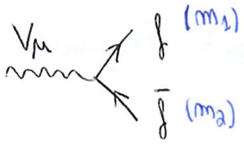
Problemas

- Check the UV-finiteness of S, T, U by explicit calculation of fermion self-energies (e.g. top-bottom loops)

We can borrow some of the results from arxiv hep-ph/0512342 (Wells)

For a generic vertex 

given by $iA\gamma_\mu(v - a\gamma_5)$

the self-energy $\Pi^{\mu\nu}_{VV'}$ is given by (in terms of Passarino - Veltman integrals)

$$\Pi^{\mu\nu}_{VV'} = \frac{AA'}{4\pi^2}\Big\{(vv' + aa')\Big[2p^\mu p^\nu (B_{21} - B_1) + g^{\mu\nu}\big(-2B_{22} - p^2 B_{21} + p^2 B_1\big)\Big] + m_1 m_2 (vv' - aa') g^{\mu\nu} B_0\Big\}$$

- As discussed in the lecture, we are only interested here in the $\Pi^{\mu\nu}_{VV'}$ piece proportional to $g^{\mu\nu}$

- The divergent pieces of the Passarino - Veltman integrals are given by

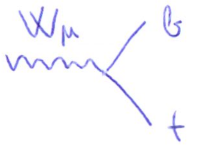
$$B_{22} = \left(\frac{m_1^2 + m_2^2}{4} - \frac{p^2}{12}\right)\Delta \quad ; \quad B_{21} = \frac{\Delta}{3} \quad ; \quad B_1 = \frac{\Delta}{2}$$

$$B_0 = \Delta$$

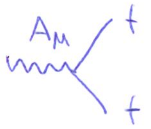
$$\Delta \equiv \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi$$

Then $\Pi^{(\Delta)\mu\nu}_{VV'} = g^{\mu\nu} \cdot \frac{AA'}{4\pi^2} \left\{ (bb' + aa') \left(-\frac{(m_1^2 + m_2^2)}{2} + \frac{p^2}{3} \right) + (bb' - aa')\, m_1 m_2 \right\} \Delta$

Now, for the top + bottom quark contribution to the self-energies (the divergent part) the relevant Feynman rules are:

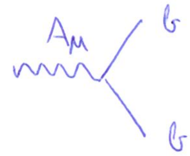


$\frac{ig}{2\sqrt{2}} \gamma_\mu (1 - \gamma_5)$

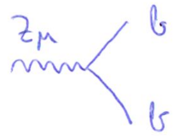


$ie\,\frac{2}{3}\,\gamma_\mu$

$\frac{ig}{4c} \gamma_\mu \left[\left(1 - \frac{8}{3} s^2\right) - \gamma_5 \right]$



$-ie\,\frac{1}{3}\,\gamma_\mu$



$\frac{ig}{4c} \gamma_\mu \left[\left(1 + \frac{4}{3} s^2\right) + \gamma_5 \right]$

- UV-finiteness of T:

$$\Pi^{\Delta}_{ZZ}(0) = \frac{g^2}{16c^2} \cdot \frac{1}{4\pi^2} \left\{ -2m_t^2 - 2m_b^2 \right\} \Delta$$

$$\Pi^{\Delta}_{WW}(0) = \frac{g^2}{8} \frac{1}{4\pi^2} \left\{ -m_t^2 - m_b^2 \right\} \Delta$$

$$\frac{\Pi^{\Delta}_{WW}(0)}{m_W^2} - \frac{\Pi^{\Delta}_{ZZ}(0)}{m_Z^2} = \frac{g^2}{4\pi^2} \Delta \left\{ \frac{-m_t^2 - m_b^2}{8 \underbrace{m_Z^2 c^2}_{m_W^2}} - \frac{(-m_t^2 - m_b^2)}{8 m_Z^2 c^2} \right\} = 0$$

UV - finiteness of U :

Note : $\Pi^{(\Delta)\mu\nu}_{VV'}(p^2 = m_X^2) - \Pi^{(\Delta)\mu\nu}_{VV'}(p^2=0) =$

$$\frac{\Delta}{4\pi^2}\, g^{\mu\nu} \cdot AA'\,(vv' + aa')\,\frac{m_X^2}{3}$$

- $$\frac{\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2} = \frac{\Delta}{4\pi^2}\,\frac{1}{3}\,\frac{g^2}{8}\cdot(2)$$

- $$c^2\left(\frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}\right) = \frac{\Delta}{4\pi^2}\,\frac{1}{3}\,c^2\,\frac{g^2}{16\,c^2}\left\{\left(1-\tfrac{8}{3}s^2\right)^2 + \left(1+\tfrac{4}{3}s^2\right)^2 + 2\right\}$$

- $$s^2\,\frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2} = \frac{\Delta}{4\pi^2}\,\frac{1}{3}\,g^2 s^4\left(\frac{4}{9}+\frac{1}{9}\right)$$

- $$2sc\left(\frac{\Pi_{\gamma Z}(m_Z^2) - \Pi_{\gamma Z}(0)}{m_Z^2}\right) = \frac{\Delta}{4\pi^2}\,\frac{1}{3}\,\frac{g^2 s^2}{2}\left\{\left(1-\tfrac{8}{3}s^2\right)\tfrac{2}{3} - \tfrac{1}{3}\left(1+\tfrac{4}{3}s^2\right)\right\}$$