Spectral gaps in PEPS:
the possible and the impossible

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QUITEMAD Quantum Information Technologies in Madrid

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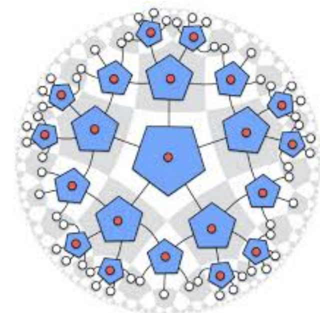
Project 648913

Bulk-Boundary correspondences



Maldacena's AdS - CFT

Li - Haldane's Entanglement Spectrum

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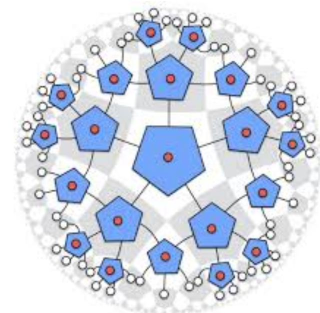

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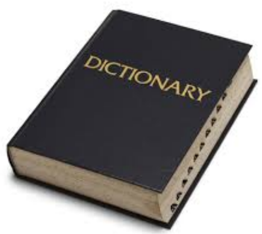

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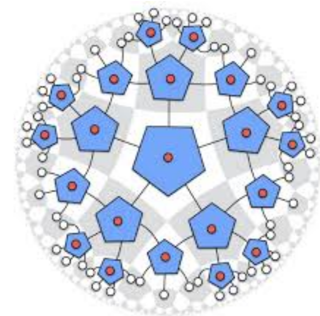

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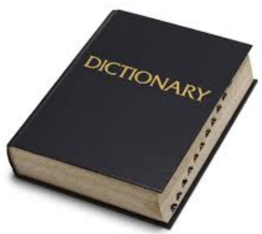

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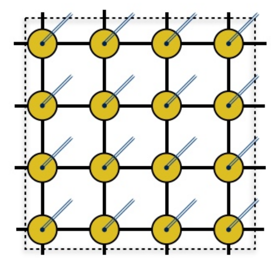
In PEPS there is a very natural bulk - boundary correspondence

Cirac, Poilblanc, Schuch, Verstraete, PRB **83**, 245134, 2011



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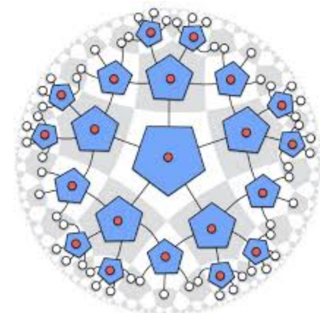

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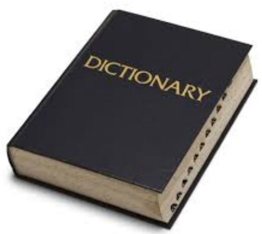

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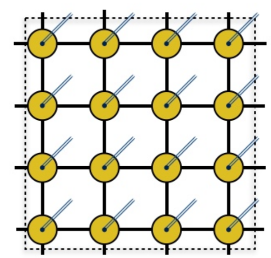
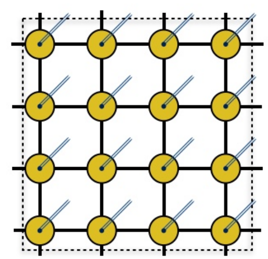


Bulk gap ⟷ locality of boundary Hamiltonian

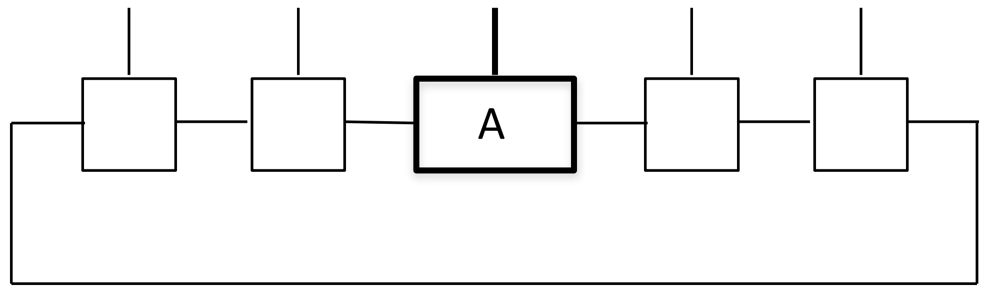
Bulk-Boundary correspondence in PEPS

Cirac et al 2011.

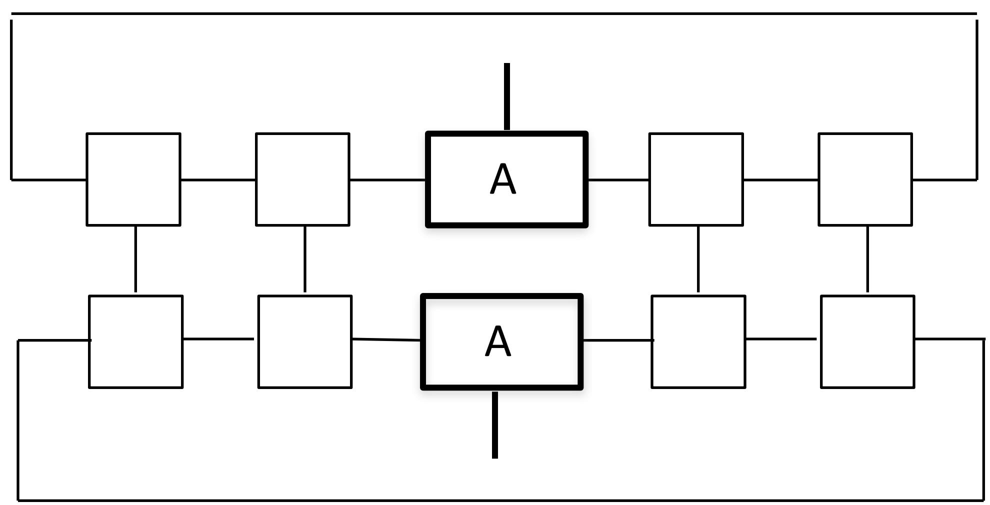


Bulk-boundary. Illustration in 1D

$|\psi\rangle =$



$\rho_A =$



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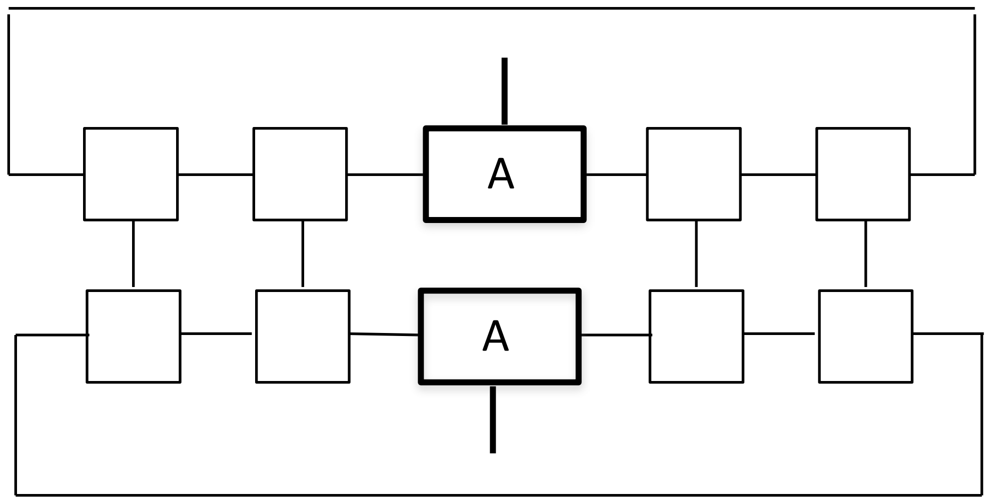
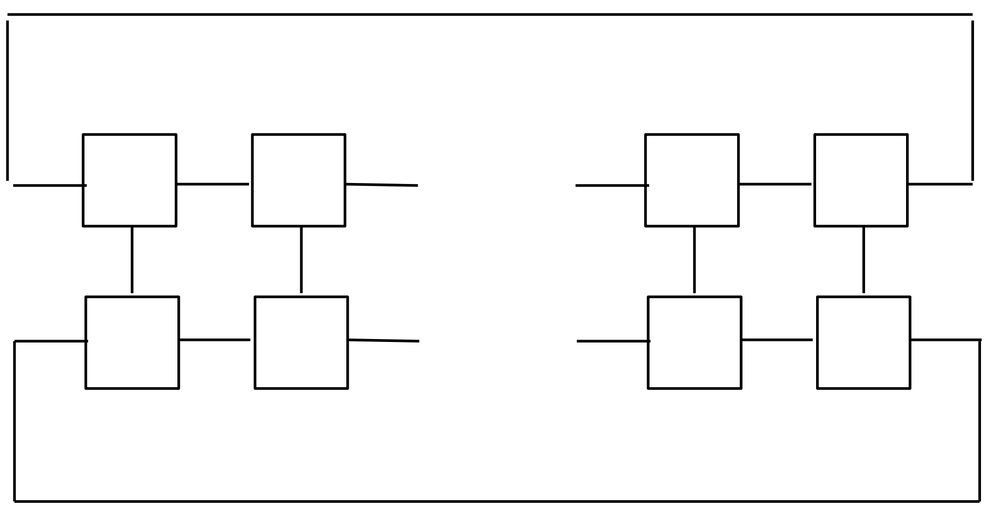


$\rho_{\partial A^c} =$

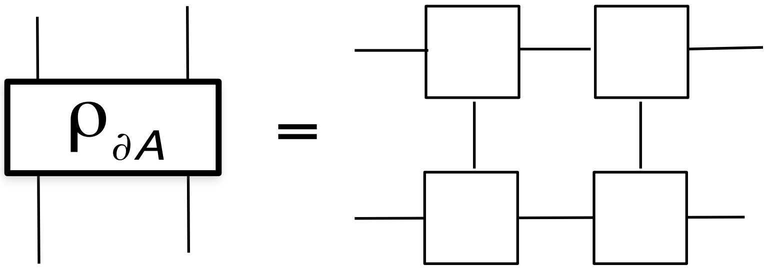


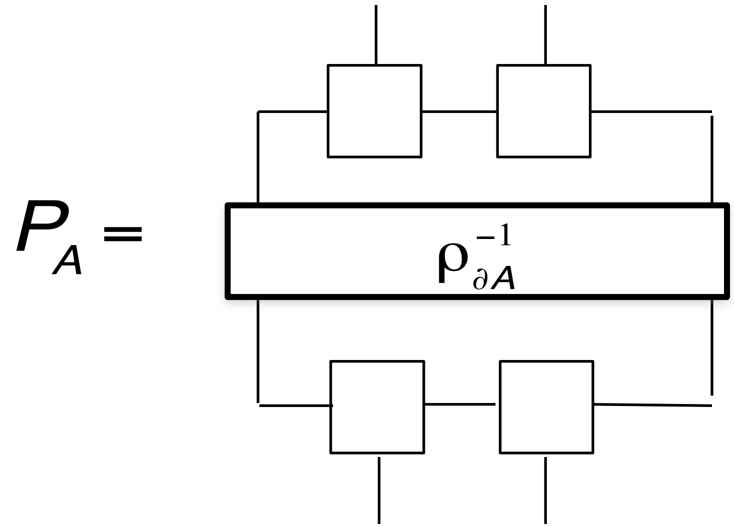
Boundary state

Lives on the virtual d.o.f connecting A & A^c

Encodes the correlations of the system

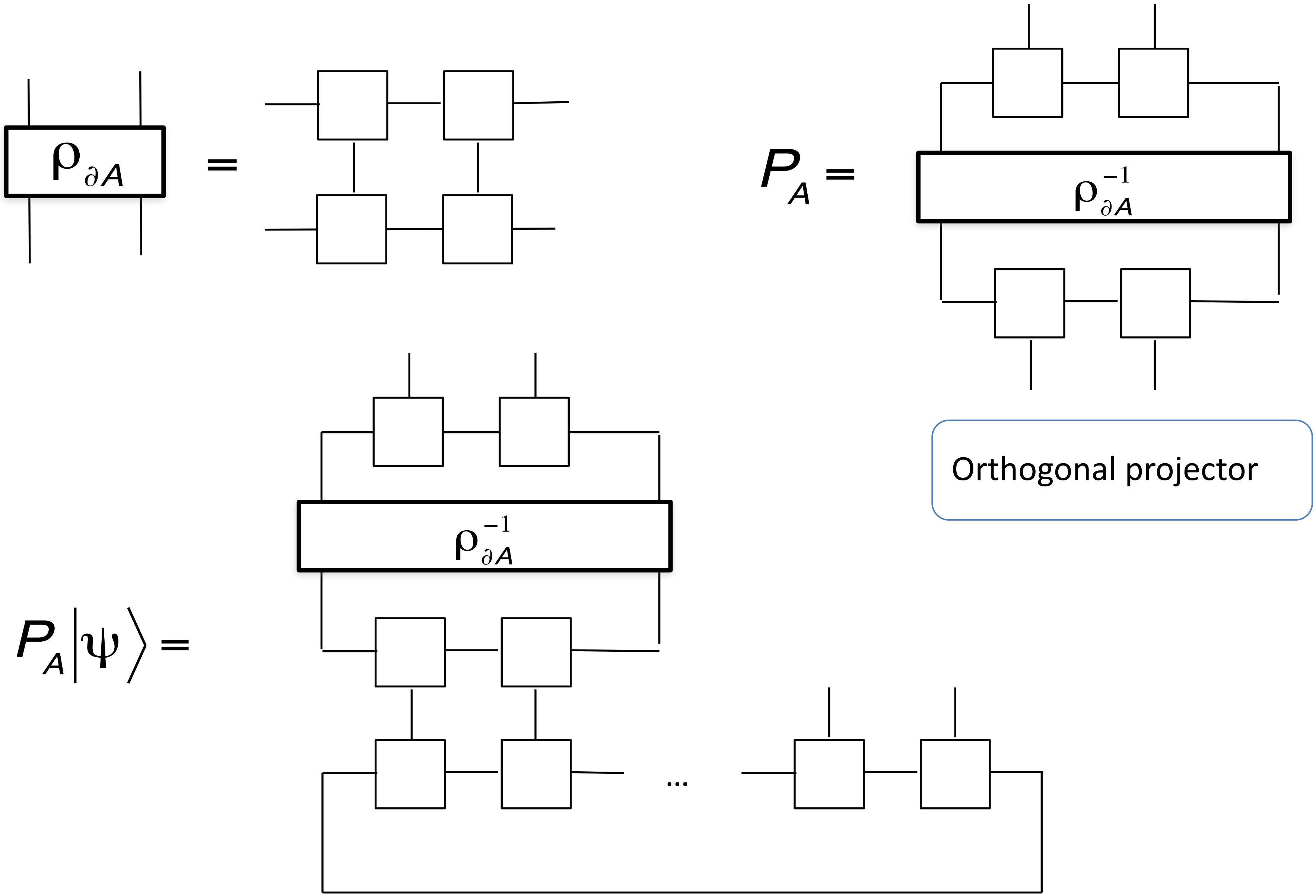
Boundary state





Orthogonal projector

Boundary state



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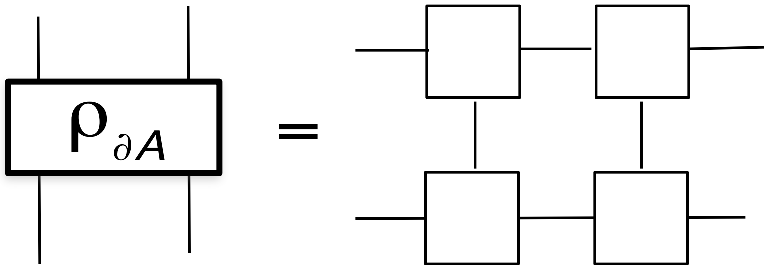

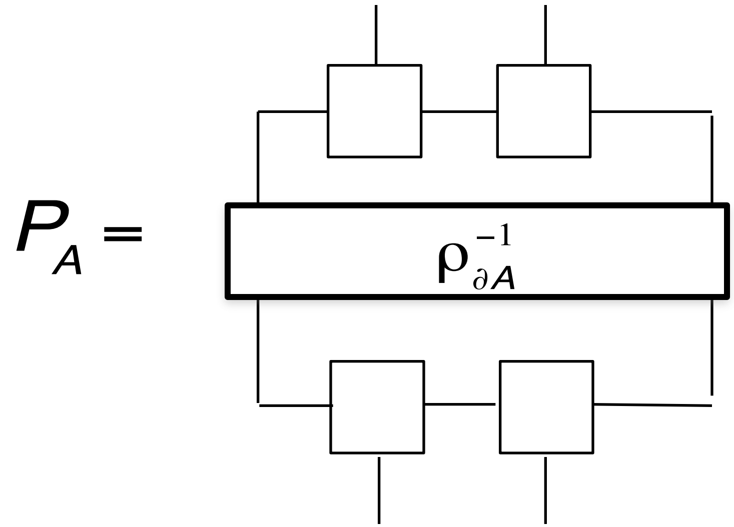

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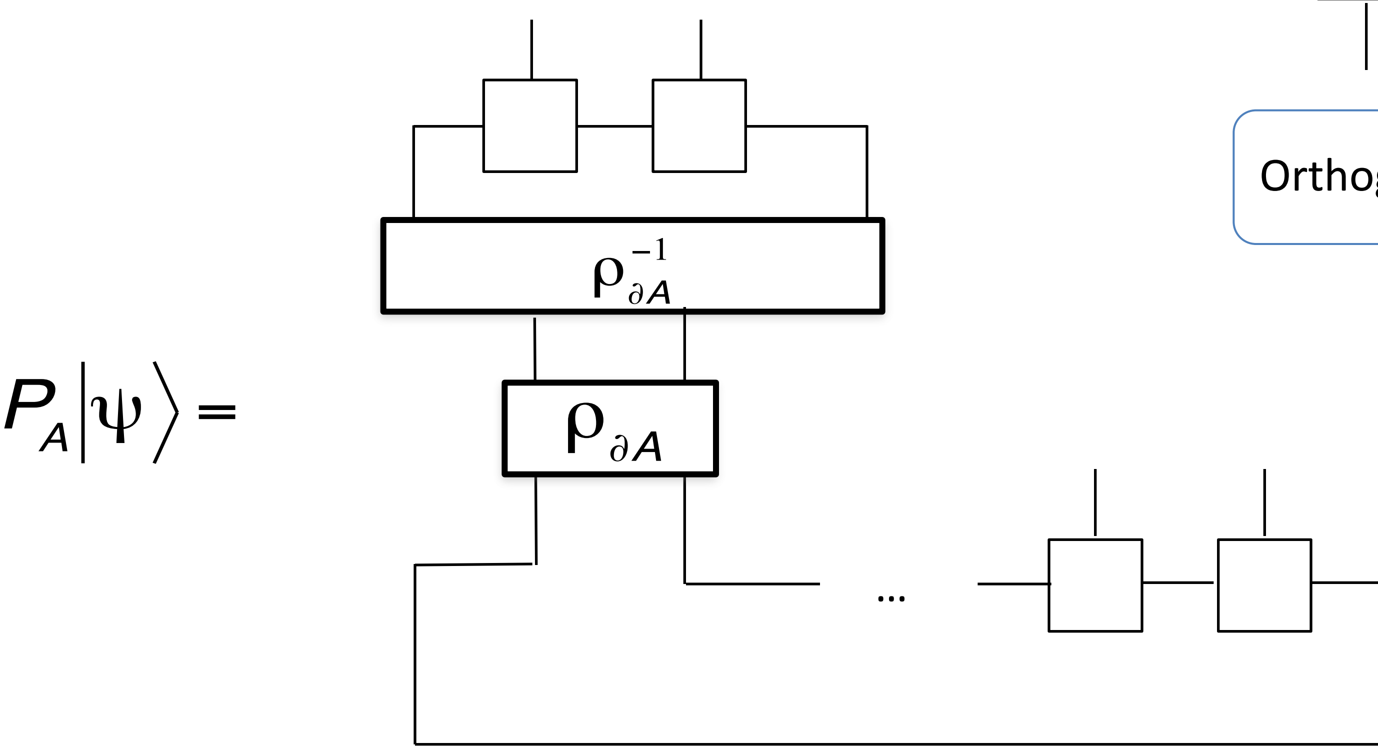

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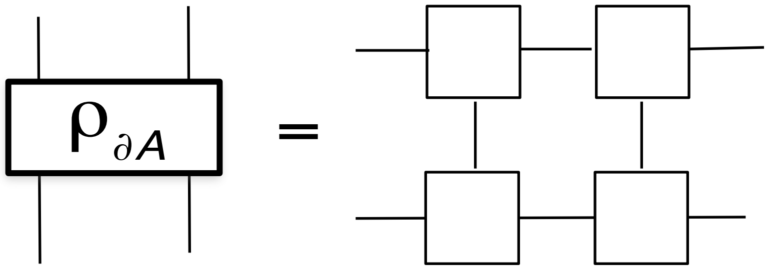

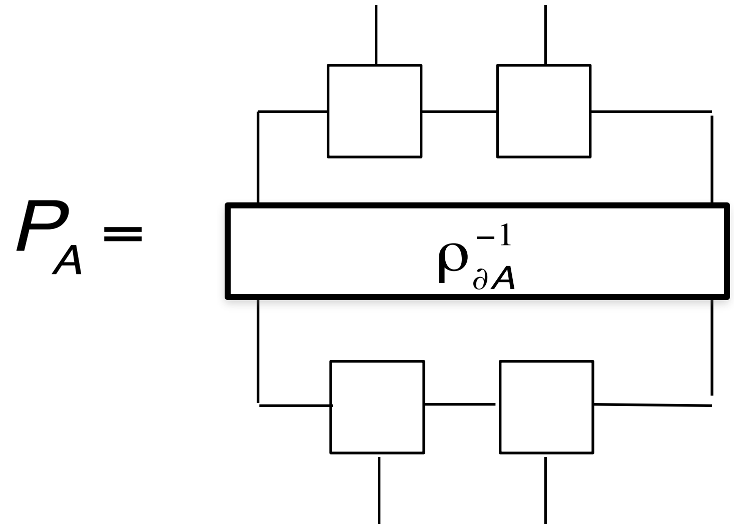

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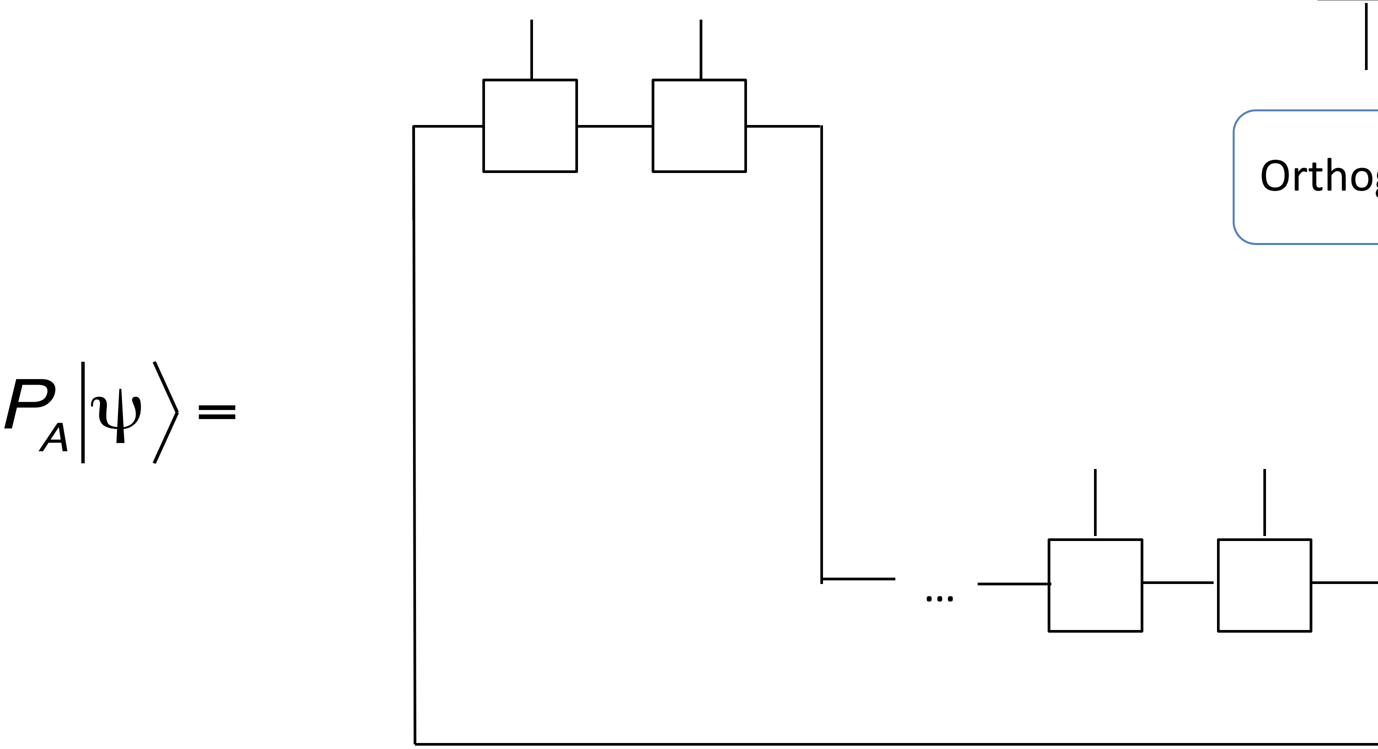

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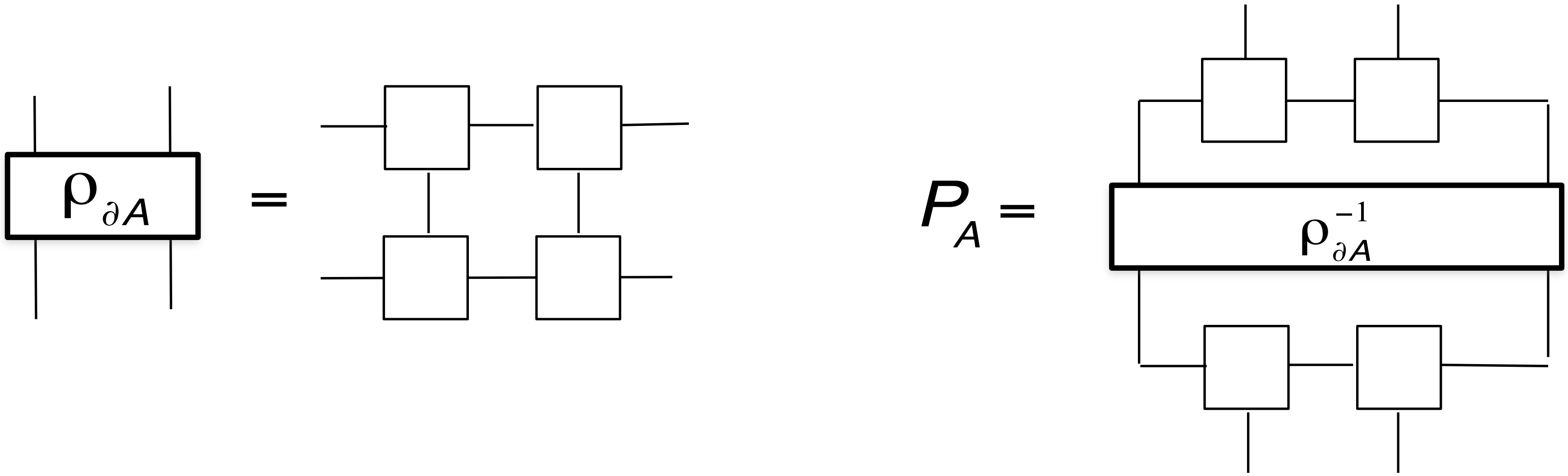

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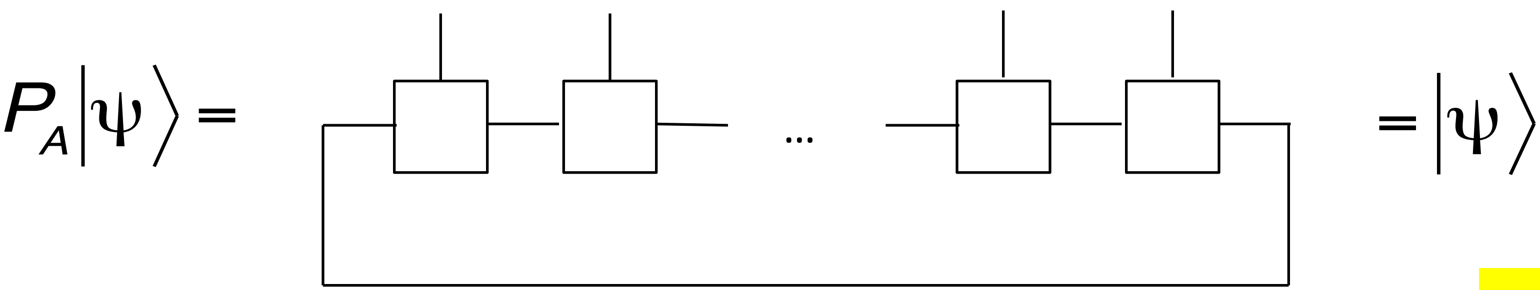
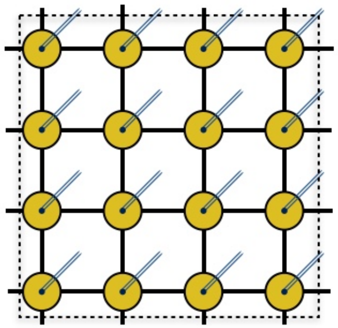
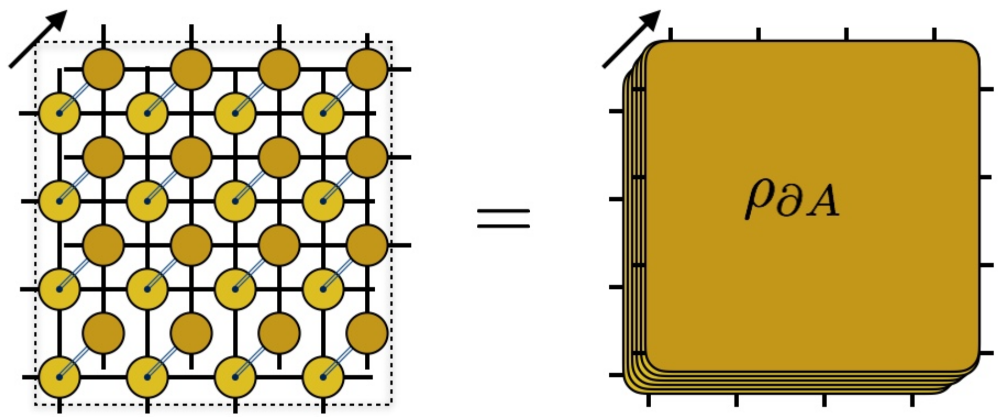


$$H = \sum_i (1 - P_i)$$

Boundary state





It is a mixed 1D state living on the virtual d.o.f.
Mediates the correlations in the system
Defines the parent Hamiltonian of the state

Spectral gap via boundary state

M. Kastoryano, A. Lucia, DPG, Commun. Math. Phys. (2019) 366: 895

Spectral gap in PEPS

Conjecture Cirac et al. 2011 (numerical evidence): the parent Hamiltonian of the PEPS has gap if and only if the boundary state is the Gibbs state of a short-range Hamiltonian.

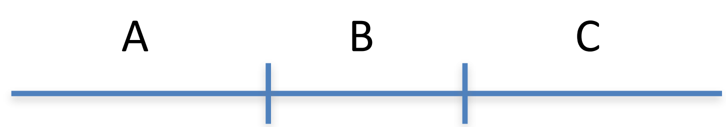
Intuition. Araki's theorem: Gibbs state of *finite range* 1D Hamiltonians have exponentially decaying correlations

Remember that boundary states mediate the correlations in a PEPS.

Spectral gap in PEPS

Theorem 1: If the boundary state is approximately factorizable, then the bulk Hamiltonian is gapped.

A 1D state is approximately factorizable if $\rho_{ABC} \approx \Lambda_{AB}\,\Omega_{BC}$



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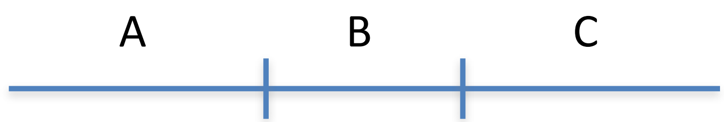

The case of *exact* factorization implies that the Hamiltonian terms ($1-P_i$) commute with each other and hence the system is gapped. (Remember boundary states define the Hamiltonian terms)

The approximate case reduces to the martingale condition of Nachtergaele.

Martingale condition is equivalent to gap (Lucia, Kastoryano)

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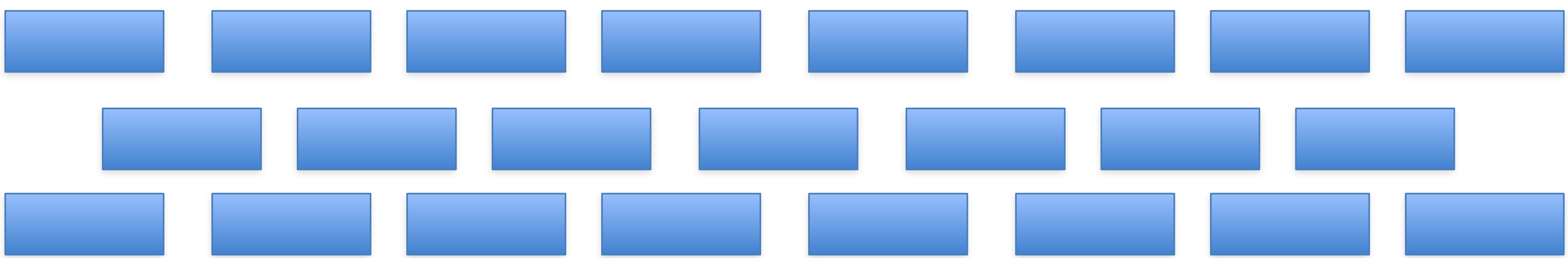
Imagine $e^{-H} \to e^{-iH} \xrightarrow[\textbf{locality}]{}$ **finite depth circuit** $\to \Lambda_{AB}\,\Omega_{BC}$



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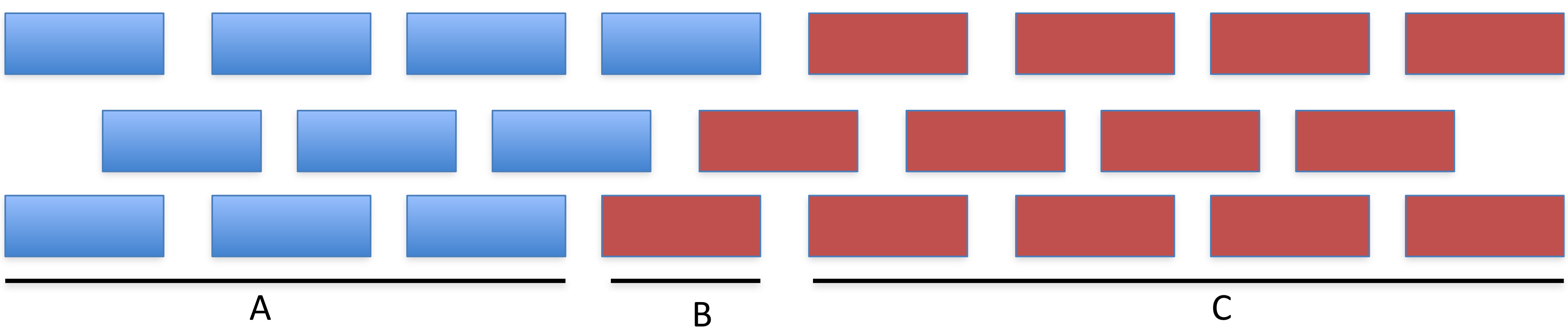

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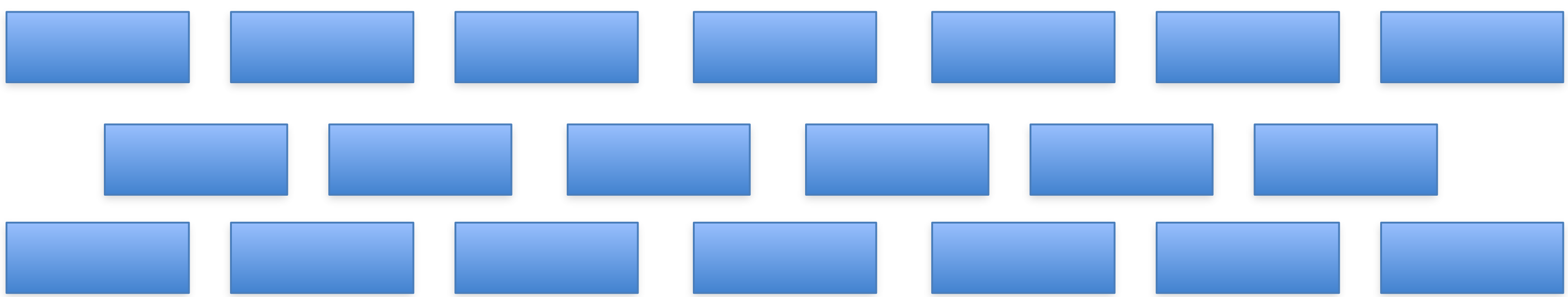
Kuwahara, Alhambra, Anshu, arXiv:2007.11174

$e^{-\beta H} \approx$



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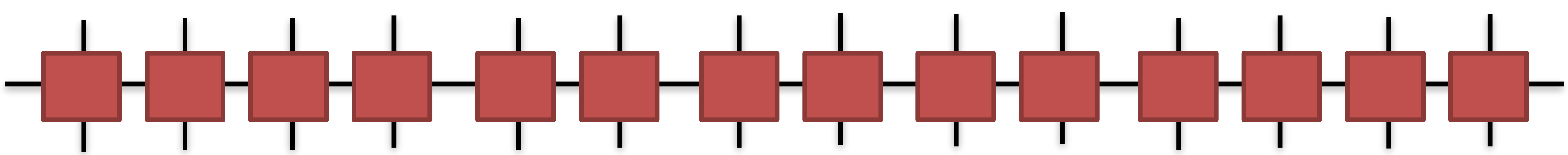

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Question: Is Araki's theorem true for exponentially-decaying interactions?

I.e, does there exist thermal phase transitions in 1D with exponentially decaying interactions?

Spectral gap in PEPS

Theorem 3 (DPG, A. Pérez-Hernández, arXiv:2004.10516):
Let H be a 1D translational invariant Hamiltonian and assume that the interaction strength decays as $\exp(-\alpha\ell)$, then both Araki's theorem and Theorem 2 are true for all $0 < \beta < \lambda$

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How sharp / useful is our gap criterion?

Lifetime of topological quantum memories

Quantum memories

Take a **2D** topological model with finite range commuting Hamiltonian H_{top}

E.g Kitaev's quantum double of a group G (Toric code for $G = \mathbb{Z}_2$).

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$$\rho_t = e^{t\,\mathscr{L}_\beta}(\rho_0)$$

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Short memory time $\Leftrightarrow$ $\mathrm{Gap}(\mathscr{L}_\beta) \geq c_\beta > 0$, for all β

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Alicki, Fannes, Horodecki 2007. For the Toric code: $\mathrm{Gap}(\mathscr{L}_\beta) \geq c_\beta > 0$, for all β

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Komar, Landon-Cardinal, Temme 2016.

For abelian models constant energy barrier implies $\mathrm{Gap}(\mathscr{L}_\beta) \geq c_\beta > 0$, for all β

What about the non-abelian case?

Quantum memories

Theorem 4 (A. Lucia, DPG, A. Pérez-Hernández, in preparation):
For all (even **non abelian**) quantum double models $\mathrm{Gap}(\mathscr{L}_\beta) \geq c_\beta > 0$, for all β

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At each site we do a partial transposition: $|\cdot\rangle\langle\cdot| \to |\cdot\rangle|\cdot\rangle$

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Apply our condition ("easy" to check it in this case).

Thank you for your attention