Scalar Mesons and Chiral Symmetry

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(with E. Eichten and H. Thacker)

Abstract

I explore the nature of scalar mesons in quenched lattice QCD. The lattice realization of chiral symmetry has an important impact on the structure of the scalar correlators.

- Introduction.

The long distance behavior of QCD focuses on the interactions of the pseudoscalar mesons, the Goldstone boson degrees of freedom associated with the dynamical breaking of the light quark chiral symmetries.

It is important to understand the interactions of these light mesons before the dynamics of the heavier states can be clearly revealed. We see a clear example of this in our study of scalar mesons using the lattice version of QCD.

- Chiral Lagrangian.

The dynamics of pseudoscalar mesons is dictated by the chiral symmetries and to lowest order is incorporated in a nonlinear chiral Lagrangian,

$$\begin{aligned} L = {} & \frac{f^2}{4} tr\left(\partial_\mu U^+ \partial^\mu U\right) + \frac{f^2}{4} tr\left(\chi^+ U + U^+ \chi\right) \\ & + L_5 tr\left(\partial_\mu U^+ \partial^\mu U\left(\chi^+ U + U^+ \chi\right)\right) \\ & + L_8 tr\left(\chi^+ U \chi^+ U + U^+ \chi U^+ \chi\right) + L_{hairpin} \end{aligned}$$

where $U = \exp(i\lambda \cdot \phi)$ is the nonlinear chiral field and $L_{hairpin}$ includes terms reflecting the explicit breaking of the anomalous U(1) chiral symmetries in QCD. (We exhibit only those terms needed for our analysis.)

$$L_{hairpin} = -\frac{1}{2} m_o^2 \frac{f^2}{8}\left(i tr \ln\left(U^+ / U\right)\right)^2$$

To leading order in the large Nc expansion of the quark-gluon theory only valence quarks are included as internal quark loops and nonplanar gluon diagrams are suppressed. In the meson picture, the tree-level chiral Lagrangian describes the leading order of the large Nc expansion and the meson loop expansion corresponds to an expansion in powers of 1/Nc.

- Quenched lattice QCD.

The quenched lattice version of QCD is similar to the large Nc expansion as internal quark loops are also suppressed. However, there are important differences related to the nonplanar gluon interactions that are retained in quenched lattice QCD.

In the quenched lattice theory meson loops are generated by insertions of the hairpin vertex operator according to rules formulated by Sharpe and Bernard and Golterman. These meson loops are visible through the chiral logs that modify correlation functions of the pseudoscalar mesons.

- MQA, exceptional configurations.

To study light quark masses with Wilson fermions, we encounter the problem of exceptional configurations where one eigenvalue of a single configuration can dominate the ensemble average. This arises because the real eigenvalues of the Wilson-Dirac operator are not confined to zero, the value in the chiral limit, but have a spread which can make the quark propagators singular even for positive values of the quark mass.

We define a modified quark propagator by making a compensated shift of the most sensitive real eigenvalues to restore, in part, the chiral structure of the real eigenvalues. We use this MQA method to explore the chiral limit of quenched QCD.

- Lattice computations.

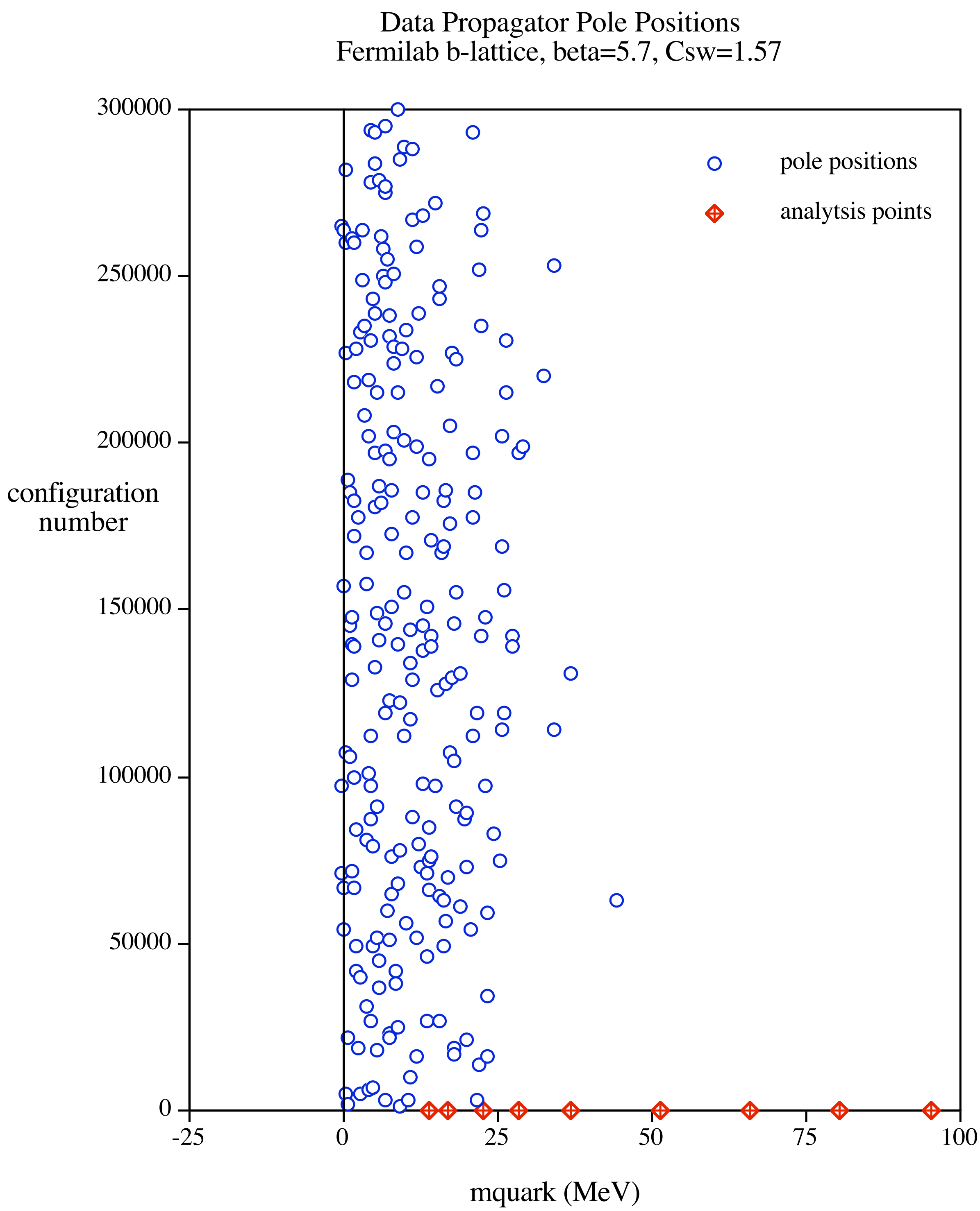
We have analyzed these long distance effects using two sets of lattice QCD configurations of different lattice spacings and clover fermion actions.

The Fermilab b-lattice consists of 300 configurations at β=5.7 on a 12^3X24 lattice with clover fermions, Csw=1.57, and seven quark masses corresponding to pion masses from 275 MeV to 565 Mev.

The Fermilab c-lattice consists of 350 configurations at β=5.9 on a 16^3X32 lattice with clover fermions, Csw=1.50, and six quark masses corresponding to pion masses from 330 MeV to 665 Mev.

- Chiral Lagrangian Fits.

The chiral Lagrangian parameters are determined by studying correlation functions for pseudoscalar and axial-vector densities. We measure the pseudoscalar masses, the pseudoscalar decay constants and the axial-vector decay constants for the full spectrum of quark masses. The singlet mass term, m_o, is determined from the analysis of the hairpin correlator.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mquark (MeV)
pole positions
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50
75
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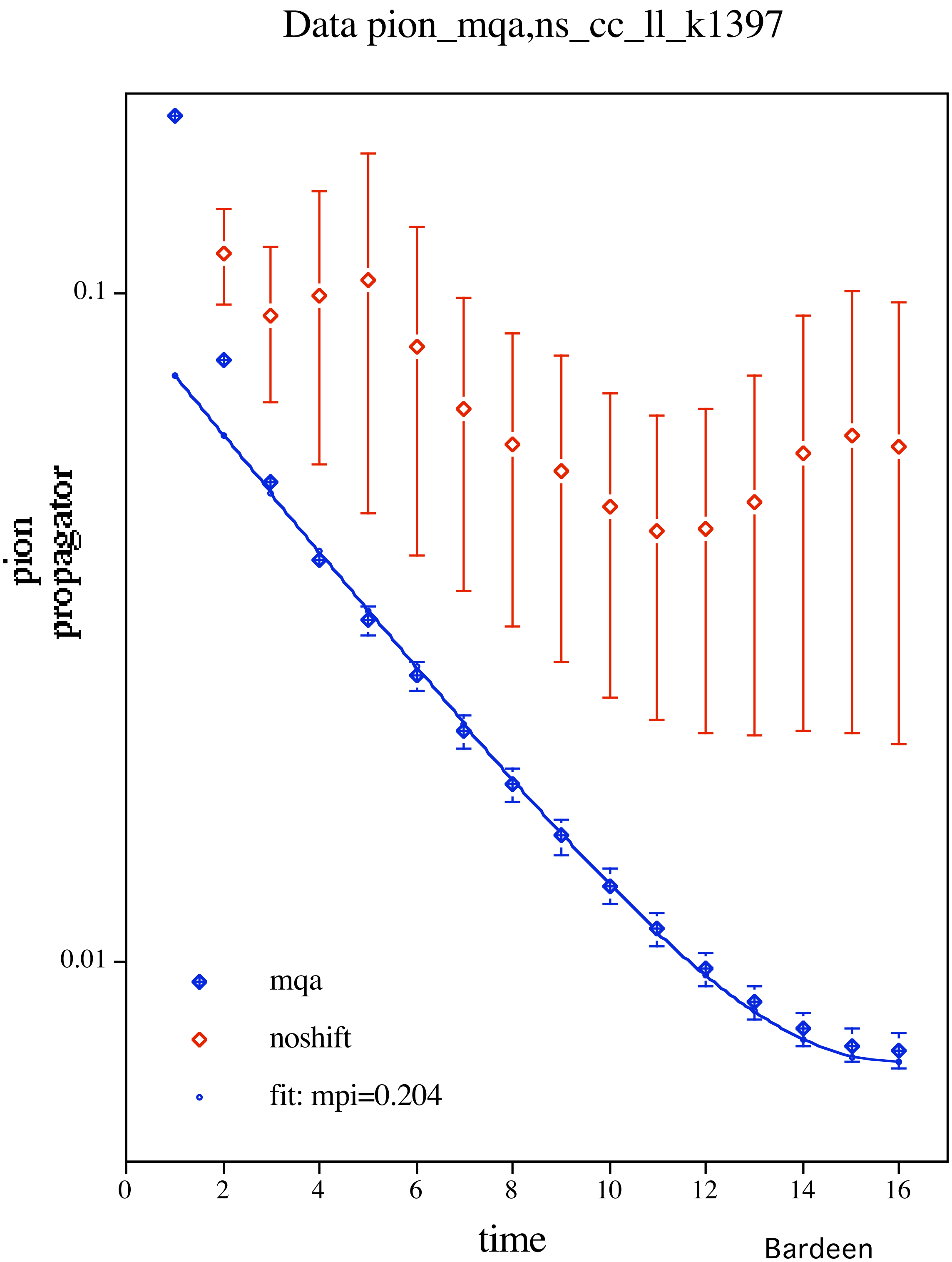
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2
4
6
8
10
12
14
16
time
mqa
noshift
fit: mpi=0.204

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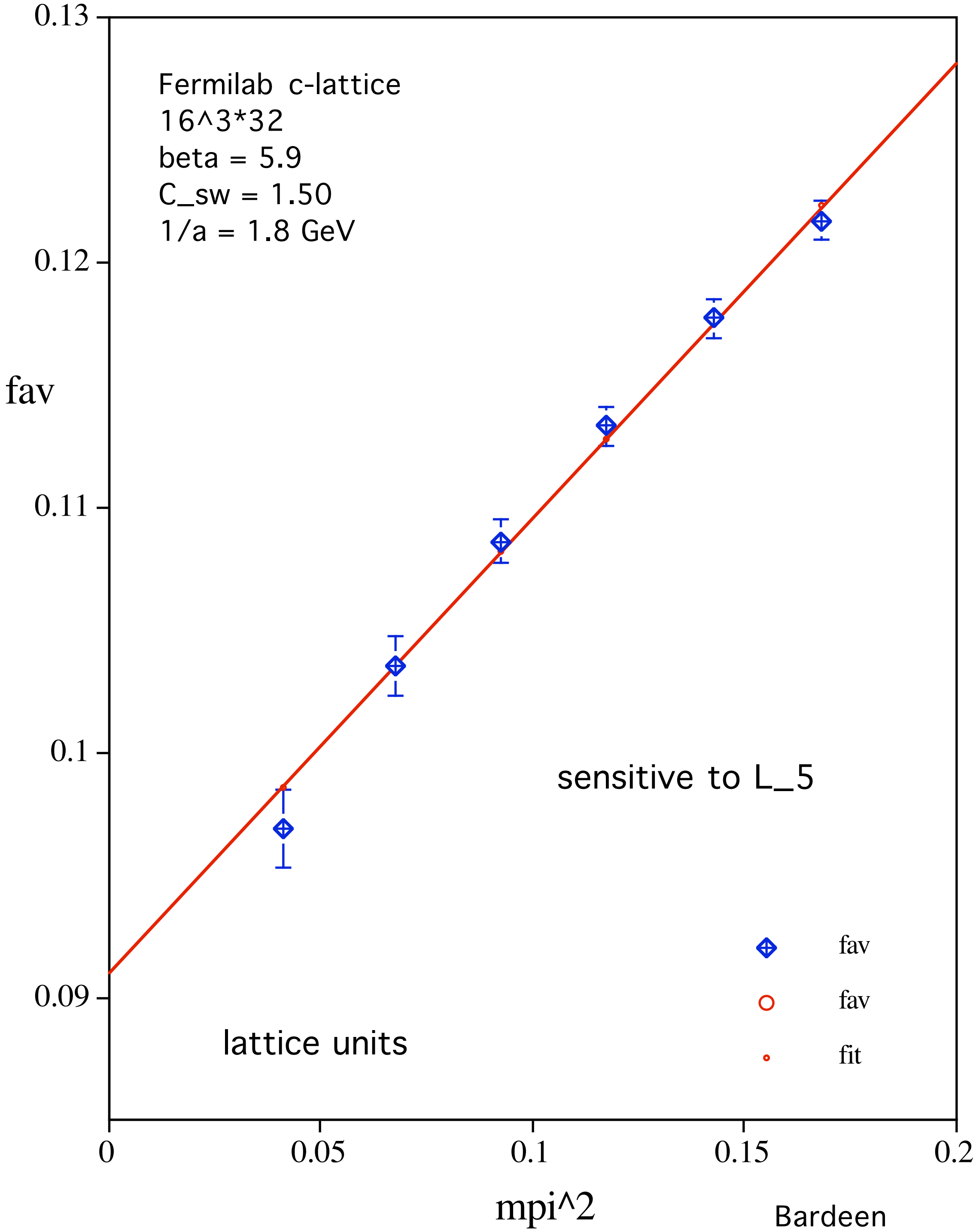
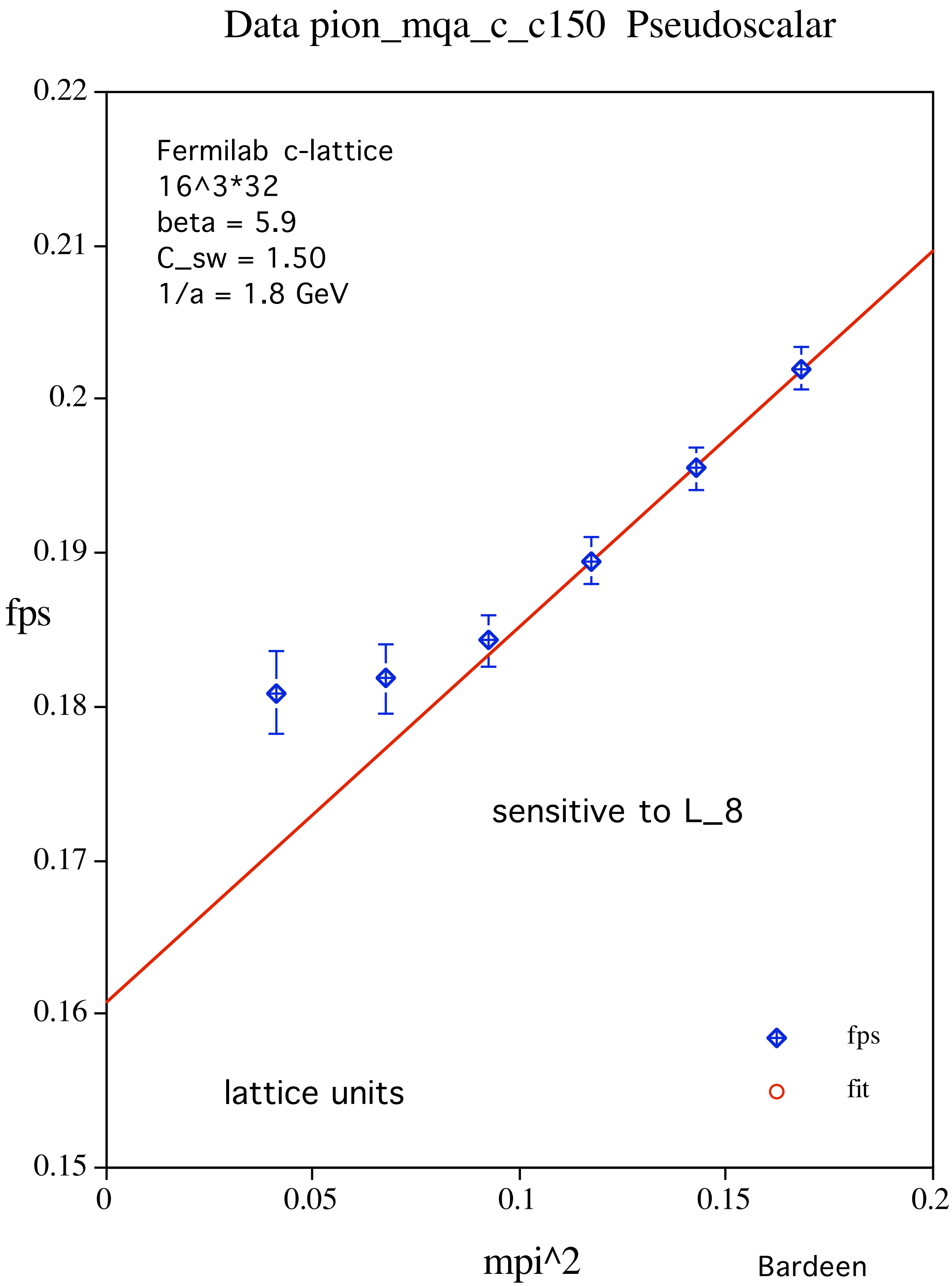
Data pion_mqa_c_c150 Axial Vector





Data pion_mqa_c_c150 Pseudoscalar
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16^3*32
beta = 5.9
C_sw = 1.50
1/a = 1.8 GeV
sensitive to L_8
lattice units
fps
fit
0.22
0.21
0.2
0.19
0.18
0.17
0.16
0.15
fps
0
0.05
0.1
0.15
0.2
mpi^2
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The expected chiral log behavior of these quantities is computed from the chiral Lagrangian. The meson masses are given by

$$M_{ij}^2 = \chi_{ij}\left(1+\delta I_{ij}\right)\left\{1+\frac{8}{f^2}\left(2L_8-L_5\right)\chi_{ij}\left[1+\delta\left(\tilde{I}_{ij}+I_{ij}\right)\right]\right.$$
$$\left.+\frac{8}{f^2}L_5\chi_{ij}\delta\tilde{J}_{ij}\right\}$$

The pseudoscalar decay constant is given by

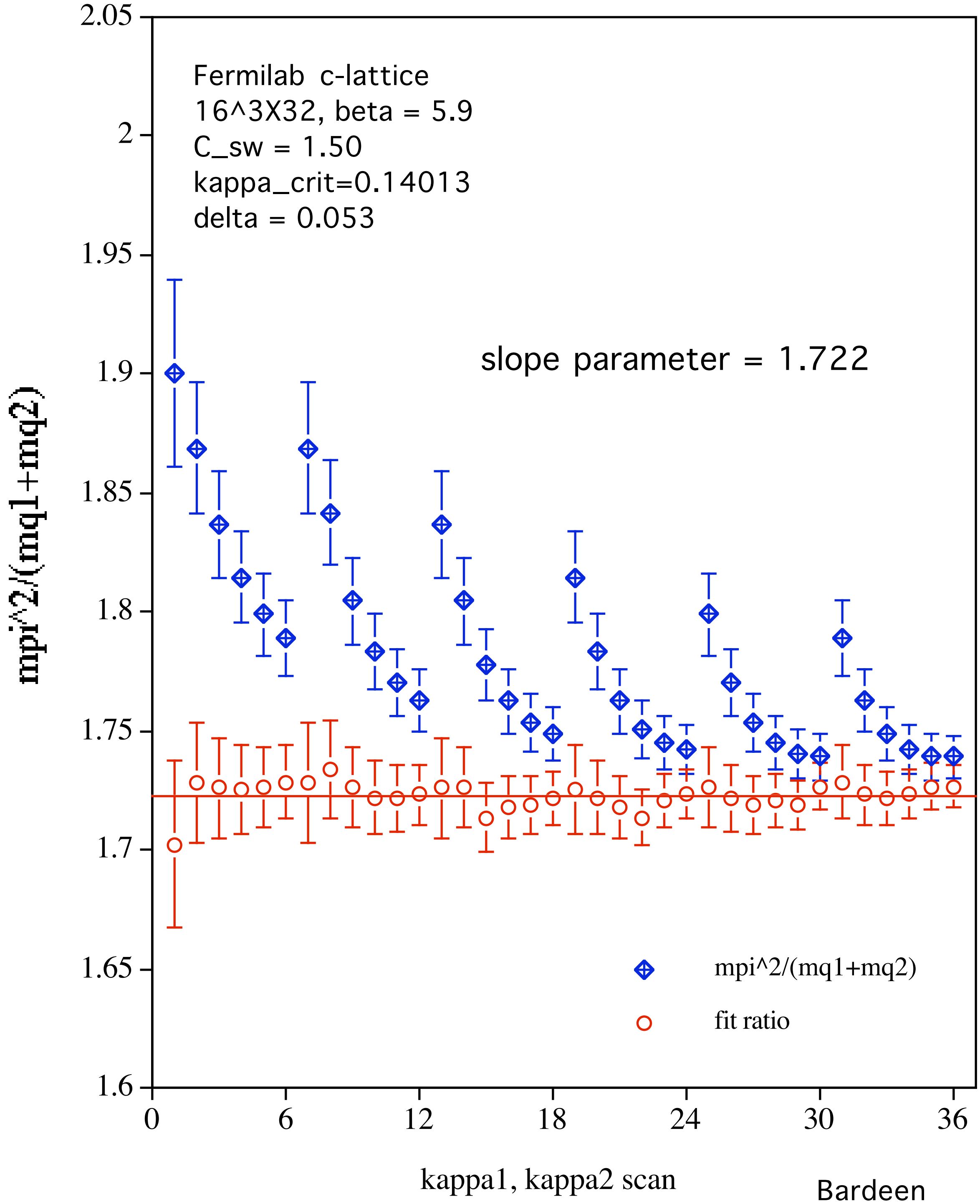
$$f_{P;ij} = \sqrt{2}fr_o\left(1+\frac{1}{4}\delta\left(I_{ii}+I_{jj}+2I_{ij}\right)\right)$$
$$\bullet\left\{1+\frac{4}{f^2}\left(4L_8-L_5\right)\chi_{ij}\left(1+\delta\left(\tilde{I}_{ij}+I_{ij}\right)\right)\right.$$
$$\left.-\frac{4}{f^2}L_5\delta\chi_{ij}\left[\frac{1}{2}\tilde{I}_{ij}+I_{ij}-\tilde{J}_{ii}-\tilde{J}_{jj}\right]\right\}$$

The axial-vector decay constant is given by

$$f_{A;ij} = \sqrt{2}f\left(1+\frac{1}{4}\delta\left(I_{ii}+I_{jj}-2I_{ij}\right)\right)$$
$$\bullet\left\{1+\frac{4}{f^2}\chi_{ij}L_5\left[1+\delta\left(2I_{ij}-\tilde{I}_{ij}\right)\right]\right\}$$

The chiral log parameter is defined as $\delta \equiv m_o^2 / 24\pi^2 f_A^2$ where m_o is the hairpin mass parameter.

Data combined fit - all correlators



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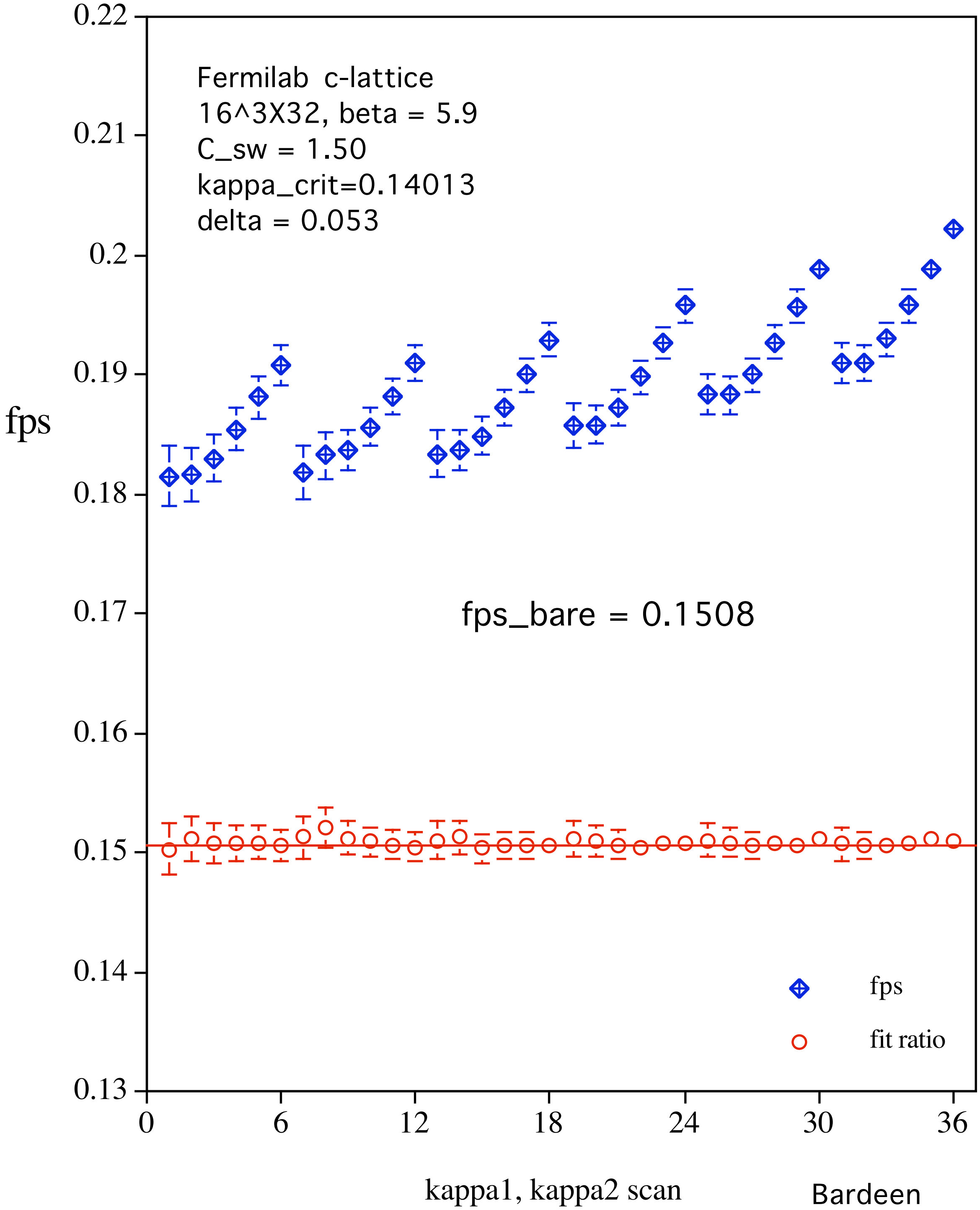
Data combined fit - all correlators



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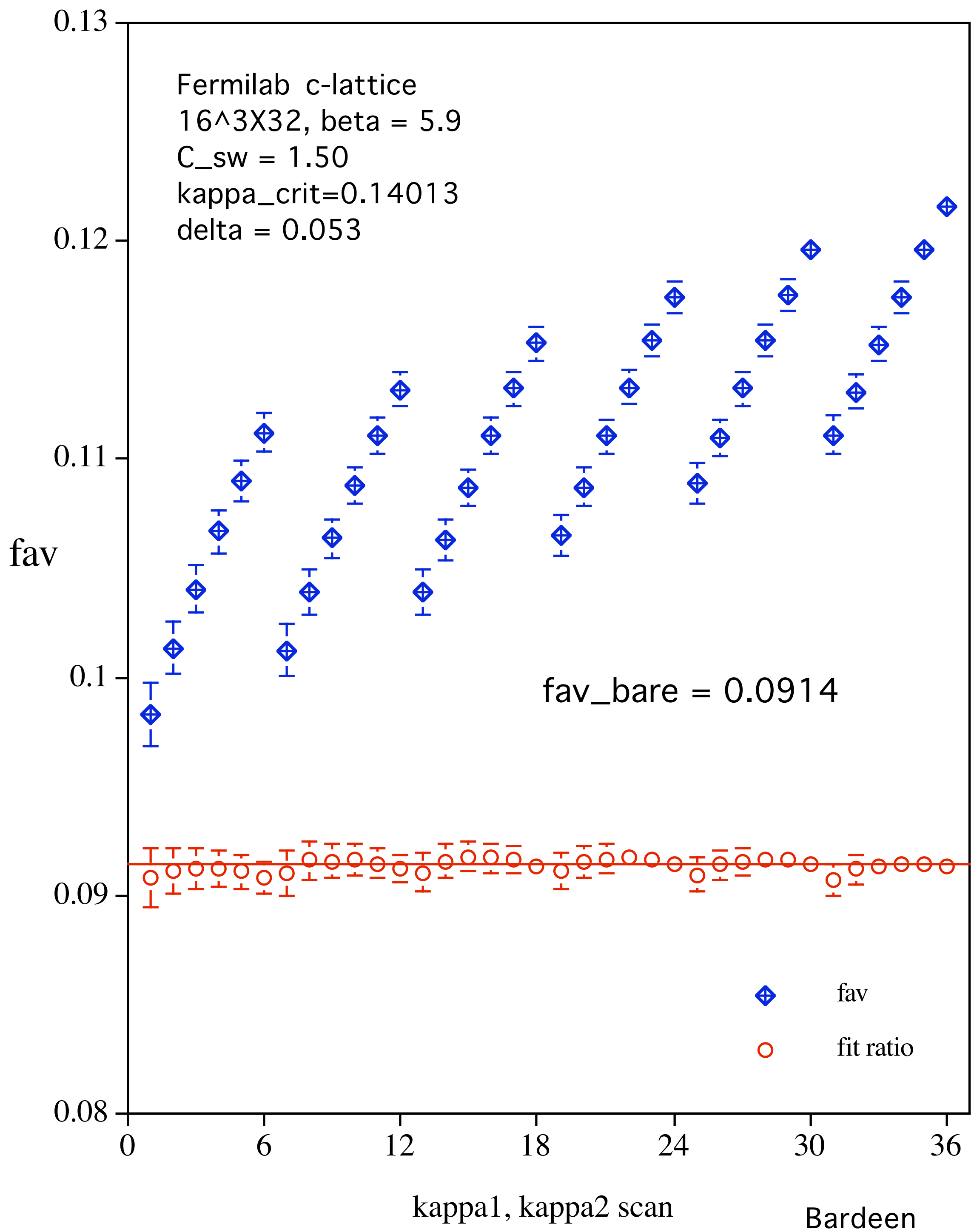
Data combined fit - all correlators



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The other quantities are defined as $\chi_{ij} = \chi_i + \chi_j$, $\chi_i = 2r_o m_i$, $m_i = \ln\left(1 + 1/2\kappa_i - 1/2\kappa_c\right)$, $\tilde{I}_{ij} = \left(I_{ii}\chi_i + I_{jj}\chi_j\right)$, $J_{ij} = \left(I_i + I_j - \left(M_{ii}^2 + M_{jj}^2\right)I_{ij}\right)/2$, and $\tilde{J}_{ij} = J_{ij}/\chi_{ij}$. The loop functions, I_i and I_{ij}, are defined by

$$I_i = \frac{1}{\pi^2}\int \frac{d^4 p}{p^2 + M_i^2} \to 16\pi^2 \sum_p \left(D(p, M_i) - reg\right)$$

$$I_{ij} = \frac{1}{\pi^2}\int d^4 p \frac{1}{p^2 + M_i^2}\frac{1}{p^2 + M_j^2}$$

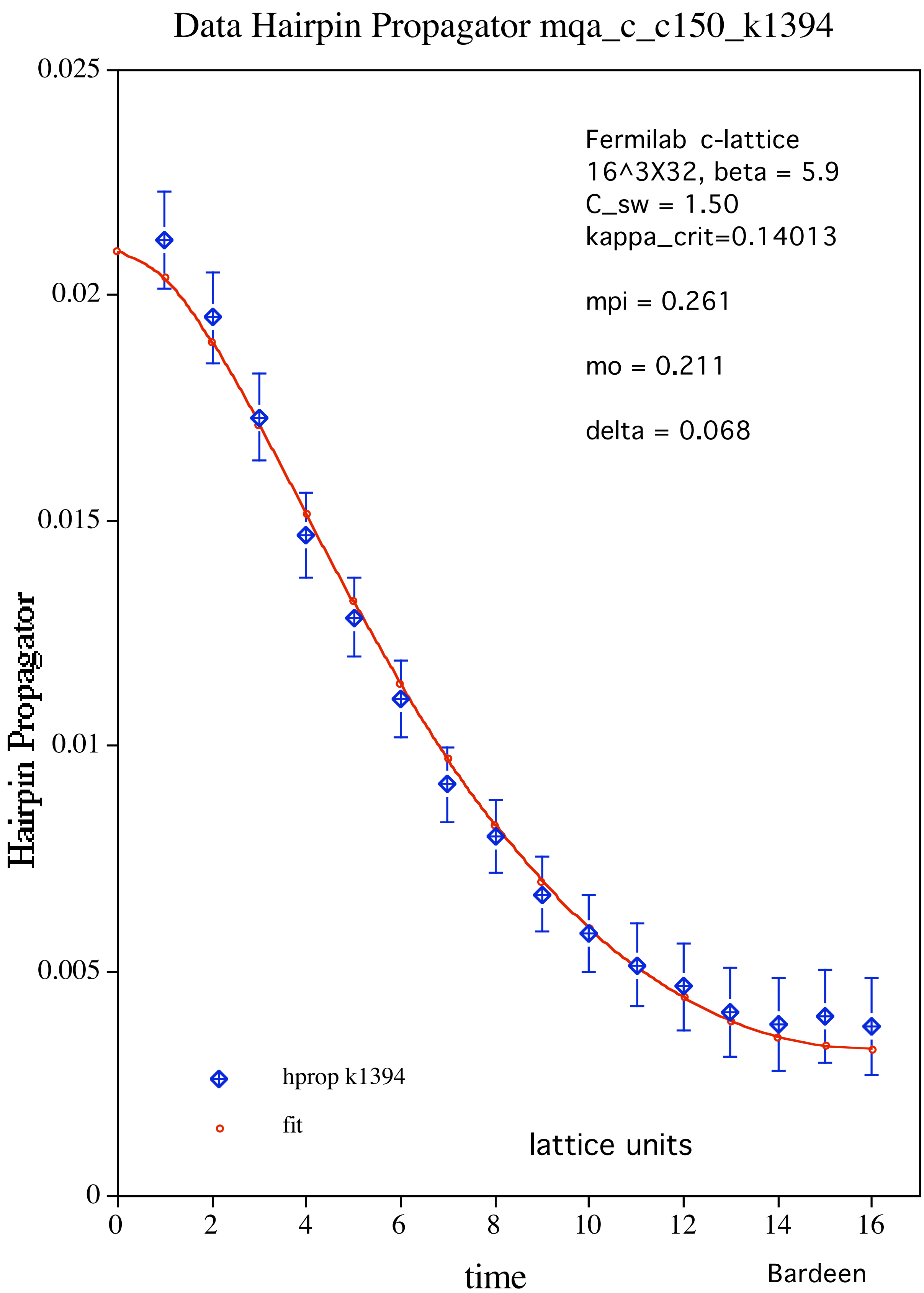
$$\to 16\pi^2 \sum_p \left(D(p, M_i)D(p, M_j) - reg\right)$$

where $D(p, M_i)$ is the lattice meson propagator.

Replacing the integrals by sums accounts for some of the more obvious finite size effects.

- Hairpin correlator.

We use the all-to-all quark propagator method to compute the hairpin correlators needed for the direct determination of the singlet mass parameter, m_o. We find that this correlator is dominated by the leading double pole contribution that is proportional to m_o^2. m_o^2 is generated by topological fluctuations of the QCD vacuum and is sensitive to our treatment of the real eigenvalues of the Wilson-Dirac operator.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mpi = 0.261
mo = 0.211
delta = 0.068
hprop k1394
fit
lattice units
Hairpin Propagator
time
0.025
0.02
0.015
0.01
0.005
0
0
2
4
6
8
10
12
14
16
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- Scalar mesons in lattice QCD.

Scalar mesons pose special problems for lattice computations. In the unquenched theory, scalar mesons decay rapidly to light pions and it may be difficult to extract the resonance parameters from the scalar density correlators. At leading order in the large Nc expansion, the scalars are stable as the decays are suppressed. One might expect a similar behavior for the quenched lattice theory.

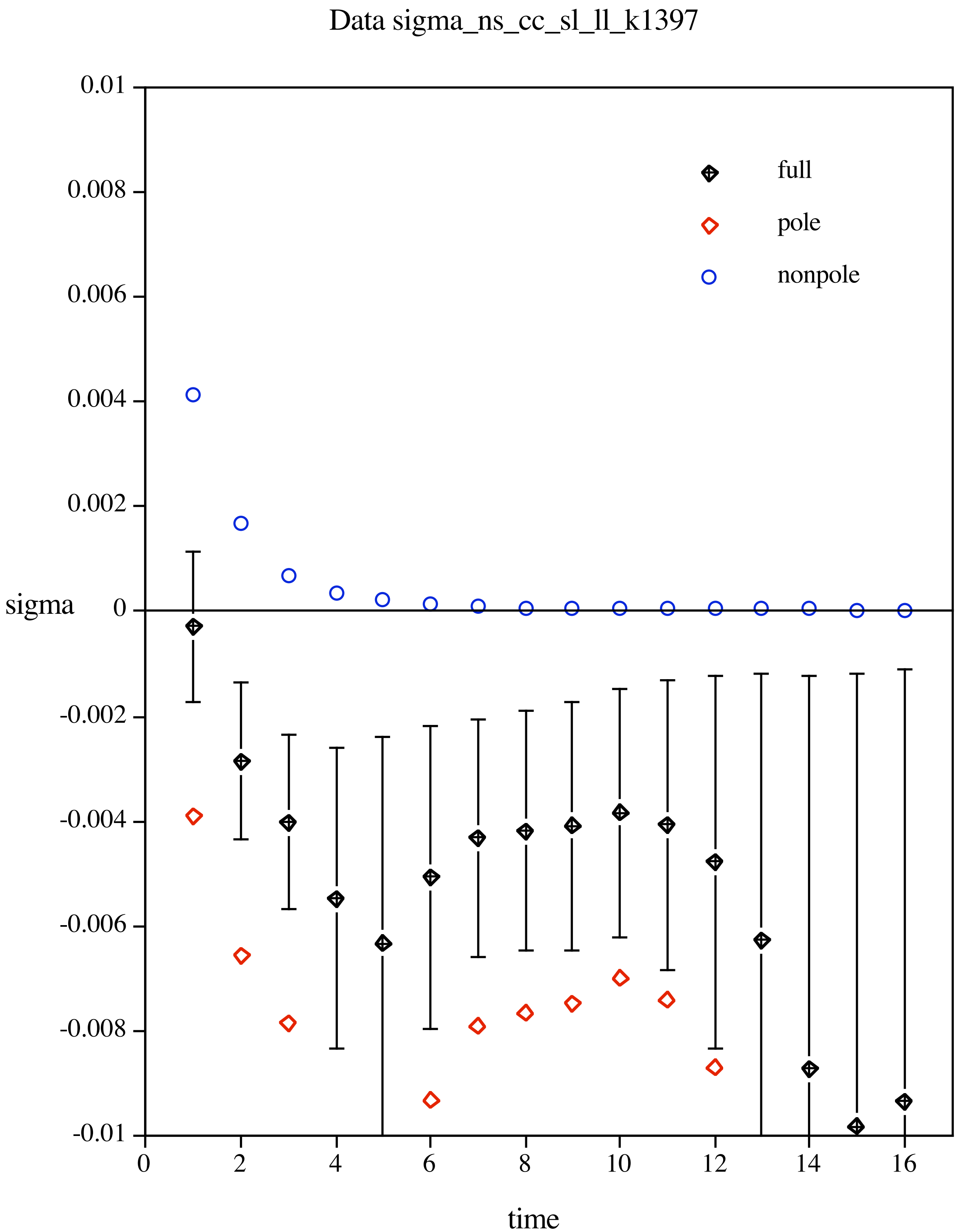
Unfortunately, this is not the case as the hairpin diagrams generate loop contributions that are even more infrared singular than the unquenched loop corrections.

Fortunately, these chiral loop corrections are completely determined by the parameters of the pseudoscalar chiral Lagrangian. The chiral dynamics of a heavy scalar meson are also described via a nonlinear chiral Lagrangian,

$$L_{scalar} = \frac{1}{4} tr\left(D_\mu \sigma D^\mu \sigma - m_s^2 \sigma^2\right) + f_s tr\left(\chi^+ \sqrt{U} \sigma \sqrt{U} + hc\right)$$

where D is a chiral covariant derivative. The structure of the second term is dictated by the chiral transformation properties of the scalar density operator.

The scalar density correlator is the sum of two terms, one is the scalar propagator and the second is an $\eta' - \pi$ loop contribution where the η' propagator is replaced by the more singular hairpin propagator. Using a somewhat simplified picture, we fit the scalar correlator as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full
pole
nonpole
sigma
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0.004
0.002
0
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-0.006
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-0.01
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2
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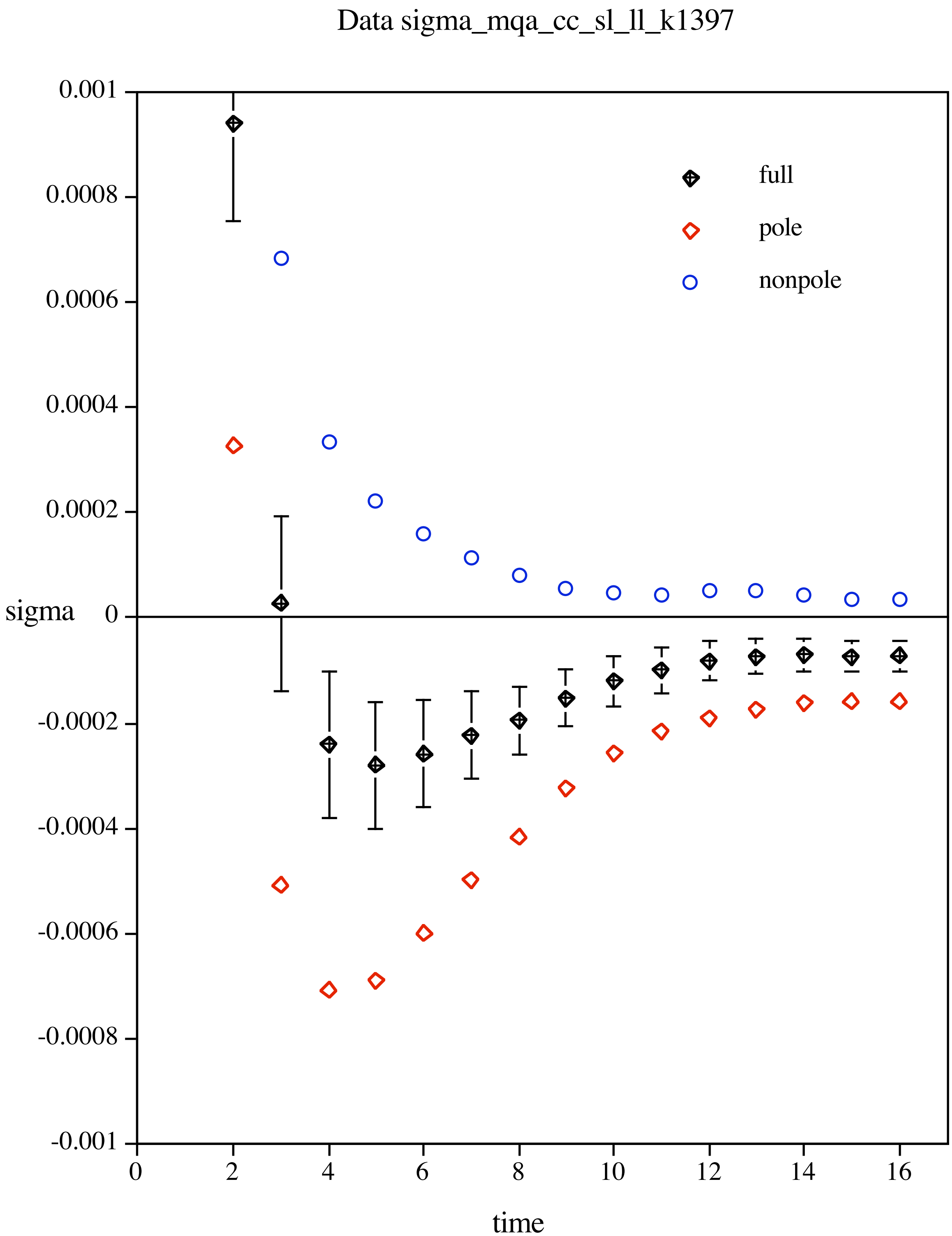
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pole
nonpole
sigma
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0.0004
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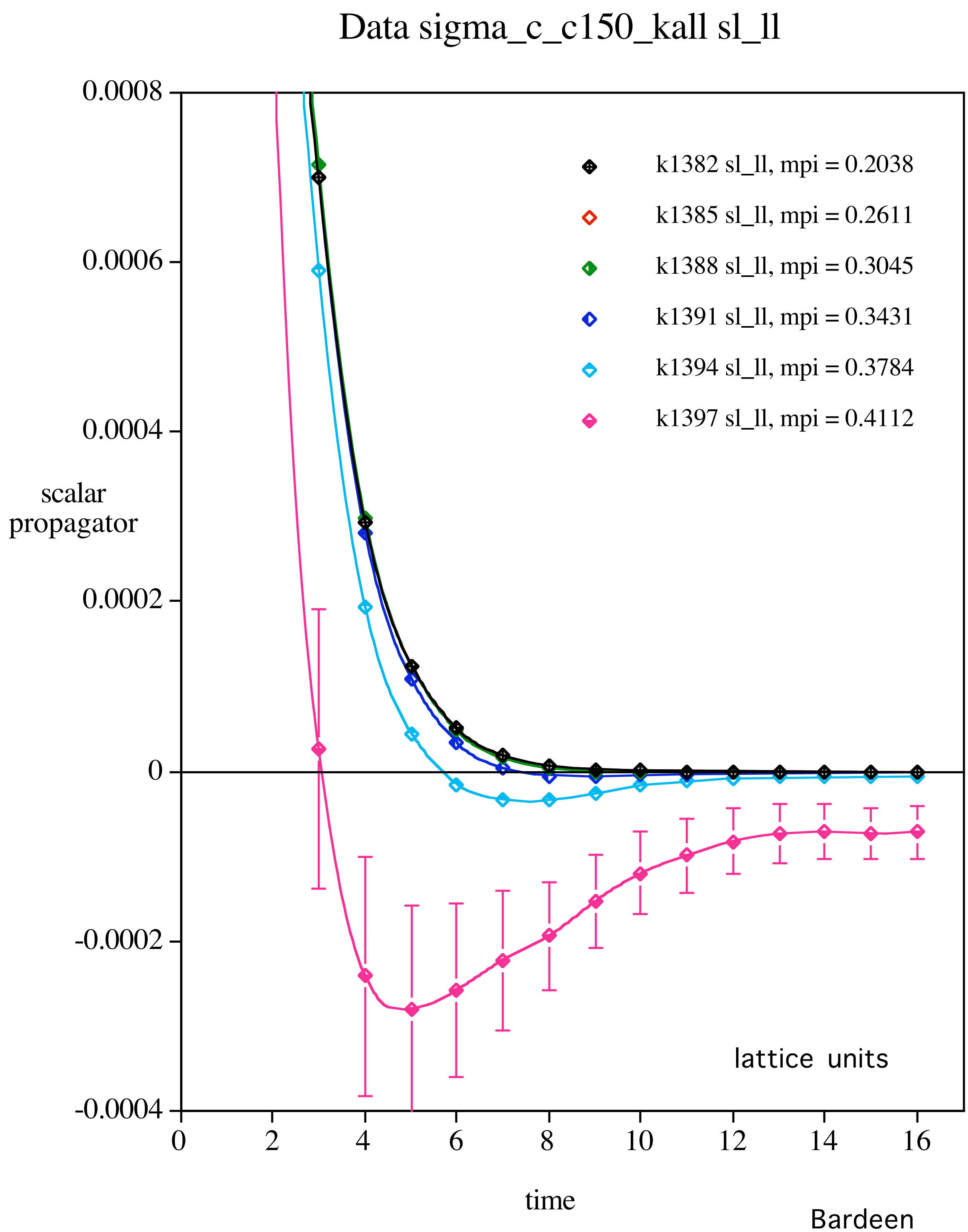
time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0.0006
0.0004
0.0002
0
-0.0002
-0.0004
scalar
propagator
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k1385 sl_ll, mpi = 0.2611
k1388 sl_ll, mpi = 0.3045
k1391 sl_ll, mpi = 0.3431
k1394 sl_ll, mpi = 0.3784
k1397 sl_ll, mpi = 0.4112
lattice units
0
2
4
6
8
10
12
14
16
time
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$$\Delta_s(t) = 32r_o^2 \frac{f_s^2}{2m_s} \exp(-m_s t) + 4r_o^2 B_{hp}(t)$$

where m_s and f_s are the scalar meson parameters and $B_{hp}(t)$ is the Fourier transform of the hairpin bubble function with $\vec{p} = 0$,

$$B_{hp}(p) = \frac{1}{VT} \sum_k \frac{1}{\left[(k+p)^2 + m_\pi^2\right]} \frac{-m_o^2}{\left(k^2 + m_\pi^2\right)^2}$$

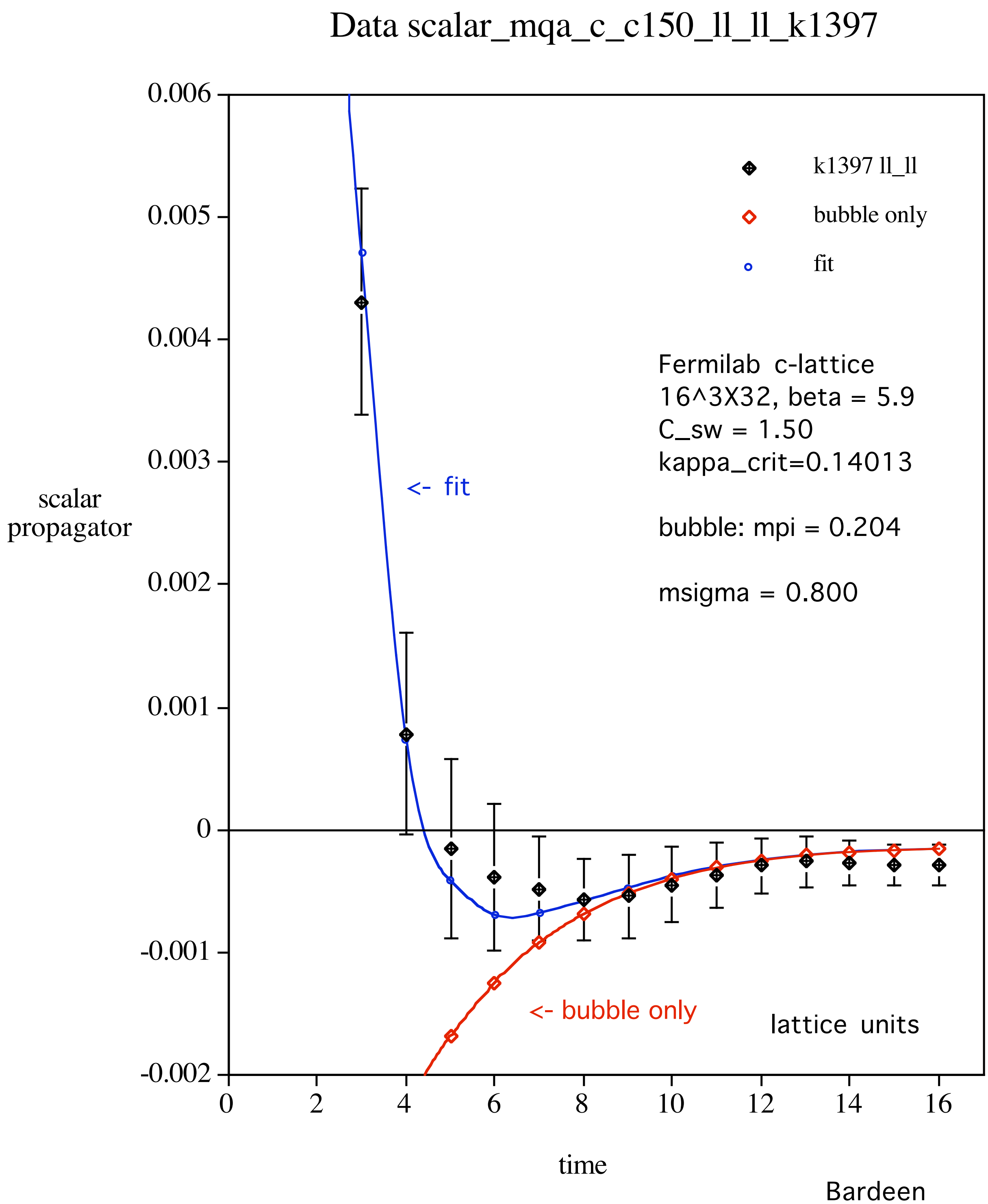
In the unquenced theory, this hairpin contribution would be resumed as part of the heavy η' propagator contribution and this bubble function would not be infrared sensitive.

At first glance, the scalar correlator data do not seem to make sense as the correlators are negative and the negative contribution increases dramatically as the pion mass decreases. However, this is just the expected behavior of the hairpin bubble function - being negative metric and infrared singular by a power of $1/m_\pi^2$.

From our lattice data we can measure the scalar meson parameters, m_s and f_s. We obtain very consistent results from both b- and c-lattices computations,

$$m_s = 1285(60) MeV (\beta = 5.7), \quad = 1326(86) MeV (\beta = 5.9)$$

$$f_s = 64(5) MeV (\beta = 5.7); \quad = 68(3) MeV (\beta = 5.9)$$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0.001
0
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-0.002
scalar propagator
0
2
4
6
8
10
12
14
16
time
k1397 ll_ll
bubble only
fit
Fermilab c-lattice
16^3X32, beta = 5.9
C_sw = 1.50
kappa_crit=0.14013
bubble: mpi = 0.204
msigma = 0.800
<- fit
<- bubble only
lattice units
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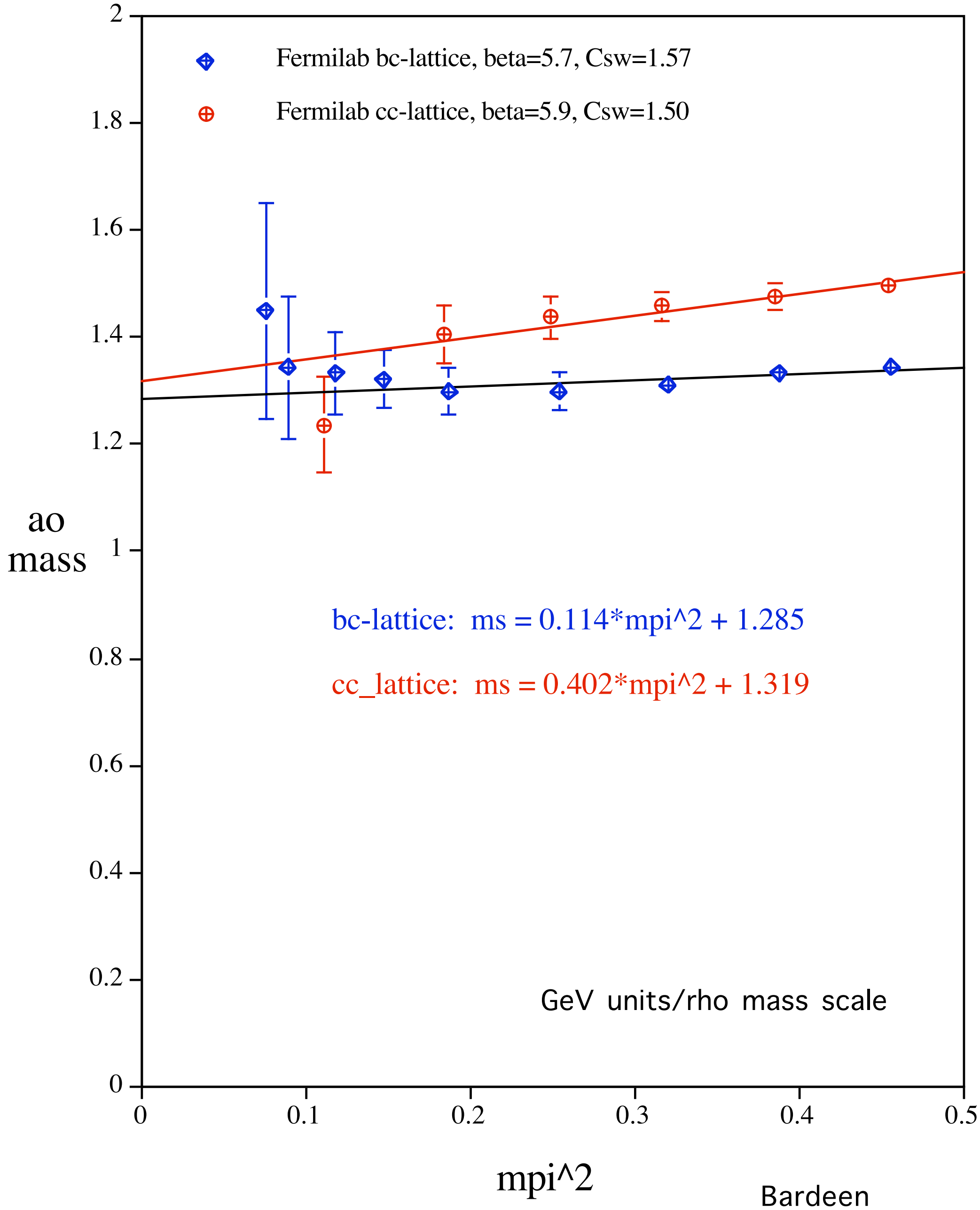
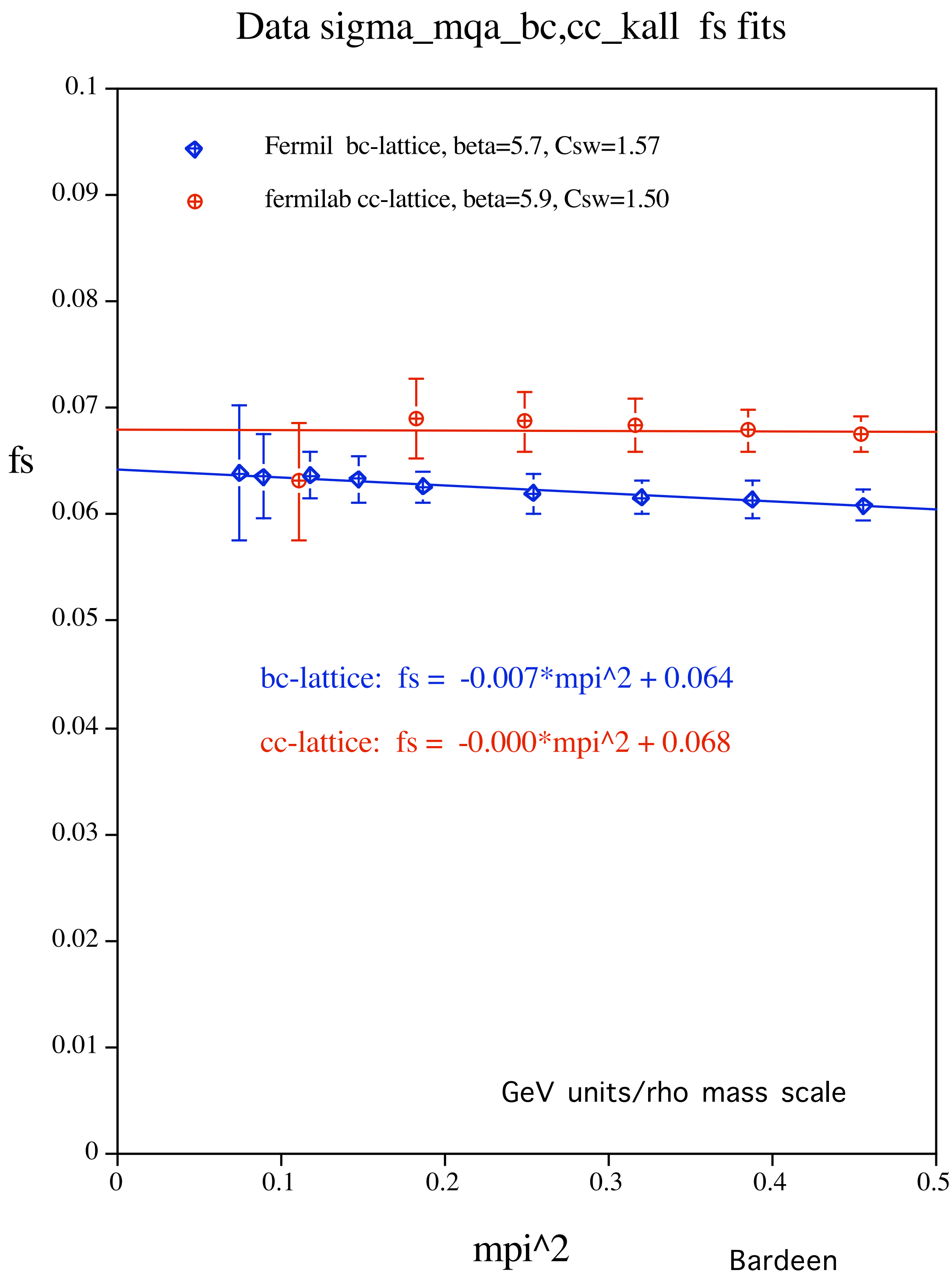
Data sigma_mqa_bc_kall ms fits

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fermilab cc-lattice, beta=5.9, Csw=1.50
bc-lattice: fs = -0.007*mpi^2 + 0.064
cc-lattice: fs = -0.000*mpi^2 + 0.068
GeV units/rho mass scale
fs
mpi^2
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.1
0.2
0.3
0.4
0.5
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- Conclusions.

The MQA modification of the standard clover formulation of lattice fermions allows us to explore the long-range behavior of quenched QCD on fixed lattices.

We measure pseudoscalar, axial-vector and hairpin correlators for the quenched lattice QCD. We match our results to the predictions of the appropriate meson chiral Lagrangians and determine the Lagrangian parameters.

We extend our results to the scalar correlators. We observe the anomalous infrared power corrections expected in the quenched lattice theory. We compute these effects using our previous determination of the pseudoscalar chiral Lagrangian and are then able to extract the scalar meson masses and coupling parameters appropriate to the chiral Lagrangian describing the dynamics of heavy scalar mesons.

Our results for the quenched lattice theory are not consistent with very light scalar mesons.

Similar quenched results are found by Prelovsek and Orginos using Domain Wall Fermions on similar size lattices. They confirm the strong mass dependence of the negative metric bubble contributions which are enhanced by the finite size effects.

New unquenched and partially quenched studies of the scalar correlator using Domain Wall Fermions with two light flavors have just been posted by Prelovsek et al. The sea quark masses range from 500 to 700 MeV while the valence quark have masses in the range 380 to 770 MeV.

The partially quenched data with $m_{valence} < m_{sea}$ clearly show the effects of the negative metric bubble contributions. These effects disappear in the unquenched analysis or give positive correlators when $m_{valence} > m_{sea}$, as expected from partially quenched chiral perturbation theory. Both sets of data are fit to the appropriate chiral Lagrangians to extract values for mass of the isovector scalar resonance with the results:
$m_{scalar} = 1.51 \pm 0.19$ (partially quenched) and
$m_{scalar} = 1.58 \pm 0.34$ (unquenched).

References in

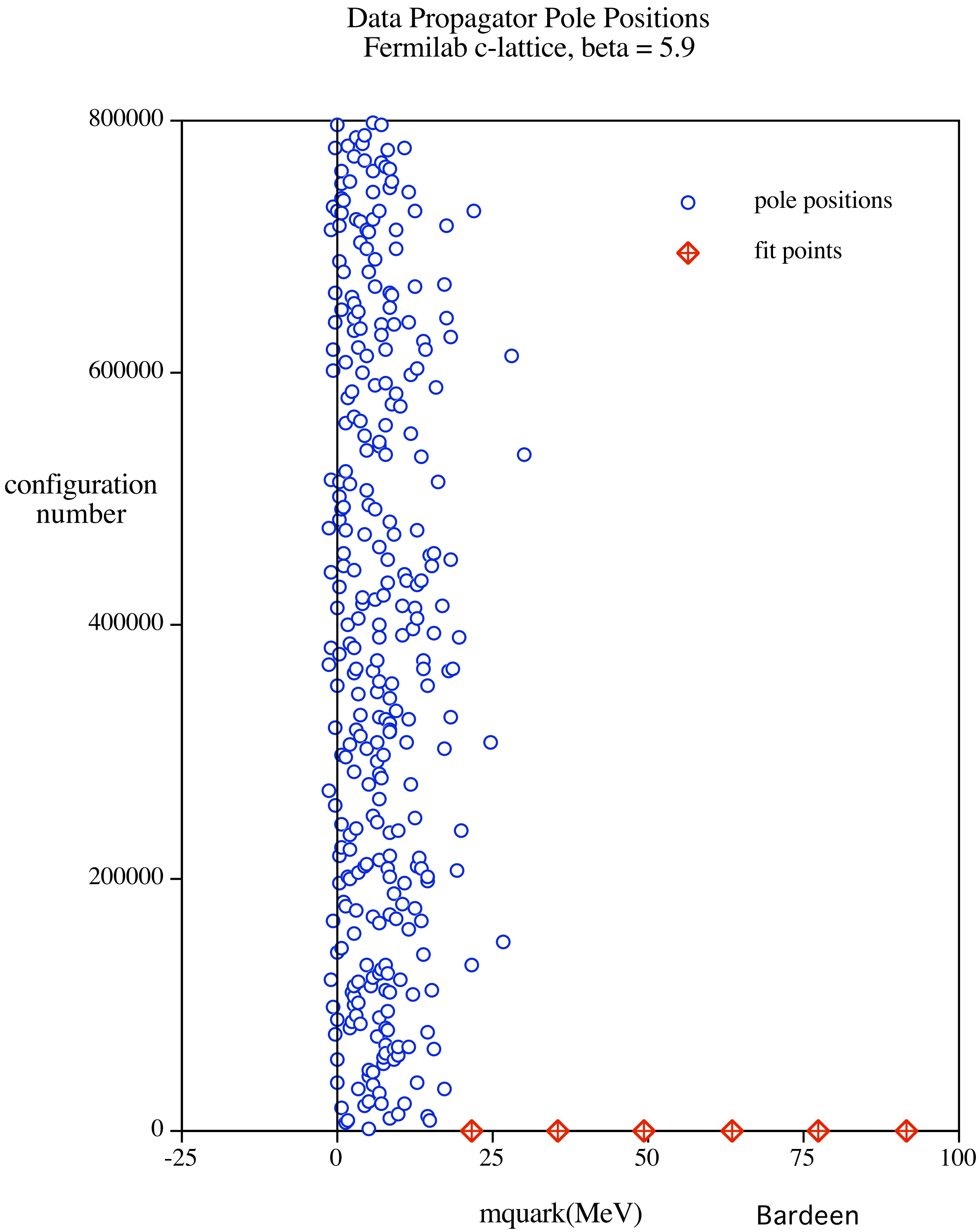
W. Bardeen, A. Duncan, E. Eicthen, N. Isgur and H. Thacker, CHIRAL LOOPS AND GHOST STATES IN THE QUENCHED SCALAR PROPAGATOR. Phys.Rev.D65:014509,2002. hep-lat/0106008.

William A. Bardeen, E. Eichten , and H. Thacker , CHIRAL LAGRANGIAN PARAMETERS FOR SCALAR AND PSEUDOSCALAR MESONS. Phys.Rev.D69:054502,2004. e-Print Archive: hep-lat/0307023.

S. Prelovsek and K. Orginos, RBC Collaboration, QUENCHED SCALAR-MESON CORRELATOR WITH DOMAIN WALL FERMIONS. Nucl. Phys. PS B119,822,2004.

S. Prelovsek et al., SCALAR MESON IN DYNAMICAL AND PARTIALLY QUENCHED TWO-FLAVOR QCD: LATTICE RESULTS AND CHIRAL LOOPS. hep-lat/0407037 (29 July 2004).

Extra Slide



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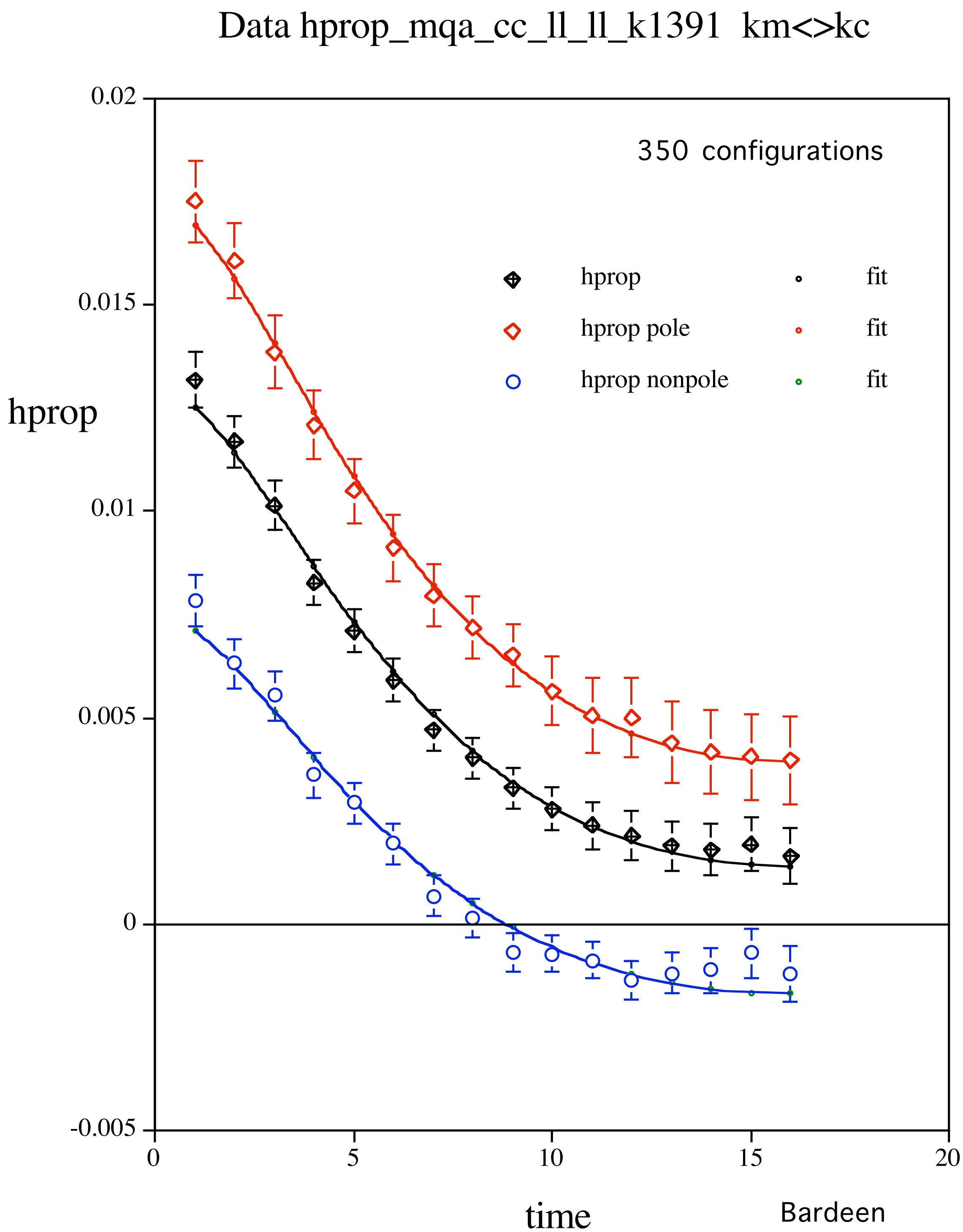

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350 configurations
hprop
fit
hprop pole
fit
hprop nonpole
fit
hprop
time
Bardeen
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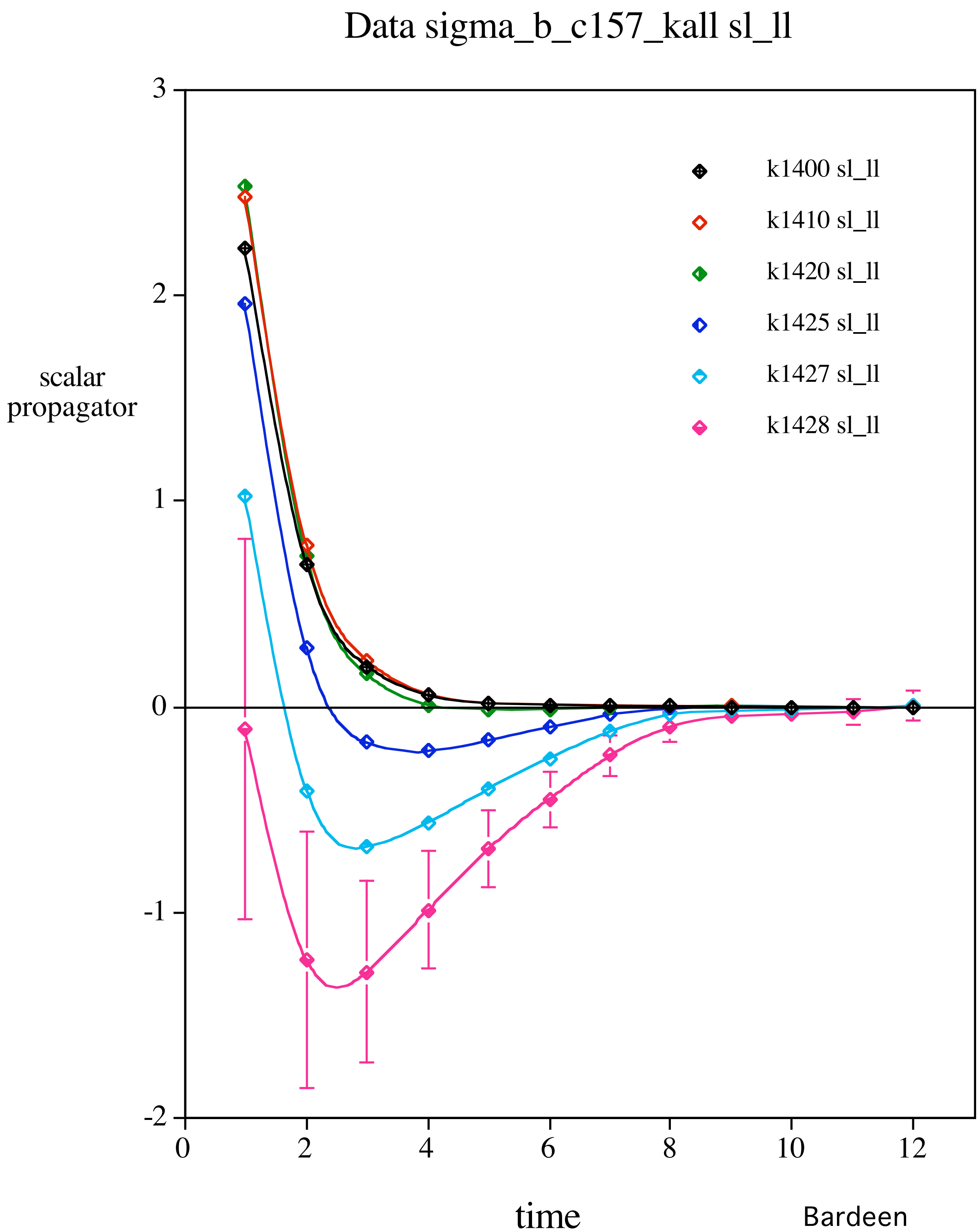
0.01
0.005
0
-0.005
0
5
10
15
20

Extra Slide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scalar
propagator
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k1410 sl_ll
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k1425 sl_ll
k1427 sl_ll
k1428 sl_ll
0
2
4
6
8
10
12
time
Bardeen