

Calculating ϵ'/ϵ using staggered fermions

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Collaboration

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$Re(A_2)$, $Re(A_0)$, and ϵ'/ϵ (Exp)

- $\langle (\pi\pi)_I | H_W | K^0 \rangle = A_I e^{i\delta_I}$
- $\omega = \frac{ReA_2}{ReA_0} = 0.045$
- $ReA_2 = 1.50 \times 10^{-8} GeV$
- $ReA_0 = 33.3 \times 10^{-8} GeV$
- $\epsilon'/\epsilon = -\frac{\omega}{\sqrt{2}|\epsilon|} \left[\frac{ImA_0}{ReA_0} - \frac{ImA_2}{ReA_2} \right]$
KTeV $\rightarrow Re(\epsilon'/\epsilon) = (20.7 \pm 2.8) \times 10^{-4}$
NA48 $\rightarrow Re(\epsilon'/\epsilon) = (15.3 \pm 2.6) \times 10^{-4}$



Main goals of the staggered ϵ'/ϵ project.

- Check the results obtained using the **quenched** Domain-Wall fermions: \rightarrow **negative** ϵ'/ϵ (CP-PACS, RBC).
- Calculate ϵ'/ϵ in the **full** QCD.
 - **Quenching** \rightarrow main systematic errors.
 - Using **the improved actions** \rightarrow **HYP** or $\overline{\text{Fat7}}$.
- Comparison with experiment \rightarrow new physics (?)



Why staggered fermions?

- Advantage:

1. Conserved $U(1)_V \otimes U(1)_A$.
2. Quark mass is protected (No Residual Mass).
3. Computationally cheap ($< 50 \times 4$).
4. Discretization error = $\mathcal{O}(a^2)$.
5. Easy to improve (no extra cost).

- Disadvantage

1. Broken $SU(4)$ Flavor (Taste) Symmetry.
2. Operator mixing matrix of 65536×65536 .
3. NPR is impractical.



The First Stage

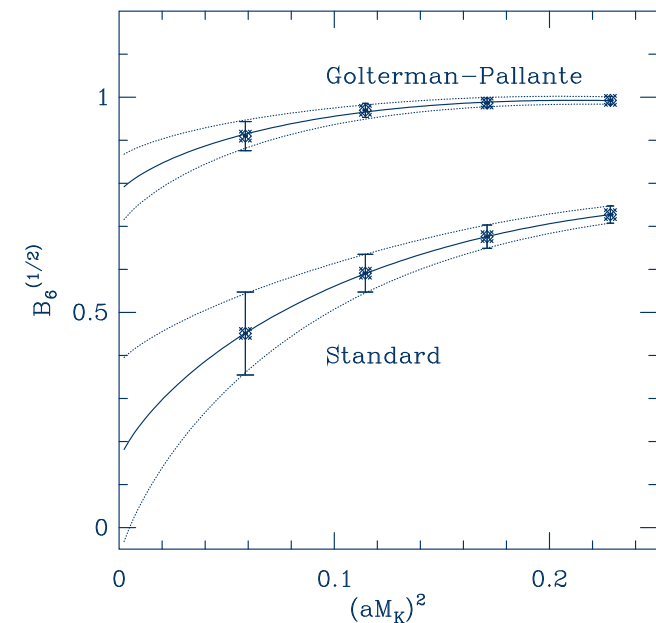
Perturbative matching for unimproved staggered operators

- Penguin diagrams (Sharpe, Patel, 1993)
→ small ($\approx 2\%$).
- Current-current diagrams (W. Lee, 2001)
→ large ($\approx 100\%$ for $[P \times P][P \times P]$).
- Hence, we must improve the staggered fermion action and operators to reduce the perturbative correction.



The Second Stage:

- Numerical study with unimproved staggered
- Check the previous work (Kilcup, Pekurovsky).
- Use fully one-loop matched operators.
- Nucl. Phys. B (Proc. Suppl.) 106, 311; hep-lat/0111004.
- Large quenching uncertainty.





The Third Stage:

Improving staggered fermions

- AsqTad: Fat7 + Lepage + Naik
- Fat7
- Fat7 + Lepage
- Hypercubic (HYP)
- Other possibilities: $\overline{\text{Fat7}}$, APE



Main goal of improvement:

Reduce perturbative corrections

- **Key idea:**

Suppress the taste changing interaction of high momentum gluons with quarks, using the Symanzik improvement program and smearing.

- **Numerical Studies find:**

- Taste symmetry breaking substantially reduced.
(Hasenfratz, Knechtli)
- Largest reduction in HYP and APE.



Bilinear Operator Renormalization

- W. Lee and S. Sharpe, PRD 66, (2002), 114501.
- Fat7 and HYP are the best in reducing the size of one-loop corrections. ($\leq 10\%$ level)
- Comparable to those for Wilson and Domain Wall Fermions.
- This leads us to choosing HYP for our numerical study.



The Fourth Stage: Study on HYP

- W. Lee, PRD 66 (2002) 114504.
- Properties:
 - Theorem 1 $SU(3)$ Projections.
 - Theorem 2 Triviality of renormalization.
 - Theorem 3 Multiple $SU(3)$ Projection.
 - Theorem 4 Uniqueness
 - Theorem 5 Equivalence ($HYP \leftrightarrow \overline{Fat7}$)
- $|C_{HYP}| \ll |C_{thin}|$, $SU(3)$ Projection \leftrightarrow T.I. for doublers.
- Renormalization simplified by $\langle A_\mu A_\nu \rangle \rightarrow \langle B_\mu B_\nu \rangle$



Current-current Diagrams for $\overline{\text{HYP/Fat7}}$

- W. Lee and S. Sharpe, PRD68, (2003) 054510; hep-lat/0306016.
- $\overline{\text{HYP/Fat7}} \approx 10\%$.
- Tadpole improvement:
 $C_N = 13.159$
 $C_H = 1.4051$

Finite correction to $(\mathcal{O}_3)_{II}$

Operators	$[S \times P][S \times P]_{II}$
NAIVE	$2 \times (95.6 - 6C_N)$
$\overline{\text{HYP/Fat7}}$	$2 \times (19.1 - 6C_H)$

Operators	$[P \times P][P \times P]_{II}$
NAIVE	$2 \times (111.3 - 2C_N)$
$\overline{\text{HYP/Fat7}}$	$2 \times (6.9 - 2C_H)$



Penguin Diagrams for HYP

- K. Choi and W. Lee: hep-lat/0309070.
- Theorem 1 (Equivalence):

At the one loop level, diagonal mixing coefficients are identical between unimproved and improved staggered operators of HYP (I), HYP (II), Fat7, Fat7+Lepage, and $\overline{\text{Fat7}}$.

- Note that **AsqTad** is **NOT** included in the list.
- Off-diagonal mixing vanishes for Fat7, HYP (II), and $\overline{\text{Fat7}}$.



The Fifth Stage:

Numerical Study with HYP staggered

- W. Lee NPB (P.S.) 128 (2004) 125, hep-lat/0310047;
T. Bhattacharya, *et al.*, hep-lat/0309105.
- $\beta = 6.0$, $16^3 \times 64$ lattice, 218 confs.
- Bare quark mass shifted by ≈ 2.5 .
 $Z_m \cong 2.5 \rightarrow Z_m \cong 1$.
- Small perturbative correction ($\approx 10\%$).
- Reduced taste symmetry breaking.



$Re(A_2)$, $Re(A_0)$, and ϵ'/ϵ (Exp)

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Standard model H_W (Theory)

- $H_W = \frac{G_F}{\sqrt{2}} V_{us} V_{ud} \sum_i \left(z_i(\mu) + \tau y_i(\mu) \right) Q_i(\mu)$
- Current-current: $Q_1, Q_2 = (\bar{s}u)_L(\bar{u}d)_L$
- QCD Penguin: $Q_3, Q_4, Q_5, Q_6 = (\bar{s}d) \sum_q (\bar{q}q)$
- EW Penguin: $Q_7, Q_8, Q_9, Q_{10} = (\bar{s}d) \sum_q e_q (\bar{q}q)$
- Group representation:
 $Q_1, Q_2, Q_9, Q_{10} \in (27_L, 1_R) + (8_L, 1_R)$
 $Q_3, Q_4, Q_5, Q_6 \in (8_L, 1_R)$
 $Q_7, Q_8 \in (8_L, 8_R)$



$Re(A_0)$ and $Re(A_2)$ (Theory)

- Standard model:

$$ReA_I = \frac{G_F}{\sqrt{2}} |V_{ud}V_{us}| \left[\sum_{i=1,2} z_i \langle Q_i \rangle_I + Re(\tau) \sum_{i=3}^{10} y_i \langle Q_i \rangle_I \right]$$

- $\langle Q_i \rangle_I \equiv \langle (\pi\pi)_I | Q_i | K^0 \rangle$
- $z_i, y_i \approx 1$ or $\alpha = 1/129$
- $Re(\tau) = -Re(\lambda_t/\lambda_u) = 0.002$
- Hence, only $\langle Q_1 \rangle_I$ and $\langle Q_2 \rangle_I$ are important in ReA_I .

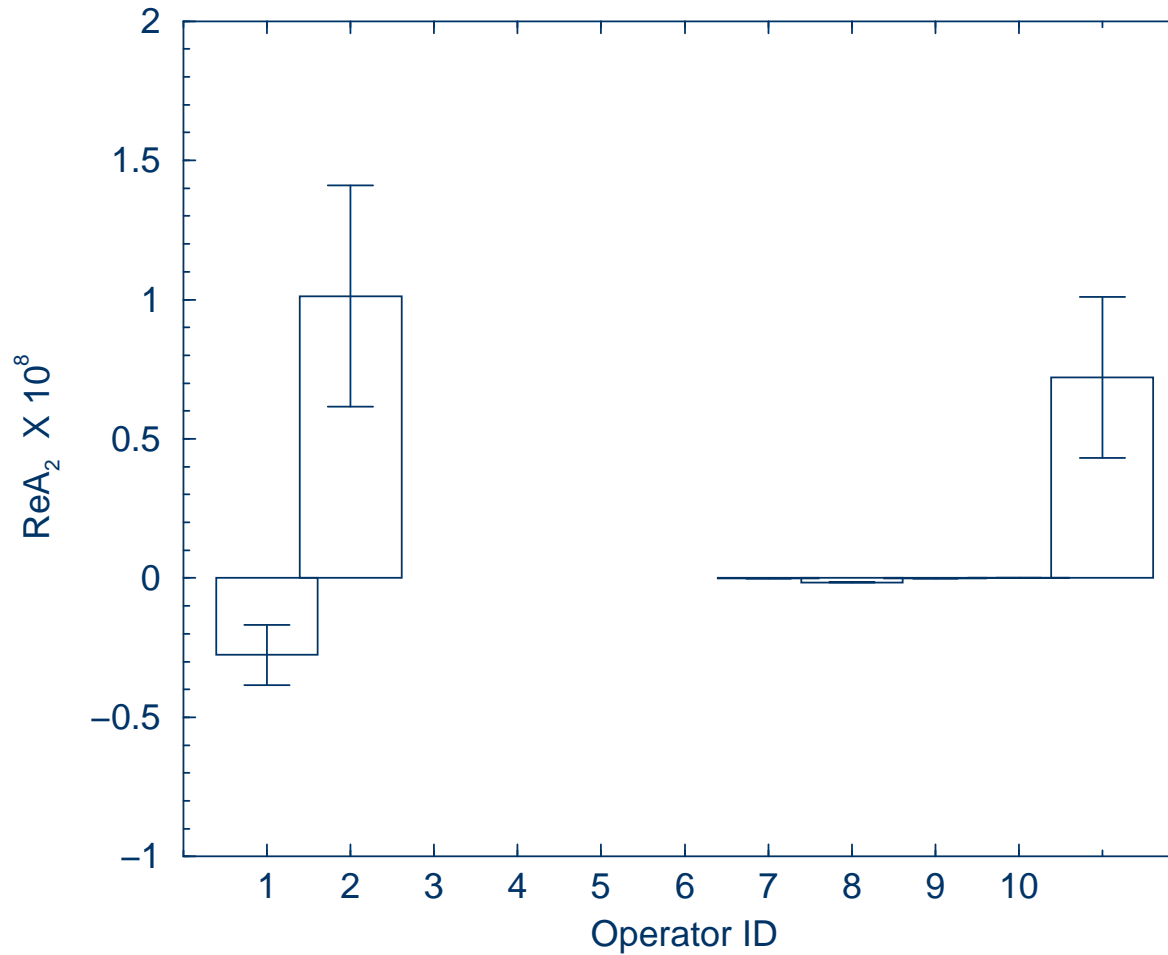


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$ReA_2 \times 10^8$

Linear fit in the chiral limit



$$ReA_2 = 1.50 \times 10^{-8} GeV \text{ (Experiment)}, \mu = 1/a$$

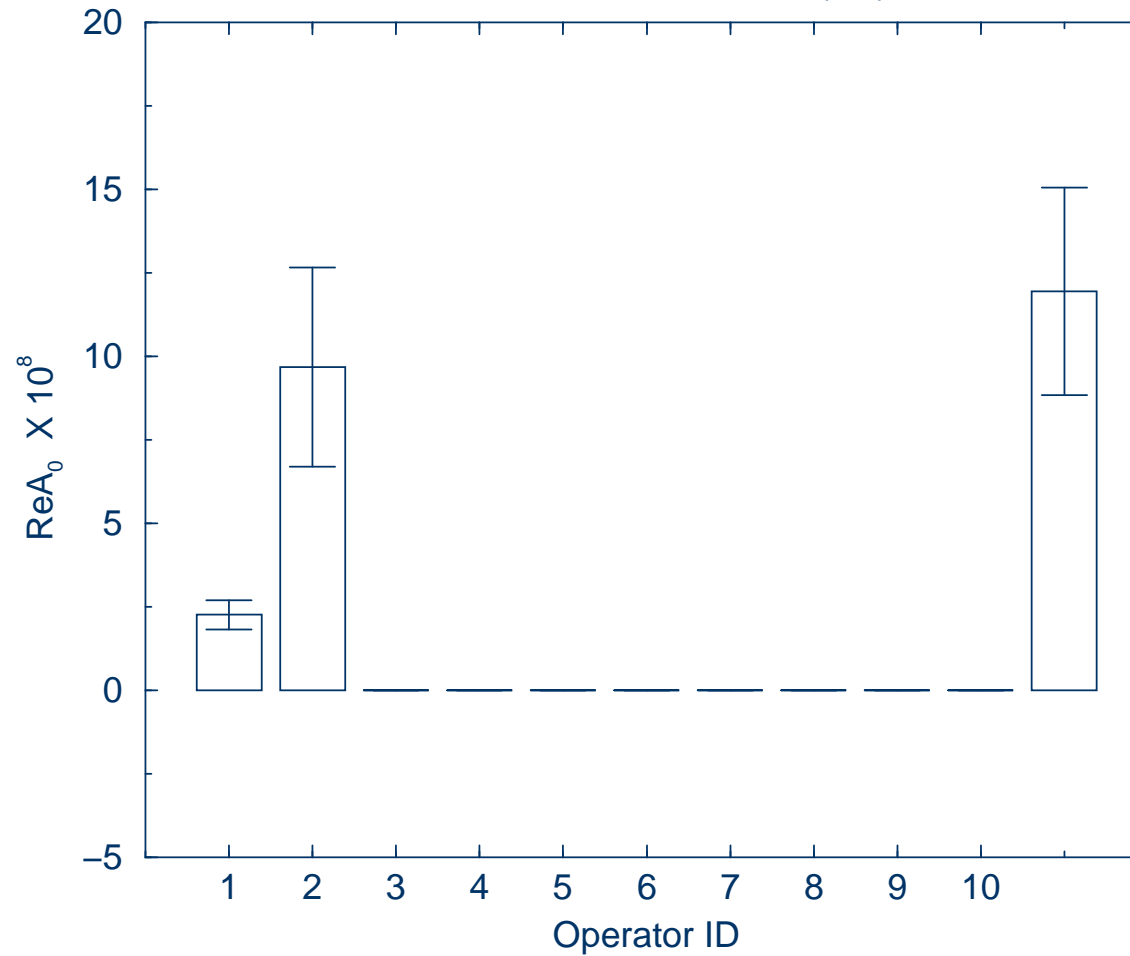


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$\text{Re}A_0 \times 10^8$

Linear fit in the chiral limit (std)



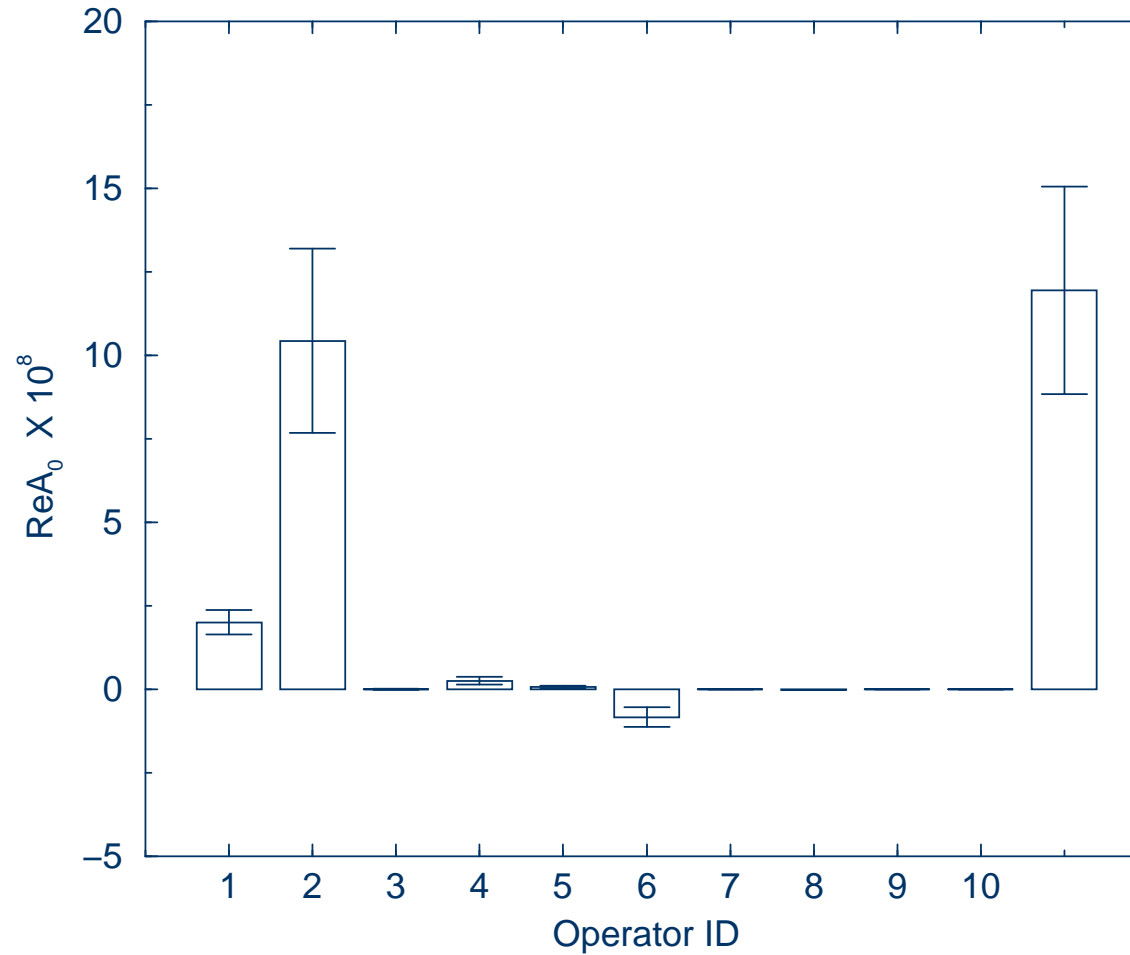
$$\text{Re}A_0 = 33.3 \times 10^{-8} \text{GeV (Experiment)}, \mu = m_c$$





$ReA_0 \times 10^8$

Linear fit in the chiral limit (std)



$$ReA_0 = 33.3 \times 10^{-8} GeV \text{ (Experiment)}, \mu = 1/a$$

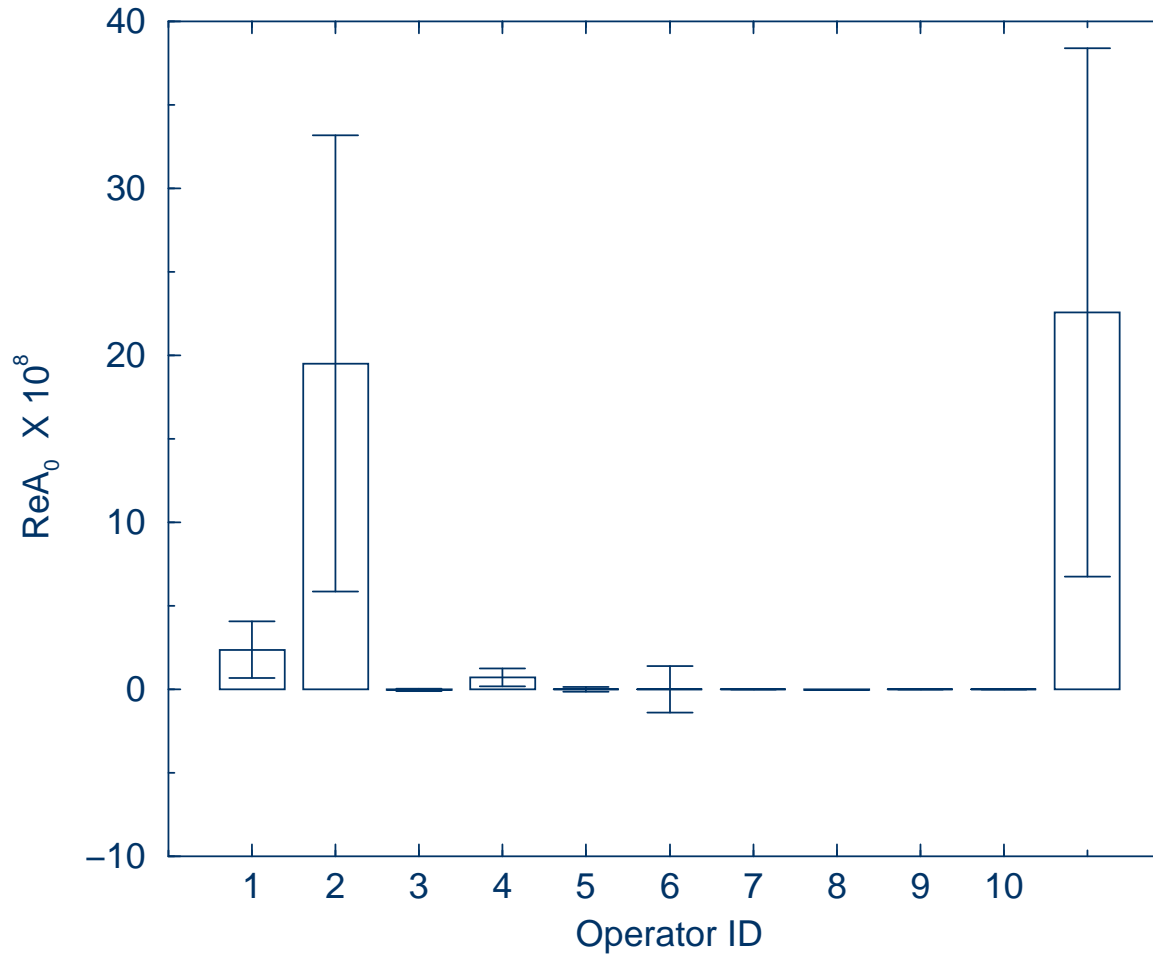


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$ReA_0 \times 10^8$

Lin-log fit in the chiral limit (std)



$$ReA_0 = 33.3 \times 10^{-8} GeV \text{ (Experiment)}, \mu = 1/a$$

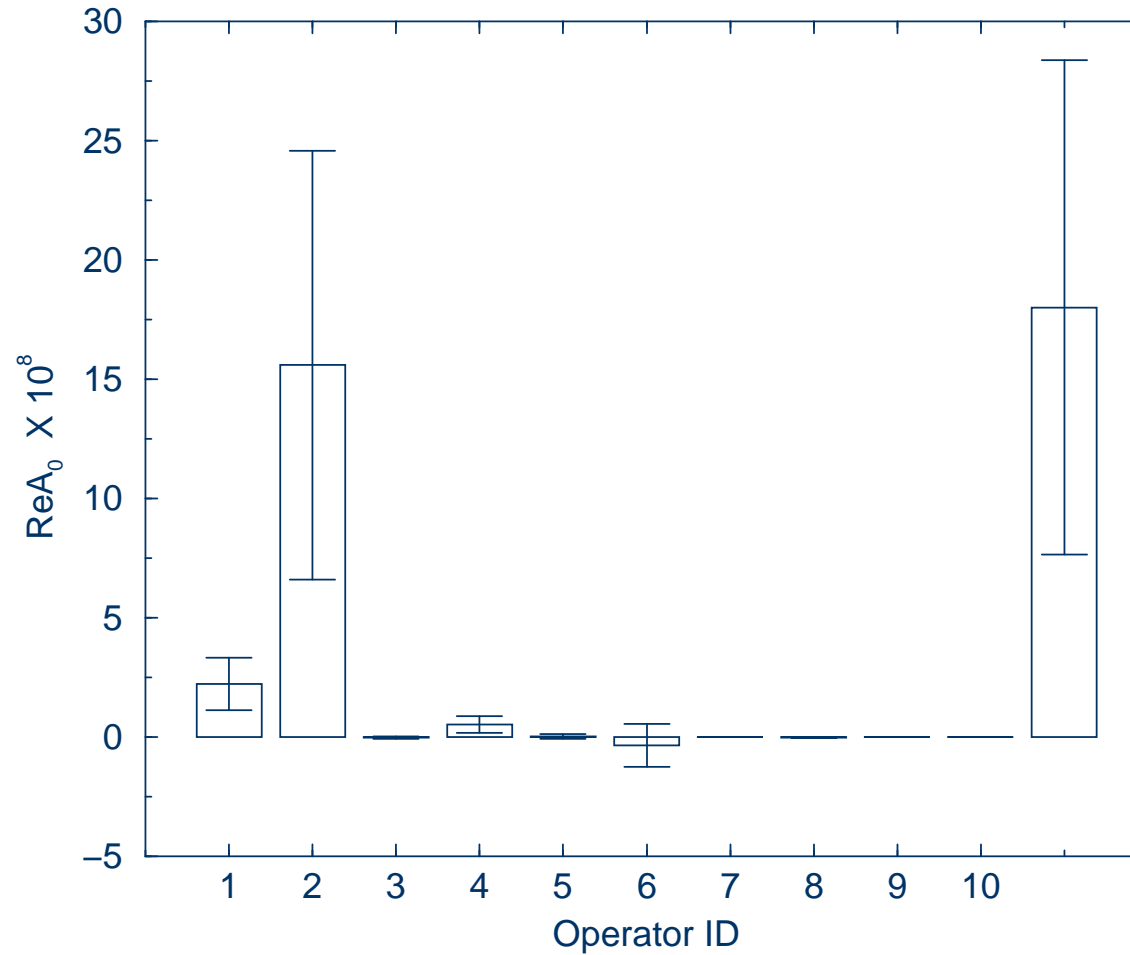


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$ReA_0 \times 10^8$

Quadratic fit in the chiral limit (std)



$$ReA_0 = 33.3 \times 10^{-8} GeV \text{ (Experiment)}, \mu = 1/a$$



ϵ'/ϵ (Theory)

- Standard model:

$$\epsilon'/\epsilon = \text{Im}(V_{ts}^* V_{td}) \left[P^{(1/2)} - P^{(3/2)} \right]$$

$$P^{(1/2)} = r \sum_{i=3}^{10} y_i(\mu) \langle O_i \rangle_0(\mu) (1 - \Omega_{\eta+\eta'})$$

$$P^{(3/2)} = \frac{r}{\omega} \sum_{i=3}^{10} y_i(\mu) \langle O_i \rangle_2(\mu)$$

$$r = \frac{G_F \omega}{2|\epsilon| \text{Re} A_0}$$



ϵ'/ϵ (Theory)

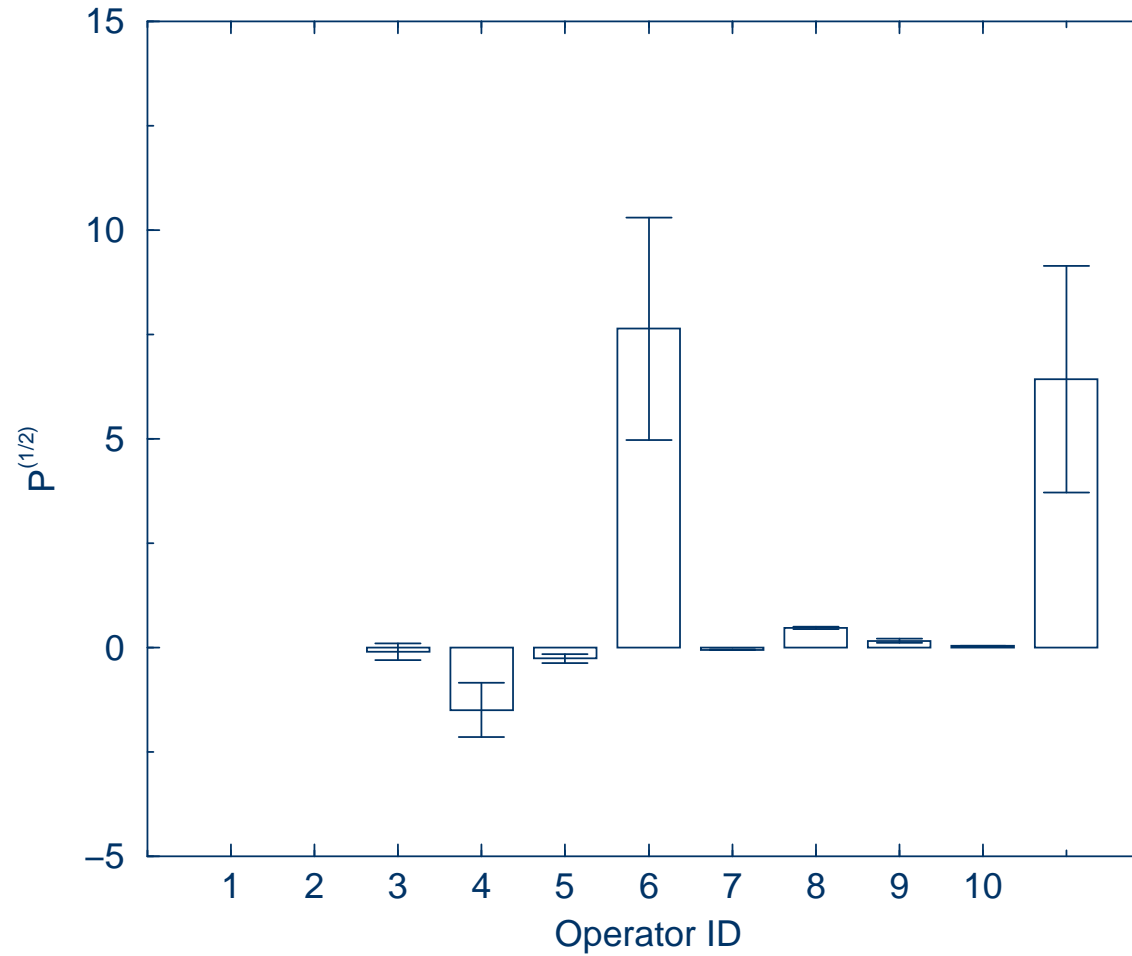
- NO contribution from $\langle Q_1 \rangle_I$ and $\langle Q_2 \rangle_I$.
- $P^{(1/2)}$ is dominated by $\langle Q_6 \rangle$.
- $P^{(3/2)}$ is dominated by $\langle Q_8 \rangle$.





$P^{(1/2)}$ (Standard op)

Linear fit in the chiral limit



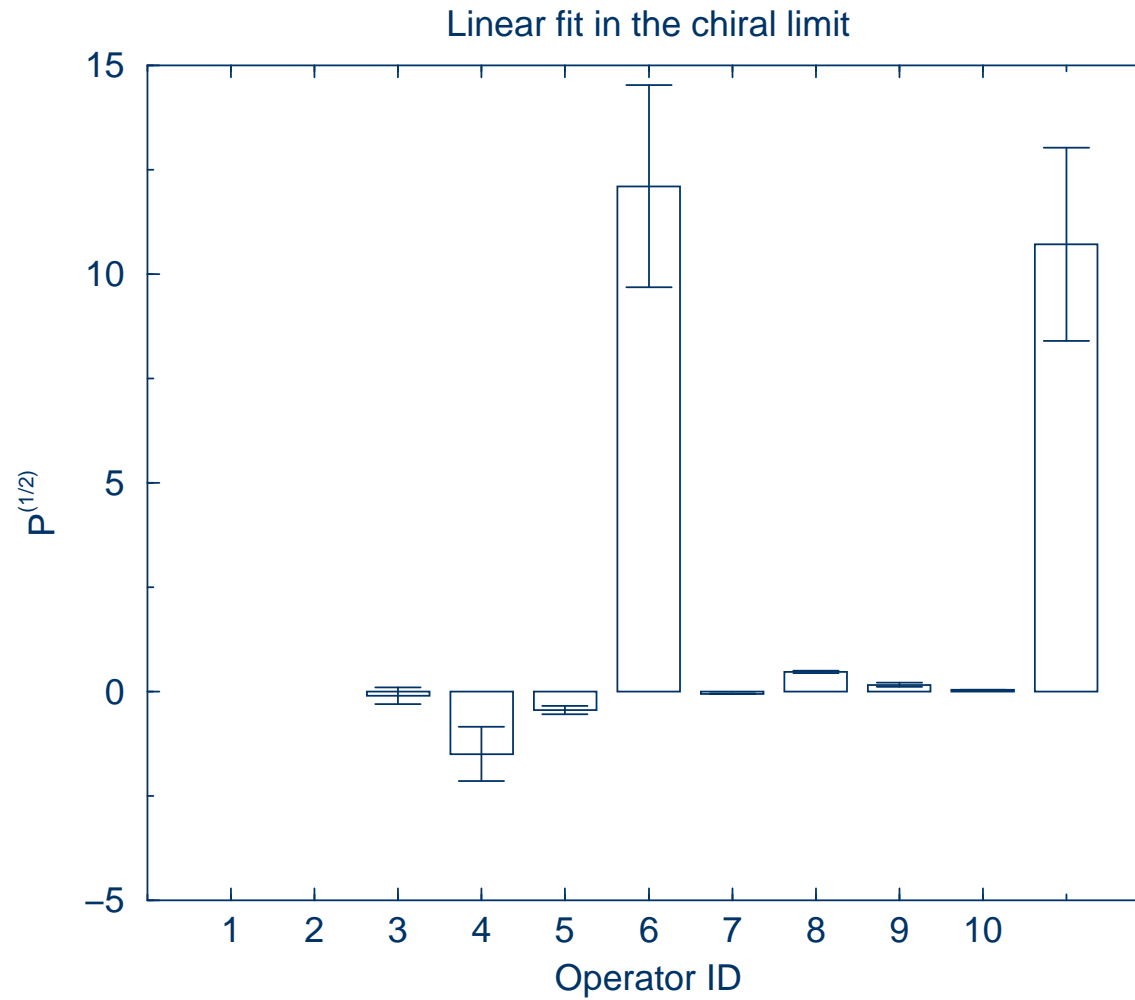
$$\mu = 1/a$$



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$P^{(1/2)}$ (Golterman–Pallante op)



$$\mu = 1/a$$

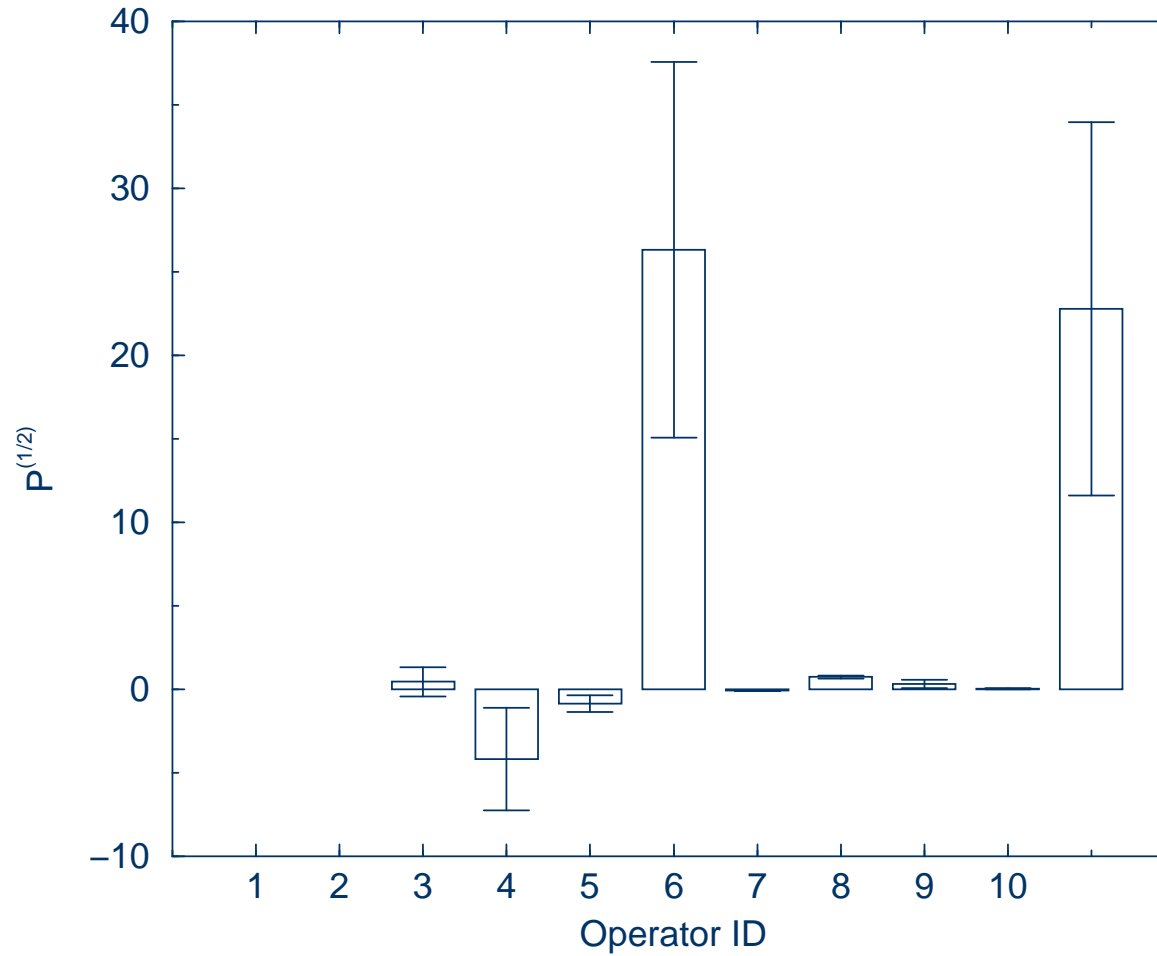


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$P^{(1/2)}$ (Golterman–Pallante op)

Lin-log fit in the chiral limit



$$\mu = 1/a$$

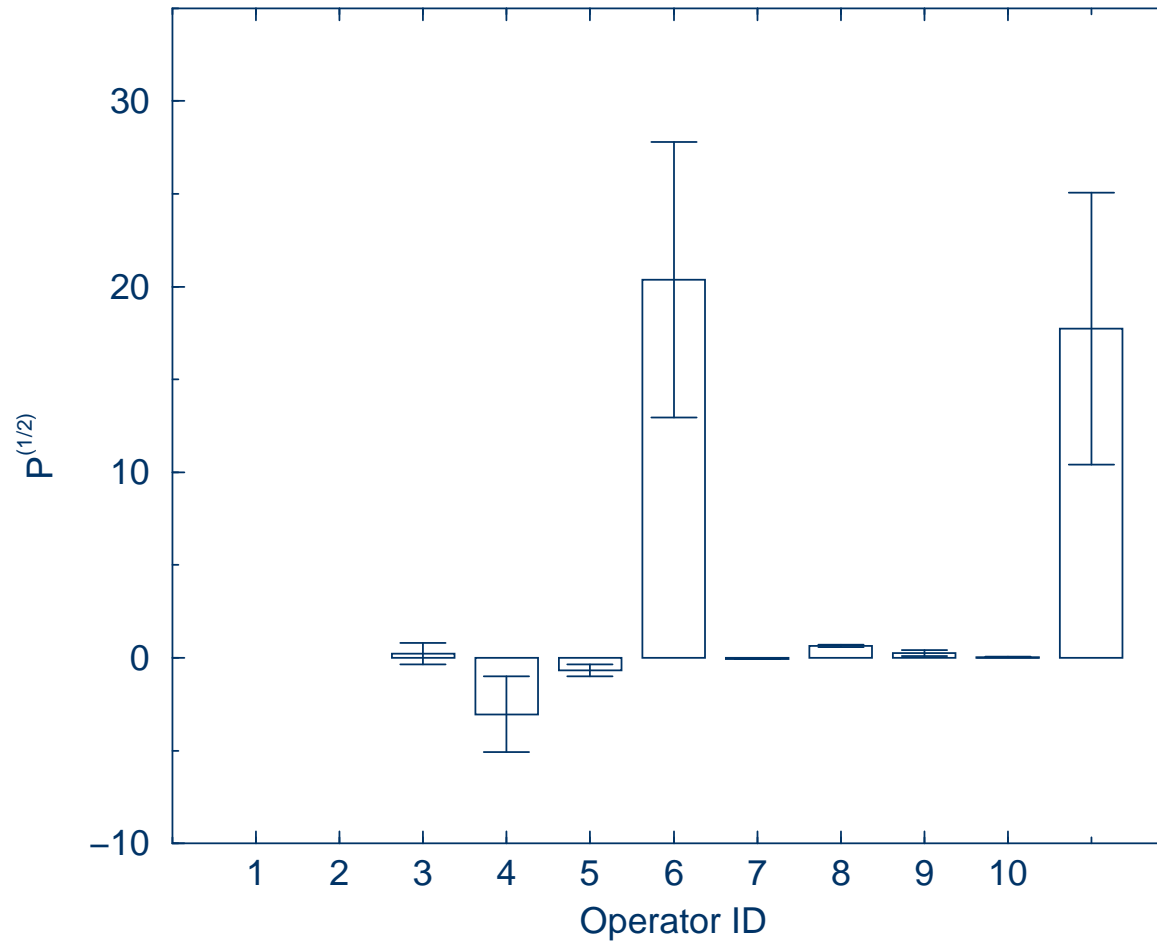


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$P^{(1/2)}$ (Golterman–Pallante op)

Quadratic fit in the chiral limit



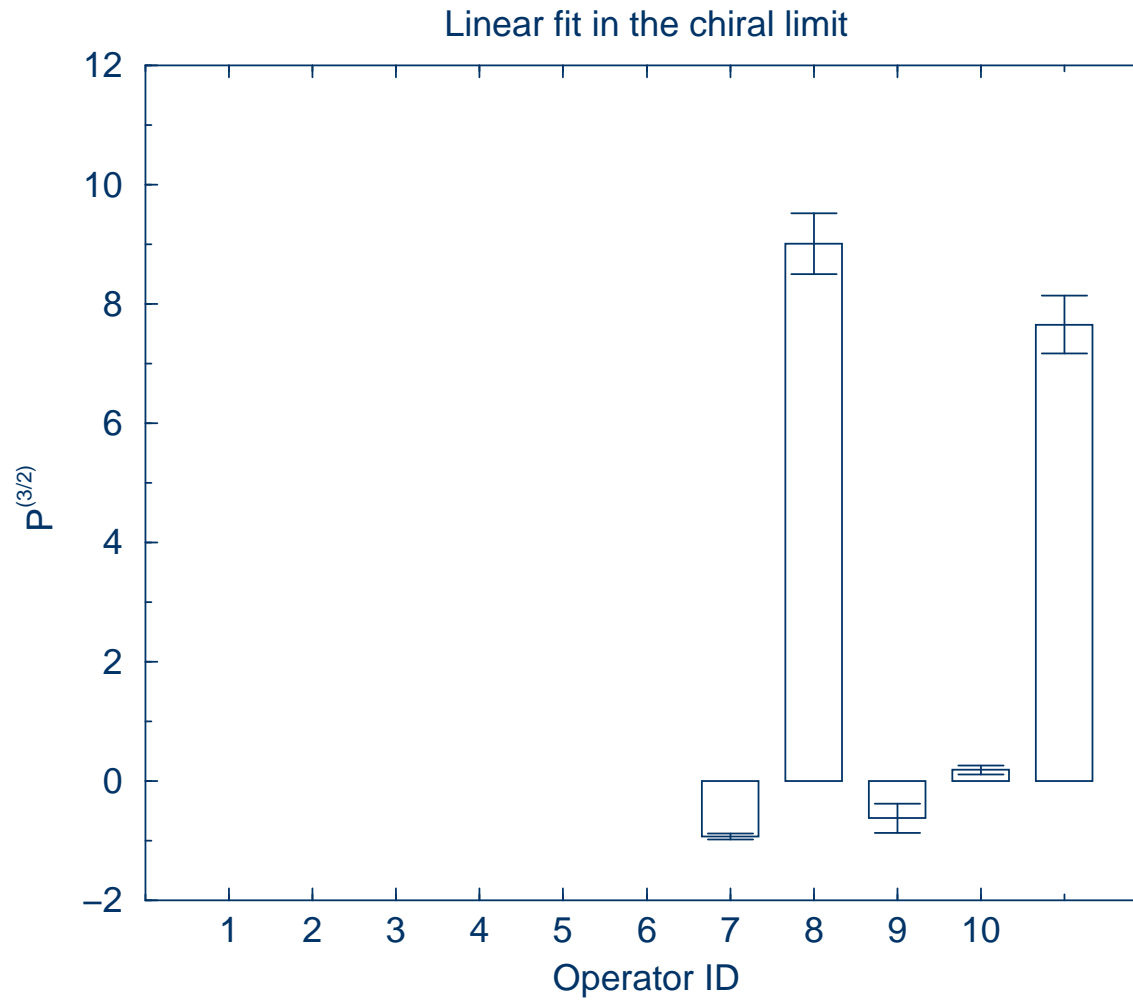
$$\mu = 1/a$$



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$P^{(3/2)}$



$$\mu = 1/a$$



ϵ'/ϵ ($\mu = 1/a$, **preliminary results**)

- Standard Operator:

$$\epsilon'/\epsilon = \begin{cases} -2.4(34) \times 10^{-4} & \text{(linear fit)} \\ -12.8(114) \times 10^{-4} & \text{(quadratic fit)} \\ -20.1(175) \times 10^{-4} & \text{(\chi-log fit)} \end{cases}$$

- Golterman-Pallante Operator:

$$\epsilon'/\epsilon = \begin{cases} +3.2(29) \times 10^{-4} & \text{(linear fit)} \\ +8.8(93) \times 10^{-4} & \text{(quadratic fit)} \\ +13.1(142) \times 10^{-4} & \text{(\chi-log fit)} \end{cases}$$



Calculating $K \rightarrow \pi\pi$ on the lattice

- Calculate $\langle \pi^+ | O_i | K^+ \rangle$ and $\langle 0 | O_i | K^0 \rangle$ on the lattice
- Use χ PT at leading order to obtain $\langle \pi^+ \pi^- | O_i | K^0 \rangle$

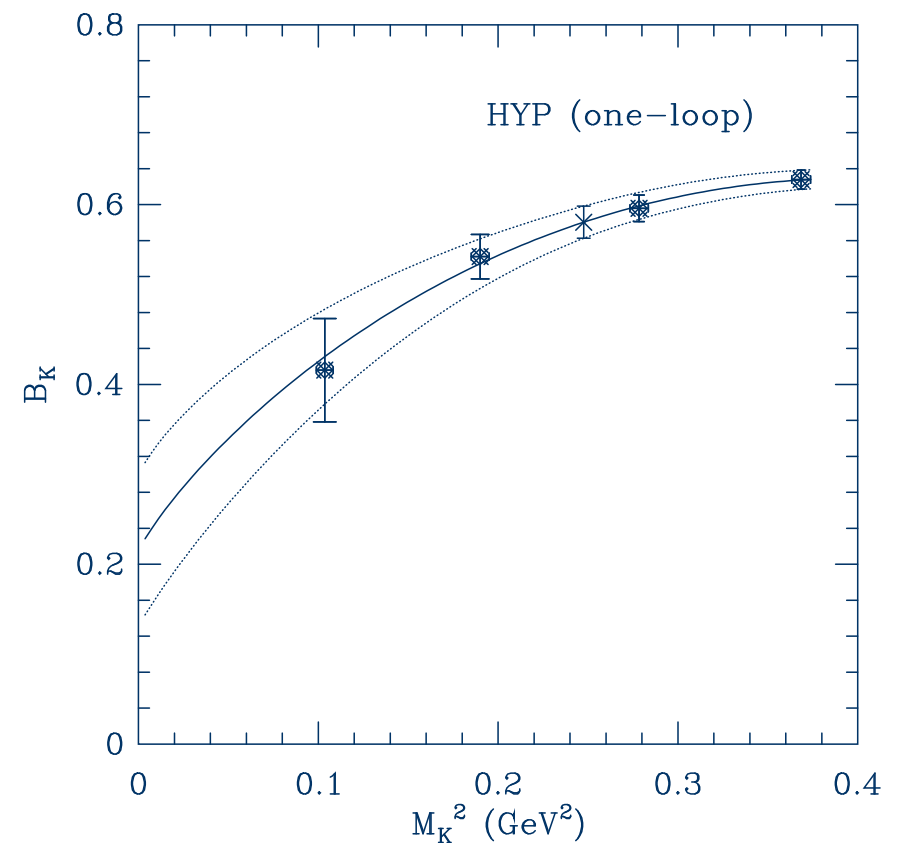
$$\left\{ \begin{array}{l} K^+ \rightarrow \pi^+ \\ K^0 \rightarrow 0 \end{array} \right\} \Longrightarrow \chi PT \Longrightarrow (K^+ \rightarrow \pi^+ \pi^-)$$

- Numerical study in quenched QCD.



B_K

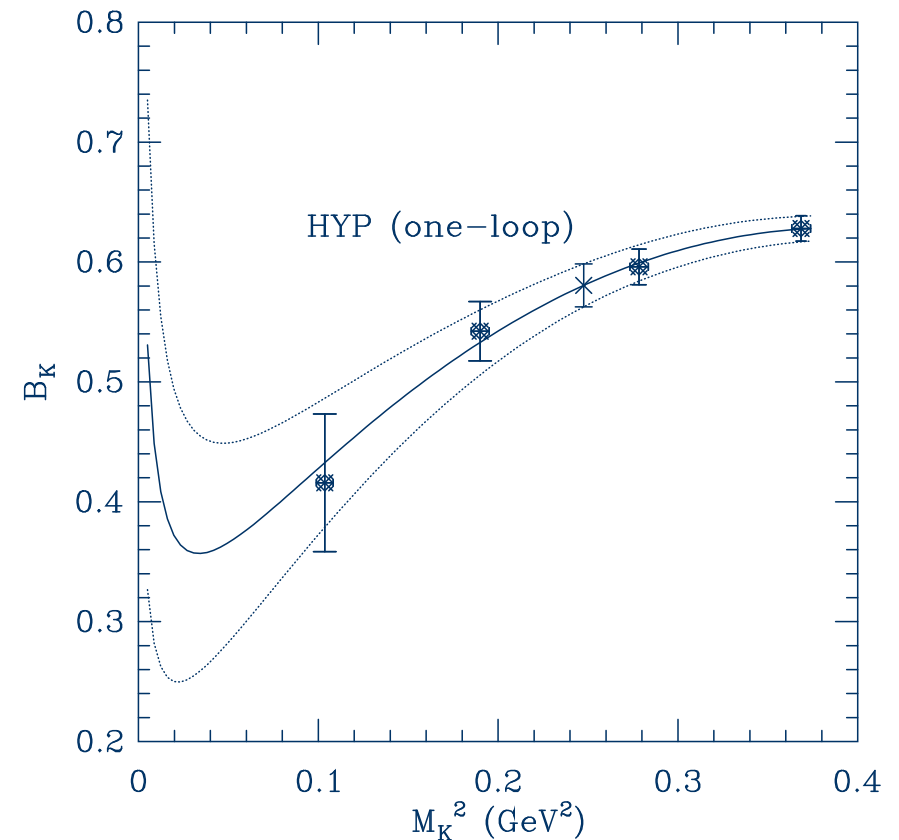
- Fit to quenched χ PT.
- $B_K(m_K, \mu = 2\text{GeV}) = 0.580(18)$
- Consistent with JLQCD results at $a = 0$.
- $B_K(0, RGI) = 0.298(117)$
- Consistent with $1/N_c$ results.





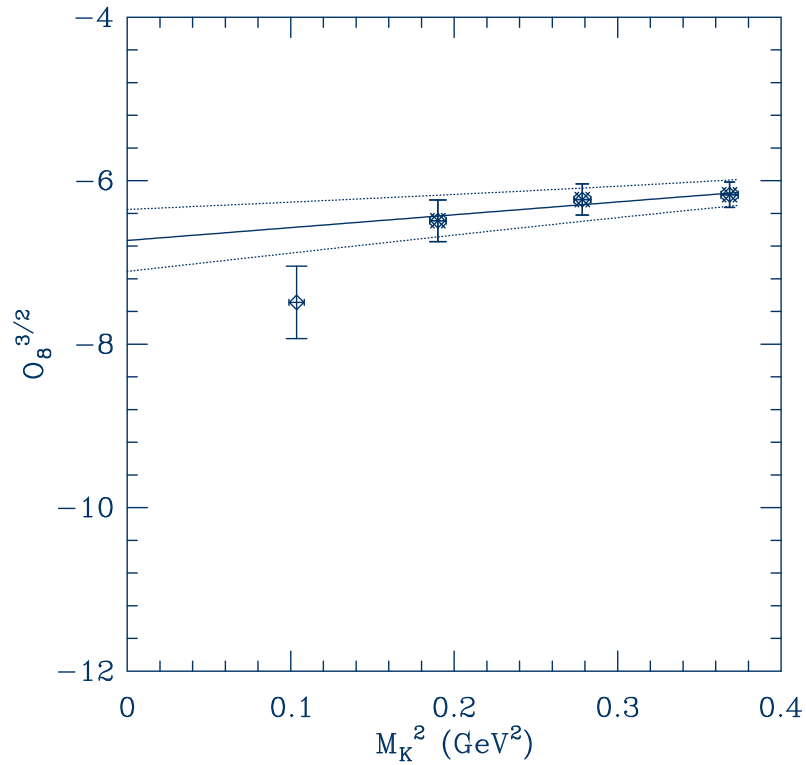
Finite volume effect on B_K

- Fit to quenched χ PT with finite volume corrections.
- $B_K(m_K, \mu = 2\text{GeV}) = 0.574(16)$
- $B_K(0, RGI) = 0.252(100)$
- Consistent with $1/N_c$ results.

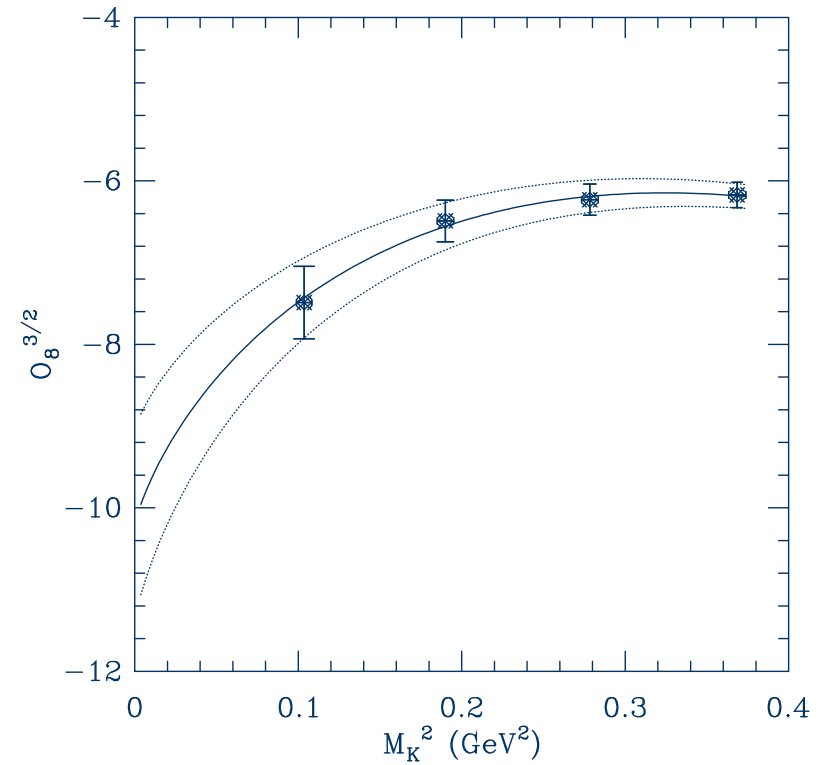




$$\langle O_8^{\Delta I=3/2} \rangle$$



Linear fit



Chiral Log fit

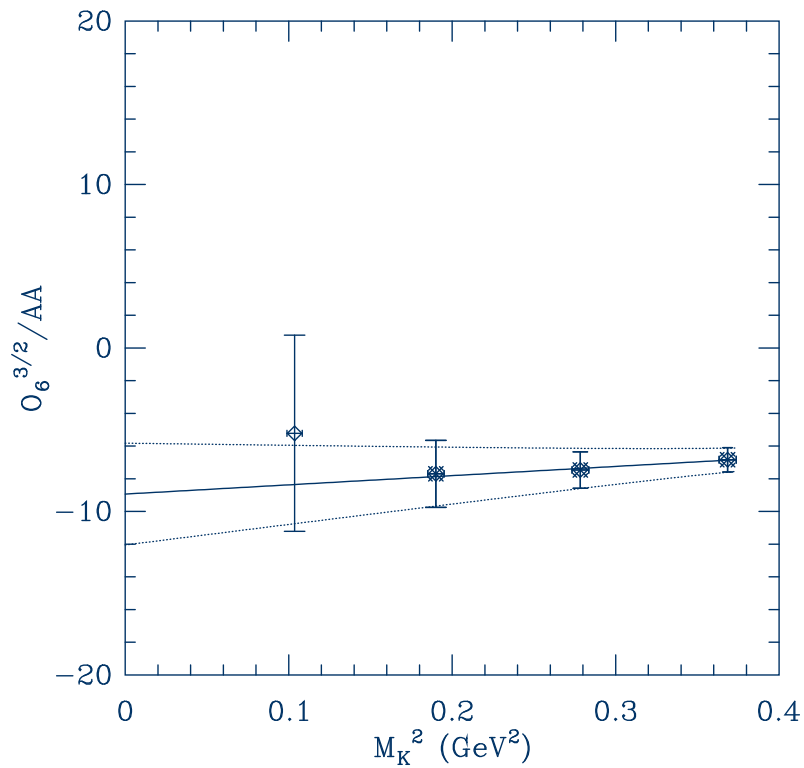


Lattice version of O_6

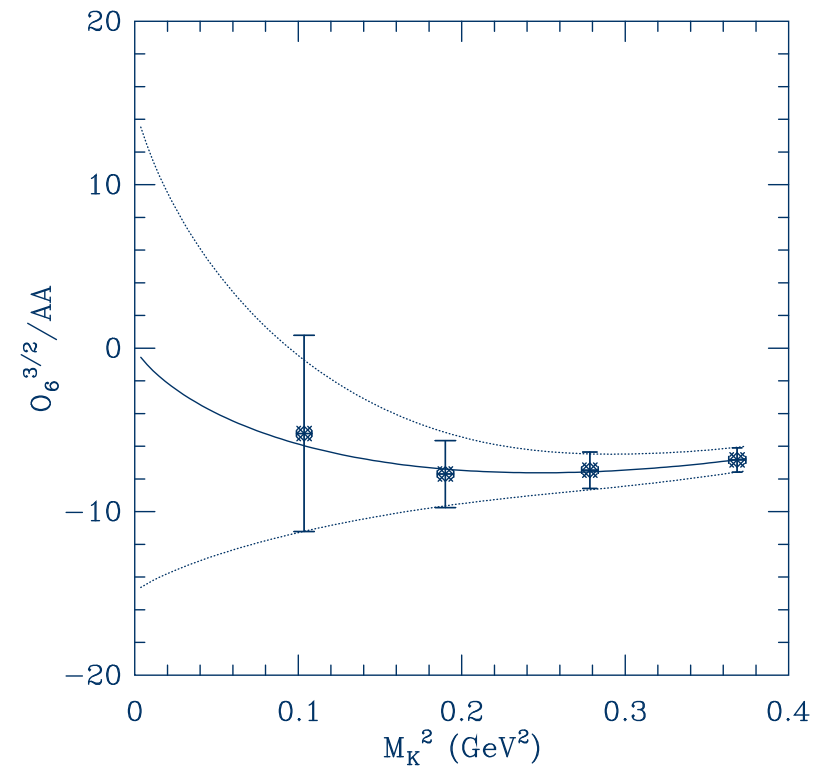
- Flavor symmetry of quenched QCD = $SU(3|3)$
- Flavor symmetry of full QCD = $SU(3)$
- Hence, one may choose a singlet in $SU(3)_R$ or a singlet in $SU(3|3)_R$ in quenched QCD.
- The standard operator $\in SU(3)_R$.
- Golterman-Pallante operator $\in SU(3|3)_R$.



$$\frac{\langle O_6^{\Delta I=1/2} \rangle}{m_K^2 f^2} \text{ (standard op)}$$



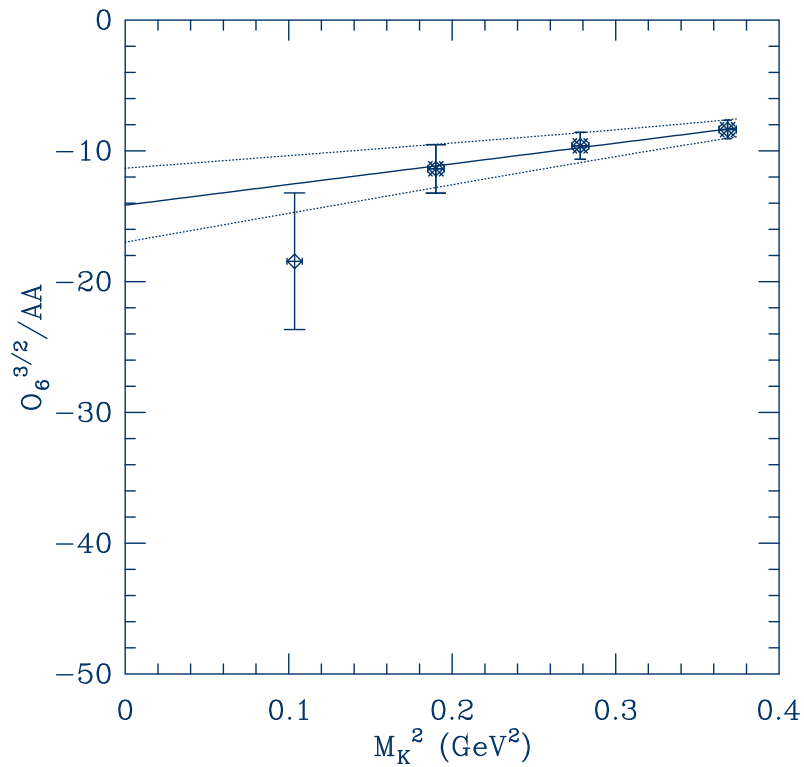
Linear fit



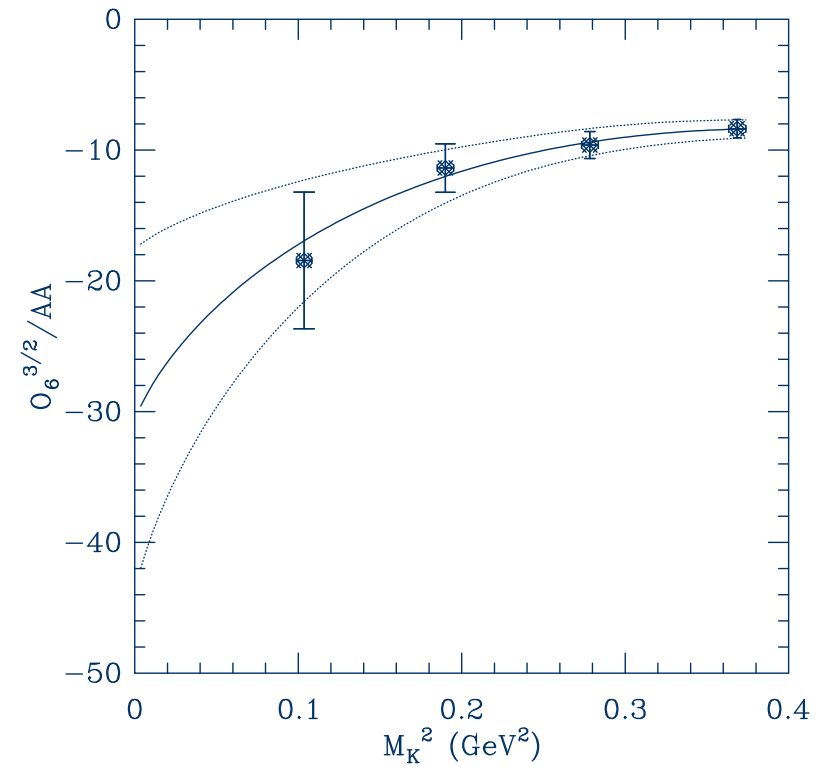
Chiral Log fit



$\frac{\langle O_6^{\Delta I=1/2} \rangle}{m_K^2 f^2}$ (Golterman-Pallante op)



Linear fit



Chiral Log fit



Summary and Conclusion

- HYP/ $\overline{\text{Fat7}}$ resolves the long-standing problem of large perturbative corrections.
- The perturbative calculation for HYP/ $\overline{\text{Fat7}}$ is extended to current-current diagrams and penguin diagrams.
- This allows full matching between lattice and continuum matrix elements.
- The Golterman-Pallante method prefers positive ϵ'/ϵ .
- The standard method prefer negative ϵ'/ϵ .
- This tells us that uncertainty from the quenched approximation is large for ϵ'/ϵ .



- We are extending the current calculation to include more quark masses and to accumulate higher statistics.
- In order to constrain the form of the fitting functions, we need results of the staggered chiral perturbation.
- We need to check the finite volume effect (1.6 fm \rightarrow 2.4 fm).
- We need to extend it to weaker gauge couplings to check the scaling violation, which is expected to be quite small.
- We need to calculate ϵ'/ϵ in partially quenched QCD and full QCD.
- We need to include the next leading order in χ PT to



check the size of NLO correction.



RG evolution for $N_f = 3$

- For $N_f = 3$, there is a removable singularity.
- $2\beta_0 + \gamma_8^{(0)} - \gamma_7^{(0)} = 0 \rightarrow$ singularity.
- Both the denominator and numerator vanish.
- Final analytical form contains

$$\ln \left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right), \quad \left[\ln \left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right) \right]^2$$