Membrane Structure of Topological Charge Fluctuations in 2D and 4D Gauge Theory

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Topological Charge Fluctuations in QCD:

- We have several different possible views of the QCD vacuum
  - 1. “Strong coupling” view of vacuum: color phases are disordered at large distances
    -- Naturally incorporates quark confinement
    -- chiral SB and $\eta'$ mass mechanism is obscure (Aoki phases…)
  - 2. Instanton liquid model populates vacuum with localized, self-dual and anti-self-dual quantized lumps of topological $F\tilde{F}$ charge.
    -- Naturally incorporates chiral SB and $\eta'$ mass.
    -- **Does not** incorporate confinement. (Too much long-range order.)
Fundamental new development in QCD: ADS/CFT duality

- 4D QCD = IIA String Theory in a black hole metric
  (See E. Witten, “Black Holes and Quark Confinement”, Current Science, V.81(2002))

  e.g. Confinement “explained” by

  QCD flux tube = holographic projection of fundamental string

  ADS/CFT also confirms Witten’s (1979) large-\(N_c\) view of topological charge:

  - Multiple vacuum states (“k-vacua”) with \(\theta_{\text{eff}} = \theta + 2\pi k\)
  - Local k-vacua separated by domain wall = membrane
  - Domain wall = fundamental 6-brane of IIA string theory wrapped around \(S_4\)
  - \(k=\text{integer}\) is a Dirac quantization of 6-brane charge in 9+1 D (Note: 6+2 = 9-1, hence 2D surface surrounds 6-brane in 9 space dimensions.)
Exactly chiral Dirac operators: A new method for studying topological charge on the lattice:

Discovery of exactly chiral Ginsparg-Wilson fermions provided a new definition of topological charge on the lattice (Hasenfratz, et al. 1998):

\[ \text{local TC} \equiv Q(x) = \frac{1}{2} \text{tr} \gamma^5 D(x, x) \]

-- Overlap Dirac operator provides an effective tool for studying local topological charge structure without modifying (e.g. cooling) the gauge field.

-- Overlap construction of Q derived from structure of the chiral anomaly on the lattice. Chiral symmetry and non-ultralocality of D lead to a smoothing of short-range fluctuations allowing the possibility of observing long-range coherent structure.
Result of first study of $Q(x)$ distribution in 4D QCD (Horvath, et al, Phys. Rev. D (2004)):

Results:
-- Only small 4D coherent structures found with sizes of $O(a)$ and integrated $Q(x) \ll 1$. (No instantons.)

-- Large coherent structures are observed which are locally 3-D sheets in 4-D space (surfaces of codimension 1), typically only ~1 or 2 lattice spacings thick in transverse direction.

-- In each configuration, two sheets of opposite charge are found, which are everywhere close to each other and “crumpled,” occupying the bulk of 4D space.

⇒ Short range, negative TCh correlator (required by spectral decomposition).

(Note: Another fundamental problem with dominance of instantons or any 4D coherent structures: Positive TCh correlator violates spectral requirement that correlator $<0$ for all nonzero separation.)
$12^4$ lattice, $a = 0.110$ fm

2D slice of $Q(x)$ distribution for 4D QCD
The ADS/CFT holographic view of topological charge in the QCD vacuum has an analog in 2D U(1) theories:

-- Multiple discrete k-vacua characterized by an effective value of $\theta$ which differs from the $\theta$ in the action by integer multiples of $2\pi$.

-- Interpretation of effective $\theta$ similar to Coleman’s discussion of 2D massive Schwinger model (Luscher (1978), Witten (1979,1998)), where $\theta = \text{background E field}$.

In 2D U(1) models (CP(N-1) or Schwinger model): Domain walls between k-vacua are world lines of charged particles:

\[ \theta = 0 \text{ vac} \quad \theta = 2\pi \text{ vac} \]
Precise analogy between U(1) in 2D and SU(N) in 4D (Luscher, 1978):

- Identify Chern-Simons currents for the two theories.

\[ A_\mu \rightarrow A_{\mu\nu\sigma} = -\text{Tr} \begin{split} \frac{1}{2} \epsilon_{\mu\nu\sigma} A_\nu A_\sigma + \frac{3}{2} A_\mu \partial_\nu A_\sigma \end{split} \]

\[ j^{CS}_\mu = \epsilon_{\mu\nu} A_\nu \rightarrow j^{CS}_\mu = \epsilon_{\mu\nu\sigma\tau} A_{\nu\sigma\tau} \]

\[ Q = \partial_\mu j^{CS}_\mu \rightarrow Q = \partial_\mu j^{CS}_\mu \]

Wilson line \rightarrow integral over 3-surface

charged particle \rightarrow charged membrane

(= domain wall) \hspace{1cm} (= domain wall)

In both cases, CS current correlator has massless pole \( \sim 1/q^2 \)

This analogy suggests that the coherent 3D structures recently found in 4D QCD should have an analog of 1D coherent structures in 2D U(1) gauge theory.
**CP(N-1) models on the lattice** (Seiberg, 1984)

\[ S = \beta \sum_{x, \mu} z^*(x) U(x, x + \mu) z(x + \mu) + h.c \]

Here \( z \) = N-component scalar, and \( U = U(1) \) gauge field.

Monte Carlo is done with a Cabibbo-Marinari heat bath for the \( z \)'s and 10-hit Metropolis for the \( U \) links.

Overlap Dirac op is defined on the \( U(1) \) field and solved exactly using the LAPACK singular value decomposition routine.

\[ \text{SVD on } D_w - 1 = U \Lambda \tilde{U} \]

Overlap is \( D = 1 + U \tilde{U} \)

**Timing:** Overlap calculation of topological charge on a single \( U(1) \) config:

- 20x20 1 minute
- 50x50 7 hours
Overlap $Q(x)$
Ultralocal $Q(x)$
Estimate Hausdorff dimension of largest structures on each configuration:

- Starting from each point on structure, draw a circle of radius $r$ and count $n(r) = \text{number of points on structure inside circle}$.
- Fit to $n(r) = \text{const} \times r^d$
- Preliminary result: $d = 1.32$

For comparison, generate spin configurations for 2D Ising model slightly above $T_c$ and adjust $T$ so that total volume of largest domains are equal to size of topological charge structures in CPN configs.

Result: Ising $d = 1.85$

i.e. Ising domains are approximately “spherical” while CPN structures are approximately one-dimensional.
Conclusions:

- In both 4D QCD and 2D CPN, topological charge excitations exhibit long range order on surfaces of dimension D-1 (codimension 1).
- Lattice results strongly support a “membraney” view of topological charge excitations in the QCD vacuum, similar to that implied by ADS/CFT duality.
- Picture of QCD vacuum as a liquid of membranes may give a unified view of how Topological Charge, Quark Confinement, and XSB are related:
  
  **XSB:** Goldstone bosons “skate” along membranes
  
  **QC:** Unlike instantons, membranes separate space into two disjoint regions, so color phases can be incoherent on opposite sides of membrane.

  ⇒  disordered vacuum  ⇒  confinement