Benasque, August 2004

How well do we understand the interaction among the pions at low energies ?

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$\pi\pi$ interaction at low energies

- Plays a crucial role whenever the strong interaction is involved at low energies
- Main experiments on $\pi\pi$ scattering were done in the seventies. What's new ?
- Significant theoretical progress, based on χPT + dispersion theory

  G. Colangelo, *Introduction to χPT*, Schleching 2003
- In the isospin limit, the scattering amplitude is characterized by a single function $A(s,t)$
  At low energies, this function can now be predicted to a remarkable degree of precision
- Theory passed one successful test:
  precision data of E865 on $K \to \pi\,\pi\,\ell\,\nu$ are in excellent agreement with the predictions

Comment on isospin breaking

- $A(s,t)$ only exists in a theoretical world where isospin is conserved. In reality, isospin is broken: $M_{\pi^0} \neq M_{\pi^+}$ etc.
- Our analysis is done at $m_u = m_d$, $e = 0$

  F_π = physical value of $F_{\pi^+} \Rightarrow \Lambda_{\rm QCD}$

  M_π = physical value of $M_{\pi^+} \Rightarrow m_u$

  M_K = physical value of $M_{K^+} \Rightarrow m_s$

  m_c, m_b, m_t = physical values
- Can establish contact with experiment only to the extent that isospin breaking is understood

  Cirigliano, Ecker, Neufeld, Pich

  $M_{\pi^+} - M_{\pi^0}$, $M_{K^+} - M_{K^0}$ $\surd$

  π^0-η-mixing, ρ-ω-mixing $\surd$

  $M_{\rho^+} - M_{\rho^0}$, $\Gamma_{\rho^+} - \Gamma_{\rho^0}$?

  Ghozzi & Jegerlehner, Davier

Chiral symmetry

- Goldstone bosons of zero momentum do not interact $\to$ $A(s,t)$ has an Adler zero
- Chiral expansion starts at $O(p^2)$

$$A(s,t) = \frac{s - M_\pi^2}{F_\pi^2} + O(p^4)$$

Weinberg 1966

- Expression is linear in s, t

$\Rightarrow$ only S- and P-waves present at $O(p^2)$

- Representation for $A(s,t)$ known to two loops, i.e. up to and including $O(p^6)$

Bijnens, Colangelo, Ecker, Gasser, Sainio 1996

- Representation is very accurate near the center of the Mandelstam triangle
- The singularities required by unitarity generate curvature, uncertainties grow with the distance from the center of the triangle

  Already at threshold (scattering lengths), the chiral representation leaves to be desired

Roy equations

- χPT is not needed for dependence on s, t
  Analyticity, unitarity and crossing determine the amplitude in terms of its imaginary part, except for the subtraction constants

- $\pi\pi$ scattering is special: crossed channels are identical

$\Rightarrow$ Re$A(s,t)$ can be represented as an integral over physical region imaginary part

S.M. Roy 1971

- Representation involves 2 subtraction constants, can identify these with 2 scattering lengths:

$$a_0^0\ ,\ a_0^2 \begin{array}{l} \leftarrow \text{isospin} \\ \leftarrow \text{angular momentum} \end{array}$$

- Representation leads to dispersion relations for the individual partial waves: *Roy equations*

- Roy equations were studied long ago

  early work reviewed by Pennington, Ann.Phys. 1975

  Main problem at that time: experimental information near threshold is meagre

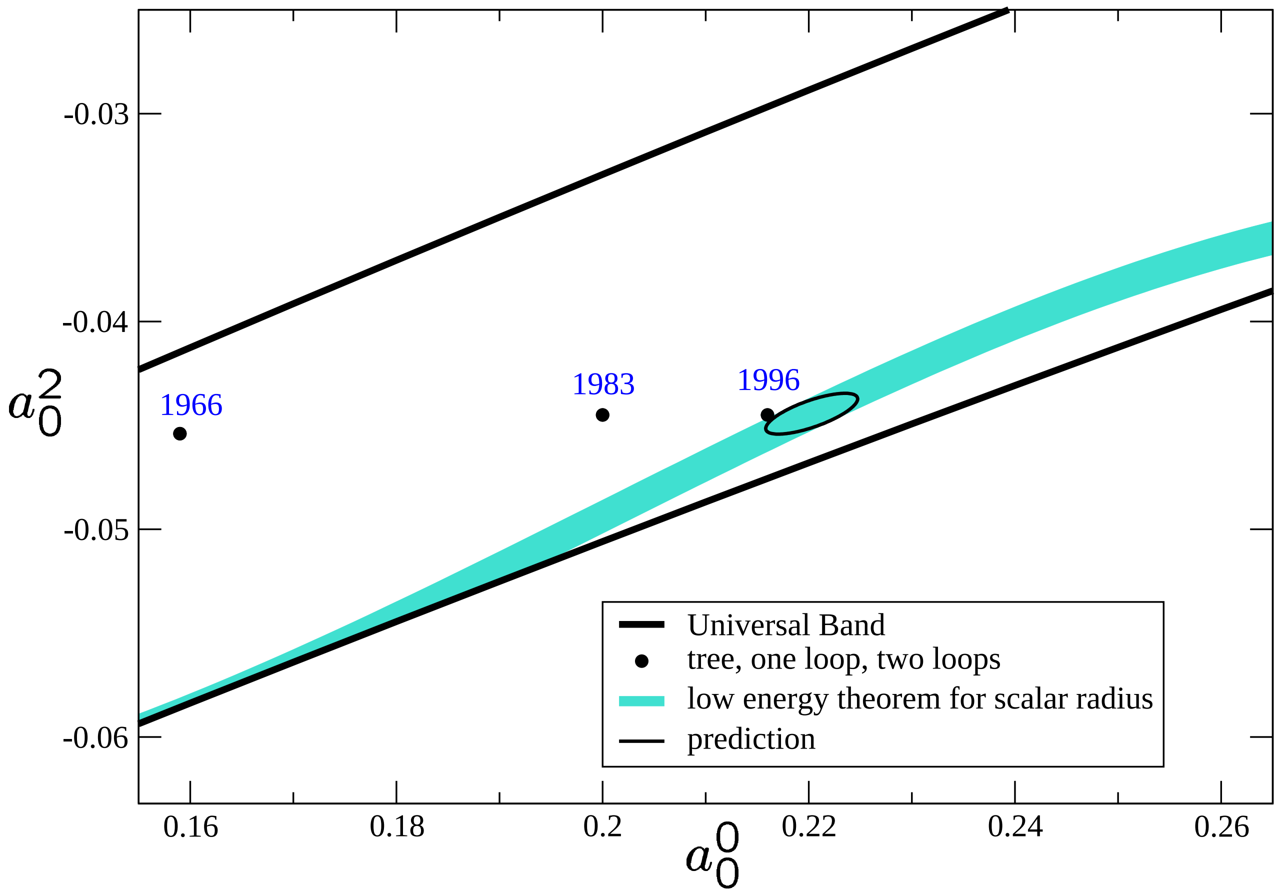
⇒ Large uncertainties in a_0^0, a_0^2

- The two subtraction constants are the essential parameters in the low energy region: given a_0^0, a_0^2, the scattering amplitude can be calculated to within very small uncertainties

  Ananthanarayan, Colangelo, Gasser & L. 2001
  Descotes, Fuchs, Girlanda & Stern 2002

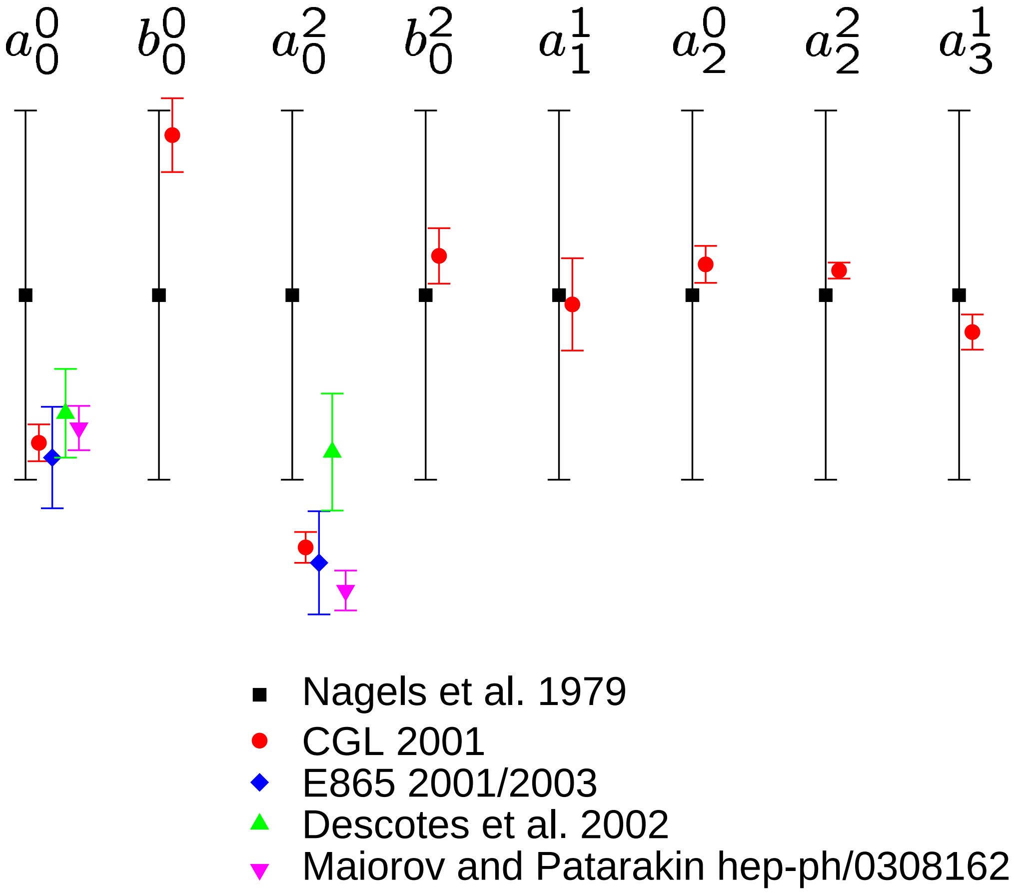
- χPT provides the missing piece: low energy theorems for a_0^0, a_0^2

- More accurate method: match the two loop and dispersive representations below threshold



Predictions for the S-wave $\pi\pi$ scattering lengths

- What difference does it make whether or not the subtraction constants are known ?



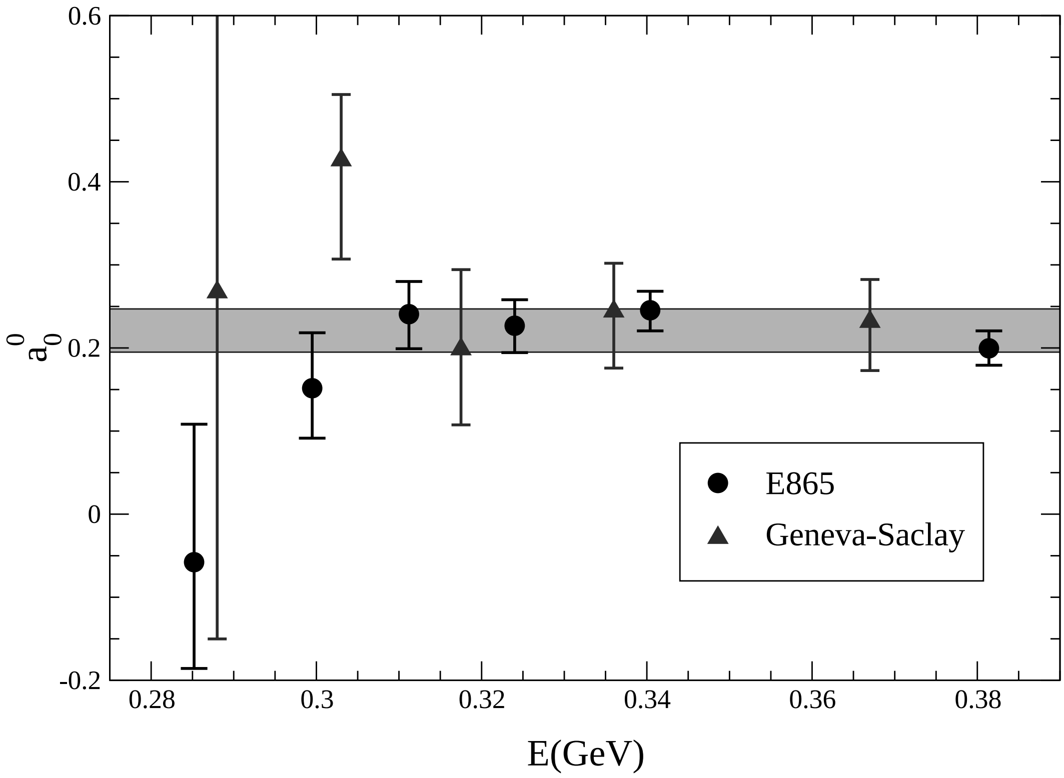
⇒ Quantum jump in low energy pion physics

In combination with the low energy theorems of χPT , the dispersion relations for the partial waves fix the $\pi\pi$ scattering amplitude to a very high degree of accuracy

- For once in strong interaction physics, theory is ahead of experiment ...

Experimental test via $K \to \pi\,\pi\,e\,\nu$

- New data from E865-collaboration at Brookhaven allow a significant test of the GMOR relation
- Final state interaction theorem: phase of the $K \to \pi\pi e\nu$ transition form factors is determined by elastic $\pi\pi$ scattering amplitude
- Conversely, can measure the phase difference $\delta_0^0 - \delta_1^1$ by means of this decay



- Fit to the data yields

$$a_0^0 = 0.216 \pm 0.013\,(\text{stat}) \pm 0.002\,(\text{syst}) \pm 0.002\,(\text{th})$$

S. Pislak et al., Phys. Rev. D67 (2003) 072004

- To be compared with the prediction of χPT

$$a_0^0 = 0.220 \pm 0.005$$

Amoros, Bijnens & Talavera, Nucl.Phys. 2000

Colangelo, Gasser & L., Phys.Lett. 2000

- The prediction only holds if the quark condensate is the leading order parameter

⇒ More than 94 % of the pion mass originates in the quark condensate term

$$M_\pi^2 \simeq (m_u + m_d) \times |\langle 0|\,\bar{q}\,q\,|0\rangle| \times \frac{1}{F_\pi^2} \;\checkmark$$

May "Generalized χPT" rest in peace

- Data analysis relies on LET for $\langle r^2 \rangle_s$
  Very important to test that prediction as well
  Descotes, Fuchs, Girlanda & Stern

- Dependence on m_s, Zweig-rule violations ?
  Ananthanarayan, Büttiker, Descotes-Genon, Fuchs, Girlanda, Jamin, Knecht, Moussallam, Oller, Pich, Stern, . . .

- Forthcoming: more precise data on $K \to \pi\,\pi\,e\,\nu_e$ from NA48/2 (CERN) and KLOE (Frascati)

- $\pi^+\pi^-$ atoms provide an ideal laboratory
- Atoms decay through the strong interaction

$$\pi^+\pi^- \to \pi^0\pi^0$$

Decay rate $\propto (a_0^0 - a_0^2)^2$

- Interference of e.m. and strong interactions in bound state and decay is now well understood

Gasser, Rusetsky et al.

$\Rightarrow$ Can reliably measure low energy properties of the $\pi\pi$ scattering amplitude in this way

- Prediction for the lifetime: $\tau = 2.9 \pm 0.1$ fs

Colangelo, Gasser & L. 2001

- DIRAC: beautiful experiment done at CERN
  Aims at a measurement of the lifetime to 10%

$\Rightarrow$ clean test of symmetry breaking due to m_u, m_d

- Using a subset of the collected data, DIRAC has achieved an accuracy of 16%:

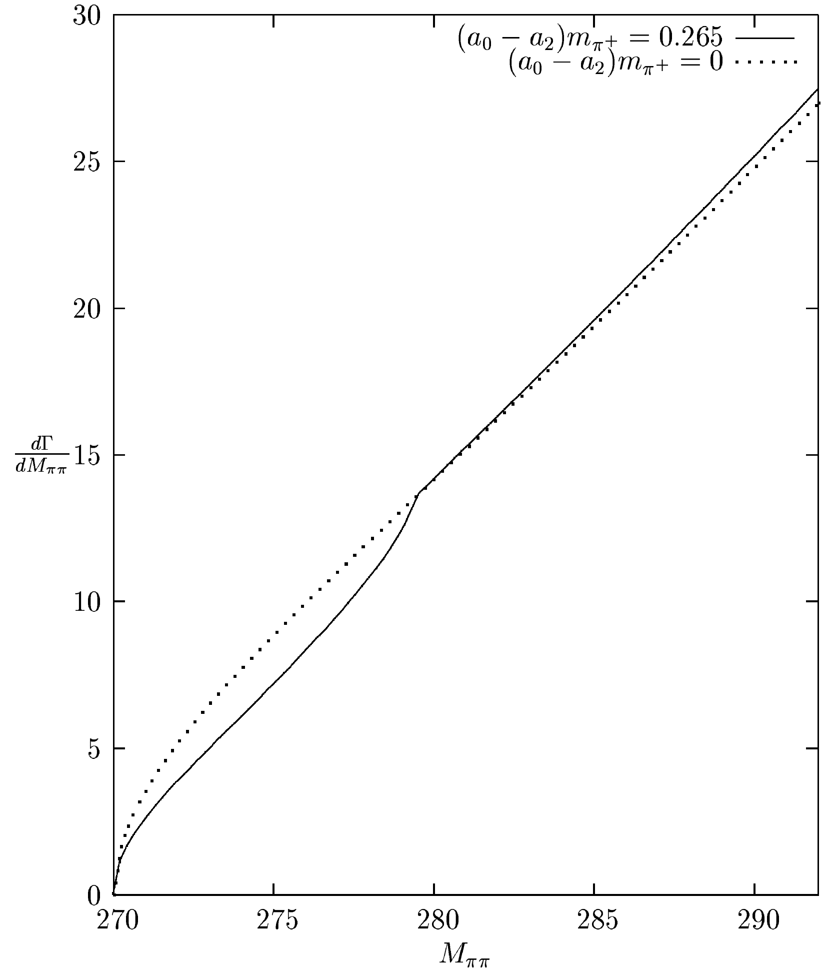
$$\tau = 2.85\,^{+0.48}_{-0.41}\ \text{fs}$$

L.Tauscher at www.lnf.infn.it/conference/dafne04/

- New idea: accurate data in the threshold region of the decay $K^+ \to \pi^+\pi^0\pi^0$ would allow a determination of $a_0^0 - a_0^2$

  Cabibbo, hep-ph/0405001

  Here, isospin breaking plays a central role. Theoretical understanding is underdeveloped



Taken from N. Cabibbo, hep-ph/0405001

Scalar form factor

$$\langle\pi|\, m_u\bar{u}u + m_d\bar{d}d\,|\pi\rangle = \sigma_\pi\, f(t)$$
$$f(t) = 1 + \tfrac{1}{6}\,\langle r^2\rangle_s\, t + O(t^2)$$

- Value at $t=0$: σ-term of the pion

$$\sigma_\pi = m_u \frac{\partial M_\pi^2}{\partial m_u} + m_d \frac{\partial M_\pi^2}{\partial m_d} \simeq M_\pi^2$$

- Slope at $t=0$: *scalar radius* $\langle r^2\rangle_s$ plays an important role in χPT , because it determines the sensitivity of F_π to m_u, m_d

$$\frac{F_\pi}{F} = 1 + \frac{1}{6} M_\pi^2 \langle r^2\rangle_s + \frac{13 M_\pi^2}{192\,\pi^2 F_\pi^2} + O(M_\pi^4)$$

- There is an analogous formula also for F_K/F If Zweig rule violating contributions are dropped

$$\frac{F_K}{F_\pi} = 1 + \frac{1}{6}(M_K^2 - M_\pi^2)\langle r^2\rangle_s + \chi\text{logs}$$
$$\Rightarrow \quad \langle r^2\rangle_s = 0.55 \pm 0.15\,\text{fm}^2$$

Gasser & L. 1985

Dispersive analysis

- Early work was motivated by the search for a very light Higgs meson

Truong & Willey 1989
Donoghue, Gasser & L. 1990

- Assume that $f(t)$ does not have zeros
$\Rightarrow$ $f(t)$ is determined by its phase $\delta_f(t)$

$$f(t) = |f(t)|e^{i\,\delta_f(t)}$$
$$f(t) = \exp \frac{t}{\pi}\int_{4M_\pi^2}^{\infty} ds\,\frac{\delta_f(s)}{s(s-t)}$$

In particular, $\langle r^2\rangle_s$ can be calculated from $\delta_f(t)$

$$\langle r^2\rangle_s = \frac{6}{\pi}\int_{4M_\pi^2}^{\infty} ds\,\frac{\delta_f(s)}{s^2}$$

- Watson theorem:

$$\delta_f(t) = \delta_0^0(t) \qquad t < 16\, M_\pi^2$$

- Phenomenology: inelasticity remains very small below $4\, M_K^2$ $\Rightarrow$ $\delta_f(t) \simeq \delta_0^0(t)$ holds for $t < 4M_K^2$

$$\frac{6}{\pi}\int_{4M_\pi^2}^{4M_K^2} ds\, \frac{\delta_f(s)}{s^2} \simeq 0.42\,\text{fm}^2$$

- $K\bar{K}$ channel dominates the inelasticity

$\Rightarrow$ two-channel version of the Omnes formula

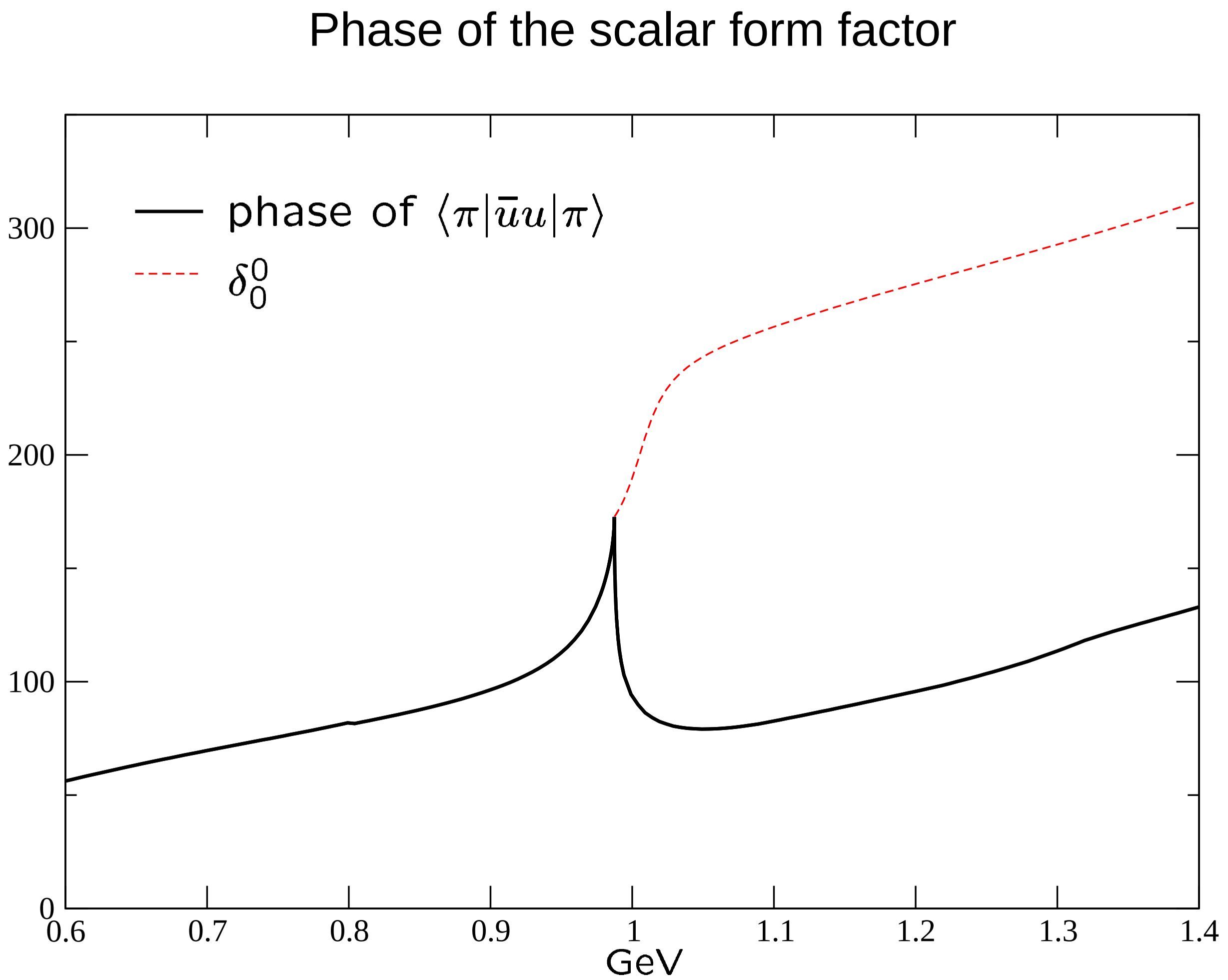
$$\langle r^2 \rangle_s = 0.61 \pm 0.04\,\text{fm}^2$$

DGL 1990, CGL 2001

- Thorough analysis of the problem, including contributions from other inelastic channels:

$$\langle r^2 \rangle_s = 0.58\,\text{to}\,0.65\,\text{fm}^2$$

Moussallam 1999



Ananthanarayan, Caprini, Gasser & L., in preparation

- Behaviour near $K\bar{K}$ threshold:
  - δ_0^0 rapidly grows
  - phase of the form factor rapidly drops

Slope of the scalar $K\pi$ form factor

$$\lambda_0 = \frac{1}{6}\, M_\pi^2 \,\langle r^2 \rangle_{K\pi}$$

- Can be measured in $K \to \pi\mu\nu$ decay
- Prediction based on χPT to one loop:

$$\langle r^2 \rangle_{K\pi} = 0.20 \pm 0.05\,\text{fm}^2$$

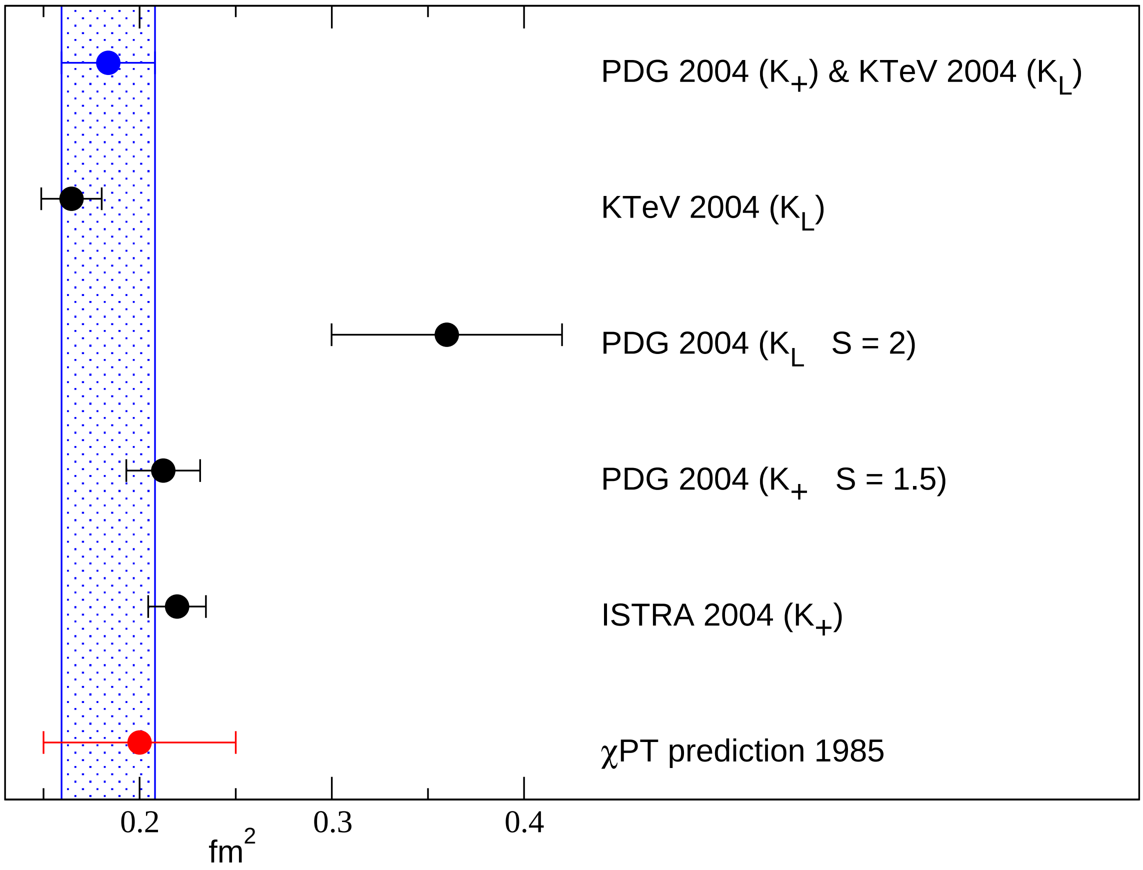
Gasser & L. 1985

- $\sim$ 3 times smaller than scalar radius of pion ! χPT predicts very strong symmetry breaking in the scalar radii, weak s.b. in the vector radii
- Experimental situation was not clear in 1985

  A high statistics experiment (Donaldson 1974) was in agreement with the theoretical expectations, but more recent ones (Clark 1977, Hill 1979, Cho 1980, Birulev 1981) were in flat contradiction with chiral symmetry.

- During the last year, the experimental situation improved very significantly

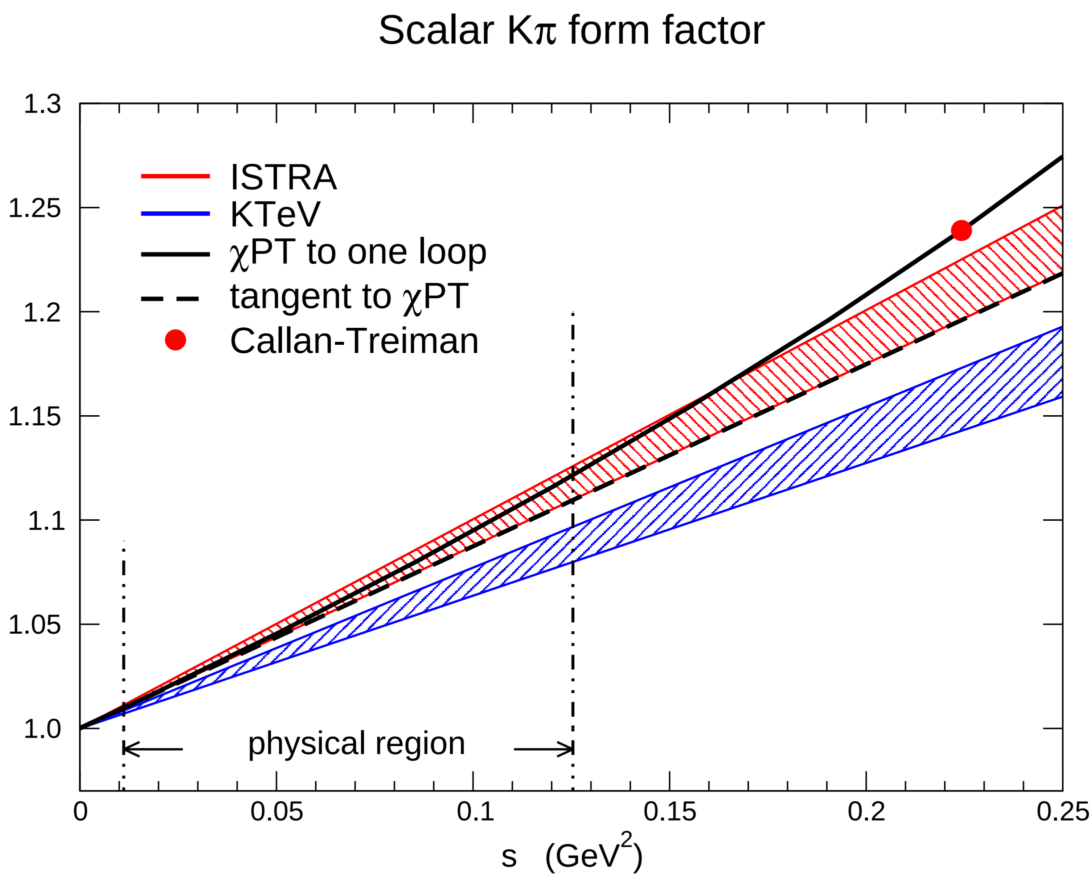
Mean square radius of the scalar Kπ form factor



- Mean of (KTeV for K_L) and (PDG for K_+), error stretched by factor of 2:

experiment: $\langle r^2 \rangle_{K\pi} = 0.184 \pm 0.024 \text{ fm}^2$

theory: $\langle r^2 \rangle_{K\pi} = 0.20 \pm 0.05 \, \text{fm}^2$



Plot shows normalized form factor $f_0(s)/f_0(0)$

- Callan-Treiman-relation:

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi} + O(m_u, m_d)$$

Correction is tiny: no term of order $M_\pi^2 \log M_\pi^2$ (leading order in SU(3)×SU(3) : 3 permille)

- Curvature not negligible at this precision, is due to $I = \frac{1}{2}$ $K\pi$ final state interaction

Progress on the theoretical side

- Form factors are now known to two loops

  Post & Schilcher, Bijnens & Talavera

- Extension of Roy analysis to $K\pi$ scattering: Roy-Steiner equations, yield reliable results for the behaviour of the phases below 1 GeV

  Estimate for the subtraction constants on the basis of the available data

  $a_0^{1/2} = 0.224 \pm 0.022 \qquad a_0^{3/2} = 0.0448 \pm 0.0077$

  Büttiker, Descotes-Genon & Moussallam

⇒ The Roy-Steiner equations can now be matched with the two loop representation of χPT

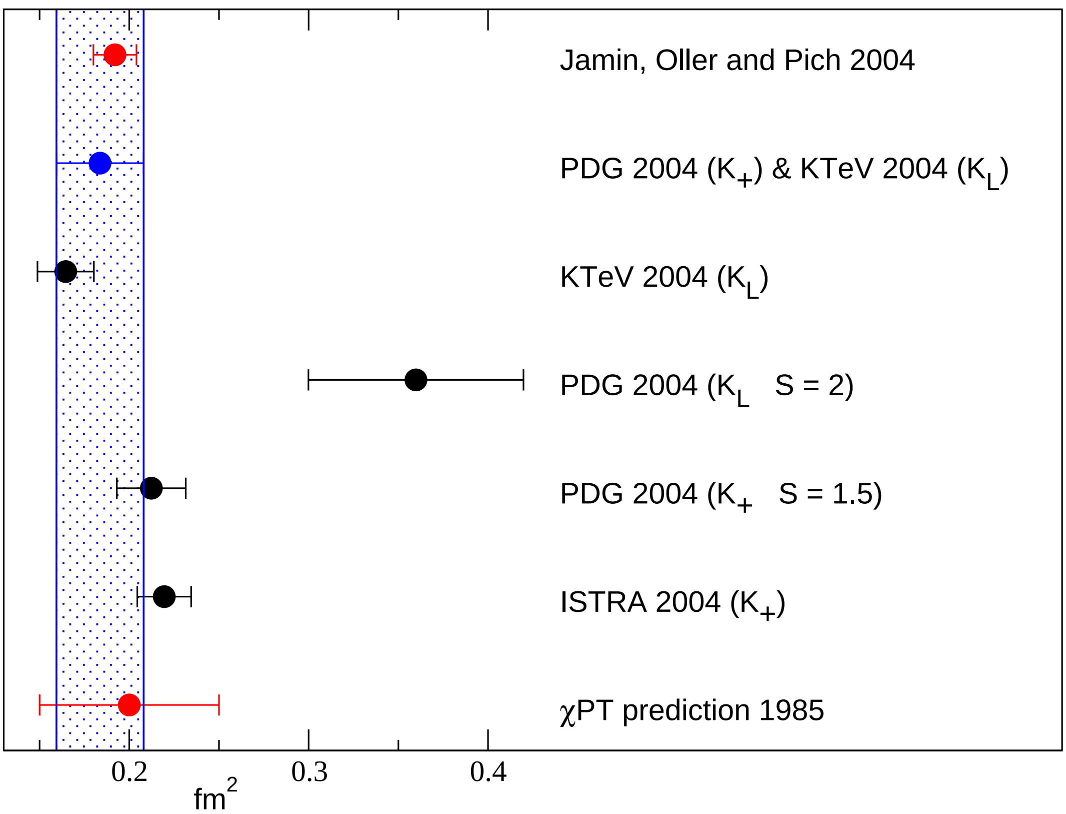
⇒ On this basis, the low energy theorems of SU(2)×SU(2) and SU(3)×SU(3) can then be analyzed in a controlled manner

- Dispersive analysis of the $K\pi$ form factors allows to determine their curvature

  The Callan-Treiman relation then leads to a much sharper prediction for the radius:

$$\langle r^2 \rangle_{K\pi} = 0.192 \pm 0.012 \text{ fm}^2$$

Jamin, Oller and Pich 2004



- Lattice methods now reach the domain where it becomes possible to make contact with χPT

$\Rightarrow$ In view of the improved experimental situation, expect significant progress in kaon physics soon

Pelaez and/or Yndurain

- Violently criticize our work in several papers
- Main difference to our approach: PY do not use the dispersion relations obeyed by the scattering amplitude, rely on phenomenology
- Claims in Phys. Rev. D68 (2003) concerning our work were shown to be untenable

Caprini et al., Phys. Rev. D68 (2003)
Colangelo, talk at Chiral Dynamics, Bonn (2003),
www-itp.unibe.ch/staff/colangelo.html

- New claim: *"robust lower bound"*

$$\langle r^2 \rangle_s > 0.70 \pm 0.06\,\mathrm{fm}^2$$

Yndurain, Phys. Lett. B578 (2004)

- Origin of the disagreement is not identified

???

Origin of the disagreement

"As implied by the experimental data on $\pi\pi$ *scattering [14b], the inelasticity is compatible with zero (indeed, the central value is almost equal to zero) for the S0 wave, within experimental errors, in the energy region 1.1 GeV* $< \sqrt{s} <$ *1.5 GeV. It thus follows that the phase of* $F_S(s)$ *must be approximately equal to* $\delta_0^0(s)$ *for 1.1 GeV* $< \sqrt{s} <$ *1.42 GeV."*

Yndurain, Phys. Lett. B568 (2004) 99

$$S_{\pi\pi}(s) = \eta_0^0(s)\, e^{2\,i\,\delta_0^0(s)}$$

$$f(s) = |f(s)|\, e^{i\,\delta_f(s)}$$

- "robust bound" is based on the claim that for energies where the scattering is approximately elastic, δ_f must be approximately equal to δ_0^0

$$\eta_0^0 \simeq 1 \quad \Rightarrow \quad \delta_f \simeq \delta_0^0$$

- Watson theorem:

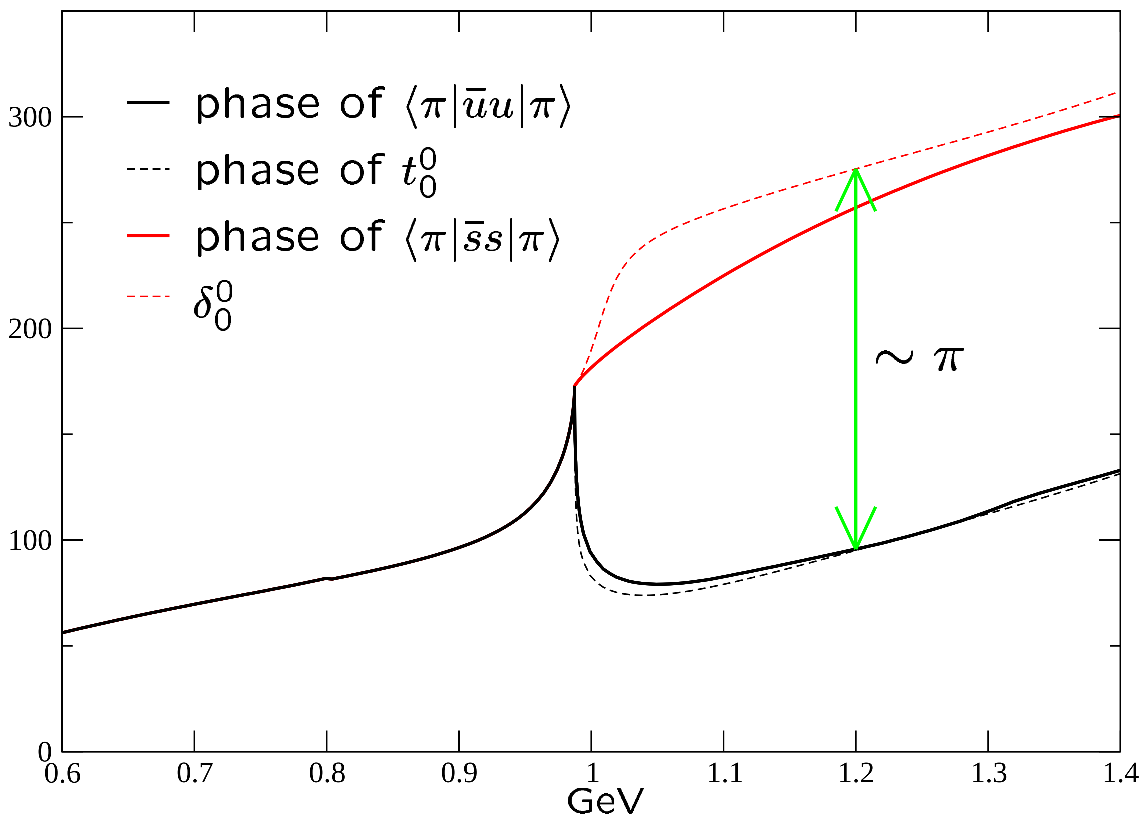
$$f(s)^\star = S^\star_{\pi\pi}(s) f(s) + S^\star_{\pi K}(s) f_K(s) + \ldots$$

If $\eta_0^0 = 1$ then $S_{\pi K}$ vanishes

$\Rightarrow$ $f(s)^\star = e^{-2\,i\,\delta_0^0}\, f(s)$

$\Rightarrow$ $e^{2\,i(\delta_f - \delta_0^0)} = 1$

$\Rightarrow$ $\eta_0^0 \simeq 1$ does not imply that δ_f is close to δ_0^0 but $\delta_f - \delta_0^0$ must be close to a multiple of π



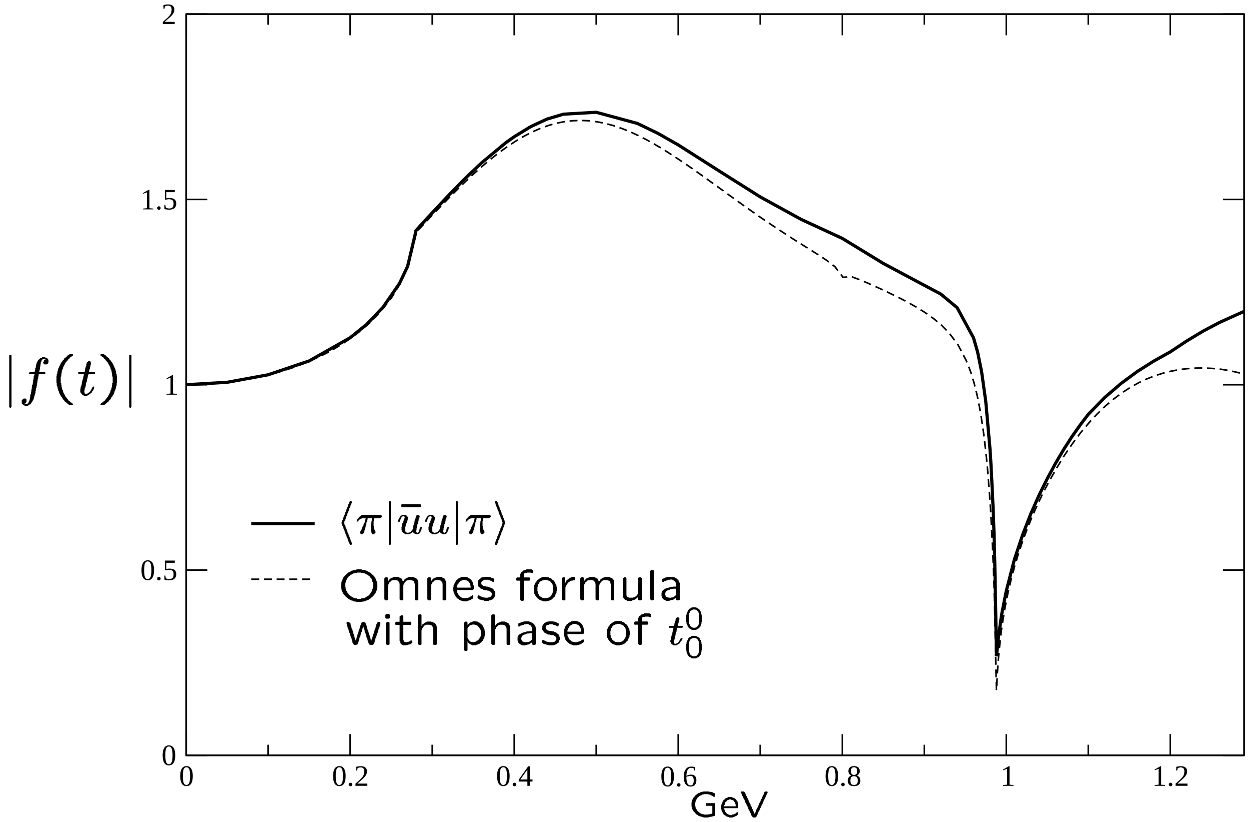
⇒ The *"robust bound"* relies on a false claim

- Form factor phase follows phase of t_0^0, not δ_0^0

$$\eta_0^0\, e^{2\,i\,\delta_0^0} = 1 + 2\,i\,v_\pi\, t_0^0$$

v_π is the pion velocity in the CMS

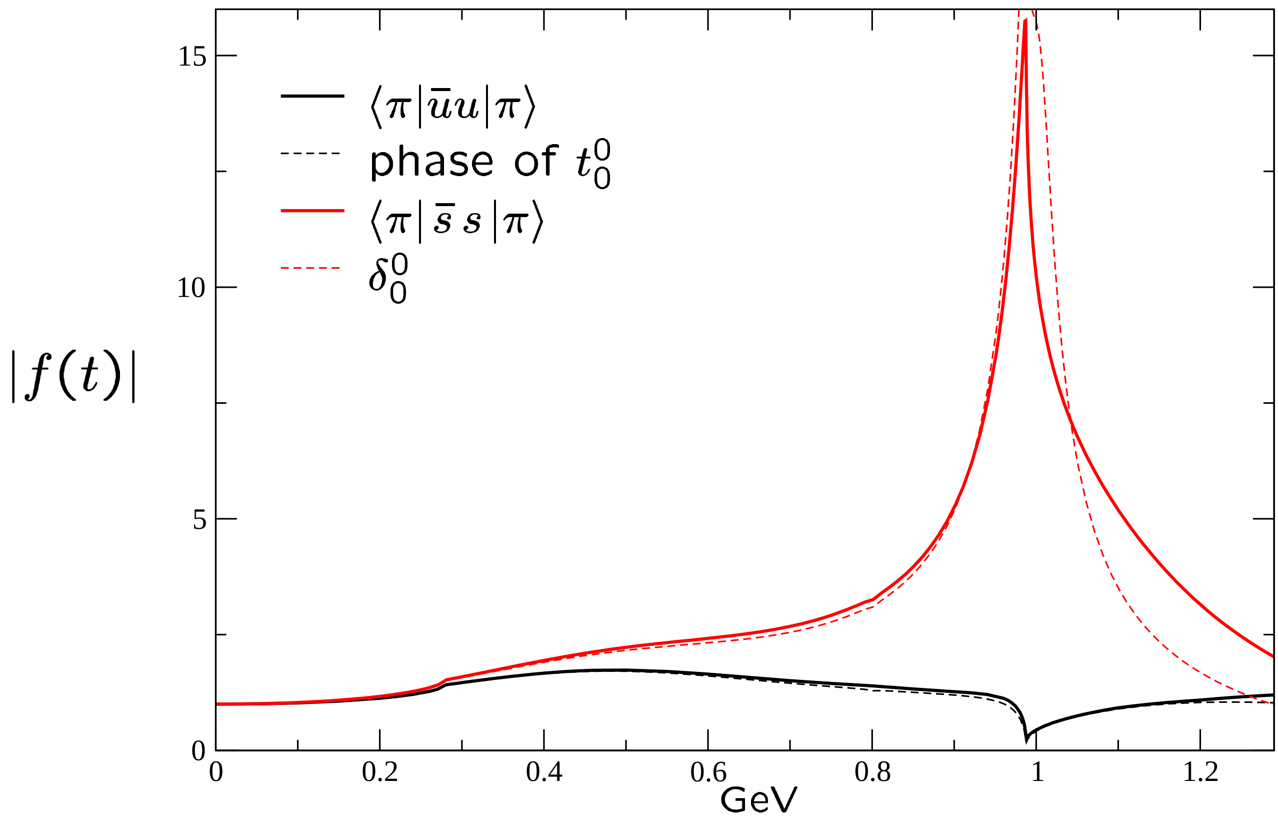
Magnitude of form factor



- Form factor follows Omnes formula evaluated with the phase of t_0^0, not δ_0^0

Morgan & Pennington 1984
Truong & Willey 1989
Donoghue, Gasser & L. 1990
Locher, Markushin & Zheng 1996
Morgan & Pennington 1998

- Model of Truong & Willey: simple analytic representation of the form factor
  - Solves the two-channel unitarity conditions
  - Useful for understanding the structure in the vicinity of $K\bar{K}$ threshold
  - Model is too crude at lower energies (a_0^0, bump from σ)



The four curves are calculated with

$$f(t) = \exp \frac{t}{\pi} \int_{4M_\pi^2}^{\infty} ds \, \frac{\delta(s)}{s(s-t)}$$

- For small values of t, the noise is very small
- Uncertainties from experimental information used above 0.8 GeV grow with the energy
- The specific curves shown are based on the T-matrix representation of Hyams et al.

Ynduráin on $\langle r^2 \rangle$

Phys. Lett. B568 (2004) 99

- The work of Moussallam is quoted together with other references and the comment *"They do not add anything essential in connection with what interests us here."*

  Moussallam's paper offers a careful study of the scalar radius, in particular of the sensitivity to the behaviour above $K\bar{K}$ threshold . . .

  Physics Letters must have consulted Santa Ignorância

- *"It is difficult to point out where lies the failure in the calculation of Donoghue, Gasser and Leutwyler, as it is of the 'black-box' type."*

???

- *"To get a value as low as that of these authors, one would have to . . . "*

???

- We stated that the behaviour of the T-matrix above 1.4 GeV does not significantly affect our results

  "Contrary to this, our explicit calculations show that the contributions from energies above 1.4 GeV are large*: of 20% for* $\langle r^2 \rangle_s$*"*

  Our central value for this contribution amounts to 19%. Not the contribution as such, but the uncertainty therein does not significantly affect our result, because the phase of the form factor must tend to π. We implement this condition with $T \rightarrow 0$.

- *"This is one of the few cases in which the PDT recommend a number difficult to believe . . . "*

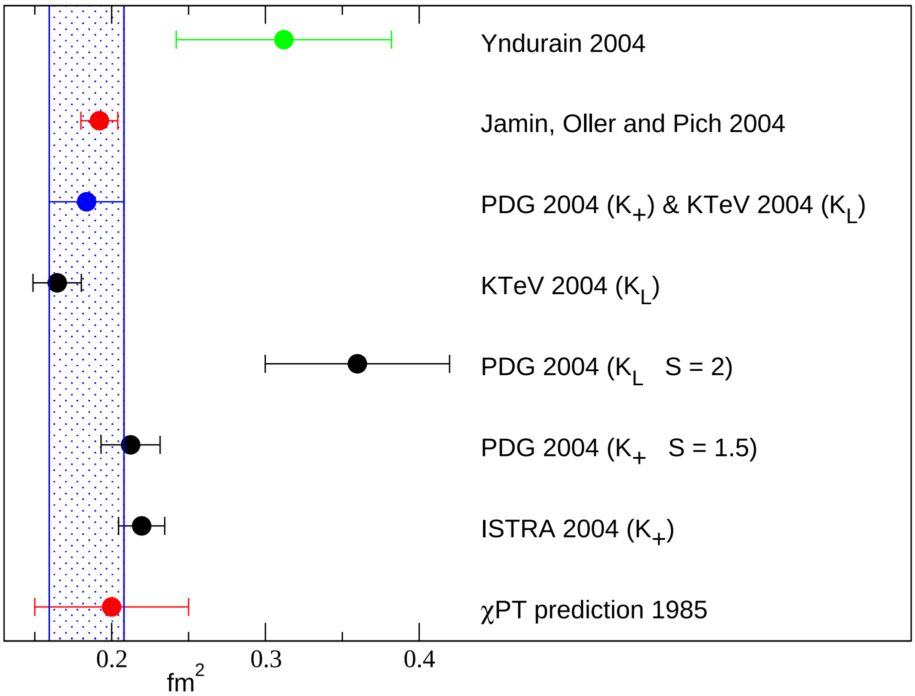
???

- The data disagree: The PDG obtains S = 1.5 or 2. According to Yndurain there is no such problem:

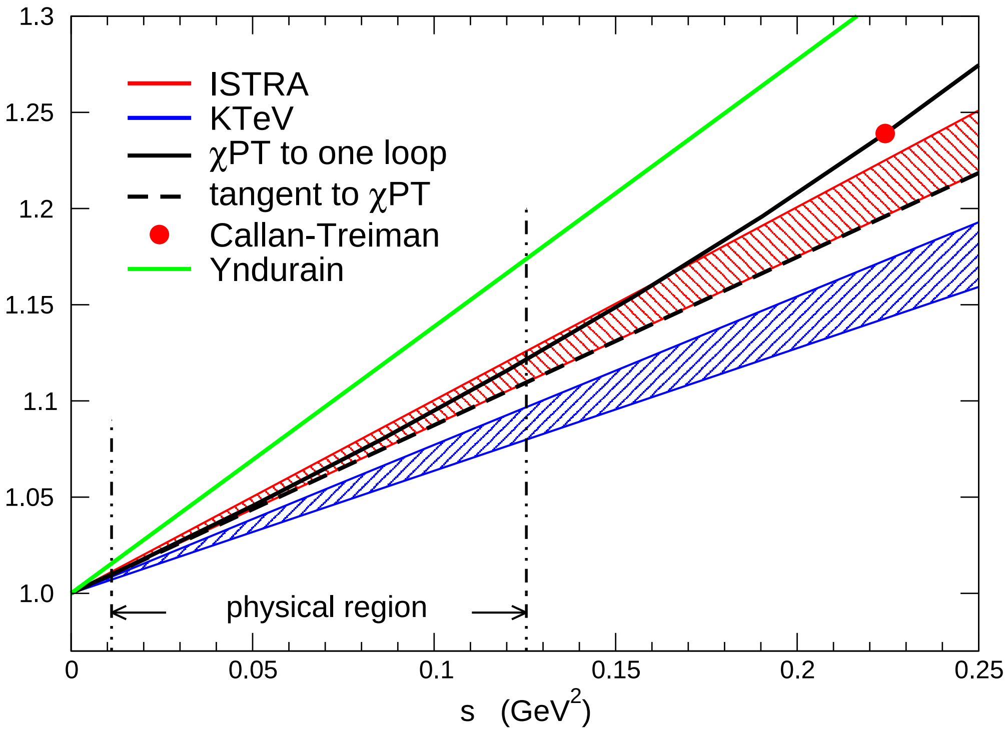
  "If we average them (exp. values for λ_0 from K_+), which is permissible since they are compatible within errors, we find ... in perfect agreement with (1.4a) (λ_0 from K_L) ... and find what we will consider the experimental value for the form factor:

$$\langle r^2 \rangle_{K\pi} = 0.312 \pm 0.070\,\text{fm}^2$$

  ... the central value lies clearly outside the error bars of the chiral theory prediction"



Scalar Kπ form factor



Regge analysis of Pelaez & Yndurain

Phys.Rev. D69 (2004) 114001

- Compared to Phys. Rev. D68 (2003) 074005, this is a significant improvement: the sources used for their Regge parametrization are now indicated
- *"For $I_t = 1$, we also take the parametrization of ref.[14]."*

  [14] is Rarita et al.(1968), but that reference does not contain a representation for $\beta_\rho^{\pi\pi}(t)$ ...
- PY tacitly assume that the position of the zeros as well as the slopes of $\beta_\rho^{\pi\pi}(t)$ and $\beta_\rho^{\pi N}(t)$ are the same: the residues are supposed to obey $\beta_\rho^{\pi\pi}(t) = \text{const} \times \beta_\rho^{\pi N}(t)$
- The constant of proportionality can then be worked out from a sum rule. Outcome for $\beta_\rho \equiv \beta_\rho^{\pi\pi}(0)$:

$$\beta_\rho = 0.94 \pm 0.14$$

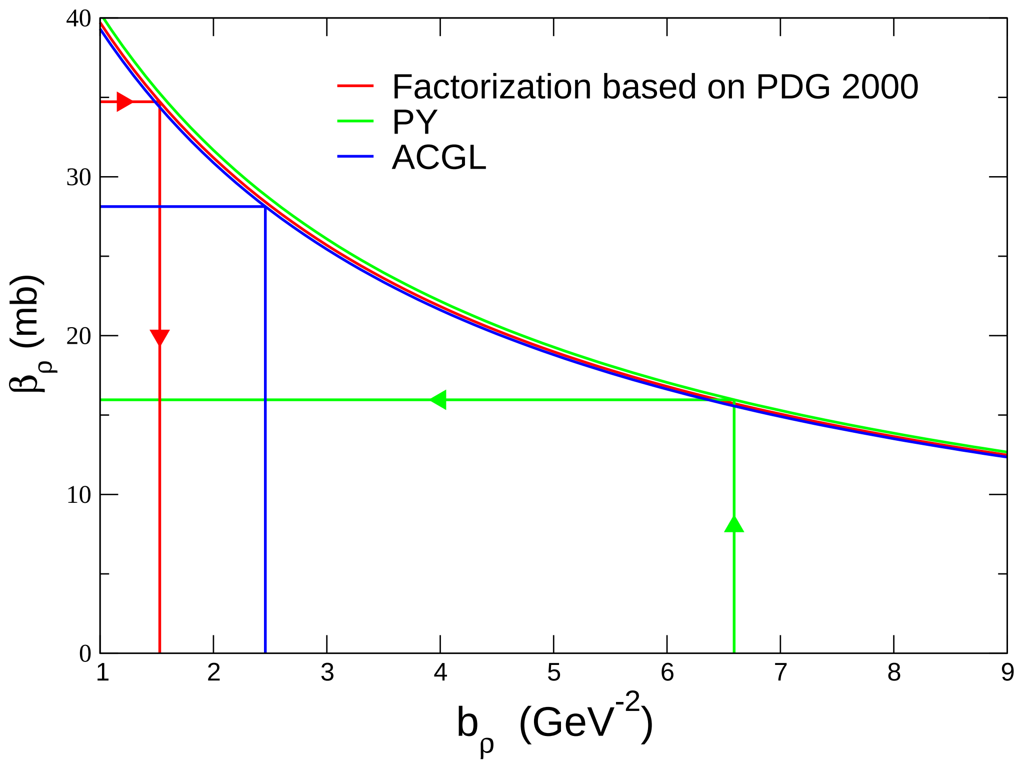
- The crucial ingredient is the assumption that the slopes

$$b_\rho = \left.\frac{\beta_\rho(t)'}{\beta_\rho(t)}\right|_{t\to 0}$$

  in $\pi\pi$ and πN scattering are the same: $\boxed{b_\rho^{\pi\pi} = b_\rho^{\pi N}}$
- The πN data indicate that $\beta_\rho^{\pi N}(t)$ has a zero rather close to the origin ($t \simeq -0.2$ GeV^2) and hence a large slope:

$$b_\rho^{\pi N} \simeq 6.6 \text{ GeV}^{-2}$$

- Sum rule correlates β_ρ with the slope b_ρ



The trajectory parameters are fixed at
$\alpha_P(0) = 1.092$, $\alpha_\rho(0) = \alpha_f(0) = 0.51$, $\alpha_\rho(0)' = 0.9$ GeV^{-2}

Standard normalization used in Regge phenomenology:

$$\sigma = \beta\,(s/s_0)^{\alpha-1}, \qquad s_0 = 1\,\text{GeV}^2$$

In these units:

$\beta_\rho = 35 \pm 5$ mb	PDG 2000
$\beta_\rho = 14.4 \pm 2.1$ mb	PY
$\beta_\rho = 28 \pm 5$ mb	ACGL

- *"Likewise, the value of* $\beta_\rho = 0.94 \pm 0.14$ *is similar to what one has in the Veneziano model* ($\beta_\rho \simeq 0.95$)*, . . ."*

  This claim is off by a factor of 3. The Veneziano model yields $\beta_\rho \simeq 45$ mb (in the normalization of PY: $\beta_\rho \simeq 3.0$)

- Rarita et al. were among the first to predict the size of the Pomeron term in $\pi\pi$ scattering: their solution 1a (this is the one PY rely on) yields

  $\frac{1}{3}\beta_P = 6.7$ mb   Rarita et al. (1968)

- Recent work (e.g. Cudell et al.) confirms this result:

  $\frac{1}{3}\beta_P = 7.5 \pm 0.3$ mb   PDG 2000

- The number

  $\frac{1}{3}\beta_P = 13.1 \pm 0.8$ mb   PY, Phys. Rev. D69 (2004)

  is about twice as large. Nevertheless, PY claim

  "Our present results are compatible with those in refs.[6,11,14]."

  [14] is Rarita et al. . . .

  ??

Conclusion on Regge analysis of PY

- The Regge parametrization of PY heavily relies on the ad hoc assumption $\boxed{b^{\pi\pi} = b^{\pi N}}$
- An attempt at justifying this relation is not made – the assumption is not even mentioned
- The representation obtained on this basis is in flat contradiction with the Regge representations in the literature – this is not mentioned either
- Nevertheless, Santa Ignorância gave her blessings for a publication in Physical Review . . .

My view

- The theoretical progress in low energy strong interaction physics has triggered precision measurements in this domain. Exciting results have already been obtained and several projects are under way.
- It matters whether or not the theory is sound. Ignoring wrong papers is not an acceptable way out for me.
- In the last two years, I spent quite a fraction of my time with the sins committed by Paco & José. With due respect for the friendly ladies who clean up the mess generated by others, this is not the occupation of my dreams.
- In Phys. Rev. D68 (2003) we explain why the statements made by PY about our work are false. A reply or an erratum closing this chapter did not appear.
- Instead of being able to discuss our recent work on $g-2$, for instance, G. Colangelo is kept busy reacting to the same false claims again and again: Bonn 2003 (Y.), Vienna 2004 (P.), Villasimius 2004 . . .

Any suggestions for how to stop this circus ?