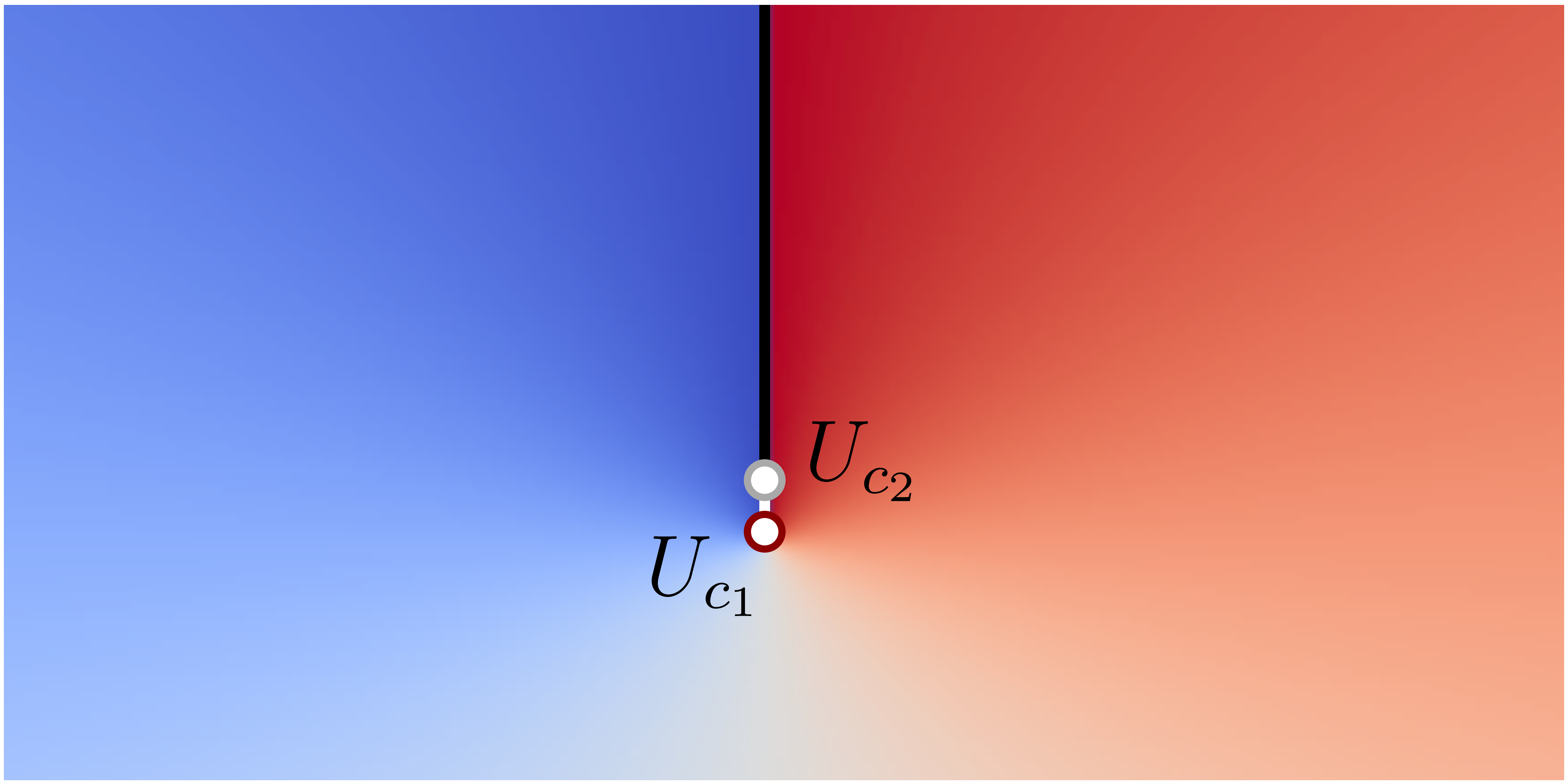
Quotient symmetry protected topological (QSPT) phenomena

Ruben Verresen, Julian Bibo, Frank Pollmann [arXiv:2102.08967]



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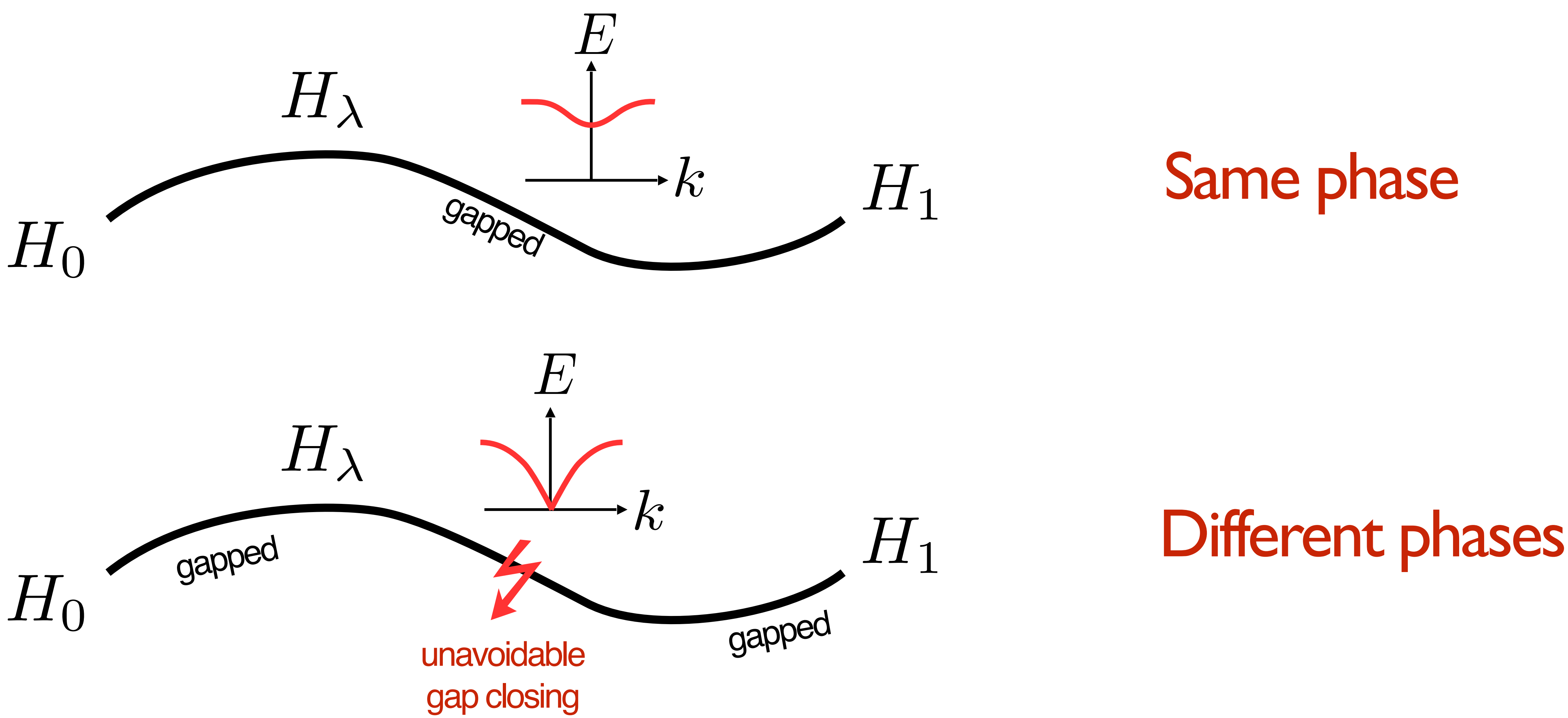
Entanglement in Strongly Correlated Systems
Feb. 15, 2021

Matter occurs in different phases

Gapped quantum phases $(T = 0)$: Two Hamiltonians are in the same phase if a path of gapped Hamiltonians connects them

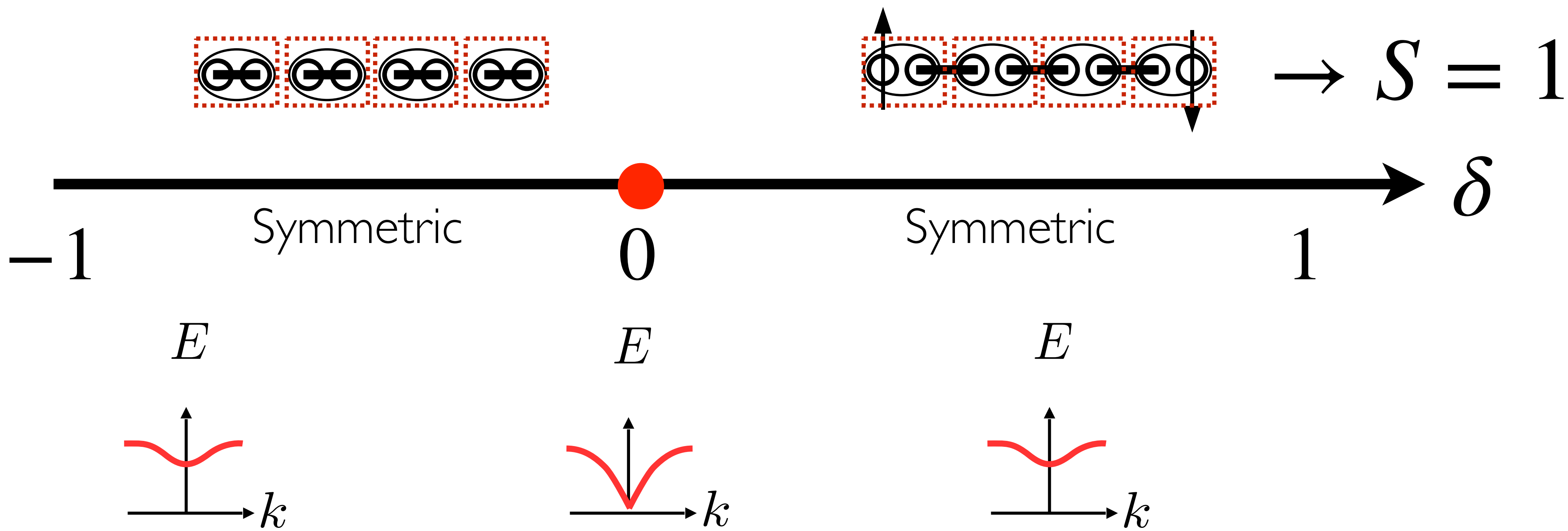


Symmetry-protected topological (SPT) phases

Haldane phase in a dimerized $S = 1/2$ chain: $Z_2 \times Z_2$, TR, …

[Haldane '83]

$$H = \sum_j (1 + (-1)^j \delta)\mathbf{S_j} \cdot \mathbf{S_{j+1}}$$

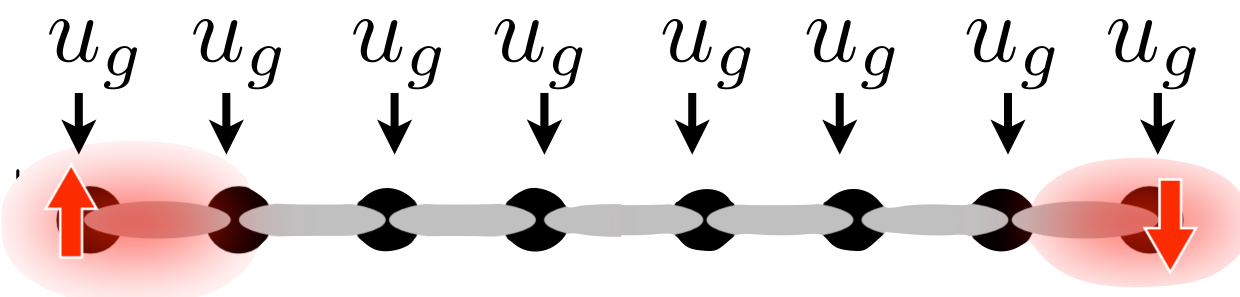


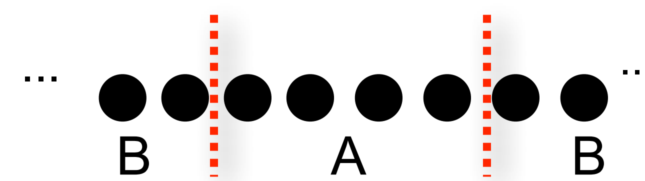
Classified by "Symmetry fractionalization"

[FP, Turner, Berg, Oshikawa '10, Chen et al. '11; Schuch et al '11]

Symmetry-protected topological (SPT) phases

Local Hamiltonian and gapped ground state $|\psi_0\rangle$:
Symmetric under $g, h \in G$





Bulk: Linear representation on unit cell $u_g u_h = u_{gh}$
(e.g., integer spin)

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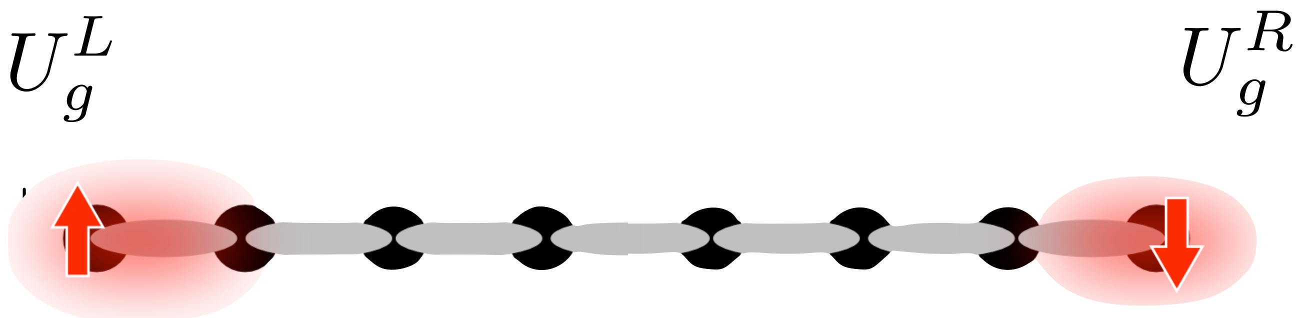
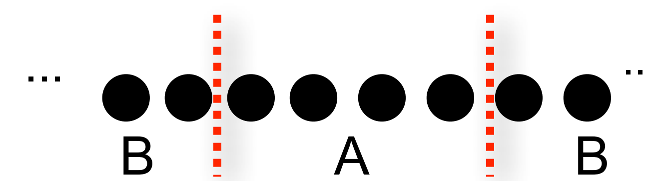




Bulk: Linear representation on unit cell $u_g u_h = u_{gh}$
(e.g., integer spin)
Boundary: Projective representations $U_g U_h = e^{i\phi(g,h)} U_{gh}$
(e.g., half-integer spin)

Classified by the **second cohomology** $H^2[G, U(1)]$ [Schur 1911]

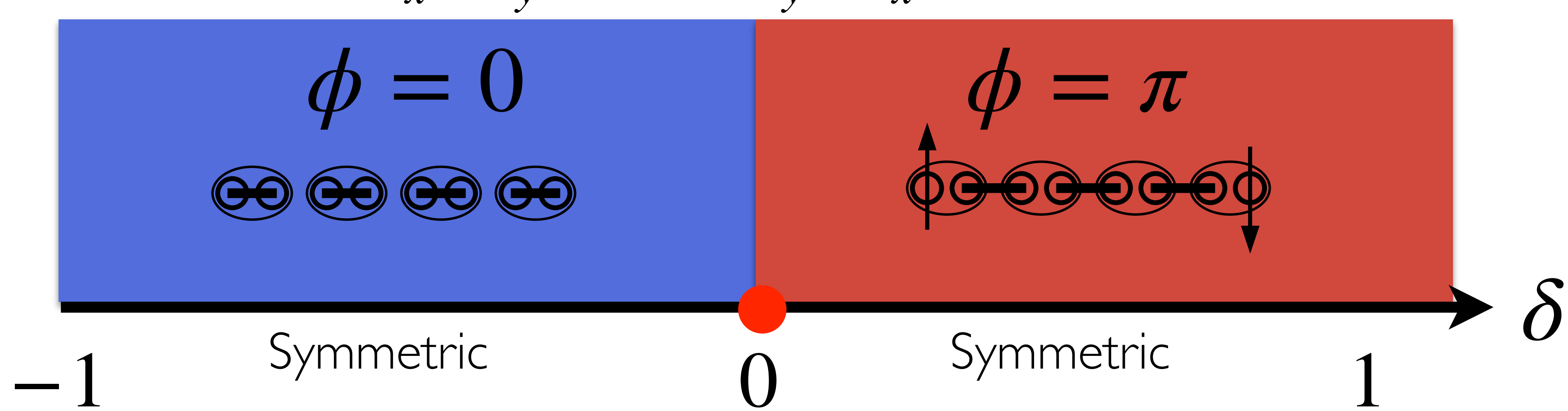
➡ Classification of **symmetry protected topological phases**
[FP, Turner, Berg, Oshikawa '10; Chen et al. '11; Schuch et al '11]

Symmetry-protected topological (SPT) phases

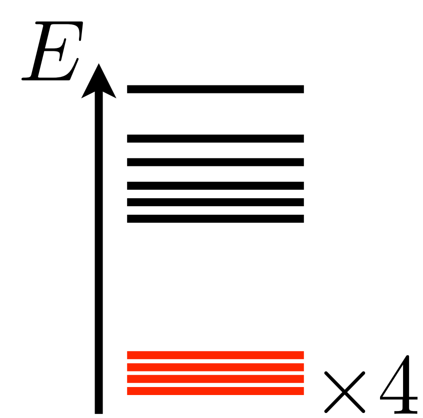
Haldane phase in a dimerized $S = 1/2$ chain: $Z_2 \times Z_2$, TR, …

$$H = \sum_j (1 + (-1)^j \delta) \mathbf{S_j} \cdot \mathbf{S_{j+1}}$$

$$Z_2 \times Z_2: \quad \hat{R}_x^{L/R} \hat{R}_y^{L/R} = e^{i\phi} \hat{R}_y^{L/R} \hat{R}_x^{L/R}$$



➡ Symmetry protected **ground state and entanglement degeneracies** due to $\hat{R}_x^{L/R} \hat{R}_y^{L/R} = -\hat{R}_y^{L/R} \hat{R}_x^{L/R}$



Symmetry-protected topological (SPT) phases

How to trivialize the Haldane phase?

(1) **Break the protecting symmetries:** All characteristic properties disappear right away (e.g, edge modes are gapped out and no SPT transition)

(2) **Extend its symmetry group!** [Anfuso and Rosch '07; Moudgalya and FP '14, Prakash et al '18,…]

To which extent are topological features parametrically stable?

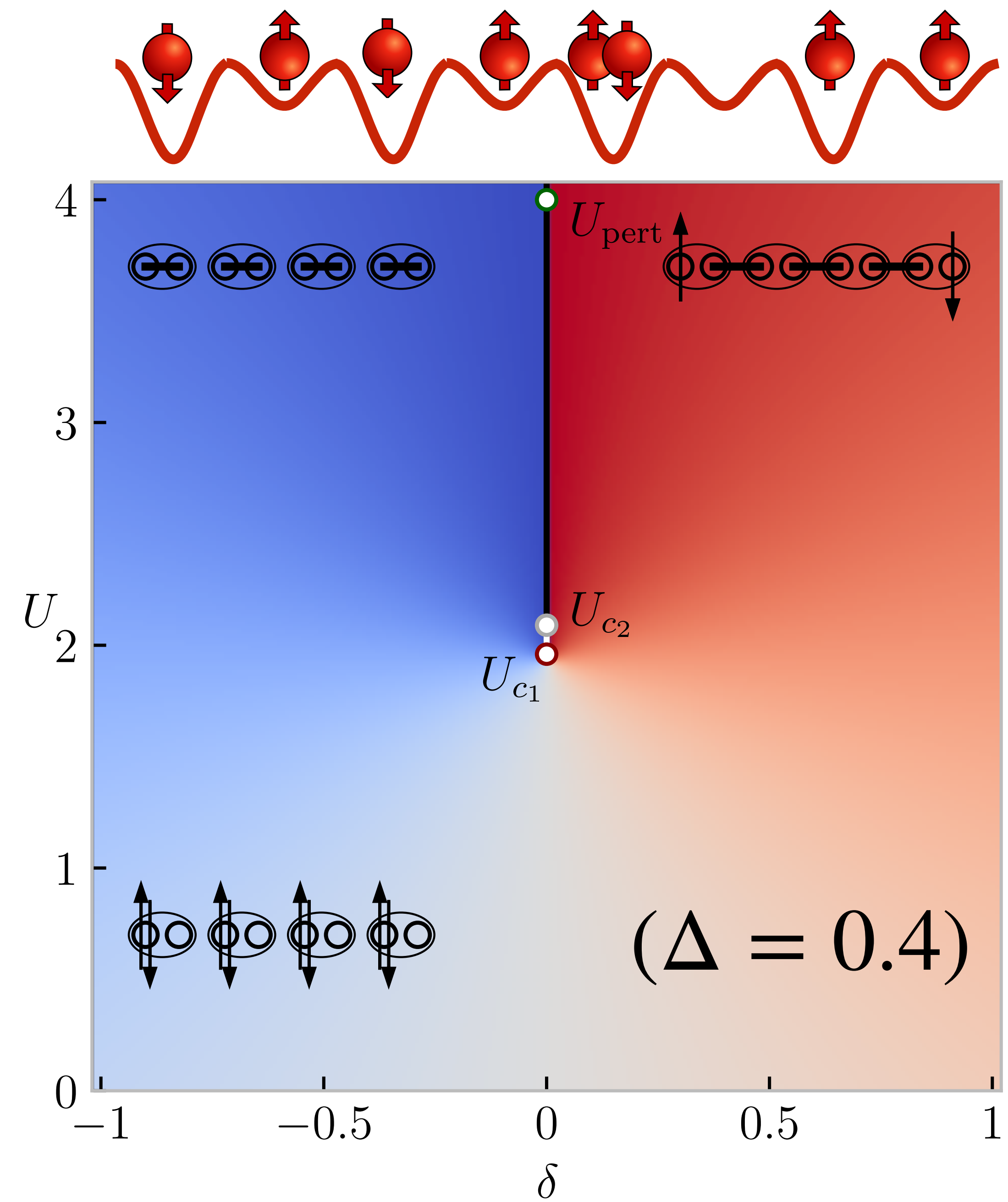
Trivialization by extending the symmetry

Ionic Hubbard model $\hat{H} = \hat{H}_\delta + \hat{H}_\Delta + \hat{H}_U$ at half filling

$$\hat{H}_\delta = -\sum_{j,s}\left[\left(1+(-1)^j\delta\right)\hat{c}^\dagger_{j+1,s}\hat{c}_{j,s} + \text{h.c.}\right]$$

$$\hat{H}_\Delta = \frac{\Delta}{2}\sum_{j,s}(-1)^j\hat{n}_{j,s}$$

$$\hat{H}_U = U\sum_j\left(\hat{n}_{j,\uparrow}-\frac{1}{2}\right)\left(\hat{n}_{j,\downarrow}-\frac{1}{2}\right)$$



Relevant symmetry

$$SU(2) \xrightarrow{U\to\infty} SO(3) = SU(2)/Z_2$$

$$H^2(SU(2), U(1)) = 0$$

➡ **Trivialized by fluctuating charge degrees of freedom**

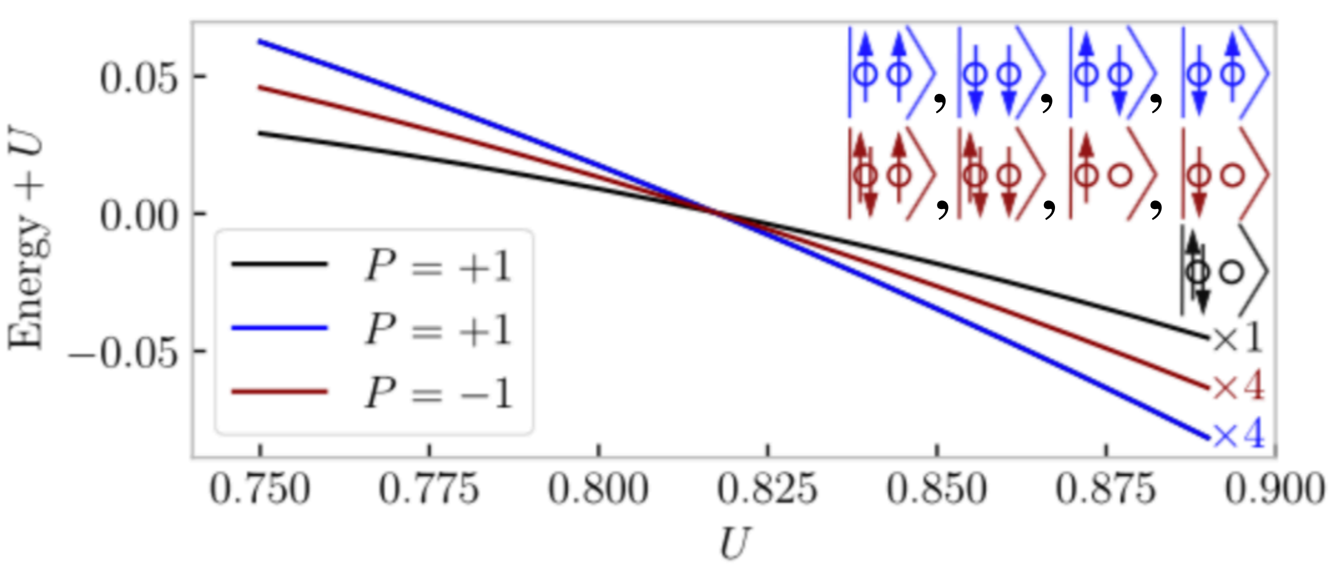
Stability of the edge modes

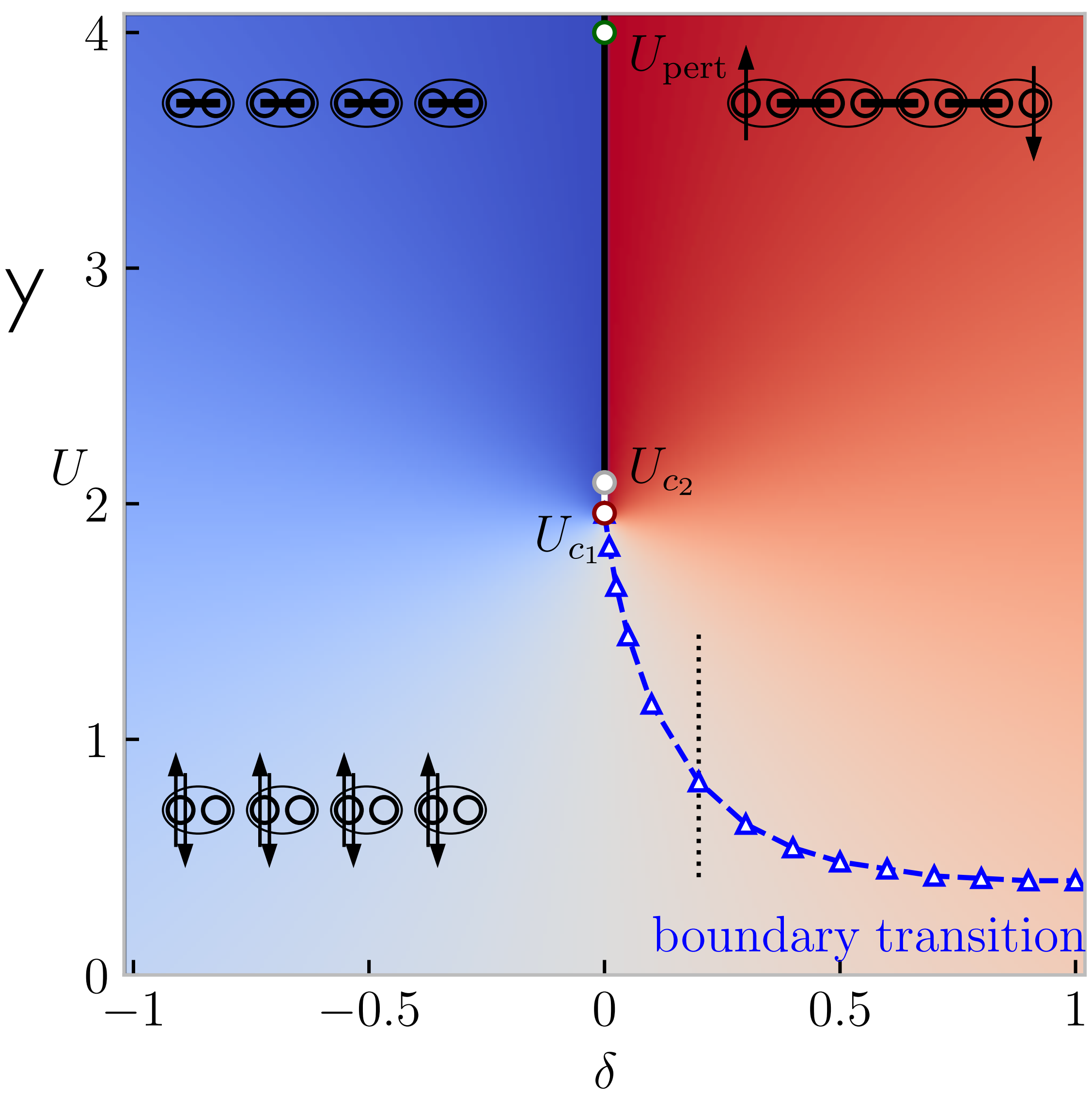
Zero-energy edge modes of the Haldane phase ($\delta > 0$) are stable until the parity gap closes

Fractionalized symmetries obey

$$\hat{R}_x^L \hat{R}_y^L = \hat{P}^L \hat{R}_y^L \hat{R}_x^L \xrightarrow{U\to\infty} \hat{R}_x^L \hat{R}_y^L = -\hat{R}_y^L \hat{R}_x^L$$

Eigenvalue of $\hat{P}^L$ cannot immediately jump: Robust for range of U





➡4-fold degeneracy robust until boundary transition occurs

Stability of the edge modes

Entanglement degeneracies of the Haldane phase ($\delta > 0$) are stable until the parity gap closes

Fractionalized symmetries obey

$$\hat{R}^L_x \hat{R}^L_y = \hat{P}^L \hat{R}^L_y \hat{R}^L_x \xrightarrow{U\to\infty} \hat{R}^L_x \hat{R}^L_y = -\hat{R}^L_y \hat{R}^L_x$$

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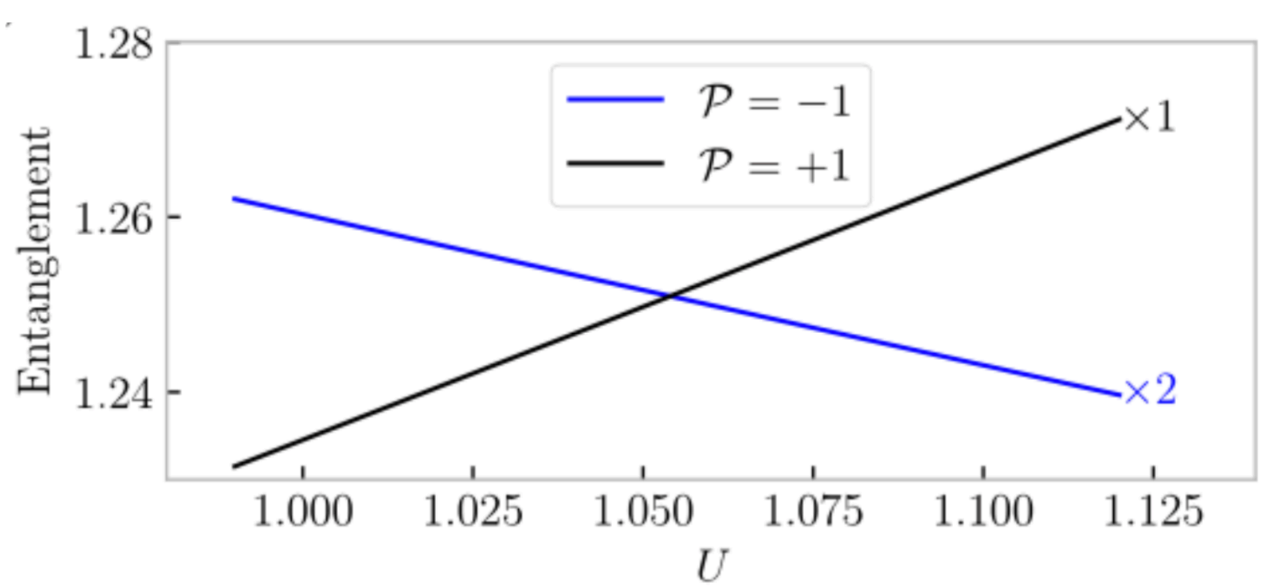
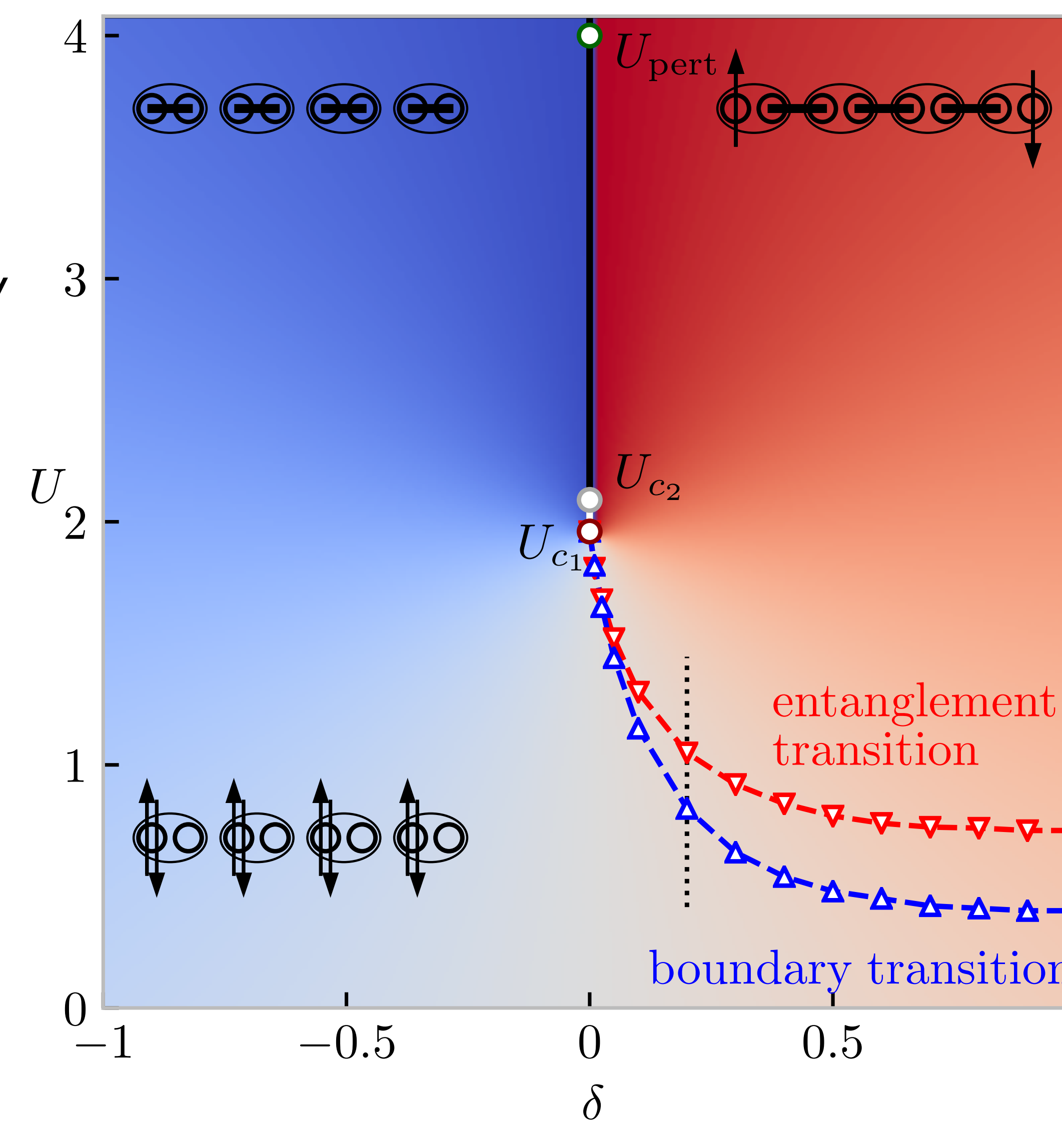
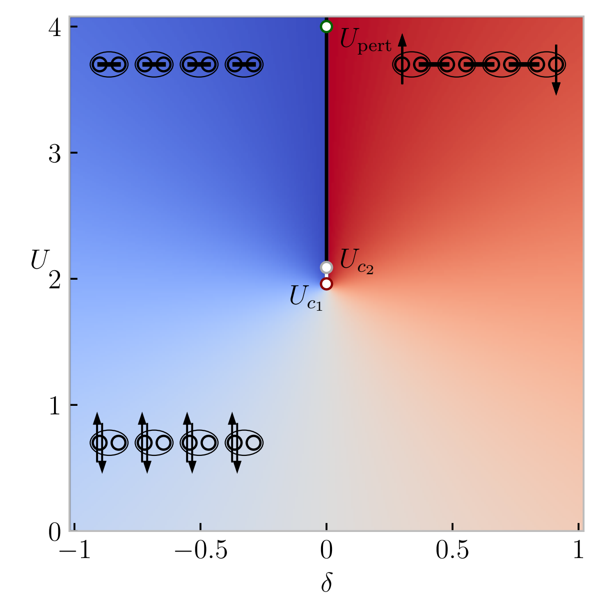




➡2-fold degeneracy robust until boundary transition occurs

Stability of the SPT phase transition

Phase transition between a trivial and SPT phase does not immediately gap out after extending the symmetry group



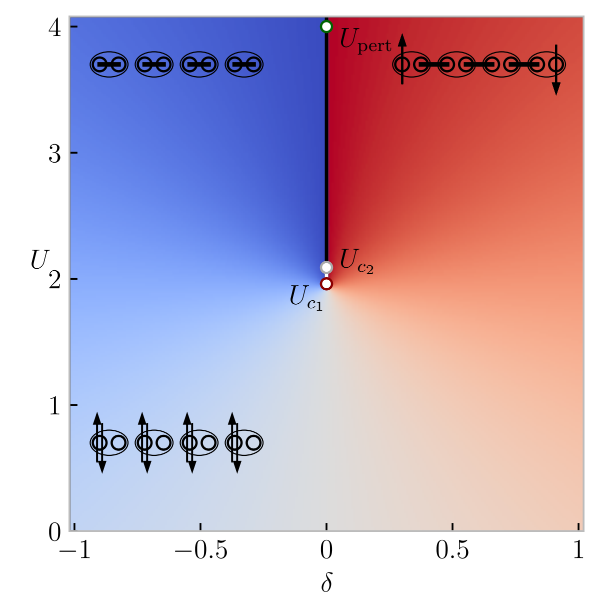
Duality symmetry $\delta \to -\delta$ and thus a direct transition has to occur at $\delta = 0$

$U \to \infty$: Lieb-Schultz-Mattis (LSM) guaranties a phase transitions at $\delta = 0$ as $\hat{R}_x \hat{R}_y \hat{R}_x^{-1} \hat{R}_y^{-1} = -1$

Finite U : Emergent LSM enforces parametric stability of the phase transition

Emergent LSM at $\delta = 0$

Fermion parity string generically has long-range as long as fermionic operators are gapped



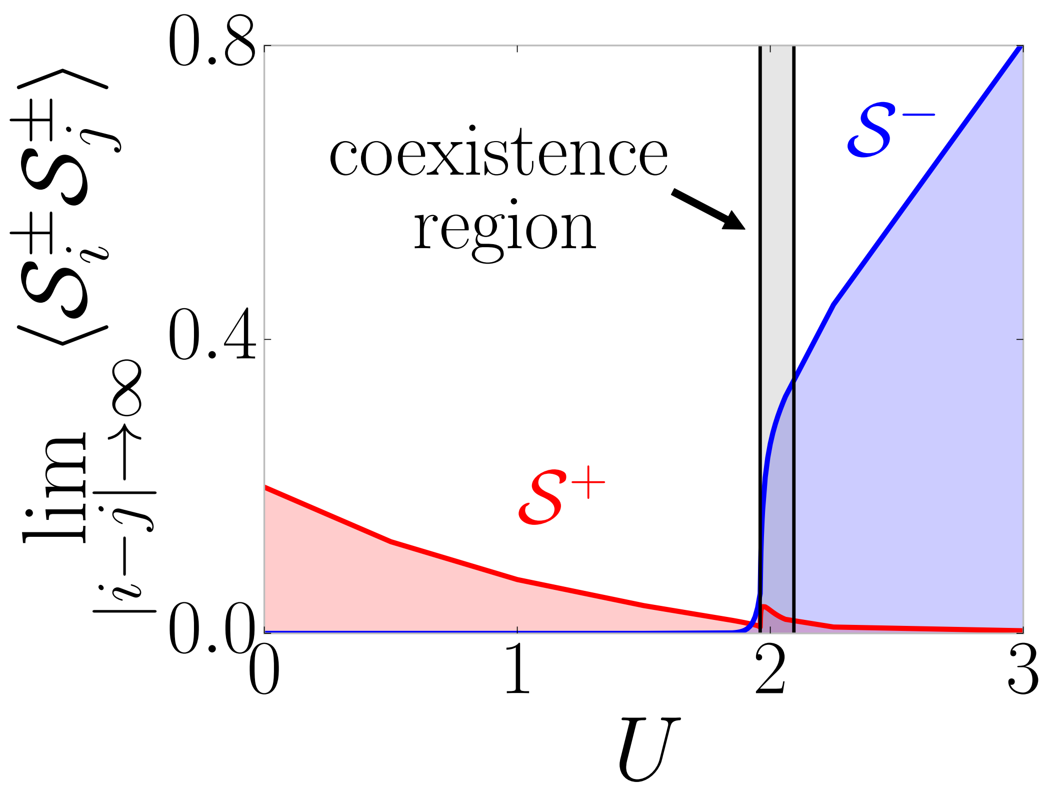
$$\langle \hat{P}_m \hat{P}_{m+1} \cdots \hat{P}_{n-1} \hat{P}_n \rangle \sim C\, e^{i\theta(n-m)}, \text{ with } \theta \in \{0,\pi\}$$

$\theta = \pi$ implies an emergent anomaly that forbids a unique gapped symmetric ground state!

$$\hat{\mathcal{S}}^{\pm}_j := \prod_{k<j} \hat{P}_k \left(\hat{P}_j \pm 1 \right)$$

String order $\mathcal{S}^{\pm}$ is non-zero iff $e^{i\theta} = \pm 1$

➡ $\theta = \pi$ for $U \gtrsim 2$



General emergent anomalies

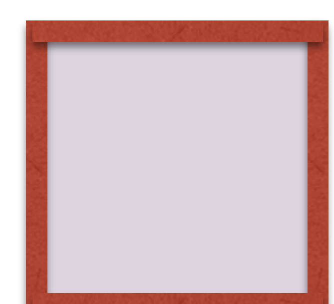
Edge modes of 1D SPTs characterized by non-trivial projective representation of a symmetry group $\tilde{G}$

Extend symmetry group to a larger group G with $\tilde{G} = G/H$

Quantum numbers of the additional symmetry group H label distinct representations and edge modes remain robust as long as excitations charged under H remain gapped!

Same concept can be generalized to higher dimensions!

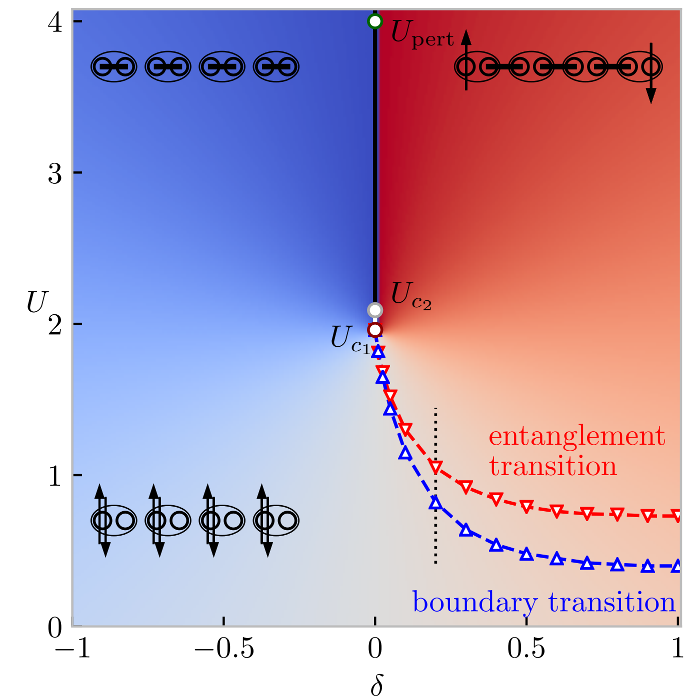
Example for "unnecessary criticality"
[Bi and Senthil '19; Jian and Xu '20]



$Z_2 \rightarrow Z_4$

Summary

Two ways of trivializing an SPT phase:
Either break or extend symmetry group



Extending the symmetry group leaves various topological phenomena intact over a finite region of the phase diagram!

Experimental relevance: Phenomena such as zero energy states are robust in the presence of charge fluctuations!

Thank you!

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