V_{us} and m_s from hadronic τ decays

Work in collaboration with:

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Introduction

- Precision determinations of Standard Model parameters are of utmost importance for many areas in particle physics.

- Investigations of hadronic τ decays already contributed tremendously for fundamental QCD parameters like α_s, the strange mass and non-perturbative condensates.

- The strange mass determination from hadronic τ decays depends sensitively on the CKM matrix element V_{us}.

- Thus the flavour-breaking difference of decay fractions can also be used to determine V_{us} and, with precise experimental data, even m_s and V_{us} simultaneously.

Unitarity relation

The unitarity relation for the first row of the CKM matrix reads:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta \,.$$

Numerically, the present PDG04 average is represented by:

$$V_{ud} = \begin{cases} 0.9740 \pm 0.0005 & \text{nuclear } \beta\text{-decay} \\ 0.9725 \pm 0.0013 & \text{neutron } \beta\text{-decay} \end{cases}$$

$$\Rightarrow \quad \boxed{V_{ud} = 0.9738 \pm 0.0005}$$

$V_{us} = 0.2200 \pm 0.0027$ K_{e3} decays (Leutwyler, Roos 1984)

$$\Rightarrow \quad \boxed{\delta = (3.3 \pm 1.5) \cdot 10^{-3}}$$

$\approx 2.2\,\sigma$ difference !

Theoretically, the K_{e3}-decay rate is given by:

$$\Gamma[K \to \pi e \nu_e(\gamma)] = \frac{G_F^2 M_K^5}{192\pi^3} C^2 I_K S_{\rm EW} |V_{us}|^2 f_+^2(0)$$

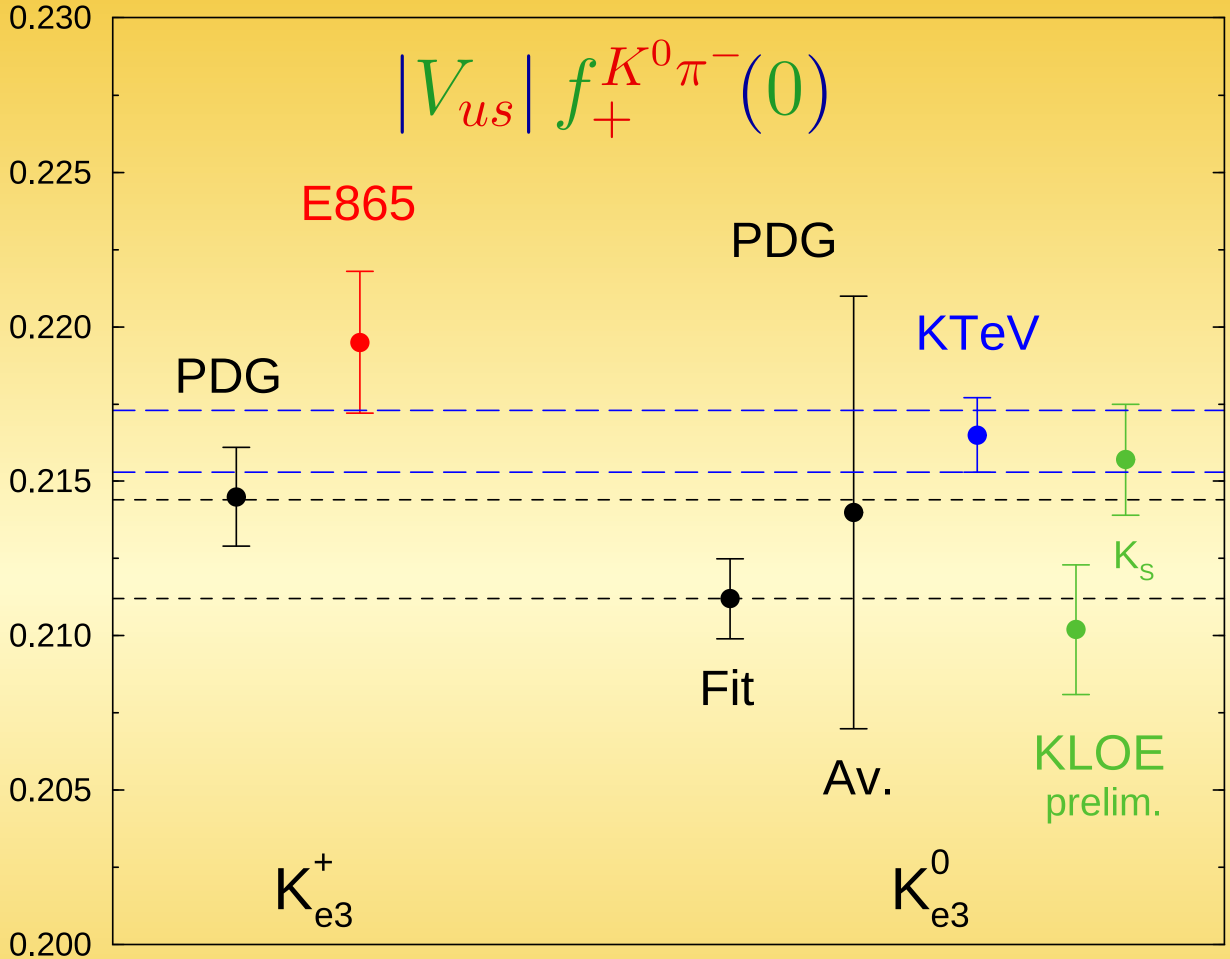
where with $t = (p - p')^2$:

$$\langle \pi^-(p') | \bar{s}\gamma_\mu u | K^0(p) \rangle = (p+p')_\mu f_+(t) + (p-p')_\mu f_-(t)$$

The largest theoretical uncertainty resides in the hadronic form factor $f_+(0)$. $\Rightarrow$ First determine: $|V_{us}|\, f_+(0)$

In the last two years there have been new measurements by E865 and KTeV. Soon new results by KLOE and NA48.

V_{us} from K_{e3}-decays



$\Rightarrow$ Average: $|V_{us}|\, f_+^{K^0\pi^-}(0) = 0.2163 \pm 0.0010$

The hadronic form factor $f_+(0)$ can be calculated in the framework of chiral perturbation theory. (Gasser, Leutwyler 1985) (Bijnens, Talavera 2003)

$$f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)(M_K^2 - M_\pi^2)^2 \,,$$

where

$$\Delta(0) = -\,0.080 \pm 0.057 \text{ [loops]} \pm 0.028 \; [L_i^r] \,.$$

Precisely the same chiral couplings C_{12}^r and C_{34}^r appear in the full scalar form factor $f_0(t)$ at order p^6 in χPT.

The scalar $K\pi$ form factor has been determined from a coupled-channel dispersion relation analysis, which as an input employed S-wave $K\pi$ phase-shifts as fitted in the RχPT framework with a unitarisation prescription. (M.J., Oller, Pich, 2000/01/02)

From this, slope and curvature of $f_0(t)$ can be predicted:

$$f_0'(0) = 0.804 \pm 0.048 \text{ GeV}^{-2}, \quad f_0''(0) = 1.67 \pm 0.10 \text{ GeV}^{-4},$$

the slope corresponding to: $\lambda_0 = 0.016 \pm 0.001$.

Employing these findings, we obtain: (M.J., Oller, Pich 2004)

$$(C_{12}^r + C_{34}^r)(M_\rho) = (3.2 \pm 1.5) \cdot 10^{-6}\,.$$

$$\Rightarrow \quad f_+^{K^0\pi^-}(0) = 1 - 0.023 - 0.003 = 0.974 \pm 0.011\,.$$

$$\Rightarrow \quad \boxed{|V_{us}| = 0.2221 \pm 0.0027}$$

And for the unitarity test: $\delta = (2.4 \pm 1.6) \cdot 10^{-3}$.

From the ratio $\Gamma[K\to l\nu_l(\gamma)]/\Gamma[\pi\to l\nu_l(\gamma)]$, we can predict:

(Marciano 2004)

$$\frac{|V_{us}|}{|V_{ud}|}\,\frac{F_K}{F_\pi} = 0.27569 \pm 0.00047\,.$$

Together with the recent lattice result (MILC 2004)

$$\frac{F_K}{F_\pi} = 1.210 \pm 0.014\,,$$

this leads to:

$$|V_{us}| = 0.2219 \pm 0.0026$$

Consider the physical quantity R_τ: (Braaten, Narison, Pich (1992))

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \text{hadrons}\,\nu_\tau(\gamma))}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau(\gamma))} = 3.642 \pm 0.012\,.$$

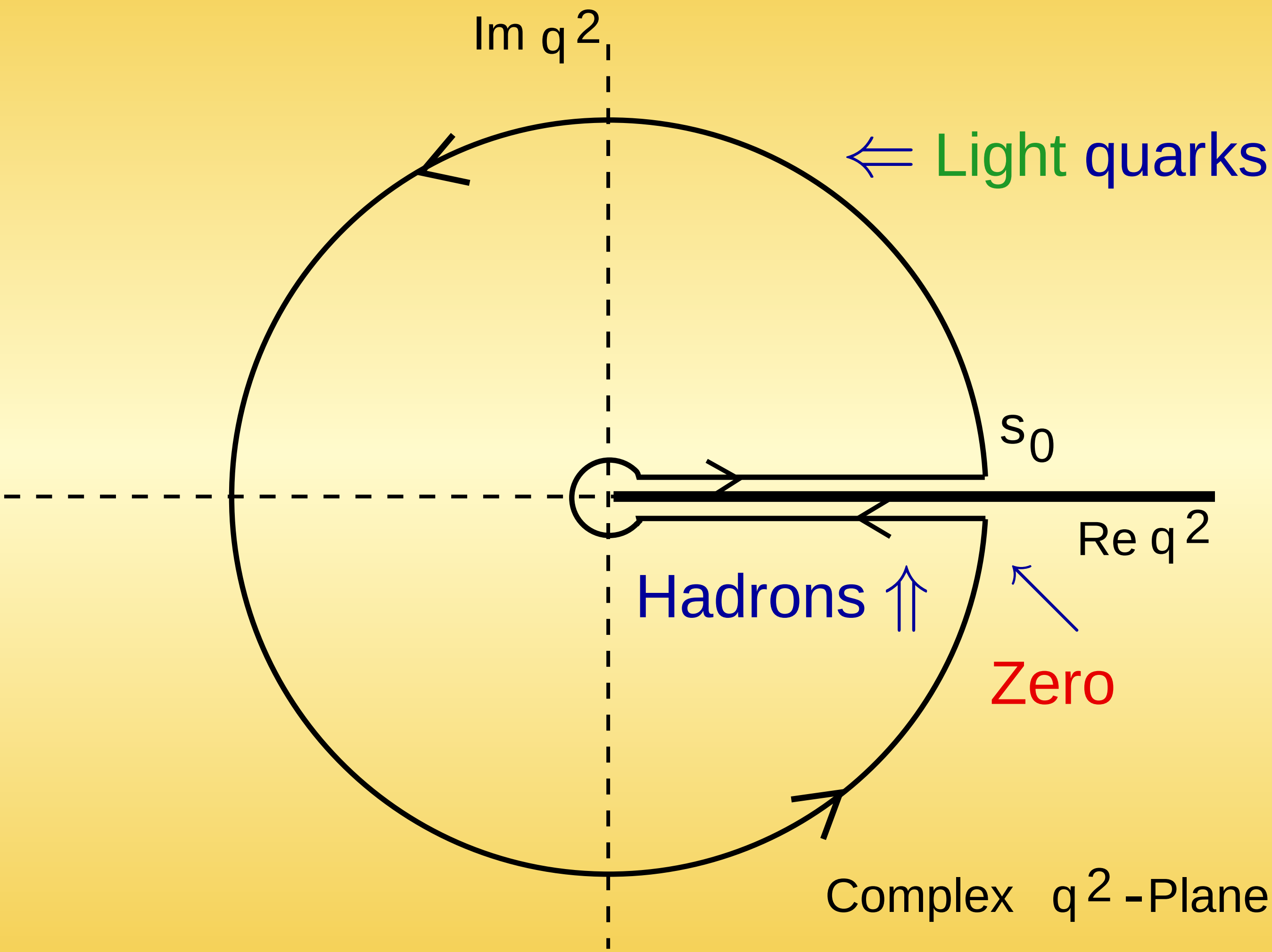
R_τ is related to the QCD correlators $\Pi^{T,L}(z)$: $(z \equiv s/M_\tau^2)$

$$R_\tau = 12\pi \int\limits_0^1 dz(1-z)^2\Big[(1+2z)\,\text{Im}\Pi^T(z) + \text{Im}\Pi^L(z)\Big]\,,$$

with the appropriate combinations

$$\Pi^J(z) = |V_{ud}|^2\Big[\Pi_{ud}^{V,J} + \Pi_{ud}^{A,J}\Big] + |V_{us}|^2\Big[\Pi_{us}^{V,J} + \Pi_{us}^{A,J}\Big]\,.$$

Hadronic τ decay rate



Additional information can be inferred from the moments

$$R_\tau^{kl} \equiv \int_0^1 dz\,(1-z)^k z^l\,\frac{dR_\tau}{dz} = R_{\tau,NS}^{kl} + R_{\tau,S}^{kl}\,.$$

Theoretically, R_τ^{kl} can be expressed as:

$$R_\tau^{kl} = N_c\,S_{\rm EW}\Big\{(|V_{ud}|^2+|V_{us}|^2)\Big[1+\delta^{kl(0)}\Big] + \sum_{D\geq 2}\Big[|V_{ud}|^2\delta_{ud}^{kl(D)}+|V_{us}|^2\delta_{us}^{kl(D)}\Big]\Big\}\,.$$

$\delta_{ud}^{kl(D)}$ and $\delta_{us}^{kl(D)}$ are corrections in the Operator Product Expansion, the most important ones being $\sim m_s^2$ and $m_s\langle\bar{q}q\rangle$.

The sensitivity to the strange quark mass can be enhanced by considering the flavour SU(3)-breaking difference:

(Pich, Prades; ALEPH (1998))

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,NS}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = 3\, S_{\rm EW} \sum_{D\geq 2} \Big(\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \Big) .$$

Flavour independent uncertainties drop out in the difference.

In previous analyses a sizeable part of the theoretical error was due to large α_s corrections in the longitudinal contribution.

This uncertainty could be greatly reduced by replacing badly behaved scalar/pseudoscalar correlators with phenomenology.

Defining a sum rule for the longitudinal contribution alone:

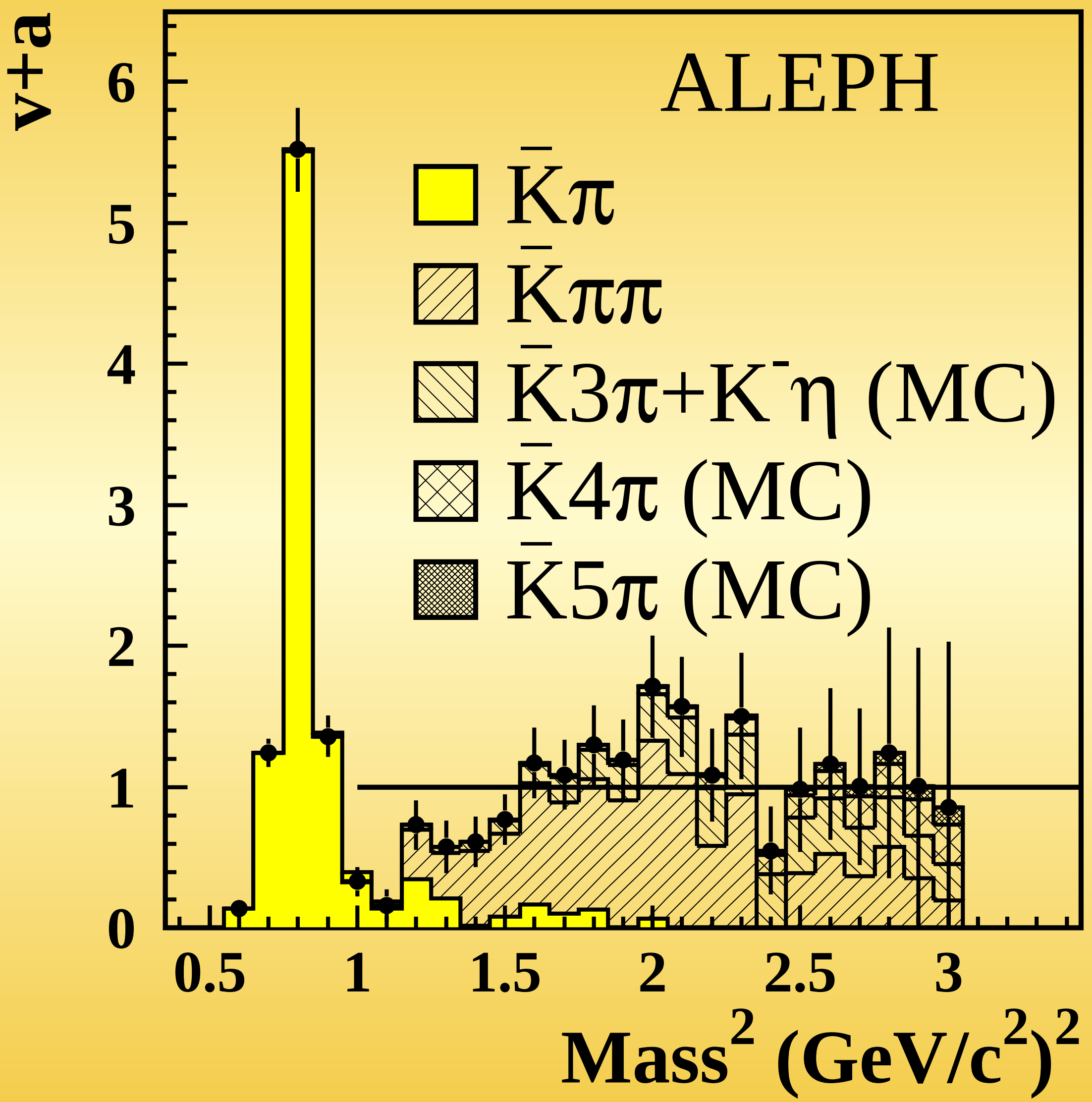
$$R^{kl,L}_{ij,V/A} \equiv -24\pi^2 \int_0^1 dz\,(1-z)^{2+k}\,z^{l+1}\,\rho^{V/A,L}_{ij}(z)\,.$$

The dominant contribution stems from the pseudoscalar (us) spectral function, $\rho = \mathrm{Im}\Pi/\pi$, which can be parameterised as:

$$s^2\rho^{A,L}_{us} = 2f_K^2 M_K^4 \delta(s-M_K^2) + 2f_{K(1460)}^2 M_{K(1460)}^4 BW(s)\,.$$

	$R^{00,L}_{us,A}$	$R^{00,L}_{us,V}$	$R^{00,L}_{ud,A}$ $[10^{-3}]$
Theo:	-0.144 ± 0.024	-0.028 ± 0.021	-7.79 ± 0.14
Phen:	-0.135 ± 0.003	-0.028 ± 0.004	-7.77 ± 0.08

Strange spectral function



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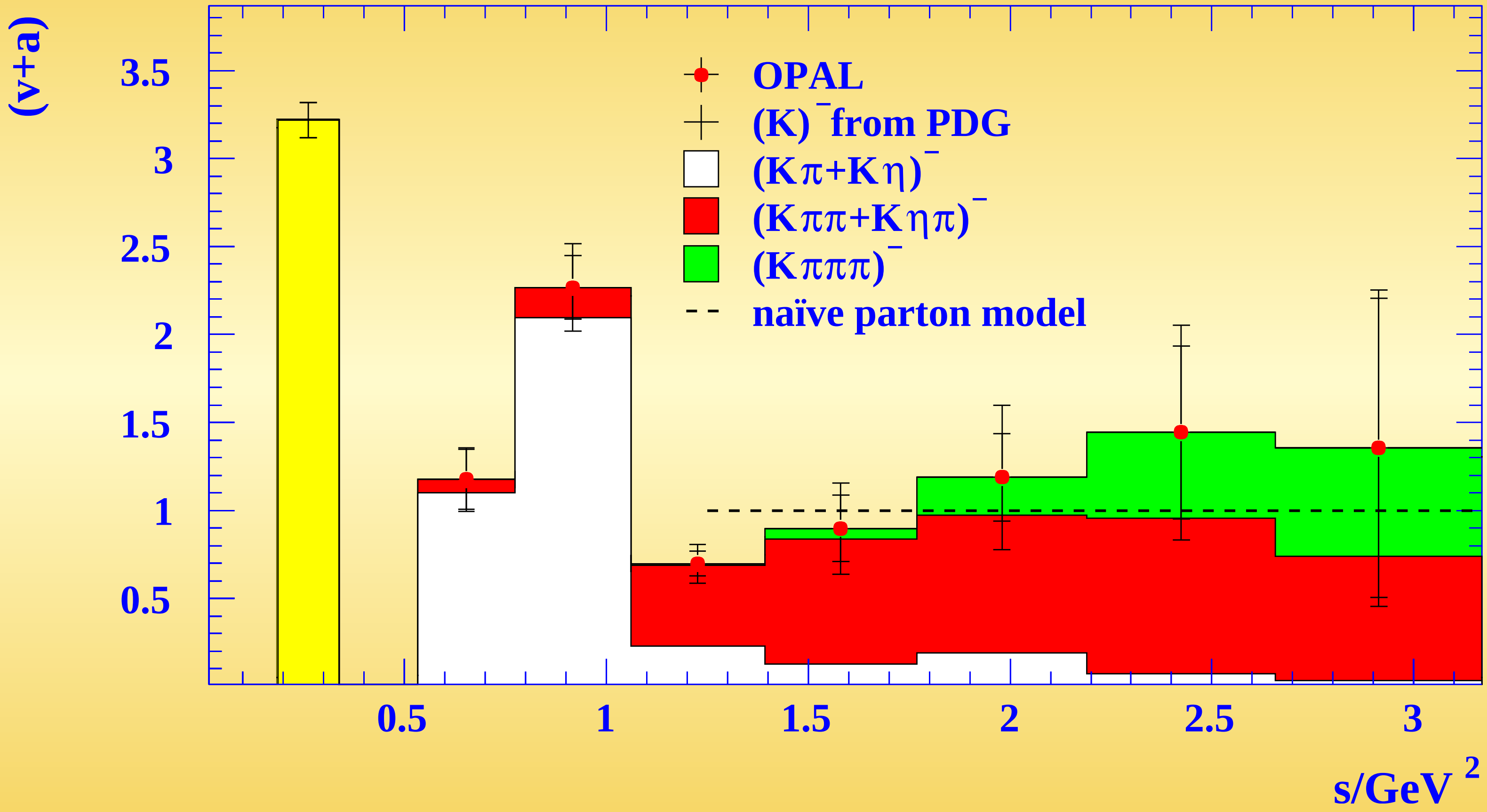
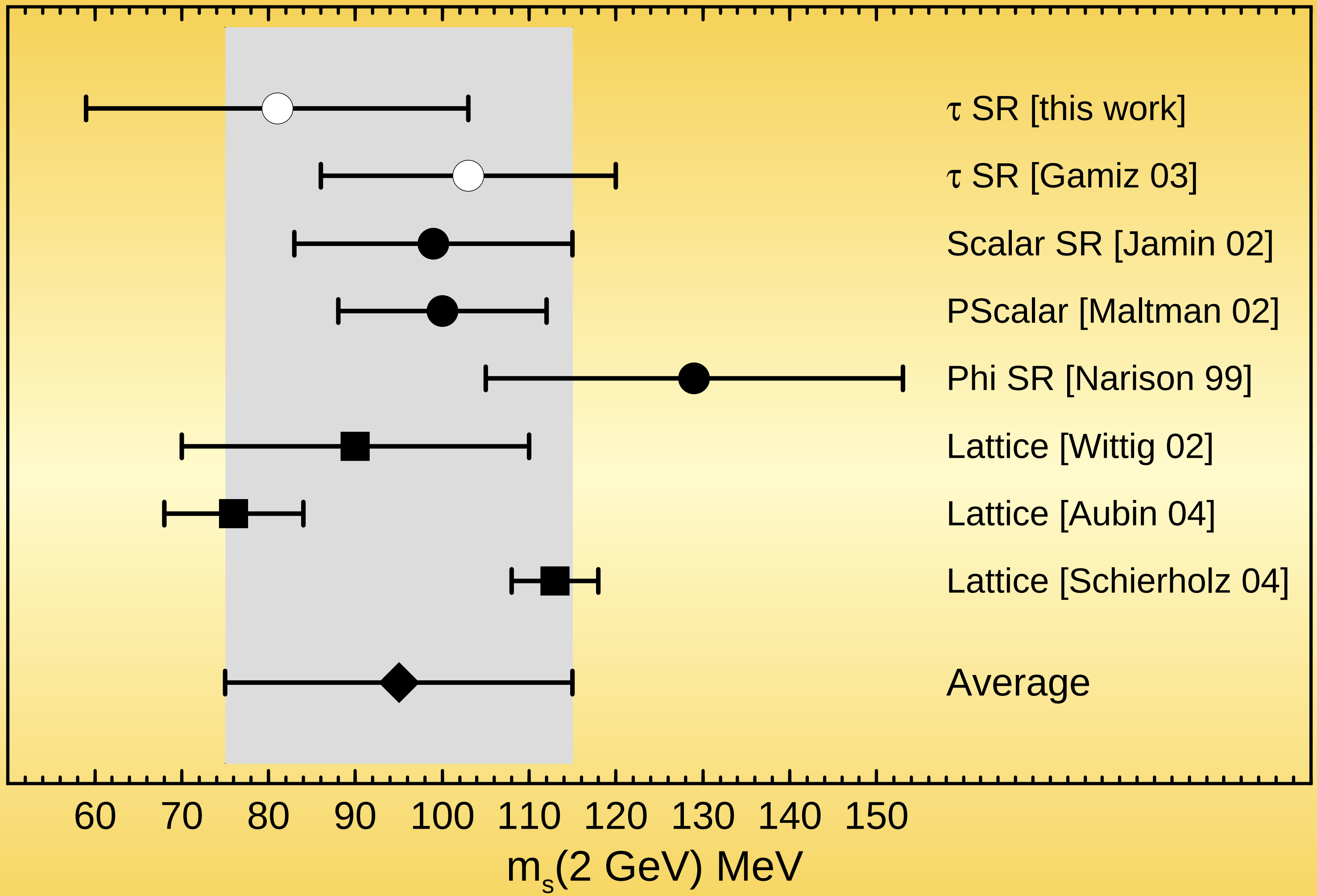


Strange quark mass



$\Rightarrow$ Average: $m_s(2\,\text{GeV}) = 95 \pm 20$ MeV

Given m_s, we are in a position to predict δR_τ^{kl} from theory. Theoretically, the uncertainty is smallest for the $(0,0)$ moment:

$$\delta R_{\tau,th} = 0.162 + 6.12\, m_s^2 - 7.78\, m_s^4 = 0.218 \pm 0.026\,.$$

Let us now reconsider the equation for δR_τ:

$$|V_{us}| = \sqrt{\frac{R_{\tau,S}}{\underbrace{R_{\tau,NS}/|V_{ud}|^2}_{\approx 3.658} - \delta R_{\tau,th}}}$$

Thus the theoretically derived quantity $\delta R_{\tau,th}$ only gives a small correction to experimentally measured quantities.

V_{us} from R_τ

Together with the experimental results $R_{\tau,NS} = 3.469 \pm 0.014$ as well as $R_{\tau,S} = 0.1677 \pm 0.0050$, V_{us} can be determined:

$$|V_{us}| = 0.2208 \pm 0.0033_{\rm exp} \pm 0.0009_{\rm th} = \boxed{0.2208 \pm 0.0034}$$

The uncertainty on V_{us} is dominated by the experimental error on $R_{\tau,S}$. The theoretical error by our knowledge of m_s.

In the near future, it should be possible to reduce the uncertainty with the τ-data sets from BABAR and BELLE.

If the experimental value $B(\tau \to K \nu_\tau) = (0.686 \pm 0.023)$% is replaced by the theoretical prediction (0.715 ± 0.004)% based on $K_{\mu 2}$ decays, one finds $|V_{us}| = 0.2219 \pm 0.0034$.

Composition of experimental and theoretical uncertainties:

Parameter	Value	(2,0)	(3,0)	(4,0)
$m_s(M_\tau)$		93.2	86.3	79.2
$R^{kl}_{\tau,NS}$	OPAL	$+5.1$ -5.4	$+3.6$ -3.7	$+2.8$ -2.9
$R^{kl}_{\tau,S}$	OPAL	-30.9 $+23.3$	-19.5 $+15.8$	-13.9 $+11.6$
V_{us}	0.2208 ± 0.0034	$+21.7$ -29.8	$+14.6$ -18.7	$+10.6$ -13.0
$\mathcal{O}(\alpha_s^3)$	$2\times\mathcal{O}(\alpha_s^3)$ no $\mathcal{O}(\alpha_s^3)$	-4.0 $+4.6$	-5.3 $+6.5$	-6.1 $+7.8$
ξ	1.5 0.75	$+2.7$ $+2.3$	$+4.7$ -0.2	$+6.3$ -2.2
$\alpha_s(M_\tau)$	0.334 ± 0.022	$+0.7$ $+0.1$	-0.7 $+1.3$	-1.6 $+2.2$
$\langle\bar{s}s\rangle/\langle\bar{u}u\rangle$	0.8 ± 0.2	-8.7 $+7.7$	-9.9 $+8.9$	-10.6 $+9.6$
f_K	113 ± 2 MeV	-1.8 $+1.7$	-1.4 $+1.4$	-1.2 $+1.1$
Total		$+33.6$ -44.3	$+25.0$ -29.5	$+21.3$ -23.0

$\Rightarrow$ Average: $m_s(M_\tau) = 84 \pm 23$ MeV

Strange mass from R_τ

Taking a weighted average of the strange mass extractions, for the $(2{,}0)$ to $(4{,}0)$ moments, we obtain

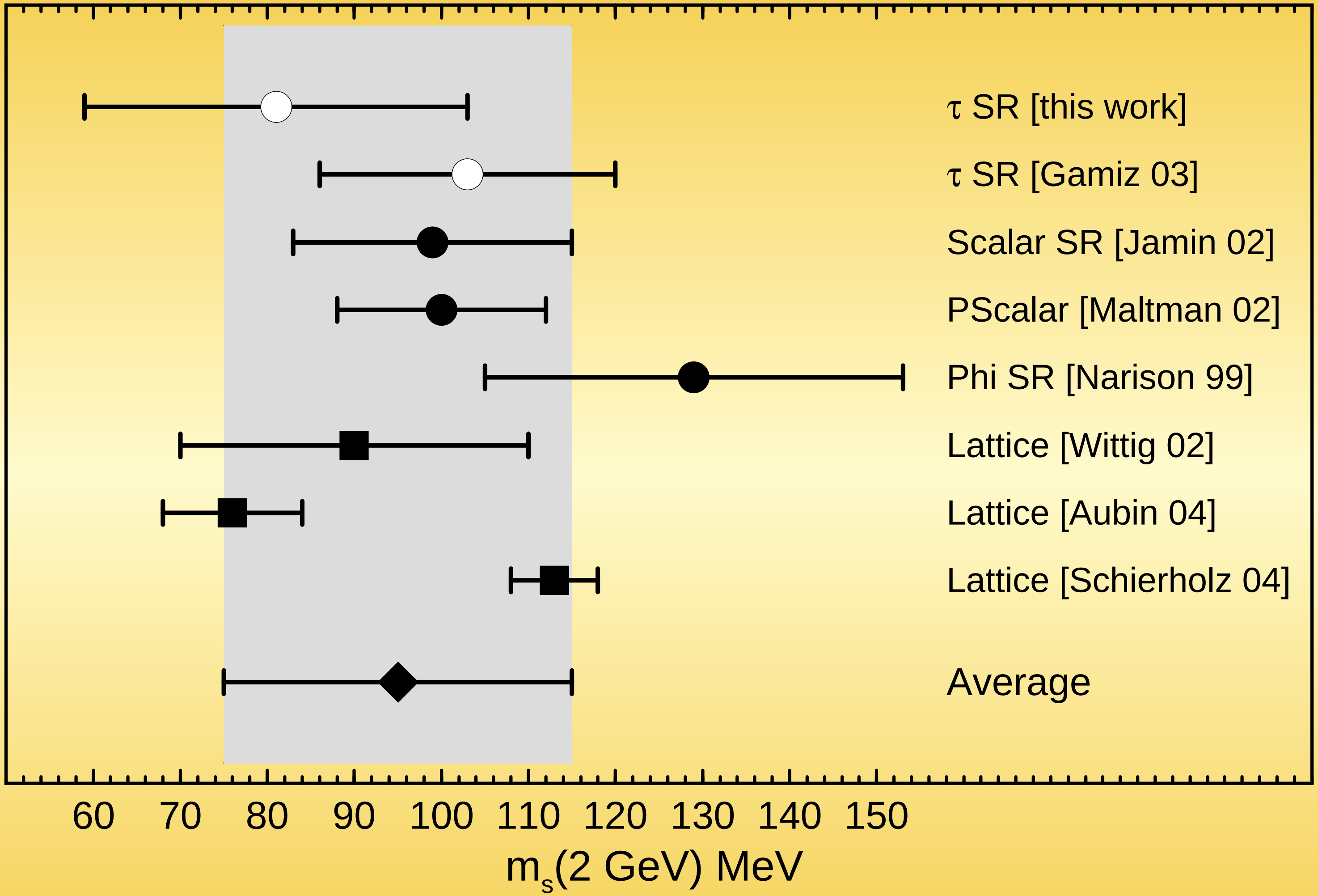
$$m_s(M_\tau) = 84 \pm 23 \text{ MeV} \quad \Rightarrow \quad \boxed{m_s(2\,\text{GeV}) = 81 \pm 22 \text{ MeV}}$$

The strong k-dependence, of m_s present in the analysis of the ALEPH data, is much reduced for the OPAL spectral function.

This is mainly due to a larger contribution in the higher energy range of the spectrum, namly the $\tau^- \to K^- \pi^+ \pi^- \nu_\tau$ mode.

Performing a simultaneous fit for V_{us} and m_s to the first five $(k{,}0)$ moments, we find: $|V_{us}| = 0.2196$ and $m_s = 76$ MeV.

Strange quark mass



$\Rightarrow$ Average: $m_s(2\,\text{GeV}) = 95 \pm 20$ MeV

Conclusions

The uncertainties in the V_{us}-determination from K_{e3}-decays is dominated by the theoretical error on $f_+(0)$.

$$\Rightarrow \qquad |V_{us}| = 0.2221 \pm 0.0027$$

It is also possible to determine the CKM-element V_{us} from the SU(3)-breaking difference of the hadronic τ-decay rate.

$$\Rightarrow \qquad \boxed{|V_{us}| = 0.2208 \pm 0.0034}$$

This result is dominated by experimental uncertainties and will be improvable in the near future by BABAR and BELLE.

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Thank You for Your attention !