

Heavy meson χ PT in finite volume

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Based on

Daniel Arndt and C-JDL, Phys Rev **D70**, 014503 (2004)

Outline

- Introduction – CKM Matrix and Lattice QCD.
- Heavy meson χ PT (HM χ PT).
- HM χ PT in finite volume – physical picture.
- HM χ PT in finite volume – technical aspect.
- Volume effects in $B^0-\bar{B}^0$ mixing and their impact on the global CKM fit.
- Conclusion.

Introduction

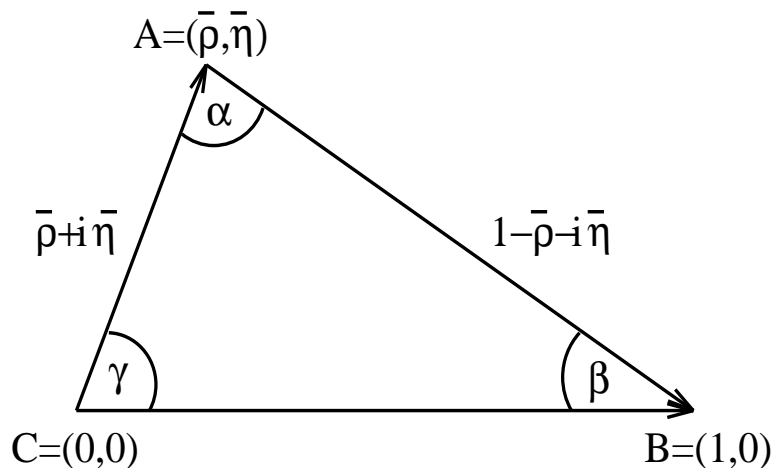
The CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

The Wolfenstein parameterisation

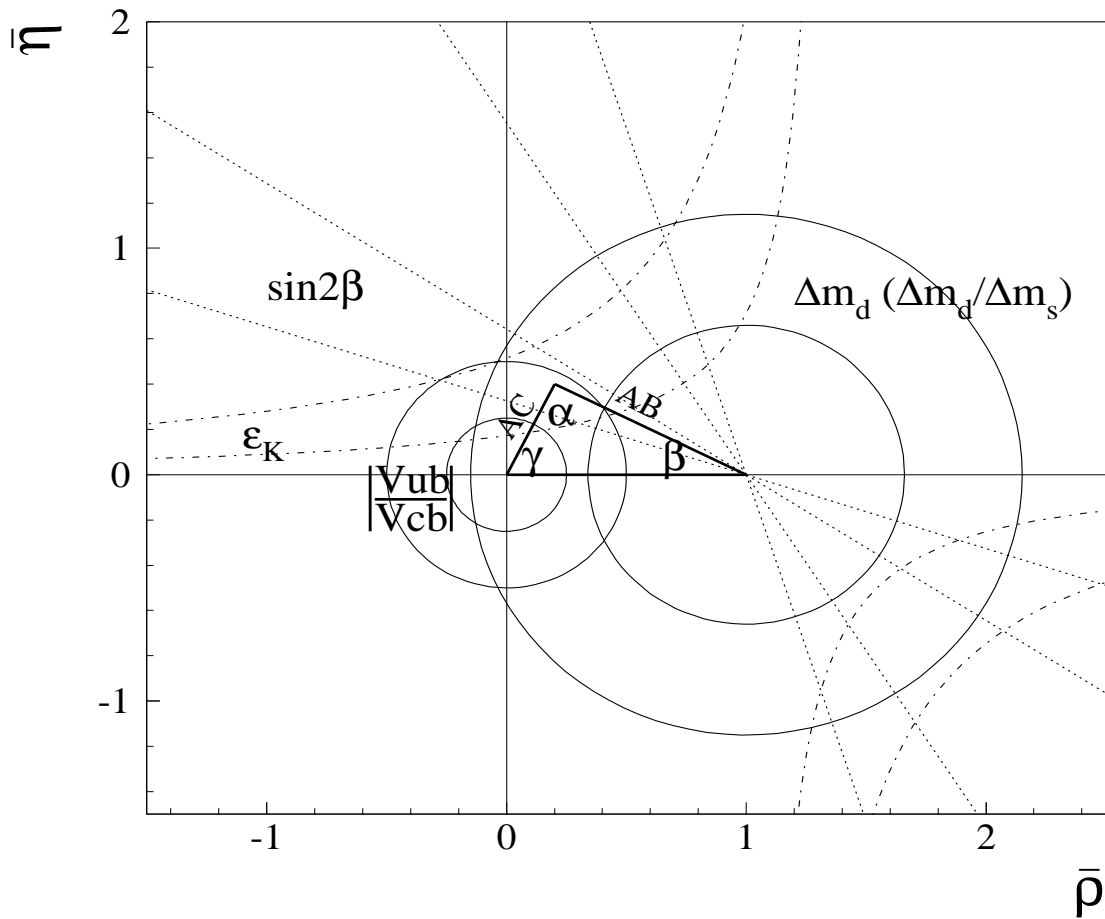
$$\hat{V}_{\text{CKM}} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4).$$

Unitarity implies $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$.



Introduction

The global CKM fit

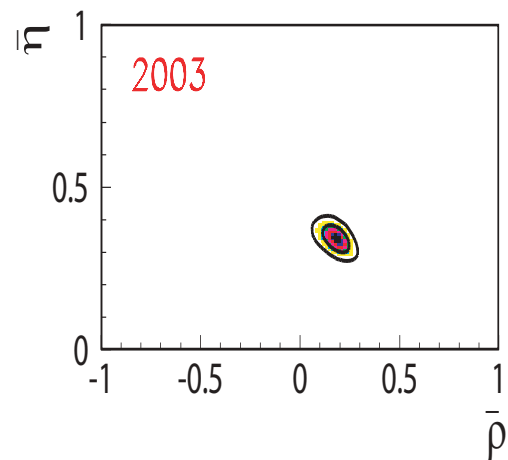
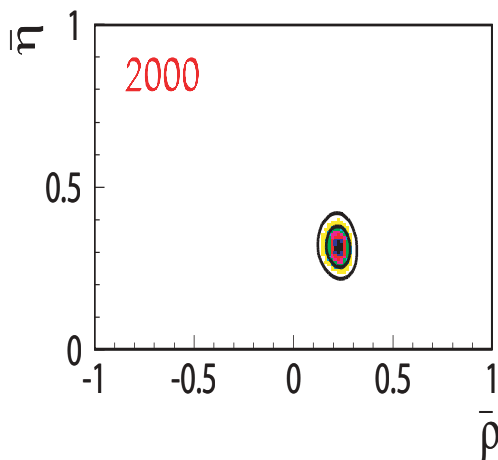
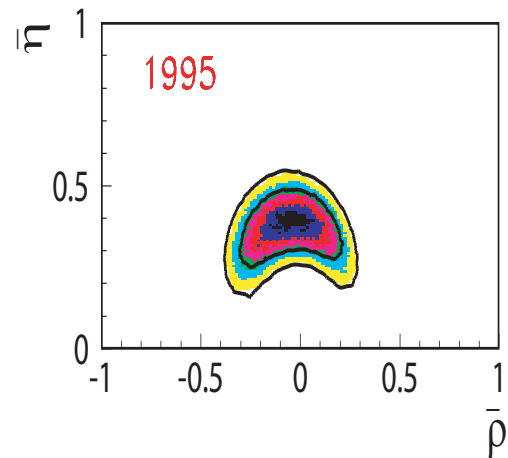
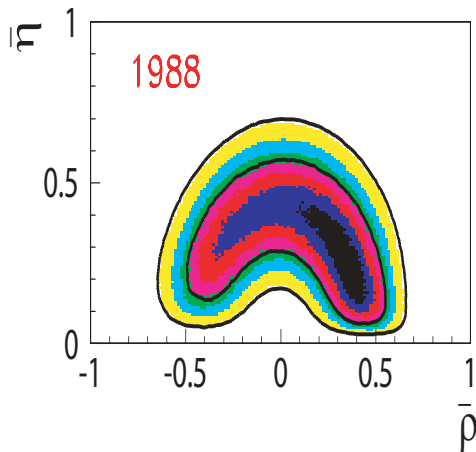


Experimental result = V_{ij} (short-distance QCD effect)

× (long-distance QCD matrix element)

Lattice QCD

Working with experiments

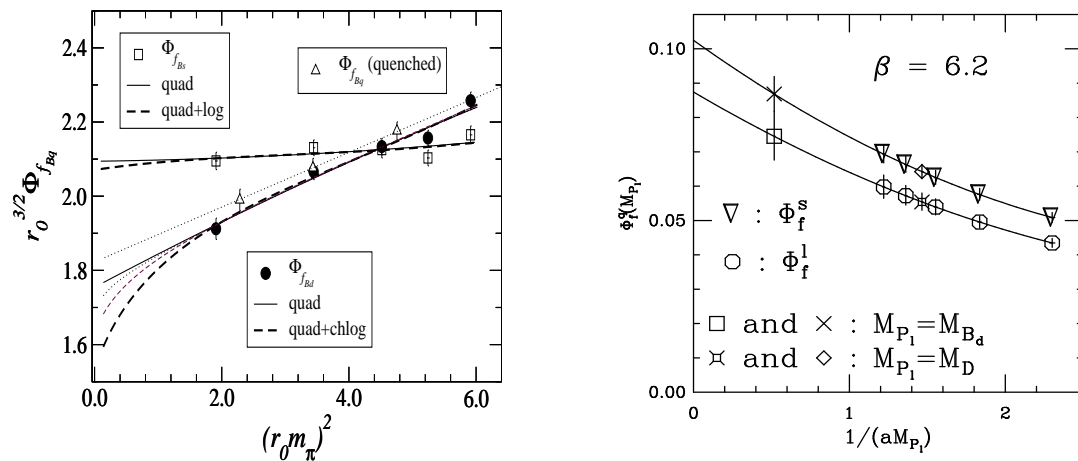


- Impressive progress.
- On-going efforts pursuing high precision.

Lattice QCD

Systematic errors in lattice QCD

- Unrealistic quark masses in the simulation



- Volume effects often depend on the meson masses
→ are they seriously amplified in the mass extrapolations?
- Other systematic effects include finite lattice spacing, (partial) quenching, etc.

HM χ PT

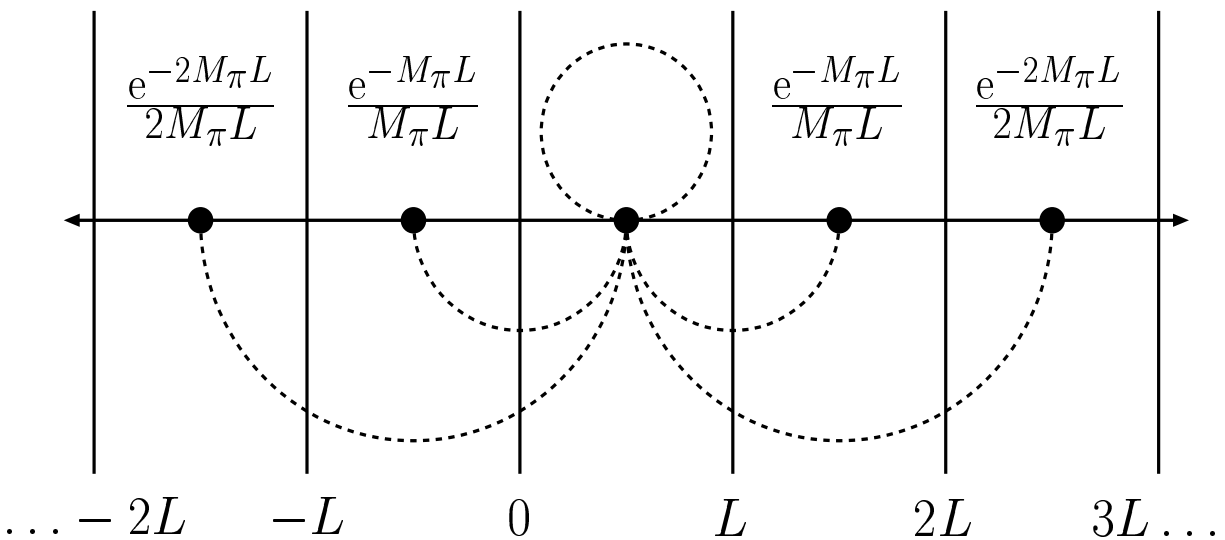
The EFT in infinite volume

- Chiral and heavy-quark spin symmetries:
 - B, B^* : static sources emitting/absorbing π .
 - $p_\mu = M_B v_\mu + k_\mu \rightarrow$ Velocity superselection rule.
- Leading order:
 - Propagators: $\underbrace{\frac{i}{2(v \cdot k + i\epsilon)}}_{B_{(s)}}, \underbrace{\frac{-i(g_{\mu\nu} - v_\mu v_\nu)}{2(v \cdot k + i\epsilon)}}_{B_{(s)}^*}$
 $\xrightarrow{\text{position space}} \sim \theta(t)\delta^{(3)}(\vec{x})$
 - $B-B^*-\pi$ and $B^*-B^*-\pi$ couplings g .
- NLO in $1/M_B$ and M_π [discard $\mathcal{O}(M_\pi/M_B)$]:
 - Mass splittings from two counterterms
 1. $\Delta = M_{B^*} - M_B \sim \lambda_2/M_B$ (indep. of M_π).
 2. $\delta_s = M_{B_s} - M_B \sim \lambda_1(m_s - m_d)$ (indep. of M_B).
 - $\underbrace{\frac{-i(g_{\mu\nu} - v_\mu v_\nu)}{2(v \cdot k - \Delta + i\epsilon)}}_{B^*}, \underbrace{\frac{i}{2(v \cdot k - \delta_s + i\epsilon)}}_{B_s}, \underbrace{\frac{-i(g_{\mu\nu} - v_\mu v_\nu)}{2(v \cdot k - \Delta - \delta_s + i\epsilon)}}_{B_s^*}$.
 - Additional counterterms.
- In reality and on the lattice, Δ and δ_s are of $\mathcal{O}(M_\pi)$.

HM χ PT in (large) finite volume

The physical picture

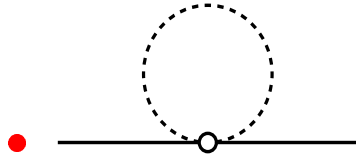
$T \rightarrow \infty$, finite L with periodic BC, B meson rest frame



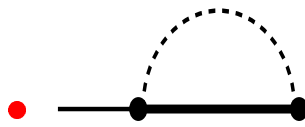
- In the limit $M_B \rightarrow \infty$, FV effects come entirely from pion wrapping around the world $\rightarrow e^{-nM_\pi L} / (nM_\pi L)$.
- When $M_B \not\rightarrow \infty$, $\Delta = M_{B^*} - M_B \sim \mathcal{O}(M_\pi)$ alters FV effects by bringing the B^* off-shell, with $\delta t \sim 1/\Delta$:
 - FV effects decrease as Δ increases.
 - The alteration is controlled by M_π/Δ .
 - Notice $\Delta \sim 1/M_B$.
 - $\delta_s = M_{B_s} - M_B$ has similar effects.

HM χ PT in finite volume

Momentum sums in the EFT



$$\text{FV}_{\text{tadpole}} \sim \sum_{\vec{n} \neq \vec{0}} \frac{1}{nL} \int_0^\infty dk \frac{k \sin(nkL)}{\sqrt{k^2 + M_\pi^2}} \quad M_\pi L \gg 1 \quad \sim \sum_{\vec{n} \neq \vec{0}} \frac{e^{-nM_\pi L}}{nM_\pi L}$$



□ Need the integral/sum:

$$-i \frac{1}{L^3} \sum_{\vec{k}} \int \frac{dk_0}{2\pi} \frac{1}{(v_0 k_0 - \Delta + i\epsilon)(k^2 - M_\pi^2 + i\epsilon)}$$

$$\text{FV}_{\text{sunset}} \sim \sum_{\vec{n} \neq \vec{0}} \frac{1}{nL} \int_0^\infty dk \frac{k \sin(nkL)}{\sqrt{k^2 + M_\pi^2} (\sqrt{k^2 + M_\pi^2} + \Delta)}$$

$$M_\pi L \gg 1 \quad \sim \sum_{\vec{n} \neq \vec{0}} \frac{e^{-nM_\pi L}}{nM_\pi L} \mathcal{A}(\Delta).$$

\mathcal{A} is the alteration of volume effects due to the mass splittings amongst the heavy-light mesons.

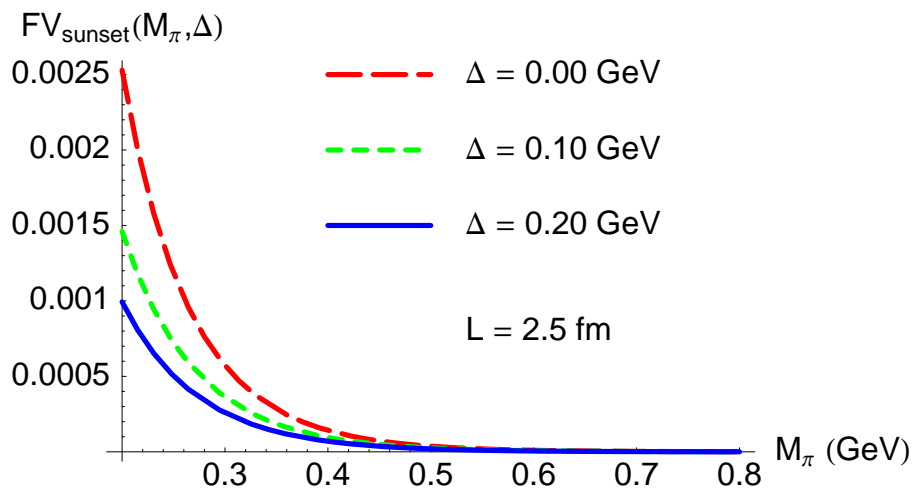
HM χ PT in finite volume Reproducing the physical picture

$$\mathcal{A} = \underbrace{\exp(z^2)}_{\mathcal{A}_0} [1 - \text{Erf}(z)] + \sum_{j=1}^{\infty} \mathcal{A}_j(z) \left(\frac{1}{nM_\pi L} \right)^j,$$

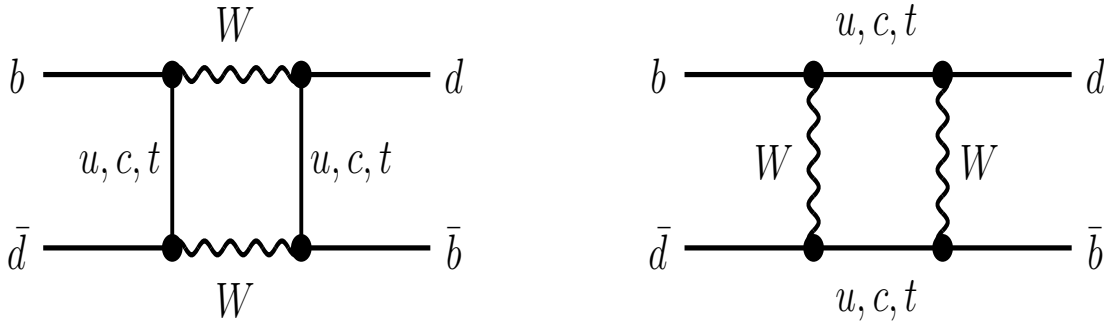
$$z = \frac{\Delta}{M_\pi} \sqrt{\frac{nM_\pi L}{2}}.$$

\mathcal{A} has the properties:

1. $\mathcal{A}_j(z) \rightarrow$ polynomials in z and $Z^m \mathcal{A}_0$, such that,
2. $\mathcal{A} = 1$ when $\Delta = 0$ (No alteration).
3. $\mathcal{A} \approx \exp(z^2) [1 - \text{Erf}(z)]$ when $z \ll 1$.
4. $\mathcal{A} \approx 1/z$ when $\Delta \sim M_\pi$.



$B^0-\bar{B}^0$ mixing Phenomenology



- The top quark dominates the box $\rightarrow |V_{td}|$.
- $\Delta m_d = \frac{G_F}{8\pi^2} M_W^2 |V_{td} V_{tb}^*|^2 \eta_B S_0 \left(\frac{m_t^2}{M_W^2} \right) C_B(\mu) |\langle \bar{B}_d | \mathcal{O}_d^{\Delta B=2}(\mu) | B_d \rangle| / M_B$.

- The operator $\mathcal{O}_d^{\Delta B=2} = [\bar{b}\gamma^\mu(1 - \gamma_5)d][\bar{b}\gamma^\mu(1 - \gamma_5)d]$.
- Cleaner theory prediction in

$$\frac{\Delta m_s}{\Delta m_d} = \left| \frac{V_{ts}}{V_{td}} \right|^2 \left(\frac{M_{B_d}}{M_{B_s}} \right) \left| \frac{\langle \bar{B}_s | \mathcal{O}_s^{\Delta B=2}(\mu) | B_s \rangle}{\langle \bar{B}_d | \mathcal{O}_d^{\Delta B=2}(\mu) | B_d \rangle} \right|.$$

\rightarrow Accurate Δm_s from Tevatron and LHC.

- Conventionally $\langle \bar{B}_d | \mathcal{O}_d^{\Delta B=2}(\mu) | B_d \rangle = \frac{8}{3} M_{B_d}^2 f_{B_d}^2 B_{B_d}(\mu)$, and

$$\left| \frac{\langle \bar{B}_s | \mathcal{O}_s^{\Delta B=2}(\mu) | B_s \rangle}{\langle \bar{B}_d | \mathcal{O}_d^{\Delta B=2}(\mu) | B_d \rangle} \right| = \left(\frac{M_{B_s}}{M_{B_d}} \right)^2 \xi^2 = \left(\frac{M_{B_s}}{M_{B_d}} \right)^2 \xi_f^2 \xi_B.$$

$B^0-\bar{B}^0$ mixing

Analysis of finite volume effects

- Ensure $M_\pi L > 2.5$.
- Analyse the lattice setup (isospin limit):
 1. simulation performed at $(m_s)_{\text{latt}} = (m_s)_{\text{phys}}$.
 2. $(m_d)_{\text{latt}}$ is varied.

Therefore need to express $M_{K,\eta}$, in χPT , as

$$M_K^2 = \frac{M_{s\bar{s}}^2 + M_\pi^2}{2},$$
$$M_\eta^2 = \frac{2M_{s\bar{s}}^2 + M_\pi^2}{3},$$

- In this scenario, $((m_s)_{\text{phys}}, m_d) \rightarrow ((M_{s\bar{s}})_{\text{phys}}, M_\pi)$:
 1. Fix $(M_{s\bar{s}})_{\text{phys}}^2 = (2M_K^2 - M_\pi^2)_{\text{phys}}$
 2. In $\text{HM}\chi\text{PT}$, the $\text{SU}(3)$ breaking effect

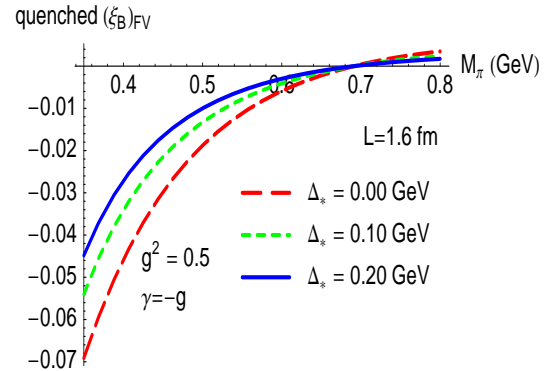
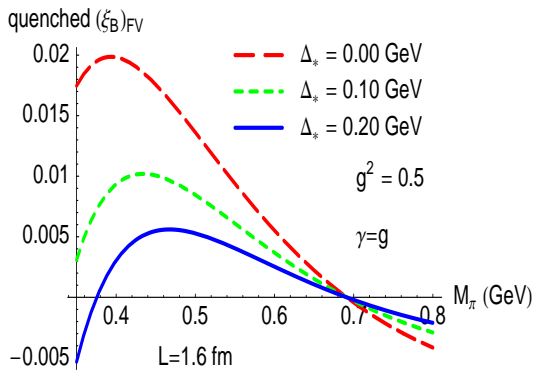
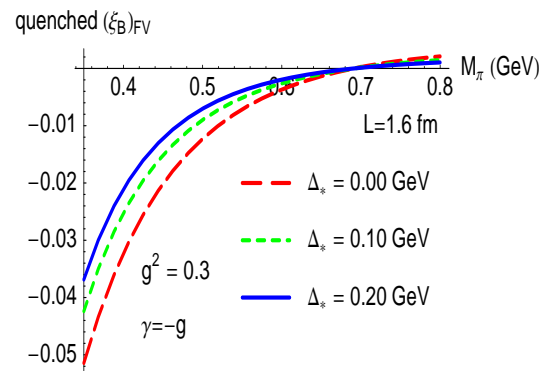
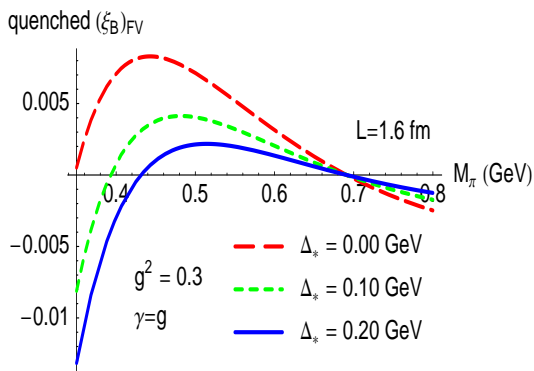
$$M_{B_s} - M_B \equiv \delta_s = \lambda_1 (M_{s\bar{s}}^2 - M_\pi^2),$$

→ Use physical M_{B_s} , M_B , $M_{s\bar{s}}$ and M_π to fix λ_1 .

$B^0-\bar{B}^0$ mixing

Plots for FV effects in B_{B_s}/B_{B_d}

$(\xi_B)_{FV}$ in quenched QCD.



γ is the $\eta'-B-B^*$ coupling.

It is clear that volume effects can be amplified in both light-quark and heavy-quark mass scalings.

$B^0-\bar{B}^0$ mixing

Impact on the global CKM fit

- Lattice 2003 review (A. Kronfeld, 2003) quotes

$$\xi = 1.23 \pm 0.10$$

- Our work on FV effects shows that in the **parameter space** of near past/future simulations:
 - QQCD data have $\sim 5-7\%$ FV effects.
 - PQQCD data have $\sim 3-4\%$ FV effects.
- **Amplified** in both light and heavy quark mass extrapolations/interpolations.
- They have **not** been taken into account in all the lattice calculations hitherto.

Conclusions and outlook

- The mass splittings amongst heavy-light mesons can alter FV effects.
- These effects can be amplified in both light and heavy quark mass extrapolations in lattice calculations.
- They can significantly enlarge the quoted error on ξ , hence have an impact on the global CKM fit.
- Same techniques in **baryon systems**
 - * Nucleon mass and magnetic moment

S.R.Beane, hep-lat/0403015.

- * Baryon axial charge

S.R.Beane and M.J.Savage, hep-ph/0404131.

- * Twist-2 operators in proton SF

W.Detmold and C-JDL, to appear.