c\,G^d_\mu\,G^e_\nu$$



- **Gluon Self – interactions** $\quad G^3\ ,\ G^4$
- **Universal Coupling** $\quad g_s$ $\quad$ **(No Colour Charges)**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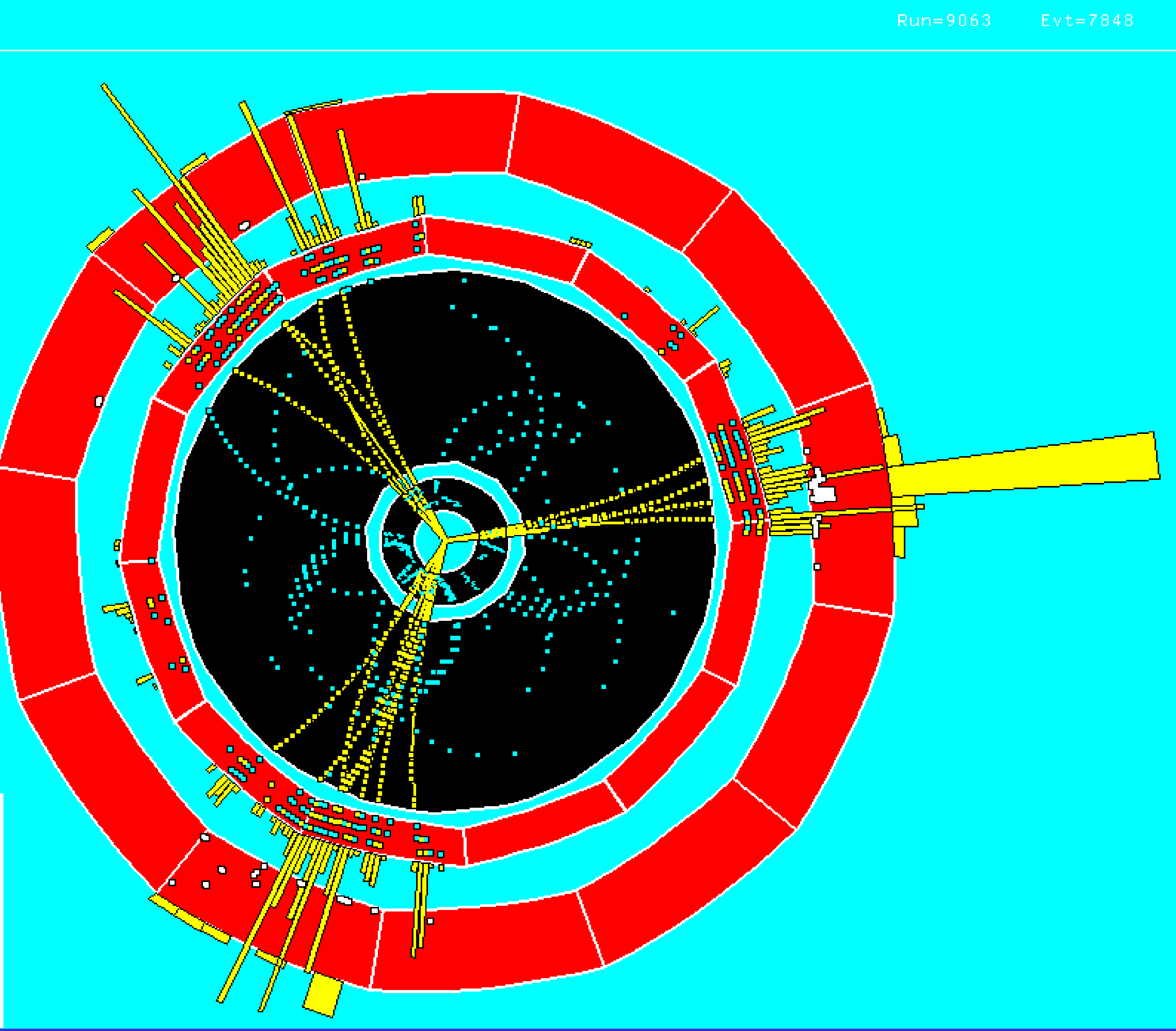
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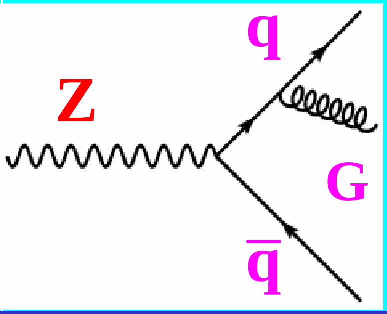
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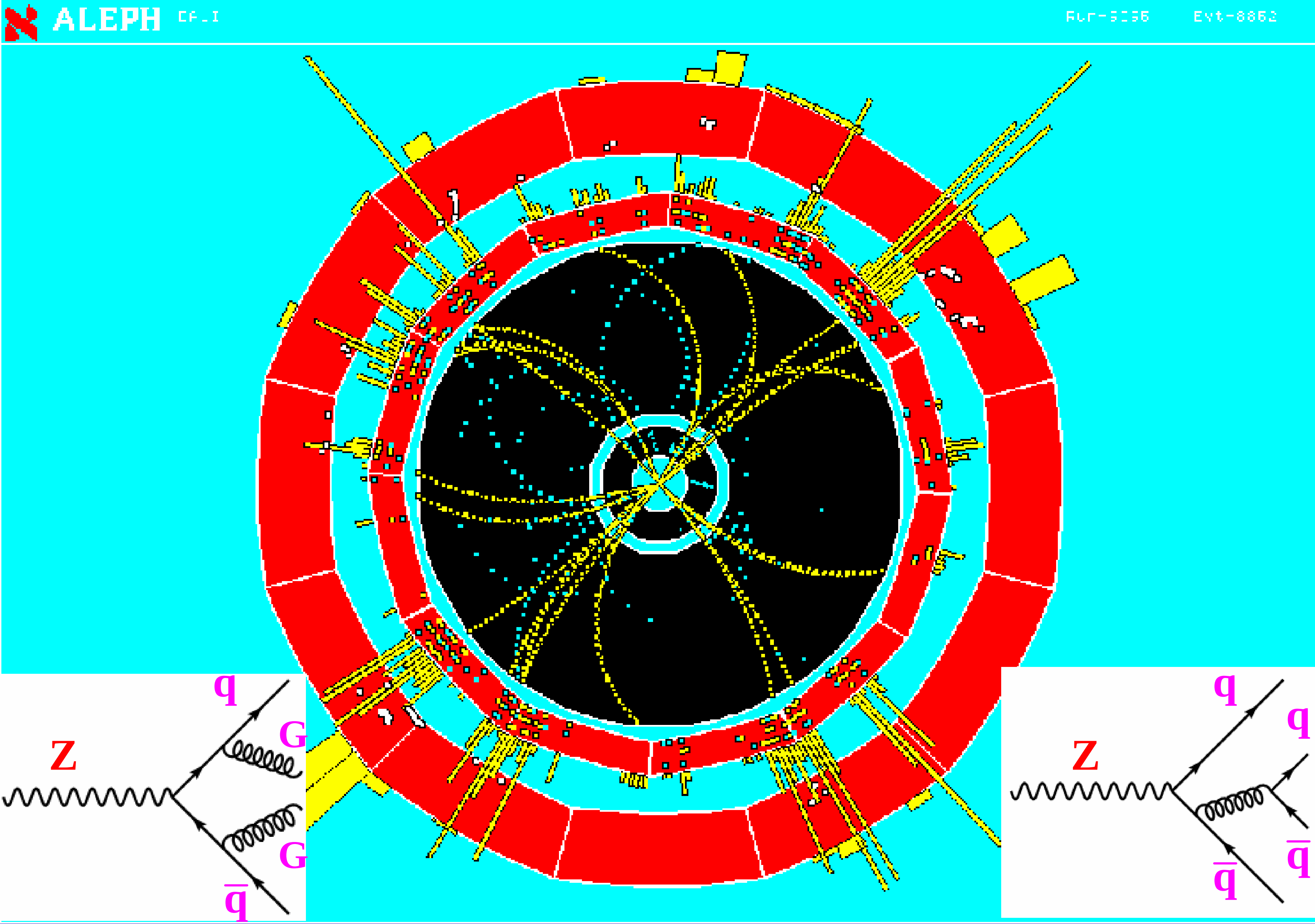
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GAUGE FIXING

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi} \left(\partial^\mu G_\mu^a\right) \left(\partial_\nu G_a^\nu\right)$$

GAUGE FIXING

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$$\langle 0|\, T\left[G_a^\mu(x)\, G_b^\nu(y)\right] |0\rangle \;\equiv\; i\,\delta_{ab}\, D_F^{\mu\nu}(x-y) \;=$$

$$i\,\delta_{ab}\int\frac{d^4k}{(2\pi)^4}\;\frac{\mathrm{e}^{-ik\cdot(x-y)}}{k^2+i\,\epsilon}\left\{-g^{\mu\nu} + (1-\xi)\,\frac{k^\mu k^\nu}{k^2+i\,\epsilon}\right\}$$

GAUGE FIXING

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}\,\left(\partial^\mu G^a_\mu\right)\,\left(\partial_\nu G^\nu_a\right)$$

$$\langle 0|\, T\,[G^\mu_a(x)\, G^\nu_b(y)]\,|0\rangle \;\equiv\; i\,\delta_{ab}\, D_F^{\mu\nu}(x-y) \;=$$

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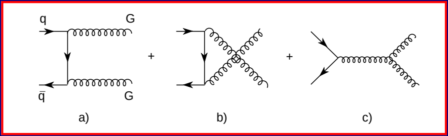

$$T \;=\; J^{ab}_{\mu\nu}\;\; \varepsilon^{\mu *}_{a,r}\,\varepsilon^{\nu *}_{b,s}$$

GAUGE FIXING

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}\,\left(\partial^\mu G^a_\mu\right)\,\left(\partial_\nu G^\nu_a\right)$$

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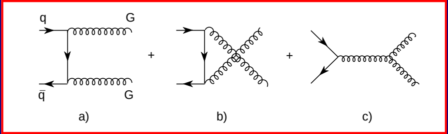


$$T \;=\; J^{ab}_{\mu\nu}\;\; \varepsilon^{\mu *}_{a,r}\,\varepsilon^{\nu *}_{b,s}$$

$$\mathcal{P}_T \sim \frac{1}{2}\, J^{ab}_{\mu\nu}\, (J^{cd}_{\rho\sigma})^\dagger\; \sum_{r=1}^{2} \varepsilon^{\mu *}_{a,r}\,\varepsilon^{\rho}_{c,r}\; \sum_{s=1}^{2} \varepsilon^{\nu *}_{b,s}\,\varepsilon^{\sigma}_{d,s}$$

$$\mathcal{P}_C \sim \frac{1}{2}\, J^{ab}_{\mu\nu}\, (J^{cd}_{\rho\sigma})^\dagger\;\; g^{\mu\rho}\;\; g^{\nu\sigma}$$

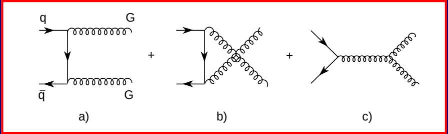
GAUGE FIXING

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}\left(\partial^\mu G_\mu^a\right)\left(\partial_\nu G_a^\nu\right)$$

$$\langle 0|\, T\left[G_a^\mu(x)\, G_b^\nu(y)\right]|0\rangle \;\equiv\; i\,\delta_{ab}\, D_F^{\mu\nu}(x-y) \;=$$

$$i\,\delta_{ab}\int\frac{d^4k}{(2\pi)^4}\,\frac{\mathrm{e}^{-ik\cdot(x-y)}}{k^2+i\,\epsilon}\left\{-g^{\mu\nu}+(1-\xi)\,\frac{k^\mu k^\nu}{k^2+i\,\epsilon}\right\}$$

Probability problem:



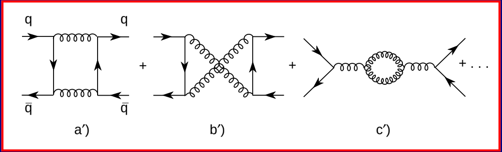
$$T \;=\; J^{ab}_{\mu\nu}\;\varepsilon^{\mu *}_{a,r}\,\varepsilon^{\nu *}_{b,s}$$

$$\mathcal{P}_T \sim \frac{1}{2}\, J^{ab}_{\mu\nu}\,\left(J^{cd}_{\rho\sigma}\right)^\dagger\;\sum_{r=1}^{2}\varepsilon^{\mu *}_{a,r}\,\varepsilon^{\rho}_{c,r}\;\sum_{s=1}^{2}\varepsilon^{\nu *}_{b,s}\,\varepsilon^{\sigma}_{d,s}$$

$$\mathcal{P}_C \sim \frac{1}{2}\, J^{ab}_{\mu\nu}\,\left(J^{cd}_{\rho\sigma}\right)^\dagger\; g^{\mu\rho}\;\; g^{\nu\sigma}$$

$$\mathcal{P}_C > \mathcal{P}_T$$

Same problem for virtual gluons:

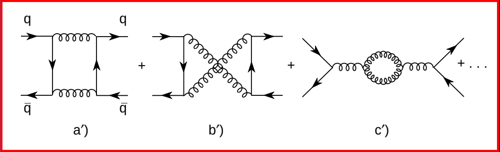


$$\mathrm{Im}\left[T(q\,\bar{q} \to q\,\bar{q})\right] \sim \left|T(q\,\bar{q} \to G\,G)\right|^2$$

Unphysical contribution from non-transverse gluon polarizations (c,c')

Gauge fixing: $\partial_\mu G_a^\mu = 0$ ➡ $k_\mu\, J^{\mu\nu} \neq 0$

Same problem for virtual gluons:



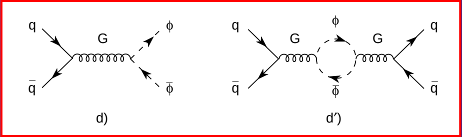
$$\mathrm{Im}\,[T(q\,\bar{q} \to q\,\bar{q})] \sim |T(q\,\bar{q} \to G\,G)|^2$$

Unphysical contribution from non-transverse gluon polarizations (c,c′)

Gauge fixing: $\partial_\mu G^\mu_a = 0$ ⟶ $k_\mu\, J^{\mu\nu} \neq 0$

GHOSTS:

(Faddeev–Popov)



$$\mathcal{L}_{\mathrm{FP}} = -\partial_\mu \bar{\phi}_a\, D^\mu \phi^a$$

$$D^\mu \phi^a \equiv \partial^\mu \phi^a - g_s f^{abc} \phi^b G^\mu_c$$

Wrong (Fermi) Statistics ⟶ **Probability** < 0