Problema 1)

Given the experimental measurements

$$\hat{\alpha}^{-1} = 137.035999180(10)$$

$$\hat{G}_F = 1.1663788(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$$\hat{m}_Z = 91.1876(21)\ \mathrm{GeV}$$

$$\hat{m}_W = 80.377(12)\ \mathrm{GeV}$$

$$\hat{s}^2_{eff} = 0.23148(33)$$

$$\hat{\Gamma}^{(Z)}_{e^+e^-} = 83.942(85)\ \mathrm{MeV}$$

Show explicitly via a χ^2 fit that the SM EW sector at tree-level is unable to successfully fit the above set of measurements.

$$\chi^2 = \sum_{\substack{i \\ \text{(observables)}}} \frac{(\hat{O}_i - O_i)^2}{(\Delta \hat{O}_i)^2}$$

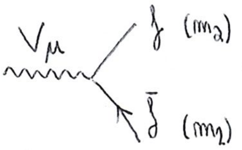
Problema 2)

Given the following combinations of (1-loop) self-energies

- $$\frac{\Pi_{WW}(p^2=0)}{m_W^2} - \frac{\Pi_{ZZ}(p^2=0)}{m_Z^2}$$

- $$\frac{\Pi_{WW}(p^2=m_W^2) - \Pi_{WW}(p^2=0)}{m_W^2} - c^2\left(\frac{\Pi_{ZZ}(p^2=m_Z^2) - \Pi_{ZZ}(p^2=0)}{m_Z^2}\right)$$
$$- s^2\,\frac{\Pi_{\gamma\gamma}(p^2=m_Z^2)}{m_Z^2} - 2sc\left(\frac{\Pi_{\gamma Z}(p^2=m_Z^2) - \Pi_{\gamma Z}(p^2=0)}{m_Z^2}\right)$$

Check the UV-finiteness of them by explicit calculation of fermion self-energies (specifically, top & bottom quark loops)

$\Rightarrow$ For a generic vertex 

with Feynman rule given by $i A \gamma_\mu (v - a\gamma_5)$, the self-energy $\Pi^{\mu\nu}_{VV'}$ is given in terms of Passarino-Veltman integrals by

$$\Pi^{\mu\nu}_{VV'} = \frac{AA'}{4\pi^2}\Big\{(vv'+aa')\Big[2p^\mu p^\nu (B_{21}-B_1) + g^{\mu\nu}\left(-2B_{22} - p^2 B_{21} + p^2 B_1\right)\Big] + (vv'-aa')\, g^{\mu\nu} m_1 m_2 B_0\Big\}$$

⇒ The divergent pieces of the Passarino-Veltman integrals B_{22}, B_{21}, B_1, B_0 are:

$\Delta = 1/(4-d) - \gamma_E + \ln 4\pi$

$$B_{22} = \left(\frac{m_1^2 + m_2^2}{4} - \frac{p^2}{12} \right) \Delta$$

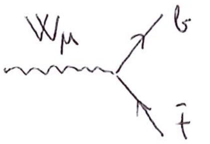
$$B_{21} = \Delta/3$$

$$B_1 = \Delta/2$$

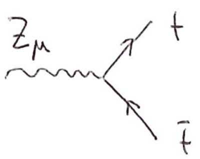
$$B_0 = \Delta$$

⇒ Remember we are only interested in the part of $\Pi^{\mu\nu}_{VV'}$ proportional to $g^{\mu\nu}$

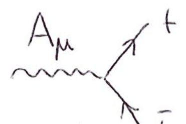
⇒ The relevant SM Feynman rules for the top + bottom quark contribution to the self-energies are:



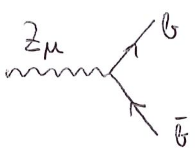
$$\frac{ig}{\sqrt{2}\cdot 2}\gamma_\mu(1-\gamma_5)$$



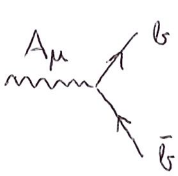
$$\frac{ig}{4c}\gamma_\mu\left[\left(1-8/3\,s^2\right)-\gamma_5\right]$$



$$ieQ_t\gamma_\mu$$



$$\frac{ig}{4c}\gamma_\mu\left[\left(1+4/3\,s^2\right)+\gamma_5\right]$$



$$ieQ_b\gamma_\mu$$