Benasque WS (2021.02.22-28, Zoom)

Constructing Linearized Superoperator from HOTRG

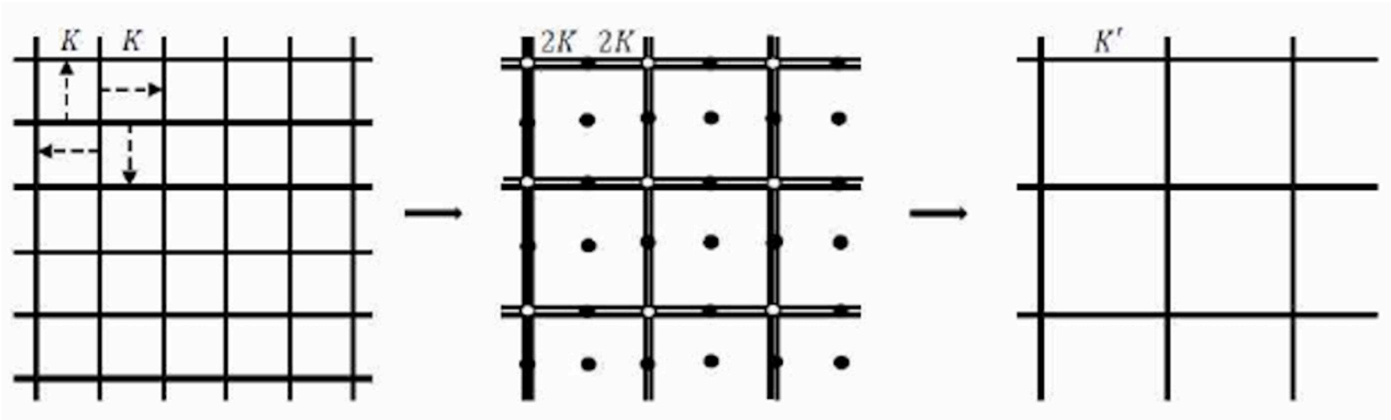
Xinliang LYU (ISSP)

(Lyu, Xu and NK: arXiv:2102.08136)

2021.02.24 Naoki KAWASHIMA (ISSP)

東京大学
物性研究所

The textbook real-space RG --- Migdal-Kadanoff RG



$$y_t \equiv 1/\nu \approx \log 1.676/\log 2 \approx 0.747$$ (for 2d Ising)

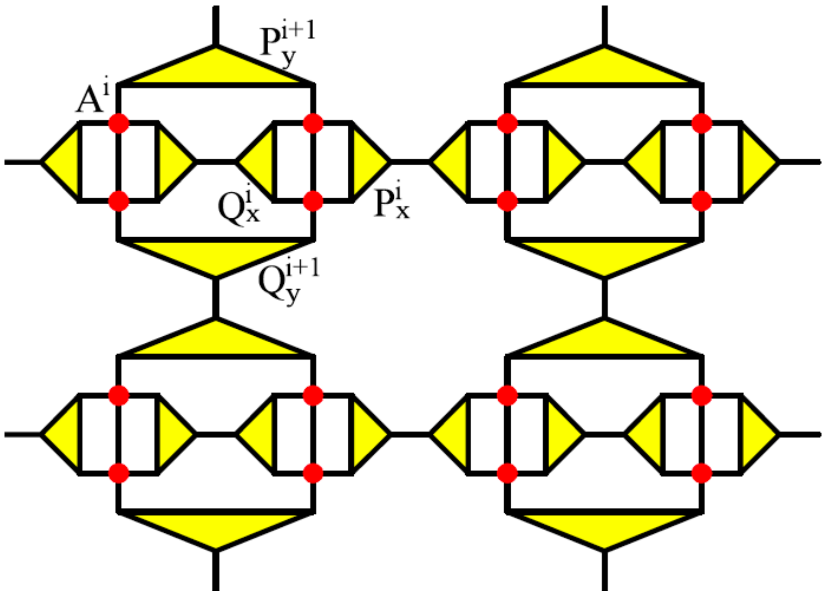
A nice starting-point approximation. However,

not clear how to improve.

Modern Generalization of MKRG --- HOTRG

Xie, Chen, Qin, Zhu, Yang, and Xiang: Phys. Rev. B 86, 045139 (2012).

- Simple
- Easy extension to higher dimensions
- **Systematically improvable (!)**
- **Short-range contribution hides the fixed point.**
- **No direct estimators for scaling dimensions.**

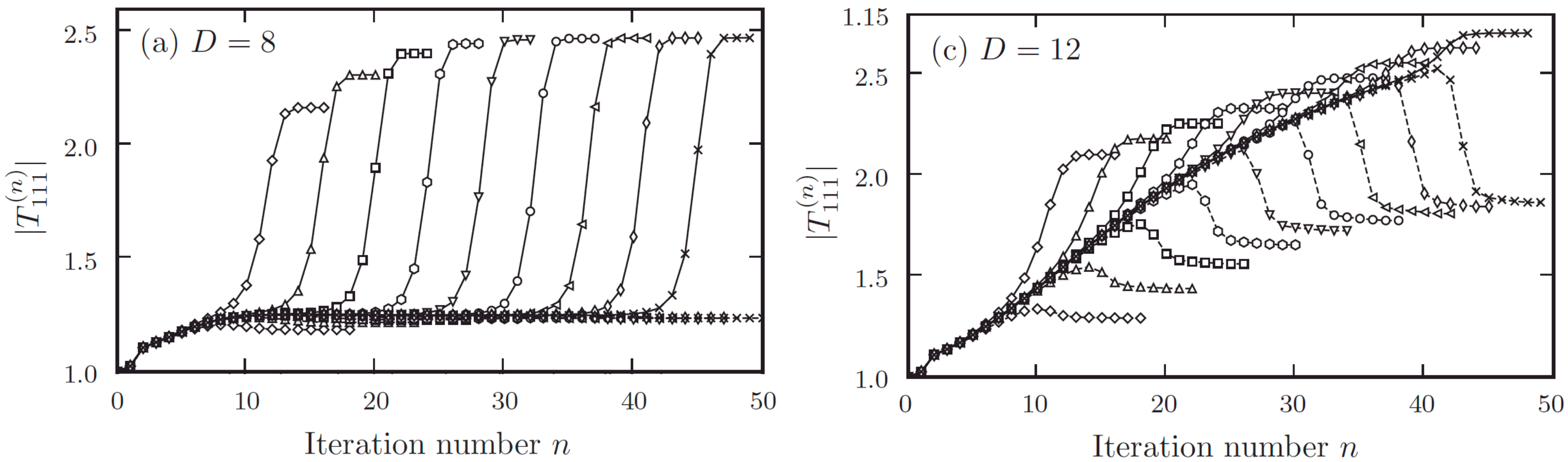


Picture taken from Zhao et al PRB93 125115 (2016)

Convergence to fixed point

<u>TRG (tensor network renormalization a la Levin and Nave)</u>

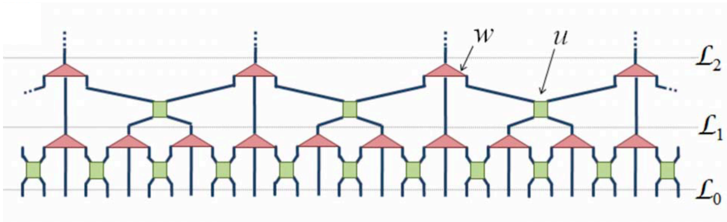
Hinczewski and Berker: PRE 77 011104 (2008)

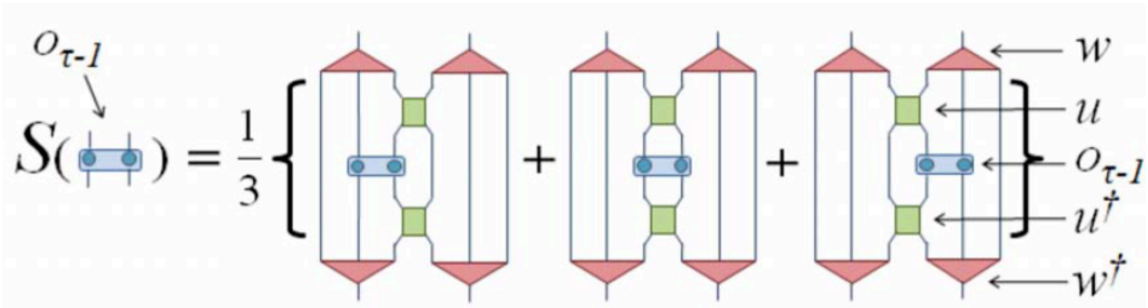


We don't reach the fixed point tensor for large bond dimensions.
(Something similar happens to HOTRG.)

Lineaized Super-operator with MERA
(for quantum systems)

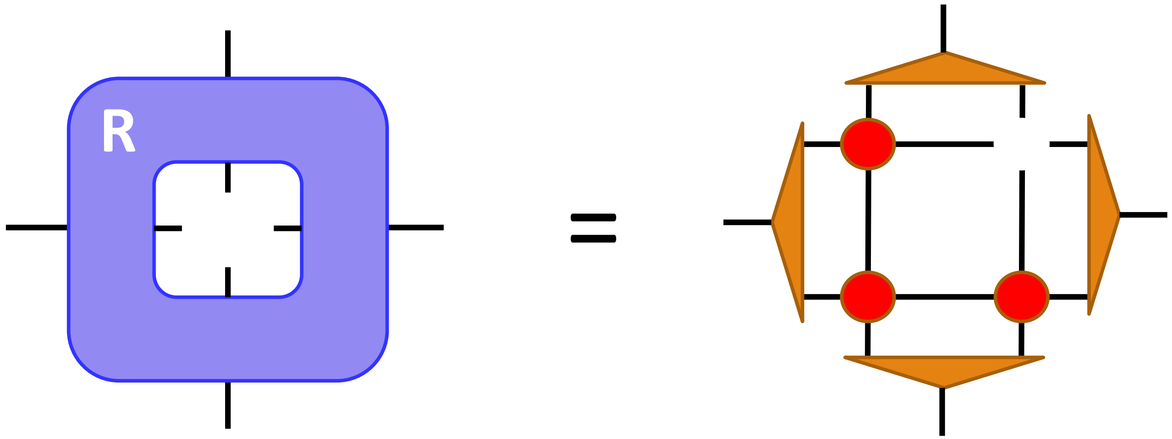
Pfeifer, Evenbly and Vidal: PRA79 040301(R) (2009)





$$\mathcal{S}(\phi_\alpha) = \lambda_\alpha \phi_\alpha, \quad \Delta_\alpha \equiv -\log_3 \lambda_\alpha$$

Naive Construction of Super-operator with HOTRG

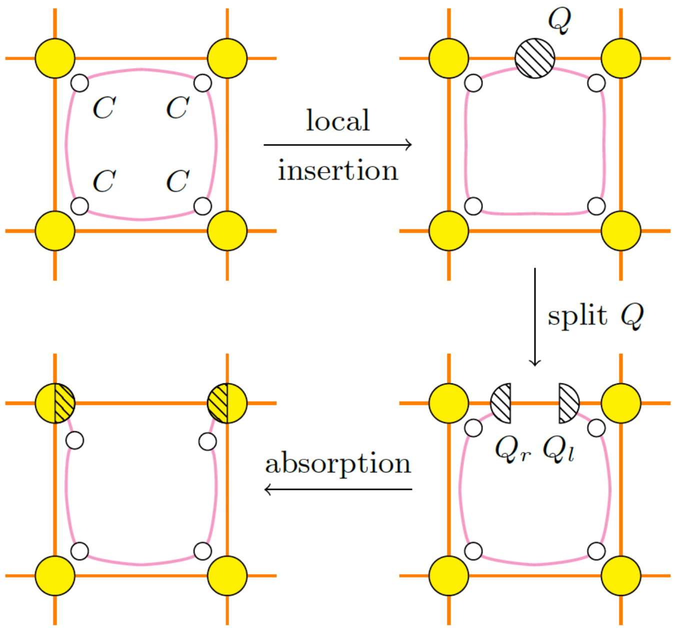


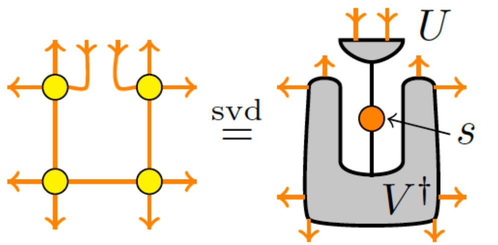
- ✓ Short-range correlation stays no matter how much we repeat RG steps.
- ✓ The gauge is not automatically fixed like the MERA construction.

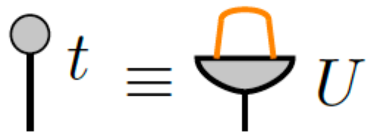
Removal of the short-range correlation

GILT

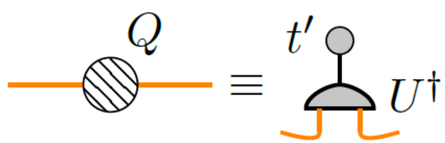
Hauru, Delcamp and Mizera: PRB97 045111 (2018)







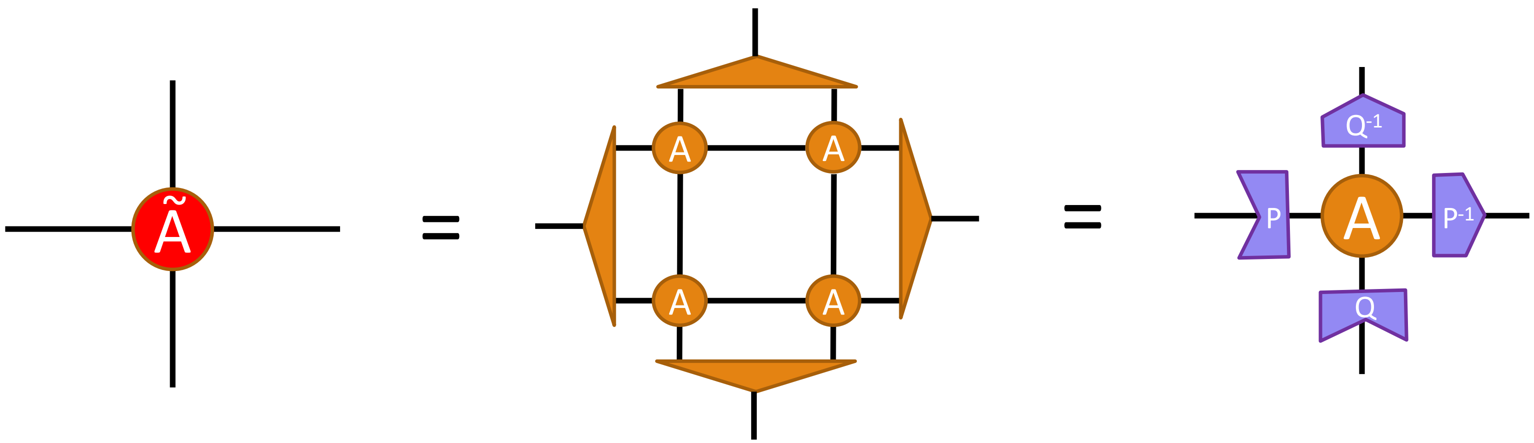
$$t'_i = t_i \frac{s_i^2}{s_i^2 + \epsilon_{\text{gilt}}^2}$$



Q filters out the redundant loops
of short-range correlations

Gauge problem

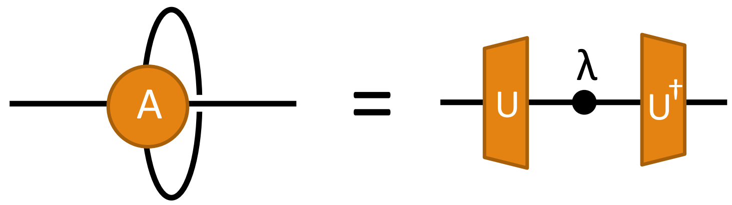
Even after we've reached the "fixed point", we are not sure wether it is the stable solution of our recursive equation.
(Generally we have to expect some gauge change.)



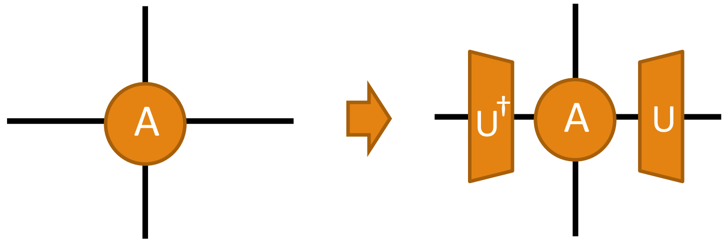
While the whole tensor network may have reached the stable solution, the may still be fluctuating by P and Q.

Gauge Fixing --- How?

1. Construct the transfer matrix, and eigenvalue decomose it, to obtain the prefered gauge ``U''.



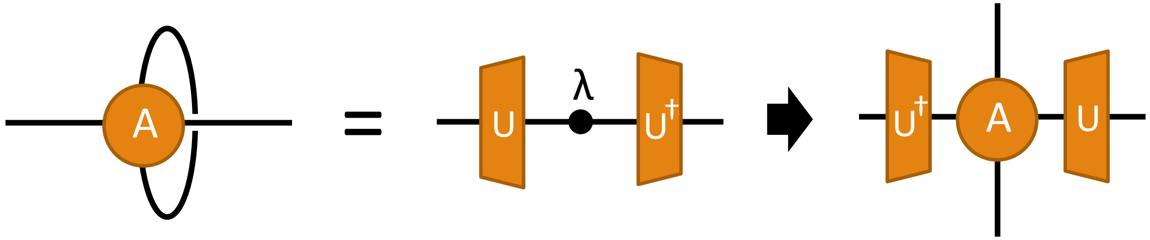
2. Replace A by its orthogonal transform by U.

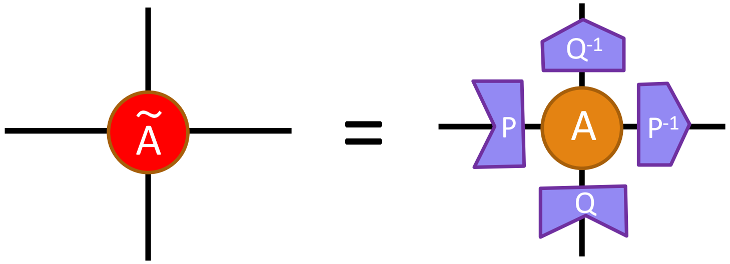


3. Do the same for the vertical direction.

Gauge Fixing --- Why?

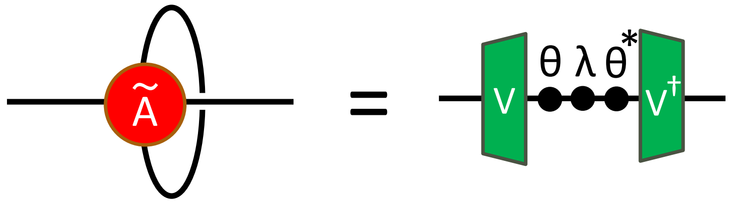
Gauge fixing procedure

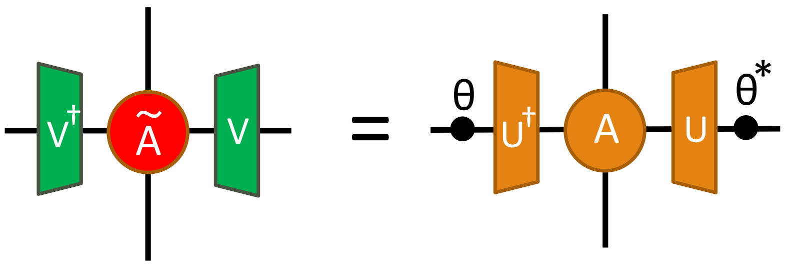




The result of the RG process may have a different gauge than the initial one. (P,Q: not necessarily unitary)

However, with the inversion symmetry, we can assume:





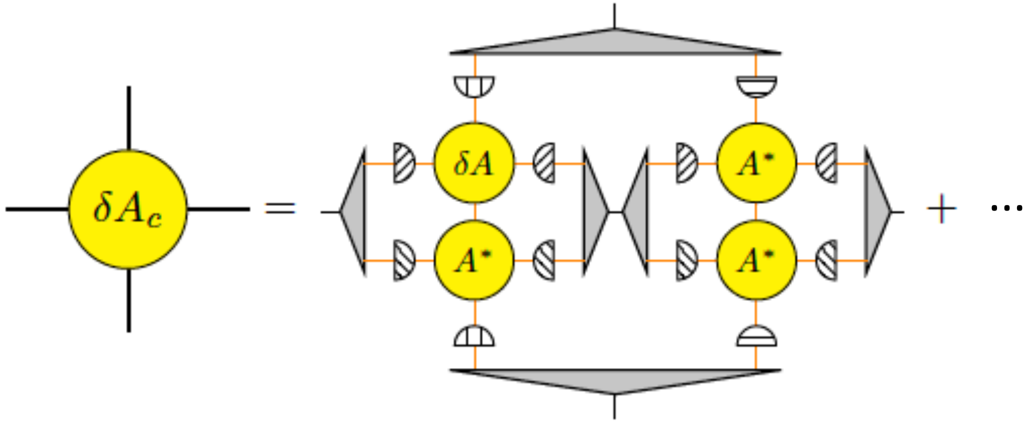
Then, the result of the GF procedure doesn't depend on the initial gauge except the phase factors.

Linearized Super-operator

$$A' = R(A)$$

$$\delta A' = S\,\delta A$$

$$S \equiv \frac{\partial R(A)}{\partial A}$$



* We've used automatic differentiation for the actual computation.

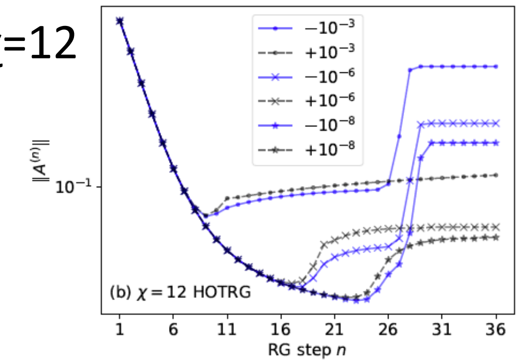
Benchmark (2d Ising)

Step dependence of the tensor norm ratio

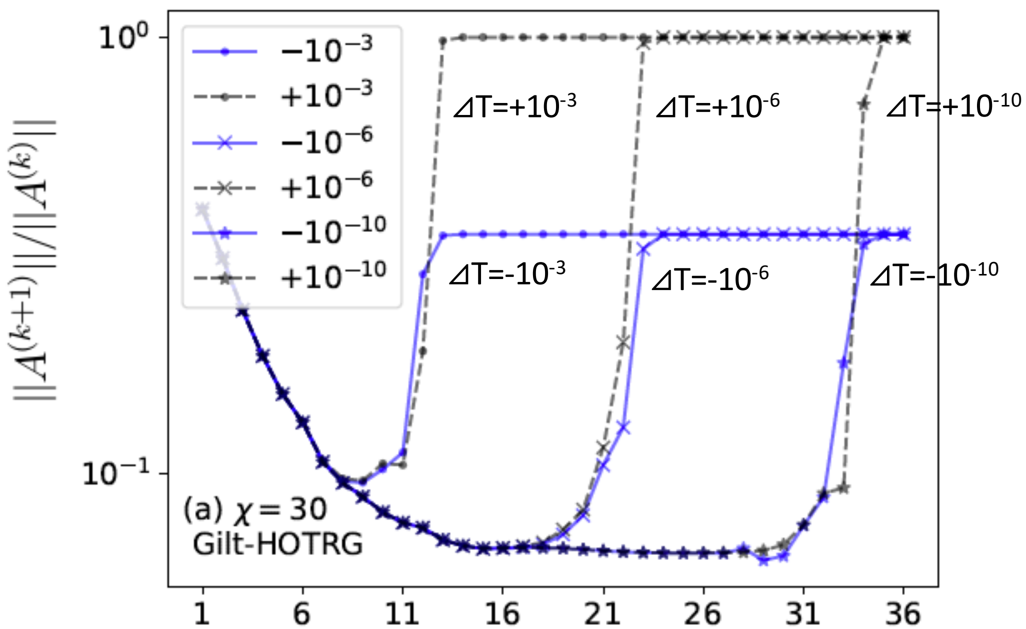
$$\Delta T \equiv \frac{T - T_c^{[\chi=30]}}{T_c}$$

- depending on the temperature, curves start to deviate from the critical curve.
- χ=30 shows longer flat bottom compared to χ=12, indicating approach to the true fixed point as we increase χ.

χ=12



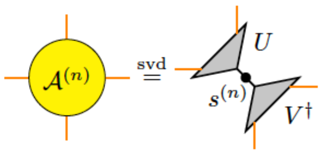
χ=30

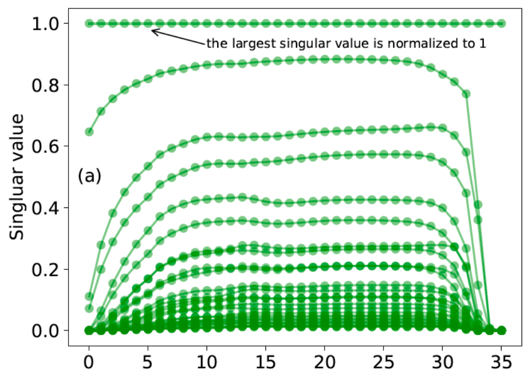


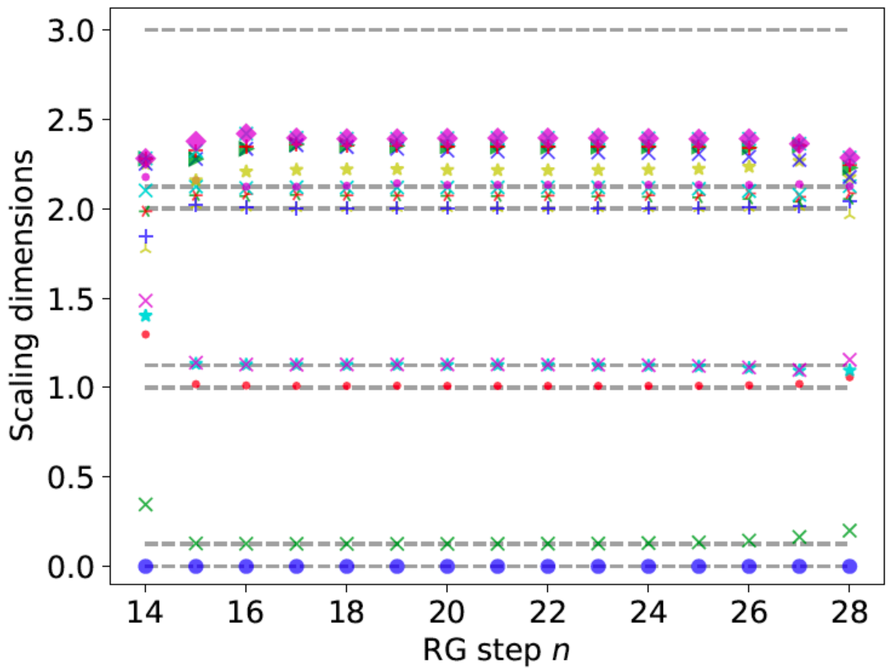
Benchmark (2d Ising)

- The method yields at least 4 digits of scaling dimensions for the most relevant ones.
- While the tensor themselves are not very stable, the scaling dimensions are.

Exact	0.125	1	1.125	1.125	2	2	2	2
RG pres.	0.127	1.009	1.125	1.128	2.002	2.004	2.068	2.073
Trans. mat.	0.125	1.009	1.130	1.148	1.313	1.457	1.558	1.654







Concluding Remarks

- We propose a systematic generalization of Migdal-Kadanoff real-space RG, based on HOTRG.
- By constructing the linearized super-operator the scaling dimensions can be obtained as its eigenvalues.
- Use of GILT to get rid of short-renge contributions.
- Systematically improvable.
- Converges to the correct fixed point.
- The bench-mark on 2d Ising confirms that it works and accurate up to 4 digits.
- The computational comlexity is the same as the HOTRG.