The Intersection of Kaons, Lattices and Accuracy

Robert D. Mawhinney
Columbia University
RBC Collaboration

Matching Light Quarks to Hadrons
Benasque, Spain
August 2, 2004

1. Some lattice considerations
2. Domain wall fermions and chiral symmetry
3. B_K
4. $\Delta I = 1/2$ rule, Q_8 and ϵ'/ϵ

RBRC-BNL-CU (RBC) Collaboration
July 2004

RBRC	BNL	Columbia
Blum, Tom	Creutz, Mike	Aubin, Christopher
Dawson, Chris	Laiho, Jack	Christ, Norman
Doi, Takumi	Jung, Chulwoo	Cohen, Saul
Izubuchi, Taku	Petreczky, Peter	Clark, Michael
(Kanazawa)	Petrov, Konstantin	Kim, Changhoan
Ohta, Shigemi (KEK)	Soni, Amarjit	Levkova, Ludmila
Yamada, Norikazu		Li, Sam
Yamazaki, Takeshi		Lin, Meifeng
		Lin, HueyWen
Aoki, Yasumichi		Mawhinney, Robert
Nemoto, Yukio		
Noaki, Jun-Ichi		
Orginos, Kostas		

Improved Fermion Actions

	$SU_V(N_f)$	$SU_A(N_f)$	$U_V(1)$	$U_A(1)$
Wilson clover	$\surd$	$\times$ $\mathcal{O}(a^2)$	$\surd$	$\times$ $\mathcal{O}(a^2)$
ASQTAD staggered	$\times$ $\mathcal{O}(a^2)$ discrete subgroup	$\times$ $\mathcal{O}(a^2)$ $U(1)$ subgroup	$\surd$	$\times$ $\mathcal{O}(a^2)$
	($4N_f$ flavors on lattice from fermion doubling)			
domain wall	$\surd$	$\surd$ $\mathcal{O}\left(ae^{-\alpha L_s}\right)$	$\surd$	$\surd$ $\mathcal{O}\left(ae^{-\alpha L_s}\right)$
	(for modes bound to 4-d walls)			

- Wilson clover fermions markedly improves chiral symmetry.
- ASQTAD staggered fermions have much smaller $\mathcal{O}(a^2)$ flavor breaking.
- DWF also gives off-shell improvement.

Domain Wall Fermion Operator

- Introduce extra dimension, labeled by s

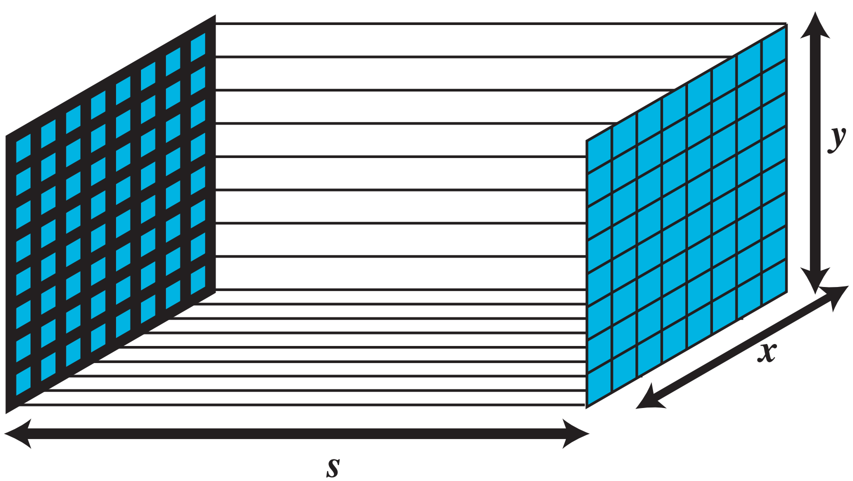
$$D_{x,s;x',s'} = \delta_{s,s'} D^{\parallel}_{x,x'} + \delta_{x,x'} D^{\perp}_{s,s'}$$

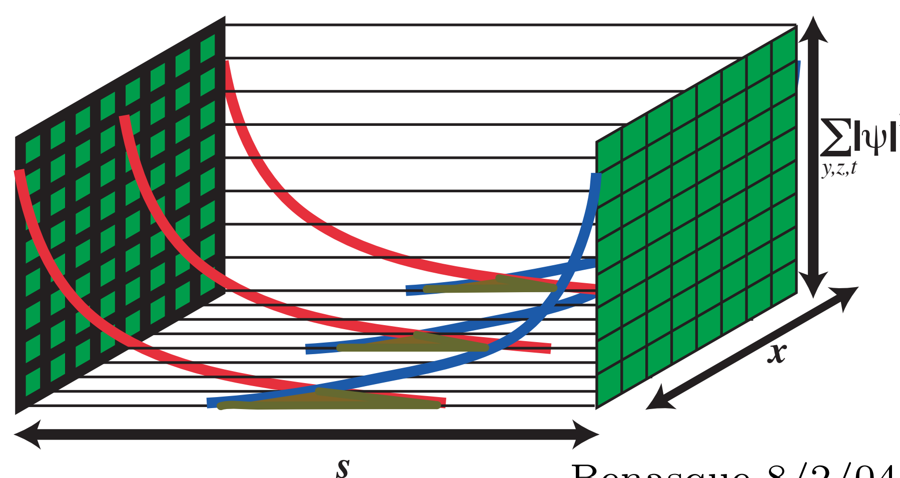
- $D^{\parallel}_{x,x'}$ is a Wilson Dirac operator with an opposite sign for the mass term.

$$D^{\parallel}_{x,x'} = \frac{1}{2} \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_{x,\mu} \delta_{x+\hat{\mu},x'} + (1+\gamma_\mu) U^{\dagger}_{x',\mu} \delta_{x-\hat{\mu},x'} \right] + (M_5 - 4)\delta_{x,x'}$$

- $D^{\perp}_{s,s'}$ couples in fifth dimension, distinguishing left- and right-handed fermions

$$D^{\perp}_{s,s'} = P_L \delta_{s+1,s'} + P_R \delta_{s-1,s'} - \delta_{s,s'} - m_f \left[P_l \delta_{s,L_s-1} \delta_{0,s'} + P_R \delta_{s,0} \delta_{L_s-1,s'} \right]$$





Residual Chiral Symmetry Breaking for DWF

- Consider introducing in action a $SU(N_f)$ matrix Ω through term at $l \equiv L_s/2$

$$-\sum_x \left\{ \overline{\Psi}_{x,l-1} P_L \left(\Omega^\dagger - 1\right) \Psi_{x,l} + \overline{\Psi}_{x,l} P_R \left(\Omega - 1\right) \Psi_{x,l-1} \right\} \qquad \Omega \to U_R \, \Omega \, U_L^\dagger$$

- Conventional DWF recovered by $\Omega \to 1$
- QCD chiral Lagrangian $\mathcal{L}^{(2)}_{\mathrm{QCD}}$, with $\Sigma \equiv \exp\left[2i\phi^a t^a/f\right]$ and mass matrix M is:

$$\frac{f^2}{8} \mathrm{Tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + v \, \mathrm{Tr}\left[M\Sigma + (M\Sigma)^\dagger\right] + v' \, \mathrm{Tr}\left[\Omega\Sigma + (\Omega\Sigma)^\dagger\right] + v'' \, \mathrm{Tr}\left[\Omega M^\dagger + \Omega^\dagger M\right]$$

- For modes bound to walls of fifth dimension, Ω enters Green's functions as

$$\Omega e^{-\alpha L_s} \quad \Rightarrow \quad v', v'' \sim e^{-\alpha L_s}$$

- Chiral condensate from differentiating w.r.t. mass, m_π^2 from expanding Σ

$$-\langle \overline{q} q \rangle (m_f = 0, L_s) \sim v + v'' \qquad v = \frac{f^2 m_{\pi^+}^2}{4(m_u + m_d + 2m_{\mathrm{res}})} \quad m_{\mathrm{res}} \equiv v'/v$$

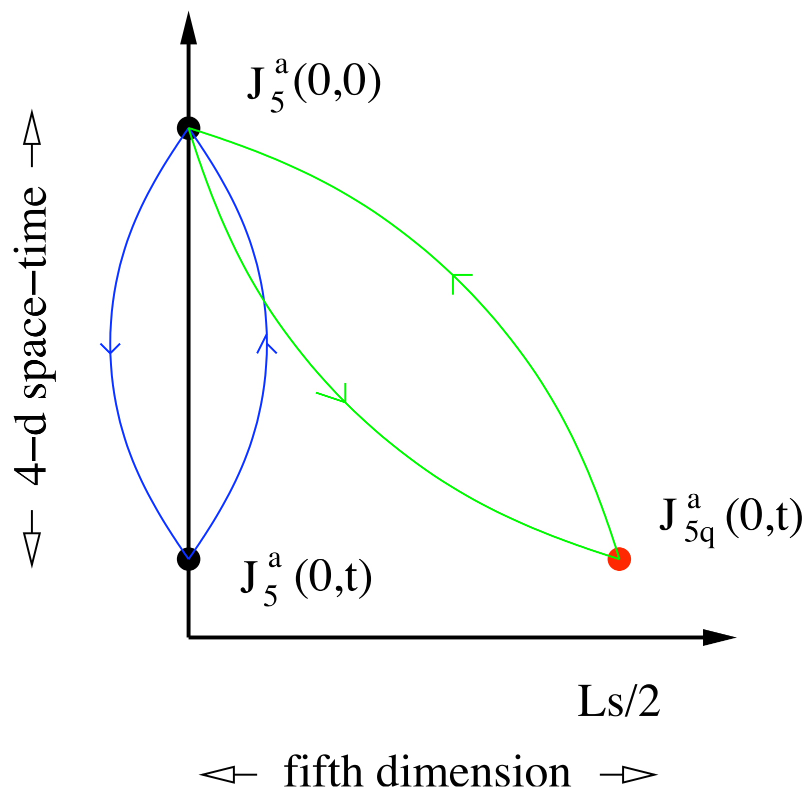
$N_f = 0$, $N_f = 2$ and $N_f = 3$ DWF Calculations by the RBC

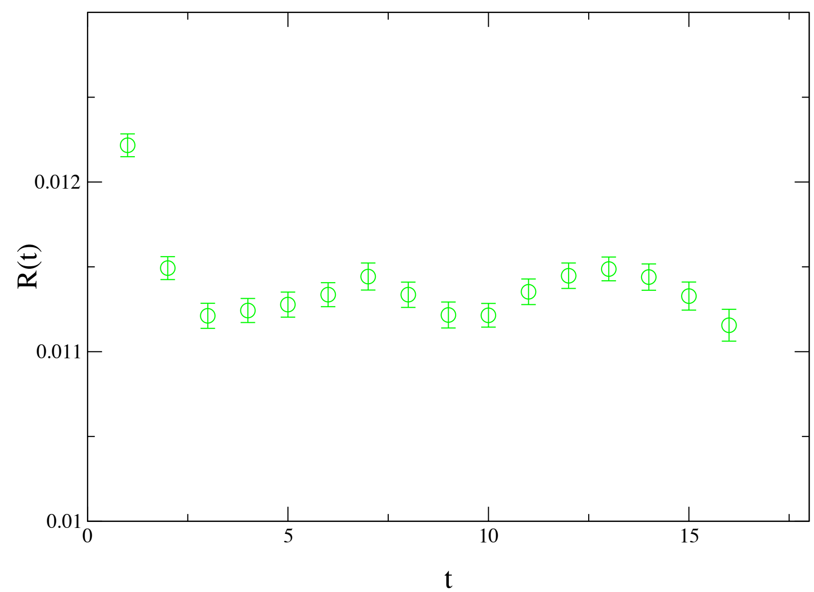
Parameter	$N_f = 0$	$N_f = 0$	$N_f = 2$	$N_f = 3$
Gauge action	Wilson	DBW2	DBW2	DBW2
β	6.0	1.04	0.80	0.72
Volume	$16^3 \times 32$	$16^3 \times 32$	$16^3 \times 32$	$16^3 \times 32$
L_s	16	16	12	8
a^{-1} (GeV)	1.92(4)	1.98(2)	1.70(5)	≈ 1.7
$m_{\rm res}$	$1.24(5) \times 10^{-3}$	$1.85(12) \times 10^{-5}$	$1.37(2) \times 10^{-3}$	$1.17(1) \times 10^{-2}$
Dyn. masses	–	–	0.02, 0.03, 0.04	0.04
Algorithm	HB	HB + OR	HMC	R
Trajectories	–	–	5361 (0.02) 6195 (0.03) 5605 (0.04)	1525

Measuring the residual mass $m_{\rm res}$ for $N_f = 3$

- Simplest use of divergence of axial curent: $\Delta_\mu \mathcal{A}^a_\mu(x) = 2m_f J^a_5(x) + 2J^a_{5q}(x)$
- Compare pion propagation along $s = 0$ and $L_s - 1$ with propagation to $L_s/2$

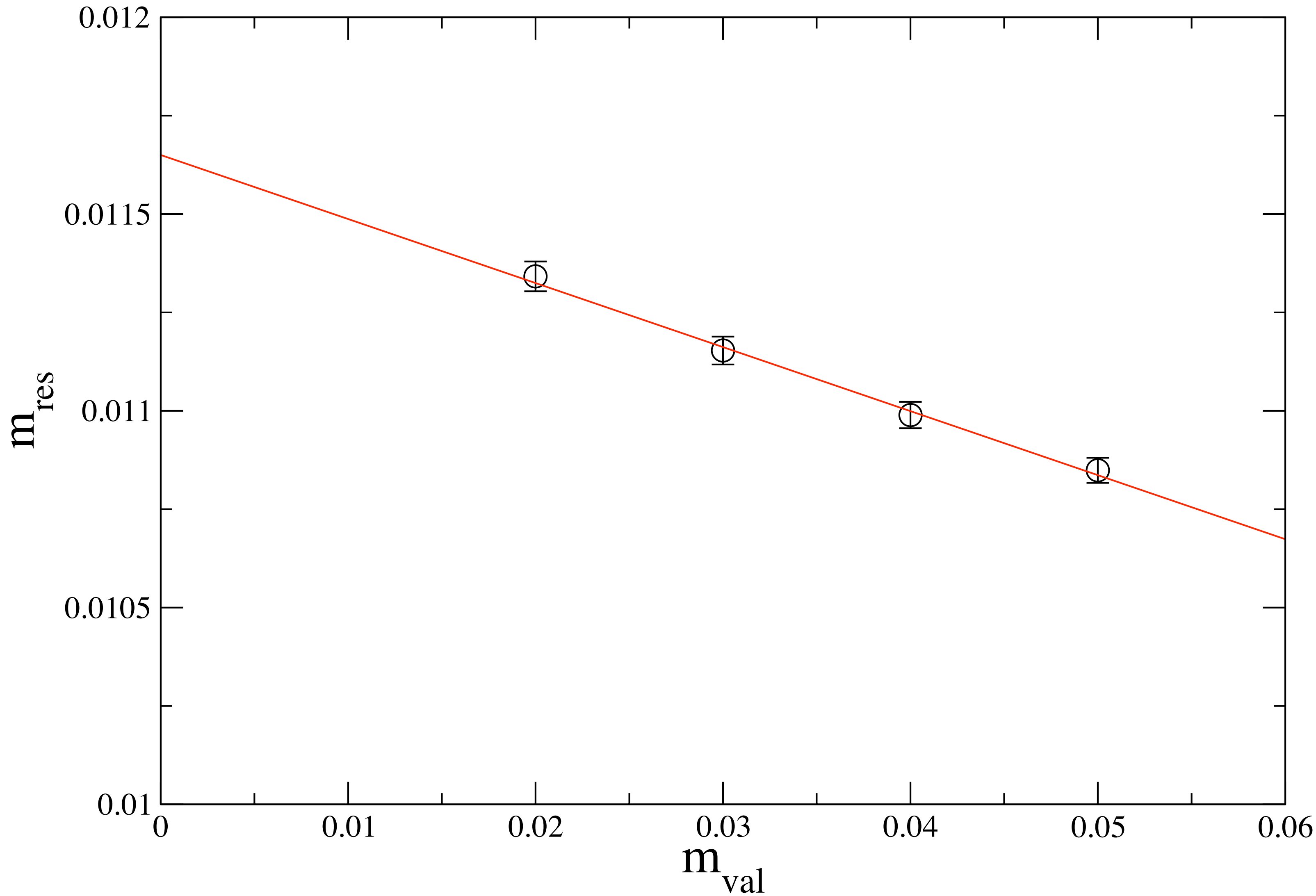
$$R(t) = \frac{\sum_{\vec{x}} \langle J^a_{5q}(\vec{x},t) J^a_5(0,0)\rangle}{\sum_{\vec{x}} \langle J^a_5(\vec{x},t) J^a_5(0,0)\rangle} \qquad m_{\rm res} \equiv \frac{1}{N}\sum_t R(t) \quad \text{if plateau}$$



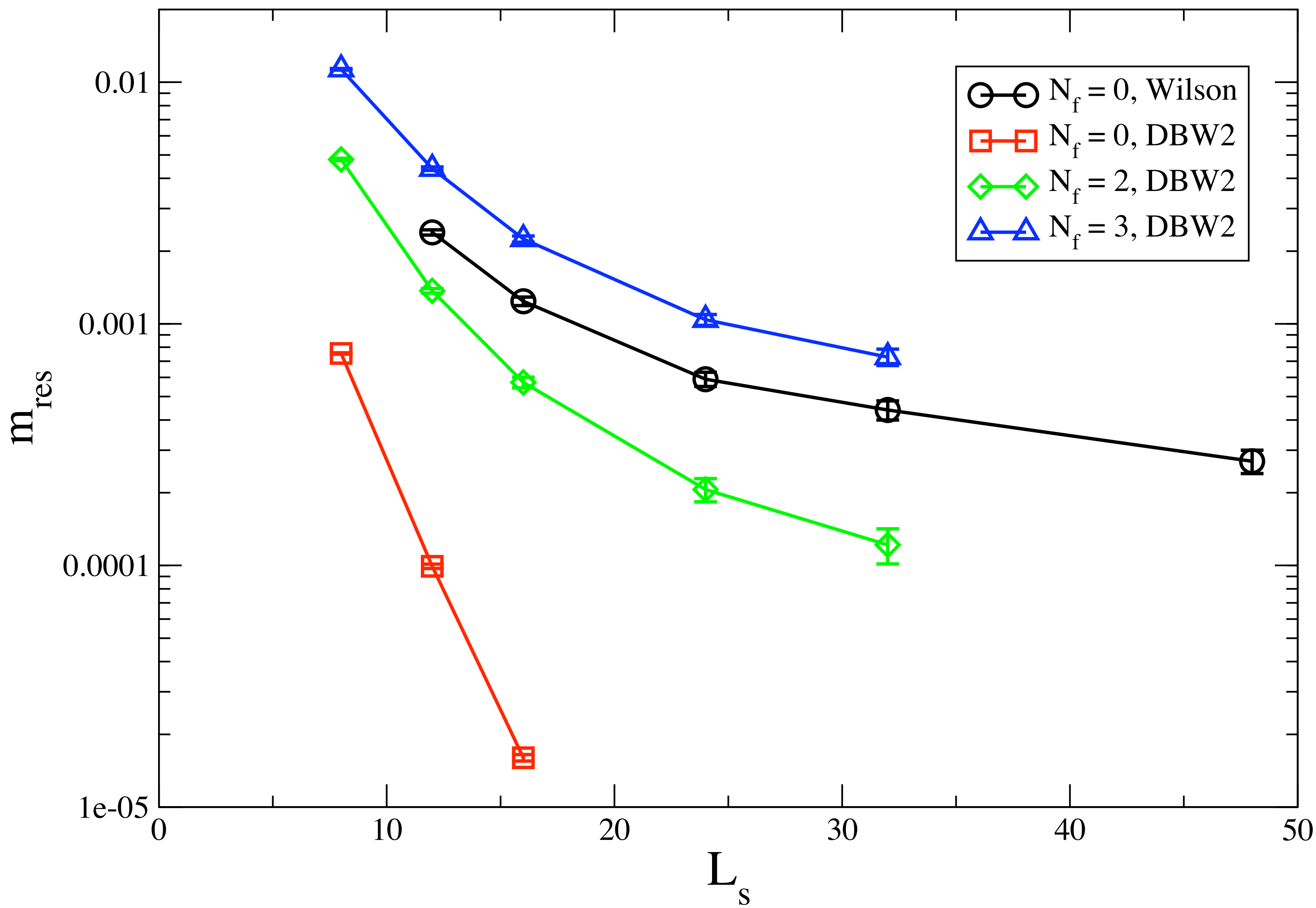


Residual Mass for $N_f = 3$ versus $m_{\rm val}$

- Extrapolating to $m_f = 0$ gives $m_{\rm res} = 0.0117(1)$



m_{res} versus L_s for $N_f = 0, 2$ and 3



CP Violation in the Kaon System

- Two amplitudes determine ϵ and ϵ'

$$\eta_{+-} = \frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} = \epsilon + \epsilon' \qquad \eta_{00} = \frac{A(K_L^0 \to \pi^0\pi^0)}{A(K_S^0 \to \pi^0\pi^0)} = \epsilon - 2\epsilon'$$

- SM: $\overline{K}^0 - K^0$ mixing via $Q^{(\Delta S=2)} = (\bar{s}_\alpha d_\alpha)_{V-A}\,(\bar{s}_\beta d_\beta)_{V-A}$ defines B_K as;

$$\langle \overline{K}^0 | Q^{(\Delta S=2)}(\mu) | K^0 | \rangle \equiv \frac{8}{3} B_K(\mu) f_K^2 m_K^2$$

- RGI parameter $\hat{B}_K \equiv B_K(\mu)\left[\alpha_s^{(3)}(\mu)\right]^{-2/9}\left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_3\right]$ relates SM and ϵ

$$\epsilon = \hat{B}_K\, \mathrm{Im}\lambda_t \frac{G_F^2 f_K^2 m_K M_W^2}{12\sqrt{2}\pi^2 \Delta M_K} \left\{\mathrm{Re}\lambda_c \left[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)\right] - \mathrm{Re}\lambda_t\, \eta_2 S_0(x_t)\right\} \exp(i\pi/4)$$

- Defining $A(K^0 \to \pi\pi(I)) \equiv A_I e^{(i\delta_I)}$, $P_2 \equiv \mathrm{Im}A_2/\mathrm{ReA}_2$, $P_0 \equiv \mathrm{Im}A_0/\mathrm{ReA}_0$:

$$\epsilon' = \frac{ie^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left(\frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0}\right)\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right) \qquad w \equiv \frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} \approx 22$$

Operator Mixing and Chiral Symmetry

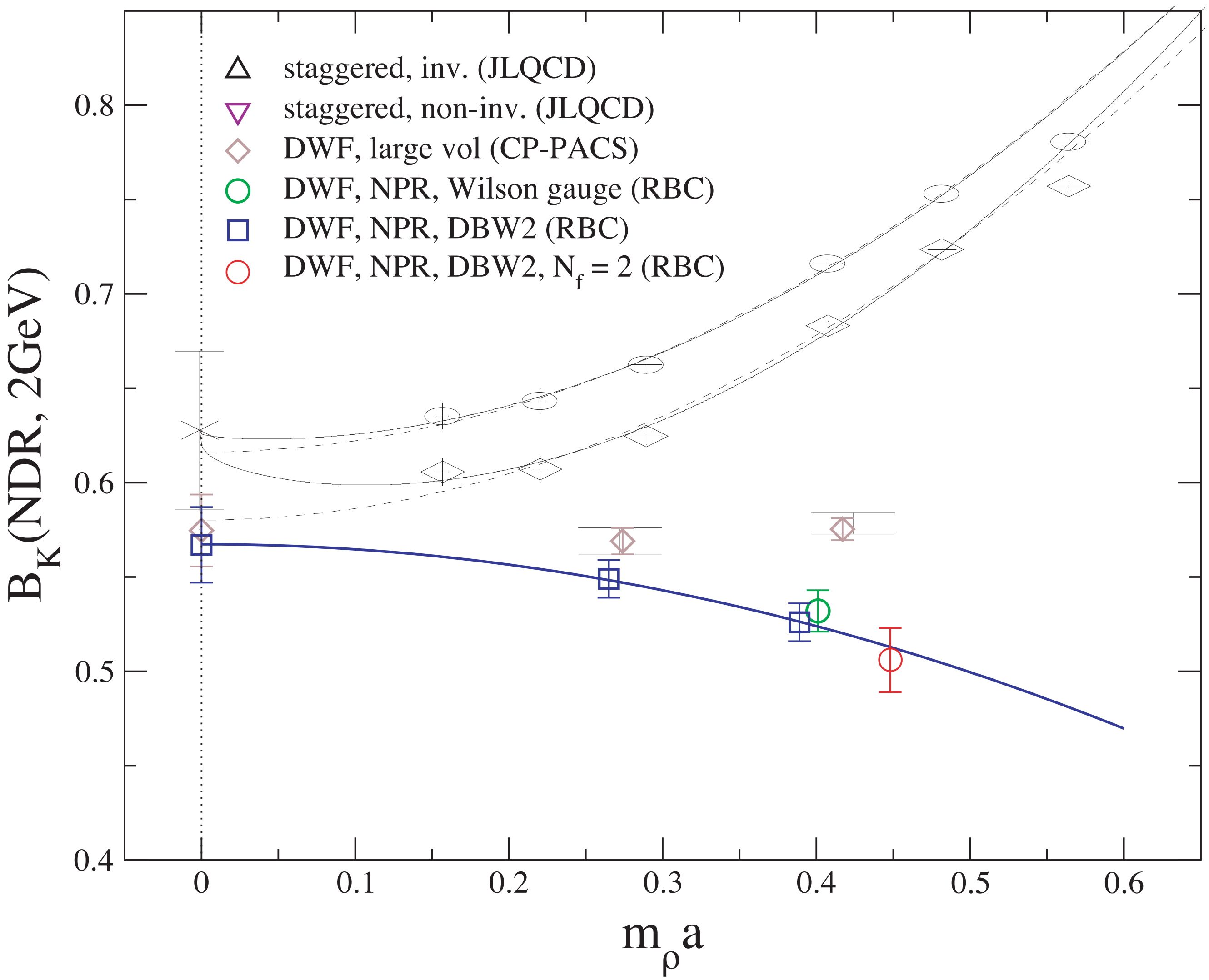
- Presence of lattice chiral symmetry markedly helps operator mixing
- Consider $Q^{(\Delta S=2)}$ as an example

$$
\begin{aligned}
\bar{s}^{\text{lat}}\gamma_\mu(1-\gamma_5)d^{\text{lat}}\ \bar{s}^{\text{lat}}\gamma_\mu(1-\gamma_5)d^{\text{lat}} &\equiv (\bar{s}^{\text{lat}}d^{\text{lat}})_{V-A}\ (\bar{s}^{\text{lat}}d^{\text{lat}})_{V-A} \\
&= Z_1(\mu a)(\bar{s}d)_{V-A}\ (\bar{s}d)_{V-A} \\
&+ Z_2(\mu a)(\bar{s}d)_{V+A}\ (\bar{s}d)_{V+A} \\
&+ Z_3(\mu a)(\bar{s}d)_{P-S}\ (\bar{s}d)_{P-S} \\
&+ Z_4(\mu a)(\bar{s}d)_{P+S}\ (\bar{s}d)_{P+S} \\
&+ Z_5(\mu a)(\bar{s}d)_{T}\ (\bar{s}d)_{T}
\end{aligned}
$$

- For DWF, Z_2, Z_3, Z_4, Z_5 are $\mathcal{O}(m_{\text{res}}^2)$, so small
- For DWF, use non-perturbative renormalization (NPR) (Rome-Southampton)
- Only reliance on continuum perturbation theory in OPE

The Kaon B Parameter, $B_K^{\overline{\rm MS}}(\mu = 2\ \text{GeV})$

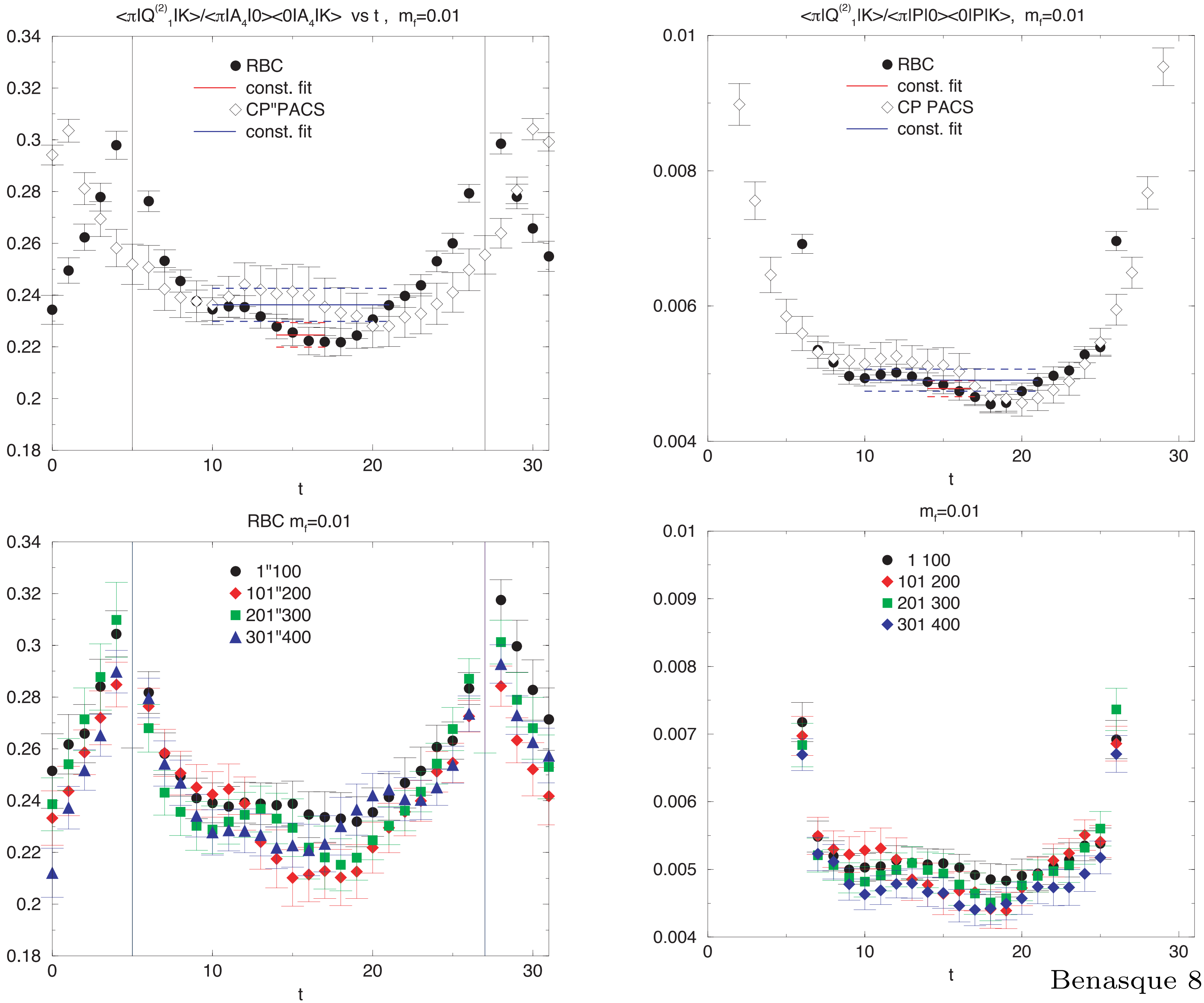
	quenched $a \to 0$			dyn. $a^{-1} = 1.7$ GeV
PDG	JLQCD (stag)	CP-PACS (DWF)	RBC (DWF)	RBC (2f DWF)
0.65 ± 0.15	0.628 ± 0.042	0.575 ± 0.019	0.570 ± 0.020	0.492 ± 0.018



Systematic and Statistical Errors in Lattice QCD

- Finite lattice spacing and volume: systematic
- Light quarks with unphysical masses: systematic
  (Can be minimized or removed with chiral extrapolations.)
- Insufficient sampling of gauge configuration space: statistical
- Observables from the same lattice can be strongly correlated: statistical
  (unreliable χ^2)
- Fitting ranges for masses and matrix elements: systematic
- Increasing statistical precision requires more care of fitting ranges.

Comparison of RBC and CP-PACS lattice data (Noaki)

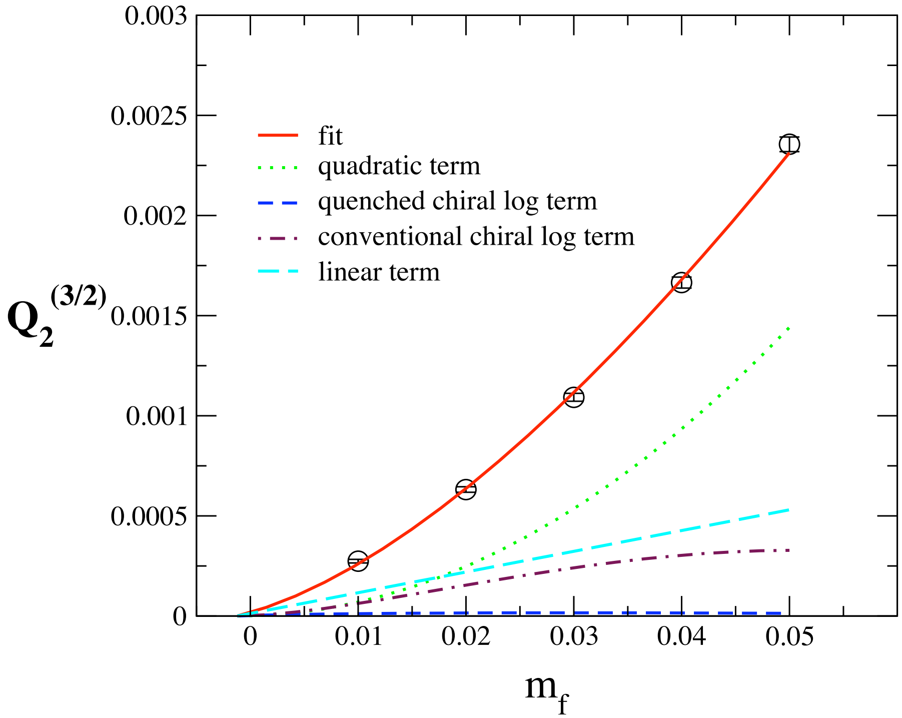


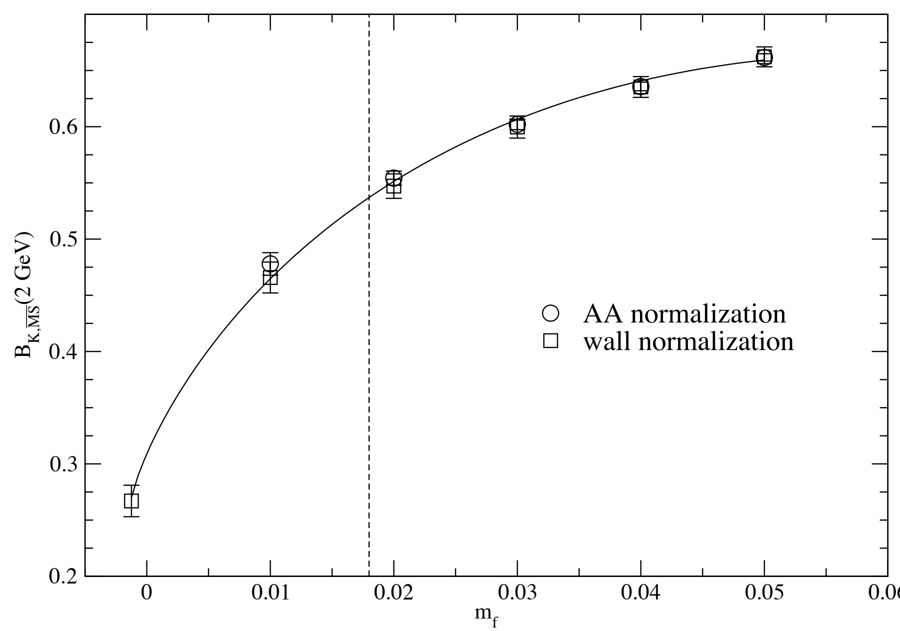
Quenched Chiral Extrapolation for (27,1) Operator

- Fit with known continuum chiral logarithm for quenched theory

$$1 - \frac{6m_M^2}{(4\pi f)^2} \ln(m_M^2/\Lambda^2)$$

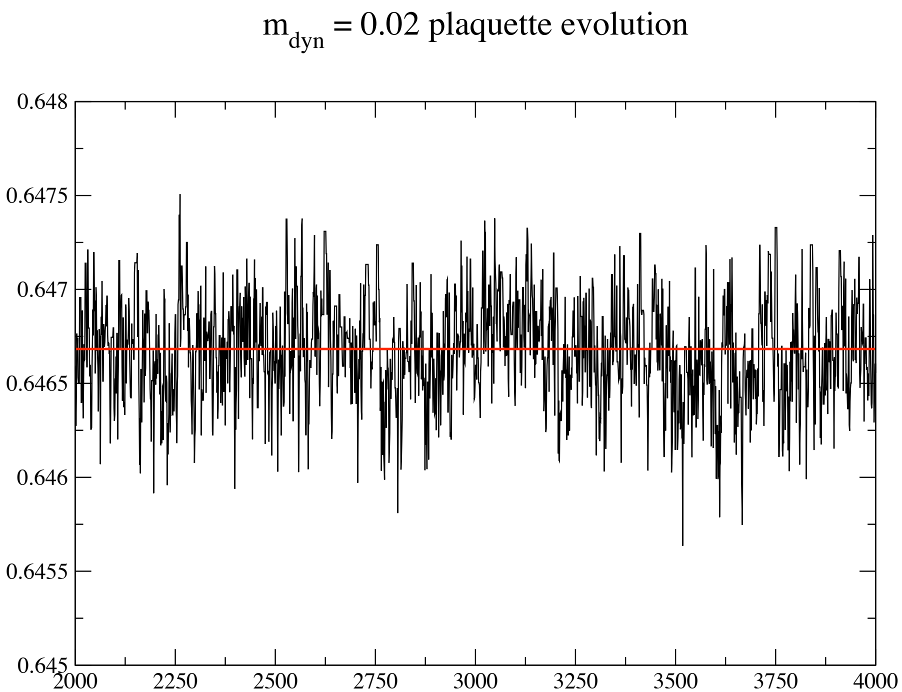
- Fit determines LO and NLO order constants (2 parameters)
- Good description of data, but 400 MeV $\leq m_{\rm PS} \leq$ 800 MeV.

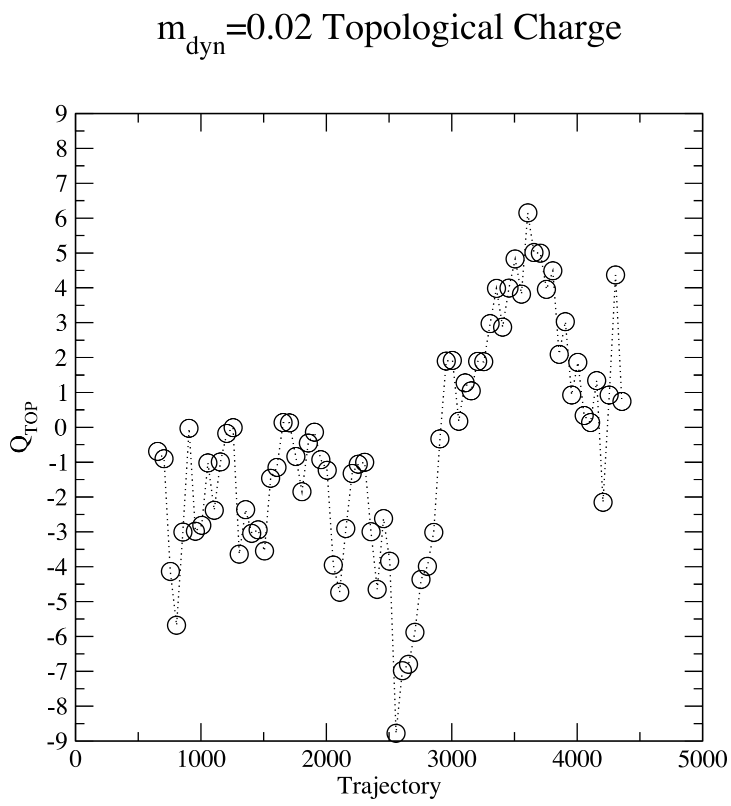




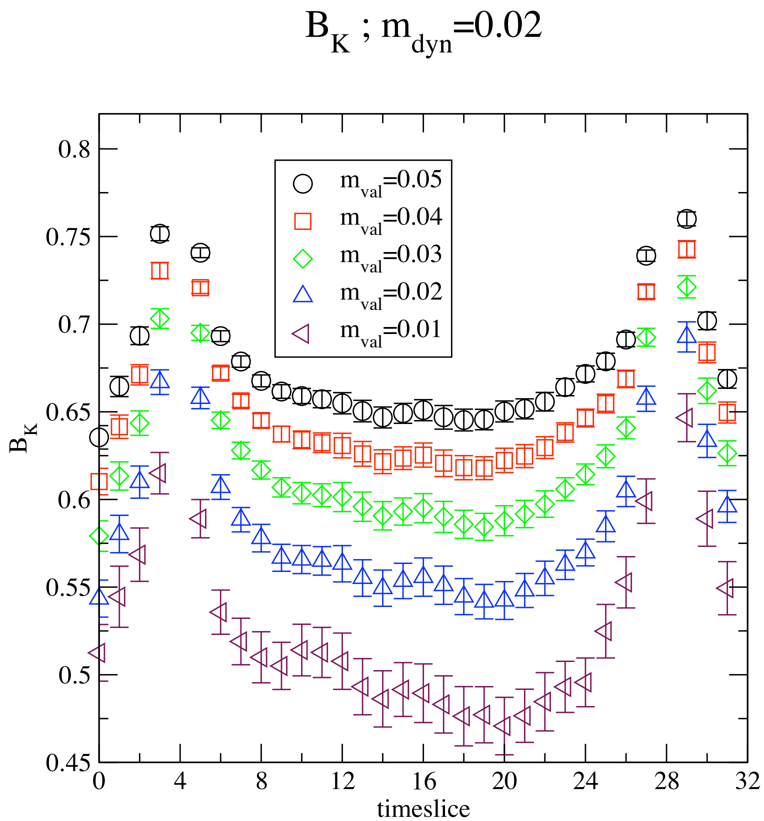
Evolution of $N_f = 2$ Lattices with $m_{\text{dyn}} \approx m_{\text{strange}}/2$

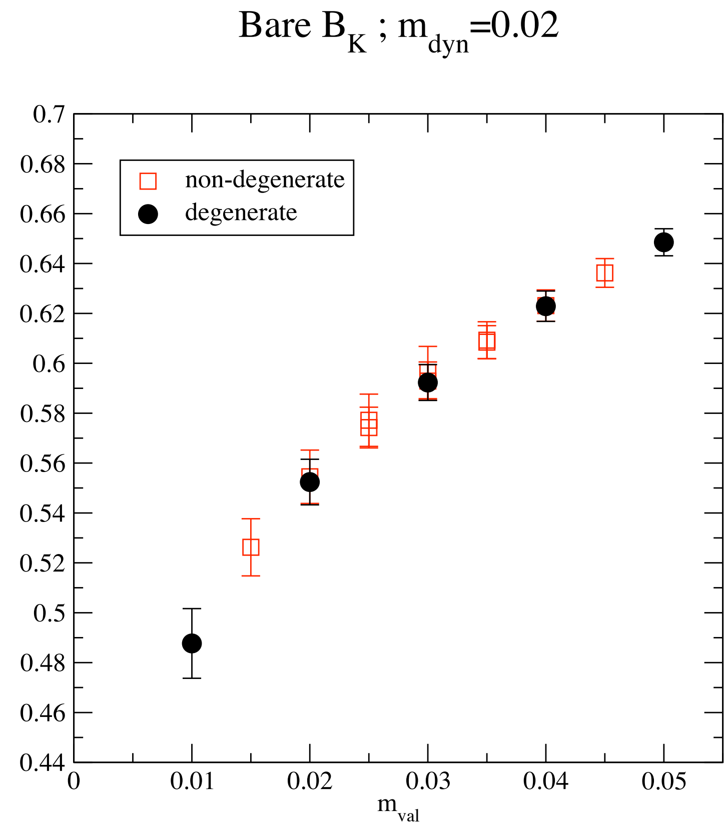
- Long autocorrelation times can yield underestimation of errors
- Topological fluctuations correctly weighted for DWF, if evolutions are long enough to sample phase space.





Improving B_K Determinations - Matrix Element Plateaus



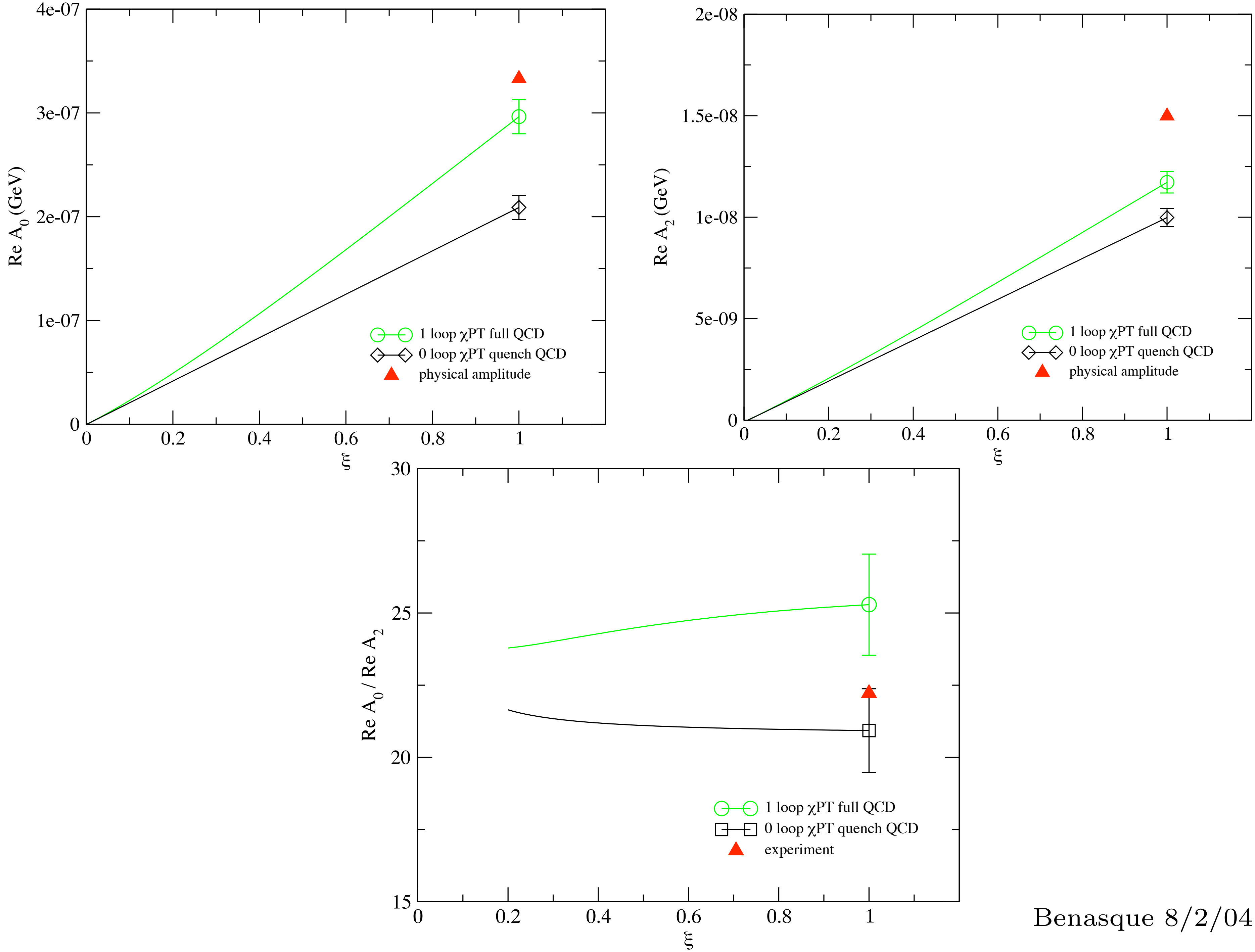


$m_{\text{dyn}} = 0.02$. The left-hand graph shows the plateau quality (degrades with decreasing valence quark mass). The right-hand graph shows the extracted bare B_K for all the combinations of masses.

$\langle \pi\pi | Q_i | K \rangle$ Matrix Elements

- Direct approaches to lattice measurements
  - Lellouch-Luscher finite volume approach
  - Kim-Christ variant of Lellouch-Luscher.
    Use finite volume with antiperiodic or G-parity quark boundary conditions.
    $\Delta I = 3/2$ matrix elements, in quenched theory with physical kinematics, within reach of QCDOC at $a^{-1} = 1.3$ GeV.
  - Sachradja talk tomorrow
- Chiral PT approaches
  - Limited by chiral perturbation theory errors at m_{strange}.
  - Numerically challenging to determine all required constants.
  - Quenched theory has different χPT than full theory (Golterman-Pallante)
    Talk by Jack Laiho this afternoon.
- Still substantial room for theoretical improvements.
- Determination of lowest-order constants for $Q^{(27,1)}$ $Q^{(8,1)}$, $Q^{(8,8)}$ for full QCD appears achievable in 1-2 year time.

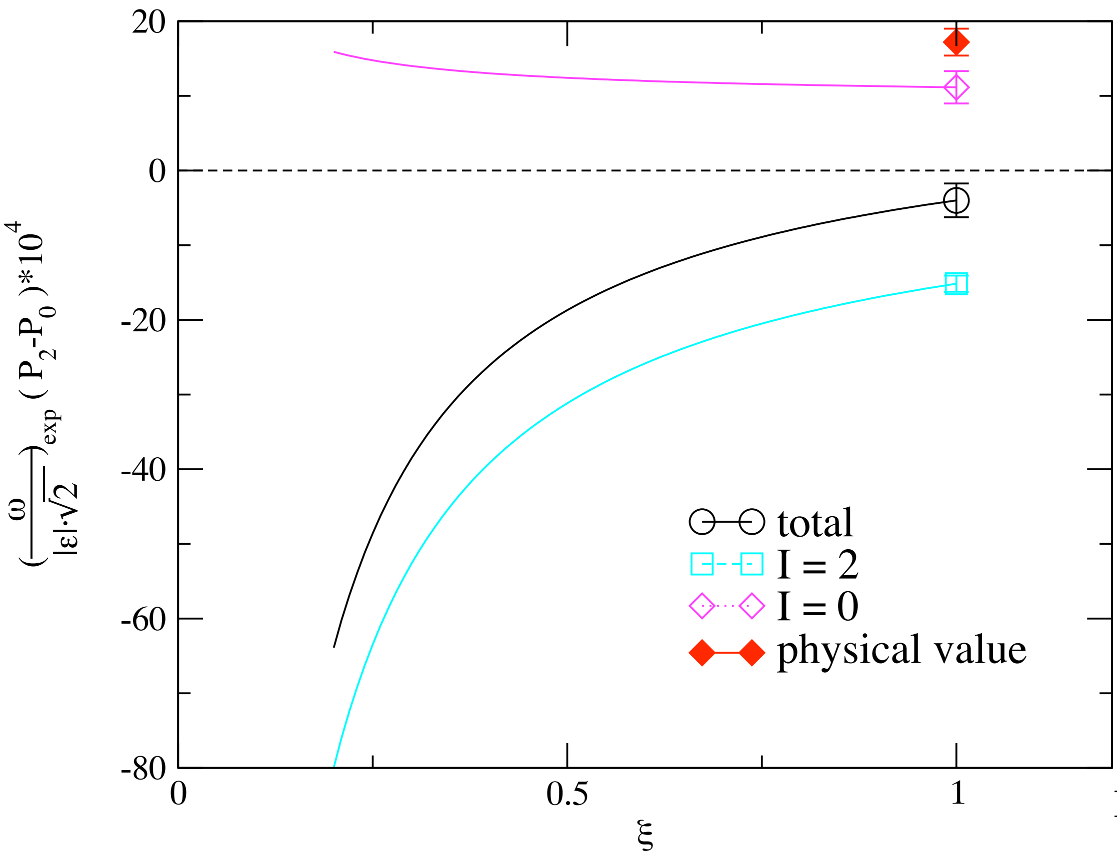
Real $K \to \pi\pi$ Amplitudes from Quenched QCD and χPT



ϵ'/ϵ from Quenched QCD and χPT

- Dominant contribution: Q_2 to Re A_2 and Re A_0, Q_6 to Im A_0, Q_8 to Im A_2.
- Contributions depend on renormalization scale GeV
- Schematic formula for ϵ'/ϵ

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \left(\frac{\omega}{\sqrt{2}|\epsilon|}\right)_{\rm exp} \left\{ \left[\frac{\alpha_{\rm W}\alpha_8}{\alpha_{\rm W}\alpha_8 + \alpha_2 m_{K^0}^2 \xi}\right]^{(3/2)} - \left[\frac{\alpha_{\rm W}\alpha_8 + \alpha_{\rm S}\alpha_6 m_{K^0}^2 \xi}{\alpha_{\rm W}\alpha_8 + \alpha_2 m_{K^0}^2 \xi}\right]^{(1/2)} \right\}$$



Estimating NLO ϵ'/ϵ from Quenched QCD and χPT

- Calculation determines all $K \to \pi\pi$ amplitudes to lowest order.
- ϵ'/ϵ not determined to consistent order in χPT
- Need NLO for $\Delta I = 3/2$. For physically relevant strange quark masses have:

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \left(\frac{\omega}{\sqrt{2}|\epsilon|}\right)_{\mathrm{exp}} \left\{ \left[\frac{\alpha_{\mathrm{W}}\alpha_8(1+c^{(8,8)}m_q + \mathrm{logs})}{\alpha_2 m_{K^0}^2 \xi(1+c^{(27,1)}m_q + \mathrm{logs})}\right]^{(3/2)} - \left[\frac{\alpha_{\mathrm{S}}\alpha_6}{\alpha_2}\right]^{(1/2)} \right\}$$

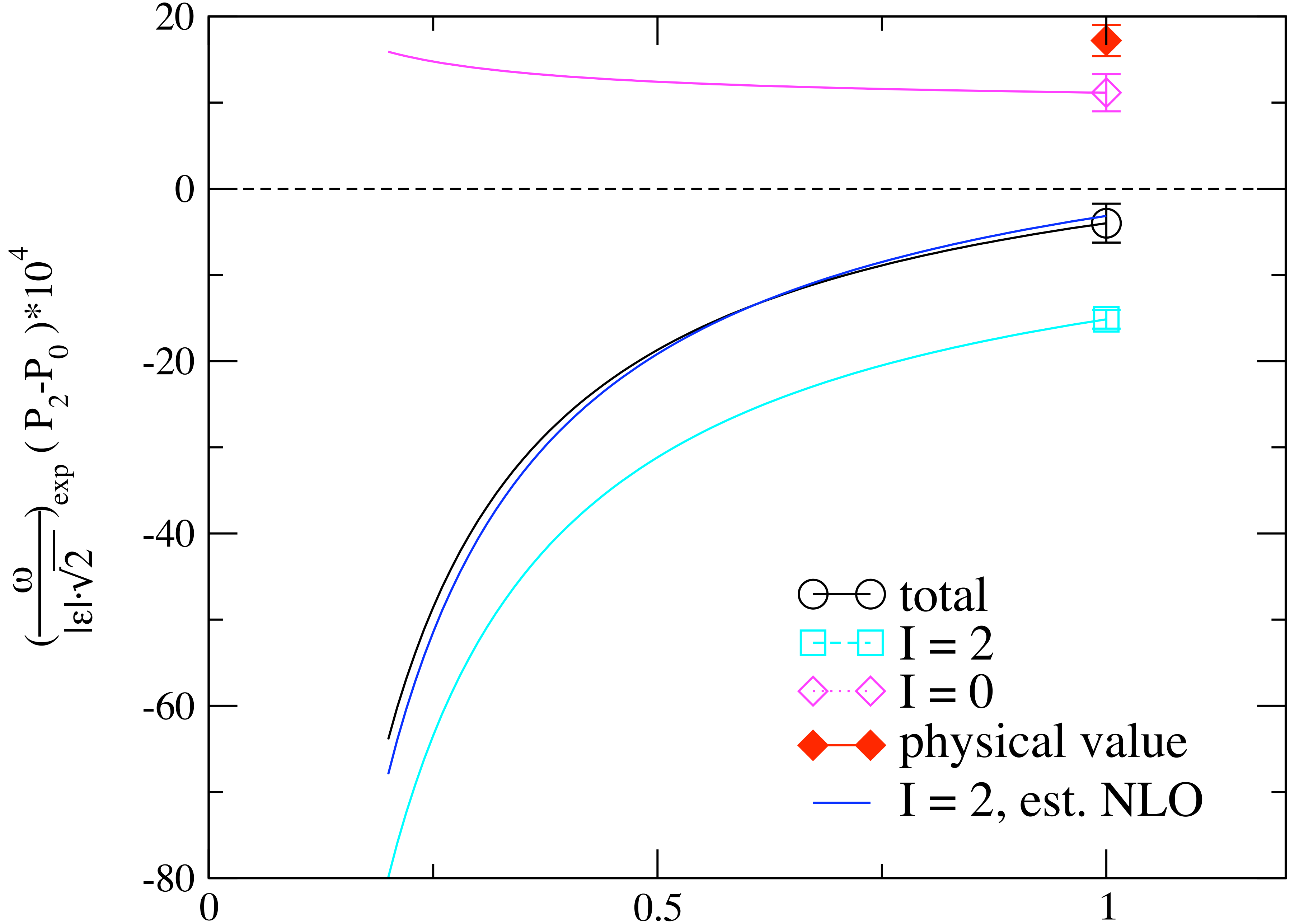
- For $K \to \pi$ matrix elements, log and m_q^2 corrections at m_{strange} to $Q_2^{(3/2)}$ similar to leading m_q term. Barring unexpected cancellation, $c^{(27,1)}m_q$ should be a considerable positive contribution to ϵ'/ϵ.

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \left(\frac{\omega}{\sqrt{2}|\epsilon|}\right)_{\mathrm{exp}} \left\{ \left[\frac{\alpha_{\mathrm{W}}\alpha_8}{\alpha_2 m_{K^0}^2}\left(1-\mathcal{O}(c^{(27,1)}m_q)\right)\right]^{(3/2)} - \left[\frac{\alpha_{\mathrm{S}}\alpha_6}{\alpha_2}\right]^{(1/2)} \right\}$$

- Putting in values from quenched simulation gives

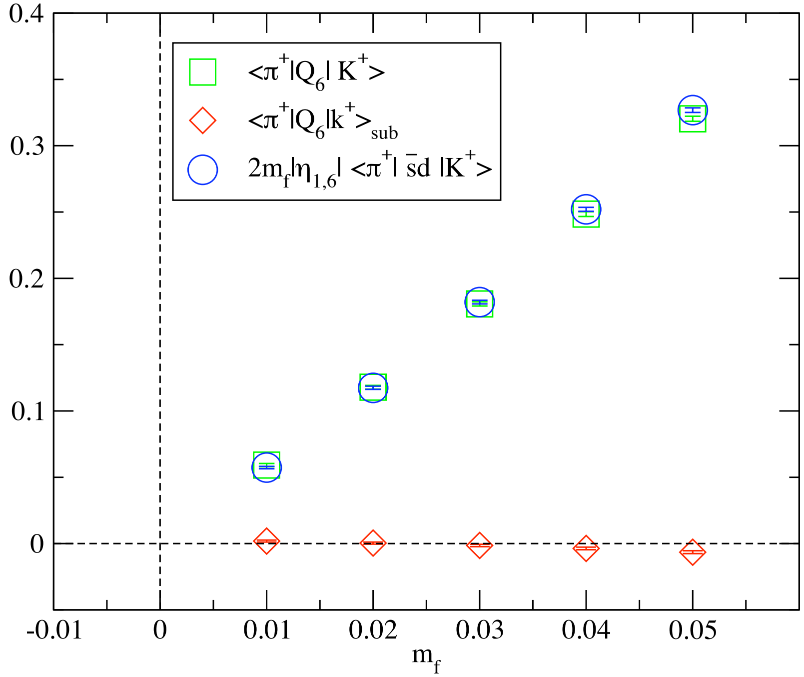
$$\mathrm{Re}(\epsilon'/\epsilon) \approx -13 \times 10^{-4}(1-\mathcal{O}(0.95)) - (-14 \times 10^{-4})$$

Graph of Estimated NLO ϵ'/ϵ from Quenched QCD and χPT

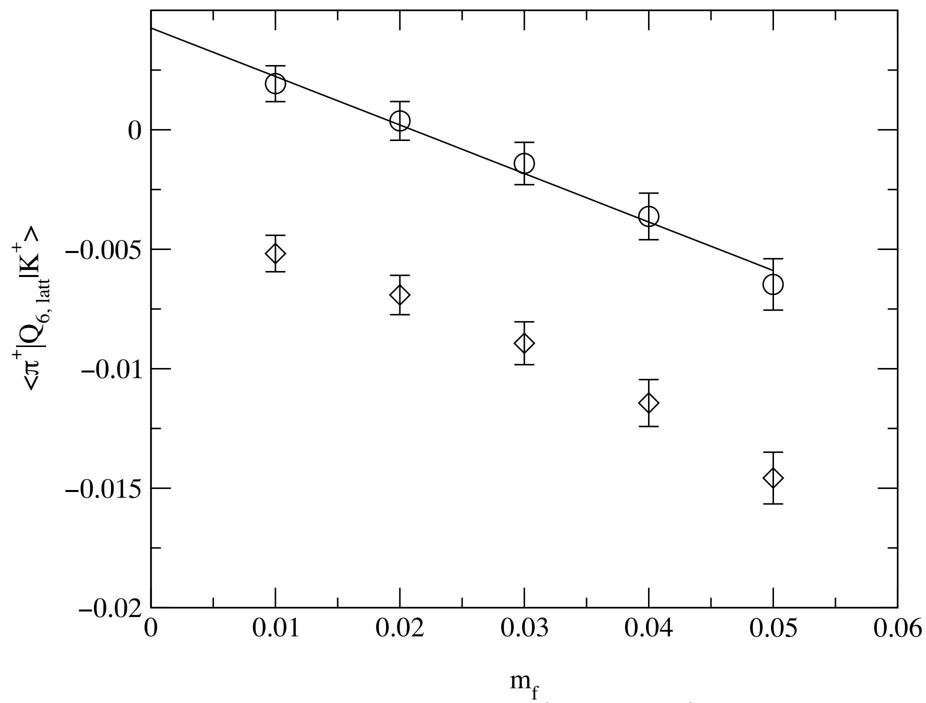


$\langle \pi^+ | Q_6 | K^+ \rangle$

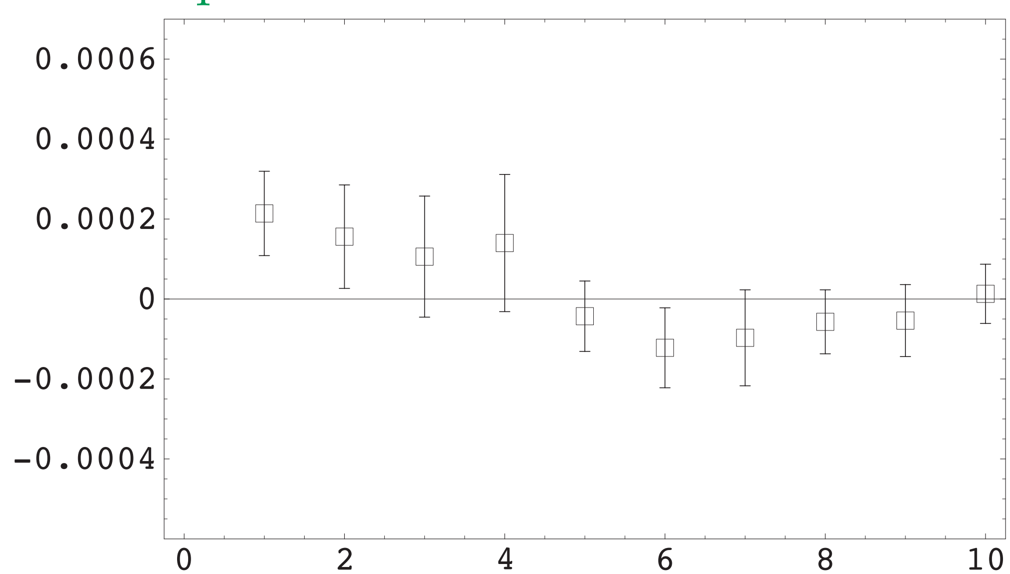
Large power divergent subtraction



Only slope relevant in subtracted ME



Unsubtracted operator deviates little from linear (Laiho)



Conclusions

- Dynamical DWF simulations already done with $m_{\rm dyn} \approx m_{\rm strange}/2$ at single a.
- QCDOC promises larger volumes and smaller $m_{\rm dyn}$
- For B_K, no open theoretical issues. Precision requires careful control of systematics and good statistics.
- For $K \to \pi$ matrix elements, LO constants should be accessible in full QCD.
- For ϵ'/ϵ, only $K \to \pi$ matrix elements of (27,1) operator show large NLO corrections in quenched simulation.
- $K \to \pi\pi$ for (27,1) and Q_8 at physical kinematics achievable quenched on coarse lattices.
- For full QCD direct calculations of $K \to \pi\pi$, extrapolation/interpolation/χPT will be needed with current machines and understanding.