Membrane Structure of Topological Charge Fluctuations in 2D and 4D Gauge Theory

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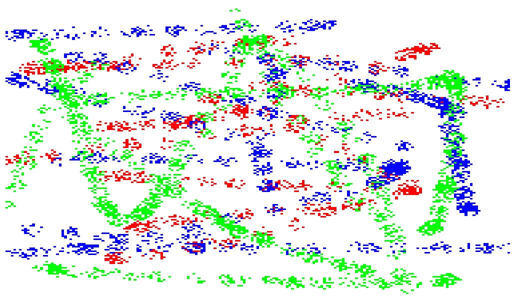
Topological Charge Fluctuations in QCD:

- We have several different possible views of the QCD vacuum
  - 1. “Strong coupling” view of vacuum: color phases are disordered at large distances

    -- Naturally incorporates quark confinement

    -- chiral SB and η' mass mechanism

    is obscure (Aoki phases…)



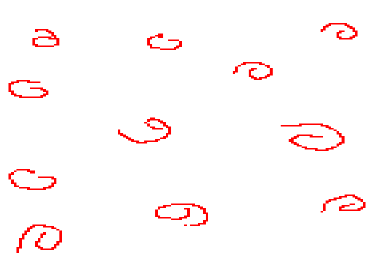
  - 2. Instanton liquid model populates vacuum with localized, self-dual and anti-self-dual quantized lumps of topological $F\widetilde{F}$ charge.

    -- Naturally incorporates chiral SB and η' mass.

    -- Does not incorporate

    confinement. (Too much

    long-range order.)



Fundamental new development in QCD: ADS/CFT duality

- **4D QCD = IIA String Theory in a black hole metric**

(See E. Witten, “Black Holes and Quark Confinement”, Current Science ,V.81(2002))

e.g. Confinement “explained” by

QCD flux tube = holographic projection of fundamental string

ADS/CFT also confirms Witten’s (1979) large-N_c view of topological charge:

- Mutiple vacuum states (“k-vacua”) with $\theta_{\text{eff}} = \theta + 2\pi k$
- Local k-vacua separated by domain wall = membrane
- Domain wall = fundamental 6-brane of IIA string theory wrapped around S_4
- k=integer is a Dirac quantization of 6-brane charge in 9+1 D (Note: 6+2 = 9-1, hence 2D surface surrounds . 6-brane in 9 space dimensions.)

Exactly chiral Dirac operators: A new method for studying topological charge on the lattice:

Discovery of exactly chiral Ginsparg-Wilson fermions provided a new definition of topological charge on the lattice (Hasenfratz, et al. 1998):

$$\text{local TC} \equiv Q(x) = \frac{1}{2} tr\gamma^5 D(x,x)$$

-- Overlap Dirac operator provides an effective tool for studying local topological charge structure without modifying (e.g cooling) the gauge field.

-- Overlap construction of Q derived from structure of the chiral anomaly on the lattice. Chiral symmetry and non-ultralocality of D lead to a smoothing of short-range fluctuations allowing the possibility of observing long-range coherent structure.

Result of first study of Q(x) distribution in 4D QCD (Horvath, et al, Phys. Rev. D (2004)):

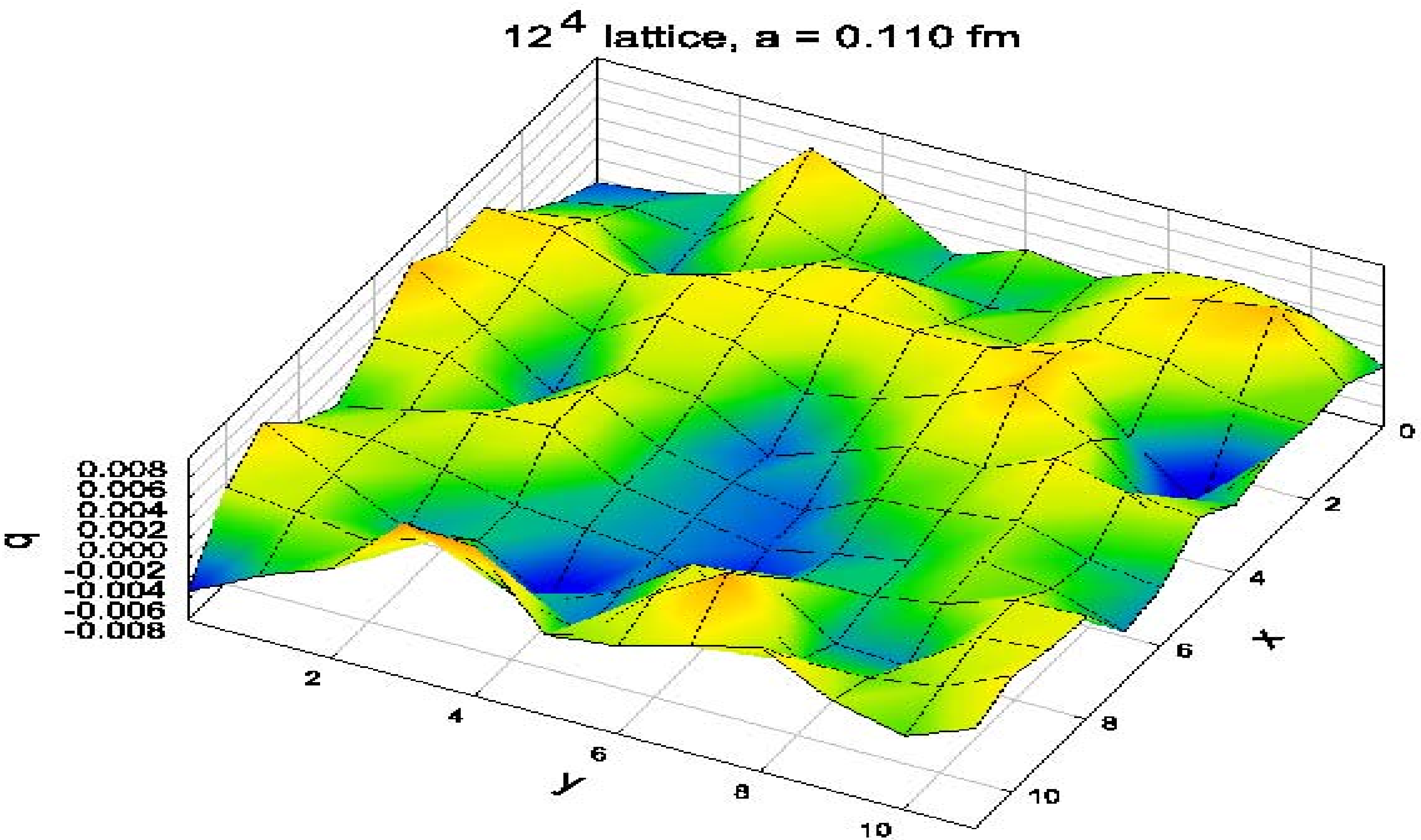
Results:
-- Only small 4D coherent structures found with sizes of O(a) and integrated Q(x)<<1. (No instantons.)

-- Large coherent structures are observed which are locally 3-D sheets in 4-D space (surfaces of codimension 1), typically only ~1 or 2 lattice spacings thick in transverse direction.

-- In each configuration, two sheets of opposite charge are found, which are everywhere close to each other and "crumpled," occupying the bulk of 4D space.

$\Rightarrow$ Short range, negative TCh correlator (required by spectral decomposition).

(Note: Another fundamental problem with dominance of instantons or any 4D coherent structures: Positive TCh correlator violates spectral requirement that correlator <0 for all nonzero separation.)



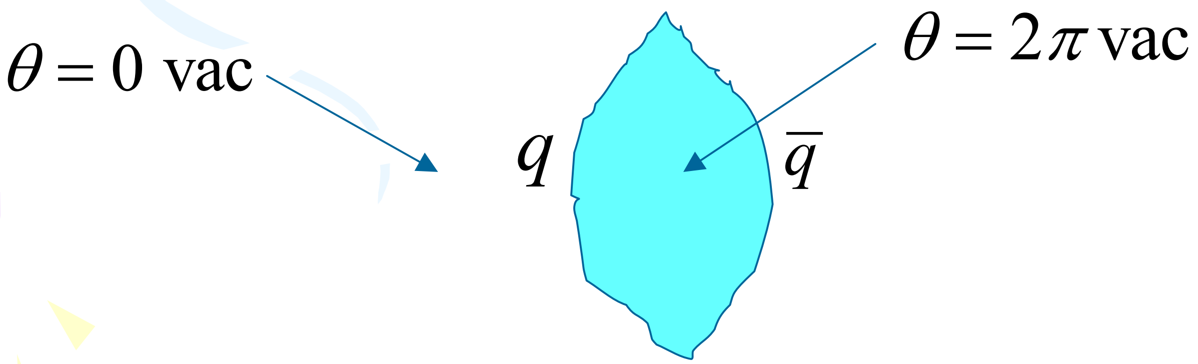
2D slice of Q(x) distribution for 4D QCD

The ADS/CFT holographic view of topological charge in the QCD vacuum has an analog in 2D U(1) theories:

--Multiple discrete k-vacua characterized by an effective value of θ which differs from the θ in the action by integer multiples of 2π.

-- Interpretation of effective θ similar to Coleman's discussion of 2D massive Schwinger model (Luscher (1978), Witten (1979,1998)), where $\theta =$ background E field.

In 2D U(1) models (CP(N-1) or Schwinger model): Domain walls between k-vacua are world lines of charged particles:



Precise analogy between U(1) in 2D and SU(N) in 4D (Luscher, 1978):

◆ **Identify Chern-Simons currents for the two theories.**

$$A_\mu \quad \rightarrow \quad A_{\mu\nu\sigma} \equiv -Tr\left\{ A_\mu A_\nu A_\sigma + \frac{3}{2} A_{[\mu}\partial_\nu A_{\sigma]} \right\}$$

$$j_\mu^{CS} = \varepsilon_{\mu\nu} A_\nu \quad \rightarrow \quad j_\mu^{CS} = \varepsilon_{\mu\nu\sigma\tau} A_{\nu\sigma\tau}$$

$$Q = \partial_\mu j_\mu^{CS} \quad \rightarrow \quad Q = \partial_\mu j_\mu^{CS}$$

Wilson line → integral over 3-surface

charged particle → charged membrane

(= domain wall) (= domain wall)

In both cases, CS current correlator has massless pole $\sim 1/q^2$

This analogy suggests that the coherent 3D structures recently found in 4D QCD should have an analog of 1D coherent structures in 2D U(1) gauge theory.

CP(N-1) models on the lattice (Seiberg, 1984)

$$S = \beta\sum_{x,\hat{\mu}} z^*(x)U(x,x+\hat{\mu})z(x+\hat{\mu}) + h.c$$

Here z = N-component scalar, and U = U(1) gauge field.

Monte Carlo is done with a Cabibbo-Marinari heat bath for the z's and 10-hit Metropolis for the U links.

Overlap Dirac op is defined on the U(1) field and solved exactly using the LAPACK singular value decomposition routine.

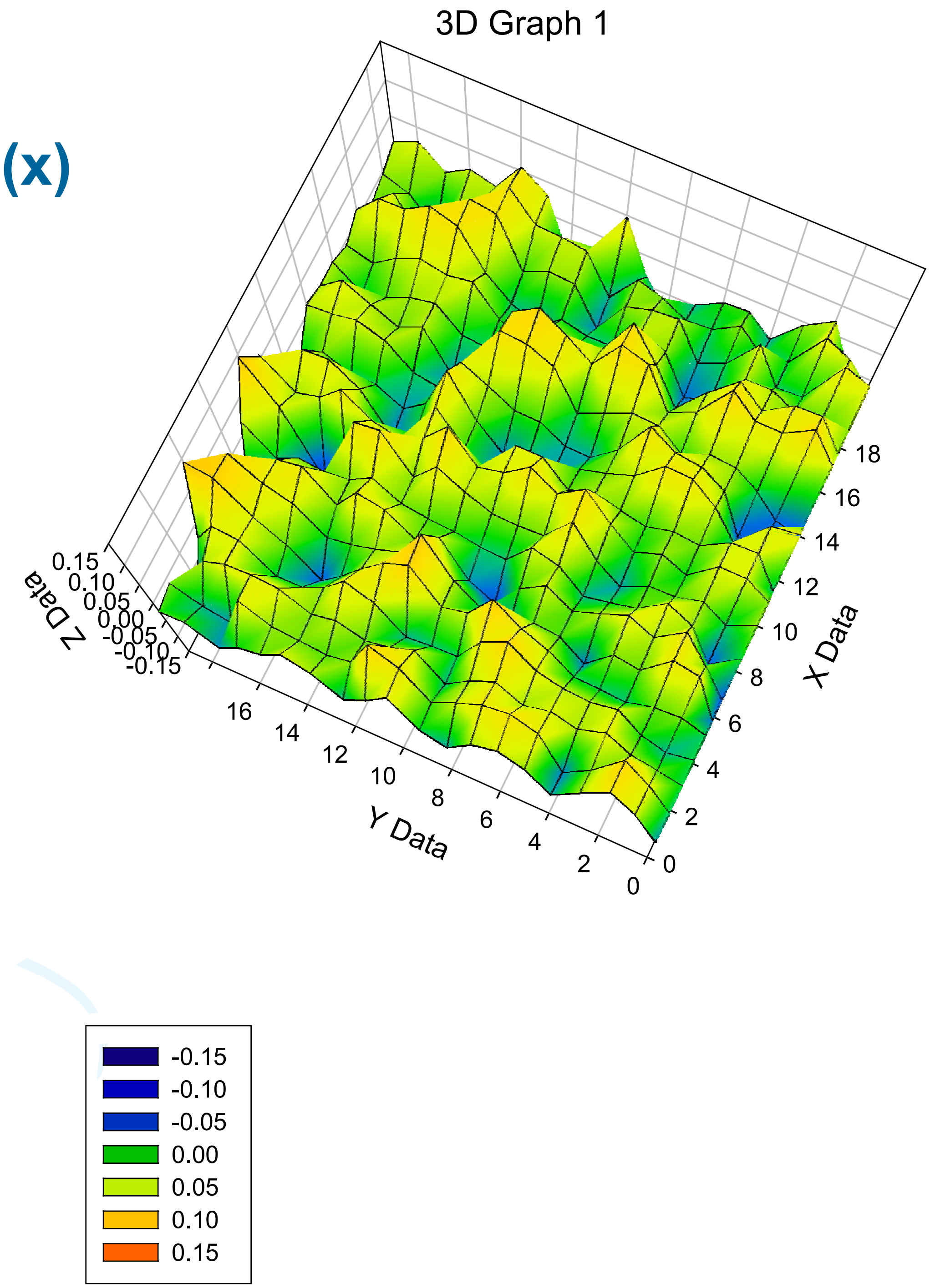
$$\text{SVD on } D_W - 1 = U\Lambda\tilde{U}$$

$$\text{overlap is } \quad D = 1 + U\tilde{U}$$

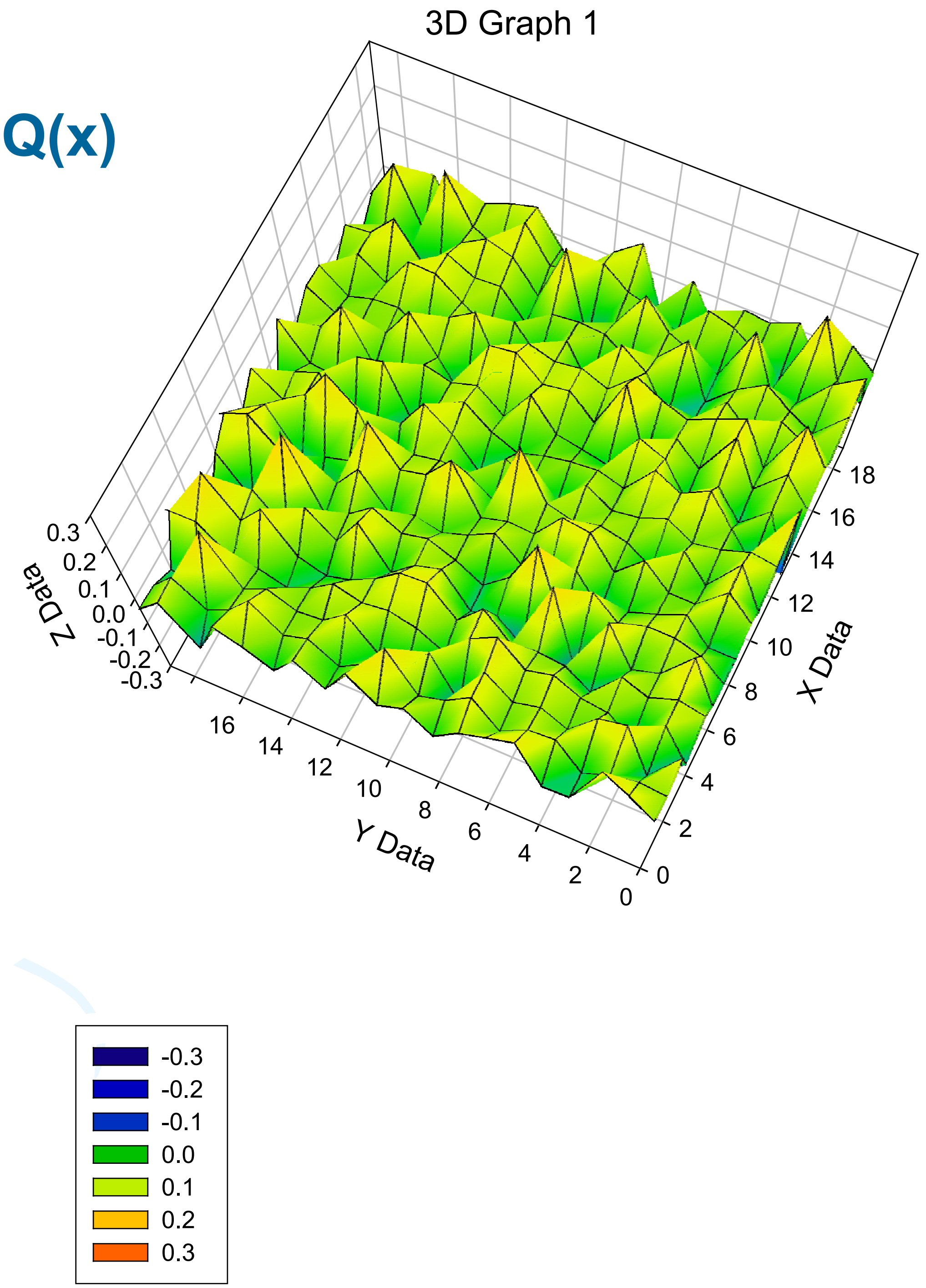
Timing: Overlap calculation of topological charge on a single U(1) config:

20x20	1 minute
50x50	7 hours

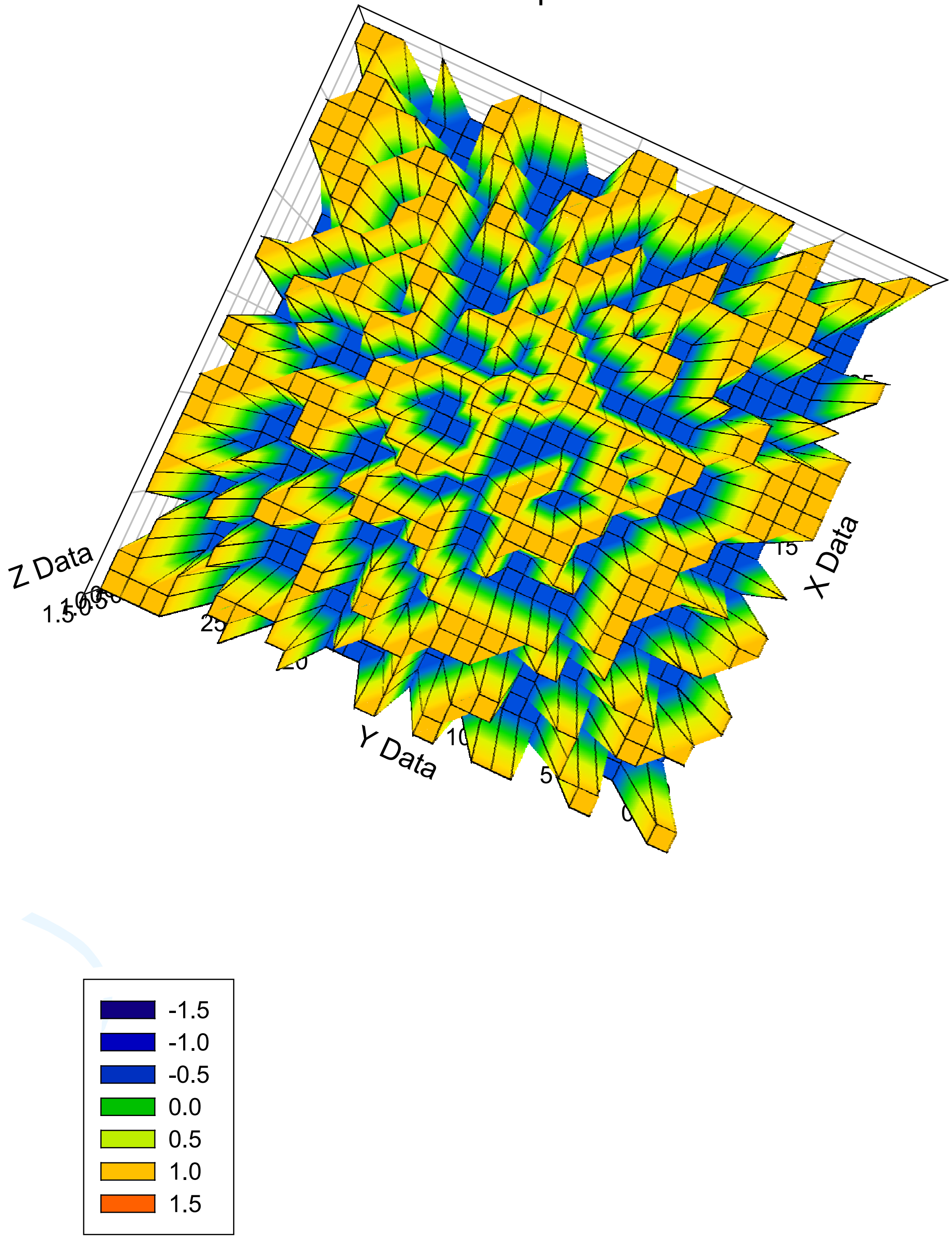
Overlap Q(x)

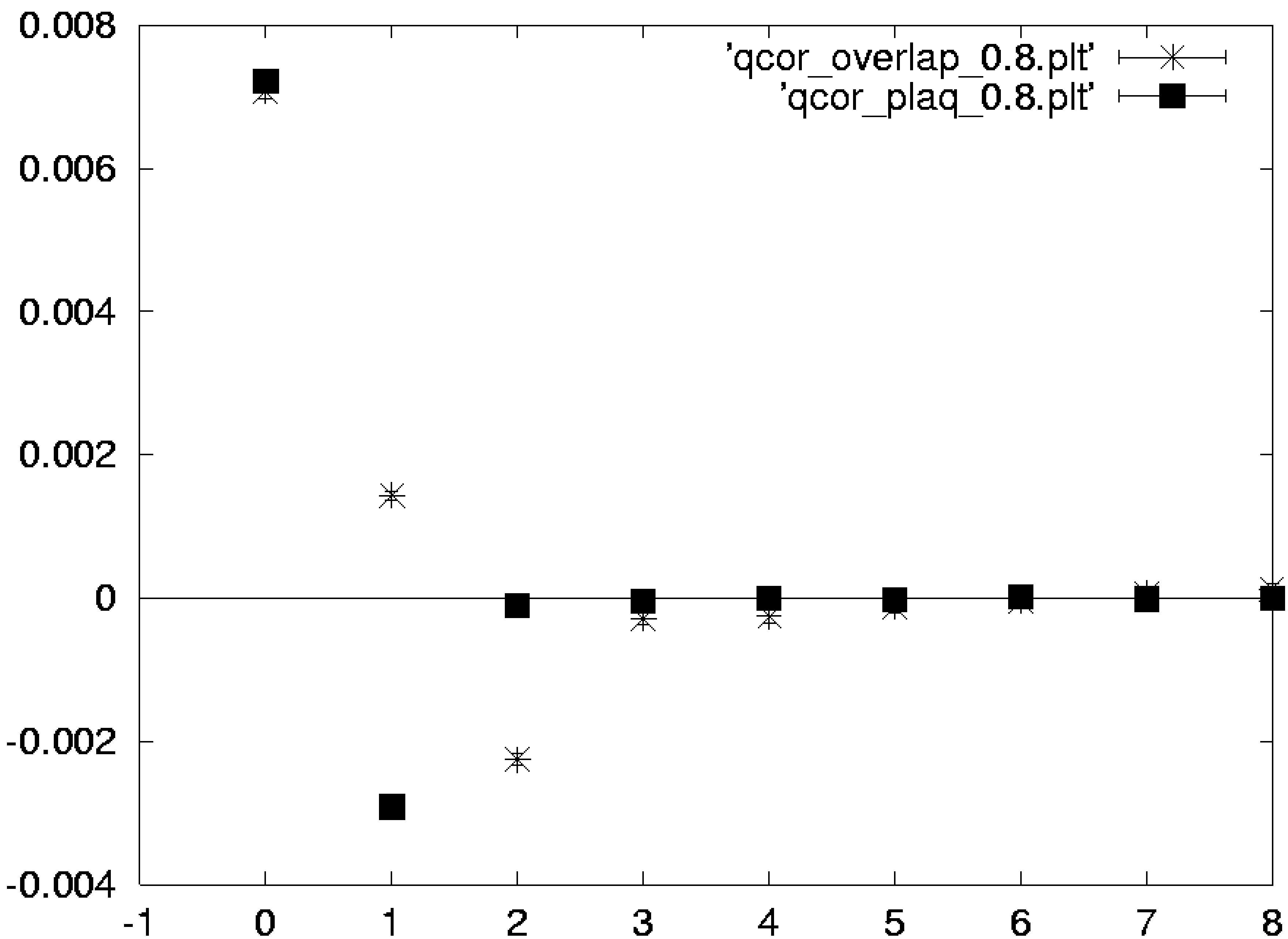

3D Graph 1
Z Data
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
Y Data
0
2
4
6
8
10
12
14
16
X Data
0
2
4
6
8
10
12
14
16
18
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15

Ultralocal Q(x)



3D Graph 1

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'qcor_plaq_0.8.plt'
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-1
0
1
2
3
4
5
6
7
8

Estimate Hausdorff dimension of largest structures on each configuration:

- Starting from each point on structure, draw a circle of radius r and count n(r) = number of points on structure inside circle.
- Fit to $n(r) = \text{const.} \times r^{d}$
- Preliminary result: d=1.32

For comparison, generate spin configurations for 2D Ising model slightly above T_c and adjust T so that total volume of largest domains are equal to size of topological charge structures in CPN configs.

Result: Ising d=1.85

i.e. Ising domains are approximately “spherical” while CPN structures are approximately one-dimensional.

Conclusions:

- In both 4D QCD and 2D CPN, topological charge excitations exhibit long range order on surfaces of dimension D-1 (codimension 1).

- Lattice results strongly support a "membraney" view of topological charge excitations in the QCD vacuum, similar to that implied by ADS/CFT duality.

- Picture of QCD vacuum as a liquid of membranes may give a unified view of how Topological Charge, Quark Confinement, and XSB are related:

  XSB: Goldstone bosons "skate" along membranes

  QC: Unlike instantons, membranes separate space into two disjoint regions, so color phases can be incoherent on opposite sides of membrane.

  $\Rightarrow$ disordered vacuum $\Rightarrow$ confinement