DUALITY, etc, OPE

INFORMAL COMMENTS

$$i\int d^4x\, e^{iq\cdot x}\langle 0|T\, V_3^\mu(x) V_3^\nu(0) - A_3^\mu(x) A_3^\nu(0)\,|0\rangle$$

$$= (q^\mu q^\nu - q^2 g^{\mu\nu})\,\Pi_{V-A}(q^2) - q^\mu q^\nu\, \Pi^{(0)}_{A,3}(q)$$

$$\Pi_{V-A}(Q^2) = \sum_n \frac{a_n}{Q^n} \qquad (n = \cdots, 6, 8, 10\cdots) \qquad Q^2 = -q^2$$

- $a_6 \sim \langle\pi\pi|\mathcal{O}_{EWP}|K\rangle$ NOT THE FOCUS HERE

→ LOOK AT $a_8, a_{10}\cdots$ AND DUALITY VIOLATION

JOHN DONOGHUE
BENASQUE
2004

EXTRACTION OF OPE COEFFICIENTS

LOTS OF WORK ON a_s, PLUS

DAVIER, GIRLANDA, HOCKER, STERN
IOFFE, ZYABLYUK
BIJNENS, GAMIZ, PRADES
KNECHT, PERIS, de RAFAEL
CIRIGLIANO, DONOGHUE, GOLOWICH, MALTMAN
CIRIGLIANO, GOLOWICH, MALTMAN
DOMINGUEZ, SCHILCHER
ROJO, LATORRE
ZYABLYUK
CIULLI, SEBU, SCHILCHER, SPIESBERGER

SAMPLES OF EXTRACTED a_d

FROM ROJO, LATORRE

	Reference	$\langle \mathcal{O}_6 \rangle \times 10^3 \mathrm{GeV}^6$	$\langle \mathcal{O}_8 \rangle \times 10^3 \mathrm{GeV}^8$
DGHS	Ref. [23]	-6.4 ± 1.6	8.7 ± 2.4
IZ	Ref. [24]	-6.8 ± 2.1	7 ± 4
BCP	Ref. [26]	-3.2 ± 2.0	-12.4 ± 9.0
KPR	Ref. [27]	-9.5 ± 3	16.2 ± 5
CGM	Ref. [29]	-4.45 ± 0.7	-6.2 ± 3.2
DS	Ref. [30]	-4 ± 1	-1.2 ± 6
RL	This work	-4 ± 2	$-12\ ^{+7}_{-11}$

REASONABLE AGREEMENT

NOTE SIGN ISSUE

FROM CGM

$$a_6 = -(4.54 \pm 0.83 \pm 0.18) \times 10^{-3}\ \mathrm{GeV}^6$$
$$a_8 = -(5.70 \pm 3.72 \pm 0.64) \times 10^{-3}\ \mathrm{GeV}^8$$
$$a_{10} = (4.82 \pm 1.02 \pm 0.20) \times 10^{-2}\ \mathrm{GeV}^{10}$$
$$a_{12} = -(1.60 \pm 0.26 \pm 0.05) \times 10^{-1}\ \mathrm{GeV}^{12}$$
$$a_{14} = (4.26 \pm 0.62 \pm 0.14) \times 10^{-1}\ \mathrm{GeV}^{14}$$
$$a_{16} = -(1.03 \pm 0.14 \pm 0.03)\ \mathrm{GeV}^{16}\ .$$

"DUALITY VIOLATION" IS A MISNOMER

- IF WE COULD CALCULATE BETTER WITH QUARKS GET RIGHT ANSWER

DON'T REALLY HAVE A THEORY OF DUALITY VIOLATION

- WHAT IS JUST WISHFUL THINKING ?
- LOOK TO DATA AND MODELS
- GET AS CLOSE TO DATA AS POSSIBLE

CLEAR THAT DUALITY VIOLATION IS NOT A LEADING EFFECT

- BUT WE ARE NO LONGER WORRIED ABOUT LEADING
- AT WHAT SUB-LEADING ORDER IS IT RELEVANT ?

OUR FAVORITE LABORATORY

$$\Pi_{V-A} \sim i\int d^4x\, e^{iq\cdot x} \langle 0 | T(V^\mu(x) V^\nu(0) - A^\mu(x) A^\nu(0)) | 0 \rangle$$

$$= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_{V-A}(q^2) - q^\mu q^\nu \Pi_A^{(0)}(q^2)$$

$$\Pi_{V-A}(Q^2) = -\frac{F_\pi^2}{Q^2} + \int_{4m_\pi^2}^{\infty} ds \frac{1}{s+Q^2} \rho_{V-A}(s)$$

$$= \frac{1}{Q^4} \int_0^\infty ds \frac{s^2}{s+Q^2} \rho_{V-A}(s) \qquad \text{(CHIRAL LIMIT)}$$

$$= \sum_d \frac{1}{Q^d} \left[a_d(\mu) + b_d(\mu) \ln \frac{Q^2}{\mu^2} \right] \qquad d = 4, 6, 8.$$

$Q^2 > 0$ EUCLIDEAN

$q^2 = -Q^2 = s > 0$ MINKOWSKI

CHIRAL SUM RULES

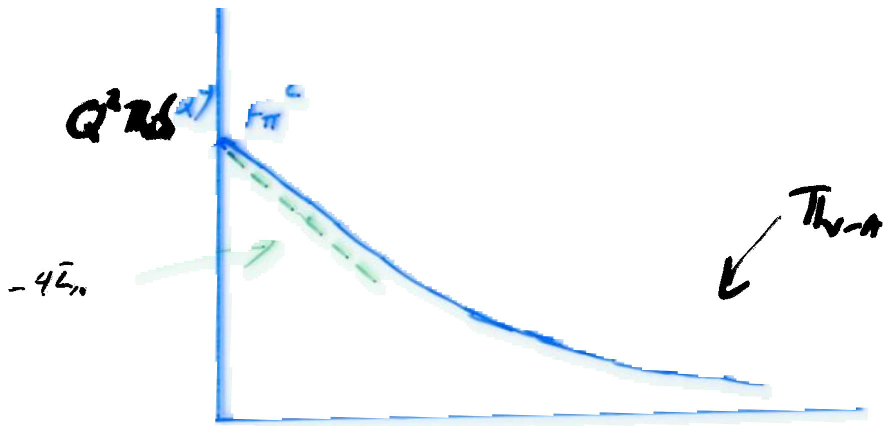
$$-4\bar{L}_{10} = \int_{4m_\pi^2}^{\infty} ds\, \frac{1}{s}\, \rho_{V-A}(s)$$

$$F_\pi^2 = \int_{4m_\pi^2}^{\infty} ds\, \rho_{V-A}(s)$$

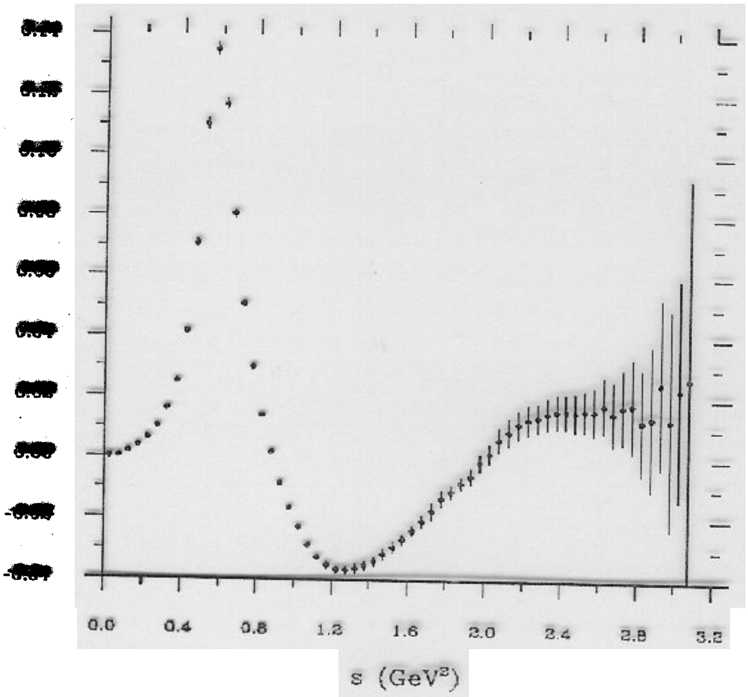
$$\text{"}0\text{"} = \int_0^{\infty} ds\, s\, \rho_{V-A}(s)$$ CHIRAL LIMIT

$$-\frac{16\pi^2 F_\pi^2}{3e^2}\, \text{"}(m_{\pi^\pm}^2 - m_{\pi^0}^2)\text{"} = \int_0^{\infty} ds\, s \ln s\, \rho_{V-A}(s)$$ CHIRAL LIMIT

GENERAL STRUCTURE



$\Pi_{V-A}(Q^2) \sim \frac{1}{Q^6}$ IN CHIRAL LIMIT s)



?

THE O. P. E.

d = 2 $a_2, b_2 \propto \mathcal{O}(m_{u,d}^2)$ NEGLECT

d = 4 $$\left[\Pi_{V-A}(Q^2)\right]_{d=4} = \left(\frac{8}{3}\frac{\alpha_s(Q^2)}{\pi} + \frac{59}{3}\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\right)\left(-\frac{F_\pi^2 m_\pi^2}{Q^4}\right)$$ UNIMPORTANT

d = 6 IN NDR

$$a_6(\mu) = 2\left[2\pi\langle\alpha_s O_8\rangle_\mu + \frac{25}{4}\langle\alpha_s^2 O_8\rangle_\mu + 2\langle\alpha_s^2 O_1\rangle_\mu\right]$$

$$b_6(\mu) = 2\left[-1\langle\alpha_s O_8\rangle_\mu + \frac{8}{3}\langle\alpha_s^2 O_1\rangle_\mu\right]$$

$$\langle O_1\rangle = \langle 0|\bar q\gamma_\mu\frac{\tau_3}{2}q\,\bar q\gamma^\mu\frac{\tau_3}{2}q - (V\to A)|0\rangle$$

$$\langle O_8\rangle = \langle 0|\bar q\gamma_\mu\lambda^A\frac{\tau_3}{2}q\,\bar q\gamma^\mu\lambda^A\frac{\tau_3}{2}q - (V\to A)|0\rangle$$

Related to Q_7, Q_8 in weak OPE

NAIVE DUALITY

1) $\rho_V(s) = \rho_A(s) + O(m_{q,d}^2)$ IN PERTURBATIVE REGION

$$\Longrightarrow \rho_{V-A}(s) = \text{“}0\text{”}$$

2) IF WE CONTINUE LOG

$$\Pi(Q^2) = \cdots + \frac{1}{Q^6}\left[a_6 + b_6 \ln Q^2/\mu^2\right]$$

FOR $s = -Q^2$, $\ln(-Q^2) \to \ln s + i\pi$

$$\Longrightarrow \rho(s) \sim \frac{b_6 \pi}{s^3}$$

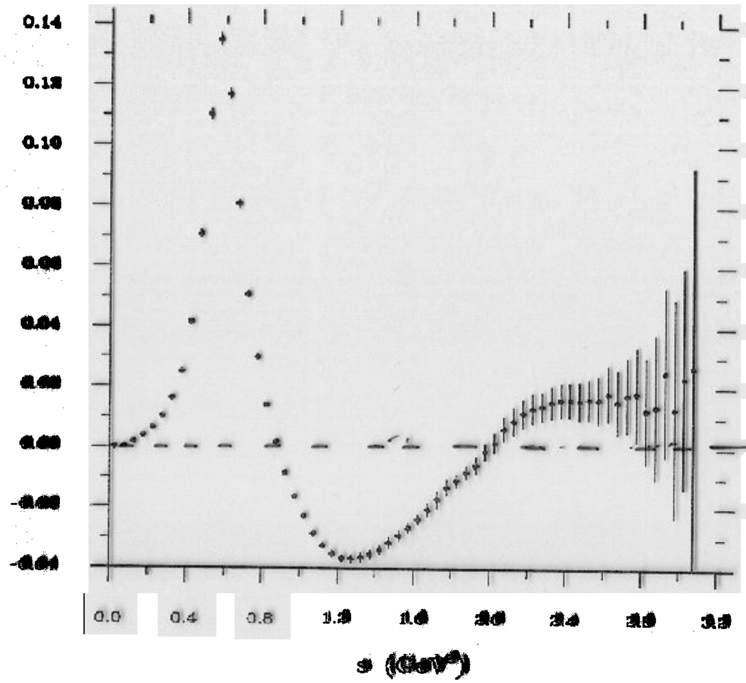
BUT NUMERICALLY SMALL

$$b_6 = -2\langle \alpha_s O_8 \rangle_\mu + \cdots$$

INCLUDED IN SOME OF FOLLOWING WORK

$\Longrightarrow$ NAIVE DUALITY $\rho_{V-A}(s) \sim 0$ AT LARGE

NAIVE DUALITY



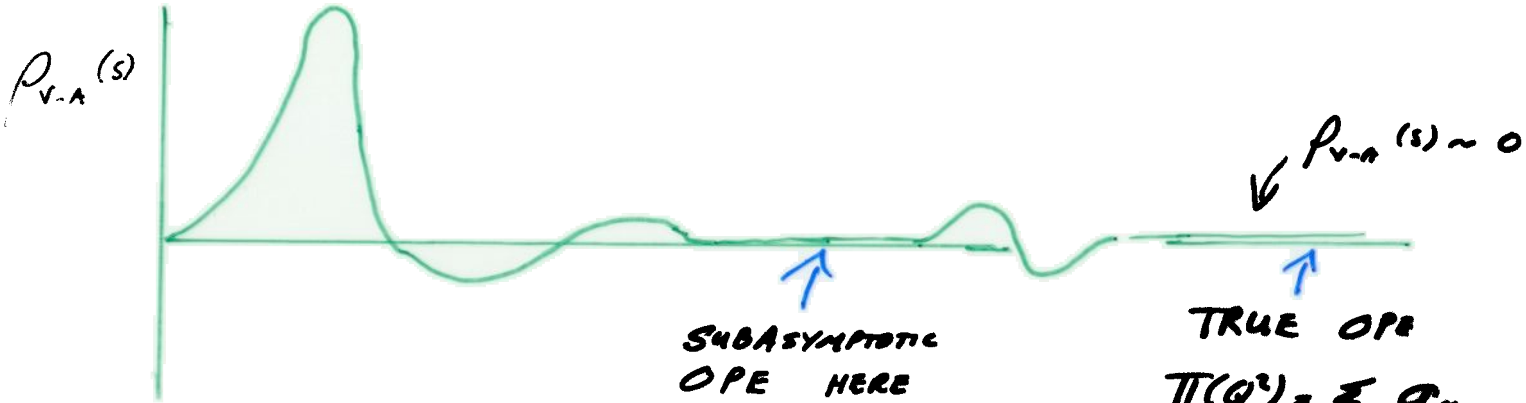
$\rho = 0$

$a_\rho = \frac{b_0 \pi}{s^3}$

Two Types of "Duality Violation"

1) **Subasymptotic**

– $\rho_{V-A}(s) \sim 0$ for $s > s_0$, but we work at $s < s_0$



SUBASYMPTOTIC OPE HERE

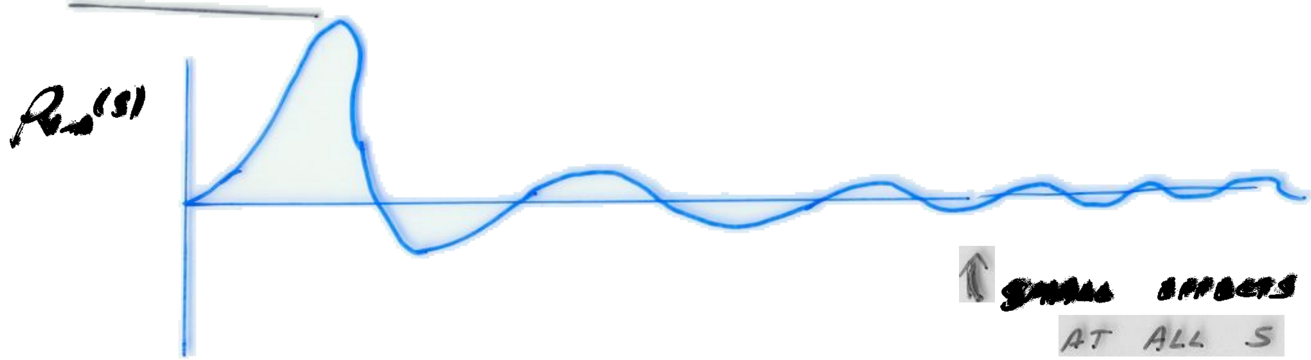
TRUE OPE

$$\Pi(Q^2) = \sum_n \frac{a_n}{Q^n}$$

$$\Pi(Q^2) = \sum \frac{C_n}{Q^n}$$

But $a_n \neq C_n$

2) "Intrinsic"



SMALL EFFECTS AT ALL s

RELATED BUT NOT THE SAME

SUBASYMPTOTIC OPE

- IMAGINE GAP IN DATA $\rho \sim 0$ FROM $s_1 \cdots s_2$
- PUT $s_1 < Q^2 < s_2$

$$\Pi(Q^2) = \int^{s_1} ds \frac{\rho(s)}{s+Q^2} + \int_{s_2}^{\infty} ds \frac{\rho(s)}{s+Q^2} - F_\pi^2/Q^2$$

$$= \int^{s_1} ds\, \rho(s) \left[\frac{1}{Q^2} - \frac{s}{Q^4} + \frac{s^2}{Q^6} + \frac{s^3}{Q^8} + \cdots \right]$$

NO OPE IN GENERAL VALID FOR $Q^2 < s_2$

ALWAYS GET OPE

NOTE WEIGHTING FOR HIGH d α_d

BUT NOT TRUE OPE

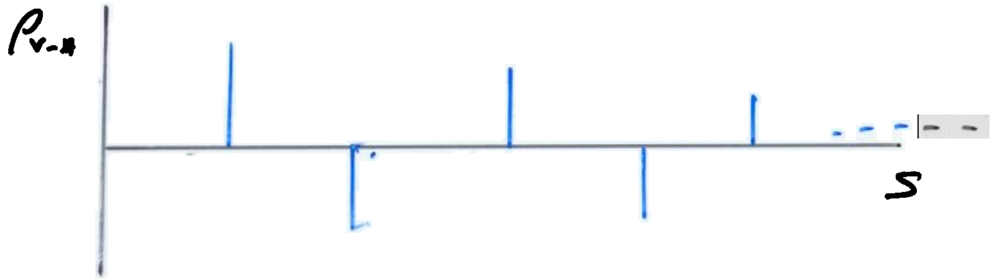
OPE IS AUTOMATIC ONCE $\rho(s) \approx 0$

" " INTERESTING WHEN $\rho(s > |Q^2|) \neq 0$

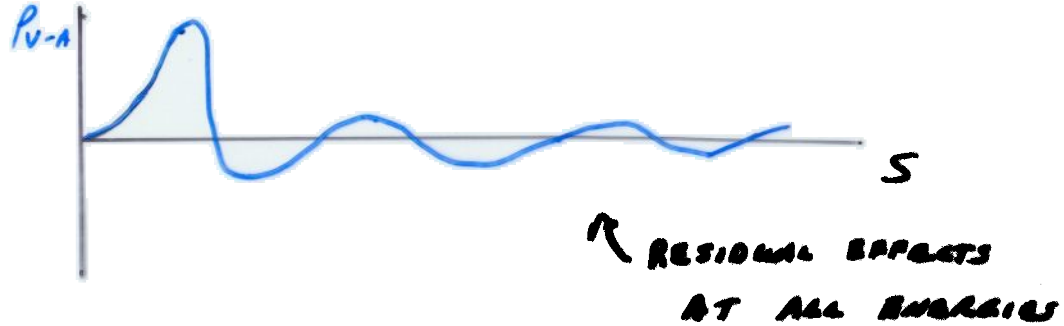
GOLTERMAN
PERIS

"INTRINSIC" IS MORE INTERESTING

LARGE N_c



REAL QCD



KEY FEATURE

$$\rho(s) \sim \frac{1}{s^n} \sin a(s-s_0) \Longrightarrow \Pi(Q^2) \sim \frac{1}{Q^{2n}} e^{-aQ^2}$$

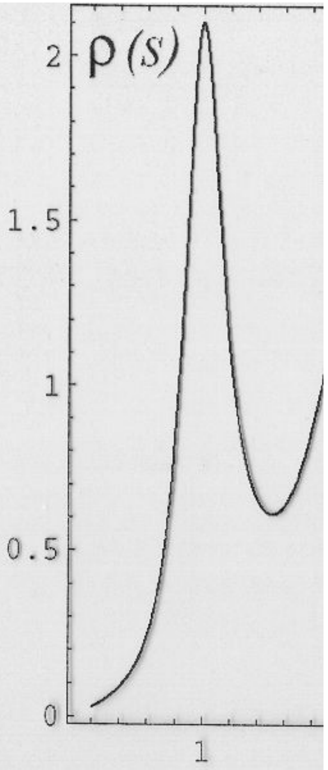
$$\sim \frac{e^{-bs}}{s^n} \sin a(s-s_0) \Longrightarrow \Pi(Q^2) \sim \frac{e^{-aQ^2}}{Q^{2n}} + \text{POWERS IN OPE}$$

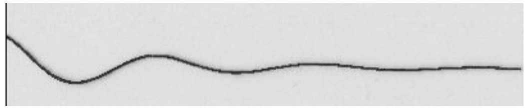
$\Longrightarrow$ NOT PART OF OPE

$$\frac{\cdots}{q^2 - M_n^2 + i\Gamma_n M_n}.$$

$$M_n^2 = M_0^2 + \sigma^2 n$$

$$\Gamma_n = \frac{B}{N_c} M_n$$





$$1 + \text{power corr.} + 2\exp\left(-\frac{2\pi s B}{\sigma^2 N}\right)\cos\left(\frac{2\pi s}{\sigma^2}\right)$$

OVERALL:

$$\Pi_{V-A}(Q^2) = \sum_d \frac{a_d + b_d \ln Q^2/\mu^2}{Q^d} + \sum_n \frac{c_n e^{-a_n Q^2}}{Q^n} + \sum_n \frac{d_n e^{-b_n Q}}{Q^m} + \cdots$$

ENERGY DEPENDENT

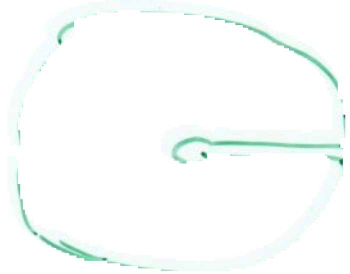
"DUALITY VIOLATION"

$$\rho(s) = \text{"}0\text{"} + \sum e^{-as} \sin bs \ \ldots\ldots$$

FESR EXTRACTION

IN GENERAL

$$\int_{s_{th}}^{s_0} ds\, \rho(s)\, W(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} ds\, \Pi(s)\, W(s)$$



↑ ANALYTIC WEIGHT FUNCTION

IF WE USE OPE

$$\Pi(Q^2) = \sum_d \frac{a_d}{Q^d} \qquad (\text{neglect } b_d)$$

AND

$$-\frac{1}{2\pi i} \oint_{|s|=s_0} ds\, \frac{a_d}{Q^d} \left(\frac{s}{s_0}\right)^k = (-1)^k\, \delta_{k,\, d-1}\, \frac{a_d}{s_0^k}$$

↑ s^d

THEN FOR POLYNOMIAL $W(s)$, WE CAN OBTAIN

$$\int_{s_{th}}^{s_0} ds\, \rho(s)\, \frac{s}{s_0}\left[1 + \alpha_1 \frac{s}{s_0} + \alpha_2 \left(\frac{s}{s_0}\right)^2 + \ldots\right] = \text{"}0\text{"} + \frac{\alpha_1 a_4}{s_0^2} - \frac{\alpha_2 a_8}{s_0^3} + \ldots$$

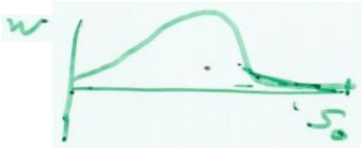
A STRATEGY

CGM

1) CHOOSE DOUBLY PINCHED WEIGHTS

$$W(s=s_0)=0$$

$$W'(s=s_0)=0$$



$\Rightarrow$ STRONGLY SUPPRESSED WHERE DATA IS WEAK

$$W_N = \frac{s}{s_0}\left[1-\left(\frac{N}{N-1}\right)\frac{s}{s_0}+\frac{1}{N-1}\left(\frac{s}{s_0}\right)^N\right]$$

2) CALCULATE

$$\int_{s_{th}}^{s_0} ds\, \rho(s)\, W_N(s) = f_N(s_0)$$

FOR RANGE OF s_0 $(\sim 2 \to 3\ \mathrm{GeV}^2)$

Ex $f_{W_1} = \frac{7a_6}{s_0^2} + \frac{3a_8}{s_0^3}$

3) FIT FOR a_d

MODELLING

VARIOUS COMPLETIONS OF SPECTRAL FUNCTIONS

- USE DATA WHERE IT IS GOOD
- ADD HIGH ENERGY COMPONENT

$$\frac{(e^{-as})\,\sin b(s-s_0)}{s^{3,4}} \quad \text{or} \quad \sin b(\sqrt{s}-\sqrt{s_0})$$

- MATCH TO DATA
- APPLY CHIRAL SUM RULES (WITHIN ERRORS)

$\Longrightarrow$ KNOW THE TRUE OPE FOR THESE FUNCTIONS

MATH QUESTION:

- CAN FESR METHOD (OR OTHERS) UNCOVER THE TRUE OPE COEFF. ?

PHYSICS QUESTION:

- IS DUALITY VIOLATION LIKELY LARGE ENOUGH TO UPSET EXTRACTION OF α_s ?

NMHA EXERCISE

— APPLY WEINBERG SUM RULES

2 POLES

$$Q^2\Pi(Q^2) = \frac{-F_\pi^2 m_V^2 m_A^2}{(Q^2+m_V^2)(Q^2+m_A^2)}$$

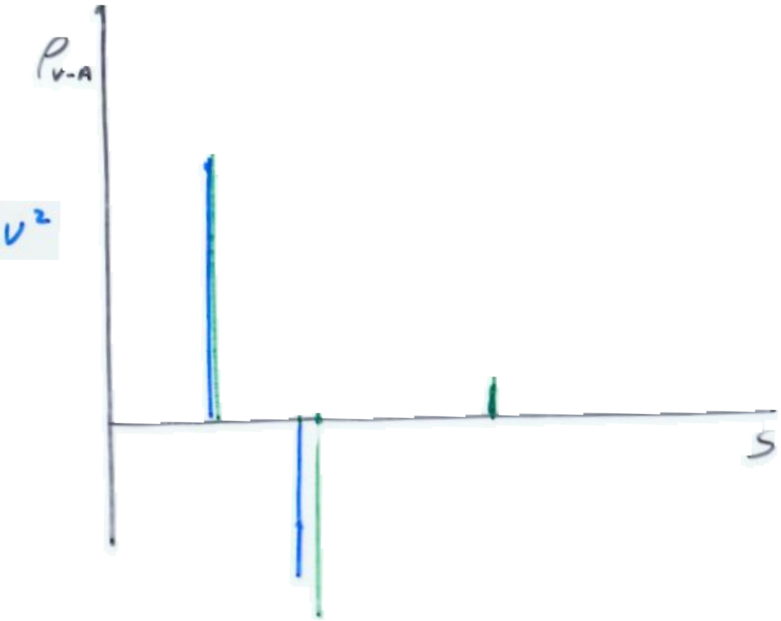
$F_A^2 = 0.62 F_V^2$

$m_A^2 = 0.953 \text{ GeV}^2$

$a_6 = F_V^2 m_V^4 - F_A^2 m_A^4 = -F_\pi^2 m_V^2 m_A^2$

$a_8 = F_V^2 m_V^6 - F_A^2 m_A^6$

$\frac{a_8}{a_6} = -1.54 \text{ GeV}^2$



3 POLES

$$Q^2\Pi(Q^2) = -\frac{|a_6|Q^2 + F_\pi^2 m_V^2 m_A^2 m_{V'}^2}{(Q^2+m_V^2)(Q^2+m_A^2)(Q^2+m_{V'}^2)}$$

$a_6 = F_V^2 m_V^4 - F_A^2 m_A^4 + F_{V'}^2 m_{V'}^4$

$a_8 = F_V^2 m_V^6 - F_A^2 m_A^6 + F_{V'}^2 m_{V'}^6$

CHOOSE $F_{V'}^2 = \frac{1}{10} F_V^2$; $m_{V'}^2 = 2 \text{ GeV}^2$

THEN $F_A^2 = 0.72 F_V^2$; $m_A^2 = 1.1 \text{ GeV}^2$

$\frac{a_8}{a_6} = +0.5 \text{ GeV}^2$

SENSITIVE

OPE WORKS AFTER $Q^2 \approx 2 \text{GeV}^2$

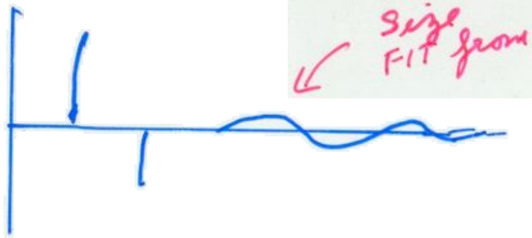
I'M NOT ADVOCATING THIS FOR PHENOMENOLOGY

- IF V', THEN SHOULD ADD A'
- GO TO OTHER PARAMETRIZATIONS – ALTERNATING

FOR a_d DIRECTLY AT s_0

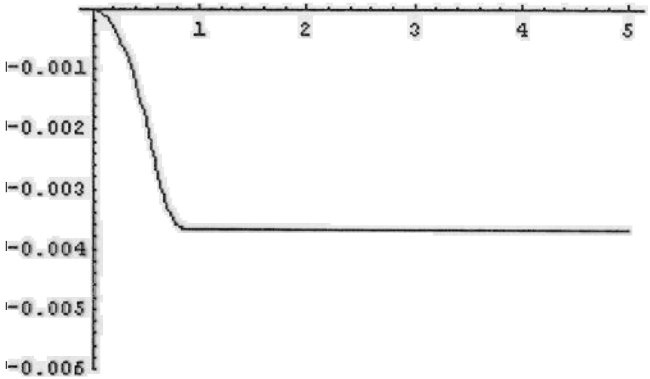
– POSSIBLE IF YOU KNOW F_π^2, a_6, ..

ρ_{V-A}



size FIT from data

Plain VMD

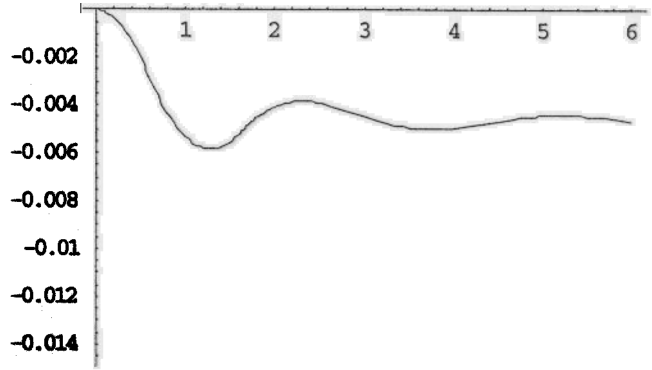


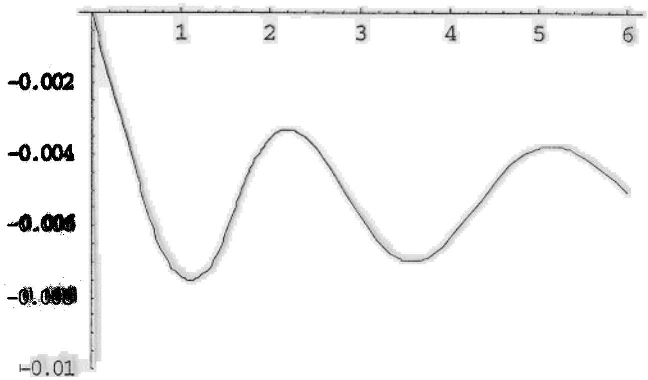
a_6

VMD plus:

average to $\sim 1/s^3$

rhoc = Sin[a (s-sc)] / s^2

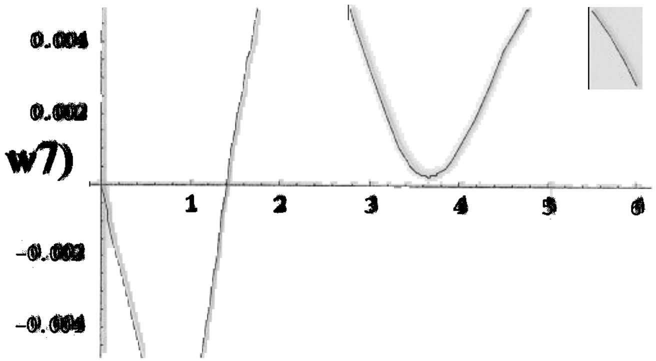




a_8

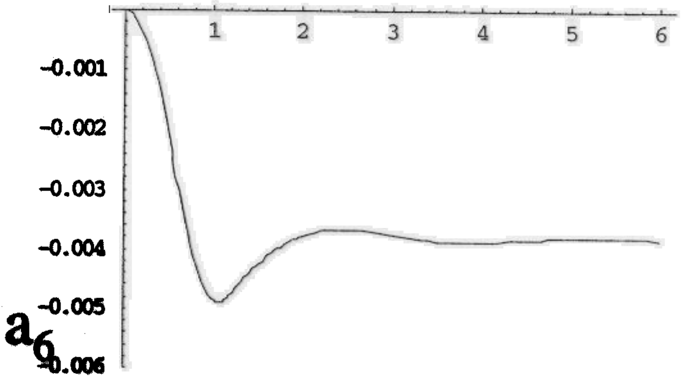
a_{10}

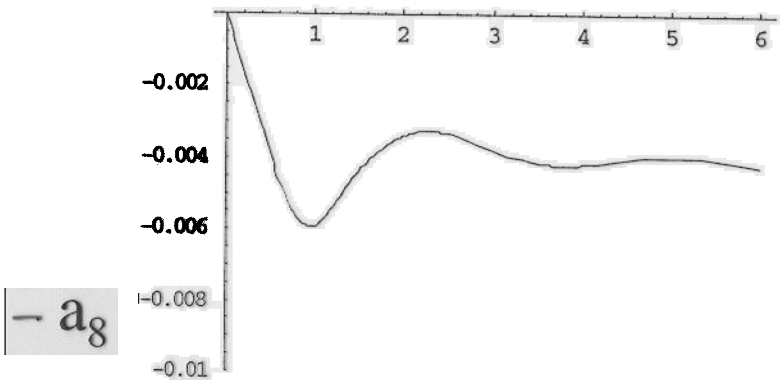
(using w7)



More rapidly fall-off

$\frac{\sin a(s-s_0)}{s^3}$





FESR FITTING SCHEME
– DESCRIBED / ILLUSTRATED LATER WITH BETTER MODEL

s^{-2} model

$a_6^{true} = -0.00447$ 80%

$a_6^{fit} = -0.0036$

$a_8^{true} = 0.005$ −20%

$a_8^{fit} = -0.001$

s^{-3} model

$a_6^{true} = -0.0039$

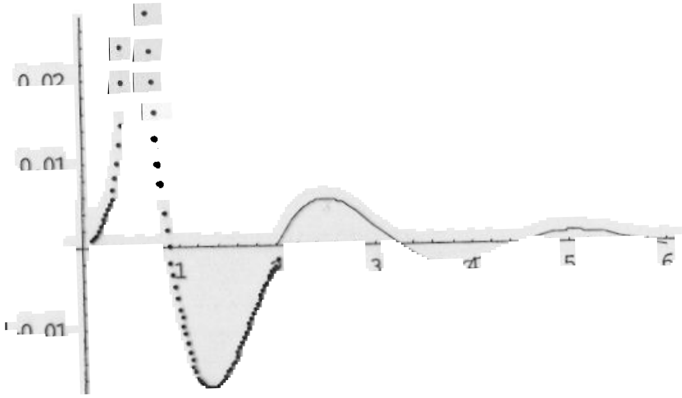
$a_6^{fit} = -0.0030$

$a_8^{true} = 0.0045$

$a_8^{fit} = -0.0009$

A better model – data plus $\frac{\sin(a(s-s_0))}{s^2}$

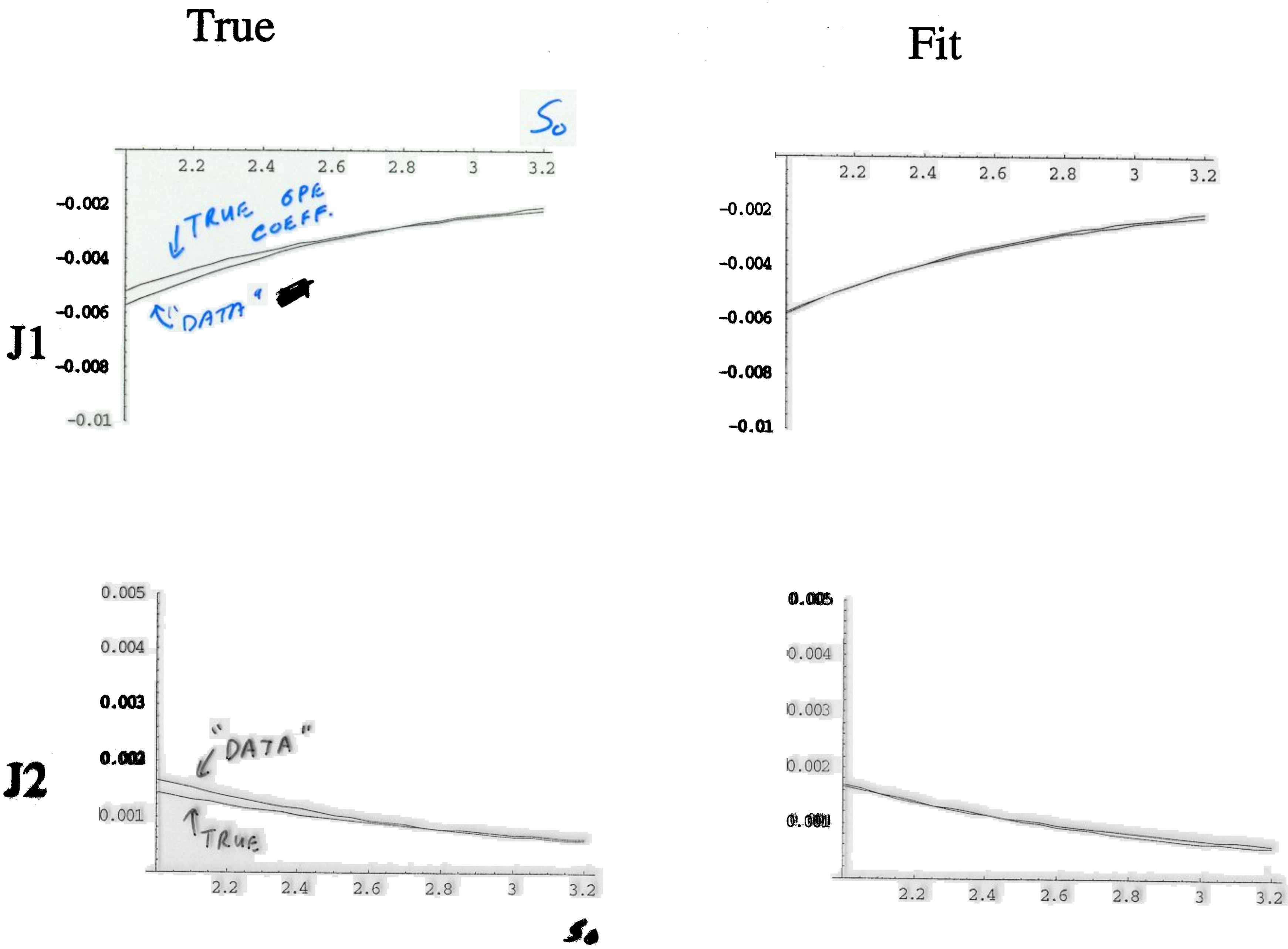
OLDER DATA



$$\frac{0.00850418}{q2} - 1.6075\times 10^{-7}\left(\frac{1}{q2}\right)^2 - 0.00353732\left(\frac{1}{q2}\right)^3 + 0.00264307\left(\frac{1}{q2}\right)^4 + 0.00218915\left(\frac{1}{q2}\right)^5 - 0.00652396\left(\frac{1}{q2}\right)^6 + O\left[\frac{1}{q2}\right]^7$$

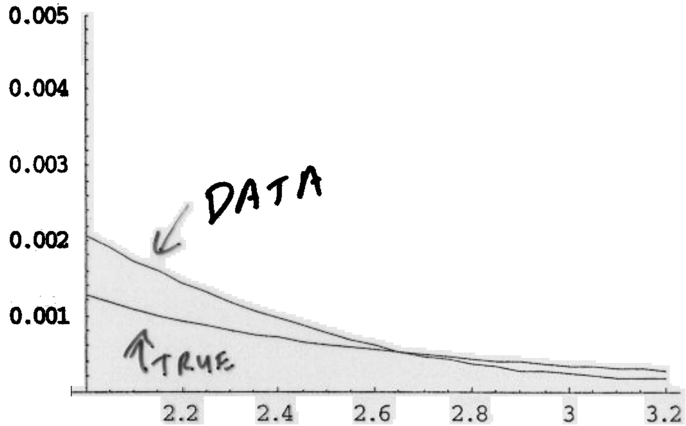
True	Fit	Ratio
a6 = -0.00353732	a_6f =-0.0029	f6=0.82
a8 = 0.00264307	a_8f = -0.002	f8= -0.76
a10 = 0.00218915	a_10f = 0.02	f10=10.0
a12 = -0.00652396	a_12f= -0.076	f12=11.6

Fit a_6, a_8

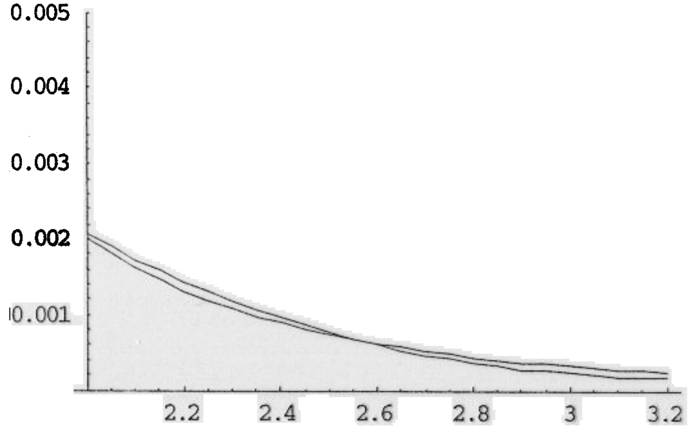


True
Fit
J1
J2
s_0
2.2
2.4
2.6
2.8
3
3.2
-0.002
-0.004
-0.006
-0.008
-0.01
TRUE OPE COEFF.
"DATA"
0.005
0.004
0.003
0.002
0.001
"DATA"
TRUE

J7



FIT



OVERALL, EXCELLENT FIT TO FESR INTEGRALS

— BY ADJUSTING "OPE COEFFICIENTS"

<u>BUT</u>, FIT "OPE COEFFICIENTS" ARE <u>NOT</u> THE TRUE OPE COEFF.

DIFFERENCE ARISES FROM ENERGY DEPENDENCE FROM OSCILLATING TAIL OF

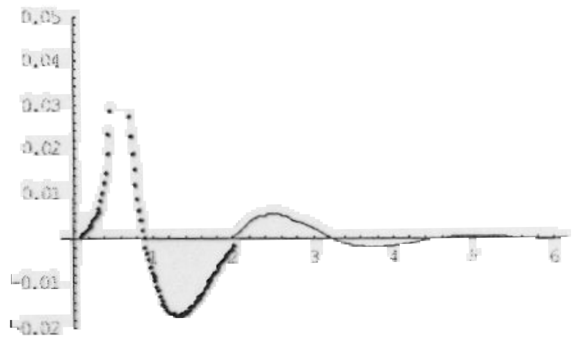
Another model $1/s^3$ tail:

```
a6quart = -0.00381024
a8quart = 0.0035428
a10quart = 0.00167786
a12quart = -0.012112
```

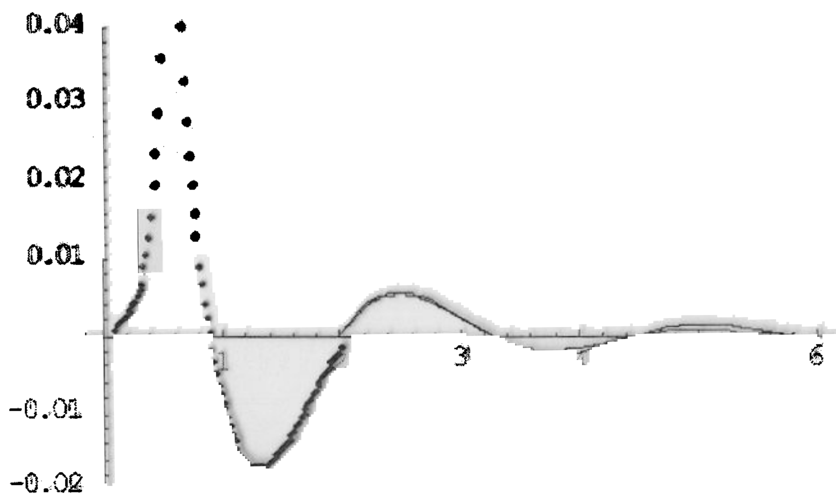
```
fact6q= .78
fact8q= -.13
fact10q = 11
fact12q = 6
```

↑ RATIO OF FIT COEFF TO TRUE COEFF

Exponentially falling duality violation



chosen to resemble other model



```
Sin[ a (t - sc) ] Exp[-a (t - sc) / 3]
```

$$\frac{0.0084761 - 7.26962\times10^{-20}\,i}{q2}$$

$$(2.68643\times10^{-8} - 1.62747\times10^{-19}\,i)\left(\frac{1}{q2}\right)^2$$

$$(0.00345551 + 3.06886\times10^{-18}\,i)\left(\frac{1}{q2}\right)^3 +$$

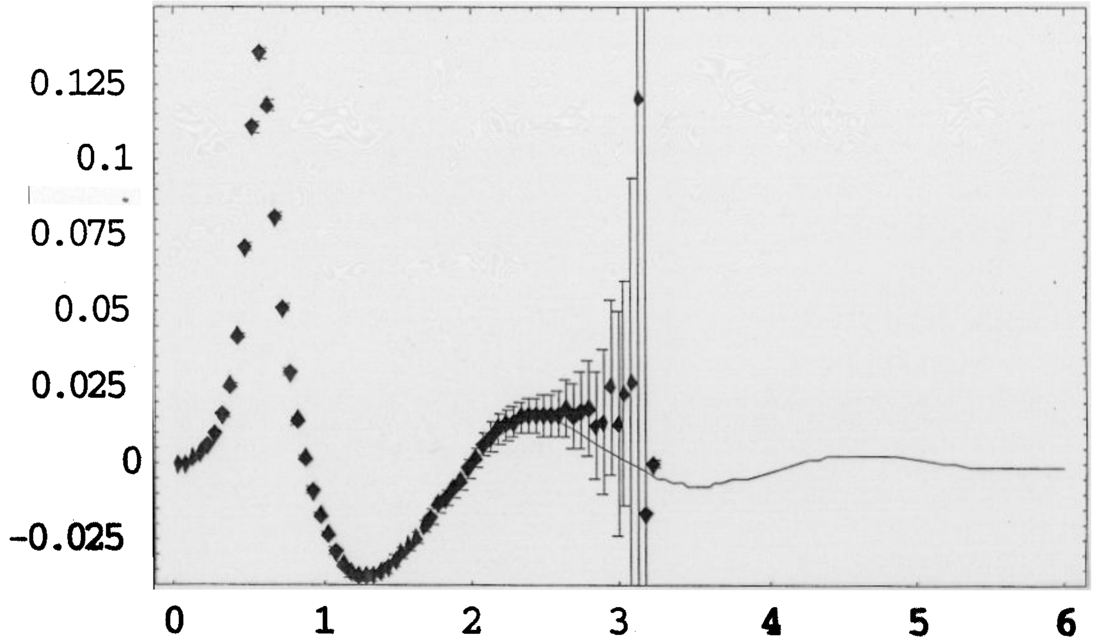
$$(0.00264962 + 6.91512\times10^{-19}\,i)\left(\frac{1}{q2}\right)^4$$

$$(0.000293731 + 4.21332\times10^{-18}\,i)\left(\frac{1}{q2}\right)^5 +$$

$$(0.00557648 + 2.3354\times10^{-18}\,i)\left(\frac{1}{q2}\right)^6 + O\left[\frac{1}{q2}\right]^7$$

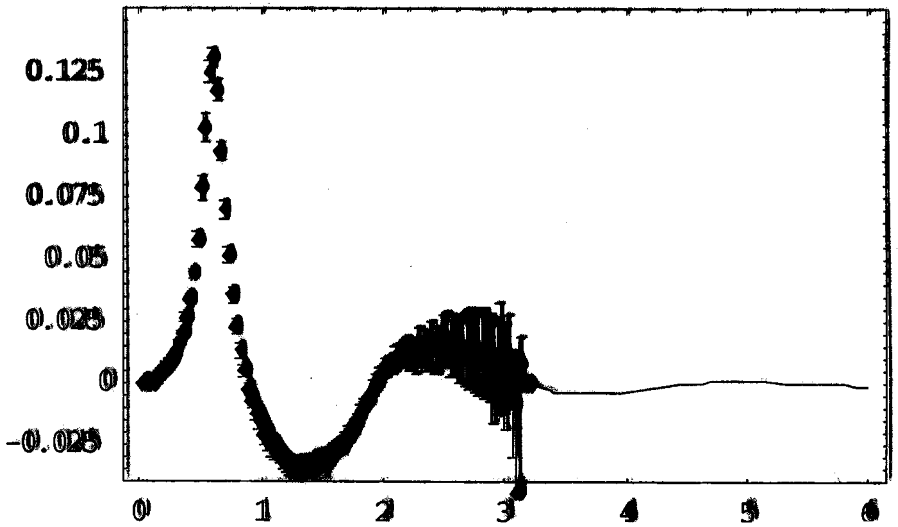
TRUE	FIT
$a_6 = -0.0034$	a_6f = -0.0029
$a_8 = +0.0026$	a_8f = -0.002
$a_{10} = +0.0029$	a_10f = 0.02
$a_{12} = +0.0036$	a_12f = -0.076

Aleph



rexp = Sin[a (t- sc)] Exp[-a (t- sc) / 4]

Opal



NOTE: FACTOR OF 2 DIFF. IN NORMALIZATION CONVENTION HERE, COMPARED TO EARLIER EXAMPLES

$$\frac{0.0170334 + 0.\, i}{q2} + (3.18305 \times 10^{-8} - 1.96626 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^2 -$$

$$- (0.00703691 - 5.14902 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^3 +$$

$$+ (0.00540206 + 8.56672 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^4 + \; -$$

$$+ (0.00337969 + 3.54031 \times 10^{-18}\, i) \left(\frac{1}{q2}\right)^5 - \; -$$

$$- (0.00991835 + 3.01181 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^6 - \; -$$

$$- (0.00908984 - 1.47482 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^7 + \; -$$

$$(0.0642836 + 9.07057 \times 10^{-18}\, i) \left(\frac{1}{q2}\right)^8 -$$

$$(0.0283016 + 2.68287 \times 10^{-17}\, i) \left(\frac{1}{q2}\right)^9 -$$

$$(0.49208 - 6.33585 \times 10^{-17}\, i) \left(\frac{1}{q2}\right)^{10} +$$

$$(1.17086 - 2.22309 \times 10^{-15}\, i) \left(\frac{1}{q2}\right)^{11} +$$

$$(3.57498 - 3.59222 \times 10^{-15}\, i) \left(\frac{1}{q2}\right)^{12} -$$

$$(23.1179 - 1.27702 \times 10^{-14}\, i) \left(\frac{1}{q2}\right)^{13} -$$

$$(14.3633 - 5.67342 \times 10^{-14}\, i) \left(\frac{1}{q2}\right)^{14} +$$

$$(496.194 - 7.19615 \times 10^{-13}\, i) \left(\frac{1}{q2}\right)^{15} -$$

$$(871.28 + 1.0816 \times 10^{-12}\, i) \left(\frac{1}{q2}\right)^{16} + O\left[\frac{1}{q2}\right]^{17}$$

Aleph

$$\frac{0.0168301 - 5.3788 \times 10^{-20}\, i}{q2} -$$

$$(0.0000705715 + 7.10628 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^2 +$$

$$\left((-0.0072375 - 2.16981 \times 10^{-18}\, i) + 0.\, \mathrm{Log}\left[\frac{1}{q2}\right]\right) \left(\frac{1}{q2}\right)^3 +$$

$$+ (0.00766026 - 2.79627 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^4 -$$

$$- (0.00746088 + 3.43296 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^5 +$$

$$+ (0.0305726 + 9.99635 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^6 -$$

$$- (0.12681 + 1.40024 \times 10^{-18}\, i) \left(\frac{1}{q2}\right)^7 +$$

$$(0.299203 + 7.78777 \times 10^{-19}\, i) \left(\frac{1}{q2}\right)^8 -$$

$$(0.158541 - 2.95163 \times 10^{-18}\, i) \left(\frac{1}{q2}\right)^9 -$$

$$(1.44184 + 1.58284 \times 10^{-18}\, i) \left(\frac{1}{q2}\right)^{10} +$$

$$(2.75065 - 5.08815 \times 10^{-15}\, i) \left(\frac{1}{q2}\right)^{11} +$$

$$(21.1208 - 2.15509 \times 10^{-14}\, i) \left(\frac{1}{q2}\right)^{12} -$$

$$(124.433 + 7.27469 \times 10^{-14}\, i) \left(\frac{1}{q2}\right)^{13} -$$

$$(56.9819 - 1.5221 \times 10^{-13}\, i) \left(\frac{1}{q2}\right)^{14} +$$

$$(3350.93 + 2.28372 \times 10^{-12}\, i) \left(\frac{1}{q2}\right)^{15} -$$

$$(10188.9 - 1.59351 \times 10^{-12}\, i) \left(\frac{1}{q2}\right)^{16} + O\left[\frac{1}{q2}\right]^{17}$$

OPAL

COMPARISON FOR OPAL DATA

TRUE

$a_6 = -7.2 \times 10^{-3}$

$a_8 = +7.8 \times 10^{-3}$

$a_{10} = -7.5 \times 10^{-3}$

$a_{12} = +3.1 \times 10^{-2}$

$a_{14} = -1.2 \times 10^{-1}$

$a_{16} = +.30$

FIT

$$
\begin{aligned}
a_6 &= -(5.06 \pm 0.89 \pm 0.12) \times 10^{-3}\ \mathrm{GeV}^6 \\
a_8 &= -(3.12 \pm 3.82 \pm 0.45) \times 10^{-3}\ \mathrm{GeV}^8 \\
a_{10} &= \ \ (3.87 \pm 1.06 \pm 0.10) \times 10^{-2}\ \mathrm{GeV}^{10} \\
a_{12} &= -(1.32 \pm 0.27 \pm 0.03) \times 10^{-1}\ \mathrm{GeV}^{12} \\
a_{14} &= \ \ (3.54 \pm 0.66 \pm 0.06) \times 10^{-1}\ \mathrm{GeV}^{14} \\
a_{16} &= -(0.85 \pm 0.15 \pm 0.02)\ \mathrm{GeV}^{16}\ .
\end{aligned}
$$

<u>ALSO</u> NOTE THAT THE OPE FOR ALEPH AND OPAL "COMPLETIONS" ARE VERY DIFFERENT AT HIGHER d

NO METHOD, USING DATA, COULD UNCOVER THESE a_d

Rojo, Latorre

COMPLETION TO DATA USING NEURAL NETWORKS

"TRUE" COEFF FOR COMPLETED OPE

$$\langle \mathcal{O}_6 \rangle = (-4.0 \ \pm 2.0)\ 10^{-3}\ \mathrm{GeV}^6$$

$$\langle \mathcal{O}_8 \rangle = \left(-12\ {}^{+7}_{-11}\right) 10^{-3}\ \mathrm{GeV}^8$$

$$\langle \mathcal{O}_{10} \rangle = (7.8 \pm 2.4)\ 10^{-2}\ \mathrm{GeV}^{10}\ ,$$

$$\langle \mathcal{O}_{12} \rangle = (-2.6 \pm 0.8)\ 10^{-1}\ \mathrm{GeV}^{12}\ .$$

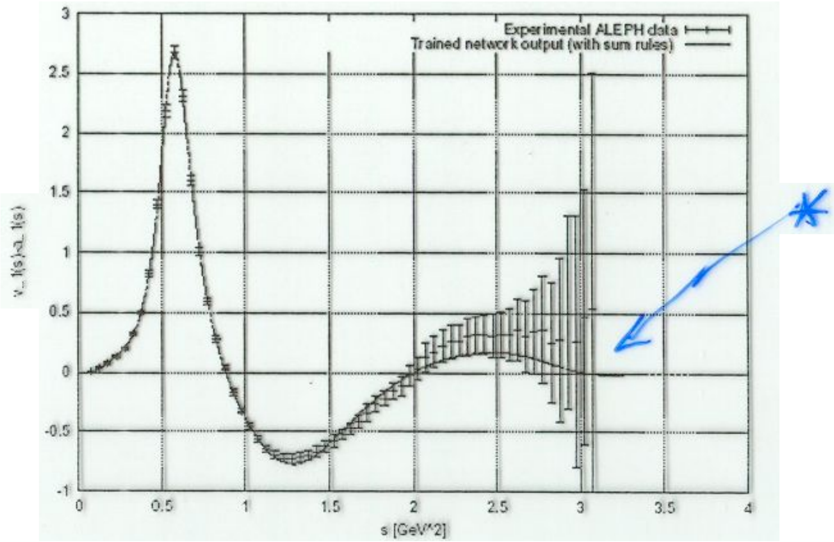
FESR FIT TO DATA

$\langle O_6 \rangle = -5.06 \times 10^{-3}$

$\langle O_8 \rangle = -3.1 \pm 3.5 \times 10^{-3}$

$\langle O_{10} \rangle = +3.9 \pm 1.0 \times 10^{-2}$

$\langle O_{12} \rangle = -1.3 \pm 0.3 \times 10^{-1}$



NOTE THAT ALL "FIT" COEFF. ARE SIMILAR

– CORRECTING FOR FACTOR OF 2 NORMALIZATION

$\frac{\sin}{s^2}$	$\sin/s^3$	"OPAL"	·ALEPH
$a_6 = -.0058$	$a_6 = -.0059$	$a_6 = -.0051$	$a_6 = -.0045$
$a_8 = -.004$	$a_8 = -.0009$	$a_8 = -.0031$	$a_8 = -.0057$
$a_{10} = +.04$	$a_{10} = +0.037$	$a_{10} = +0.039$	$a_{10} = +0.048$
$a_{12} = -.15$	$a_{12} = -0.14$	$a_{12} = -0.13$	$a_{12} = -0.16$

NOTE: IF $\left|\frac{a_{16}}{a_6}\right| \equiv (S_*)^5$

THEN $S_* = 2.9\ GeV^2$

BUT, "TRUE" COEFF. ARE NOT SIMILAR

$\frac{\sin}{s^2}$	$\sin/s^3$	"ALEPH"	"OPAL"	Rojo-Latorre
$a_6 = -0.0070$	-0.0072	-0.0070	-0.0072	-0.004
$a_8 = +$[illegible]	$+0.0070$	$+0.0054$	$+0.0077$	-0.012
$a_{10} =$ [illegible]	$+0.0033$	$+0.0034$	-0.0074	$+0.078$
$a_{12} = -0.013$	-0.024	-0.010	$+0.021$	-0.26

CONCLUSIONS / COMMENTS

IN THESE MODELS, DUALITY VIOLATION IS SUBDOMINANT

D.V. LEADS TO POTENTIAL MODIFICATION OF $\Pi(Q^2)$ OPE

$$\Pi(Q^2) = \sum_d \frac{a_d}{Q^d} + \sum_d \frac{c_d\, e^{-b_d Q}}{Q^d} + \cdots$$

D.V. COMPARABLE TO SUB-LEADING a_d

SEEMS SIGNIFICANT IN EXTRACTION OF a_8

— OVERWHELMING FOR $a_{10}, a_{12} \cdots$

NO METHOD, USING DATA REGION ONLY, IS CAPABLE OF EXTRACTING TRUE a_d OF THESE MODELS (MATH)

IS THERE SOME PHYSICAL REQUIREMENT FOR DUALITY VIOLATION THAT COULD REDUCE VARIATION BETWEEN MODELS?