Encoding Binary Variables For Efficient Use In A Quantum Computer

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Quantum computers in the NISQ era (noisy, intermediate-scale, quantum) still offer a relatively small number of qubits. The largest quantum computers so far, dedicated to binary optimization, do not surpass a few thousand qubits. We nevertheless are willing and able to probe such computers in real-life tasks with their high demand in number of variables to optimize over.

We tackle binary optimization based on a real-world problem of selecting photos to be taken from a satellite among the many photo requests made. We discuss the issues around its conversion to a QUBO (quadratic unconstrained binary optimization), which is the kind of problem treated by quantum annealers such as the D-Wave quantum computers. The two main issues to be tackled are the use of penalties and quadratization.

Penalties are necessary to treat real-world problems, which are almost always constrained, in hardware designed for unconstrained problems. Considering a minimization problem, the basic logic is that of adding terms that increase the value of the optimized function whenever any constraint is broken, but preserve its value when constraints are obeyed. A proper choice of penalty can have severe impact on performance.

Quadratization amounts to converting higher-order binary polynomials into quadratic. Naturally, this cannot be done in a literal sense, but it amounts to finding an equivalent polynomial that is quadratic and that shares a solution with the original one. It often requires the addition of extra variables, which is quite problematic given the tight restrictions on the number of qubits available.

Different encodings of the original problem's variables problem are discussed, and, as one can imagine, there are advantages to replacing an encoding for a denser one (in computation time, in the number of variables, in capability). However, surprisingly, using the densest possible encoding into binary variables becomes counterproductive after a certain degree in the context of QUBO optimization (Fig.1). We show how this result comes about in the context of different forms of quadratization and expose their advantages and disadvantages for use in a quantum computer. In particular, the interplay of choice of penalty coefficients and quadratization will be discussed.

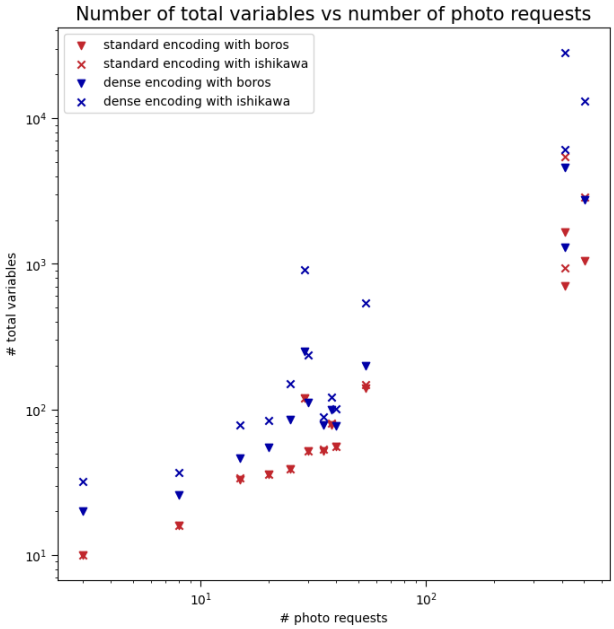

Figure 1 – Number of variables for different encodings of the problem of choosing photos over all requests. Counterintuitively, the denser encoding requires more variables (hence, more qubits) in total.