# Unitarity Triangle Analysis: Past, Present, Future

• **INTRODUCTION**: quark masses, weak couplings and $\not{CP}$ in the Standard Model

• Unitary Triangle Analysis: **PAST**

• **PRESENT**

• **FUTURE**

C, CP and CPT and their violation are related to the foundations of modern physics (Relativistic quantum mechanics, Locality, Matter-Antimatter properties, Cosmology etc.)

Although in the Standard Model (SM) all ingredients are present, new sources of $\not{CP}$ beyond the SM are necessary to explain quantitatively the BAU

**Almost all New Physics Theories generate new sources of $\not{CP}$**

# Quark Masses, Weak Couplings and CP Violation in the Standard Model

In the Standard Model the quark mass matrix, from which the CKM Matrix and $\not{CP}$ originate, is determined by the Yukawa Lagrangian which couples fermions and Higgs

$$\mathcal{L}^{\text{quarks}} = \mathcal{L}^{\text{kinetic}} + \mathcal{L}^{\text{weak int}} + \mathcal{L}^{\text{yukawa}}$$

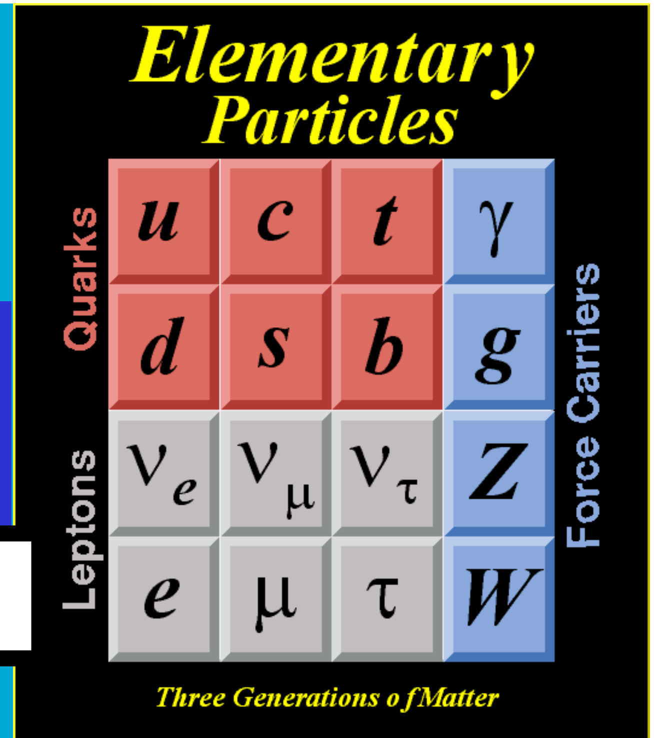
CP invariant

$\not{CP}$ and symmetry breaking are closely related !

## QUARK MASSES ARE GENERATED BY DYNAMICAL SYMMETRY BREAKING

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad H^C = i\tau_2 H^*$$

$$\phi^+ \to 0 \qquad \phi^0 \to \frac{V}{\sqrt{2}}$$




Charge +2/3

$$\mathcal{L}^{\mathrm{yukawa}} \equiv \Sigma_{\mathbf{i,k=1,N}} [\ Y_{\mathbf{i,k}} (q^{\mathbf{i}}_{\mathbf{L}} H^C)\ U^{\mathbf{k}}_{\mathbf{R}} + X_{\mathbf{i,k}} (q^{\mathbf{i}}_{\mathbf{L}} H)\ \mathbf{D}^{\mathbf{k}}_{\mathbf{R}} + \mathrm{h.c.}\ ]$$

Charge -1/3

$$\Sigma_{\mathbf{i,k=1,N}} [\ m^{\mathbf{u}}_{\mathbf{i,k}} (\bar{u}^{\mathbf{i}}_{\mathbf{L}}\, u^{\mathbf{k}}_{\mathbf{R}}) + m^{\mathbf{d}}_{\mathbf{i,k}} (\bar{d}^{\mathbf{i}}_{\mathbf{L}}\, d^{\mathbf{k}}_{\mathbf{R}}) + \mathrm{h.c.}\ ]$$

# Diagonalization of the Mass Matrix

Up to singular cases, the mass matrix can always be diagonalized by 2 unitary transformations

$u^i_L \rightarrow \mathbf{U}^{ik}_L u^k_L \qquad u^i_R \rightarrow \mathbf{U}^{ik}_R u^k_R$

$\mathbf{M}' = \mathbf{U}^{\dagger}_L \mathbf{M} \mathbf{U}_R \qquad (\mathbf{M}')^{\dagger} = \mathbf{U}^{\dagger}_R (\mathbf{M})^{\dagger} \mathbf{U}_L$

$$\mathcal{L}^{\mathbf{mass}} \equiv m_{\mathbf{up}} (\bar{u}_{\mathbf{L}} u_{\mathbf{R}} + \bar{u}_{\mathbf{R}} u_{\mathbf{L}}) + m_{\mathbf{ch}} (\bar{c}_{\mathbf{L}} c_{\mathbf{R}} + \bar{c}_{\mathbf{R}} c_{\mathbf{L}}) + m_{\mathbf{top}} (\bar{t}_{\mathbf{L}} t_{\mathbf{R}} + \bar{t}_{\mathbf{R}} t_{\mathbf{L}})$$

$$L_{CC}^{weak\,int} = \frac{g_W}{\sqrt{2}} \left( J^-_\mu W^+_\mu + h.c. \right) \rightarrow \frac{g_W}{\sqrt{2}} \left( \bar{u}_L \mathbf{V}^{CKM} \gamma_\mu d_L W^+_\mu + \ldots \right)$$

N(N-1)/2 angles and (N-1)(N-2) /2 phases

N=3 3 angles + 1 phase KM

the phase generates complex couplings i.e. CP violation;

6 masses +3 angles +1 phase = 10 parameters

| | | |
|---|---|---|
| $V_{ud}$ | $V_{us}$ | $V_{ub}$ |
| $V_{cd}$ | $V_{cs}$ | $V_{cb}$ |
| $V_{tb}$ | $V_{ts}$ | $V_{tb}$ |

**NO Flavour Changing Neutral Currents (FCNC) at Tree Level (FCNC processes are good candidates for observing NEW PHYSICS)**

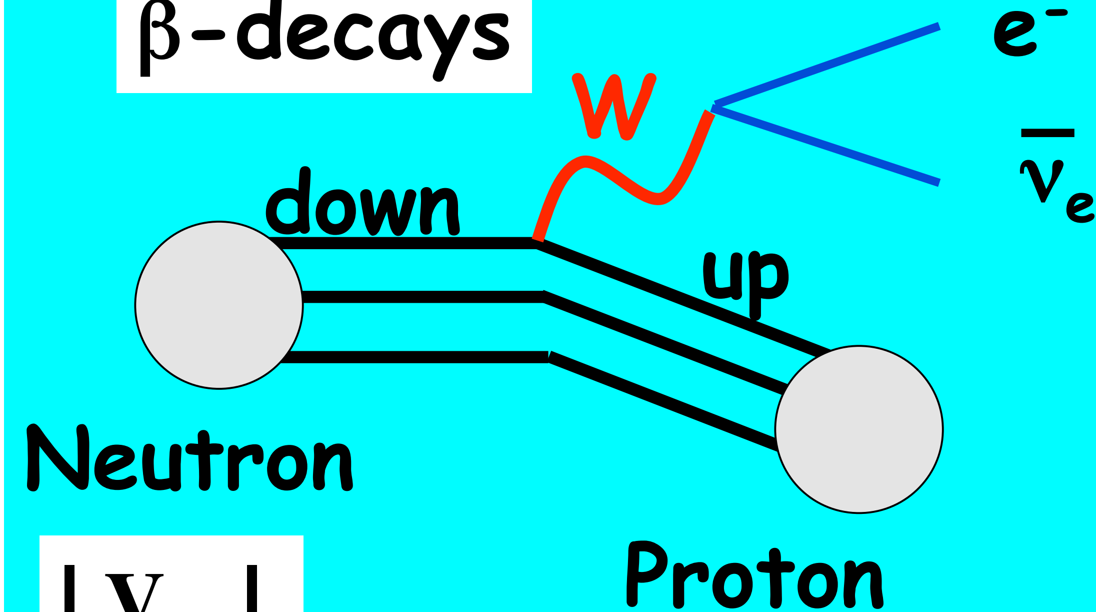
**CP Violation is natural with three quark generations (Kobayashi-Maskawa)**

**With three generations all CP phenomena are related to the same unique parameter ( $\delta$ )**

| | | |
|---|---|---|
| $V_{ud} \sim 1$ | $V_{us} \sim \lambda$ | $V_{ub} \sim \lambda^3$ |
| $V_{cd} \sim \lambda$ | $V_{cs} \sim 1$ | $V_{cb} \sim \lambda^2$ |
| $V_{tb} \sim \lambda^3$ | $V_{ts} \sim \lambda^2$ | $V_{tb} \sim 1$ |

# Quark masses & Generation Mixing

β-decays




$| V_{ud} |$

$| V_{ud} | = 0.9735(8)$

$| V_{us} | = 0.2196(23)$

$| V_{cd} | = 0.224(16)$

$| V_{cs} | = 0.970(9)(70)$

$| V_{cb} | = 0.0406(8)$

$| V_{ub} | = 0.00363(32)$

$| V_{tb} | = 0.99(29)$
(0.999)

| | | |
|---|---|---|
| $c_{12}\, c_{13}$ | $s_{12}\, c_{13}$ | $s_{13}\;\; e^{-i\delta}$ |
| $-s_{12}c_{23}$ $-c_{12}s_{23}s_{13}\, e^{i\delta}$ | $c_{12}c_{23}$ $-s_{12}s_{23}s_{13}\, e^{i\delta}$ | $s_{23}\, c_{13}$ |
| $s_{12}s_{23}$ $-c_{12}c_{23}s_{13}\, e^{i\delta}$ | $-c_{12}s_{23}$ $-s_{12}c_{23}s_{13}\, e^{i\delta}$ | $c_{23}\, c_{13}$ |

$c_{ij} = \text{Cos}\ \theta_{ij}$ $s_{ij} = \text{Sin}\ \theta_{ij}$ $c_{ij} \geq 0$ $s_{ij} \geq 0$

$0 \leq \delta \leq 2\pi$ $|s_{12}| \sim \text{Sin}\ \theta_c$

for small angles $|s_{ij}| \sim |V_{ij}|$

# The Wolfenstein Parametrization

| | | |
|---|---|---|
| $1 - 1/2\,\lambda^2$ | $\lambda$ | $A\,\lambda^3(\rho - i\,\eta)$ |
| $-\lambda$ | $1 - 1/2\,\lambda^2$ | $A\,\lambda^2$ |
| $A\,\lambda^3 \times (1 - \rho - i\,\eta)$ | $-A\,\lambda^2$ | $1$ |

$+ O(\lambda^4)$

$\lambda \sim 0.2 \quad A \sim 0.8$

$\eta \sim 0.2 \quad \rho \sim 0.3$

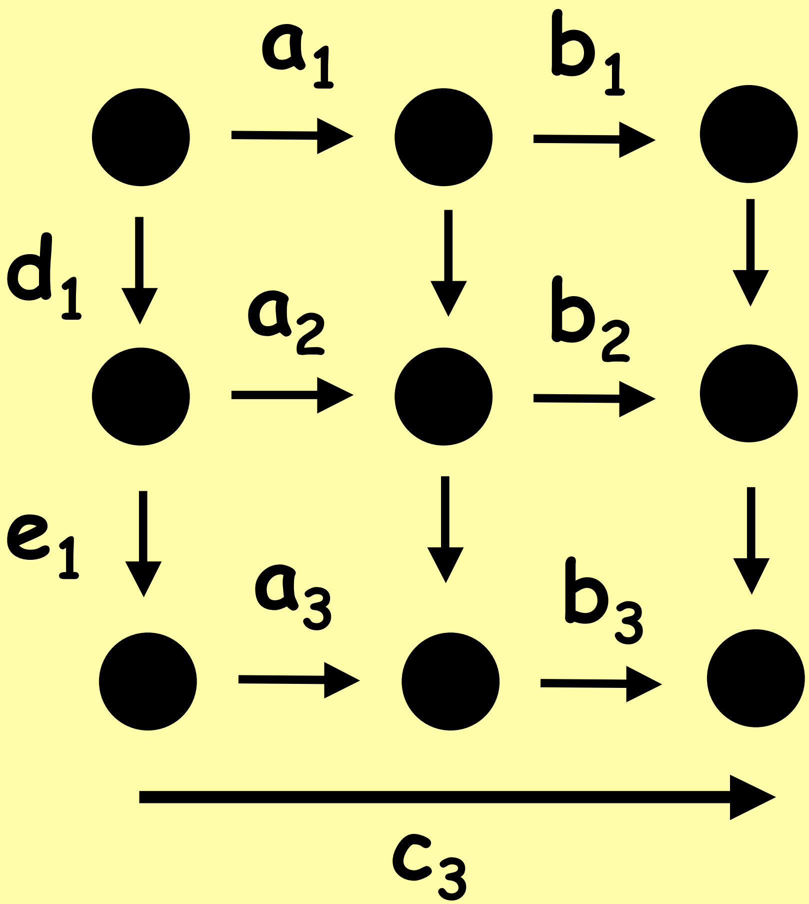
$\sin\theta_{12} = \lambda$

$\sin\theta_{23} = A\,\lambda^2$

$\sin\theta_{13} = A\,\lambda^3(\rho - i\,\eta)$

# The Bjorken-Jarlskog Unitarity Triangle

**$|V_{ij}|$ is invariant under phase rotations**



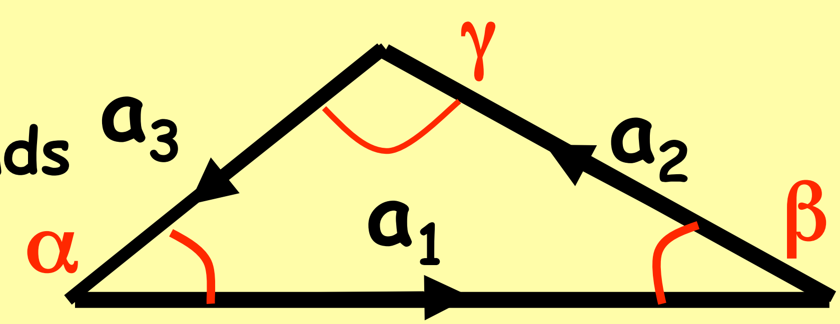

$a_1 = V_{11} V_{12}^* = V_{ud} V_{us}^*$

$a_2 = V_{21} V_{22}^* \quad a_3 = V_{31} V_{32}^*$

$a_1 + a_2 + a_3 = 0$

$(b_1 + b_2 + b_3 = 0 \text{ etc.})$

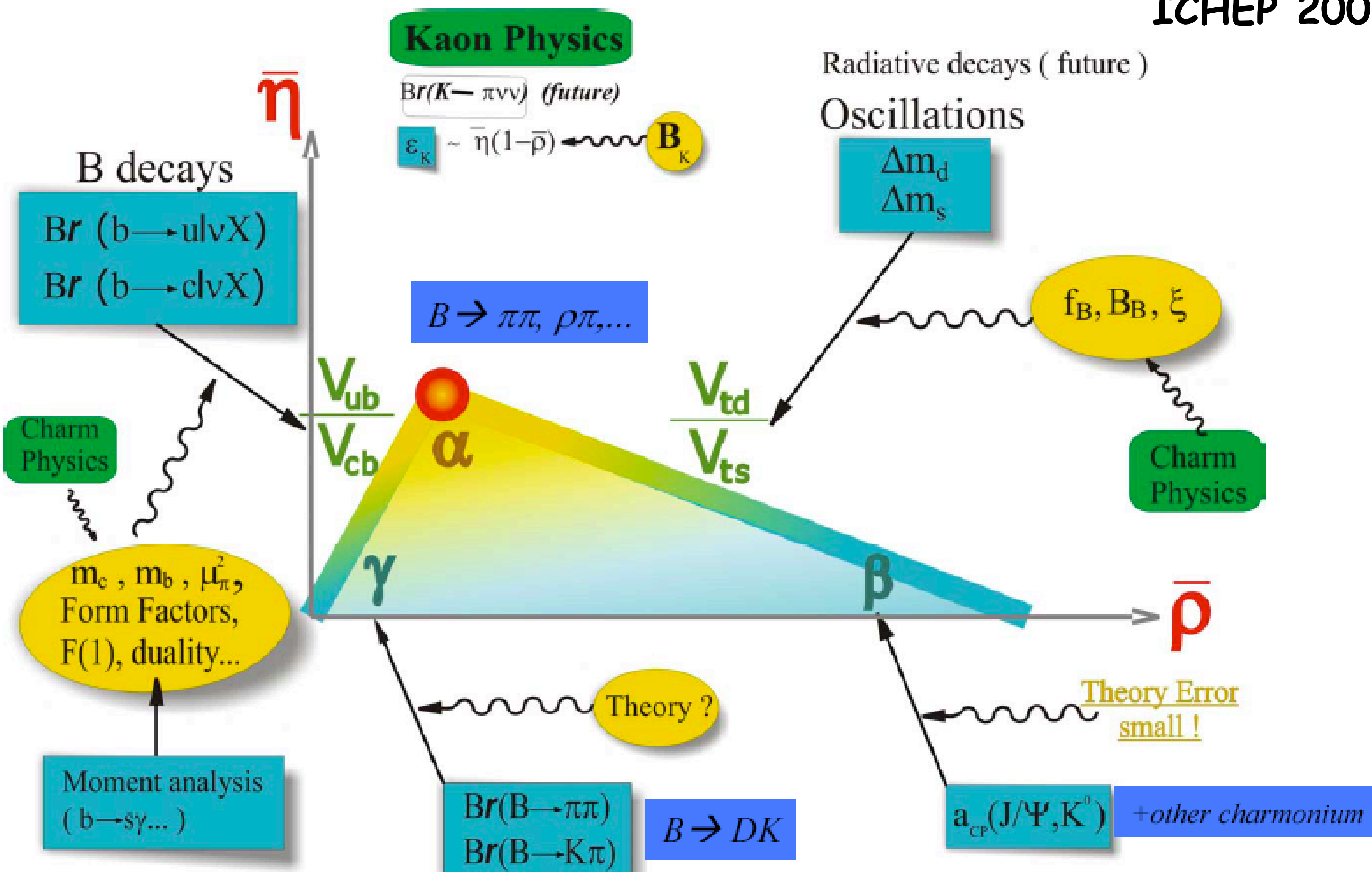
Only the orientation depends on the phase convention



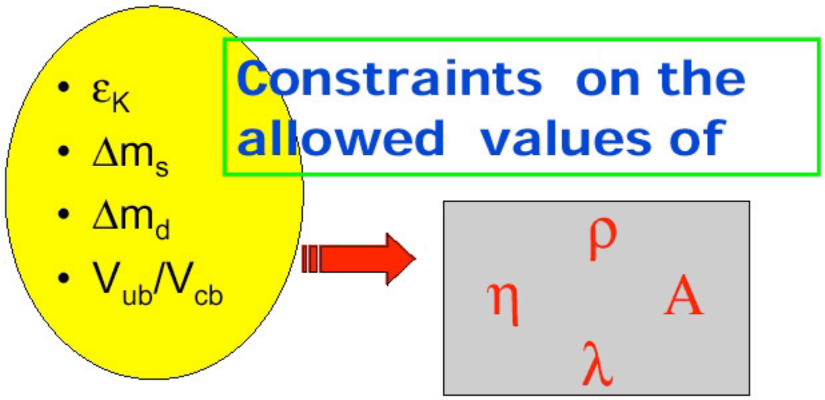
# Visualization of the unitarity of the CKM matrix

## Unitarity Triangle in the (ρ–η) plane

From
A. Stocchi
ICHEP 2002



**SEVERAL UNITARITY TRIANGLE ANALYSES, USING METHODS BASED ON THE "BAYESIAN" APPROACH, HAVE BEEN MADE DURING THE LAST DECADE**




| Measure | $V_{CKM}$ | Other NP parameters |
|---|---|---|
| $\Gamma(b \to u)/\Gamma(b \to c)$ | $\bar{\rho}^2 + \bar{\eta}^2$ | $\bar{\Lambda}, \lambda_1, F(1), \ldots$ |
| $\varepsilon_K$ | $\eta\left[(1-\bar{\rho}) + \ldots\right]$ | $B_K$ |
| $\Delta m_d$ | $(1-\bar{\rho})^2 + \bar{\eta}^2$ | $f_{B_d}^2 B_{B_d}$ |
| $\Delta m_d/\Delta m_1$ | $(1-\bar{\rho})^2 + \bar{\eta}^2$ | $\xi$ |
| $A_{CP}(B_d \to J/\psi K_s)$ | $\sin 2\beta$ | – |

sin 2β is measured directly from B → J/ψ $K_s$ decays at Babar & Belle

$$\mathcal{A}_{J/\psi K_s} = \frac{\Gamma(B_d^0 \to J/\psi K_s, t) - \Gamma(\bar{B}_d^0 \to J/\psi K_s, t)}{\Gamma(B_d^0 \to J/\psi K_s, t) + \Gamma(\bar{B}_d^0 \to J/\psi K_s, t)}$$

$$\mathcal{A}_{J/\psi K_s} = \sin 2\beta \quad \sin(\Delta m_d t)$$

# DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

1) First class quantities, with reduced or negligible uncertainties

$$\mathcal{A}_{CP}(B \to J/\psi\, K_s)$$
$$K^+ \to \pi^+ \nu\bar{\nu}$$

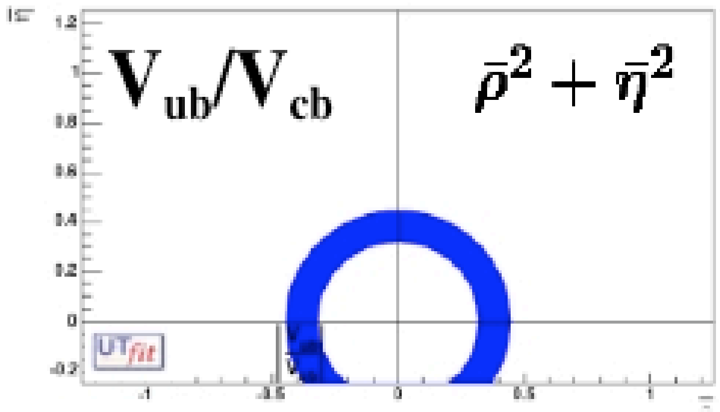
2) Second class quantities, with theoretical errors of O(10%) or less that can be reliably estimated

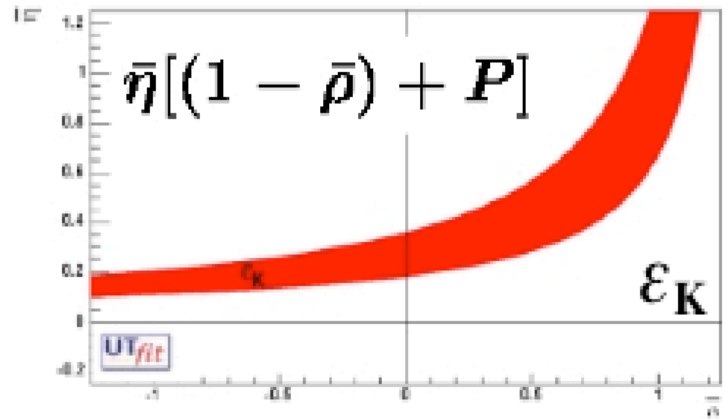
$$\varepsilon_K\,,\qquad \Delta m_{d,s}$$
$$\Gamma(b \to c, u)$$
$$\gamma\ from\ B \to D\,K$$

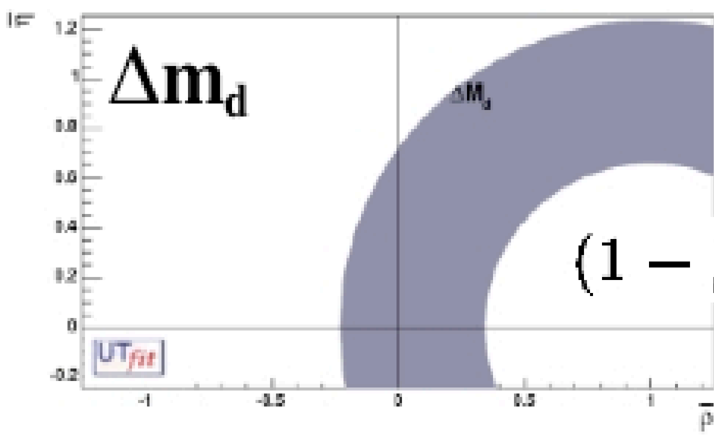
3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)

In case of discrepacies we cannot tell whether is <u>*new physics or we must blame the model*</u>

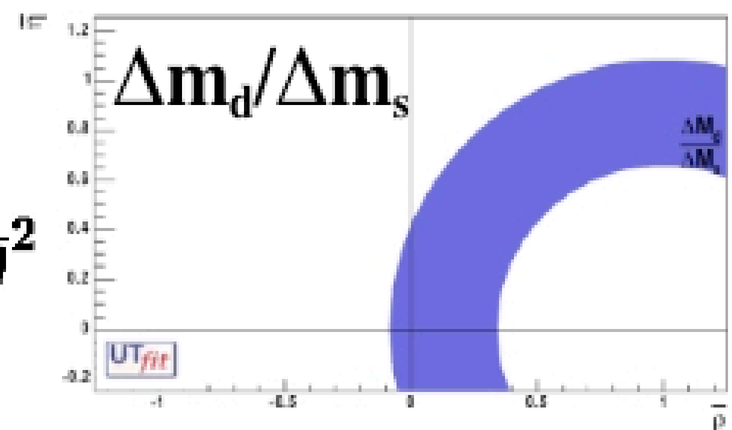
$$B \to K\,\pi \qquad B \to \pi^0\,\pi^0$$
$$B \to \phi\, K_s$$

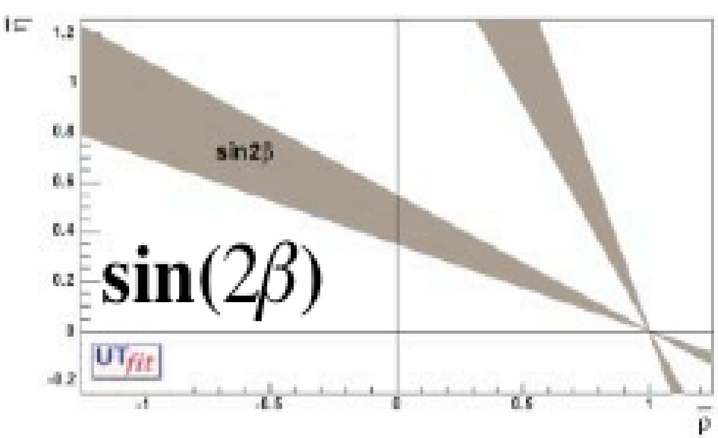



$(1-\bar{\rho})^2 + \bar{\eta}^2$




**Quantities used in the Standard UT Analysis**

# THE COLLABORATION

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NEW 2004 ANALYSIS IN PREPARATION

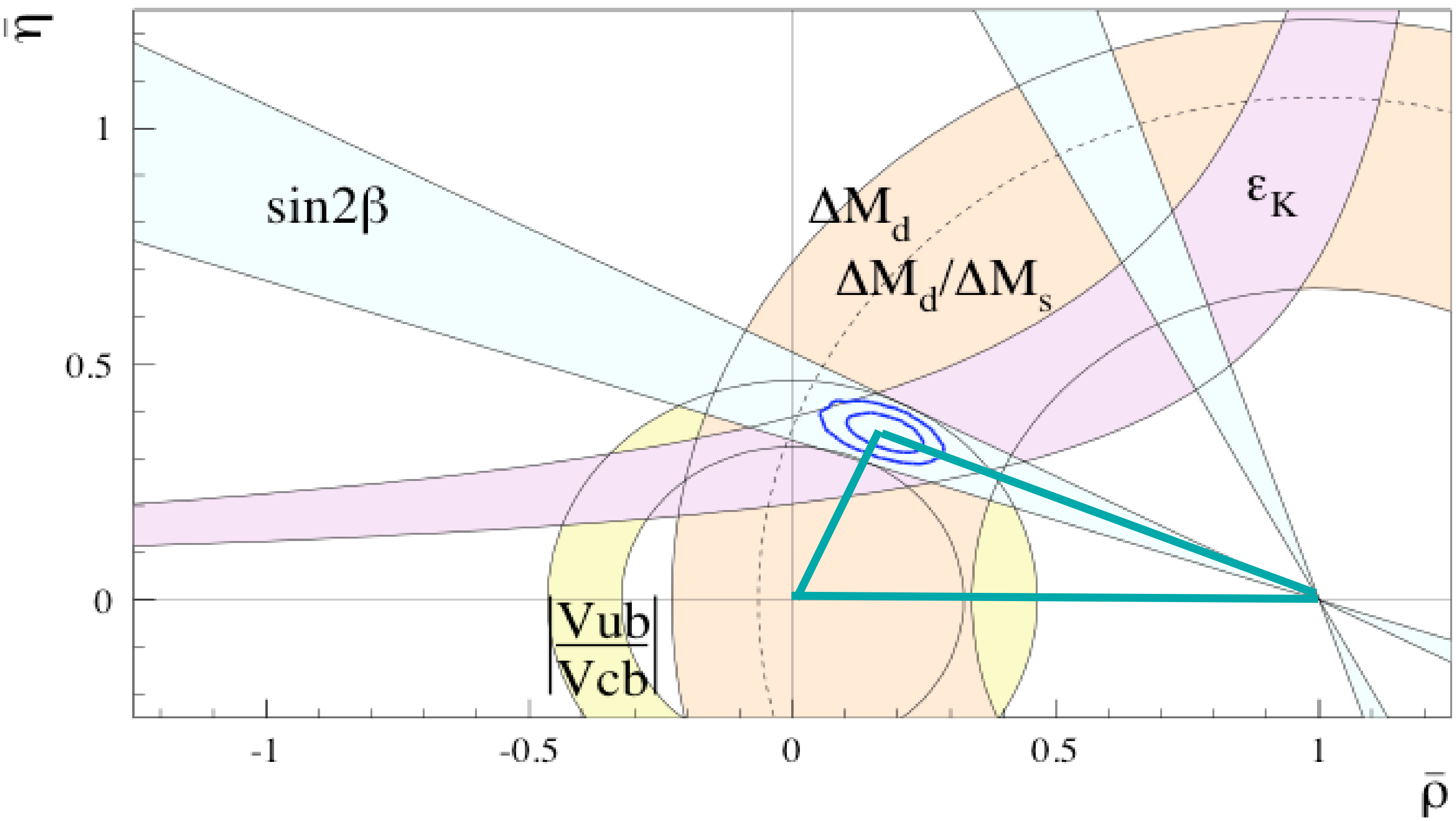
- New quantities e.g. B -> DK will be included
- Upgraded experimental numbers after Bejing

**www.utfit.org**

# PAST and PRESENT (the Standard Model)

| Constraints, Parameters | Value | Gauss Error | Flat Error | Comments |
|---|---|---|---|---|
| $\sin 2\beta$ | 0.739 | 0.048 | - | |
| $\lambda$ | 0.2240 | 0.0036 | - | |
| $\|V_{cb}\|$ ($10^{-3}$) | 42.1 | 2.1 | - | Average of exclusive |
| $\|V_{cb}\|$ ($10^{-3}$) | 41.4 | 0.7 | 0.6 | Average of inclusive |
| $\|V_{ub}\|$ $10^{-4}$ (excl.) | 33.0 | 2.4 | 4.6 | For the moment -> only CLEO |
| $\|V_{ub}\|$ $10^{-4}$(incl.) | 40.9 | 4.6 | 3.6 | For the moment --> LEP + CLE0 end-point |
| $m_b$ (GeV/$c^2$) | 4.21 | 0.08 | - | |
| $m_c$ (GeV/$c^2$) | 1.3 | 0.1 | - | |
| $\Delta(m_d)$ ($ps^{-1}$) | 0.503 | 0.006 | - | WA (CDF/CLEO/LEP/Babar/Belle) |
| $\Delta(m_s)$ ($ps^{-1}$) | > 14.5 @ 95 % C.L. | - | - | Sensitivity at 18.3 (CDF/LEP/SLD)<br>The Likelihood Ratio is used. |
| $m_t$ (GeV/$c^2$) | 167 | 5 | - | (CDF/D0) |
| $f_{Bs}\sqrt{B_{Bs}}$ (MeV) | 276 | 38 | - | Lattice QCD |
| $\xi$ | 1.24 | 0.04 | 0.06 | Lattice QCD |
| $\eta_b$ | 0.55 | 0.01 | - | |
| $\|\varepsilon_K\|10^{-3}$ | 2.280 | 0.013 | - | |
| $B_K$ | 0.86 | 0.06 | 0.14 | Lattice QCD |
| $\eta_1$ | 1.38 | 0.53 | - | |
| $\eta_2$ | 0.574 | 0.004 | - | |
| $\eta_3$ | 0.47 | 0.04 | - | |
| $f_K$ (GeV) | 0.161 | - | - | |
| $\Delta(m_K)$ ($10^{-2}$ $ps^{-1}$) | 0.5301 | - | - | |
| $\alpha_s$ | 0.119 | 0.003 | - | |

# Results for ρ and η & related quantities




With the constraint from $\Delta m_s$

contours @ 68% and 95% C.L.

| | | |
|---|---|---|
| $\bar{\rho} = 0.174 \pm 0.048$ | $\bar{\eta} = 0.344 \pm 0.027$ | |
| [ 0.085 - 0.265] | [ 0.288 - 0.397] | at 95% C.L. |

$\sin 2\alpha = -0.14 \pm 0.25$ [ -0.62 - +0.33]

$\sin 2\beta = 0.697 \pm 0.036$ [ 0.636 - 0.779]

## Comparison of $\sin 2\beta$ from direct measurements (Aleph, Opal, Babar, Belle and CDF) and UTA analysis

$$\sin 2\beta_{measured} = 0.739 \pm 0.048$$

$$\sin 2\beta_{UTA} = 0.685 \pm 0.047$$

$$\sin 2\beta_{UTA} = 0.698 \pm 0.066$$

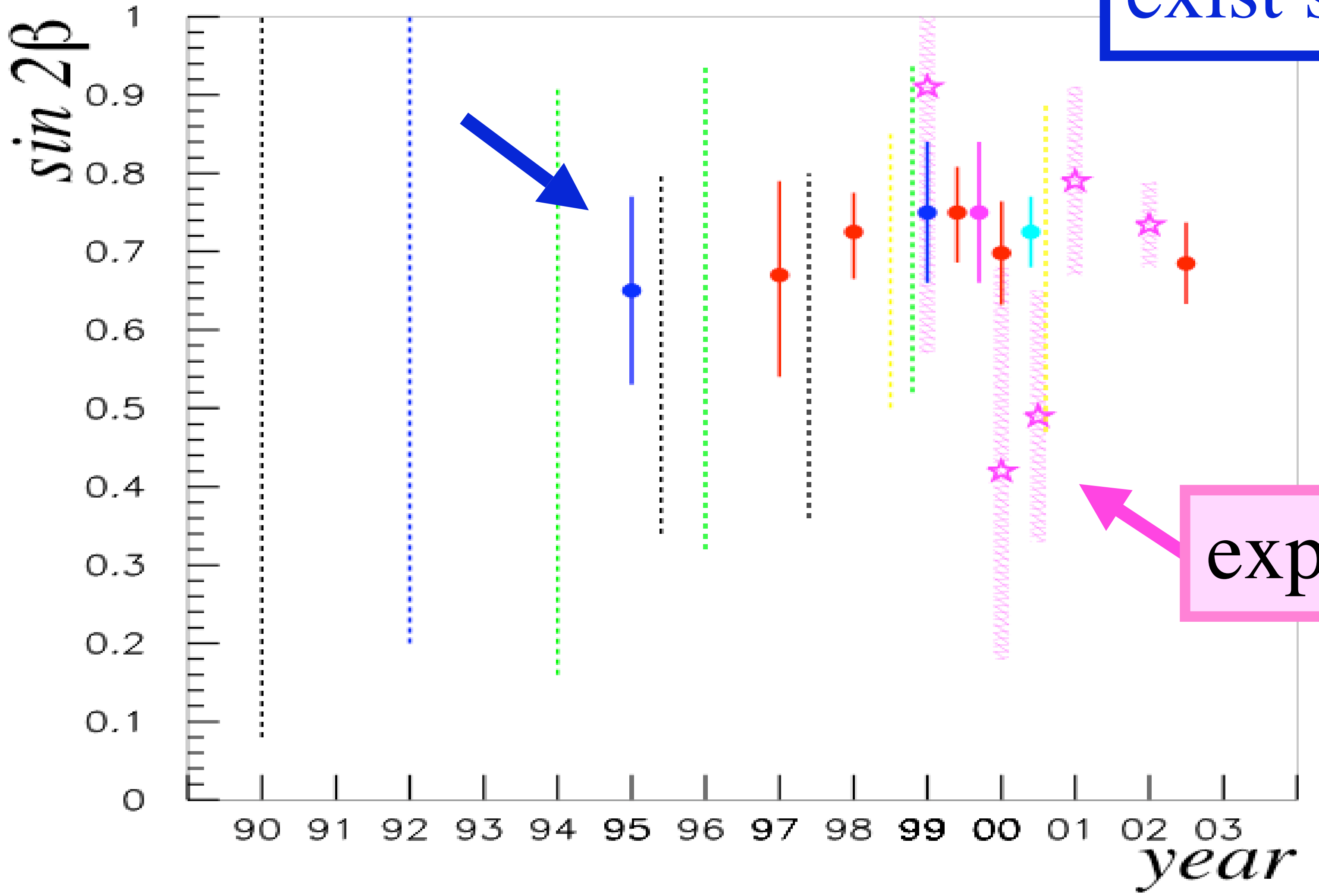
prediction from Ciuchini et al. (2000)

**Very good agreement**

**no much room for physics beyond the SM !!**

# Theoretical predictions of Sin 2 β in the years

predictions exist since '95

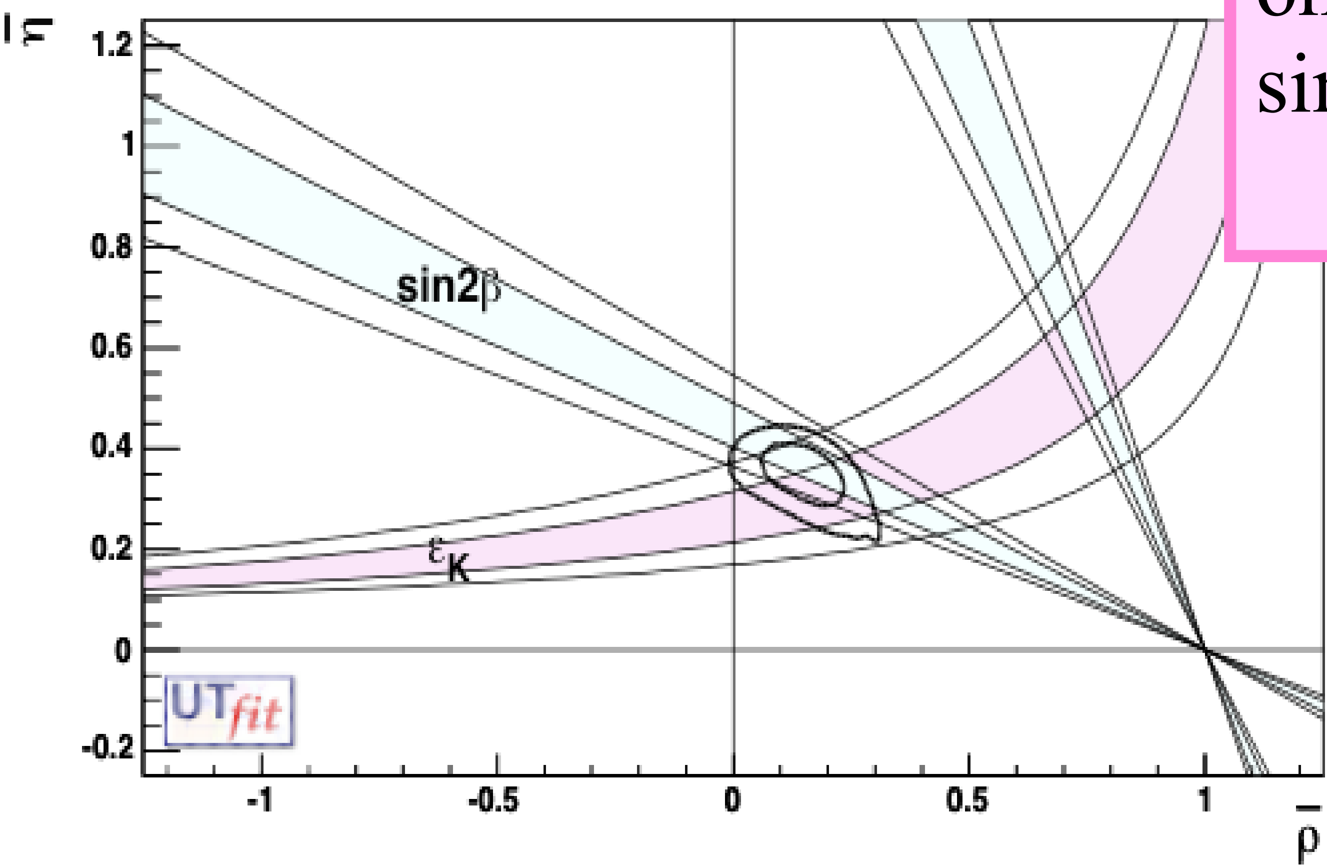



experiments

# Crucial Test of the Standard Model Triangle Sides (Non CP) compared to $\sin 2\beta$ and $\varepsilon_K$

From the sides only

$\sin 2\beta = 0.715 \pm 0.050$

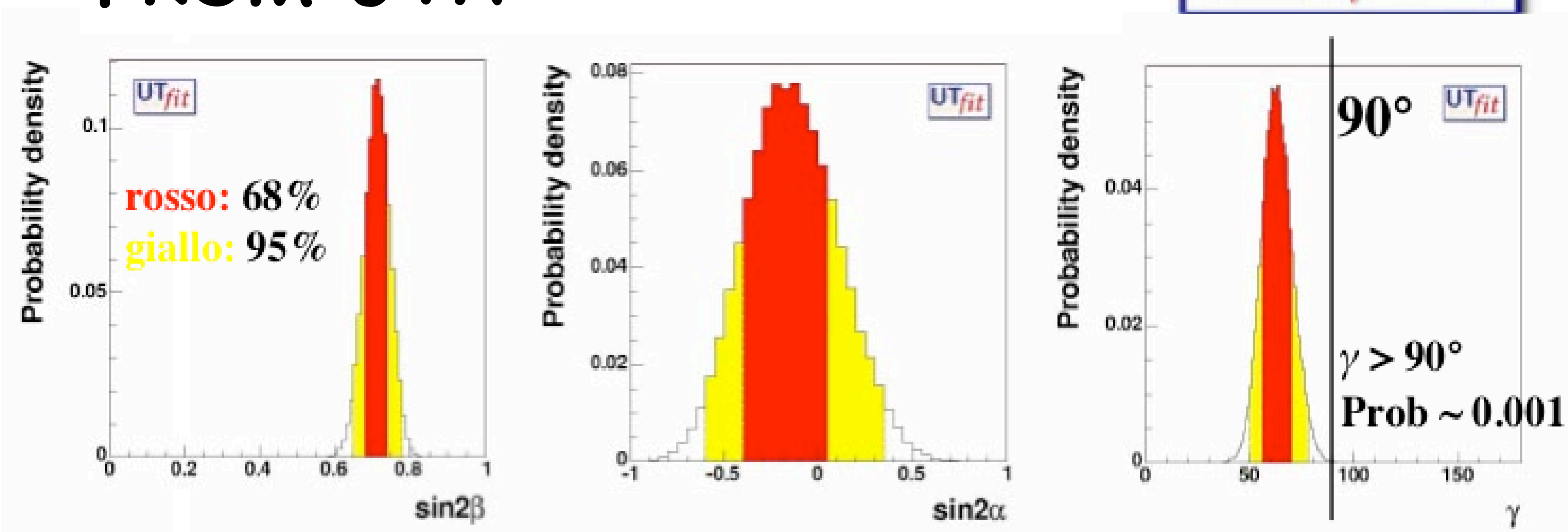


# PRESENT: sin 2α from B -> ππ & ρρ

# γ and (2β+γ) from B -> DK & B -> D(D*) π

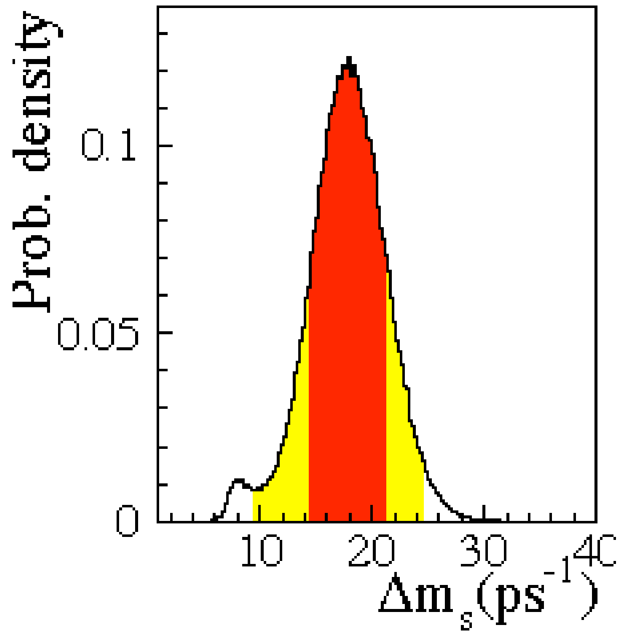
**A posteriori per $\sin 2\alpha$, $\sin 2\beta$ e $\gamma$:**

**FROM UTA**




| | | |
|---|---|---|
| $\sin 2\beta = 0.715^{+0.25}_{-0.35}$ <br> [0.65, 0.78] @ 95% CL | $\sin 2\alpha = -0.13^{+0.18}_{-0.28}$ <br> [-0.60, 0.35] @ 95% CL | $\gamma = 64° \pm 7°$ <br> [50, 78] @ 95% CL |

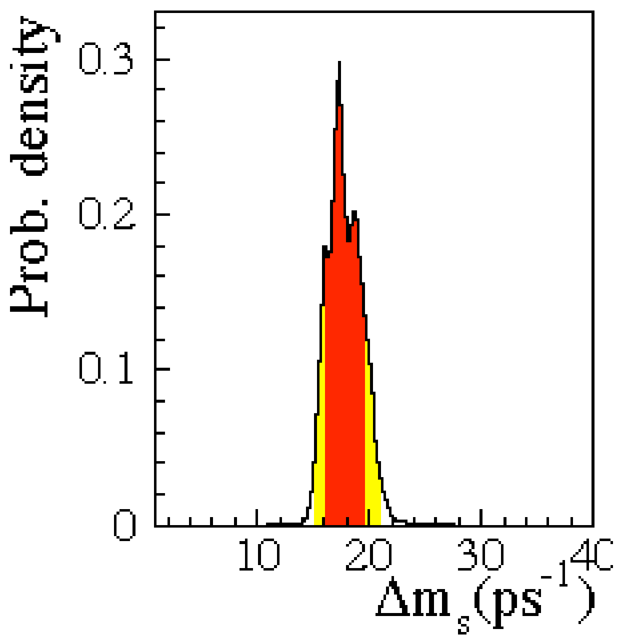
# $\Delta m_s$ Probability Density




Without the constraint from $\Delta m_s$

$\Delta m_s = (20.6 \pm 3.5)\ ps^{-1}$

[ 14.2 - 28.1] $ps^{-1}$ at 95% C.L.

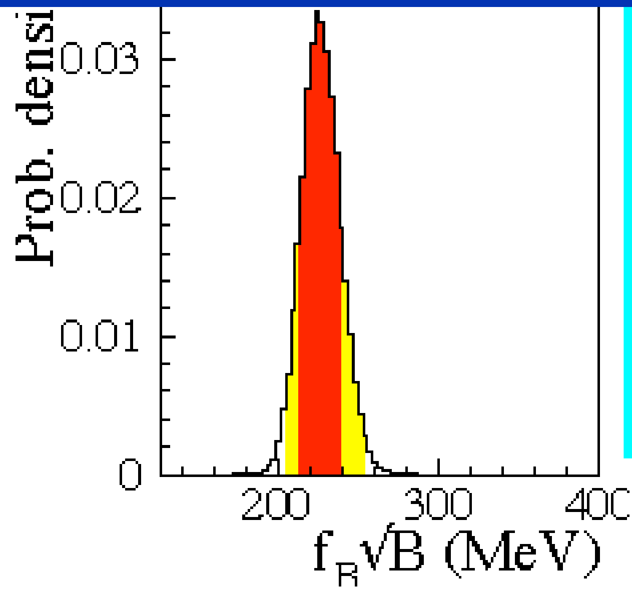


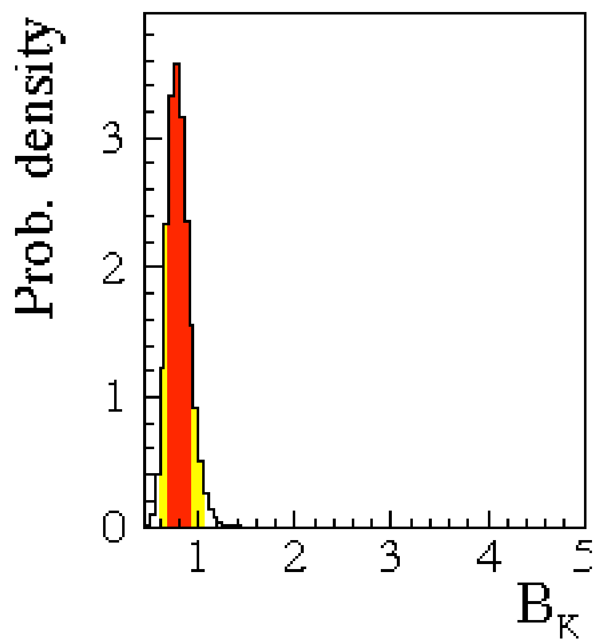

With the constraint from $\Delta m_s$

$\Delta m_s = (18.3^{+1.7}_{-1.5})\ ps^{-1}$

[ 15.6 - 22.2] $ps^{-1}$ at 95% C.L.

$$\xi = f_{B_s}\sqrt{B_{B_s}} / f_{B_d}\sqrt{B_{B_d}}$$




# Hadronic parameters

$$f_{Bs}\sqrt{B_{Bs}} = 276 \pm 38 \text{ MeV} \quad 14\%$$

**lattice**

$$f_{Bs}\sqrt{B_{Bs}} = 279 \pm 21 \text{ MeV} \quad 8\%$$

**UTA**

**6%** → $\xi = 1.24 \pm 0.04 \pm 0.06$ *Lattice*

**4%** → $\xi = 1.22 \pm 0.05$ *UTA*

$$f_{Bd}\sqrt{B_{Bd}} = 223 \pm 33 \pm 12 \text{ MeV}$$

**lattice**

$$f_{Bd}\sqrt{B_{Bd}} = 217 \pm 12 \text{ MeV}$$

**UTA**

$$B_K = 0.86 \pm 0.06 \pm 0.14$$

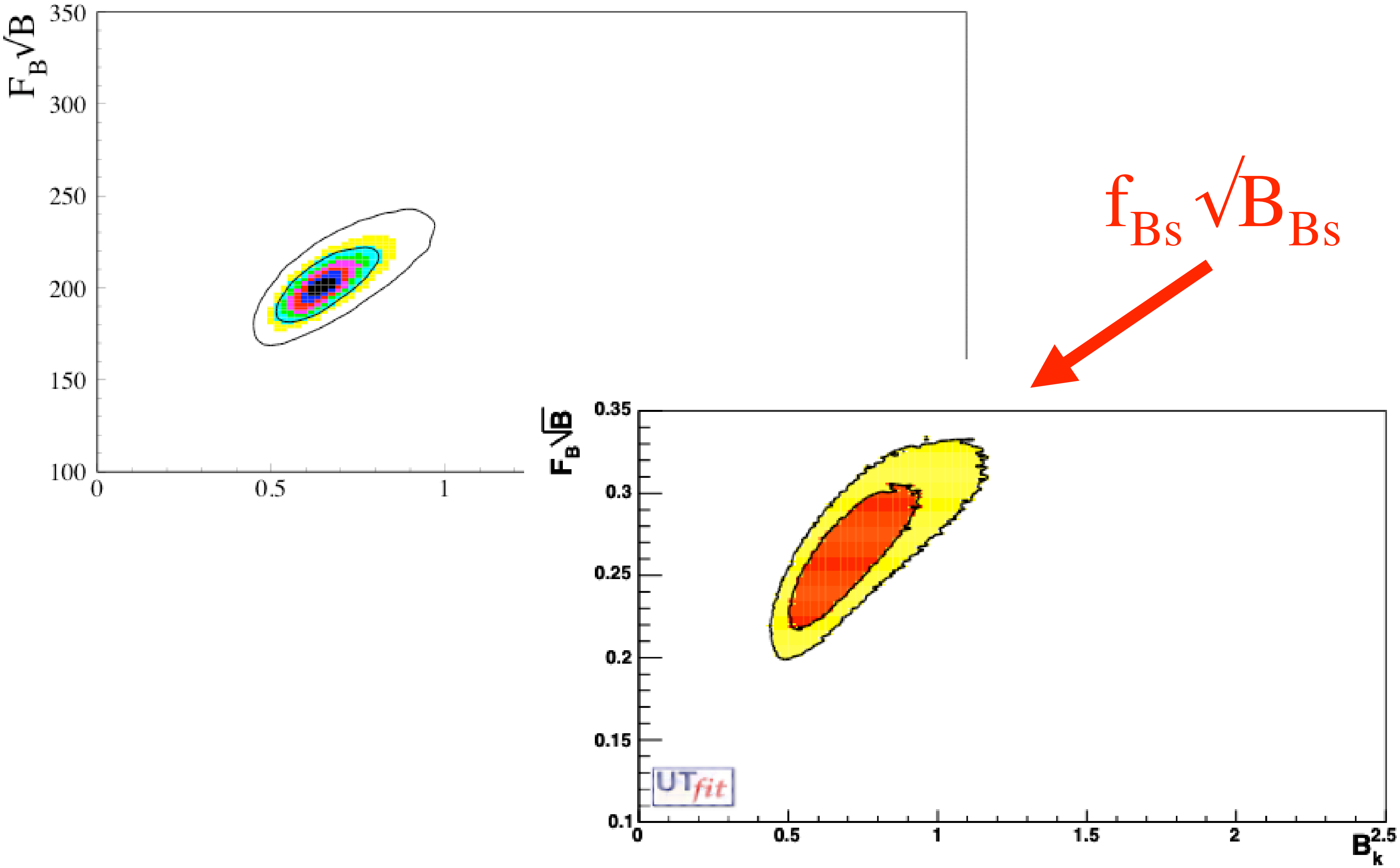
**lattice**

$$B_K = 0.69^{(+0.13)}_{(-0.08)}$$

**UTA**

# Limits on Hadronic Parameters

$f_{Bs}\sqrt{B}_{Bs}$



## Summary of the Results

| Parameter | Value ± Error | 95% probability | 99% probability |
|---|---|---|---|
| $\bar{\eta}$ | 0.344 ± 0.027 | [0.291, 0.396] | [0.272, 0.415] |
| $\bar{\rho}$ | 0.174 ± 0.048 | [0.076, 0.260] | [0.045, 0.293] |
| $\sin 2\beta$ | 0.697 ± 0.036 | [0.637, 0.761] | [0.619, 0.781] |
| $\sin 2\alpha$ | -0.14 ± 0.25 | [-0.62, 0.34] | [-0.73, 0.50] |
| $\gamma(^\circ)$ | 61.9 ± 7.9 | [48.6, 76.0] | [43.2, 82.9] |
| $\mathrm{Im}\,\lambda_t\ [10^{-5}]$ | 13.1 ± 1.0 | [11.2, 15.0] | [10.6, 15.6] |
| $\Delta(m_s)\ (ps^{-1})$ | 20.5 ± 3.2 | [14.4, 27.1] | [13.1, 29.5] |
| $f_{Bs}\sqrt{B_{Bs}}$ (MeV) | 279 ± 21 | [239, 320] | [228, 332] |
| $\xi$ | 1.22 ± 0.05 | [1.10, 1.33] | [1.09, 1.34] |
| $B_K$ | 0.65 ± 0.10 | [0.52, 0.91] | [0.46, 1.05] |

The parameters of the unitarity triangle ($\bar{\rho}$, $\bar{\eta}$, $\sin 2\beta$, $\sin 2\alpha$, $\gamma$ and $\mathrm{Im}\,\lambda_t$) have been determined including all constraints. In addition the values of the parameters entering in other constraints ($\Delta m_s$, $f_{Bd}\sqrt{B_{Bd}}$ and $B_K$) are given after having removed, in turn, each of the corresponding constraint.

# PRESENT
# (the Standard Model)

# NEW MEASUREMENTS

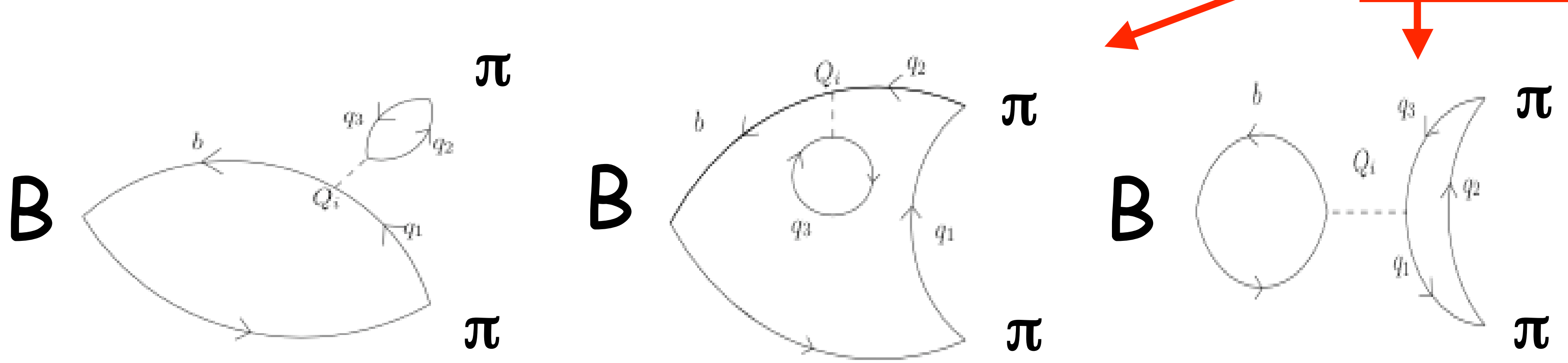
# $\sin 2\alpha$ from $B \to \pi\pi$

$$\mathcal{A}_{CP} = \frac{Prob(B^0_{phys}(\Delta t)\to f) - Prob(\overline{B}^0_{phys}(\Delta t)\to f)}{Prob(B^0_{phys}(\Delta t)\to f) + Prob(\overline{B}^0_{phys}(\Delta t)\to f)}$$
$$= C_f \cos\Delta m_d\, \Delta t + S_f \sin\Delta m_d\, \Delta t$$

$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\, ;\; S_f = -\frac{2\, Im\lambda_f}{1+|\lambda_f|^2}$$

$$\Delta I = \frac{3}{2}, \frac{1}{2} \quad V_{ub}V^*_{ud} \sim O(\lambda^3) \qquad \Delta I = \frac{1}{2} \quad V_{tb}V^*_{td} \sim O(\lambda^3)$$

$\Delta\mathcal{A}_t$



# sin 2α from B -> ππ

$$\lambda_{\pi\pi} = e^{-2i\alpha}\left[\frac{1+\tau^{*}\,\Delta\mathcal{A}_t}{1+\tau\,\Delta\mathcal{A}_t}\right]$$

$$\tau = -\frac{1-\rho-i\eta}{\rho+i\eta}$$

$$Arg[\lambda_{\pi\pi}] = \sin 2\alpha_{eff} \neq \sin 2\alpha$$

$$\frac{Im[\lambda_{\pi\pi}]}{|\lambda_{\pi\pi}|} = \sin(2\alpha+\phi)$$

$\phi$ could be extracted by measuring

$\mathcal{A}(B_d^0 \to \pi^+\pi^-), \mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-), \mathcal{A}(B_d^0 \to \pi^0\pi^0),$

$\mathcal{A}(\bar{B}_d^0 \to \pi^0\pi^0), \mathcal{A}(B_d^+ \to \pi^+\pi^0),$

# sin 2α from B -> ππ

**analisi dipendente dal tempo $\pi\pi$ e $\rho\rho$:**

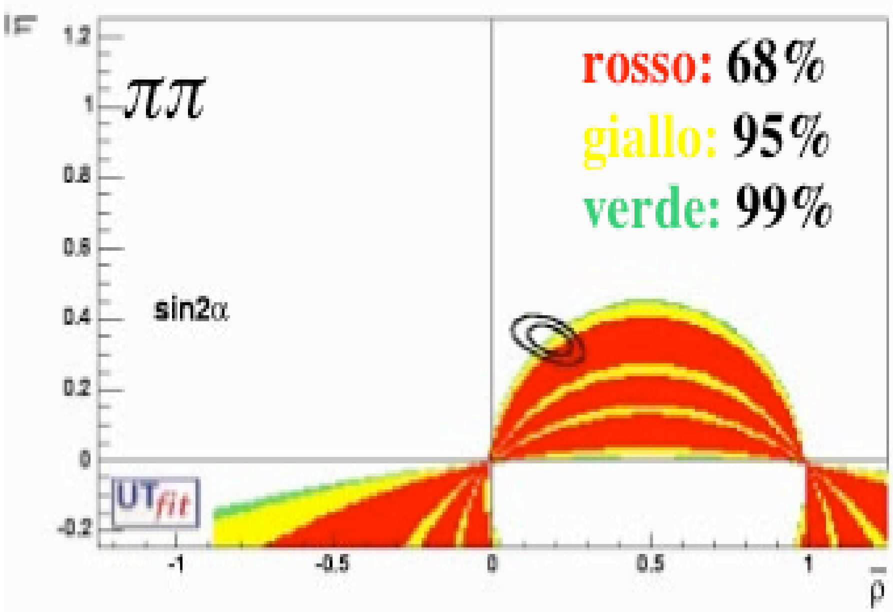
**misure di $2\alpha_{eff} = 2\alpha + \delta$**

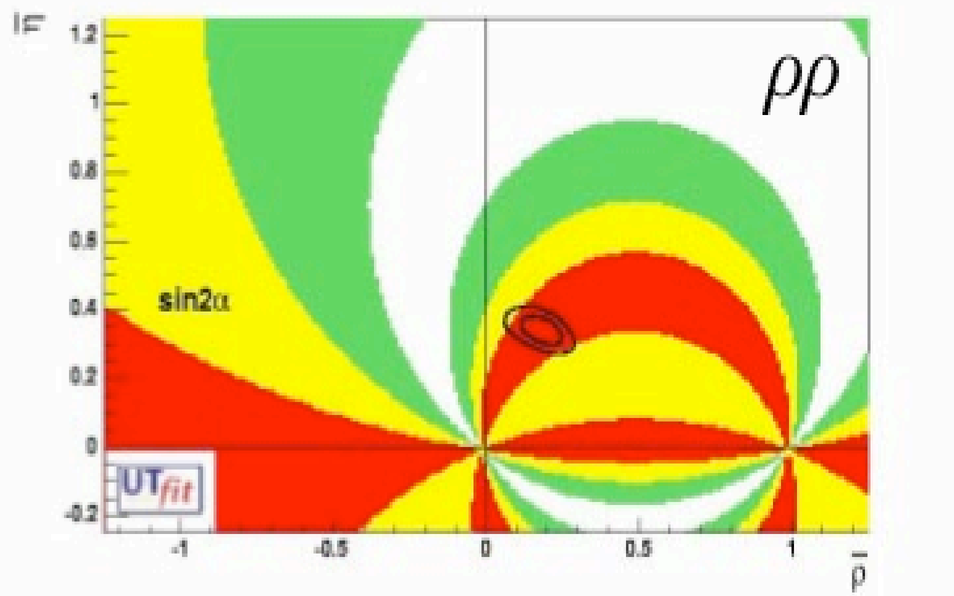
$|\alpha_{eff}-\alpha|_{\pi\pi} < 43.0°$ @ 95% CL

$|\alpha_{eff}-\alpha|_{\rho\rho} < 17.0°$

**Grossman-Quinn bound:**

$$\sin^2\delta \le \frac{BR(B^0 \to \pi^0\pi^0) + BR(\bar{B}^0 \to \pi^0\pi^0)}{BR(B^+ \to \pi^+\pi^0) + BR(B^- \to \pi^-\pi^0)}$$



# PRESENT & NEAR FUTURE

**I cannot resist to show the next 3 trasparencies**

# MAIN TOPICS

- Factorization (see M. Neubert talk)
- What really means to test Factorization
- $B \to \pi\pi$ and $B \to K\pi$ decays and the determination of the CP parameter $\gamma$
- Results including non-factorizable contributions
- Asymmetries
- Conclusions & Outlook

From g.m. qcd@work martinafranca 2001

# CIUCHINI ET AL 2001

Contrary to factorization we predict large Asymmetries for several of the particle-antiparticle BRs, in particular BR($B^+ \to K^+\pi^0$) and BR($B^0 \to K^+\pi^-$). This open new perspectives for the study of CP violation in B systems.

From g.m. qcd@work martinafranca 2001

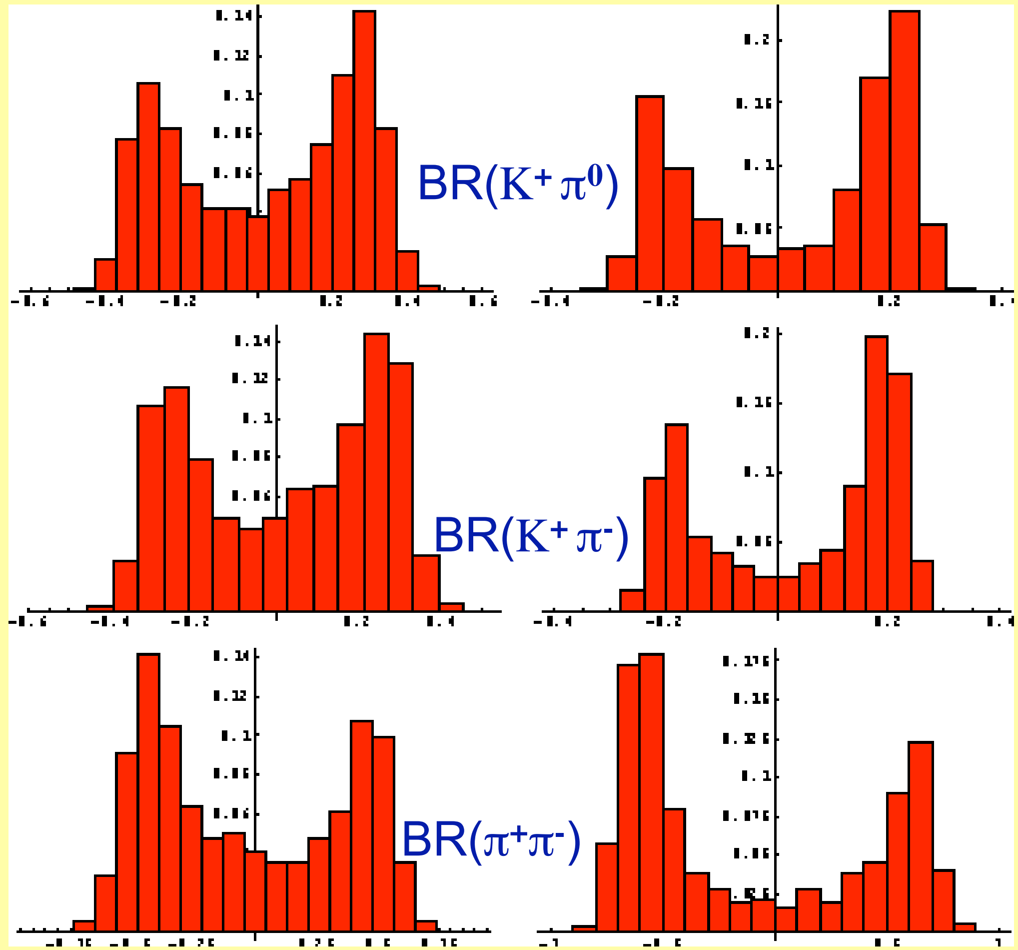
# CHARMING PENGUINS GENERATE LARGE ASYMMETRIES

$$\mathcal{A} = \frac{BR(\bar{B}) - BR(B)}{BR(\bar{B}) + BR(B)}$$

typical $\mathcal{A} \approx 0.2$

(factorized 0.03)

Large uncertainties




**From g.m. qcd@work martinafranca 2001**

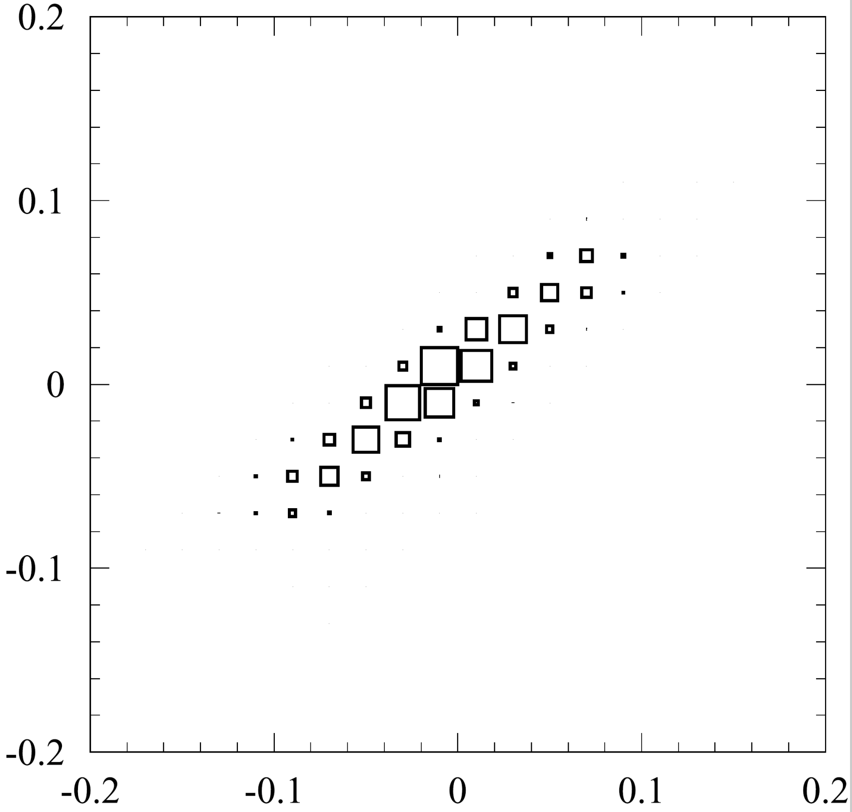
BELLE

In this study, the partial rate asymmetry $\mathcal{A}_{CP}(K^-\pi^+)$ is found to be $-0.088 \pm 0.035 \pm 0.013$, which is $2.4\sigma$ from zero. The corresponding 90% confidence level (C.L.) interval is $-0.15 < \mathcal{A}_{CP}(K^-\pi^+) < -0.03$. Our central value is similar to that reported by BaBar, $\mathcal{A}_{CP}(K^-\pi^+) = -0.107 \pm 0.041 \pm 0.013$ [17], indicating that the partial rate asymmetry may be negative. Theoretical predictions from different approaches suggest that $\mathcal{A}_{CP}(K^-\pi^+)$ and $\mathcal{A}_{CP}(K^+\pi^0)$ should have the same sign. The uncertainty in our result for $\mathcal{A}_{CP}(K^+\pi^0)$, is large enough for it to be consistent with this expectation. We set a 90% C.L. interval of $-0.04 < \mathcal{A}_{CP}(K^+\pi^0) < 0.16$. Since no evidence of direct $CP$ violation is observed in the

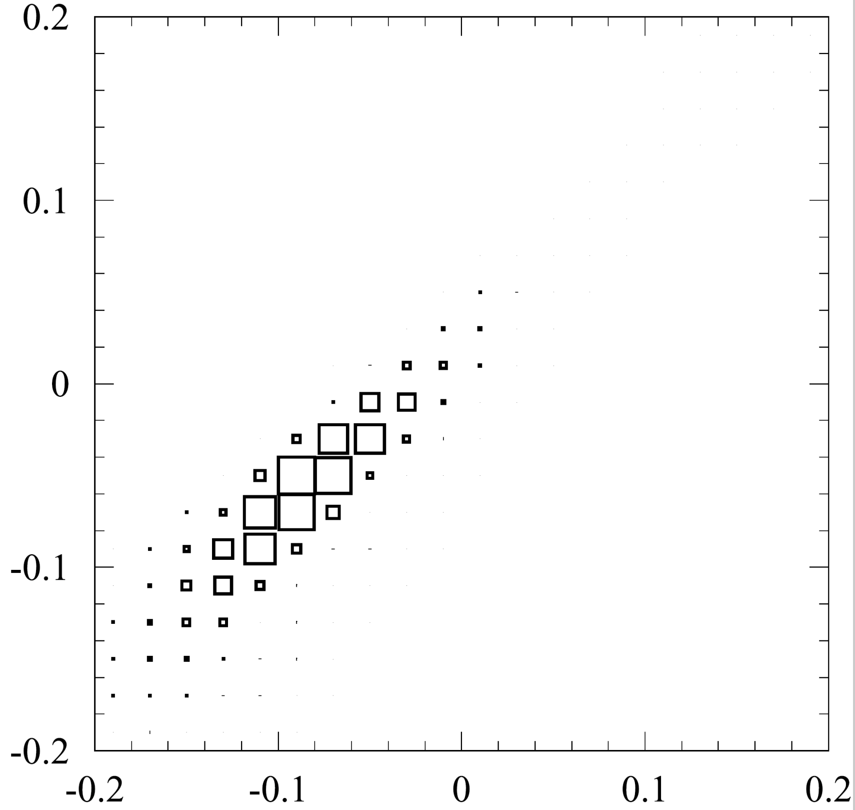
**BABAR -0.130 ± 0.030 ± 0.009**

**4.2 sigma effect (last Monday !!)**

# B -> Kπ DECAYS (III)




$A_{CP}(B_d \to K^0\pi^+)$ vs $A_{CP}(B_d \to K^0\pi^0)$



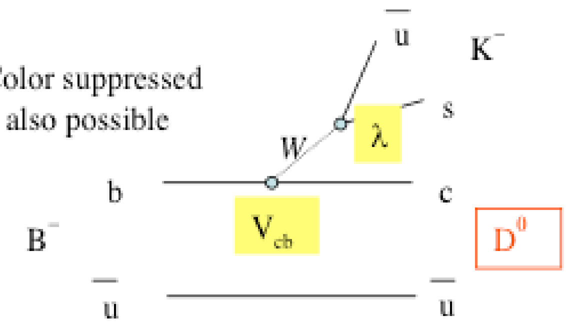

$A_{CP}(B_d \to K^+\pi^0)$ vs $A_{CP}(B_d \to K^+\pi^-)$

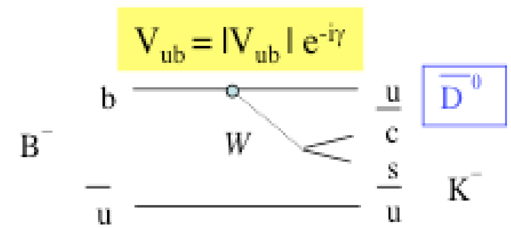
**Direct CP violation** occurs because there are two different ways of reaching the same final state

In this particular case sensitive to $\gamma$

$D^0$ and $\overline{D}^0$ are involved

Color suppressed also possible




$$A(B^- \to D^0 K^-) = A_B$$

$$A(B^+ \to \overline{D}^0 K^+) = A_B$$

$$A(B^- \to \overline{D}^0 K^-) = A_B r_B e^{i(\delta_B - \gamma)}$$

$$A(B^+ \to D^0 K^+) = A_B r_B e^{i(\delta_B + \gamma)}$$

$A_B$ — strong amplitude (the same for $V_{ub}$ and $V_{cb}$ mediated transitions

$\delta_B = \delta_1 - \delta_2$ — strong phase difference between $V_{ub}$ and $V_{cb}$ mediated transitions

$$r_B = \left| \frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right|$$

**GLW** (Gronau,London,Wyler) Method

$$|D^0_{CP\pm}\rangle = \frac{1}{\sqrt{2}}\left( |D^0\rangle \pm |\overline{D}^0\rangle \right)$$

Look at $D^0$(CP) states

$$\sqrt{2}A(B^+ \to D^0_{CP+} K^+) = A(B^+ \to D^0 K^+) + A(B^+ \to \overline{D}^0 K^+)$$

$$\sqrt{2}A(B^- \to D^0_{CP+} K^-) = A(B^- \to D^0 K^-) + A(B^- \to \overline{D}^0 K^-)$$

$$\sqrt{2}A(B^+ \to D^0_{CP-} K^+) = A(B^+ \to D^0 K^+) - A(B^+ \to \overline{D}^0 K^+)$$

$$\sqrt{2}A(B^- \to D^0_{CP-} K^-) = A(B^- \to D^0 K^-) - A(B^- \to \overline{D}^0 K^-)$$

**ADS** (Atwood, Dunietz, Soni) Method

$D^0$ and $\overline{D}^0 \to f$

$D^0$ and $\overline{D}^0$ give the same final

**GLW (Gronau,London,Wyler) Method**

$$A_{CP\pm} = \frac{\Gamma(B^+ \to D^0_{CP\pm}K^+) - \Gamma(B^- \to D^0_{CP\pm}K^-)}{\Gamma(B^+ \to D^0_{CP\pm}K^+) + \Gamma(B^- \to D^0_{CP\pm}K^-)} = \frac{\pm 2 r_B \sin\gamma \sin\delta_B}{1 + r_B{}^2 \pm 2 r_B \cos\gamma \cos\delta_B}$$

$$R_{CP\pm} = \frac{\Gamma(B^+ \to D^0_{CP\pm}K^+) + \Gamma(B^- \to D^0_{CP\pm}K^-)}{\Gamma(B^+ \to \overline{D}^0 K^+) + \Gamma(B^- \to D^0 K^-)} = 1 + r_B{}^2 \pm 2 r_B \cos\gamma \cos\delta_B$$

---

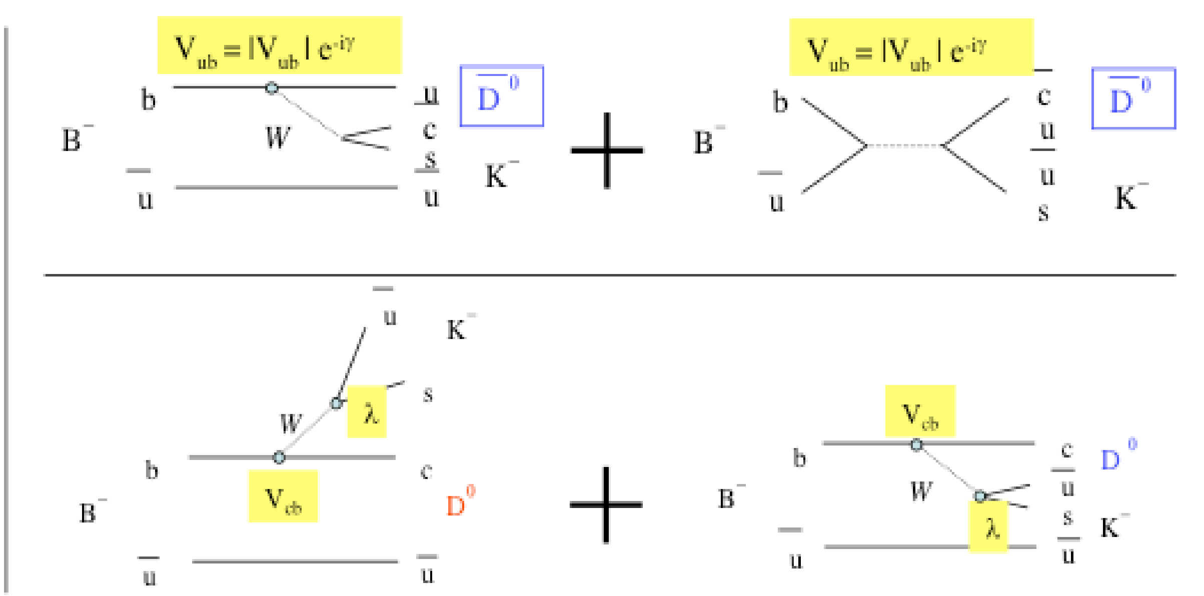
**ADS (Atwood, Dunietz, Soni) Method (only Babar)**

$$R_{ADS} = \frac{\Gamma(B^+ \to (K^-\pi^+)_D K^+) - \Gamma(B^- \to (K^+\pi^-)_D K^-)}{\Gamma(B^+ \to (K^+\pi^-)_D K^+) + \Gamma(B^- \to (K^-\pi^+)_D K^-)} = r_B{}^2 + r_{DCS}^2 + 2 r_B\, r_{DCS} \cos\gamma \cos(\delta_B + \delta_D)$$

$$r_{DCS} \equiv \sqrt{\frac{BR(D^0 \to K^-\pi^+)}{BR(D^0 \to K^+\pi^-)}} \rightarrow (3.62 \pm 0.29)10^{-3}$$

$r_B$ is a crucial parameter. It drives the sensitivity on $\gamma$

What about $r_B$ ? $\quad r_B = \left| \frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right|$




$$= \left| \frac{C + A}{T + \overline{C}} \right|$$

$$r_B = |RB \times RCT| = \left| \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right| \left| \frac{C + A}{T + \overline{C}} \right| = \sqrt{\overline{\eta}^2 + \overline{\rho}^2} \left| \frac{C + A}{T + \overline{C}} \right| \qquad RB = 0.36 \pm 0.04$$

Evaluation can be done if Annihilation diagram is neglected $\quad RCT \approx \sqrt{\frac{Br(\overline{B}^0 \to D^0 \overline{K}^0)}{Br(B^- \to D^0 K^-)}} = 0.34 \pm 0.10 \quad$ $r_B = 0.12 \pm 0.04$

Beyond this approx. If |A/C|~0.3 (max?) (+- 30% according to the interference between A and C)

$$r_B = 0.12 \pm 0.04(stat) \pm 0.04(theo.)$$

Conclusions : should be measured on data

GLW

$A_{CP+} = 0.07 \pm 0.13$ $\quad 2r \sin\gamma \sin\delta / R_{CP+}$

$A_{CP-} = -0.19 \pm 0.18$ $\quad -2r \sin\gamma \sin\delta / R_{CP+}$

$R_{CP+} = 1.09 \pm 0.16$ $\quad 1 + r^2 + 2r \cos\gamma \cos\delta_B$

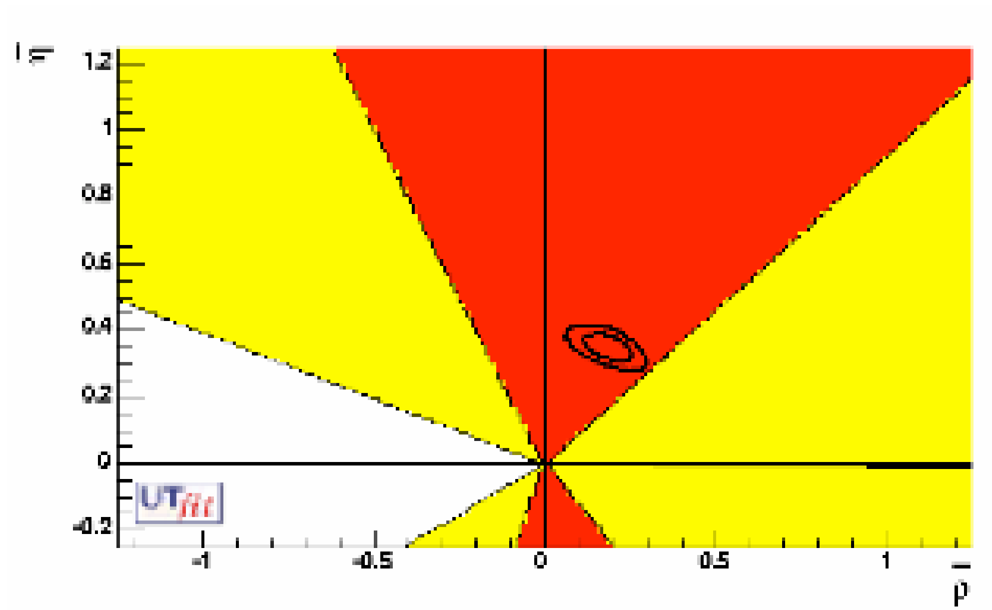
$R_{CP-} = 1.30 \pm 0.25$ $\quad 1 + r^2 - 2r \cos\gamma \cos\delta_B$

$CP+ \quad K^+\pi^- \quad \pi^+\pi^-$

$CP- \quad K_s\pi^0 \quad K_s\phi \quad K_s\omega \quad K_s\eta'$

$R_{ADS} = 0.0054 \pm 0.0124$ $\quad r_{DCS}^2 + r_B^2 + 2r_B r_{DCS} \cos\gamma \cos(\delta_B + \delta_D)$ ADS

Drawn into the ρ−η plane




$\gamma = (81 \pm 35)^o$

**Tree level diagrams, not influenced by new physics**

$\gamma = (61.9 \pm 7.9)^0$

*UTA*

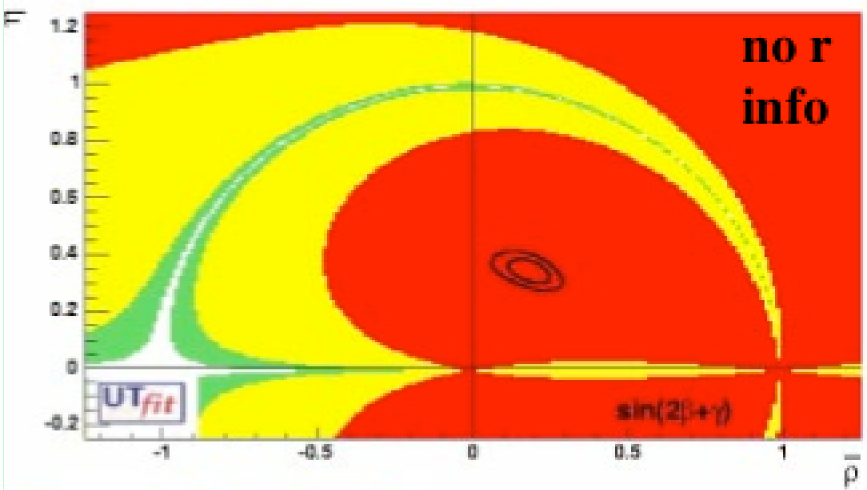
# Nuovi Input: $\sin(2\beta+\gamma)$

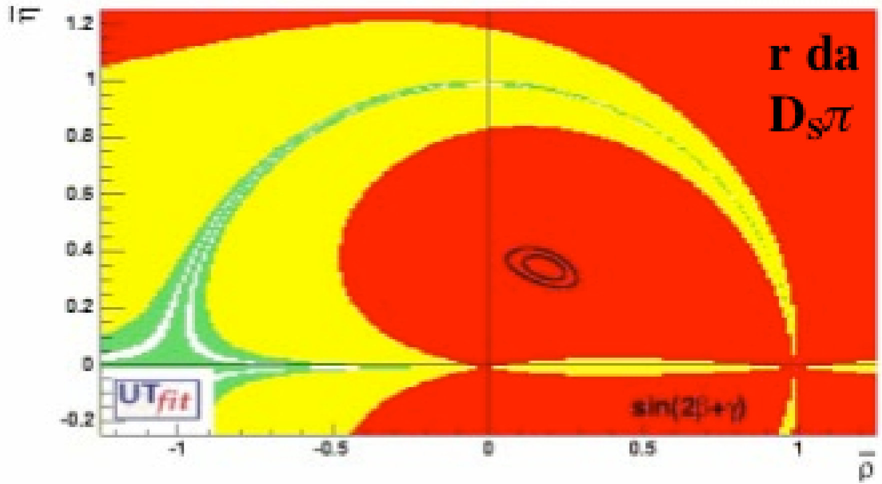
$B^0 \rightarrow D^{\pm(*)}\pi^{\pm}$

$a^{(*)} = 2r^{(*)}\sin(2\beta+\gamma)\cos\delta^{(*)}$

$c^{(*)} = 2r^{(*)}\cos(2\beta+\gamma)\sin\delta^{(*)}$ (tag leptonico)

valori medi da HFAG



# FUTURE:

# FCNC &
# CP Violation
# beyond
# the Standard Model

# $\not{C}\not{P}$ beyond the SM (Supersymmetry)

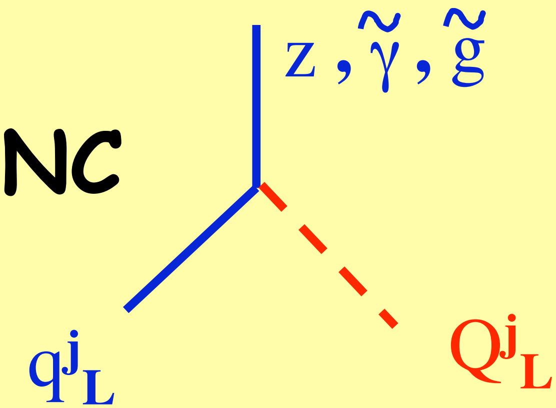
| | |
|---|---|
| Spin 1/2 — Quarks: $q_L$ , $u_R$ , $d_R$ <br> Leptons: $l_L$ , $e_R$ | Spin 0 — SQuarks: $Q_L$ , $U_R$ , $D_R$ <br> SLeptons: $L_L$ , $E_R$ |
| Spin 1 — Gauge bosons: $W$ , $Z$ , $\gamma$ , $g$ | Spin 1/2 — Gauginos: $\tilde{w}$ , $\tilde{z}$ , $\tilde{\gamma}$ , $\tilde{g}$ |
| Spin 0 — Higgs bosons: $H_1$ , $H_2$ | Spin 1/2 — Higgsinos: $\tilde{H}_1$ , $\tilde{H}_2$ |

In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case

We may either

Diagonalize the SMM

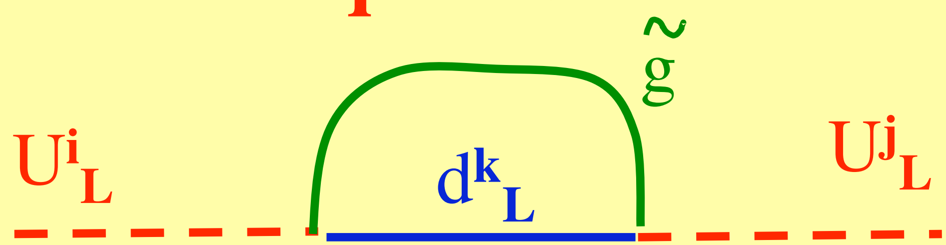
FCNC



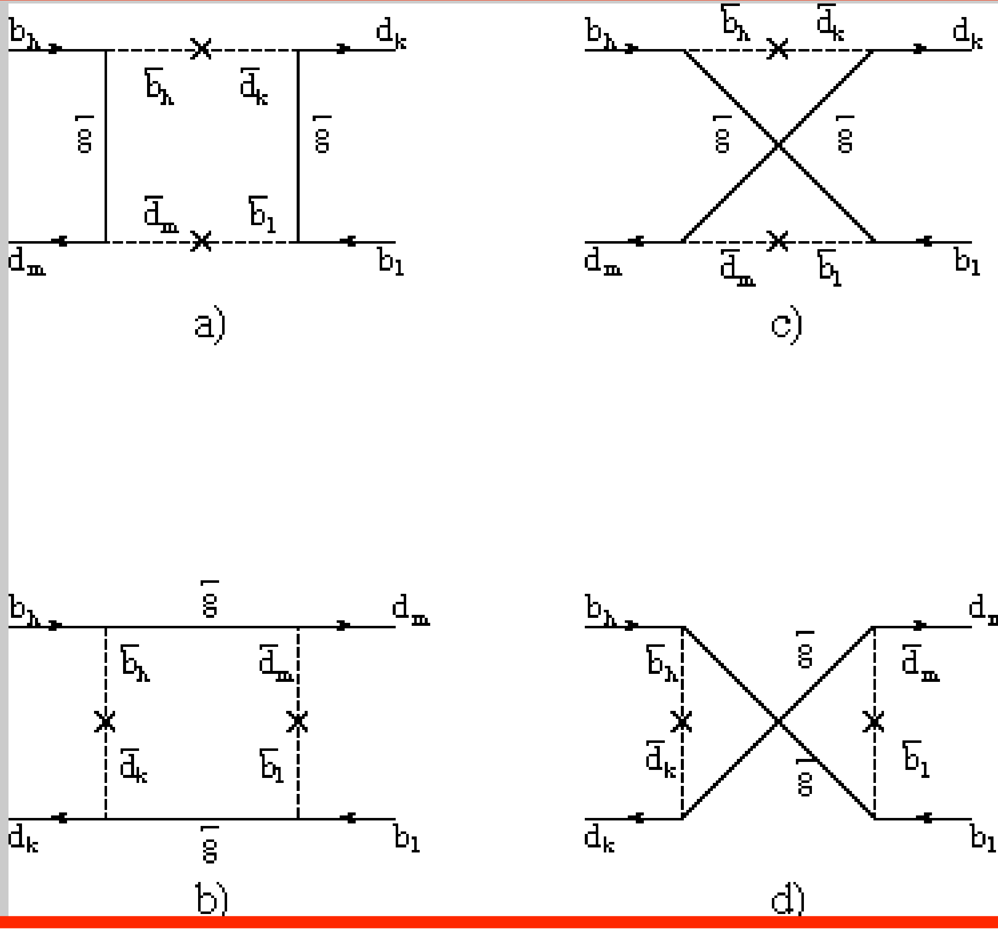

or

Rotate by the same matrices the SUSY partners of the u- and d- like quarks

$(Q^j_L)' = U^{ij}_L \, Q^j_L$



# In the latter case the Squark Mass Matrix is not diagonal




$$(m^2_Q)_{ij} = m^2_{average}\, \mathbf{1}_{ij} + \Delta m_{ij}^2 \qquad \delta_{ij} = \Delta m_{ij}^2 / m^2_{average}$$

# Deviations from the SM ?

Model independent analysis:

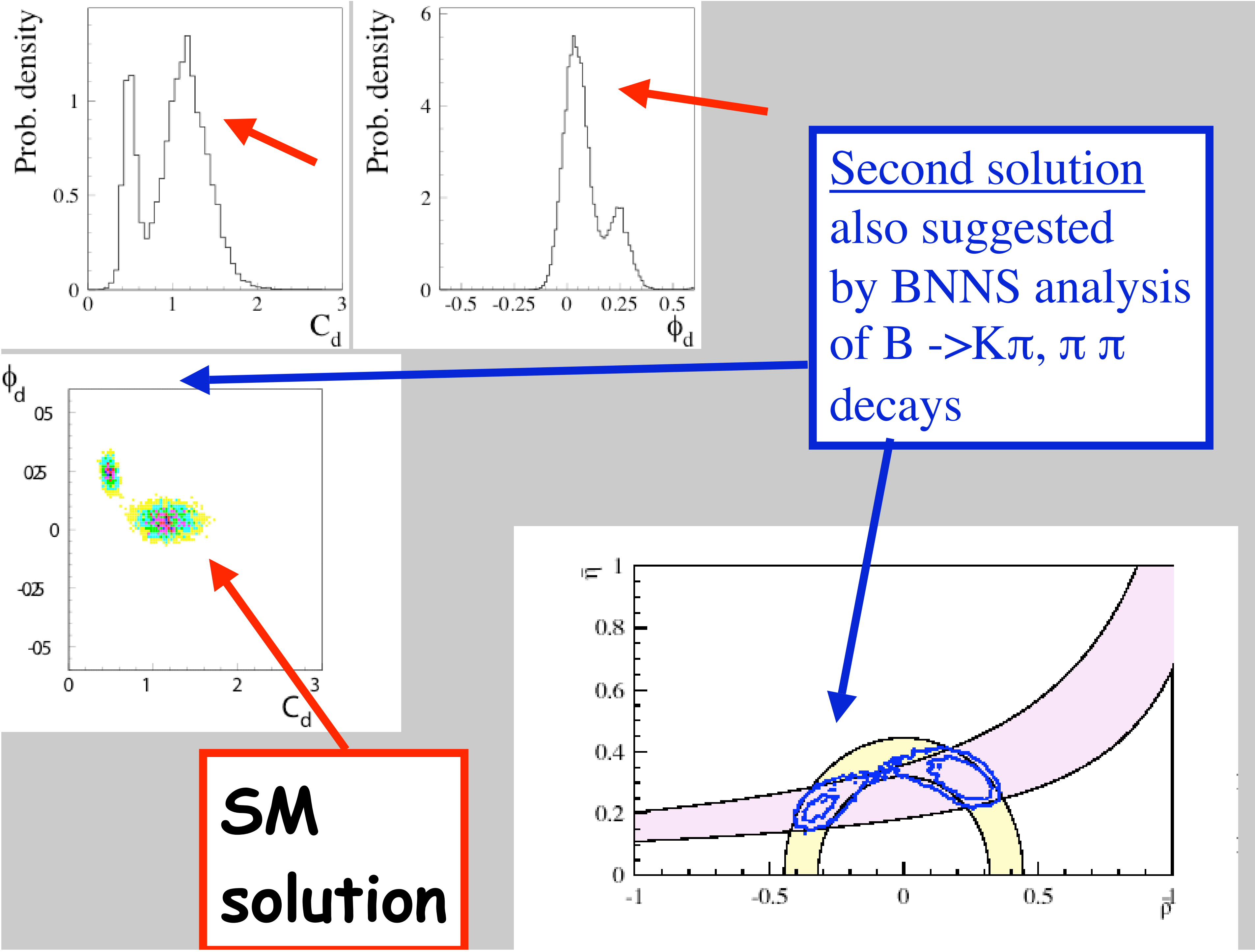
Example $B^0$-$\overline{B}^0$ mixing

(M.Ciuchini et al. hep-ph/0307195)

$$C_q\, e^{2i\phi_q} = \frac{\langle \bar{B}_q^0 | \mathcal{H}_{eff}^{full} | B_q^0 \rangle}{\langle \bar{B}_q^0 | \mathcal{H}_{eff}^{SM} | B_q^0 \rangle}$$

$$\Delta m_d = C_d\, \Delta m_d^{SM} \qquad (B_d^0 - \bar{B}_d^0 \text{ mixing})$$

$$A_{CP}(J/\Psi\ K_s) = \sin 2(\beta + \phi_d)$$


Prob. density
0
0.5
1
0
1
2
3
$C_d$
Prob. density
0
2
4
6
-0.5
-0.25
0
0.25
0.5
$\phi_d$
Second solution
also suggested
by BNNS analysis
of B ->Kπ, π π
decays
$\phi_d$
05
025
0
-025
-05
0
1
2
3
$C_d$
SM
solution
$\bar{\eta}$
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
$\bar{\rho}$

# TYPICAL BOUNDS FROM $\Delta M_K$ AND $\varepsilon_K$

$x = m^2_{\tilde{g}} / m^2_{\tilde{q}}$

$x = 1$ $\qquad m_{\tilde{q}} = 500$ GeV

$$\sqrt{|\mathrm{Re}\,(\delta_{12}^2)_{LL}|} < 3.9 \times 10^{-2}$$

$$\sqrt{|\mathrm{Re}\,(\delta_{12}^2)_{LR}|} < 2.5 \times 10^{-3}$$

$$\sqrt{|\mathrm{Re}\,(\delta_{12})_{LL}\,(\delta_{12})_{RR}|} < 8.7 \times 10^{-4}$$


from $\Delta M_K$

# from $\varepsilon_K$

$x = 1 \quad m_{\tilde{q}} = 500$ GeV

$$\sqrt{|\,\mathrm{Im}\,(\delta_{12}{}^2)_{LL}\,|} \;<\; 5.8 \times 10^{-3}$$

$$\sqrt{|\,\mathrm{Im}\,(\delta_{12}{}^2)_{LR}\,|} \;<\; 3.7 \times 10^{-4}$$

$$\sqrt{|\,\mathrm{Im}\,(\delta_{12})_{LL}\,(\delta_{12})_{RR}\,|} \;<\; 1.3 \times 10^{-4}$$

$\Delta M_B$ and $\mathcal{A}(B \to J/\psi\, K_s)$

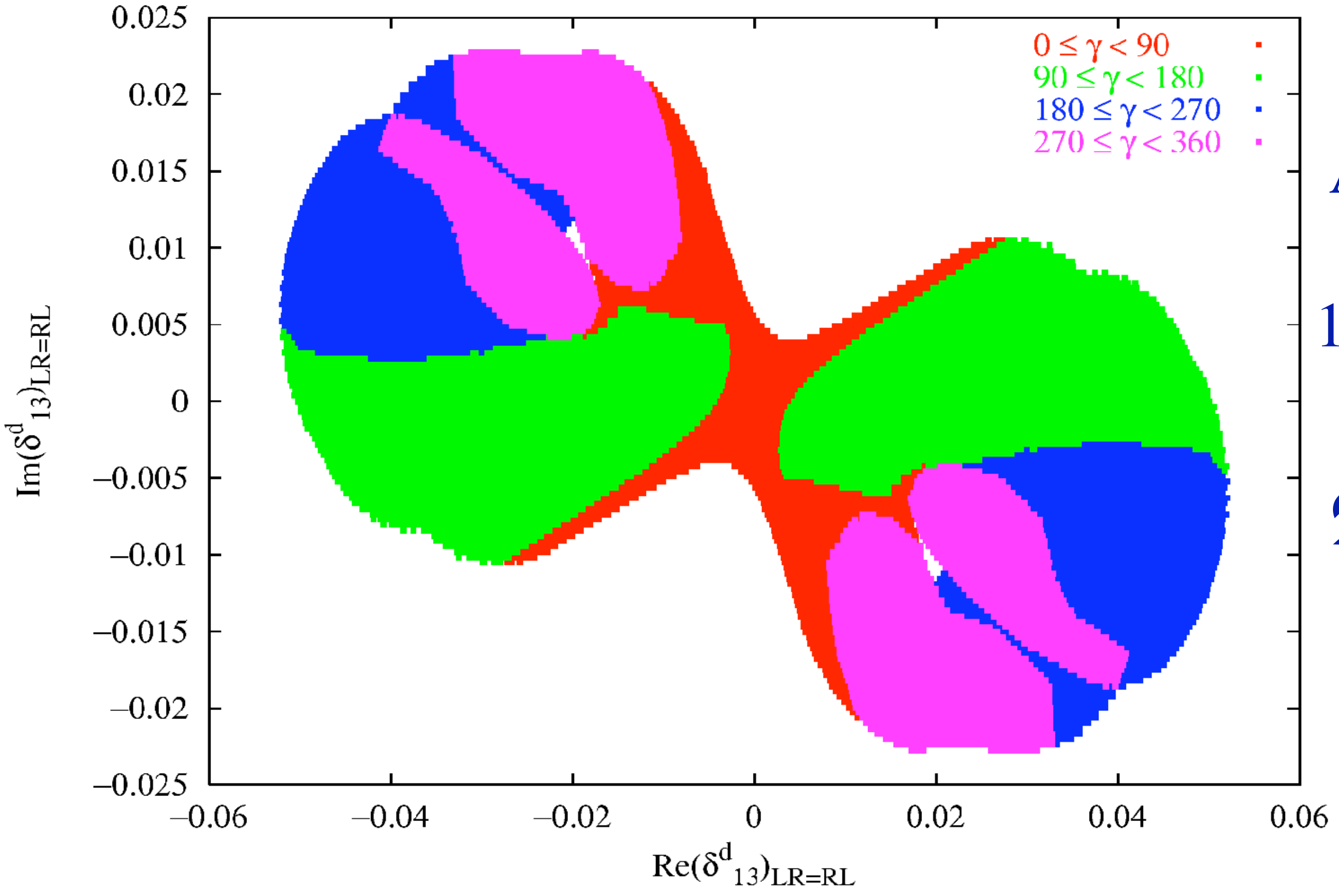
$$\Delta M_{B_d} = 2\,\mathrm{Abs}\,|\langle \overline{B}_d | \mathcal{H}_{eff}^{\Delta B=2} | B_d \rangle|$$

$$\mathcal{A}(B \to J/\psi\, K_s) = \sin 2\beta_{eff} \quad \sin \Delta M_{B_d} t$$

$$2\beta_{eff} = \mathrm{Arg}\,|\langle \overline{B}_d | \mathcal{H}_{eff}^{\Delta B=2} | B_d \rangle|$$

$\sin 2\beta = 0.734 \pm 0.054$ from exps
BaBar & Belle & others

# TYPICAL BOUNDS ON THE δ-COUPLINGS




A, B =LL, LR, RL, RR

1,3 = generation index

$$\mathcal{A}_{SM} = \mathcal{A}_{SM}(\delta_{SM})$$

$$\langle B^0 | \mathcal{H}_{eff}^{\Delta B=2} | B^0 \rangle = \mathrm{Re}\,\mathcal{A}_{SM} + \mathrm{Im}\,\mathcal{A}_{SM}$$

$$+ \mathcal{A}_{SUSY}\,\mathrm{Re}(\delta_{13}{}^{d})_{AB}{}^{2} + i\,\mathcal{A}_{SUSY}\,\mathrm{Im}(\delta_{13}{}^{d})_{AB}{}^{2}$$

# TYPICAL BOUNDS ON THE δ-COUPLINGS

$\langle B^0 | \mathcal{H}_{eff}^{\Delta B=2} | B^0 \rangle = \mathrm{Re}\,\mathcal{A}_{SM} + \mathrm{Im}\,\mathcal{A}_{SM}$

$+ \mathcal{A}_{SUSY}\,\mathrm{Re}(\delta_{13}{}^{d})_{AB}{}^{2} + i\,\mathcal{A}_{SUSY}\,\mathrm{Im}(\delta_{13}{}^{d})_{AB}{}^{2}$

Typical bounds:

$\mathrm{Re,Im}(\delta_{13}{}^{d})_{AB} \leq 1 \div 5 \times 10^{-2}$

Note: in this game $\delta_{SM}$ is not determined by the UTA

From Kaon mixing: $\mathrm{Re,Im}(\delta_{12}{}^{d})_{AB} \leq 1 \times 10^{-4}$

SERIOUS CONSTRAINTS ON SUSY MODELS

# CP Violation beyond the Standard Model

Strongly constrained for $b \to d$ transitions,

Much less for $b \to s$ :

$BR(B \to X_s \gamma) = (3.29 \pm 0.34) \times 10^{-4}$

$\mathcal{A}_{CP}(B \to X_s \gamma) = -0.02 \pm 0.04$

$BR(B \to X_s l^+ l^-) = (6.1 \pm 1.4 \pm 1.3) \times 10^{-6}$

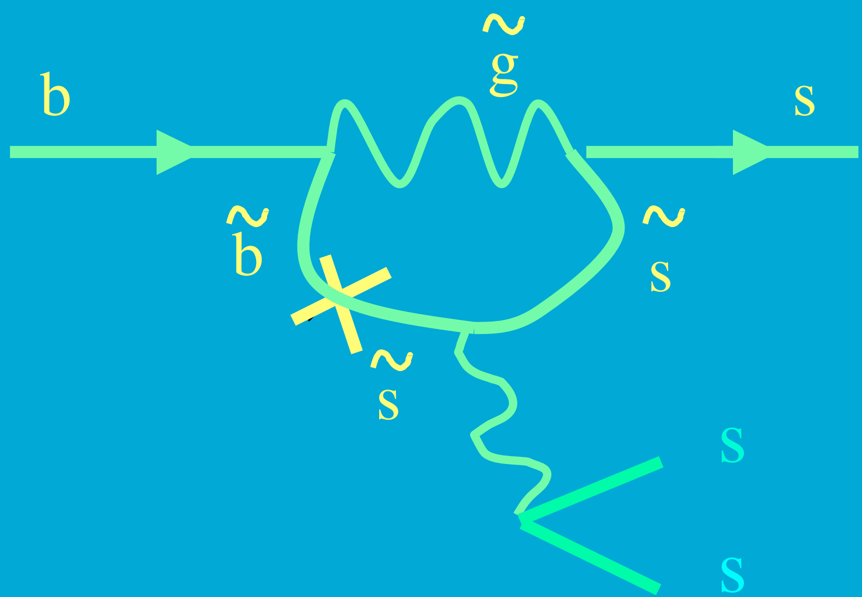
The lower bound on $B^0_s$ mixing $\Delta m_s > 14\ ps^{-1}$

# SM Penguins

# SUSY Penguins

b W s t

Recent analyses by G. Kane et al., Murayama et al.and Ciuchini et al.




Also Higgs (h,H,A) contributions

## $\mathcal{A}_{CP}(B_d \to \phi K_s)$ (2002 results)

| Observable | BaBar | Belle | Average | SM prediction |
|---|---|---|---|---|
| BR (in $10^{-6}$) | $8.1^{+3.1}_{-2.5} \pm 0.8$ | $8.7^{+3.8}_{-3.0} \pm 1.5$ | $8.7^{+2.5}_{-2.1}$ | $\sim 5$ |
| $S_{\phi Ks}$ | $-0.19^{+0.52}_{-0.50} \pm 0.09$ | $-0.73 \pm 0.64 \pm 0.09$ | $-0.39 \pm 0.41$ | $+0.734 \pm 0.054$ |
| $C_{\phi Ks}$ | – | $0.56 \pm 0.41 \pm 0.12$ | $0.56 \pm 0.43$ | $-0.08$ |

$$\mathcal{A}_{\phi Ks} = -C_{\phi Ks} \cos(\Delta m_B t) + S_{\phi Ks} \sin(\Delta m_B t)$$

[$\mathcal{A}_{CP}(B_d \to \pi K)$ do not give significant constraints ]

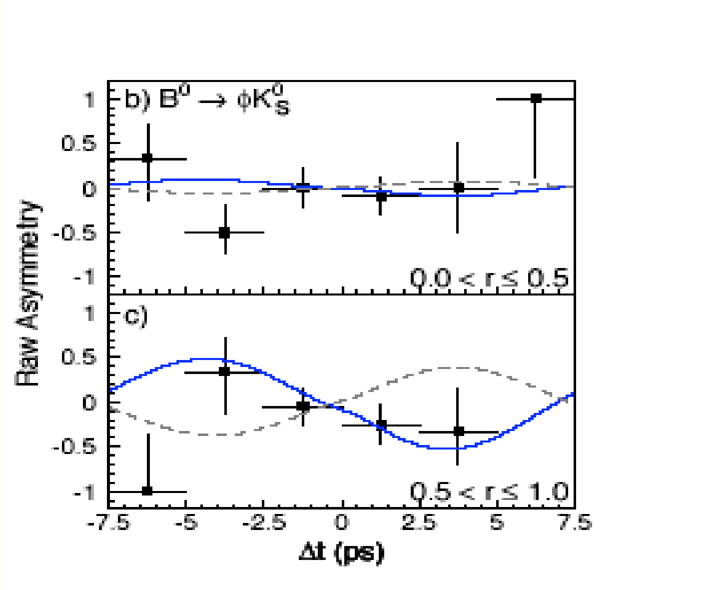
One may a also consider $B_s \to \mu\mu$
(for which there is an upper bound from Tevatron, CDF
$BR < 2.6\ 10^{-6}$)

## $\mathcal{A}_{CP}$ ($B_d$ -> $\phi$ $K_s$ ) (2003 results)

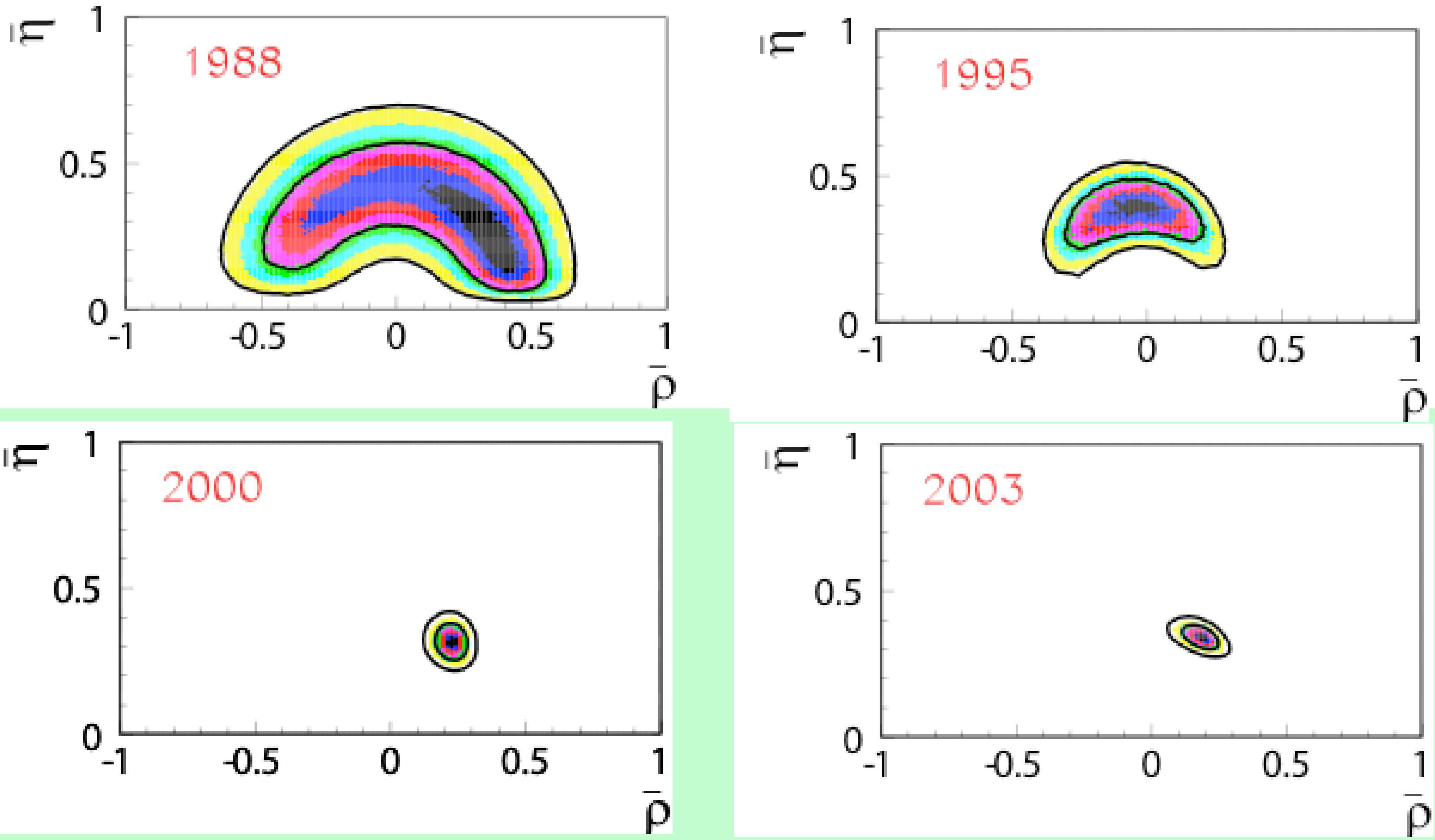
| Observable | BaBar | Belle | Average | SM prediction |
|---|---|---|---|---|
| $S_{\phi Ks}$ | $+0.47 \pm 0.34^{+0.08}_{-0.06}$ | $-0.96 \pm 0.50^{+0.09}_{-0.11}$ | | **+0.73±0.07** |

see M. Ciuchini et al. Presented at Moriond 2004 by L. Silvestrini

$$\mathcal{A}_{\phi Ks} = - C_{\phi Ks} \cos(\Delta m_B t) + S_{\phi Ks} \sin(\Delta m_B t)$$



# PROGRESS SINCE 1988



# FUTURE:
# who knows ?
# This is what makes it Interesting !

# WHY RARE DECAYS ?

Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

| | |
|---|---|
| Proton decay | baryon and lepton number conservation |
| $\mu \rightarrow e + \gamma$<br>$\nu_i \rightarrow \nu_k$ | lepton flavor number |

# RARE DECAYS WHICH ARE ALLOWED IN THE STANDARD MODEL

**FCNC:**

$q_i \to q_k + \nu\,\bar{\nu}$

$q_i \to q_k + l^+ l^-$

$q_i \to q_k + \gamma$

these decays occur only via loops because of GIM and are suppressed by CKM

THUS THEY ARE SENSITIVE TO

NEW PHYSICS

## Why we like $K \to \pi \nu \bar{\nu}$ ?

**For the same reason as $A_{J/\psi K_S}$ :**

**1) Dominated by short distance dynamics**

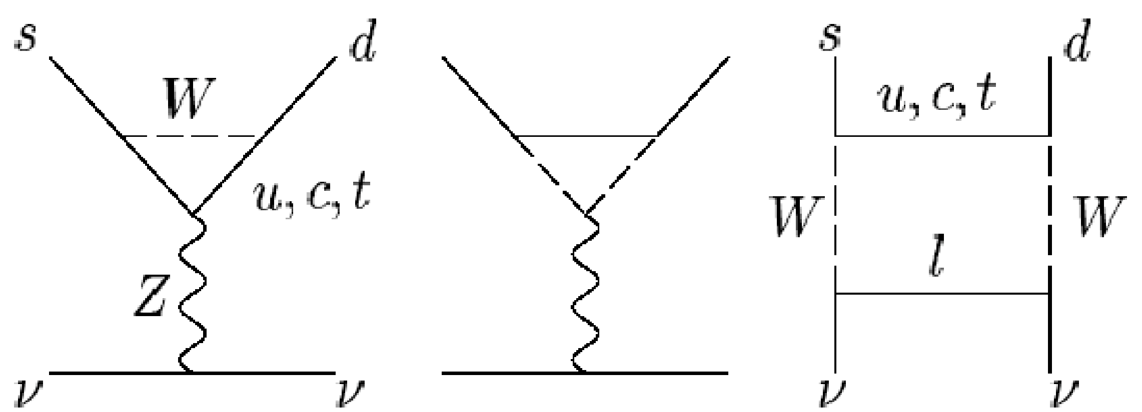
(hard GIM suppression, calculable in pert. theory )

**2) Negligible hadronic uncertainties**

(matrix element known)

$O(G^2_F)$ Z and W penguin/box $s \to d \nu \bar{\nu}$ diagrams

**SM Diagrams**



$$\mathcal{H}_{eff} = G^2_F \alpha / (2\sqrt{2}\pi\, s^2_W)[\, V_{td} V_{ts}^* X_t + V_{cd} V_{cs}^* X_c\,] \times (\bar{s}\gamma_\mu (1 - \gamma_5)\, d)\,(\bar{\nu}\gamma^\mu (1 - \gamma_5)\,\nu)$$

☺ NLO QCD corrections to $X_{t,c}$ and $O(G^3_F m^4_t)$ contributions known

☺ the hadronic matrix element $\langle \pi \,|\, s\, \gamma_\mu (1 - \gamma_5)\, d \,|\, K \rangle$ is known with very high accuracy from Kl3 decays

☺ sensitive to $V_{td} V_{ts}^*$ and expected large $\not{C}\!P$

$A(s \to d \nu \bar{\nu})$

$O(\lambda^5 m_t^2) + i\, O(\lambda^5 m_t^2)$ CKM suppressed

$O(\lambda m_c^2) + i\, O(\lambda^5 m_c^2)$

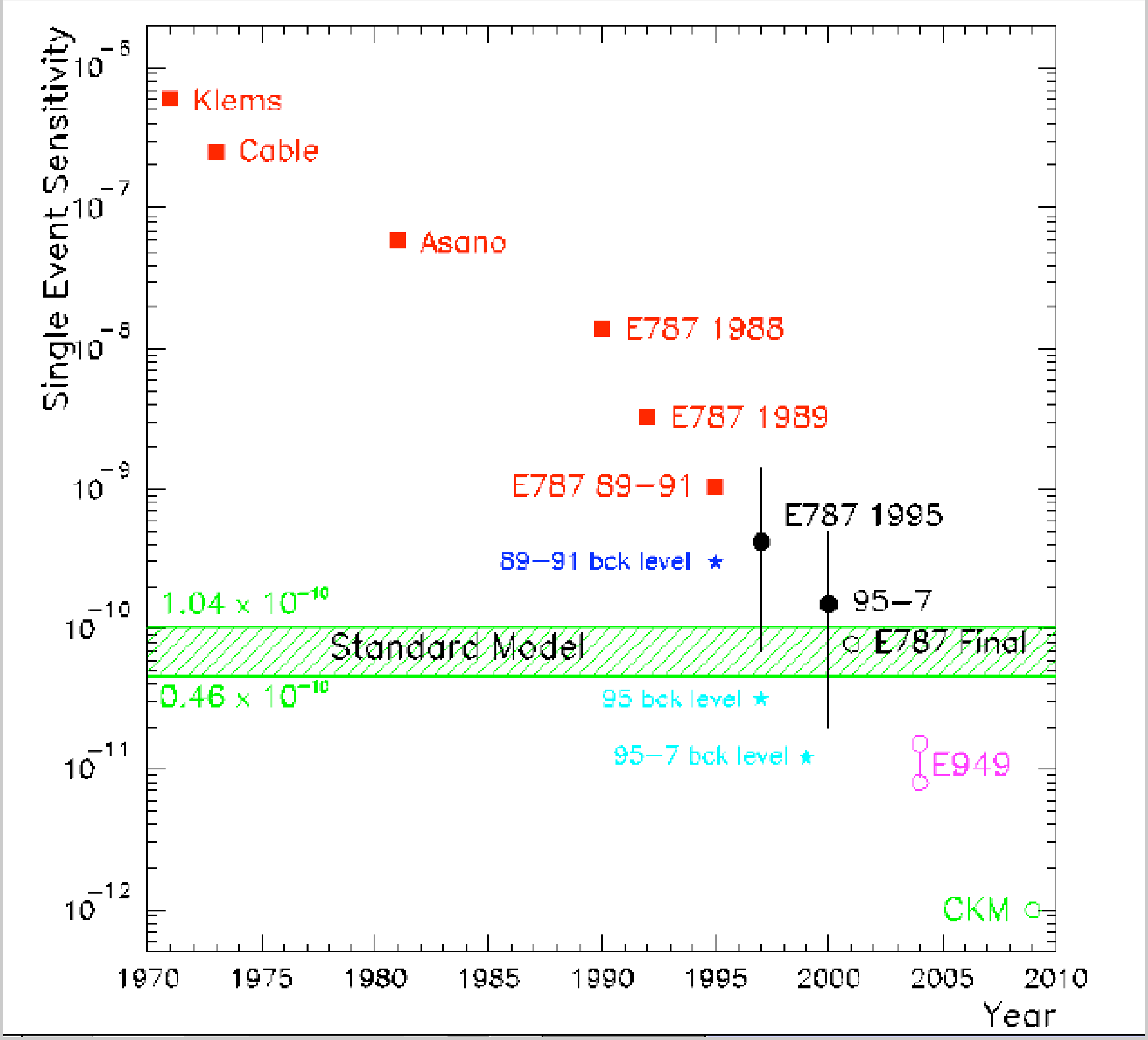
$O(\lambda \Lambda_{QCD}^2)$ GIM

---

CP conserving: error of $O(10\%)$ due to NNLO corrections in the charm contribution and CKM uncertainties **$BR(K^+)_{SM} = (7.2 \pm 2.0) \times 10^{-11}$**

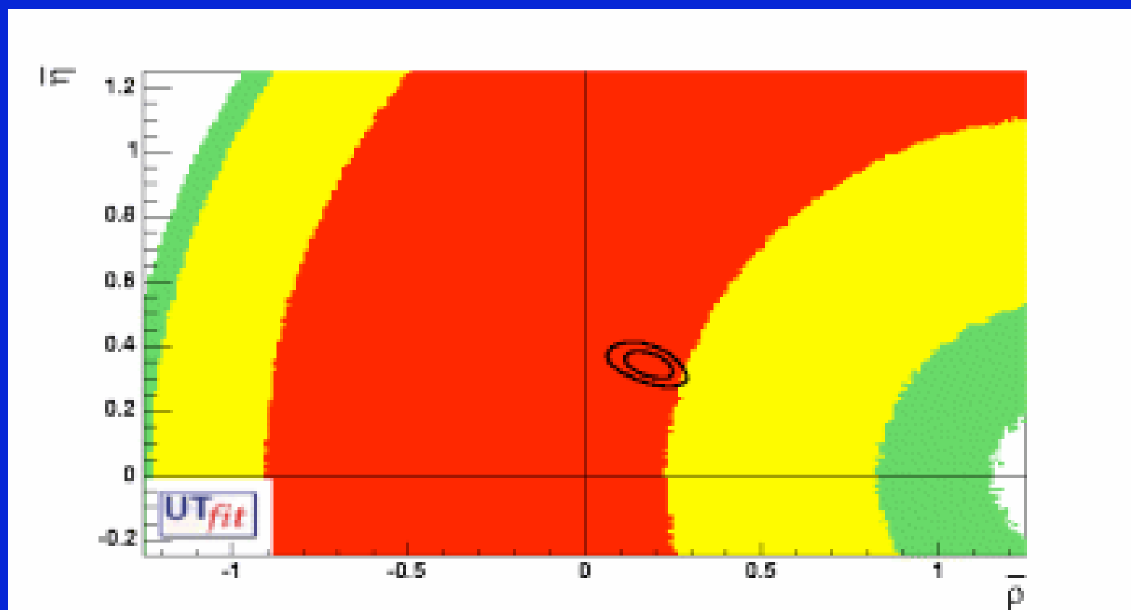
**$BR(K^+)_{EXP} = (15.7^{+17.5}_{-8.2}) \times 10^{-11}$**

- 2 events observed by E787
- central value about 2 the value of the SM
- E949 10-20 events in 2 years



Single Event Sensitivity
$10^{-6}$
$10^{-7}$
$10^{-8}$
$10^{-9}$
$10^{-10}$
$10^{-11}$
$10^{-12}$
Klems
Cable
Asano
E787 1988
E787 1989
E787 89−91
E787 1995
89−91 bck level
95−7
E787 Final
$1.04 \times 10^{-10}$
Standard Model
$0.46 \times 10^{-10}$
95 bck level
95−7 bck level
E949
CKM
1970
1975
1980
1985
1990
1995
2000
2005
2010
Year

# $K^+ \rightarrow \pi^+ \nu\nu$



# CP Violating

## $K_L \to \pi^0 \nu \bar{\nu}$

$O(\lambda^5 m^2_t) + i\, O(\lambda\, m^2_t)$

$O(\lambda\, m^2_c) + i\, O(\lambda^5 m^2_c)$

$O(\lambda\, \Lambda^2_{QCD})$

dominated by the top quark contribution -> short distances (or new physics)

theoretical error ~ 2 %

$BR(K^+)_{SM} = 4.30 \times 10^{-10} (m_t(m_t)/170GeV)^{2.3} \times (Im(V_{ts}^* V_{td})/\lambda^5)^2 = (2.8 \pm 1.0) \times 10^{-11}$

Using $\Gamma(K_L \to \pi^0 \nu\bar{\nu}) < \Gamma(K^+ \to \pi^+ \nu\bar{\nu})$

One gets $BR(K_L \to \pi^0 \nu\bar{\nu}) < 1.8 \times 10^{-9}$ (90% C.L.)

2 order of magnitude larger than the SM expectations

$$\mathrm{Im}\lambda_t = \lambda\, V_{cb}^2\, \overline{\eta} = (13.0 \pm 1.0)\ 10^{-5} \qquad (11.2 - 15.0)\ 10^{-5} \text{ at } 95\% \text{ C.L.}$$

$$BR(K_L^0 \to \pi^0 \nu \bar{\nu}) = r(K_L) \frac{\tau_{K_L}}{\tau_{K^+}} \frac{3\alpha^2}{2\pi^2} \frac{Br(K^+ \to \pi^0 e^+ \nu)}{sin^4\theta_W} (\eta_X X_0(x_t))^2 |V_{cb}|^4 \overline{\eta}^2$$

where,

$$X_0(x_t) = \frac{x}{8}\left( (\frac{x+2}{x-1}) + \frac{3x-6}{(x-1)^2} ln(x) \right)$$

$$Br(K \to \pi \nu \bar{\nu}) = (1.75 \pm 0.3)\ 10^{-11} \qquad (1.25 - 2.44)\ 10^{-11} \text{ at } 95\% \text{ C.L.}$$