The hidden SUSY face of QCD

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based on work done with

Adi Armoni and Misha Shifman

(hep-th/0302163, 0307097, 0309013)

and reviewed in

ASV, hep-th/0403071

NB: mostly from a QFT perspective

Outline

1. Large-N exp.^{ns} in QCD: need one more?
2. QCD_F vs. QCD_{OR}
3. Planar equivalence in PT and beyond
4. SUSY relics in N_f=1 QCD
5. Analytic estimate of $< \underline{\psi}\psi >$ in QCD
6. Extensions

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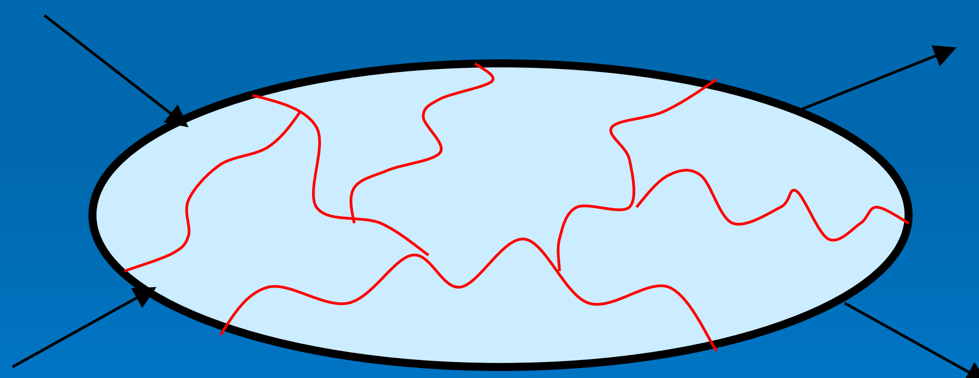
Unrelated Appendix:
Isospin mixing of tetra and pentaquark states?

Large-N expansions in QCD: who needs one more?

- Planar + quenched limit ('tHooft, 1974) = $1/N_c$ expansion @ fixed $\lambda = g^2 N_c$ and N_f

Leading diagrams



Corrections: $O(N_f/N_c)$ from q-loops, $O(1/N_c^2)$ from non-planar diagrams

Properties at leading order

1. Resonance have zero width +
2. U(1) problem not solved, WV holds @ N LO -?
3. Multiparticle production not allowed -

Theoretically, if not phenomenologically, appealing: should give the tree-level of some string theory (reason for the "accidental" discovery of the string theory we are now considering as a TOE?)

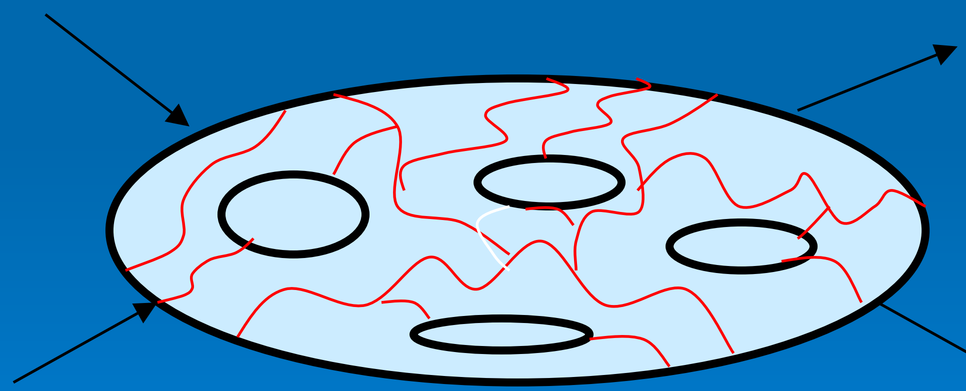
Proved hard to solve, except in D=2….

- Planar unquenched limit (GV '74--'76) = TE = 1/N expansion @ fixed $\lambda = g^2N$ and N_f/N_c

Corrections

$O(1/N^2)$ from non-planar diagrams

Leading diagrams include "empty" q-loops



Properties

1. Widths become O(1) -
2. U(1) problem solved to leading order, no reason for WV to be good *) +?
3. Multiparticle production allowed

   => Bare Pomeron & Gribov's RFT +

Perhaps phenomenologically more appealing than 'tHooft's but even harder to solve...

*) Better justified through a small-N_f expansion?

We can generalize QCD to an SU(N) gauge group in different ways by playing with matter rep.

The conventional way, QCD_F, is to keep the quarks in fundamental + antifundamental (N +N*)

The one we shall consider is called, for stringy reasons, QCD_{OR} (OR for Orientifold: see e.g. P.Di Vecchia et al. hep-th/0407038)

Put quarks in the 2-index-antisymmetric (AS)-tensor rep. of SU(N) (+ its complex conjugate)

As in 'tHooft's expansion (and unlike in TE) N_f fixed

NB. For N=3 this is still ordinary QCD

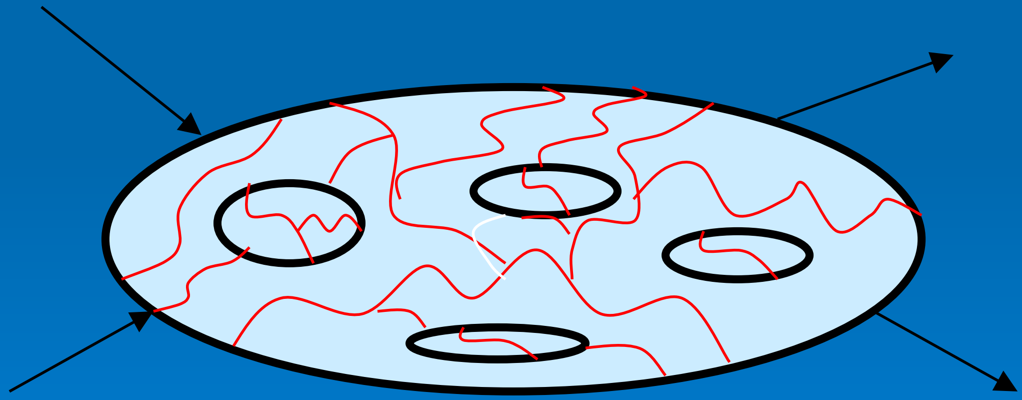
NB: In string construction: both S and AS tensor reps. are possible, but the former is never ~ QCD

Leading diagrams are planar, include "filled" q-loops since there are $O(N^2)$ quarks

Widths are zero, U(1) problem solved, no p.pr.

Phenomenologically interesting?

Theoretically manageable? Yes, I claim.



QCD_F vs. QCD_{OR}

th / coeff	YM	QCD_F	QCD_{OR}	Large-N, $N_f=1$
β_0	$11N/3$	$(11N-2N_f)/3$	$(11N-2(N-2)N_f)/3$	$3N$
β_1	$17N^2/3$	$17N^2/3 - N_f\,(13N/6 - 1/2N)$	$17N^2/3 - N_f\,(N-2) \times (5N + 3(N-2)(N+1)/N)/3$	$3N^2$
γ_0	X	$3(N^2-1)/2N$	$3(N-2)(N+1)/N$	$3N$

QCD_{OR} as an interpolating theory:

@ N=2 it coincides with pure YM (fermions decouple)

@ N=3 it coincides with QCD

… and at large N?

ASV claim of Planar Equivalence

In the large-N limit the bosonic sector of QCD_{OR} is equivalent to that of QCD_{Adj} i.e. of QCD with N_f Majorana fermions in the adjoint representation

Important corollary

For $N_f = 1$ and $m = 0$, QCD_{OR} is planar-equivalent to Supersymmetric Yang-Mills (SYM) theory

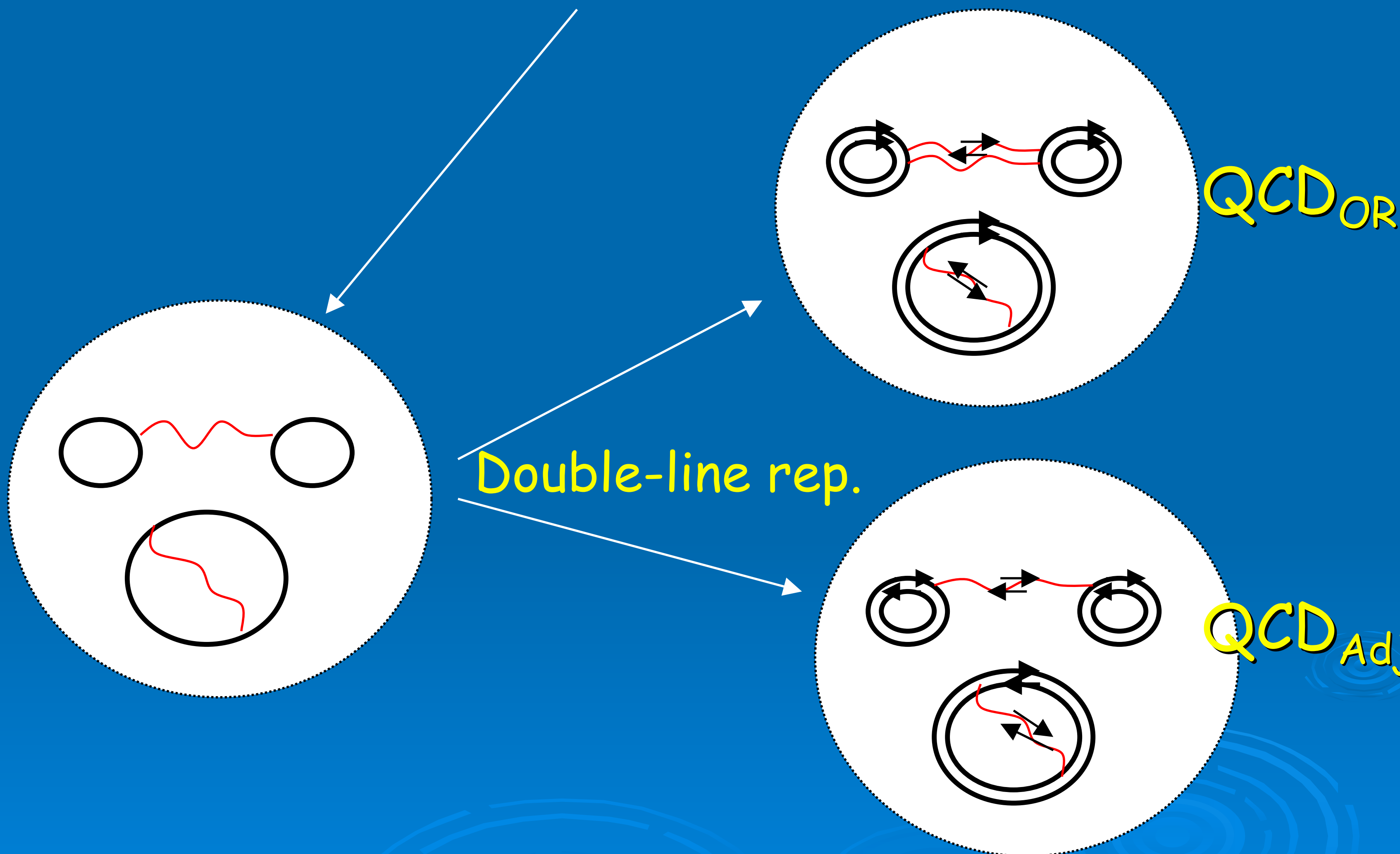
Some properties of the latter should show up in $N_f = 1$ QCD … if N=3 is large enough

NB: Expected accuracy is only 1/N…

NB': For us $N_f = 1$, $m = 0$ defines a rather special point in parameter space… unlike for Creutz..

Perturbative Argument

Draw a planar diagram on sphere



Differ by an even number of - signs…

Non-perturbative Argument (sketch)

- Integrate out fermions (after having included masses, bilinear sources)
- Use gauge invariance of det($\not{D}$+m+J) to express it in terms of Wilson-loops
- Use large-N factorization to write adjoint and OR Wilson loop as product of fundamental and/or antifundamental Wilson loops
- Use equality of fundamental and antifundamental Wilson loops

Before moving to SUSY..

It would be interesting to check numerically what happens to QCD_{OR} and to QCD_{Adj} as we increase N even for

- $m \neq 0$, $N_f \neq 1$,
- quenched limit

The two theories should approach each other

Another numerical (analytic?) check could be comparing fermionic determinants in both theories as N is increased

SUSY relics in QCD_{OR}

- Approximate parity doublets. Indeed:

  SYM: $m_S = m_P = m_F$ => OR: $m_S \sim m_P \ll m_F$

  Looks OK if can we make use of:

  a) Experiments for m_S (σ @ 600MeV) ,

  b) WV for m_P ($m_P \sim \sqrt{2}(180)^2/95$ MeV ~ 480 MeV excluding quark masses)

- Related to this: approximate absence of "activity in certain chiral correlators

$$<\psi_R\psi_L(x)\,\psi_R\psi_L(y)> \sim \text{constant}$$

while $<\overline{\psi}_R\psi_L(x)\,\overline{\psi}_L\psi_R(y)>$ has much activity

- A mass gap, no Nambu-Goldstone bosons (the only continuous axial symmetry is broken by anomaly/instantons even @ large N)
- N + O(1) distinct vacua characterized by the phase of the quark condensate. Indeed one expects N-2 distinct vacua. Except at N=3, there is an enhanced symmetry @ m=0!
- Vanishing cosmological constant at leading order in spite of the fact that the planar spectrum of the OR theory is purely bosonic

Dulcis in fundo..

An analytic estimate of the quark condensate

The quark condensate in $N_f=1$ QCD

Claim (ASV, hep-th/0309013):

$$<\bar{\psi}\psi>_\mu = -\frac{3}{2\pi^2}\mu^3 \lambda_\mu^{-1578/961} exp(-\frac{27}{31\lambda_\mu})k(1/3)$$

where (all in <u>MS</u>)

$$\lambda_\mu = \alpha_s(\mu)N/2\pi \,,\; k(0) = 1 \,,\; k(1/3) \sim 1 \pm 0.30$$

Sketch of argument: define

$$\Lambda^* = \mu exp\left[-\frac{N}{\beta_0}(1/\lambda_\mu - 1/\lambda^*)\right]\left(\frac{\lambda_\mu}{\lambda^*}\right)^{\beta_1/\beta_0^2}$$

$$< \bar{\psi}\psi >^* = \left(\frac{\lambda_\mu}{\lambda^*}\right)^{\gamma_0/\beta_0} < \bar{\psi}\psi >_\mu = < \bar{\psi}\psi >_{\Lambda^*}$$

$$R^* = < \bar{\psi}\psi >^* / \Lambda^{*3}$$

both in QCD_{OR} and in SYM. We want to compute the former from the latter

Ratio of ratios, R*(OR) / R*(SYM), is a function of 1/N , K(1/N, λ*) w/ K(0, λ*) = 1

R*(SYM) is exactly known either from weak-coupling instanton calculations or from softly broken SW:

$$R^*(SYM) = -\frac{N^2}{2\pi^2\lambda^{*2}} exp(-1/\lambda^*)$$

Using previous formulae we get

$$<\bar{\psi}\psi>_\mu^{OR} = -\frac{N^2\mu^3}{2\pi^2} exp(-\frac{3N}{\beta_0\lambda_\mu})\lambda_\mu^{-3\beta_1/\beta_0^2-\gamma_0/\beta_0} K(1/N,\lambda^*) f(\lambda^*)$$

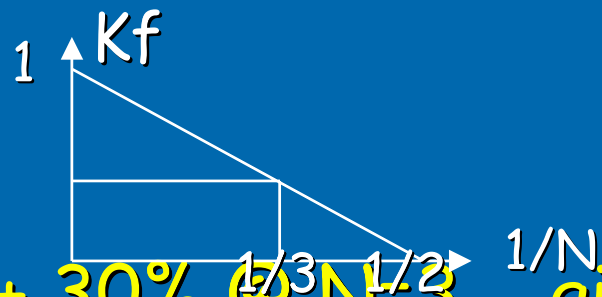
$$\frac{3N}{\beta_0} = \frac{1}{1+4/9N} \Rightarrow 1 \ , \ \frac{\gamma_0}{\beta_0} = \frac{(1+1/N)(1-2/N)}{1+4/9N} \Rightarrow 1$$

$$3\frac{\beta_1}{\beta_0^2} = \frac{(1+19/9N-4/3N^3)}{(1+4/9N)^2} \Rightarrow 1$$

$$f(\lambda^*) = exp\left[-\frac{1}{\lambda^*}(1-\frac{3N}{\beta_0})\right](\lambda^*)^{3\beta_1/\beta_0^2+\gamma_0/\beta_0-2} \Rightarrow 1$$

Making the assumption that Kf = (1-2/N)k(1/N)

(NB: @ N=2 fermion decouples..)



with k ~ 1 ± 30% @ N=3 gives the quoted result.

This can also be written as

$$< (g^2)^{12/31}\bar{\psi}\psi > = -1.1k(1/3)\Lambda_{st}^3$$

$$\Lambda_{st} = \mu \, exp\left[-\frac{N}{\beta_0\lambda_\mu}\right]\left(\frac{2N}{\beta_0\lambda_\mu}\right)^{\beta_1/\beta_0^2}$$

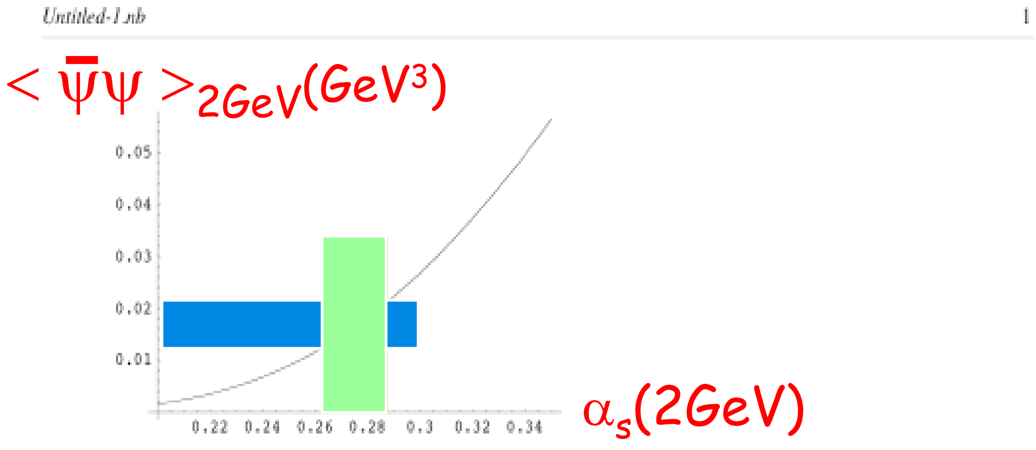
Comparison with "data"

- There are of course no real data on N_f = 1 QCD
- Unfortunately there are no fake data either…PLEASE..
- We can try to argue about relation between N_f = 1 condensate and the one of real QCD (from phenomenology or quenched lattice calculations). A typical value of latter is

to b

$$<\bar{\psi}\psi>_{2GeV} = -0.016 \pm 0.005 GeV^3$$

$$<\bar{\psi}\psi>_{\mu} = -\frac{3}{2\pi^2}\mu^3(\lambda_\mu)^{-1.63} exp(-1/\lambda_\mu)$$



Quark condensate at 2 GeV vs $\alpha_s(2GeV)$

Agreement reached for $\alpha_s(2GeV) \sim 0.275 \pm 0.015$ (see graph) a value compatible with experiments… Encouraging but dedicated lattice calculation appears to be needed

Extension #1: N_f =3

- Take OR theory and add to it two flavours in N+N*
- At N=3 this is N_f=3 QCD, at N=2 it's N_f=2 QCD
- At large N it cannot be distinguished from OR, fits SYM β-functions better at finite N
- Correlators for which dictionary can be established between the two theories should still coincide in large-N limit. These do not include F-condensate
- Vacuum manifold, NG bosons etc. are different!
- May still work for AS condensate: if so the result used above for N_f=3 QCD is justified…

Extension #2: More SUSY

- Natural question to ask is whether we can play a similar trick in order to relate N=1 and N=2 theories
- Example: add to SYM a chiral multiplet in the AS+AS* representation
- If, at large N, this is like adding an adjoint one should recover SW theory and could copy exact results from the latter for $SQCD_{OR}$ (e.g. get Kahler of N=1 theory from N=2 pre-potential) or, at N=3, for SQCD tout court.

- Also in this case moduli spaces are different. One has to work with softly broken N=2 and compare it with N = 1 with massive matter
- Chiral and large-N limit do not commute
- If due care is taken, the comparison of the results for the condensates (here known exactly in both theories) is quite instructive and provides an example where our procedure for SYM and QCD would be fully justified including the factor (1-2/N) here due to the ratio of the Konishi anomaly coeff.s in the two theories

Another possible application: "techni-orientifold"

- Recent work by F. Sannino and K. Tuominen (hep-ph/0405209)
- Instead of making TC similar to QCD_F they propose to make it similar to QCD_{OR}
- However, in order to make it most unlike QCD_F , they chose to work with S, rather than AS, reps.
- Can have walking TC with fewer flavours (possibly solving the FCNC problem while keeping small corrections to S,T parameters…)

NB: NO obvious SUSY limit for this theory!

Conclusions, part I

- The orientifold large-N expansion is arguably the first example where large-N considerations lead to quantitative predictions in non-SUSY, D=4, strongly coupled gauge theories
- More work is needed, particularly on
  - Tightening the NP proof of planar equivalence
  - Estimating 1/N corrections
  - Providing numerical checks
  - Extending the equivalence in various directions

Part II: Isospin mixing in narrow multiquark states?

1. Isosping mixing of narrow pentaquark states

G. C. Rossi and G. Veneziano, hep-ph/0404262

2. Has isosping mixing being seen in decay of $D_{sJ}(2632)$?

L. Maiani et al. hep-ph/0407025

1. Isosping mixing of narrow pentaquark states

G. C. Rossi and G. Veneziano, hep-ph/0404262

- Extend to pentaquarks old work on I-mixing in narrow tetraquark (baryonium) states,
  Phys. Lett. 70B (1980) 507
- Warning: Experiemental status of pentaquarks is still unclear...

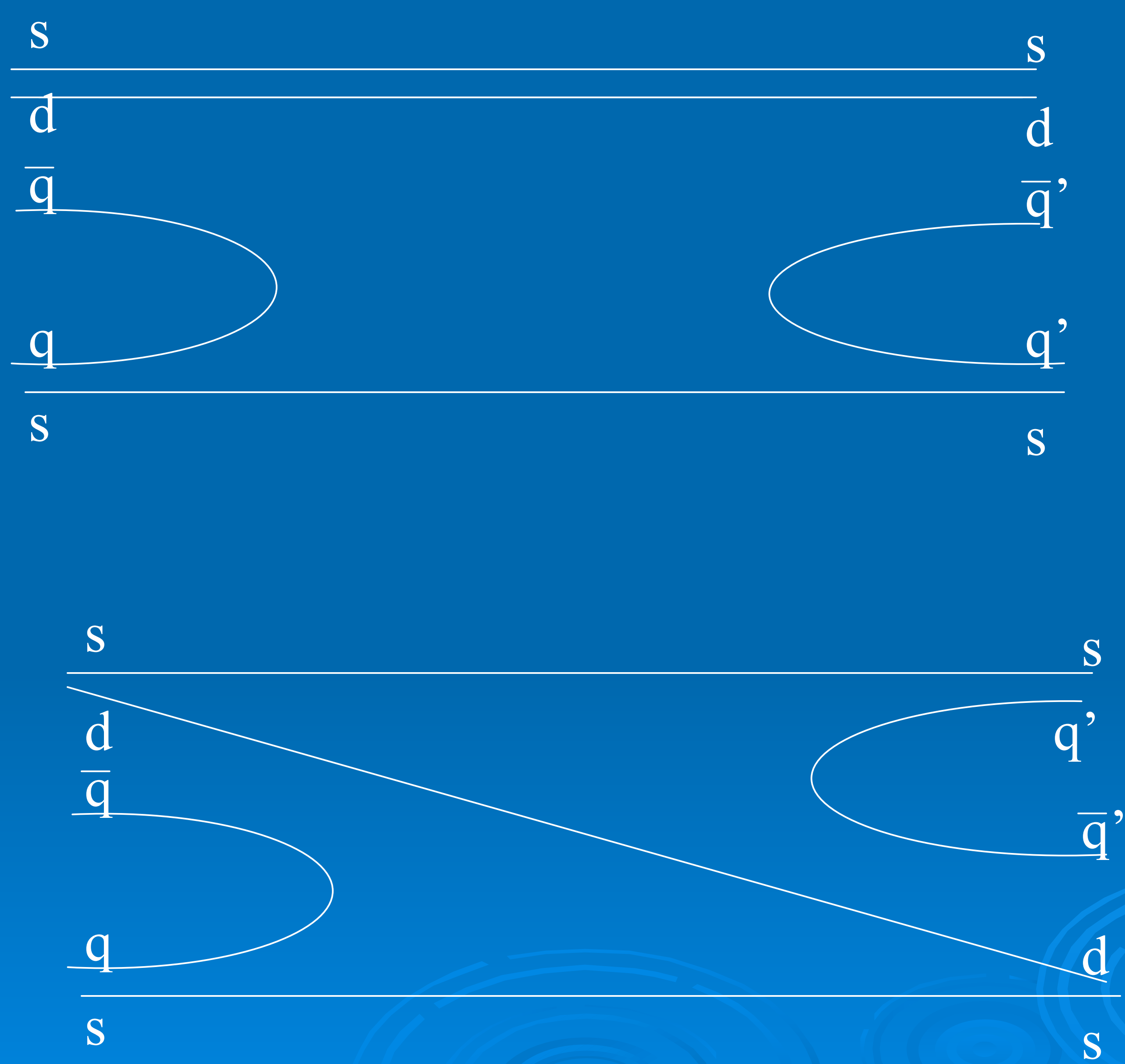
Use (for the sake of illustration) the Jaffe-Wilczek (Nussinov) assignement of Pentaquark states to an ideally mixed $8+\overline{10}$

=> six S=-2 (Ξ) states filling an I=1/2 and an I=3/2 multiplet

If they are narrow and degenerate to within a few MeV, large I-mixing can arise (from quark masses and EM effects) in the Q=-1 and Q=0 sectors (two states mix in each Q sector)

Splitting and mixing angles can be computed from a small number of parameters => predictions

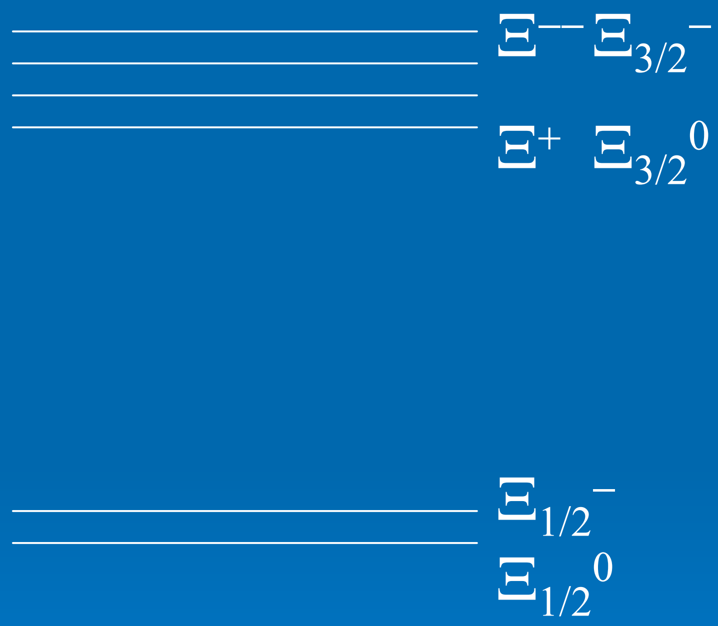
OZI violating diagrams prefer/split pure I-eigenstates



δm and O(α) effects prefer/split pure-flavour eigenstates

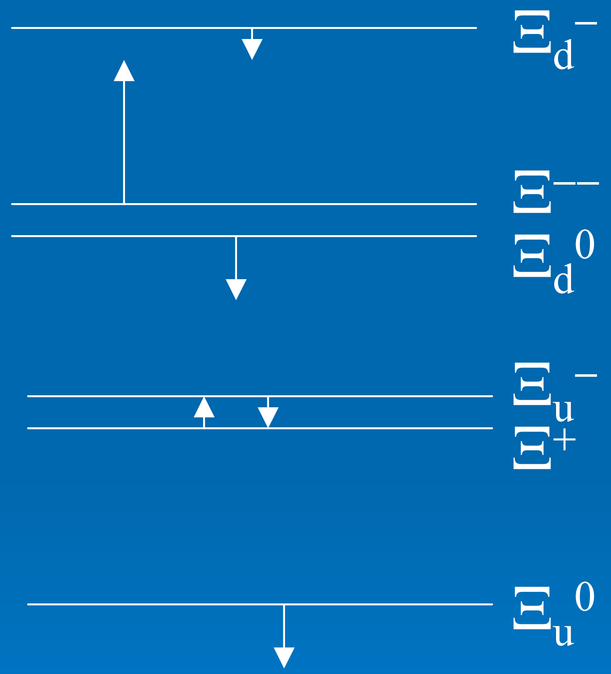
Two extreme pictures of Ξ spectrum

Small I-mixing



(a)

Nearly "ideal" I-mixing



(b)

L. Maiani, F. Piccinini, AD Polosa & V.Riquer, hep-ph/0407025

- Interpret $D_{sJ}(2632)$ as a $[cd][\bar{d}\bar{s}]$ tetraquark state that is relatively unmixed with $[cu][\bar{u}\bar{s}]$
- Explains why this state does not like to decay into D^0K^+
- Predictions for D^+K^0 , $D_s\pi^0$ channels

A striking example of a phenomenon suggested 24 years ago ?