Guy Bouchitté (U. Toulon, France): *Kantorovich-Rubinstein duality for the Hessian and convex order.*

**Abstract:** The classical Kantorovich-Rubinstein duality theorem established a significant connection between Monge optimal transport and the maximization of a linear form on 1-Lipschitz functions. This result has been widely used in various research areas, particularly to demonstrate a bridge between Monge transport theory and some class of optimal design problems in mechanics. The aim of this talk is to present a similar theory when the linear form is maximized over all real $C^{1,1}$ functions with a Hessian matrix spectral norm not exceeding one. It turns out that this new maximization problem can be viewed as the dual of a specific optimal transport problem. The task is to find a minimal three-point plan with given first two marginals, where the third is assigned to be larger than both in the sense of convex order. The existence of optimal plans allows to express solutions of the underlying Beckman problem as a combination of rank-one tensor measures supported by a graph. In the context of two-dimensional mechanics, this graph encodes the optimal location of a grillage to support a given bending load.

Giuseppe Buttazzo (U. of Pisa, Italy): *Optimal coefficients for elliptic PDEs.*

**Abstract:** The goal is to present an optimization problem related to elliptic PDEs of the form $-\mathrm{div}(a(x)\nabla u) = f$ with Dirichlet boundary condition on a given domain $\Omega$. The coefficient $a(x)$ has to be determined, in a suitable given class of admissible choices, in order to optimize a given criterion. We first deal with the case when the cost is the so-called elastic compliance, and then we discuss the more general case when the problem is written as an optimal control problem.

Sylvain Ervedoza (U. Bordeaux, France): *Global $L^p$ Carleman Estimates for the Laplace operator and applications.*

**Abstract:** In this talk, I will present some recent results on $L^p$ Carleman Estimates for the Laplace operator and its application to the quantification of the unique continuation with respect to lower order terms. Starting with the motivation, I will then briefly give some ideas of the proof, trying to avoid being too technical (except if I am asked for it !). This work has been obtained in collaboration with Belhassen Dehman and Lotfi Thabouti.

Jan Giesselmann (TU Darmstadt, Germany): *(Non)-synchronization of boundary observers for wave equations.*

**Abstract:** We consider the (linear) wave equation as a strongly simplified model for gas transport on networks. We construct observers based on measuring density and velocity at boundary nodes. We show that the state of such observers converges exponentially to the state of the original system if the network is a tree (even if all pipes are very long). We call such a convergence "synchronization". In contrast, synchronization does not hold (in general) for networks with cycles.

This result can be interpreted to say that if we wish to guarantee synchronization we need to add one (full state) measurement in each cycle the network contains. Finally, we demonstrate that even without friction finite time synchronization does not hold in general on tree shaped networks.

Alexander Keimer (FAU Erlangen, Germany): *The current state in the field of nonlocal conservation laws.*

**Abstract:** We will give an overview of what has been established in the field of nonlocal conservation laws, covering existence and uniqueness of solutions as well as numerical approximations. We then look into the singular limit problem, i.e., the problem whether the solution of the nonlocal conservation law converges to the entropy solution of the corresponding local conservation law when the nonlocal kernel converges to a Dirac distribution. Generalizations to balance laws and different nonlocalities and the corresponding singular limit will be discusses as well. We conclude with some open problems.

Martin Lazar (U. Dubrovnik, Croatia): *Eigenvalue decay bounds for the Gramian operator of the heat equation.*

**Abstract:** The talk deals with the eigenvalue decay of solutions to operator Lyapunov equations, a relevant topic in the context of model reduction for parabolic control problems. We mainly focus on the Gramian operator that arises in the context of control and observation of heat processes in infinite time. By improving existing energy and observability estimates for parabolic equations, we obtain both upper and lower bounds on the convergence rate of the eigenvalues of the Gramian operator towards zero. Both bounds follow the same polynomial decay rate, up to a multiplicative constant, which ensures their optimality. This confirms the slow decay of the eigenvalues and limits the efficiency of model reduction. The theoretical findings are supported by numerical results.

Pierre Lissy (ParisTech, France): *Luenberger observers for the linearized water-wave equation (LWWE).*

**Abstract:** The aim of a Luenberger observer is to reconstruct asymptotically the state of a system, based on partial measurements. This is a crucial tool in control theory, notably in order to stabilize a system by state feedback. Here, we will explain the difficulties that appear if we want to implement a Luenberger observer for the full LWWE on the torus, when the measurement is done on a subinterval of the torus. Then, we will give a remedy that enables to reconstruct the low frequencies with an explicit exponential decay rate. This is a work in progress with Lucas Perrin.

Kang Liu (FAU Erlangen, Germany): *Representation and Regression Problems in Neural Networks: mean-field relaxation, generalization, and numerics.*

**Abstract:** In this presentation, we investigate three non-convex optimization problems related to shallow neural networks (NN), including exact and approximate representation and regression. On the theoretical side, we convexify these problems using a "mean-field" technique and demonstrate that there are no relaxation gaps with the help of a "representer theorem". We also derive a generalization bound of shallow NN that depends on the value of these problems and their hyperparameters. This bound provides insight into the choice of optimal hyperparameters for the primal problems. On the numerical side, we describe the discretization approach for the convexified problems and provide their convergence rates. The discretized problems can be addressed using the simplex method, which leads to sharp approximate solutions of the primal problems.

Lorenzo Liverani (FAU Erlangen, Germany): *SA-NODEs and the Universal Approximation of Dynamical Systems.*

**Abstract:** In this talk, I will introduce the novel framework of semi-autonomous neural ordinary differential equations (SA-NODEs), a variation of vanilla NODEs employing fewer parameters. This is achieved by making the coefficients of the SA-NODEs independent of time. Despite this apparent simplification, I will demonstrate that SA-NODEs retain all the strong approximation properties of Vanilla NODEs, both from a theoretical and a numerical perspective. Specifically, SA-NODEs are able to learn the global flow of a dynamical system and track the entire trajectory over a finite (but arbitrary) time horizon. I will conclude the talk by presenting several numerical experiments, showing that SA-NODEs perform well for various systems and significantly outperform vanilla NODEs. This is joint work with Z. Li, K. Liu, and E. Zuazua.

Morgan Morancey (U. Aix-Marseille, France): *Controllability of parabolic problems by the moment method in higher dimension.*

**Abstract:** The moment method is very efficient to study controllability of 1D parabolic problems. First I will explain why this method can also be adapted in higher dimension. Then, I will present a recent result - obtained in collaboration with F. Ammar Khodja, A. Benabdallah, M. Gonzalez-Burgos and L. de Teresa - where we construct and estimate biorthogonal families in cylindrical geometries. Our setting mostly reduces to nice 1D assumption in the tangent variable and a spectral inequality for eigenvectors of the transverse operator.

Arick Shao (Queen Mary University of London, UK): *Control of parabolic equations with inverse square infinite potential wells.*

**Abstract:** We consider heat operators on a bounded domain, with a critically singular potential diverging as the inverse square of the distance to the boundary of the domain. We establish boundary control results for such operators in all spatial dimensions. First, we prove null controllability for a range of parameters

representing the strength of the singular potential, and with control imposed on the entire boundary. Next, we prove a weaker approximate control result, but for the full range of parameters with a viable well-posedness theory, and with the control localised to any open subset of the boundary. This is joint work with Alberto Enciso (ICMAT) and Bruno Vergara (Brown).

Michael Schuster (FAU Erlangen, Germany): *Location Problem for Compressor Stations in Pipeline Networks.*

**Abstract:** In the operation of pipeline networks, compressors play a crucial role in ensuring the networks functionality for various scenarios. In this contribution we address the important question of finding the optimal location of the compressors. This problem is of a novel structure, since it is related with the gas dynamics that governs the network flow. That results in non-convex mixed integer stochastic optimization problems with probabilistic constraints. Using a steady state model for the gas flow in pipeline networks including compressor control and uncertain loads given by certain probability distributions, the problem of finding the optimal location for the control on the network, s.t. the control cost is minimal and the gas pressure stays within given bounds, is considered. In the deterministic setting, explicit bounds for the pipe length and the inlet pressure, s.t. a unique optimal compressor location with minimal control cost exists, are presented. In the probabilistic setting, an existence result for the optimal compressor location is presented and the uniqueness of the solution is discussed depending on the probability distribution. For Gaussian distributed loads a uniqueness result for the optimal compressor location is presented. Further the problem of finding the optimal compressor locations on networks including the number of compressor stations as variable is considered. Results for the existence of optimal locations on a graph in both, the deterministic and the probabilistic setting, are presented and the uniqueness of the solutions is discussed depending on probability distributions and graph topology. The paper concludes with an illustrative example demonstrating that the compressor locations determined using a steady state approach are also admissible in transient settings.