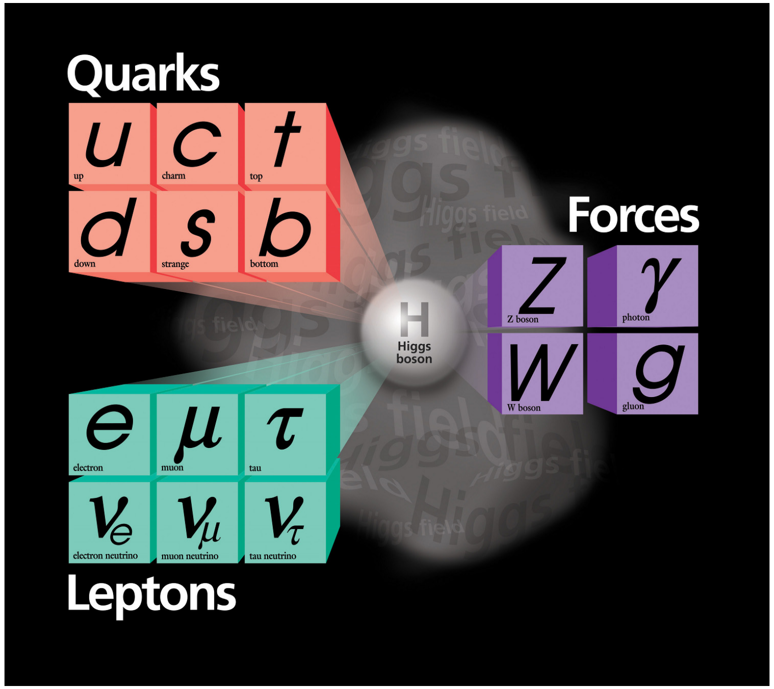
The Electroweak Standard Model



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Outline

Exercises

tools Kinematics, Loops

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1. Gauge Theories

Internal symmetry | free Lagrangian

- Lagrangian of a free fermion field $\psi(x)$:

$$\text{(Dirac)} \qquad \boxed{\mathcal{L}_0 = \overline{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi} \quad \partial\!\!\!/ \equiv \gamma^\mu \partial_\mu \,, \quad \overline{\psi} = \psi^\dagger \gamma^0$$

$\Rightarrow$ Invariant under (*spacetime*) Poincaré transformations

$\Rightarrow$ Invariant under (*internal*) global U(1) phase transformations:

$$\psi(x) \mapsto \psi'(x) = \mathrm{e}^{-\mathrm{i}q\theta}\psi(x) \,, \quad q,\ \theta \text{ (constants)} \in \mathbb{R}$$

$\Rightarrow$ By Noether's theorem: continuous symmetry implies conserved current:

$$j^\mu = q\,\overline{\psi}\gamma^\mu\psi \,, \quad \partial_\mu j^\mu = 0$$

and a Noether charge:

$$Q = \int \mathrm{d}^3x\ j^0, \quad \partial_t Q = 0$$

Internal symmetry | free Lagrangian

- A free fermion field:

$$\psi(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 \sqrt{2E_{\boldsymbol{p}}}} \sum_{s=1,2} \left(a_{\boldsymbol{p},s} u^{(s)}(\boldsymbol{p}) \mathrm{e}^{-\mathrm{i}px} + b^\dagger_{\boldsymbol{p},s} v^{(s)}(\boldsymbol{p}) \mathrm{e}^{\mathrm{i}px} \right)$$

– is a solution of the Dirac equation (Euler-Lagrange):

$$(\mathrm{i}\partial\!\!\!/ - m)\psi(x) = 0\,, \quad (p\!\!\!/ - m)u(\boldsymbol{p}) = 0\,, \quad (p\!\!\!/ + m)v(\boldsymbol{p}) = 0\,,$$

and after quantization

– is an operator from the canonical quantization rules (anticommutation):

$$\{a_{\boldsymbol{p},r}, a^\dagger_{\boldsymbol{k},s}\} = \{b_{\boldsymbol{p},r}, b^\dagger_{\boldsymbol{k},s}\} = (2\pi)^3 \delta^3(\boldsymbol{p} - \boldsymbol{k})\delta_{rs}\,, \quad \{a_{\boldsymbol{p},r}, a_{\boldsymbol{k},s}\} = \cdots = 0\,,$$

– that annihilates/creates particles/antiparticles on the Fock space of fermions

Internal symmetry | free Lagrangian

- For a quantized free fermion field:

  ⇒ The Noether charge is an operator:*

$$:Q: = q\int \mathrm{d}^3x \; :\overline{\psi}\gamma^0\psi: = q\int \frac{\mathrm{d}^3p}{(2\pi)^3} \sum_{s=1,2} \left(a^\dagger_{\boldsymbol{p},s}a_{\boldsymbol{p},s} - b^\dagger_{\boldsymbol{p},s}b_{\boldsymbol{p},s}\right)$$

$$Q\; a^\dagger_{\boldsymbol{k},s}\,|0\rangle = +q\; a^\dagger_{\boldsymbol{k},s}\,|0\rangle \;\; (\text{particle})\,, \quad Q\; b^\dagger_{\boldsymbol{k},s}\,|0\rangle = -q\; b^\dagger_{\boldsymbol{k},s}\,|0\rangle \;\; (\text{antiparticle})$$

* normal ordering prescription for fermionic operators has been introduced (subtract infinite zero-point energy):

$$:a_{\boldsymbol{p},r}a^\dagger_{\boldsymbol{q},s}: \equiv -a^\dagger_{\boldsymbol{q},s}a_{\boldsymbol{p},r}\,, \quad :b_{\boldsymbol{p},r}b^\dagger_{\boldsymbol{q},s}: \equiv -b^\dagger_{\boldsymbol{q},s}b_{\boldsymbol{p},r}$$

The gauge principle | gauge symmetry dictates interactions

- To make $\mathcal{L}_0$ invariant under local $\equiv$ gauge transformations of U(1):

$$\psi(x) \mapsto \psi'(x) = \mathrm{e}^{-\mathrm{i}q\theta(x)}\psi(x)\,, \quad \theta = \theta(x) \in \mathbb{R}$$

perform the minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu + \mathrm{i}eqA_\mu \quad \text{(covariant derivative)}$$

where a gauge field $A_\mu(x)$ is introduced transforming as:

$$A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x) \quad \Leftarrow \quad \boxed{D_\mu\psi \mapsto \mathrm{e}^{-\mathrm{i}q\theta(x)}D_\mu\psi} \quad \overline{\psi}\not{D}\psi \text{ inv.} \quad 1$$

$\Rightarrow$ The new Lagrangian contains interactions between ψ and A_μ:

$$\boxed{\mathcal{L}_{\text{int}} = -eq\,\overline{\psi}\gamma^\mu\psi A_\mu} \quad \propto \begin{cases} \text{coupling} & e \\ \text{charge} & q \end{cases}$$

$$(= -e\, j^\mu A_\mu)$$

The gauge principle | gauge invariance dictates interactions

- Dynamics for the gauge field $\Rightarrow$ add gauge invariant kinetic term:

$$\text{(Maxwell)} \qquad \boxed{\mathcal{L}_1 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}} \qquad \Leftarrow \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \mapsto F_{\mu\nu}$$

- The full U(1) gauge invariant Lagrangian for a fermion field $\psi(x)$ reads:

$$\mathcal{L}_{\text{sym}} = \overline{\psi}(\mathrm{i}\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad (= \mathcal{L}_0 + \mathcal{L}_{\text{int}} + \mathcal{L}_1) \quad \text{(QED)}$$

- The same applies to a complex scalar field $\phi(x)$:

$$\mathcal{L}_{\text{sym}} = (D_\mu \phi)^\dagger D^\mu \phi - m^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \text{(sQED)}$$

The gauge principle | non-Abelian gauge theories

- A general gauge symmetry group G is an *compact* N-dimensional Lie group

$$\mathrm{g} \in G\,, \quad \mathrm{g}(\boldsymbol{\theta}) = \mathrm{e}^{-\mathrm{i}T_a\theta^a}\,, \quad a = 1,\ldots,N$$

$$\theta^a = \theta^a(x) \in \mathbb{R}\,, \quad T_a = \text{Hermitian generators}\,, \quad [T_a, T_b] = \mathrm{i}f_{abc}T_c \quad \text{(Lie algebra)}$$

$$\mathrm{Tr}\{T_aT_b\} \equiv \tfrac{1}{2}\delta_{ab} \qquad \text{structure constants:} \quad \begin{array}{ll} f_{abc} = 0 & \text{Abelian} \\ f_{abc} \neq 0 & \text{non-Abelian} \end{array}$$

⇒ *Unitary* finite-dimensional irreducible representations:

$g(\boldsymbol{\theta})$ represented by $U(\boldsymbol{\theta})$

$$d \times d \text{ matrices :} \quad U(\boldsymbol{\theta}) \text{ [given by } \{T_a\} \text{ algebra representation]}$$

$$d\text{-multiplet :} \quad \Psi(x) \mapsto \Psi'(x) = U(\boldsymbol{\theta})\Psi(x)\,, \quad \Psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_d \end{pmatrix}$$

- Examples:

G	N	Abelian	
U(1)	1	Yes	
SU(n)	n^2-1	No	($n \times n$ unitary matrices with det $= 1$)

  - U(1): 1 generator (q), one-dimensional irreps only

  - SU(2): 3 generators

    $f_{abc} = \epsilon_{abc}$ (Levi-Civita symbol)

    * Fundamental irrep ($d = 2$): $T_a = \frac{1}{2}\sigma_a$ (3 Pauli matrices)
    * Adjoint irrep ($d = N = 3$): $(T_a^{\text{adj}})_{bc} = -\mathrm{i}f_{abc}$

  - SU(3): 8 generators

    $f^{123} = 1$, $f^{458} = f^{678} = \frac{\sqrt{3}}{2}$, $f^{147} = f^{156} = f^{246} = f^{247} = f^{345} = -f^{367} = \frac{1}{2}$

    * Fundamental irrep ($d = 3$): $T_a = \frac{1}{2}\lambda_a$ (8 Gell-Mann matrices)
    * Adjoint irrep ($d = N = 8$): $(T_a^{\text{adj}})_{bc} = -\mathrm{i}f_{abc}$

(for SU(n): f_{abc} totally antisymmetric)

- To make $\mathcal{L}_0$ invariant under local $\equiv$ gauge transformations of G:

$$\mathcal{L}_0 = \overline{\Psi}(\mathrm{i}\not{\partial} - m)\Psi\ , \qquad \Psi(x) \mapsto \Psi'(x) = U(\boldsymbol{\theta})\Psi(x)\ , \quad \boldsymbol{\theta} = \boldsymbol{\theta}(x) \in \mathbb{R}$$

substitute the covariant derivative:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g\widetilde{W}_\mu\ , \quad \widetilde{W}_\mu \equiv T_a W_\mu^a$$

where a gauge field $W_\mu^a(x)$ per generator is introduced (adjoint irrep), transforming as:

$$\widetilde{W}_\mu(x) \mapsto \widetilde{W}'_\mu(x) = U\widetilde{W}_\mu(x)U^\dagger - \frac{\mathrm{i}}{g}(\partial_\mu U)U^\dagger \quad \Leftarrow \quad \boxed{D_\mu\Psi \mapsto UD_\mu\Psi} \quad \overline{\Psi}\not{D}\Psi \text{ inv.}$$

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$\Rightarrow$ The new Lagrangian contains interactions between Ψ and W_μ^a:

$$\boxed{\mathcal{L}_{\text{int}} = g\,\overline{\Psi}\gamma^\mu T_a \Psi W_\mu^a} \quad \propto \begin{cases} \text{coupling} & g \\ \text{charge} & T_a \end{cases}$$

$$(= g\, j_a^\mu W_\mu^a)$$

The gauge principle | non-Abelian gauge theories

- Dynamics for the gauge fields $\Rightarrow$ add gauge invariant kinetic terms:

$$\text{(Yang-Mills)} \quad \boxed{\mathcal{L}_{\rm YM} = -\frac{1}{2}\mathrm{Tr}\left\{\widetilde{W}_{\mu\nu}\widetilde{W}^{\mu\nu}\right\} = -\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}}$$

$$\widetilde{W}_{\mu\nu} = T_a W^a_{\mu\nu} \equiv D_\mu \widetilde{W}_\nu - D_\nu \widetilde{W}_\mu = \partial_\mu \widetilde{W}_\nu - \partial_\nu \widetilde{W}_\mu - \mathrm{i}g[\widetilde{W}_\mu, \widetilde{W}_\nu] \;\Leftrightarrow\; \widetilde{W}_{\mu\nu} \mapsto U\widetilde{W}_{\mu\nu}U^\dagger$$

$$\Rightarrow \quad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f_{abc} W^b_\mu W^c_\nu \qquad 2$$

$\Rightarrow$ $\mathcal{L}_{\rm YM}$ contains cubic and quartic $\boxed{\text{self-interactions}}$ of the gauge fields W^a_μ:

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(\partial^\mu W^{a,\nu} - \partial^\nu W^{a,\mu})$$

$$\mathcal{L}_{\rm cubic} = -\frac{1}{2} g f_{abc}\,(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu) W^{b,\mu} W^{c,\nu}$$

$$\mathcal{L}_{\rm quartic} = -\frac{1}{4} g^2 f_{abe} f_{cde}\, W^a_\mu W^b_\nu W^{c,\mu} W^{d,\nu}$$

- The (Feynman) propagator of a scalar field: (Feynman *prescription* $\varepsilon \to 0^+$)

$$D(x-y) = \langle 0|\, T\{\phi(x)\phi^\dagger(y)\}\,|0\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\varepsilon} \mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$$

is a Green's function of the Klein-Gordon operator:

$$(\Box_x + m^2) D(x-y) = -\mathrm{i}\delta^4(x-y) \quad \Leftrightarrow \quad \widetilde{D}(p) = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\varepsilon}$$

- The propagator of a fermion field:

$$S(x-y) = \langle 0|\, T\{\psi(x)\overline{\psi}(y)\}\,|0\rangle = (\mathrm{i}\not{\partial}_x + m) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\varepsilon} \mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$$

is a Green's function of the Dirac operator:

$$(\mathrm{i}\not{\partial}_x - m) S(x-y) = \mathrm{i}\delta^4(x-y) \quad \Leftrightarrow \quad \widetilde{S}(p) = \frac{\mathrm{i}}{\not{p} - m + \mathrm{i}\varepsilon}$$

- BUT the propagator of a gauge field cannot be defined unless $\mathcal{L}$ is modified:

(e.g. modified Maxwell) $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2$$

Euler-Lagrange: $$\frac{\partial\mathcal{L}}{\partial A_\nu} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu A_\nu)} = 0 \quad \Rightarrow \quad \left[g^{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu \right] A_\mu = 0$$

– In momentum space the propagator is the inverse of:

$$-k^2 g^{\mu\nu} + \left(1 - \frac{1}{\xi}\right) k^\mu k^\nu \quad \Rightarrow \quad \widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\varepsilon}\left[-g_{\mu\nu} + (1 - \xi)\frac{k_\mu k_\nu}{k^2} \right]$$

$\Rightarrow$ Note that $(-k^2 g^{\mu\nu} + k^\mu k^\nu)$ is singular!

$\Rightarrow$ One may argue that $\mathcal{L}$ above will not lead to Maxwell equations …
unless we fix a (Lorenz) gauge where:

$$\partial^\mu A_\mu = 0 \quad \Leftarrow \quad A_\mu \mapsto A'_\mu = A_\mu + \partial_\mu \Lambda \ \text{ with } \ \partial^\mu\partial_\mu\Lambda \equiv -\partial^\mu A_\mu$$

Quantization of gauge theories | gauge fixing (Abelian case)

- The extra term is called Gauge Fixing:

$$\boxed{\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial^\mu A_\mu)^2}$$

$\Rightarrow$ modified $\mathcal{L}$ equivalent to Maxwell Lagrangian just in the gauge $\partial^\mu A_\mu = 0$

$\Rightarrow$ the ξ-dependence always cancels out in physical amplitudes

- Several choices for the gauge fixing term (simplify calculations): R_ξ gauges

$$\text{('t Hooft-Feynman gauge)} \quad \xi = 1: \quad \widetilde{D}_{\mu\nu}(k) = -\frac{\mathrm{i}g_{\mu\nu}}{k^2 + \mathrm{i}\varepsilon}$$

$$\text{(Landau gauge)} \quad \xi = 0: \quad \widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\varepsilon}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right]$$

Quantization of gauge theories | gauge fixing (non-Abelian case)

- For a non-Abelian gauge theory, the gauge fixing terms:

$$\boxed{\mathcal{L}_{\rm GF} = -\sum_a \frac{1}{2\xi_a}(\partial^\mu W_\mu^a)^2}$$

allow to define the propagators:

$$\widetilde{D}^{ab}_{\mu\nu}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2+\mathrm{i}\varepsilon}\left[-g_{\mu\nu} + (1-\xi_a)\frac{k_\mu k_\nu}{k^2}\right]$$

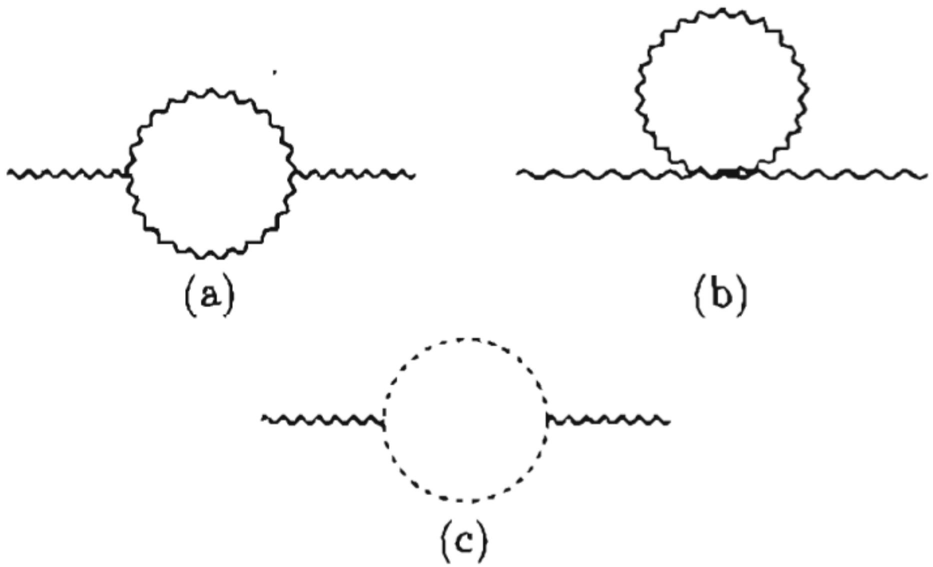
BUT, unlike the Abelian case, this is not the end of the story …

- Add Faddeev-Popov ghost fields $c_a(x)$, $a = 1, \ldots, N$:

$$\boxed{\mathcal{L}_{\text{FP}} = (\partial^\mu \bar{c}_a)(D_\mu^{\text{adj}})_{ab} c_b = (\partial^\mu \bar{c}_a)(\partial_\mu c_a - g f_{abc} c_b W_\mu^c)} \quad \Leftarrow \quad D_\mu^{\text{adj}} = \partial_\mu - \mathrm{i} g T_c^{\text{adj}} W_\mu^c$$

Computational trick: *anticommuting* scalar fields, just in loops as virtual particles

$\Rightarrow$ Faddeev-Popov ghosts needed to preserve gauge symmetry: loops 3 4



Self Energy

$$= \Pi_{\mu\nu} = \mathrm{i}(g_{\mu\nu} k^2 - k_\mu k_\nu)\Pi(k^2)$$

Ward identity: $k^\mu \Pi_{\mu\nu} = 0$

with

$$\widetilde{D}_{ab}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2 + \mathrm{i}\varepsilon}$$

[(-1) sign for closed loops! (like fermions)]

Quantization of gauge theories — complete Lagrangian

- Then the complete quantum Lagrangian is

$$\boxed{\mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}}$$

$\Rightarrow$ Note that in the case of a massive vector field

$$\text{(Proca)} \qquad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2 A_\mu A^\mu$$

it is not gauge invariant

– The propagator is:

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$$\widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M^2 + \mathrm{i}\varepsilon}\left(-g_{\mu\nu} + \frac{k^\mu k^\nu}{M^2}\right)$$

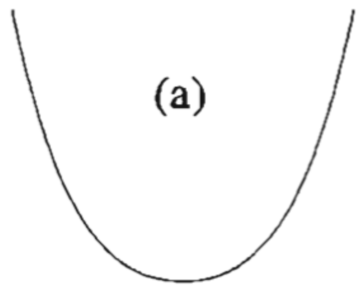
Spontaneous Symmetry Breaking — discrete symmetry

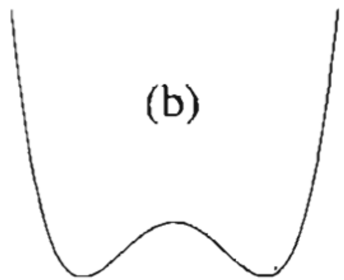
- Consider a real scalar field $\phi(x)$ with Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}\phi^4 \quad \text{invariant under} \quad \phi \mapsto -\phi$$

$$\Rightarrow \quad \mathcal{H} = \frac{1}{2}(\dot{\phi}^2 + (\nabla\phi)^2) + V(\phi)$$

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$





$\mu^2,\ \lambda \in \mathbb{R}$ (Real/Hermitian Hamiltonian) and $\lambda > 0$ (existence of a ground state)

(a) $\mu^2 > 0$: min of $V(\phi)$ at $\phi = 0$

(b) $\mu^2 < 0$: min of $V(\phi)$ at $\phi = v \equiv \pm\sqrt{\dfrac{-\mu^2}{\lambda}}$, in QFT $\langle 0|\,\phi\,|0\rangle = v \neq 0$ (VEV)

– A quantum field must have $v = 0$

$a\,|0\rangle = 0$

$$\Rightarrow \qquad \phi(x) \equiv v + \eta(x)\ , \quad \langle 0|\,\eta\,|0\rangle = 0$$

Spontaneous Symmetry Breaking | discrete symmetry

- At the quantum level, the same system is described by $\eta(x)$ with Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4 \quad \text{not invariant under} \quad \eta \mapsto -\eta$$

$$(m_\eta = \sqrt{2\lambda}\, v)$$

⇒ Lesson:

$\mathcal{L}(\phi)$ has the symmetry but the parameters can be such that the ground state of the Hamiltonian is not symmetric (Spontaneous Symmetry Breaking)

⇒ Note:

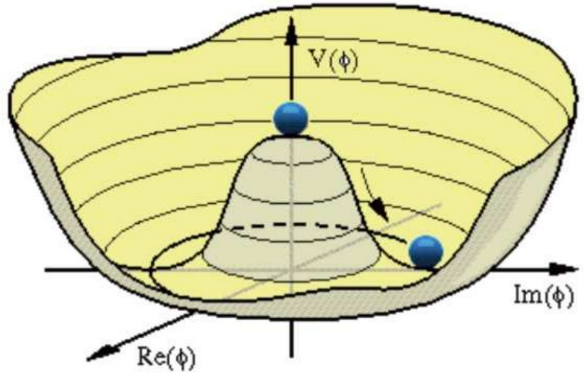
One may argue that $\mathcal{L}(\eta)$ exhibits an explicit breaking of the symmetry.
However this is not the case since the coefficients of terms η^2, η^3 and η^4 are determined by just two parameters, λ and v (remnant of the original symmetry)

Spontaneous Symmetry Breaking — continuous symmetry

- Consider a complex scalar field $\phi(x)$ with Lagrangian:

$$\mathcal{L} = (\partial_\mu\phi^\dagger)(\partial^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \quad \text{invariant under U(1):} \quad \phi \mapsto \mathrm{e}^{-\mathrm{i}q\theta}\phi$$

$$\lambda > 0,\ \mu^2 < 0: \quad \langle 0|\,\phi\,|0\rangle \equiv \frac{v}{\sqrt{2}}, \quad |v| = \sqrt{\frac{-\mu^2}{\lambda}}$$



Take $v \in \mathbb{R}^+$. In terms of quantum fields:

$$\phi(x) \equiv \frac{1}{\sqrt{2}}[v + \eta(x) + \mathrm{i}\chi(x)], \quad \langle 0|\,\eta\,|0\rangle = \langle 0|\,\chi\,|0\rangle = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) - \lambda v^2\eta^2 - \lambda v\eta(\eta^2+\chi^2) - \frac{\lambda}{4}(\eta^2+\chi^2)^2 + \frac{1}{4}\lambda v^4$$

Note: if $v\mathrm{e}^{\mathrm{i}\alpha}$ (complex) replace η by $(\eta\cos\alpha - \chi\sin\alpha)$ and χ by $(\eta\sin\alpha + \chi\cos\alpha)$

$\Rightarrow$ The actual quantum Lagrangian $\mathcal{L}(\eta,\chi)$ is not invariant under U(1)

U(1) broken $\Rightarrow$ one scalar field remains massless: $m_\eta = \sqrt{2\lambda}\,v$, $m_\chi = 0$

Spontaneous Symmetry Breaking — continuous symmetry

- Another example: consider a real scalar SU(2) triplet $\Phi(x)$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^\mathsf{T})(\partial^\mu \Phi) - \frac{1}{2}\mu^2 \Phi^\mathsf{T}\Phi - \frac{\lambda}{4}(\Phi^\mathsf{T}\Phi)^2 \quad \text{inv. under SU(2):} \quad \Phi \mapsto \mathrm{e}^{-\mathrm{i}T_a\theta^a}\Phi$$

that for $\lambda > 0,\ \mu^2 < 0$ acquires a VEV $\langle 0|\,\Phi^\mathsf{T}\Phi\,|0\rangle = v^2 \qquad (\mu^2 = -\lambda v^2)$

Assume $\Phi(x) = \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ v + \varphi_3(x) \end{pmatrix}$ and define $\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_1 + \mathrm{i}\varphi_2)$

$$\mathcal{L} = (\partial_\mu \varphi^\dagger)(\partial^\mu \varphi) + \frac{1}{2}(\partial_\mu \varphi_3)(\partial^\mu \varphi_3) - \lambda v^2 \varphi_3^2 - \lambda v(2\varphi^\dagger\varphi + \varphi_3^2)\varphi_3 - \frac{\lambda}{4}(2\varphi^\dagger\varphi + \varphi_3^2)^2 + \frac{1}{4}\lambda v^4$$

$\Rightarrow$ Not symmetric under SU(2) but invariant under U(1):

$$\varphi \mapsto \mathrm{e}^{-\mathrm{i}q\theta}\varphi \quad (q = \text{arbitrary}) \qquad\qquad \varphi_3 \mapsto \varphi_3 \quad (q = 0)$$

SU(2) broken to U(1) $\Rightarrow 3 - 1 = 2$ broken generators

$\Rightarrow$ 2 (real) scalar fields (= 1 complex) remain massless: $m_\varphi = 0$, $m_{\varphi_3} = \sqrt{2\lambda}\, v$

Spontaneous Symmetry Breaking | continuous symmetry

$\Rightarrow$ Goldstone's theorem: [Nambu '60; Goldstone '61]

The number of massless particles (Nambu-Goldstone bosons) is equal to the number of spontaneously broken generators of the symmetry

$$\text{Hamiltonian symmetric under group } G \quad \Rightarrow \quad [T_a, H] = 0\,, \quad a = 1,\dots,N$$

$$\text{By definition: } H|0\rangle = 0 \quad \Rightarrow \quad H(T_a|0\rangle) = T_a H|0\rangle = 0$$

– If $|0\rangle$ is such that $T_a|0\rangle = 0$ for all generators

$\Rightarrow$ non-degenerate minimum: *the* vacuum

– If $|0\rangle$ is such that $T_{a'}|0\rangle \neq 0$ for some (broken) generators a'

$\Rightarrow$ degenerate minimum: chose one (*true* vacuum) and $\mathrm{e}^{-\mathrm{i}T_{a'}\theta^{a'}}|0\rangle \neq |0\rangle$

$\Rightarrow$ excitations (particles) from $|0\rangle$ to $\mathrm{e}^{-\mathrm{i}T_{a'}\theta^{a'}}|0\rangle$ cost no energy: massless!

Spontaneous Symmetry Breaking | gauge symmetry

- Consider a U(1) gauge invariant Lagrangian for a complex scalar field $\phi(x)$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 , \quad D_\mu = \partial_\mu + \mathrm{i}eqA_\mu$$

inv. under $\quad \phi(x) \mapsto \phi'(x) = \mathrm{e}^{-\mathrm{i}q\theta(x)}\phi(x) , \quad A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x)$

If $\lambda > 0$, $\mu^2 < 0$, the $\mathcal{L}$ in terms of quantum fields η and χ with null VEVs:

$$\phi(x) \equiv \frac{1}{\sqrt{2}}[v + \eta(x) + \mathrm{i}\chi(x)] , \quad \mu^2 = -\lambda v^2$$

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) \\ &\boxed{-\lambda v^2\eta^2} - \lambda v\eta(\eta^2+\chi^2) - \frac{\lambda}{4}(\eta^2+\chi^2)^2 + \frac{1}{4}\lambda v^4 \\ &\boxed{+\, eqvA_\mu\partial^\mu\chi} + eqA_\mu(\eta\partial^\mu\chi - \chi\partial^\mu\eta) \\ &\boxed{+\, \frac{1}{2}(eqv)^2A_\mu A^\mu} + \frac{1}{2}(eq)^2A_\mu A^\mu(\eta^2 + 2v\eta + \chi^2)\end{aligned}$$

Comments:

(i) $m_\eta = \sqrt{2\lambda}\, v$
$\quad\; m_\chi = 0$

(ii) $M_A = |eqv|$ (!)

(iii) Term $A_\mu\partial^\mu\chi$ (?)

(iv) Add $\mathcal{L}_{\mathrm{GF}}$

Spontaneous Symmetry Breaking | gauge symmetry

- Removing the cross term and the (new) gauge fixing Lagrangian:

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial_\mu A^\mu - \xi M_A \chi)^2$$

$$\Rightarrow \quad \mathcal{L} + \mathcal{L}_{\rm GF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_A^2 A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \overbrace{M_A\partial_\mu(A^\mu\chi)}^{\substack{M_A[A_\mu\partial^\mu\chi + \partial_\mu A^\mu\chi] \\ \text{total deriv.}}}$$
$$+\frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) - \frac{1}{2}\xi M_A^2\chi^2 + \ldots$$

and the propagators of A_μ and χ are: 6

$$\widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_A^2 + \mathrm{i}\varepsilon}\left[-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi M_A^2}\right]$$
$$\widetilde{D}(k) = \frac{\mathrm{i}}{k^2 - \xi M_A^2 + \mathrm{i}\varepsilon}$$

$\Rightarrow$ χ has a gauge-dependent mass: actually it is not a physical field!

Spontaneous Symmetry Breaking | gauge symmetry

- A more transparent parameterization of the quantum field ϕ is

$$\phi(x) \equiv \mathrm{e}^{\mathrm{i}q\zeta(x)/v}\frac{1}{\sqrt{2}}[v+\eta(x)]\,, \quad \langle 0|\,\eta\,|0\rangle = \langle 0|\,\zeta\,|0\rangle = 0$$

$$\phi(x) \mapsto \mathrm{e}^{-\mathrm{i}q\zeta(x)/v}\phi(x) = \frac{1}{\sqrt{2}}[v+\eta(x)] \quad \Rightarrow \quad \zeta \text{ gauged away!}$$

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) \\ &- \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 + \frac{1}{4}\lambda v^4 \\ &+ \frac{1}{2}(eqv)^2 A_\mu A^\mu + \frac{1}{2}(eq)^2 A_\mu A^\mu(2v\eta + \eta^2)\end{aligned}$$

Comments:

(i) $m_\eta = \sqrt{2\lambda}\,v$

(ii) $M_A = |eqv|$

(iii) No need for $\mathcal{L}_{\mathrm{GF}}$

$\Rightarrow$ This is the unitary gauge ($\xi \to \infty$): just physical fields

$$\widetilde{D}_{\mu\nu}(k) \to \frac{\mathrm{i}}{k^2 - M_A^2 + \mathrm{i}\varepsilon}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right] \quad \text{and} \quad \widetilde{D}(k) \to 0$$

Spontaneous Symmetry Breaking | gauge symmetry

⇒ Brout-Englert-Higgs mechanism: [Anderson '62]

[Higgs '64; Englert, Brout '64; Guralnik, Hagen, Kibble '64]

The gauge bosons associated with the spontaneously broken generators become massive, the corresponding would-be Goldstone bosons are unphysical and can be absorbed, the remaining massive scalars (Higgs bosons) are physical (the smoking gun!)

– The would-be Goldstone bosons are 'eaten up' by the gauge bosons ('get fat') and disappear (gauge away) in the unitary gauge ($\xi \to \infty$)

⇒ Degrees of freedom are preserved

Before SSB:	2 (massless gauge boson)	+ 1 (Goldstone boson)
After SSB:	3 (massive gauge boson)	+ 0 (absorbed would-be Goldstone)

– For loops calculations, 't Hooft-Feynman gauge ($\xi = 1$) is more convenient:

⇒ Gauge boson propagators are simpler, but

⇒ Goldstone bosons must be included in internal lines

- Comments:

  – After SSB the FP ghost fields (unphysical) acquire a gauge-dependent mass, due to interactions with the scalar field(s):

$$\widetilde{D}_{ab}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2 - \xi_a M_{W^a}^2 + \mathrm{i}\varepsilon}$$

  – Gauge theories with SSB are renormalizable ['t Hooft, Veltman '72]

  UV divergences appearing at loop level can be removed by renormalization of parameters and fields of the classical Lagrangian ⇒ predictive!

2. The Standard Model

Gauge group and particle representations

[Glashow '61; Weinberg '67; Salam '68]
[D. Gross, F. Wilczek; D. Politzer '73]

- The Standard Model is a gauge theory based on the local symmetry group:

$$\underbrace{SU(3)_c}_{\text{strong}} \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{electroweak}} \to SU(3)_c \otimes \underbrace{U(1)_Q}_{\text{em}}$$

  with the electroweak symmetry spontaneously broken to the electromagnetic $U(1)_Q$ symmetry by the Brout-Englert-Higgs mechanism

- The particle (field) content: (ingredients: 12 *flavors* + 12 gauge bosons + H)

Fermions			I	II	III	Q
spin $\frac{1}{2}$	Quarks	f	uuu	ccc	ttt	$\frac{2}{3}$
		f'	ddd	sss	bbb	$-\frac{1}{3}$
	Leptons	f	ν_e	ν_μ	ν_τ	0
		f'	e	μ	τ	-1

$$Q_f = Q_{f'} + 1$$

Bosons		
spin 1	8 gluons	strong interaction
	$W^\pm, Z$	weak interaction
	γ	em interaction
spin 0	Higgs	origin of mass

Gauge group and particle representations

- The fields lay in the following representations (color, weak isospin, hypercharge):

Multiplets	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	I	II	III	$Q = T_3 + Y$
Quarks	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$ $-\frac{1}{3} = -\frac{1}{2} + \frac{1}{6}$
	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$	u_R	c_R	t_R	$\frac{2}{3} = 0 + \frac{2}{3}$
	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	d_R	s_R	b_R	$-\frac{1}{3} = 0 - \frac{1}{3}$
Leptons	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$	$0 = \frac{1}{2} - \frac{1}{2}$ $-1 = -\frac{1}{2} - \frac{1}{2}$
	$(\mathbf{1}, \mathbf{1}, -1)$	e_R	μ_R	τ_R	$-1 = 0 - 1$
	$(\mathbf{1}, \mathbf{1}, 0)$	ν_{e_R}	ν_{μ_R}	ν_{τ_R}	$0 = 0 + 0$
Higgs	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	(3 families of quarks & leptons)			

$\Rightarrow$ Electroweak (QFD): $SU(2)_L \otimes U(1)_Y$ Strong (QCD): $SU(3)_c$

The EWSM with one family (of quarks or leptons)

- Consider two massless fermion fields $f(x)$ and $f'(x)$ with electric charges $Q_f = Q_{f'} + 1$ in three irreps of $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$:

$$\mathcal{L}_F^0 = \mathrm{i}\overline{f}\not\partial f + \mathrm{i}\overline{f}'\not\partial f' \qquad f_{R,L} = \frac{1}{2}(1 \pm \gamma_5)f\,, \quad f'_{R,L} = \frac{1}{2}(1 \pm \gamma_5)f'$$

$$= \mathrm{i}\overline{\Psi}_1\not\partial\Psi_1 + \mathrm{i}\overline{\psi}_2\not\partial\psi_2 + \mathrm{i}\overline{\psi}_3\not\partial\psi_3 \quad ; \quad \Psi_1 = \underbrace{\begin{pmatrix} f_L \\ f'_L \end{pmatrix}}_{(\mathbf{2}, y_1)}, \quad \psi_2 = \underbrace{f_R}_{(\mathbf{1}, y_2)}, \quad \psi_3 = \underbrace{f'_R}_{(\mathbf{1}, y_3)}$$

- To get a Langrangian invariant under gauge transformations:

$$\Psi_1(x) \mapsto U_L(x)\mathrm{e}^{-\mathrm{i}y_1\beta(x)}\Psi_1(x), \quad U_L(x) = \mathrm{e}^{-\mathrm{i}T_i\alpha^i(x)}, \quad T_i = \frac{\sigma_i}{2} \quad \text{(weak isospin gen.)}$$

$$\psi_2(x) \mapsto \mathrm{e}^{-\mathrm{i}y_2\beta(x)}\psi_2(x)$$

$$\psi_3(x) \mapsto \mathrm{e}^{-\mathrm{i}y_3\beta(x)}\psi_3(x)$$

The EWSM with one family | gauge invariance

⇒ Introduce gauge fields $W^i_\mu(x)$ $(i=1,2,3)$ and $B_\mu(x)$ through covariant derivatives:

$$\left.\begin{aligned}
D_\mu\Psi_1 &= (\partial_\mu - \mathrm{i}g\widetilde{W}_\mu + \mathrm{i}g' y_1 B_\mu)\Psi_1\,, \quad \widetilde{W}_\mu \equiv \frac{\sigma_i}{2}W^i_\mu \\
D_\mu\psi_2 &= (\partial_\mu + \mathrm{i}g' y_2 B_\mu)\psi_2 \\
D_\mu\psi_3 &= (\partial_\mu + \mathrm{i}g' y_3 B_\mu)\psi_3
\end{aligned}\right\} \Rightarrow \boxed{\mathcal{L}_F}$$

where two couplings g and g' have been introduced and

$$\widetilde{W}_\mu(x) \mapsto U_L(x)\widetilde{W}_\mu(x)U_L^\dagger(x) - \frac{\mathrm{i}}{g}(\partial_\mu U_L(x))U_L^\dagger(x)$$

$$B_\mu(x) \mapsto B_\mu(x) + \frac{1}{g'}\partial_\mu\beta(x)$$

⇒ Add Yang-Mills: gauge invariant kinetic terms for the gauge fields

$$\boxed{\mathcal{L}_{\mathrm{YM}}} = -\frac{1}{4}W^i_{\mu\nu}W^{i,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\,, \quad W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon_{ijk}W^j_\mu W^k_\nu$$

(include self-interactions of the SU(2) gauge fields) and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

The EWSM with one family | mass terms forbidden

⇒ Note that mass terms are not invariant under $SU(2)_L \otimes U(1)_Y$, since LH and RH components do not transform the same:

$$m\bar{f}f = m(\overline{f_L} f_R + \overline{f_R} f_L)$$

⇒ Mass terms for the gauge bosons are not allowed either

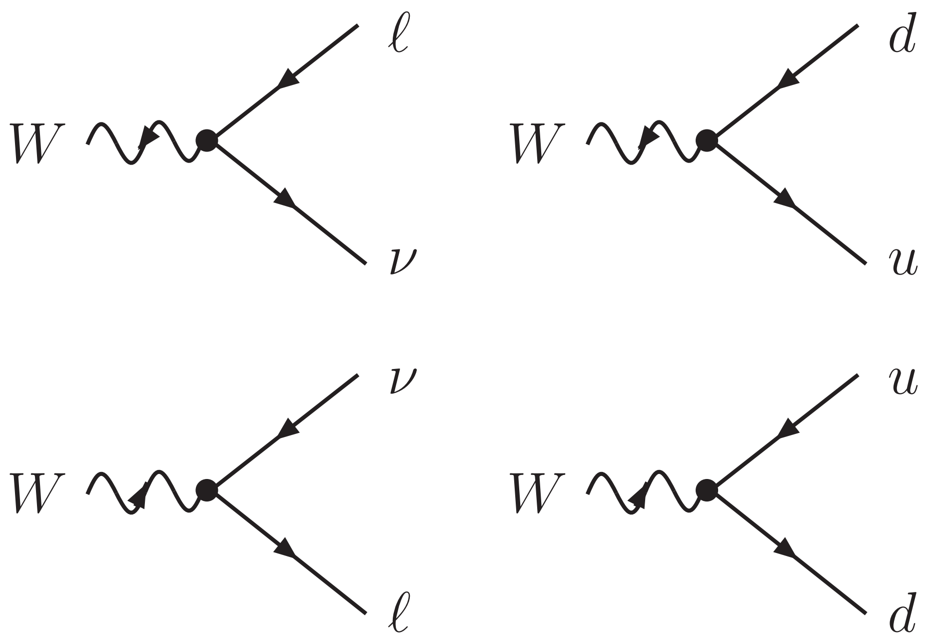
⇒ Next the different types of interactions are analyzed

The EWSM with one family | charged current interactions

- $$\mathcal{L}_F \supset g\overline{\Psi}_1\gamma^\mu \widetilde{W}_\mu \Psi_1 \,, \quad \widetilde{W}_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^\dagger \\ \sqrt{2}W_\mu & -W_\mu^3 \end{pmatrix}$$

$\Rightarrow$ charged current interactions of LH fermions with complex vector boson field W_μ:

$$\mathcal{L}_{\rm CC} = \frac{g}{2\sqrt{2}}\overline{f}\gamma^\mu(1-\gamma_5)f' W_\mu^\dagger + \text{h.c.} \,, \quad W_\mu \equiv \frac{1}{\sqrt{2}}(W_\mu^1 + \mathrm{i}W_\mu^2)$$



The EWSM with one family | neutral current interactions

- The diagonal part of

$$\mathcal{L}_F \supset g\overline{\Psi}_1\gamma^\mu \widetilde{W}_\mu \Psi_1 - g' B_\mu (y_1 \overline{\Psi}_1 \gamma^\mu \Psi_1 + y_2 \overline{\psi}_2 \gamma^\mu \psi_2 + y_3 \overline{\psi}_3 \gamma^\mu \psi_3)$$

$\Rightarrow$ neutral current interactions with neutral vector boson fields W^3_μ and B_μ

We would like to identify B_μ with the photon field A_μ but that requires:

$$y_1 = y_2 = y_3 \quad \text{and} \quad g' y_j = eQ_j \quad \Rightarrow \quad \text{impossible!}$$

$\Rightarrow$ Since they are both neutral, try a combination:

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \qquad \begin{array}{c} s_W \equiv \sin\theta_W \,, \quad c_W \equiv \cos\theta_W \\ \theta_W = \text{weak mixing angle} \end{array}$$

$$\mathcal{L}_{\rm NC} = \sum_{j=1}^{3} \overline{\psi}_j \gamma^\mu \left\{ - \left[g T_3 s_W + g' y_j c_W \right] A_\mu + \left[g T_3 c_W - g' y_j s_W \right] Z_\mu \right\} \psi_j$$

with $T_3 = \dfrac{\sigma_3}{2}$ (0) the third weak isospin component of the doublet (singlet)

- To make A_μ the photon field:

$$(1)\ \boxed{e = g s_W = g' c_W} \qquad (2)\ \boxed{Q = T_3 + Y}$$

where the electric charge operator is: $Q_1 = \begin{pmatrix} Q_f & 0 \\ 0 & Q_{f'} \end{pmatrix}\ , \quad Q_2 = Q_f\ , \quad Q_3 = Q_{f'}$

$\Rightarrow$ (1) Electroweak unification: g of SU(2) and g' of U(1) related to $e = \dfrac{gg'}{\sqrt{g^2 + g'^2}}$

$\Rightarrow$ (2) The hyperchages are fixed in terms of electric charges and weak isospin:

$$y_1 = Q_f - \frac{1}{2} = Q_{f'} + \frac{1}{2}\ , \qquad y_2 = Q_f\ , \qquad y_3 = Q_{f'}$$

$$\mathcal{L}_{\text{QED}} = -e\ Q_f \overline{f} \gamma^\mu f\ A_\mu \quad + (f \to f')$$

$\Rightarrow$ RH neutrinos are sterile: $y_2 = Q_f = 0$

The EWSM with one family — neutral current interactions

- The Z_μ is the neutral weak boson field:

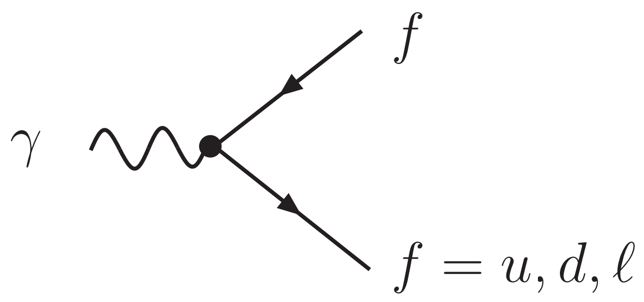
$$\mathcal{L}_{\rm NC}^{Z} = e\,\overline{f}\gamma^\mu(v_f - a_f\gamma_5)f\;Z_\mu \quad + (f \to f')$$

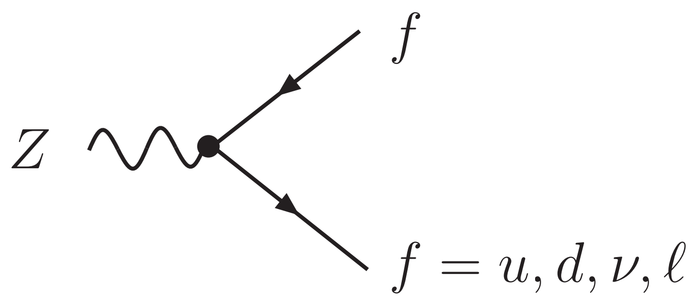
  with

$$v_f = \frac{T_3^{f_L} - 2Q_f s_W^2}{2 s_W c_W}\,, \quad a_f = \frac{T_3^{f_L}}{2 s_W c_W}$$

- The complete neutral current Lagrangian reads:

$$\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm NC}^{Z}$$





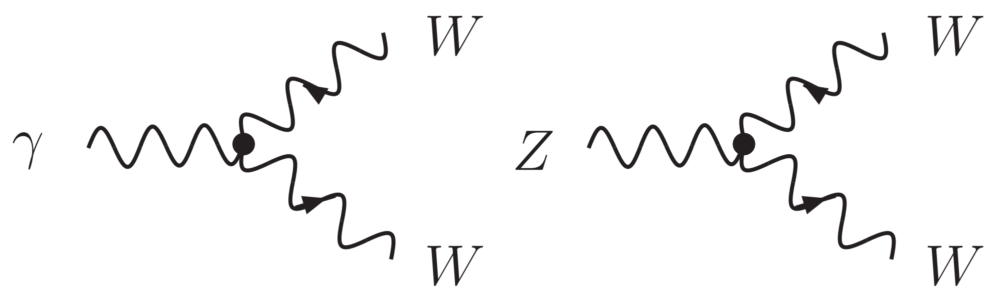
The EWSM with one family | gauge boson self-interactions

- Cubic:

$$\mathcal{L}_{\rm YM} \supset \mathcal{L}_3 = -\frac{\mathrm{i}ec_W}{s_W}\left\{W^{\mu\nu}W^\dagger_\mu Z_\nu - W^\dagger_{\mu\nu}W^\mu Z^\nu - W^\dagger_\mu W_\nu Z^{\mu\nu}\right\}$$
$$+\,\mathrm{i}e\left\{W^{\mu\nu}W^\dagger_\mu A_\nu - W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu W_\nu F^{\mu\nu}\right\}$$

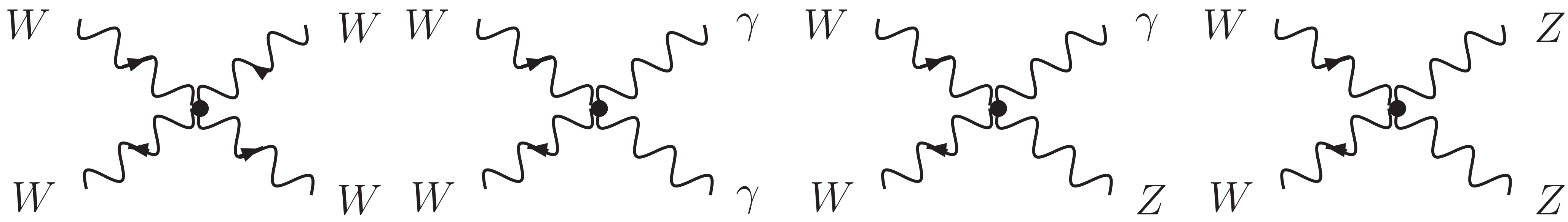
with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \qquad W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$



- Quartic:

$$\begin{aligned}
\mathcal{L}_{\rm YM} \supset \mathcal{L}_4 = & -\frac{e^2}{2s_W^2}\left\{\left(W_\mu^\dagger W^\mu\right)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu\right\} \\
& -\frac{e^2 c_W^2}{s_W^2}\left\{W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu\right\} \\
& +\frac{e^2 c_W}{s_W}\left\{2W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu\right\} \\
& - e^2\left\{W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu\right\}
\end{aligned}$$



Note: even number of W and no vertex with just γ or Z

Electroweak symmetry breaking **setup**

- Out of the 4 gauge bosons of $SU(2)_L \otimes U(1)_Y$ with generators $T_1,\ T_2,\ T_3,\ Y$ we need all to be broken except the combination $Q = T_3 + Y$ so that A_μ remains massless and the other three gauge bosons get massive after SSB

  $\Rightarrow$ Introduce a complex SU(2) Higgs doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \langle 0|\,\Phi\,|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

with gauge invariant Lagrangian ($\mu^2 = -\lambda v^2$):

$$\boxed{\mathcal{L}_\Phi} = (D_\mu\Phi)^\dagger D^\mu\Phi - \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2\,, \qquad D_\mu\Phi = (\partial_\mu - \mathrm{i}g\widetilde{W}_\mu + \mathrm{i}g' y_\Phi B_\mu)\Phi$$

$$\text{take } y_\Phi = \frac{1}{2} \quad \Rightarrow \quad (T_3 + Y)\,|0\rangle = Q\begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

$$\{T_1, T_2, T_3 - Y\}\,|0\rangle \neq 0$$

Electroweak symmetry breaking | gauge boson masses

- Quantum fields in the unitary gauge:

$$\Phi(x) \equiv \exp\left\{\mathrm{i}\frac{\sigma_i}{2v}\theta^i(x)\right\}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}$$

$$\Phi(x) \mapsto \exp\left\{-\mathrm{i}\frac{\sigma_i}{2v}\theta^i(x)\right\}\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix} \quad \Rightarrow$$

1 physical Higgs field $H(x)$

3 would-be Goldstones $\theta^i(x)$ gauged away

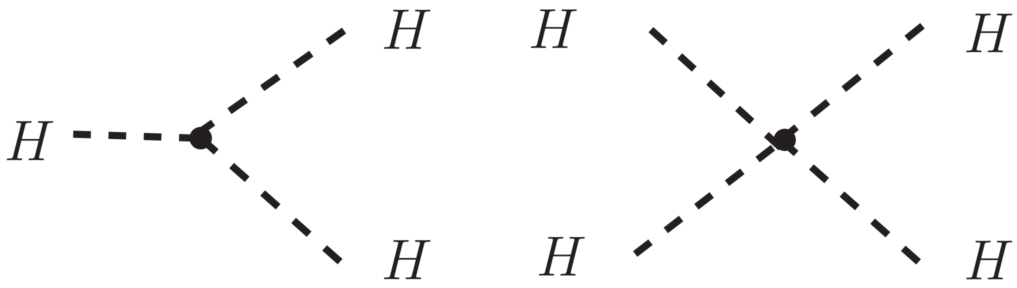
– The 3 dof apparently lost become the longitudinal polarizations of $W^\pm$ and Z that get massive after SSB:

$$\mathcal{L}_\Phi \supset \mathcal{L}_M = \underbrace{\frac{g^2v^2}{4}}_{M_W^2} W_\mu^\dagger W^\mu + \underbrace{\frac{g^2v^2}{8c_W^2}}_{\frac{1}{2}M_Z^2} Z_\mu Z^\mu \quad \Rightarrow \quad M_W = M_Z c_W = \frac{1}{2}gv$$

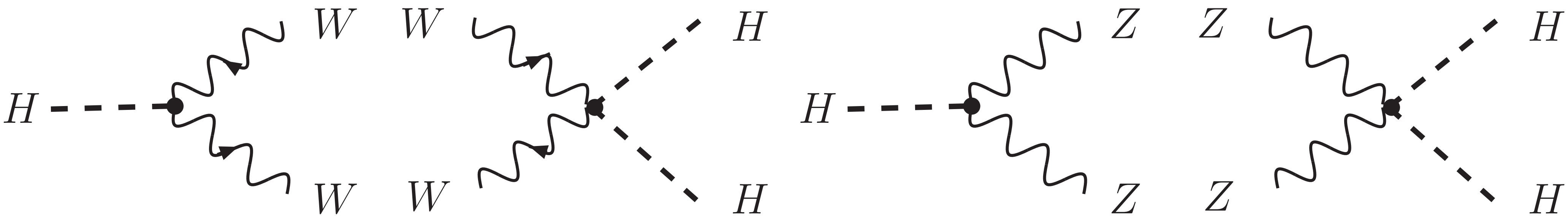
Electroweak symmetry breaking | Higgs sector

⇒ In the unitary gauge (just physical fields): $\mathcal{L}_\Phi = \mathcal{L}_H + \mathcal{L}_M + \mathcal{L}_{HV^2} + \frac{1}{4}\lambda v^4$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}M_H^2 H^2 - \frac{M_H^2}{2v}H^3 - \frac{M_H^2}{8v^2}H^4 \,, \quad M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v$$



$$\mathcal{L}_M + \mathcal{L}_{HV^2} = M_W^2 W_\mu^\dagger W^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\}$$



Electroweak symmetry breaking | Higgs sector

- Quantum fields in the R_ξ gauges:

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \equiv \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + \mathrm{i}\chi(x)] \end{pmatrix}, \quad \phi^-(x) = [\phi^+(x)]^*$$

$$\begin{aligned} \mathcal{L}_\Phi = {} & \mathcal{L}_H + \mathcal{L}_M + \mathcal{L}_{HV^2} + \frac{1}{4}\lambda v^4 \\ & + (\partial_\mu \phi^+)(\partial^\mu \phi^-) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) \\ & + \mathrm{i}M_W\ (W_\mu \partial^\mu \phi^+ - W_\mu^\dagger \partial^\mu \phi^-) + \ M_Z\ Z_\mu \partial^\mu \chi \\ & + \text{trilinear interactions [SSS, SSV, SVV]} \\ & + \text{quadrilinear interactions [SSSS, SSVV]} \end{aligned}$$

Electroweak symmetry breaking | gauge fixing

- To remove the cross terms $W_\mu \partial^\mu \phi^+$, $W^\dagger_\mu \partial^\mu \phi^-$, $Z_\mu \partial^\mu \chi$ and define propagators add:

$$\boxed{\mathcal{L}_{\rm GF}} = -\frac{1}{2\xi_\gamma}(\partial_\mu A^\mu)^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z M_Z \chi)^2 - \frac{1}{\xi_W}|\partial_\mu W^\mu + \mathrm{i}\xi_W M_W \phi^-|^2$$

$\Rightarrow$ Massive propagators for gauge and (unphysical) would-be Goldstone fields:

$$\widetilde{D}^{\gamma}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2+\mathrm{i}\varepsilon}\left[-g_{\mu\nu} + (1-\xi_\gamma)\frac{k_\mu k_\nu}{k^2}\right]$$

$$\widetilde{D}^{Z}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2-M_Z^2+\mathrm{i}\varepsilon}\left[-g_{\mu\nu} + (1-\xi_Z)\frac{k_\mu k_\nu}{k^2-\xi_Z M_Z^2}\right] \quad ; \quad \widetilde{D}^{\chi}(k) = \frac{\mathrm{i}}{k^2-\xi_Z M_Z^2+\mathrm{i}\varepsilon}$$

$$\widetilde{D}^{W}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2-M_W^2+\mathrm{i}\varepsilon}\left[-g_{\mu\nu} + (1-\xi_W)\frac{k_\mu k_\nu}{k^2-\xi_W M_W^2}\right] \quad ; \quad \widetilde{D}^{\phi}(k) = \frac{\mathrm{i}}{k^2-\xi_W M_W^2+\mathrm{i}\varepsilon}$$

('t Hooft-Feynman gauge: $\xi_\gamma = \xi_Z = \xi_W = 1$)

Electroweak symmetry breaking | Faddeev-Popov ghosts

- The SM is a non-Abelian theory $\Rightarrow$ add Faddeev-Popov ghosts $c_i(x)$ $(i=1,2,3)$

$$c_1 \equiv \frac{1}{\sqrt{2}}(u_+ + u_-)\,, \quad c_2 \equiv \frac{\mathrm{i}}{\sqrt{2}}(u_+ - u_-)\,, \quad c_3 \equiv c_W\, u_Z - s_W\, u_\gamma$$

$$\boxed{\mathcal{L}_{\mathrm{FP}}} = \underbrace{(\partial^\mu \bar{c}_i)(\partial_\mu c_i - g\epsilon_{ijk}c_j W_\mu^k)}_{\text{U kinetic + [UUV]}} + \underbrace{\text{interactions with } \Phi}_{\text{U masses + [SUU]}}$$

$\Rightarrow$ Massive propagators for (unphysical) FP ghost fields:

$$\widetilde{D}^{u_\gamma}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\varepsilon}\,, \quad \widetilde{D}^{u_Z}(k) = \frac{\mathrm{i}}{k^2 - \xi_Z M_Z^2 + \mathrm{i}\varepsilon}\,, \quad \widetilde{D}^{u_\pm}(k) = \frac{\mathrm{i}}{k^2 - \xi_W M_W^2 + \mathrm{i}\varepsilon}$$

('t Hooft-Feynman gauge: $\xi_Z = \xi_W = 1$)

Electroweak symmetry breaking — Faddeev-Popov ghosts

$$
\begin{aligned}
\mathcal{L}_{\rm FP} = \quad & (\partial_\mu \bar{u}_\gamma)(\partial^\mu u_\gamma) + (\partial_\mu \bar{u}_Z)(\partial^\mu u_Z) + (\partial_\mu \bar{u}_+)(\partial^\mu u_+) + (\partial_\mu \bar{u}_-)(\partial^\mu u_-) \\
[\mathrm{UUV}] \; & \left\{ \begin{aligned}
& + \mathrm{i}e[(\partial^\mu \bar{u}_+)u_+ - (\partial^\mu \bar{u}_-)u_-]A_\mu - \frac{\mathrm{i}e c_W}{s_W}[(\partial^\mu \bar{u}_+)u_+ - (\partial^\mu \bar{u}_-)u_-]Z_\mu \\
& - \mathrm{i}e[(\partial^\mu \bar{u}_+)u_\gamma - (\partial^\mu \bar{u}_\gamma)u_-]W^\dagger_\mu + \frac{\mathrm{i}e c_W}{s_W}[(\partial^\mu \bar{u}_+)u_Z - (\partial^\mu \bar{u}_Z)u_-]W^\dagger_\mu \\
& + \mathrm{i}e[(\partial^\mu \bar{u}_-)u_\gamma - (\partial^\mu \bar{u}_\gamma)u_+]W_\mu - \frac{\mathrm{i}e c_W}{s_W}[(\partial^\mu \bar{u}_-)u_Z - (\partial^\mu \bar{u}_Z)u_+]W_\mu
\end{aligned} \right. \\
& - \xi_Z M_Z^2\, \bar{u}_Z u_Z - \xi_W M_W^2\, \bar{u}_+ u_+ - \xi_W M_W^2\, \bar{u}_- u_- \\
[\mathrm{SUU}] \; & \left\{ \begin{aligned}
& - e\xi_Z M_Z\, \bar{u}_Z \left[\frac{1}{2 s_W c_W} H u_Z - \frac{1}{2 s_W}\left(\phi^+ u_- + \phi^- u_+\right) \right] \\
& - e\xi_W M_W\, \bar{u}_+ \left[\frac{1}{2 s_W}(H + \mathrm{i}\chi)u_+ - \phi^+ \left(u_\gamma - \frac{c_W^2 - s_W^2}{2 s_W c_W} u_Z \right) \right] \\
& - e\xi_W M_W\, \bar{u}_- \left[\frac{1}{2 s_W}(H - \mathrm{i}\chi)u_- - \phi^- \left(u_\gamma - \frac{c_W^2 - s_W^2}{2 s_W c_W} u_Z \right) \right]
\end{aligned} \right.
\end{aligned}
$$

Electroweak symmetry breaking | fermion masses

- We need masses for quarks and leptons without breaking gauge symmetry

$\Rightarrow$ Introduce Yukawa interactions:

$$\mathcal{L}_Y = -\lambda_d \begin{pmatrix} \overline{u}_L & \overline{d}_L \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - \lambda_u \begin{pmatrix} \overline{u}_L & \overline{d}_L \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} u_R$$

$$- \lambda_\ell \begin{pmatrix} \overline{\nu}_L & \overline{\ell}_L \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \ell_R - \lambda_\nu \begin{pmatrix} \overline{\nu}_L & \overline{\ell}_L \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \nu_R \quad + \text{h.c.}$$

where $\Phi^c \equiv i\sigma_2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ transforms under SU(2) like $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ 7

$\Rightarrow$ After EW SSB, fermions acquire masses ($\overline{f} f = \overline{f_L} f_R + \overline{f_R} f_L$):

$$\mathcal{L}_Y \supset -\frac{1}{\sqrt{2}}(v + H) \left\{ \lambda_d \, \overline{d} d + \lambda_u \, \overline{u} u + \lambda_\ell \, \overline{\ell} \ell + \lambda_\nu \, \overline{\nu} \nu \right\} \quad \Rightarrow \quad m_f = \lambda_f \frac{v}{\sqrt{2}}$$

Additional generations | Yukawa matrices

- There are 3 generations of quarks and leptons in Nature. They are identical copies with the same properties under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ differing only in their masses

  $\Rightarrow$ Take a general case of n_G generations and let u_i^I, d_i^I, ν_i^I, ℓ_i^I be the members of family i ($i = 1, \ldots, n_G$). Superindex I (interaction basis) was omitted so far

  $\Rightarrow$ General gauge invariant Yukawa Lagrangian:

$$\boxed{\mathcal{L}_\mathrm{Y}} = -\sum_{ij} \left\{ \begin{pmatrix} \overline{u}_{iL}^I & \overline{d}_{iL}^I \end{pmatrix} \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \lambda_{ij}^{(d)} \, d_{jR}^I + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \lambda_{ij}^{(u)} \, u_{jR}^I \right] \right.$$
$$\left. + \begin{pmatrix} \overline{\nu}_{iL}^I & \overline{\ell}_{iL}^I \end{pmatrix} \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \lambda_{ij}^{(\ell)} \, \ell_{jR}^I + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \lambda_{ij}^{(\nu)} \, \nu_{jR}^I \right] \right\} + \mathrm{h.c.}$$

  where $\lambda_{ij}^{(d)}$, $\lambda_{ij}^{(u)}$, $\lambda_{ij}^{(\ell)}$, $\lambda_{ij}^{(\nu)}$ are arbitrary Yukawa matrices

Additional generations | mass matrices

- After EW SSB, in n_G-dimensional matrix form:

$$\mathcal{L}_{\rm Y} \supset -\left(1+\frac{H}{v}\right)\left\{\overline{\mathbf{d}}_L^I\,\mathbf{M}_d\,\mathbf{d}_R^I + \overline{\mathbf{u}}_L^I\,\mathbf{M}_u\,\mathbf{u}_R^I + \overline{\mathbf{l}}_L^I\,\mathbf{M}_\ell\,\mathbf{l}_R^I + \overline{\boldsymbol{\nu}}_L^I\,\mathbf{M}_\nu\,\boldsymbol{\nu}_R^I + \text{h.c.}\right\}$$

with mass matrices

$$(\mathbf{M}_d)_{ij} \equiv \lambda_{ij}^{(d)}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_u)_{ij} \equiv \lambda_{ij}^{(u)}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_\ell)_{ij} \equiv \lambda_{ij}^{(\ell)}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_\nu)_{ij} \equiv \lambda_{ij}^{(\nu)}\frac{v}{\sqrt{2}}$$

$\Rightarrow$ Diagonalization determines mass eigenstates d_j, u_j, ℓ_j, ν_j in terms of interaction states d_j^I, u_j^I, ℓ_j^I, ν_j^I, respectively

$\Rightarrow$ Each $\mathbf{M}_f$ can be written as

$$\mathbf{M}_f = \mathbf{H}_f\,\mathcal{U}_f = \mathbf{V}_f^\dagger\,\mathcal{M}_f\,\mathbf{V}_f\,\mathcal{U}_f \quad\Longleftrightarrow\quad \mathbf{M}_f\mathbf{M}_f^\dagger = \mathbf{H}_f^2 = \mathbf{V}_f^\dagger\,\mathcal{M}_f^2\,\mathbf{V}_f$$

with $\mathbf{H}_f \equiv \sqrt{\mathbf{M}_f\mathbf{M}_f^\dagger}$ a Hermitian positive definite matrix and $\mathcal{U}_f$ unitary

– Every $\mathbf{H}_f$ can be diagonalized by a unitary matrix $\mathbf{V}_f$

– The resulting $\mathcal{M}_f$ is diagonal and positive definite

Additional generations | fermion masses and mixings

- In terms of diagonal mass matrices (mass eigenstate basis):

$$\mathcal{M}_d = \mathrm{diag}(m_d, m_s, m_b, \ldots)\,, \quad \mathcal{M}_u = \mathrm{diag}(m_u, m_c, m_t, \ldots)$$
$$\mathcal{M}_\ell = \mathrm{diag}(m_e, m_\mu, m_\tau, \ldots)\,, \quad \mathcal{M}_\nu = \mathrm{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, \ldots)$$

$$\mathcal{L}_\mathrm{Y} \supset -\left(1+\frac{H}{v}\right)\left\{\overline{\mathbf{d}}\,\mathcal{M}_d\,\mathbf{d} + \overline{\mathbf{u}}\,\mathcal{M}_u\,\mathbf{u} + \overline{\mathbf{l}}\,\mathcal{M}_\ell\,\mathbf{l} + \overline{\boldsymbol{\nu}}\,\mathcal{M}_\nu\,\boldsymbol{\nu}\right\}$$

where fermion couplings to Higgs are proportional to masses and

$$\mathbf{d}_L \equiv \mathbf{V}_d\,\mathbf{d}_L^I \qquad \mathbf{u}_L \equiv \mathbf{V}_u\,\mathbf{u}_L^I \qquad \mathbf{l}_L \equiv \mathbf{V}_\ell\,\mathbf{l}_L^I \qquad \boldsymbol{\nu}_L \equiv \mathbf{V}_\nu\,\boldsymbol{\nu}_L^I$$
$$\mathbf{d}_R \equiv \mathbf{V}_d\mathcal{U}_d\,\mathbf{d}_R^I \qquad \mathbf{u}_R \equiv \mathbf{V}_u\mathcal{U}_u\,\mathbf{u}_R^I \qquad \mathbf{l}_R \equiv \mathbf{V}_\ell\mathcal{U}_\ell\,\mathbf{l}_R^I \qquad \boldsymbol{\nu}_R \equiv \mathbf{V}_\nu\mathcal{U}_\nu\,\boldsymbol{\nu}_R^I$$

$\Rightarrow$ Neutral Currents preserve chirality
$\overline{\mathbf{f}}_L^I\,\mathbf{f}_L^I = \overline{\mathbf{f}}_L\,\mathbf{f}_L$ and $\overline{\mathbf{f}}_R^I\,\mathbf{f}_R^I = \overline{\mathbf{f}}_R\,\mathbf{f}_R$
$\Rightarrow \mathcal{L}_\mathrm{NC}$ does not change flavor

$\Rightarrow$ GIM mechanism [Glashow, Iliopoulos, Maiani '70]

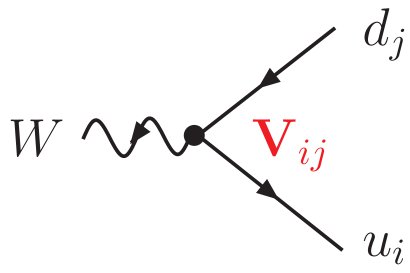
Additional generations | quark sector

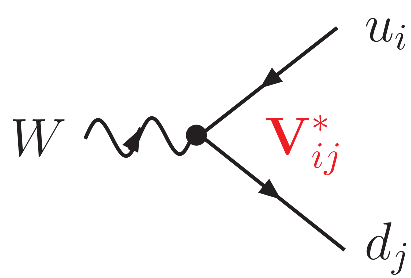
- However, in Charged Currents (also chirality preserving and only LH):

$$\overline{\mathbf{u}}_L^I\, \mathbf{d}_L^I = \overline{\mathbf{u}}_L\, \mathbf{V}_u\, \mathbf{V}_d^\dagger\, \mathbf{d}_L = \overline{\mathbf{u}}_L \mathbf{V} \mathbf{d}_L$$

with $\mathbf{V} \equiv \mathbf{V}_u\, \mathbf{V}_d^\dagger$ the (unitary) CKM mixing matrix [Cabibbo '63; Kobayashi, Maskawa '73]

$$\Rightarrow \quad \mathcal{L}_{\rm CC} = \frac{g}{2\sqrt{2}} \sum_{ij} \overline{u}_i \gamma^\mu (1-\gamma_5)\, \mathbf{V}_{ij}\, d_j\, W_\mu^\dagger + \text{h.c.}$$





$\Rightarrow$ If u_i or d_j had degenerate masses one could choose $\mathbf{V}_u = \mathbf{V}_d$ (field redefinition) and flavor would be conserved in the quark sector. But they are not degenerate

$\Rightarrow$ $\mathbf{V}_u$ and $\mathbf{V}_d$ are not observable. Just masses and CKM mixings are observable

Additional generations | quark sector

- How many physical parameters in this sector?

  – Quark masses and CKM mixings determined by mass (or Yukawa) matrices

  – A general $n_G \times n_G$ unitary matrix, like the CKM, is given by

  $$n_G^2 \text{ real parameters} = n_G(n_G-1)/2 \text{ moduli} + n_G(n_G+1)/2 \text{ phases}$$

  Some phases are unphysical since they can be absorbed by field redefinitions:

  $$u_i \to \mathrm{e}^{\mathrm{i}\phi_i}\, u_i\ , \quad d_j \to \mathrm{e}^{\mathrm{i}\theta_j}\, d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to \mathbf{V}_{ij}\, \mathrm{e}^{\mathrm{i}(\theta_j-\phi_i)}$$

  Therefore $2n_G-1$ unphysical phases and the physical parameters are:

  $$\boxed{(n_G-1)^2 = n_G(n_G-1)/2 \text{ moduli } + (n_G-1)(n_G-2)/2 \text{ phases}}$$

Additional generations — quark sector

⇒ Case of $n_G = 2$ generations: 1 parameter, the Cabibbo angle θ_C:

$$\mathbf{V} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

⇒ Case of $n_G = 3$ generations: 3 angles + 1 phase. In the standard parameterization:

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_{ud} & \mathbf{V}_{us} & \mathbf{V}_{ub} \\ \mathbf{V}_{cd} & \mathbf{V}_{cs} & \mathbf{V}_{cb} \\ \mathbf{V}_{td} & \mathbf{V}_{ts} & \mathbf{V}_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ 0 & 1 & 0 \\ -s_{13}\mathrm{e}^{\mathrm{i}\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix} \Rightarrow$$

δ only source of CP violation in the SM !

with $c_{ij} \equiv \cos\theta_{ij} \geq 0, \quad s_{ij} \equiv \sin\theta_{ij} \geq 0 \quad (i < j = 1, 2, 3)$ and $0 \leq \delta \leq 2\pi$

Additional generations | lepton sector

- If neutrinos were massless we could redefine the (LH) fields $\Rightarrow$ no lepton mixing
  But they have (tiny) masses because there are neutrino oscillations!

- Neutrinos are special:
  they *may* be their own antiparticle (Majorana) since they are neutral

- *If* they are Majorana:
  - Mass terms are different to Dirac case
    (neutrino and antineutrino *may* mix)
  - Intergenerational mixings are richer (more CP phases)

⋆ lepton sector

- About Majorana fermions

  – A Dirac fermion field is a spinor with 4 independent components: 2 LH+2 RH (left/right-handed particles and antiparticles)

  $$\psi_L = P_L\psi\ , \quad \psi_R = P_R\psi\ , \quad \psi_L^c \equiv (\psi_L)^c = P_R\psi^c\ , \quad \psi_R^c \equiv (\psi_R)^c = P_L\psi^c$$

  where $\psi^c \equiv C\overline{\psi}^{\mathsf{T}} = \mathrm{i}\gamma^2\psi^*$ (charge conjugate) with $C = \mathrm{i}\gamma^2\gamma^0$, $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$

  – A Majorana fermion field has just 2 independent components since $\psi^c \equiv \eta^*\psi$:

  $$\psi_L = \eta\psi_R^c\ , \quad \psi_R = \eta\psi_L^c$$

  where $\eta = -\mathrm{i}\eta_{\mathrm{CP}}$ (CP parity) with $|\eta|^2 = 1$. Only possible if neutral

⋆ lepton sector

- About mass terms

$$\begin{array}{llll} \overline{\psi_R}\psi_L = \overline{\psi^c_L}\psi^c_R & , \overline{\psi_L}\psi_R = \overline{\psi^c_R}\psi^c_L & & (\Delta F = 0) \\ \overline{\psi^c_L}\psi_L & , \overline{\psi_L}\psi^c_L & \left.\right\} & (|\Delta F| = 2) \\ \overline{\psi_R}\psi^c_R & , \overline{\psi^c_R}\psi_R & & \end{array}$$

$$\Rightarrow \quad -\mathcal{L}_m = \underbrace{m_D\ \overline{\psi_R}\psi_L}_{\text{Dirac term}} + \underbrace{\frac{1}{2}m_L\ \overline{\psi^c_L}\psi_L + \frac{1}{2}m_R\ \overline{\psi_R}\psi^c_R}_{\text{Majorana terms}} \quad + \text{h.c.}$$

  - A Dirac fermion can only have Dirac mass term
  - A Majorana fermion can have both Dirac and Majorana mass terms

⇒ In the SM:
- ∗ m_D from Yukawa coupling after EW SSB $\quad (m_D = \lambda_\nu\, v/\sqrt{2})$
- ∗ m_L forbidden by gauge symmetry
- ∗ m_R compatible with gauge symmetry!

⋆ lepton sector

- About mass terms (a more transparent parameterization)

  Rewrite previous mass terms introducing an array of two Majorana fermions:

$$\chi^0 = \chi^{0c} = \chi^0_L + \chi^{0c}_L \equiv \begin{pmatrix} \chi^0_1 \\ \chi^0_2 \end{pmatrix} , \qquad \begin{array}{l} \chi^0_1 = \chi^{0c}_1 = \chi^0_{1L} + \chi^{0c}_{1L} \equiv \psi_L + \psi^c_L \\ \chi^0_2 = \chi^{0c}_2 = \chi^0_{2L} + \chi^{0c}_{2L} \equiv \psi^c_R + \psi_R \end{array}$$

$$\Rightarrow \quad -\mathcal{L}_m = \frac{1}{2}\overline{\chi^{0c}_L}\,\mathbf{M}\,\chi^0_L + \text{h.c.} \quad \text{with} \quad \mathbf{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

  $\mathbf{M}$ is a square symmetric matrix ⇒ diagonalizable by a unitary matrix $\widetilde{\mathcal{U}}$:

$$\widetilde{\mathcal{U}}^\mathsf{T}\mathbf{M}\,\widetilde{\mathcal{U}} = \mathcal{M} = \text{diag}(m'_1, m'_2) \, , \quad \chi^0_L = \widetilde{\mathcal{U}}\chi_L \quad (\chi^{0c}_L = \widetilde{\mathcal{U}}^*\chi^c_L)$$

  To get real and positive eigenvalues $m_i = \eta_i m'_i$ (physical masses) take $\chi^0_L = \mathcal{U}\xi_L$:

$$\mathcal{U} = \widetilde{\mathcal{U}}\text{diag}(\sqrt{\eta_1}, \sqrt{\eta_2}) \, , \qquad \begin{array}{l} \xi_1 = \chi_{1L} + \eta_1\chi^c_{1L} \\ \xi_2 = \chi_{2L} + \eta_2\chi^c_{2L} \end{array} \quad \text{(physical fields)} \quad \eta_i = \text{CP parities}$$

⋆ lepton sector

- About mass terms (a more transparent parameterization)
  - Case of only Dirac term ($m_L = m_R = 0$)

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix} \quad \Rightarrow \quad \widetilde{\mathcal{U}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad m'_1 = -m_D, \quad m'_2 = m_D$$

Eigenstates $\Rightarrow$ Physical states

$$\chi_{1L} = \frac{1}{\sqrt{2}}(\chi^0_{1L} - \chi^0_{2L}) = \frac{1}{\sqrt{2}}(\psi_L - \psi^c_R)$$

$$\chi_{2L} = \frac{1}{\sqrt{2}}(\chi^0_{1L} + \chi^0_{2L}) = \frac{1}{\sqrt{2}}(\psi_L + \psi^c_R)$$

$$\xi_1 = \chi_{1L} + \eta_1 \chi^c_{1L} \quad [\eta_1 = -1]$$

$$\xi_2 = \chi_{2L} + \eta_2 \chi^c_{2L} \quad [\eta_2 = +1]$$

with masses $m_1 = m_2 = m_D$

$$\Rightarrow \quad -\mathcal{L}_m = \frac{1}{2} m_D (-\overline{\chi}_1 \chi_1 + \overline{\chi}_2 \chi_2) = \frac{1}{2} m_D (\overline{\xi}_1 \xi_1 + \overline{\xi}_2 \xi_2) = m_D (\overline{\psi_L} \psi_R + \overline{\psi_R} \psi_L)$$

$$\boxed{\text{One Dirac fermion = two Majorana of equal mass and opposite CP parities}}$$

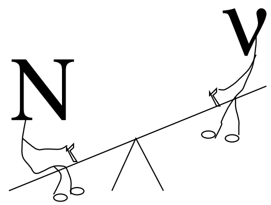
⋆ lepton sector

- About mass terms (a more transparent parameterization)
  - Case of seesaw (type I) [Yanagida '79; Gell-Mann, Ramond, Slansky '79; Mohapatra, Senjanovic '80]

    $(m_L = 0,\ m_D \ll m_R)$

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \quad \Rightarrow \quad \widetilde{\mathcal{U}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad \theta \simeq \frac{m_D}{m_R} \simeq \sqrt{\frac{m_\nu}{m_N}} \ \text{(negligible)}$$

$$m_1 \equiv m_\nu \simeq \frac{m_D^2}{m_R} \ll m_2 \equiv m_N \simeq m_R$$



$$\begin{aligned} \xi_1 &\equiv \nu = \psi_L + \eta_1 \psi_L^c \quad [\eta_1 = -1] \\ \xi_2 &\equiv N = \psi_R^c + \eta_2 \psi_R \quad [\eta_2 = +1] \end{aligned} \quad \Rightarrow \quad -\mathcal{L}_m = \frac{1}{2} m_\nu \, \overline{\nu_L^c} \nu_L + \frac{1}{2} m_N \, \overline{N_R^c} N_R + \text{h.c.}$$

> *Perhaps* the observed neutrino ν_L is the LH component of a light Majorana ν (then $\overline{\nu}$ = RH) and light because of a very heavy Majorana neutrino N

e.g. $m_D \sim v \simeq 246$ GeV, $m_R \sim m_N \sim 10^{15}$ GeV $\Rightarrow$ $m_\nu \sim 0.1$ eV ✓

Additional generations | lepton sector

- Lepton mixings
  - From Z lineshape: there are $n_G = 3$ generations of ν_L [ν_i $(i = 1, \ldots, n_G)$] (but we do not know (*yet*) if neutrinos are Dirac or Majorana fermions)
  - From neutrino oscillations: neutrinos are light, non degenerate and mix

$$|\nu_\alpha\rangle = \sum_i \mathbf{U}_{\alpha i} |\nu_i\rangle \quad \Longleftrightarrow \quad |\nu_i\rangle = \sum_\alpha \mathbf{U}^*_{\alpha i} |\nu_\alpha\rangle$$

mass eigenstates ν_i $(i = 1, 2, 3)$ / interaction states ν_α $(\alpha = e, \mu, \tau)$

$\Rightarrow$ **U** matrix is unitary (negligible mixing with heavy neutrinos) and analogous to $\mathbf{V}_u$, $\mathbf{V}_d$, $\mathbf{V}_\ell$ defined for quarks and charged leptons except for:

  - ν fields have both chiralities
  - *If* neutrinos are Majorana, **U** *may* contain two additional physical (Majorana) phases (irrelevant and therefore not measurable in oscillation experiments) that cannot be absorbed since then field phases are fixed by $\nu_i = \eta_i \nu_i^c$

Additional generations | lepton sector

- Lepton mixings

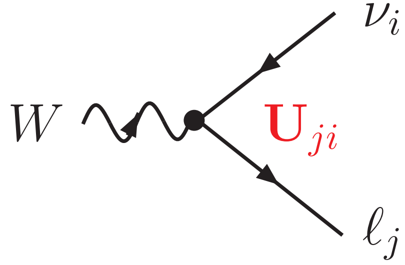
  The so called PMNS matrix **U** [Pontecorvo '57; Maki, Nakagawa, Sakata '62; Pontecorvo '68]

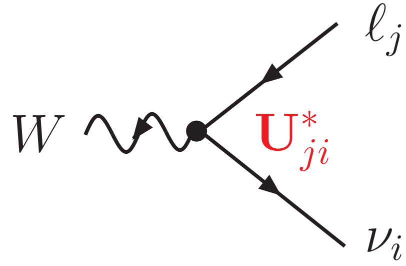
  – does not change Neutral Currents (unitarity), but

  – introduces intergenerational mixings in Charged Currents:

$$\mathcal{L}_{\rm CC} = \frac{g}{2\sqrt{2}} \sum_{\alpha i} \bar{\ell}_\alpha \gamma^\mu (1-\gamma_5)\, \mathbf{U}_{\alpha i}\, \nu_i W_\mu + \text{h.c.}$$

(basis where charged leptons are diagonal)





Additional generations | lepton sector

⇒ Standard parameterization of the PMNS matrix:

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{e1} & \mathbf{U}_{e2} & \mathbf{U}_{e3} \\ \mathbf{U}_{\mu 1} & \mathbf{U}_{\mu 2} & \mathbf{U}_{\mu 3} \\ \mathbf{U}_{\tau 1} & \mathbf{U}_{\tau 2} & \mathbf{U}_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, \mathrm{e}^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, \mathrm{e}^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, \mathrm{e}^{i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, \mathrm{e}^{i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, \mathrm{e}^{i\delta} & c_{23}\, c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e}^{i\alpha_{21}/2} & 0 \\ 0 & 0 & \mathrm{e}^{i\alpha_{31}/2} \end{pmatrix}$$

(different values than in CKM)

(Majorana phases)

[$\theta_{13} \equiv \theta_\odot$, $\theta_{23} \equiv \theta_{\rm atm}$, θ_{13} and δ measured in oscillations]

3. Phenomenology

Complete SM Lagrangian | fields and interactions

$$\boxed{\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{\text{YM}} + \mathcal{L}_\Phi + \mathcal{L}_\text{Y} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}}$$

- Fields: [F] fermions [S] scalars (Higgs and unphysical Goldstones)
  [V] vector bosons [U] unphysical ghosts

- Interactions: [FFV] [FFS] [SSV] [SVV] [SSVV]
  [VVV] [VVVV] [SSS] [SSSS]
  [SUU] [UUVV]

Complete SM Lagrangian | Feynman rules

- Feynman rules for generic couplings normalized to e (all momenta incoming):

$$
\begin{array}{lrl}
(\mathrm{i}\mathcal{L}) & [\mathrm{FFV}_\mu] & \mathrm{i}e\gamma^\mu(g_V - g_A\gamma_5) = \mathrm{i}e\gamma^\mu(g_L P_L + g_R P_R) \\
& [\mathrm{FFS}] & \mathrm{i}e(g_S - g_P\gamma_5) = \mathrm{i}e(c_L P_L + c_R P_R) \\
& [\mathrm{SV}_\mu \mathrm{V}_\nu] & \mathrm{i}eK g_{\mu\nu} \\
& [\mathrm{S}(p_1)\mathrm{S}(p_2)\mathrm{V}_\mu] & \mathrm{i}eG(p_1 - p_2)_\mu \\
& [\mathrm{V}_\mu(k_1)\mathrm{V}_\nu(k_2)\mathrm{V}_\rho(k_3)] & \mathrm{i}eJ\left[g_{\mu\nu}(k_2-k_1)_\rho + g_{\nu\rho}(k_3-k_2)_\mu + g_{\mu\rho}(k_1-k_3)_\nu\right] \\
& [\mathrm{V}_\mu(k_1)\mathrm{V}_\nu(k_2)\mathrm{V}_\rho(k_3)\mathrm{V}_\sigma(k_4)] & \mathrm{i}e^2 C_1\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right] \\
& [\mathrm{SSV}_\mu \mathrm{V}_\nu] & \mathrm{i}e^2 C_2 g_{\mu\nu} \qquad\qquad \text{also [UUVV]} \\
& [\mathrm{SSS}] & \mathrm{i}eC_3 \qquad\qquad \text{also [SUU]} \\
& [\mathrm{SSSS}] & \mathrm{i}e^2 C_4
\end{array}
$$

Note: $g_{L,R} = g_V \pm g_A$ $\qquad \partial_\mu \to -\mathrm{i}p_\mu$ $\qquad$ Attention to symmetry factors!

$c_{L,R} = g_S \pm g_P$ $\qquad$ *e.g.* $2 \times HZZ$

Complete SM Lagrangian | Feynman rules ('t Hooft-Feynman gauge)

FFV	$\bar{f}_i f_j \gamma$	$\bar{f}_i f_j Z$	$\bar{u}_i d_j W^+$	$\bar{d}_j u_i W^-$	$\bar{\nu}_i \ell_j W^+$	$\bar{\ell}_j \nu_i W^-$
g_L	$-Q_f \delta_{ij}$	$g_+^f \delta_{ij}$	$\dfrac{1}{\sqrt{2} s_W} \mathbf{V}_{ij}$	$\dfrac{1}{\sqrt{2} s_W} \mathbf{V}_{ij}^*$	$\dfrac{1}{\sqrt{2} s_W} \mathbf{U}_{ji}^*$	$\dfrac{1}{\sqrt{2} s_W} \mathbf{U}_{ji}$
g_R	$-Q_f \delta_{ij}$	$g_-^f \delta_{ij}$	0	0	0	0

$$g_\pm^f \equiv v_f \pm a_f \qquad v_f = \frac{T_3^{f_L} - 2Q_f s_W^2}{2 s_W c_W} \qquad a_f = \frac{T_3^{f_L}}{2 s_W c_W}$$

Complete SM Lagrangian **Feynman rules** ('t Hooft-Feynman gauge)

FFS	$\bar{f}_i f_j H$	$\bar{f}_i f_j \chi$	$\bar{u}_i d_j \phi^+$	$\bar{d}_j u_i \phi^-$
c_L	$-\dfrac{1}{2s_W}\dfrac{m_{f_i}}{M_W}\delta_{ij}$	$-\dfrac{\mathrm{i}}{2s_W}2T_3^{f_L}\dfrac{m_{f_i}}{M_W}\delta_{ij}$	$+\dfrac{1}{\sqrt{2}s_W}\dfrac{m_{u_i}}{M_W}\mathbf{V}_{ij}$	$-\dfrac{1}{\sqrt{2}s_W}\dfrac{m_{d_j}}{M_W}\mathbf{V}_{ij}^*$
c_R	$-\dfrac{1}{2s_W}\dfrac{m_{f_i}}{M_W}\delta_{ij}$	$+\dfrac{\mathrm{i}}{2s_W}2T_3^{f_L}\dfrac{m_{f_i}}{M_W}\delta_{ij}$	$-\dfrac{1}{\sqrt{2}s_W}\dfrac{m_{d_j}}{M_W}\mathbf{V}_{ij}$	$+\dfrac{1}{\sqrt{2}s_W}\dfrac{m_{u_j}}{M_W}\mathbf{V}_{ij}^*$

$(f = u, d, \ell)$

FFS	$\bar{\nu}_i \ell_j \phi^+$	$\bar{\ell}_j \nu_i \phi^-$
c_L	$+\dfrac{1}{\sqrt{2}s_W}\dfrac{m_{\nu_i}}{M_W}\mathbf{U}_{ji}^*$	$-\dfrac{1}{\sqrt{2}s_W}\dfrac{m_{\ell_j}}{M_W}\mathbf{U}_{ji}$
c_R	$-\dfrac{1}{\sqrt{2}s_W}\dfrac{m_{\ell_j}}{M_W}\mathbf{U}_{ji}^*$	$+\dfrac{1}{\sqrt{2}s_W}\dfrac{m_{\nu_i}}{M_W}\mathbf{U}_{ji}$

Complete SM Lagrangian **Feynman rules** ('t Hooft-Feynman gauge)

SVV	HZZ	HW^+W^-	$\phi^\pm W^\mp \gamma$	$\phi^\pm W^\mp Z$
K	$M_W/(s_W c_W^2)$	M_W/s_W	$-M_W$	$-M_W s_W/c_W$

SSV	$\chi H Z$	$\phi^\pm \phi^\mp \gamma$	$\phi^\pm \phi^\mp Z$	$\phi^\mp H W^\pm$	$\phi^\mp \chi W^\pm$
G	$-\dfrac{\mathrm{i}}{2s_W c_W}$	∓ 1	$\pm\dfrac{c_W^2 - s_W^2}{2s_W c_W}$	$\mp\dfrac{1}{2s_W}$	$-\dfrac{\mathrm{i}}{2s_W}$

VVV	γW^+W^-	ZW^+W^-
J	-1	c_W/s_W

Complete SM Lagrangian | Feynman rules ('t Hooft-Feynman gauge)

VVVV	$W^+W^+W^-W^-$	W^+W^-ZZ	$W^+W^-\gamma Z$	$W^+W^-\gamma\gamma$
C_1	$\dfrac{1}{s_W^2}$	$-\dfrac{c_W^2}{s_W^2}$	$\dfrac{c_W}{s_W}$	-1

SSVV	HHW^-W^+	$HHZZ$
C_2	$\dfrac{1}{2s_W^2}$	$\dfrac{1}{2s_W^2c_W^2}$

SSS	HHH
C_3	$-\dfrac{3M_H^2}{2M_Ws_W}$

SSSS	$HHHH$
C_4	$-\dfrac{3M_H^2}{4M_W^2s_W^2}$

– Would-be Goldstone bosons in [SSVV], [SSS] and [SSSS] omitted

– Faddeev-Popov ghosts in [SUU] and [UUVV] omitted

– All Feynman rules from FeynArts (same conventions; $\chi, \phi^\pm \to G^0, G^\pm$):

http://www.ugr.es/local/jillana/SM/FeynmanRulesSM.pdf

Input parameters

- Parameters:

17 +9 =	1	1	1	1	9 +3	4	6
formal:	g	g'	v	λ	λ_f	$\mathbf{V}_{\text{CKM}}$	$\mathbf{U}_{\text{PMNS}}$
practical:	α	M_W	M_Z	M_H	m_f		

where $g = \dfrac{e}{s_W}$ $\quad g' = \dfrac{e}{c_W}$ and

$$\underbrace{\alpha = \frac{e^2}{4\pi} \qquad M_W = \frac{1}{2} g v \qquad M_Z = \frac{M_W}{c_W}}_{g,\ g',\ v} \qquad M_H = \sqrt{2\lambda}\, v \qquad m_f = \frac{v}{\sqrt{2}} \lambda_f$$

$\Rightarrow$ Many (more) experiments

$\Rightarrow$ After Higgs discovery, for the first time *all* parameters measured!

Input parameters

- Experimental values [Particle Data Group '18]

  - Fine structure constant:

    $\alpha^{-1} = 137.035\,999\,139\,(31)$ from Harvard cyclotron (g_e)

    $\alpha^{-1} = 137.035\,999\,046\,(27)$ from Cesium atom interferometry
    (*new* anomaly in $g_e - 2$!!) [Parke et al '18]

  - The SM predicts $M_W < M_Z$ in agreement with measurements:

    $M_Z = (91.1876 \pm 0.0021)$ GeV from LEP1/SLD

    $M_W = (80.379 \pm 0.012)$ GeV from LEP2/Tevatron/LHC

  - Top quark mass:

    $m_t = (173.0 \pm 0.4)$ GeV from Tevatron/LHC

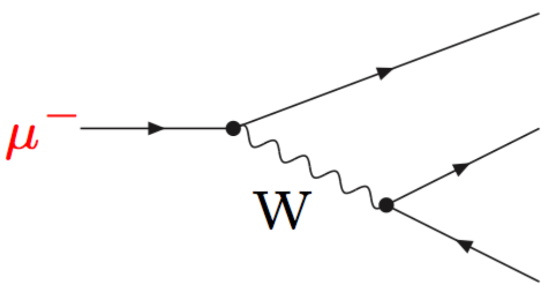
  - Higgs boson mass:

    $M_H = (125.18 \pm 0.16)$ GeV from LHC

  - ...

Observables and experiments

- Low energy observables ($Q^2 \ll M_Z^2$)
  - ν-nucleon (NuTeV) and νe (CERN) scattering:
    asymmetries CC/NC and $\nu/\bar{\nu}$ $\Rightarrow \boxed{s_W^2}$
  - Atomic parity violation (SLAC, CERN, Jefferson Lab, Mainz):
    LR asymmetries $\mathrm{e}_{R,L}N \to \mathrm{e}X$ due to Z-exchange between e and N $\Rightarrow \boxed{s_W^2}$
  - muon decay: $\mu \to e\,\overline{\nu}_e \nu_\mu$ (PSI)
    lifetime



$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} f(m_e^2/m_\mu^2)$$

$$f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \quad \Rightarrow \boxed{G_F}$$

$$= 0.99981295$$

$$\mathrm{i}\mathcal{M} = \left(\frac{\mathrm{i}e}{\sqrt{2}s_W}\right)^2 \bar{e}\gamma^\rho \nu_L \frac{-\mathrm{i}g_{\rho\delta}}{q^2 - M_W^2} \overline{\nu_L}\gamma^\delta \mu \equiv \overbrace{\mathrm{i}\frac{4G_F}{\sqrt{2}} (\bar{e}\gamma^\rho \nu_L)(\overline{\nu_L}\gamma_\rho \mu)}^{\text{Fermi theory } (-q^2 \ll M_W^2)} \; ; \quad \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2}$$

Observables and experiments

- Low energy observables

  $\Rightarrow$ Fermi constant provides the Higgs VEV (electroweak scale):

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \approx 246\,\mathrm{GeV}$$

  $\Rightarrow$ Consistency checks: e.g.

  From muon lifetime:

$$G_F = 1.166\,378\,7(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

  If one compares with (tree level result)

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2} = \frac{\pi\alpha}{2(1 - M_W^2/M_Z^2)M_W^2}$$

  using measurements of M_W, M_Z and α there is a discrepancy that disappears when *quantum corrections* are included

Observables and experiments

- $e^+e^- \to \bar{f}f$ (PEP, PETRA, TRISTAN, ..., LEP1, SLD)

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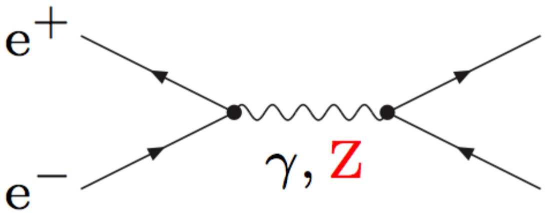


$$\frac{d\sigma}{d\Omega} = N_c^f \frac{\alpha^2}{4s}\beta_f \Big\{ \left[1+\cos^2\theta + (1-\beta_f^2)\sin^2\theta\right] G_1(s) + 2(\beta_f^2-1)\, G_2(s) + 2\beta_f \cos\theta\, G_3(s) \Big\}$$

$$G_1(s) = Q_e^2 Q_f^2 + 2Q_eQ_f v_e v_f \mathrm{Re}\chi_Z(s) + (v_e^2+a_e^2)(v_f^2+a_f^2)|\chi_Z(s)|^2$$

$$G_2(s) = (v_e^2+a_e^2)a_f^2|\chi_Z(s)|^2$$

$$G_3(s) = 2Q_eQ_f a_e a_f \mathrm{Re}\chi_Z(s) + 4v_e v_f a_e a_f |\chi_Z(s)|^2 \quad \Rightarrow \quad A_{FB}(s)$$

with $\chi_Z(s) \equiv \dfrac{s}{s-M_Z^2+\mathrm{i}M_Z\Gamma_Z}$, $N_c^f = 1$ (3) for f = lepton (quark), β_f = velocity

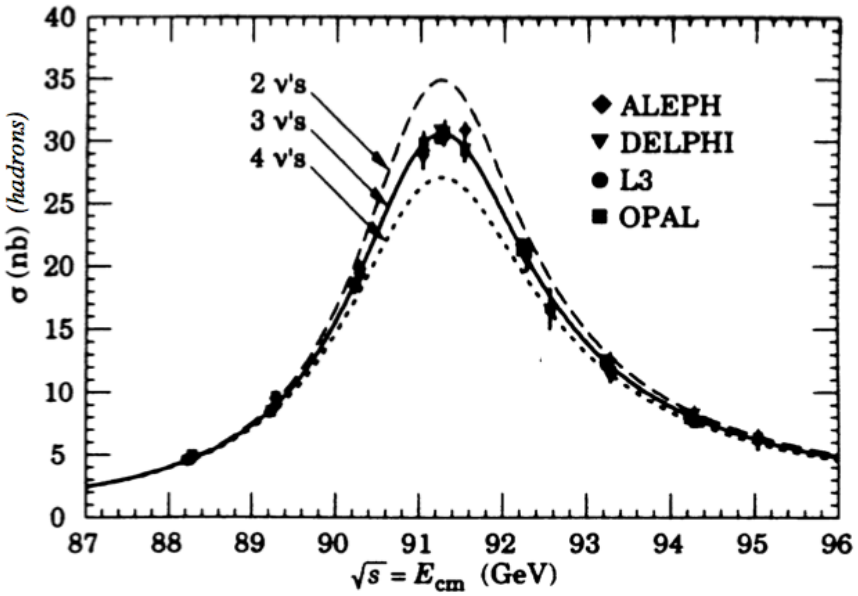
$$\sigma(s) = N_c^f \frac{2\pi\alpha^2}{3s}\beta_f \left[(3-\beta_f^2)G_1(s) - 3(1-\beta_f^2)G_2(s)\right], \quad \beta_f = \sqrt{1-4m_f^2/s}$$

Observables and experiments

- Z production (LEP1/SLD)

  $M_Z,\ \Gamma_Z,\ \sigma_{\text{had}},\ A_{FB},\ A_{LR},\ R_b,\ R_c,\ R_\ell \quad \Rightarrow \boxed{M_Z,\ s_W^2}$

  from $e^+e^- \to \bar{f}f$ at the Z pole ($\gamma - Z$ interference vanishes). Neglecting m_f:



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$$\sigma_{\text{had}} = 12\pi \frac{\Gamma(e^+e^-)\Gamma(\text{had})}{M_Z^2\Gamma_Z^2}$$

$$R_b = \frac{\Gamma(b\bar{b})}{\Gamma(\text{had})} \qquad R_c = \frac{\Gamma(c\bar{c})}{\Gamma(\text{had})} \qquad R_\ell = \frac{\Gamma(\text{had})}{\Gamma(\ell^+\ell^-)}$$

$$\left[\Gamma(Z \to f\bar{f}) \equiv \Gamma(f\bar{f}) = N_c^f \frac{\alpha M_Z}{3}(v_f^2 + a_f^2)\right]$$

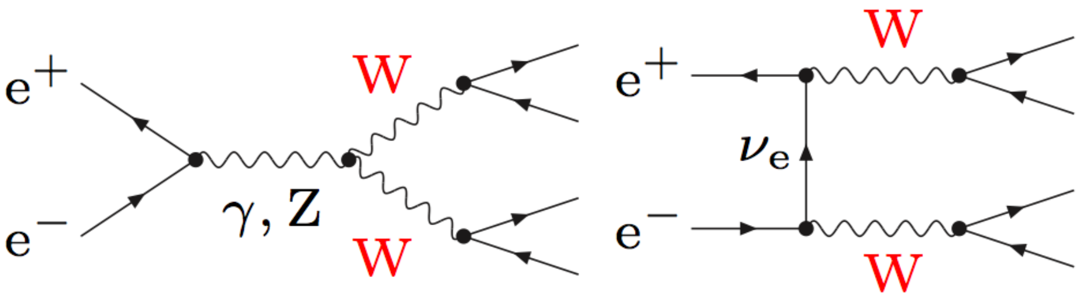
$$\Rightarrow N_\nu = 2.990 \pm 0.007$$

Forward-Backward and (if polarized e^-) Left-Right asymmetries due to Z:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_f \frac{A_e + P_e}{1 + P_e A_e} \qquad A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e P_e \quad \text{with } A_f \equiv \frac{2 v_f a_f}{v_f^2 + a_f^2}$$

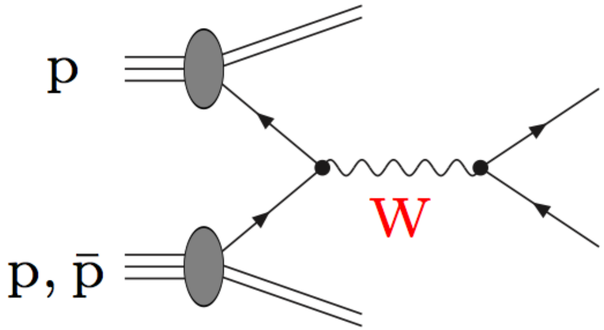
Observables and experiments

- W-pair production (LEP2)
  $e^+e^- \to WW \to 4\,f\ (+\gamma)$



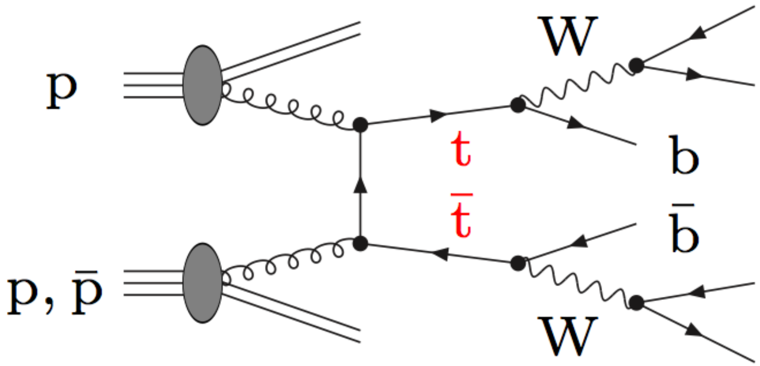
$\Rightarrow \boxed{M_W}$

- W production (Tevatron/LHC)
  $pp/p\bar{p} \to W \to \ell\nu_\ell\ (+\gamma)$



$\Rightarrow \boxed{M_W}$

- Top-quark production (Tevatron/LHC)
  $pp/p\bar{p} \to t\bar{t} \to 6\,f$



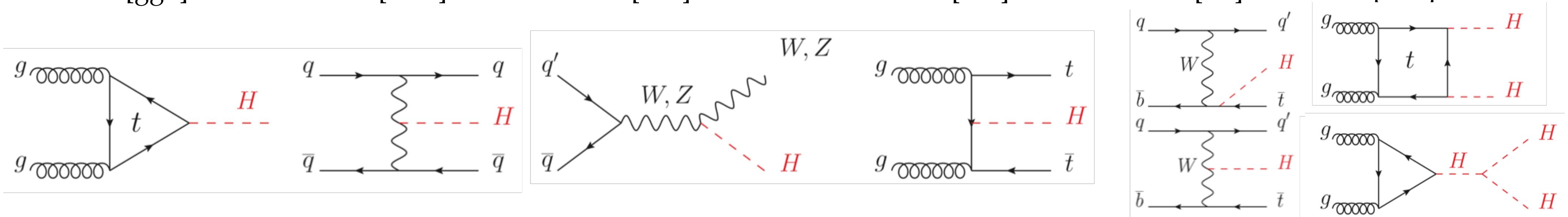
$\Rightarrow \boxed{m_t}$

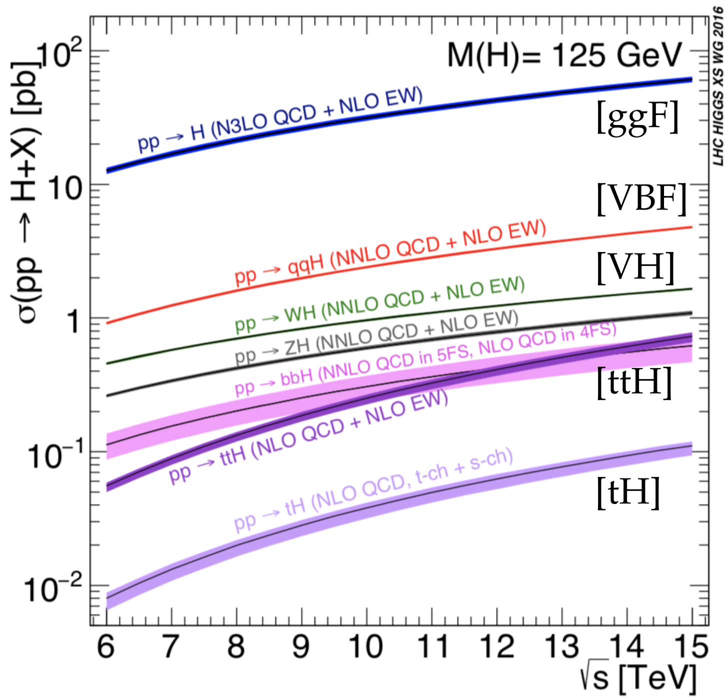
Observables and experiments

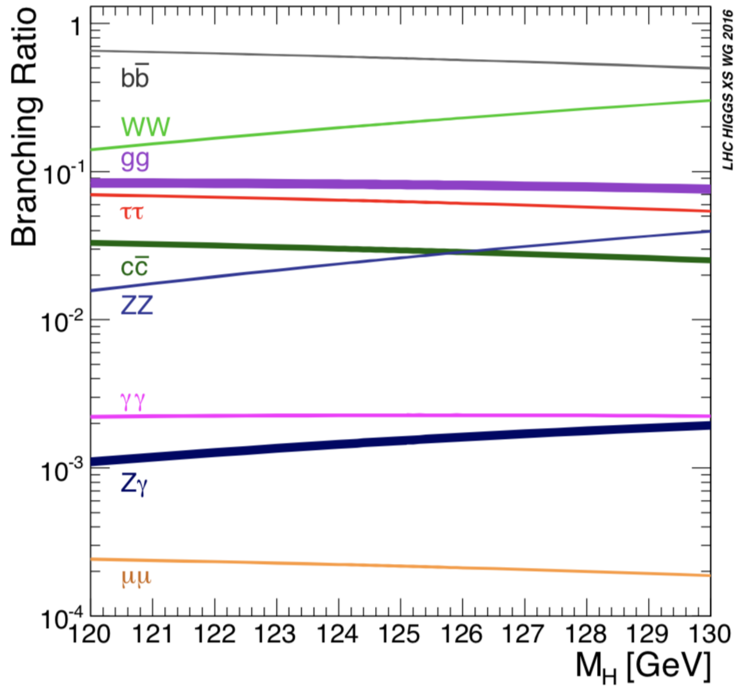
- Higgs (LHC) [PDG '18]

  Single and Double H production and decay to different channels $\Rightarrow$ $\boxed{M_H}$

[ggF] [VBF] [VH] [ttH] [tH] [HH]







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Observables and experiments

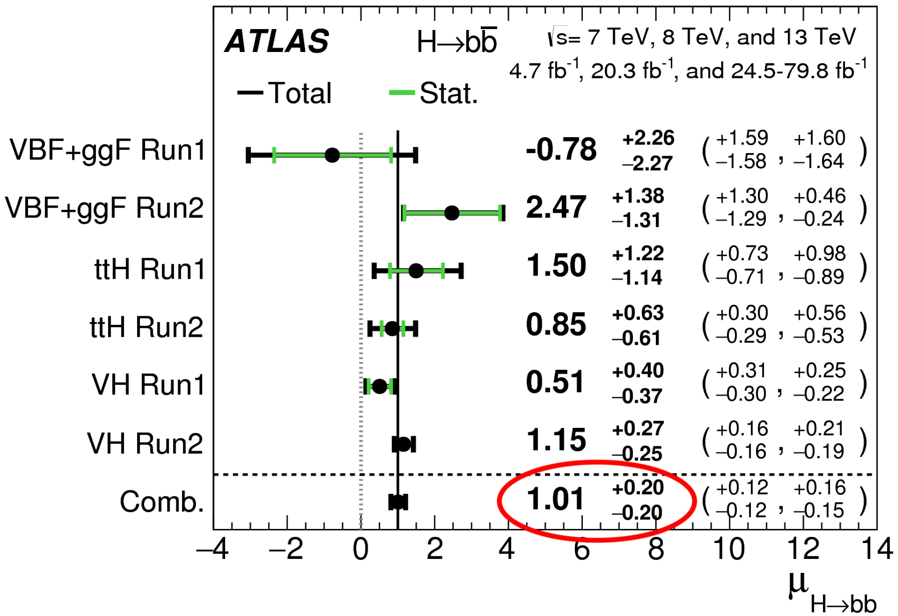
- Higgs (LHC) [PDG '18]

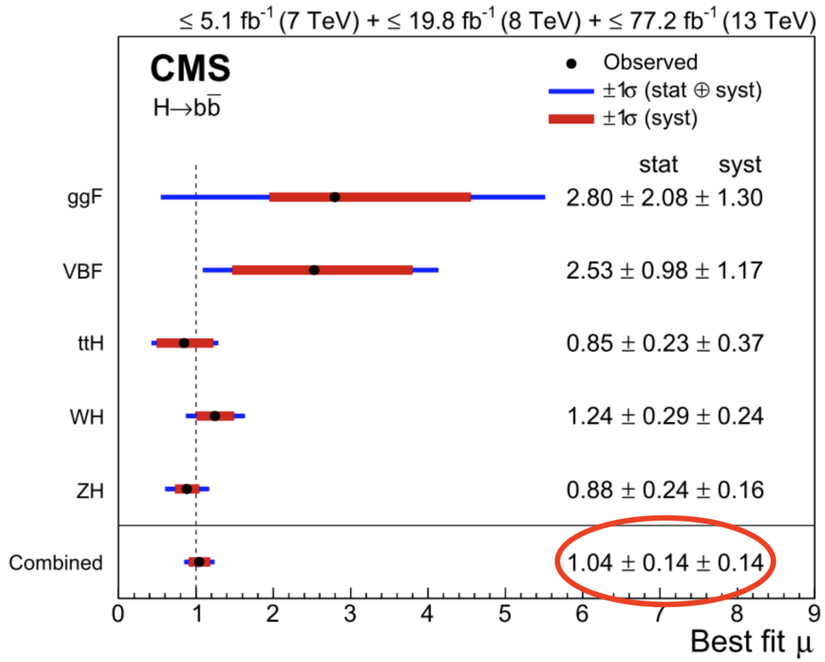
Signal strength $\mu = \dfrac{(\sigma \cdot \mathrm{BR})_{\rm obs}}{(\sigma \cdot \mathrm{BR})_{\rm SM}}$	Run 1	Run 2
ATLAS	1.17 ± 0.27	0.99 ± 0.14
CMS	$0.78^{+0.26}_{-0.23}$	$1.16^{+0.15}_{-0.14}$

Per channel:

$\gamma\gamma,\ ZZ,\ W^+W^-,\ \tau^+\tau^- > 5\sigma$

$\boxed{b\bar{b} > 5\sigma}$ at last! [LHC Seminar 28 Jul 2018]





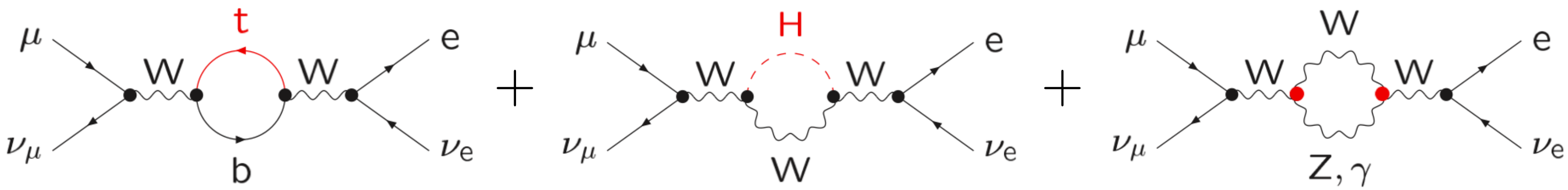
Precise determination of parameters

- Experimental precision requires accurate predictions $\Rightarrow$ quantum corrections (complication: loop calculations involve renormalization)

– Correction to G_F from muon lifetime:

$$\frac{G_F}{\sqrt{2}} \to \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2(1-M_W^2/M_Z^2)M_W^2}[1+\Delta r(m_t, M_H)]$$

when loop corrections are included:



$+ \cdots + 2$ loops

Since muon lifetime is measured more precisely than M_W, it is traded for G_F:

$$\Rightarrow M_W^2(\alpha, G_F, M_Z, m_t, M_H) = \frac{M_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}[1+\Delta r(m_t, M_H)]}\right)$$

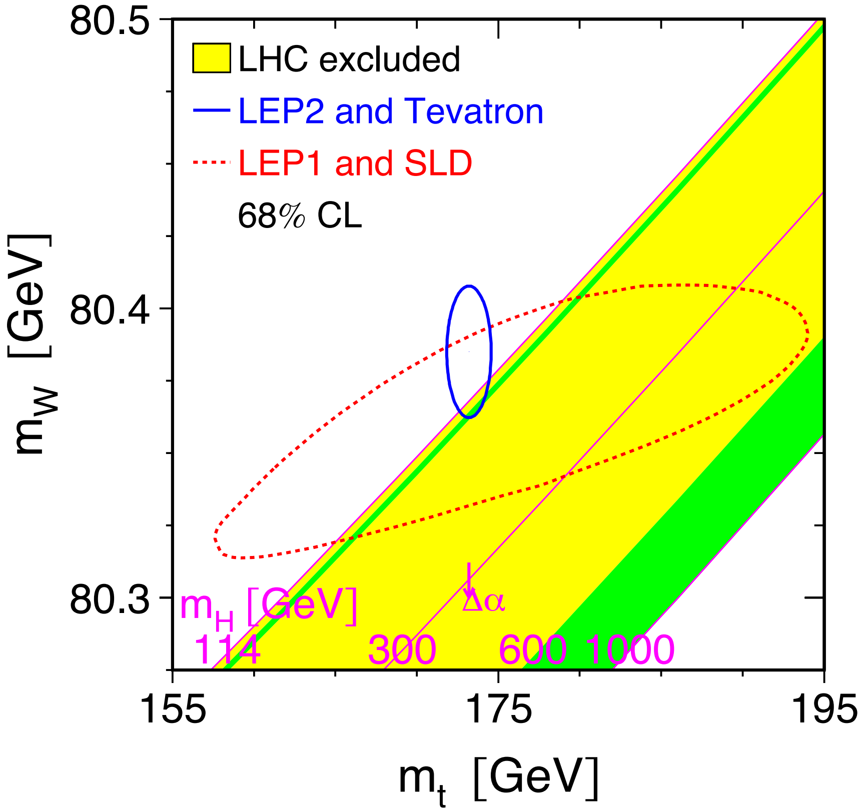
(correlation between M_W, m_t and M_H, given α, G_F and M_Z)

Precise determination of parameters

Indirect constraints from LEP1/SLD   Direct measurements from LEP2/Tevatron

$M_H(M_W, m_t)$ Allowed regions for M_H allowed by direct searches   LHC excluded

[LEPEWWG 2013]



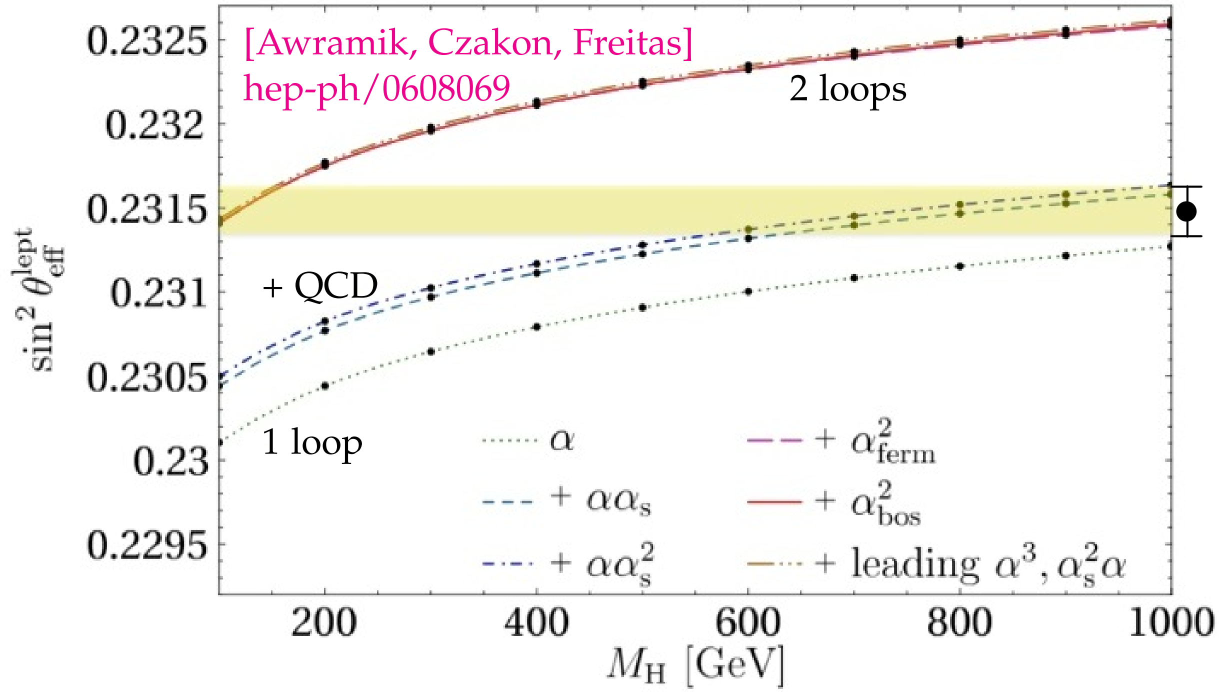
Precise determination of parameters

– Corrections to vector and axial couplings from Z pole observables:

$$v_f \to g_V^f = v_f + \Delta g_V^f \qquad a_f \to g_A^f = a_f + \Delta g_A^f$$

$$\Rightarrow \sin^2\theta_{\text{eff}}^f = \frac{1}{4|Q_f|}\left[1 - \text{Re}(g_V^f/g_A^f)\right] \equiv \overbrace{(1 - M_W^2/M_Z^2)}^{s_W^2}\ \kappa_Z^f$$

(Two) loop calculations are crucial and point to a light Higgs:



$$s_W^2 = 0.22290 \pm 0.00029 \text{ (tree)}$$

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23148 \pm 0.000017 \text{ (exp)}$$

Precise determination of parameters

- In addition, experiments and observables testing the flavor structure of the SM:
  flavor conserving: dipole moments, … flavor changing: $b \to s\gamma$, …

  $\Rightarrow$ very sensitive to new physics through loop corrections

  Extremely precise measurements are:
  - electron magnetic moment (new physics suppressed by a factor of m_e^2/m_μ^2):

$$\left.\begin{array}{rl} \text{exp:} & g_e/2 = 1.001\,159\,652\,180\,76\,(27) \\ \text{theo:} & \text{QED (5 loops!)} \end{array}\right\} \Rightarrow \alpha^{-1} = 137.035\,999\,139\,(31)$$

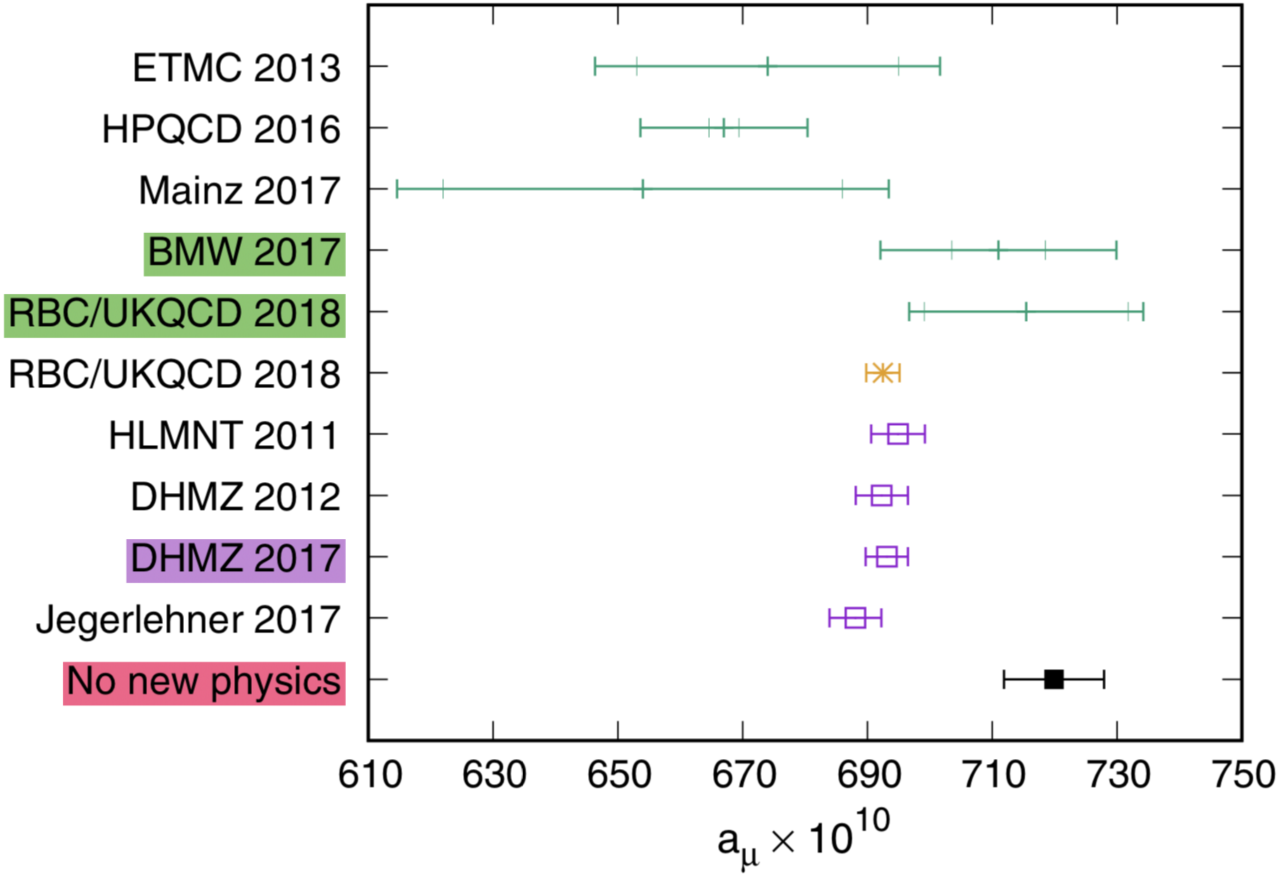
  - muon anomalous magnetic moment: $a_\mu = (g_\mu - 2)/2$

a_μ^{exp}	$= 116\,592\,089\,(63)$	$\times 10^{-11}$	[Brookhaven '06]
a_μ^{QED}	$= 116\,584\,718$	$\times 10^{-11}$	[QED: 5 loops]
a_μ^{EW}	$= 154$	$\times 10^{-11}$	[W, Z, H: 2 loops]
a_μ^{had}	$= 6\,930\,(48)$	$\times 10^{-11}$	[$e^+e^- \to$ had]
a_μ^{SM}	$= 116\,591\,802\,(49)$	$\times 10^{-11}$	

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 287\,(80) \times 10^{-11}$$

3.6σ !

- New lattice calculations of $a_\mu^{\rm had}$ [PRL **121**, 022002 & 022003, 12 July 2018]



$\leftarrow 7\,111\,(189) \times 10^{-11}$

$\leftarrow 7\,154\,(187) \times 10^{-11}$

$\leftarrow 6\,930\,(48) \times 10^{-11}$

- New Muon $g-2$ Experiment at Fermilab (running from March 2018)

Global fits

- Fit input data from a list of observables (EWPO):

$$M_H,\ M_W,\ \Gamma_W,\ M_Z,\ \Gamma_Z,\ \sigma_{\rm had},\ A_{FB}^{b,c,\ell},\ A_{b,c,\ell},\ R_{b,c,\ell},\ \sin^2\theta_{\rm eff}^{\rm lept},\ \ldots$$

finding the $\chi^2_{\rm min}$ for $n_{\rm dof} = 13\ (14)$ when M_H is included (excluded):

$$\underbrace{\alpha_s(M_Z)}_{1\ \text{(QCD)}},\ \underbrace{\Delta\alpha_{\rm had}(M_Z),\ G_F,\ M_Z,\ 9\ \text{fermion masses},\ M_H}_{17-4=13\ \text{(CKM irrelevant)}}$$

$$\alpha(M_Z) \equiv \frac{\alpha}{1-\Delta\alpha(M_Z)}$$

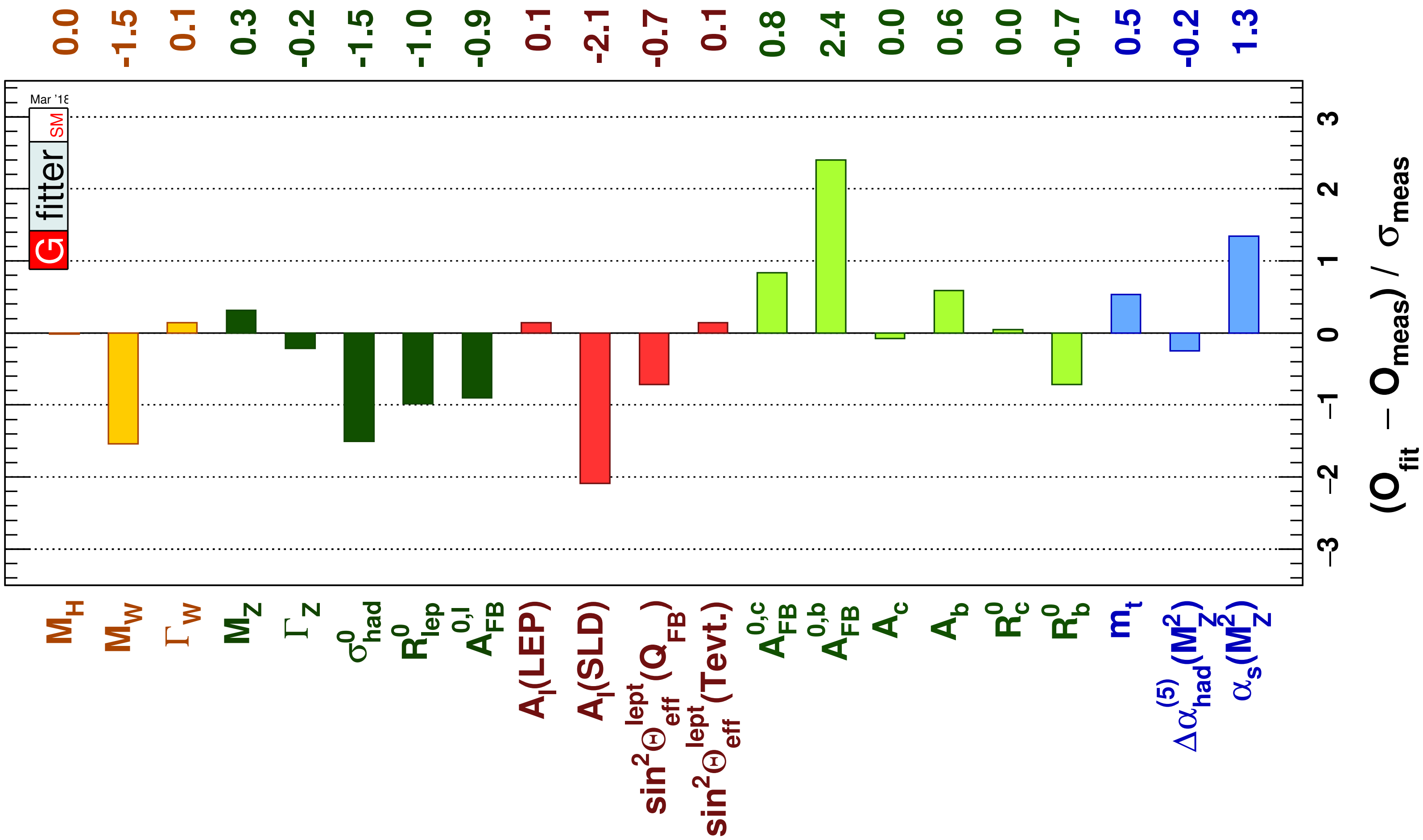
[Gfitter 2018: 1803.0853] http://gfitter.desy.de

$n_{\rm dof}$	$\chi^2_{\rm min}$	p-value
15	18.6	0.23

p-value (goodnes of fit): probability, under assumption of hypothesis H, to observe data with equal or lesser compatibility with H relative to the data we got

Global fits (Comparisons)

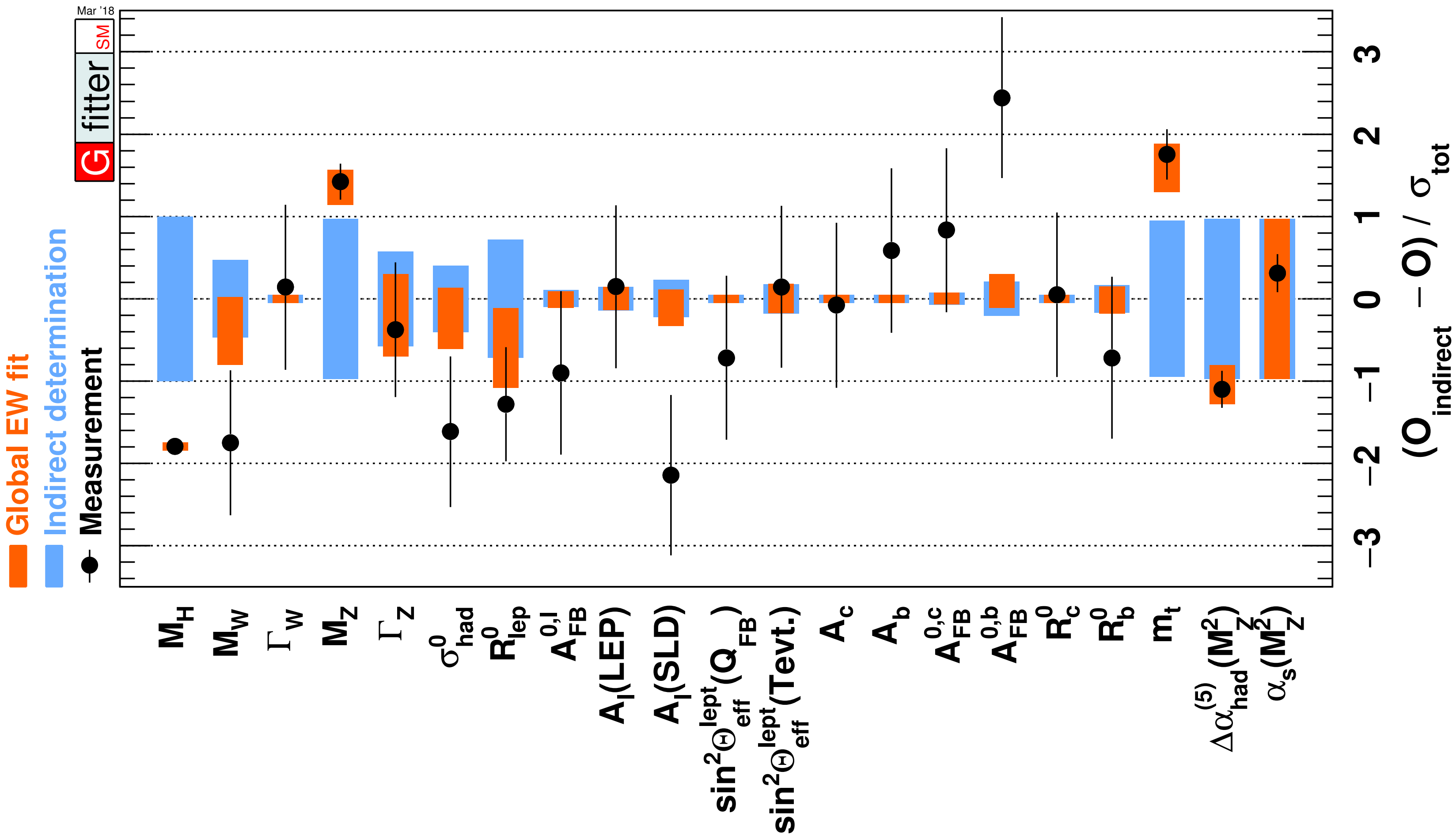
- Compare direct measurements of these observables with fit values:



$\Rightarrow$ some tensions (none above 3σ): A_ℓ(SLD), A^b_{FB}(LEP), R_b, ...

Global fits (Comparisons)

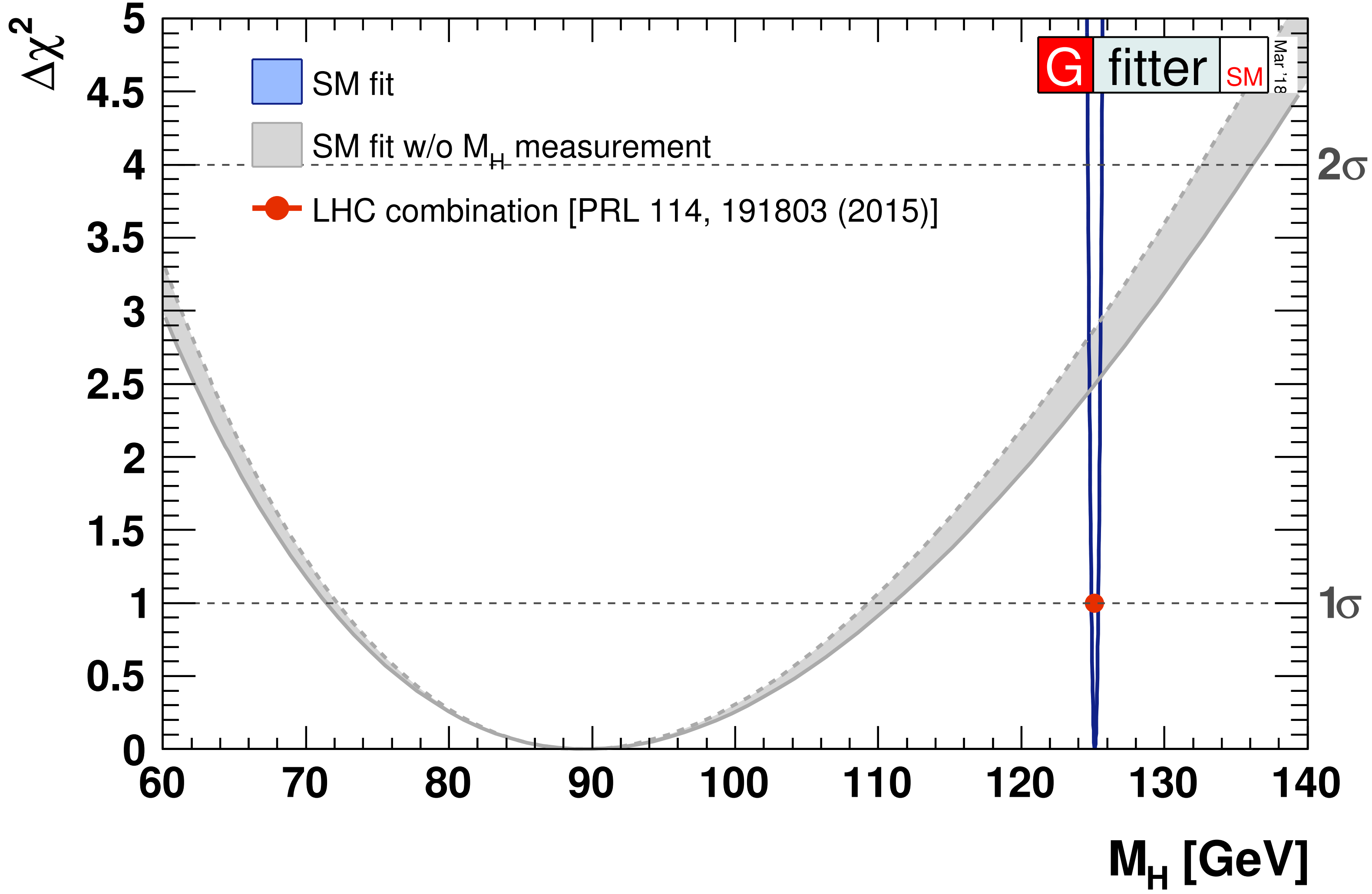
- Compare indirect determinations with fit values (error bars are direct measmts.):



[indirect determination means fit without using constraint from given direct measurement]

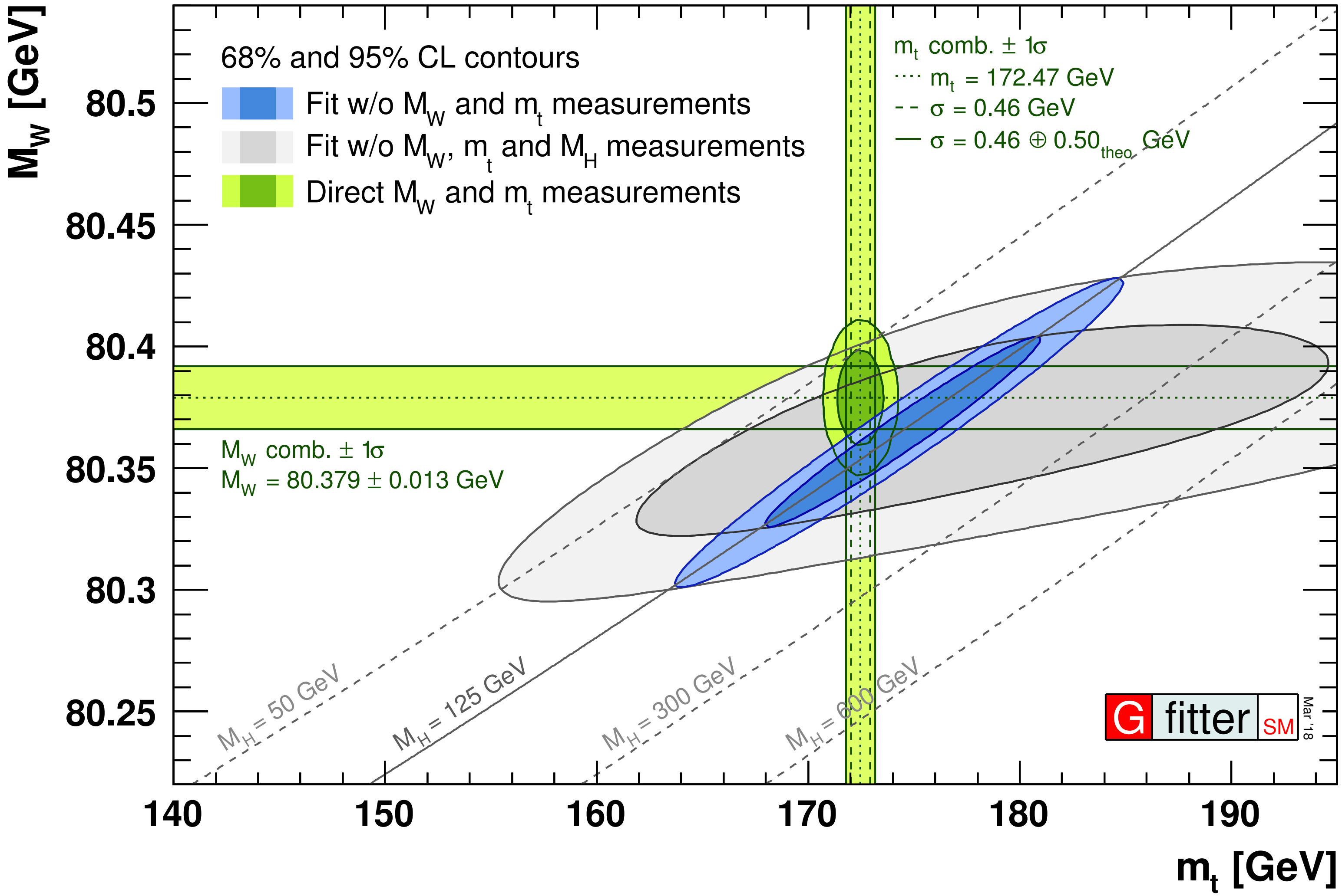
Global fits (Conclusions)

⇒ Fits prefer a somewhat lighter Higgs:



Global fits (Conclusions)

⇒ In general, impressive consistency of the SM, e.g.:

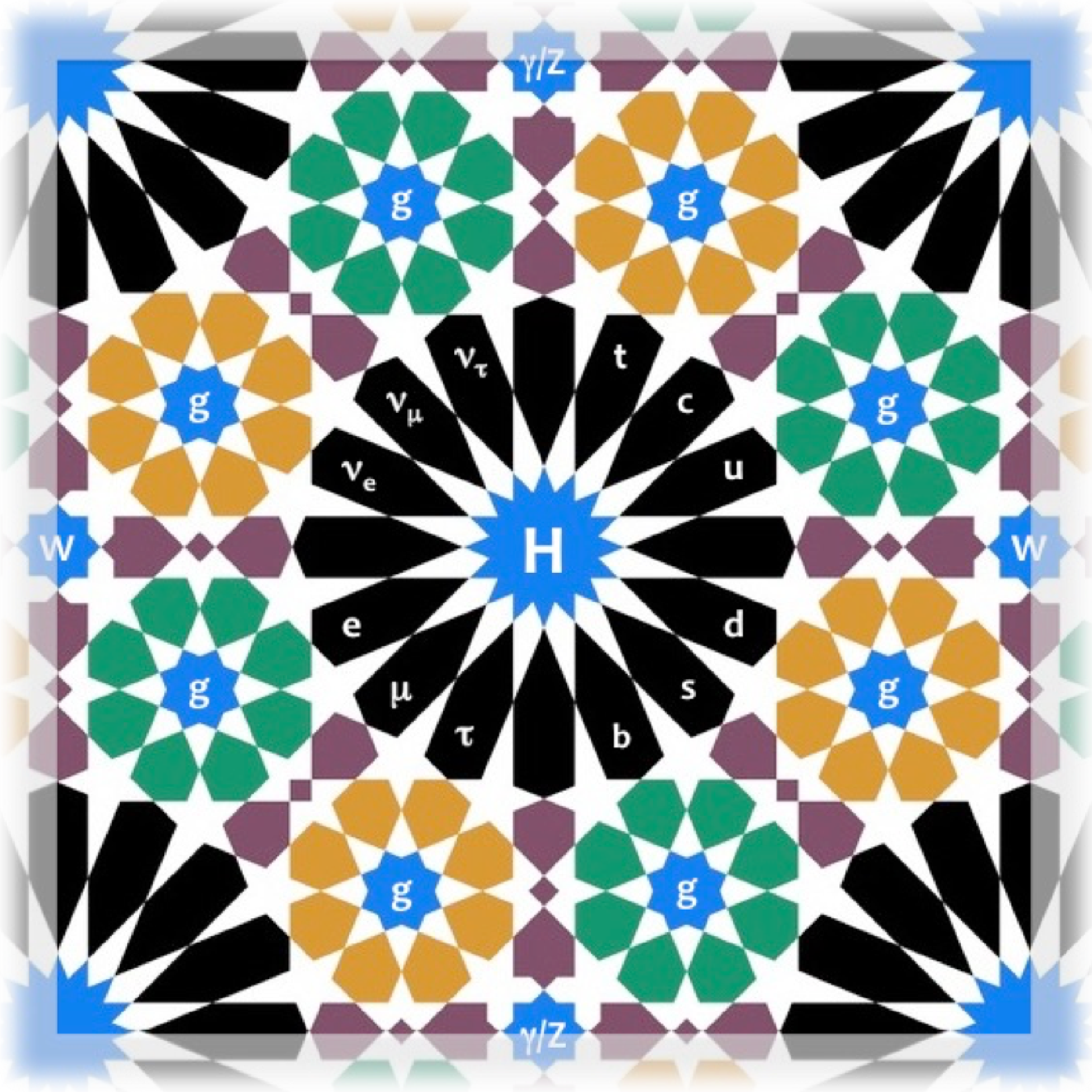


Summary

- The SM is a gauge theory with spontaneous symmetry breaking (renormalizable)
- Confirmed by many low and high energy experiments with remarkable accuracy, at the level of quantum corrections, with (almost) no significant deviations
- In spite of its tremendous success, it leaves fundamental questions unanswered:

  why 3 generations? why the observed pattern of fermion masses and mixings?
- And there are several hints for physics beyond:
  - phenomenological:
    * $(g_\mu - 2)$
    * neutrino masses
    * baryon asymmetry
    * dark matter
    * dark energy
    * cosmological constant
  - conceptual:
    * gravity is not included
    * hierarchy problem

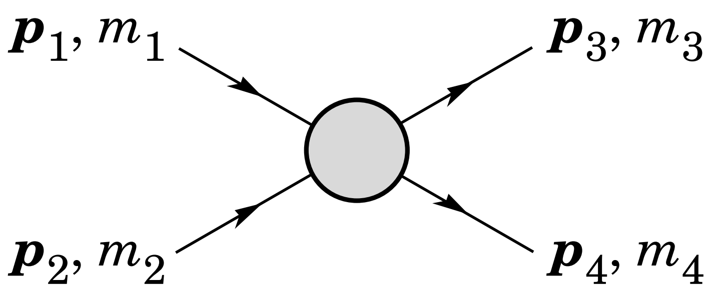
    The SM is an Effective Theory valid up to electroweak scale?



Thank You!

Kinematics

$2 \to 2$ Kinematics



$$p_1 + p_2 = p_3 + p_4$$

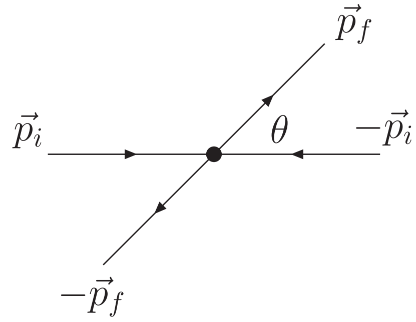
Mandelstam variables

(Lorentz invariant)

$$\begin{aligned}
s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 = m_1^2 + m_2^2 + 2(p_1 \cdot p_2) \\
t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 = m_1^2 + m_3^2 - 2(p_1 \cdot p_3) \\
u &= (p_1 - p_4)^2 = (p_2 - p_3)^2 = m_1^2 + m_4^2 - 2(p_1 \cdot p_4) \\
s + t + u &= \sum_{i=1}^{4} m_i^2 .
\end{aligned}$$

$2 \to 2$ Kinematics | Center of mass frame (CM)

Consider particular case: $m_1 = m_2 \equiv m_i, \quad m_2 = m_4 \equiv m_f$



$$
\begin{aligned}
p_1 &= (E, 0, 0, |\boldsymbol{p_i}|) \\
p_2 &= (E, 0, 0, -|\boldsymbol{p_i}|) \\
p_3 &= (E, |\boldsymbol{p_f}| \sin\theta, 0, |\boldsymbol{p_f}| \cos\theta) \\
p_4 &= (E, -|\boldsymbol{p_f}| \sin\theta, 0, -|\boldsymbol{p_f}| \cos\theta)
\end{aligned}
$$

$$
\begin{aligned}
s &= 4E^2 = E_{\text{CM}}^2 \\
t &= -\frac{s}{2}(1 - \beta_i \beta_f \cos\theta) + m_i^2 + m_f^2 \\
u &= -\frac{s}{2}(1 + \beta_i \beta_f \cos\theta) + m_i^2 + m_f^2
\end{aligned}
$$

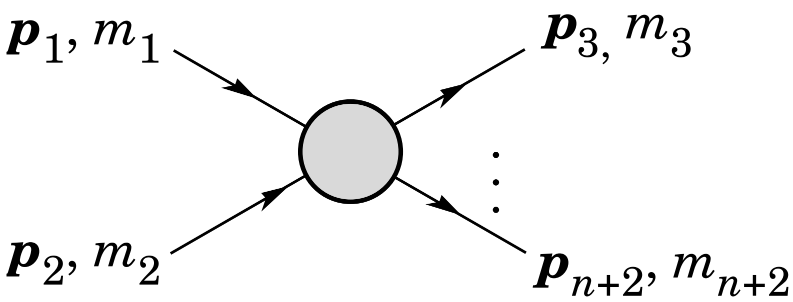
$$(s \geq \text{máx}\{4m_i^2, 4m_f^2\};\ t, u \leq -|m_i^2 - m_f^2|)$$

where $E^2 - |\boldsymbol{p}_{i,f}|^2 = m_{i,f}^2, \quad \beta_{i,f} = |\boldsymbol{p}_{i,f}|/E = \sqrt{1 - 4m_{i,f}^2/s}.$

Scalar products:

$$
\begin{aligned}
m_i^2 + (p_1 \cdot p_2) = m_f^2 + (p_3 \cdot p_4) &= 2E^2 = \frac{s}{2} \\
(p_1 \cdot p_3) = (p_2 \cdot p_4) &= E^2(1 - \beta_i \beta_f \cos\theta) = \frac{m_i^2 + m_f^2 - t}{2} \\
(p_1 \cdot p_4) = (p_2 \cdot p_3) &= E^2(1 + \beta_i \beta_f \cos\theta) = \frac{m_i^2 + m_f^2 - u}{2}
\end{aligned}
$$

Cross-section

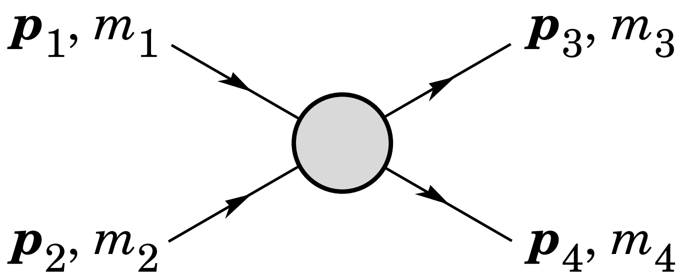


$$\mathrm{d}\sigma(i \to f) = \frac{1}{4\left\{(p_1 p_2)^2 - m_1^2 m_2^2\right\}^{1/2}} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_i - p_f) \prod_{j=3}^{n+2} \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j}$$

- ▷ Sum over initial polarizations and/or average over final polarizations if the initial state is unpolarized and/or the final state polarization is not measured
- ▷ Divide the total cross-section by a symmetry factor $S = \prod_i n_i!$ if there are n_i identical particles of species i in the final state

Cross-section | case $2 \to 2$ in CM frame

$$\boldsymbol{q} = \boldsymbol{p}_1 = -\boldsymbol{p}_2$$



$$\boldsymbol{p} = \boldsymbol{p}_3 = -\boldsymbol{p}_4$$

$$\Rightarrow \int \mathrm{d}\Phi_2 \;\equiv\; (2\pi)^4 \int \delta^4(p_1 + p_2 - p_3 - p_4) \frac{\mathrm{d}^3 p_3}{(2\pi)^3 2E_3} \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2E_4} = \int \frac{|\boldsymbol{p}|\mathrm{d}\Omega}{16\pi^2 E_{\text{CM}}}$$

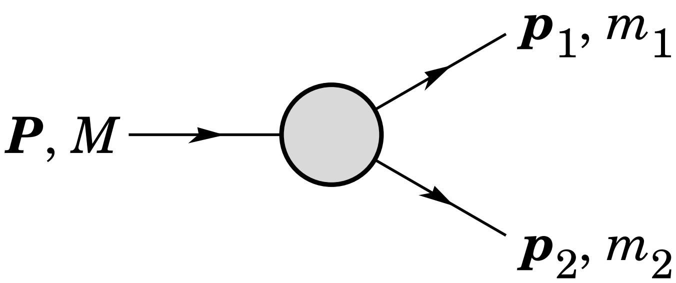
and if $m_1 = m_2 \quad \Rightarrow \quad 4\left\{(p_1 p_2)^2 - m_1^2 m_2^2\right\}^{1/2} = 4E_{\text{CM}}|\boldsymbol{q}|$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(1,2 \to 3,4) = \frac{1}{64\pi^2 E_{\text{CM}}^2} \frac{|\boldsymbol{p}|}{|\boldsymbol{q}|} |\mathcal{M}|^2}$$

Decay width

$$\mathrm{d}\Gamma(i \to f) = \frac{1}{2M} |\mathcal{M}|^2 (2\pi)^4 \delta^4(P - p_f) \prod_{j=1}^{n} \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j}$$

case $1 \to 2$



$$\boxed{\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega}(i \to 1, 2) = \frac{1}{32\pi^2} \frac{|\boldsymbol{p}|}{M^2} |\mathcal{M}|^2}$$

▷ Note that masses M, m_1 and m_2 fix final energies and momenta:

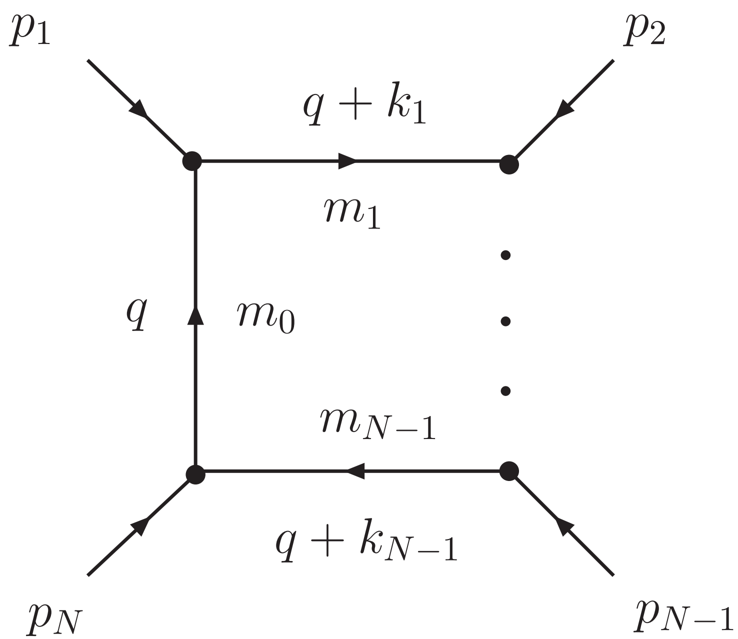
$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} \qquad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

$$|\boldsymbol{p}| = |\boldsymbol{p}_1| = |\boldsymbol{p}_2| = \frac{\left\{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]\right\}^{1/2}}{2M}$$

Loop calculations

Structure of one-loop amplitudes

- Consider the following generic one-loop diagram with N external legs:



$$k_1 = p_1, \quad k_2 = p_1 + p_2, \quad \ldots \quad k_{N-1} = \sum_{i=1}^{N-1} p_i$$

- It contains general integrals of the kind:

$$\frac{\mathrm{i}}{16\pi^2} T^N_{\mu_1\ldots\mu_P} \equiv \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q_{\mu_1} \cdots q_{\mu_P}}{[q^2 - m_0^2][(q+k_1)^2 - m_1^2] \cdots [(q+k_{N-1})^2 - m_{N-1}^2]}$$

Structure of one-loop amplitudes

- D dimensional integration in dimensional regularization
- Integrals are symmetric under permutations of Lorentz indices
- Scale μ introduced to keep the proper mass dimensions
- P is the number of q's in the numerator and determines the tensor structure of the integral (scalar if $P = 0$, vector if $P = 1$, etc.). Note that $P \leq N$
- Notation: A for T^1, B for T^2, etc. For example, the scalar integrals A_0, B_0, etc.
- The tensor integrals can be decomposed as a linear combination of the Lorentz covariant tensors that can be built with $g_{\mu\nu}$ and a set of linearly independent momenta [Passarino, Veltman '79]
- The choice of basis is not unique

  Here we use the basis formed by $g_{\mu\nu}$ and the momenta k_i, where the the tensor coefficients are totally symmetric in their indices [Denner '93]

  This is the basis used by the computer package LoopTools [www.feynarts.de/looptools]

Structure of one-loop amplitudes

- We focus here on:

$$
\begin{aligned}
B_\mu &= k_{1\mu} B_1 \\
B_{\mu\nu} &= g_{\mu\nu} B_{00} + k_{1\mu} k_{1\nu} B_{11} \\
C_\mu &= k_{1\mu} C_1 + k_{2\mu} C_2 \\
C_{\mu\nu} &= g_{\mu\nu} C_{00} + \sum_{i,j=1}^{2} k_{i\mu} k_{j\nu} C_{ij} \\
C_{\mu\nu\rho} &= \ldots
\end{aligned}
$$

- We will see that the scalar integrals A_0 and B_0 and the tensor integral coefficients B_1, B_{00}, B_{11} and C_{00} are divergent in $D = 4$ dimensions (ultraviolet divergence, equivalent to take cutoff $\Lambda \to \infty$ in q)

- It is possible to express every tensor coefficient in terms of scalar integrals (scalar reduction) [Denner '93]

Explicit calculation

- Basic ingredients:

– Euler Gamma function:

$$\Gamma(x+1) = x\Gamma(x)$$

Taylor expansion around poles at $x = 0, -1, -2, \ldots$:

$$\begin{aligned} x=0: \quad \Gamma(x) &= \frac{1}{x} - \gamma + \mathcal{O}(x) \\ x=-1: \quad \Gamma(x) &= -\frac{1}{(x+1)} + \gamma - 1 + \cdots + \mathcal{O}(x+1) \end{aligned}$$

where $\gamma \approx 0.5772\ldots$ is Euler-Mascheroni constant

– Feynman parameters:

$$\frac{1}{a_1 a_2 \cdots a_n} = \int_0^1 \mathrm{d}x_1 \cdots \mathrm{d}x_n \, \delta\left(\sum_{i=1}^{n} x_i - 1\right) \frac{(n-1)!}{[x_1 a_1 + x_2 a_2 + \cdots x_n a_n]^n}$$

Explicit calculation

– The following integrals (with $\varepsilon \to 0^+$) will be needed:

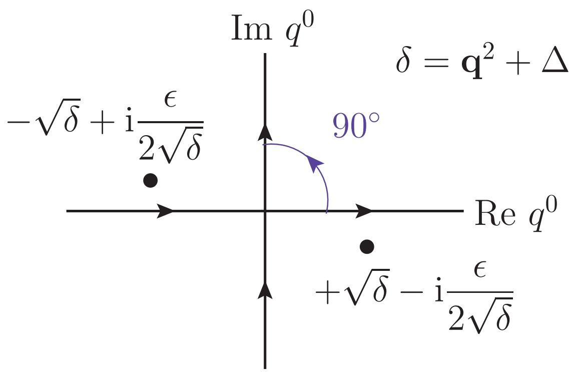
$$\int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q^2 - \Delta + \mathrm{i}\varepsilon)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{D/2}} \frac{\Gamma(n - D/2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-D/2}$$

$$\Rightarrow \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q^2}{(q^2 - \Delta + \mathrm{i}\varepsilon)^n} = \frac{(-1)^{n-1} \mathrm{i}}{(4\pi)^{D/2}} \frac{D}{2} \frac{\Gamma(n - D/2 - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-D/2-1}$$

▷ Let's solve the first integral in Euclidean space: $q^0 = \mathrm{i}q_E^0$, $\boldsymbol{q} = \boldsymbol{q}_E$, $q^2 = -q_E^2$,

$$\int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q^2 - \Delta + \mathrm{i}\varepsilon)^n} = \mathrm{i}(-1)^n \int \frac{\mathrm{d}^D q_E}{(2\pi)^D} \frac{1}{(q_E^2 + \Delta)^n}$$

(equivalent to a Wick rotation of 90°). The second integral follows from this one



Explicit calculation

In D-dimensional spherical coordinates:

$$\int \frac{\mathrm{d}^D q_E}{(2\pi)^D} \frac{1}{(q_E^2+\Delta)^n} = \int \mathrm{d}\Omega_D \int_0^\infty \mathrm{d}q_E q_E^{D-1} \frac{1}{(q_E^2+\Delta)^n} \equiv \mathcal{I}_A \times \mathcal{I}_B$$

where
$$\mathcal{I}_A = \int \mathrm{d}\Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$$

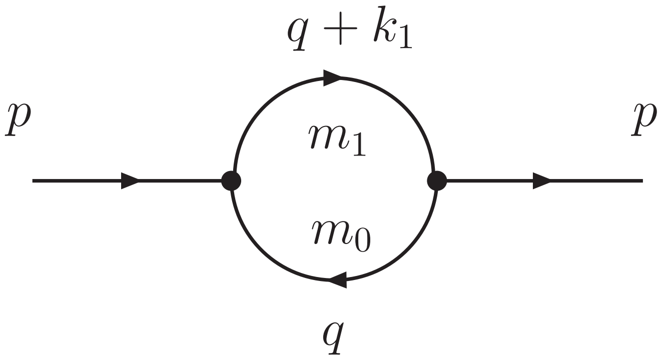
since
$$\begin{aligned}(\sqrt{\pi})^D &= \left(\int_{-\infty}^{\infty} \mathrm{d}x\ \mathrm{e}^{-x^2}\right)^D = \int \mathrm{d}^D x\ \mathrm{e}^{-\sum_{i=1}^D x_i^2} = \int \mathrm{d}\Omega_D \int_0^\infty \mathrm{d}x\ x^{D-1}\mathrm{e}^{-x^2} \\ &= \left(\int \mathrm{d}\Omega_D\right) \frac{1}{2}\int_0^\infty \mathrm{d}t\ t^{D/2-1}\mathrm{e}^{-t} = \left(\int \mathrm{d}\Omega_D\right)\frac{1}{2}\Gamma(D/2)\end{aligned}$$

and, changing variables: $t = q_E^2$, $z = \Delta/(t+\Delta)$, we have

$$\mathcal{I}_B = \frac{1}{2}\left(\frac{1}{\Delta}\right)^{n-D/2} \int_0^1 \partial z\ z^{n-D/2-1}(1-z)^{D/2-1} = \frac{1}{2}\left(\frac{1}{\Delta}\right)^{n-D/2} \frac{\Gamma(n-D/2)\Gamma(D/2)}{\Gamma(n)}$$

where Euler Beta function was used: $B(\alpha,\beta) = \displaystyle\int_0^1 \mathrm{d}z\ z^{\alpha-1}(1-z)^{\beta-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

Explicit calculation | Two-point functions



$$\frac{\mathrm{i}}{16\pi^2}\{B_0,\ B^\mu,\ B^{\mu\nu}\}\,(\text{args}) = \mu^{4-D}\int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{\{1,\ q^\mu,\ q^\mu q^\nu\}}{(q^2-m_0^2)\left[(q+p)^2-m_1^2\right]}$$

▷ $k_1 = p$

▷ The integrals depend on the masses m_0, m_1 and the invariant p^2:

$$(\text{args}) = (p^2; m_0^2, m_1^2)$$

Explicit calculation | Two-point functions

- Using Feynman parameters,

$$\frac{1}{a_1 a_2} = \int_0^1 \mathrm{d}x \frac{1}{[a_1 x + a_2(1-x)]^2}$$

$$\Rightarrow \frac{\mathrm{i}}{16\pi^2}\{B_0,\ B^\mu,\ B^{\mu\nu}\} = \mu^{4-D}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1,\ -A^\mu,\ q^\mu q^\nu + A^\mu A^\nu\}}{(q^2-\Delta_2)^2}$$

with

$$\Delta_2 = x^2 p^2 + x(m_1^2 - m_0^2 - p^2) + m_0^2$$

$$\begin{aligned} a_1 &= (q+p)^2 - m_1^2 \\ a_2 &= q^2 - m_0^2 \end{aligned}$$

and a loop momentum shift to obtain a perfect square in the denominator:

$$q^\mu \to q^\mu - A^\mu, \quad A^\mu = x p^\mu$$

Explicit calculation — Two-point functions

- Then, the scalar function is:

$$\frac{\mathrm{i}}{16\pi^2}B_0 = \mu^{4-D}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{1}{(q^2-\Delta_2)^2}$$

$$\Rightarrow \quad B_0 = \Delta_\epsilon - \int_0^1 \mathrm{d}x \; \ln\frac{\Delta_2}{\mu^2} + \mathcal{O}(\epsilon) \qquad [D=4-\epsilon]$$

where $\Delta_\epsilon \equiv \frac{2}{\epsilon} - \gamma + \ln 4\pi$ and the Euler Gamma function was expanded around $x=0$ for $D=4-\epsilon$, using $x^\epsilon = \exp\{\epsilon \ln x\} = 1 + \epsilon \ln x + \mathcal{O}(\epsilon^2)$:

$$\mu^{4-D}\frac{\mathrm{i}\Gamma(2-D/2)}{(4\pi)^{D/2}}\left(\frac{1}{\Delta_2}\right)^{2-D/2} = \frac{\mathrm{i}}{16\pi^2}\left(\Delta_\epsilon - \ln\frac{\Delta_2}{\mu^2}\right) + \mathcal{O}(\epsilon)$$

- Comparing with the definitions of the tensor coefficientes we have:

$$\frac{\mathrm{i}}{16\pi^2}B^\mu = -\mu^{4-D}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{A^\mu}{(q^2-\Delta_2)^2}$$

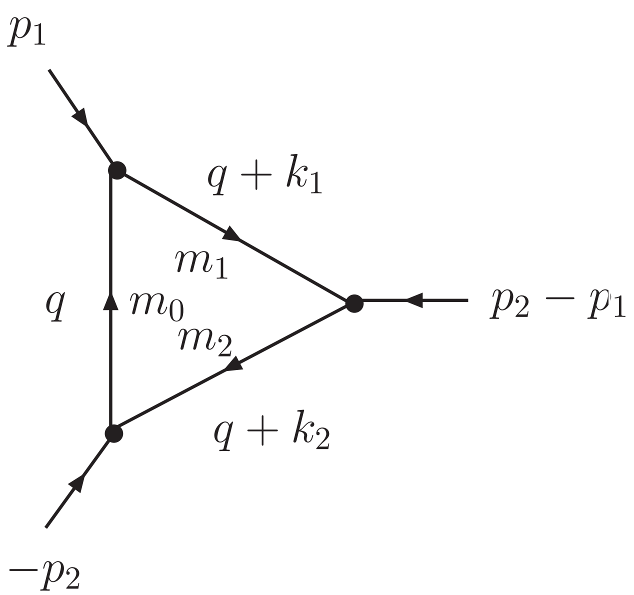
$$\Rightarrow \quad B_1 = -\frac{1}{2}\Delta_\epsilon + \int_0^1 \mathrm{d}x \; x\ln\frac{\Delta_2}{\mu^2} + \mathcal{O}(\epsilon) \qquad [D=4-\epsilon]$$

and

$$\begin{aligned}
\frac{\mathrm{i}}{16\pi^2} B^{\mu\nu} &= \mu^{4-D} \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{(q^2/D) g^{\mu\nu} + A^\mu A^\nu}{(q^2 - \Delta_2)^2} \\
\Rightarrow \quad B_{00} &= -\frac{1}{12}(p^2 - 3m_0^2 - 3m_1^2)\Delta_\epsilon + \mathcal{O}(\epsilon) \qquad [D = 4-\epsilon] \\
B_{11} &= \frac{1}{3}\Delta_\epsilon - \int_0^1 \mathrm{d}x\ x^2 \ln\frac{\Delta_2}{\mu^2} + \mathcal{O}(\epsilon) \qquad [D = 4-\epsilon]
\end{aligned}$$

where $q^\mu q^\nu$ have been replaced by $(q^2/D)g^{\mu\nu}$ in the integrand and the Euler Gamma function was expanded around $x = -1$ for $D = 4 - \epsilon$:

$$-\mu^{4-D} \frac{\mathrm{i}\Gamma(1 - D/2)}{(4\pi)^{D/2} 2\Gamma(2)} \left(\frac{1}{\Delta_2}\right)^{1-D/2} = \frac{\mathrm{i}}{16\pi^2} \frac{1}{2} \left(\Delta_\epsilon - \ln\frac{\Delta_2}{\mu^2} + 1\right) \Delta_2 + \mathcal{O}(\epsilon)$$



$$\frac{\mathrm{i}}{16\pi^2}\{C_0,\ C^\mu,\ C^{\mu\nu}\}\,(\text{args}) = \mu^{4-D}\int\frac{\mathrm{d}^D q}{(2\pi)^D}\frac{\{1,\ q^\mu,\ q^\mu q^\nu\}}{(q^2-m_0^2)\left[(q+p_1)^2-m_1^2\right]\left[(q+p_2)^2-m_2^2\right]}$$

▷ It is convenient to choose the external momenta so that:

$$k_1 = p_1, \quad k_2 = p_2.$$

▷ The integrals depend on the masses m_0, m_1, m_2 and the invariants:

$$(\text{args}) = (p_1^2, Q^2, p_2^2; m_0^2, m_1^2, m_2^2), \quad Q^2 \equiv (p_2-p_1)^2.$$

Explicit calculation | **Three-point functions**

- Using Feynman parameters,

$$\frac{1}{a_1 a_2 a_3} = 2\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \frac{1}{[a_1 x + a_2 y + a_3(1-x-y)]^3}$$

$$\Rightarrow \frac{\mathrm{i}}{16\pi^2}\{C_0,\ C^\mu,\ C^{\mu\nu}\} = 2\mu^{4-D}\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1,\ -A^\mu,\ q^\mu q^\nu + A^\mu A^\nu\}}{(q^2-\Delta_3)^3}$$

with

$$\Delta_3 = x^2 p_1^2 + y^2 p_2^2 + xy(p_1^2 + p_2^2 - Q^2) + x(m_1^2 - m_0^2 - p_1^2) + y(m_2^2 - m_0^2 - p_2^2) + m_0^2$$

$$\begin{aligned} a_1 &= (q+p_1)^2 - m_1^2 \\ a_2 &= (q+p_2)^2 - m_2^2 \\ a_3 &= q^2 - m_0^2 \end{aligned}$$

and a loop momentum shift to obtain a perfect square in the denominator:

$$q^\mu \to q^\mu - A^\mu, \quad A^\mu = x p_1^\mu + y p_2^\mu$$

- Then the scalar function is:

$$
\begin{aligned}
\frac{\mathrm{i}}{16\pi^2} C_0 &= 2\mu^{4-D} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q^2-\Delta_3)^3} \\
\Rightarrow \quad C_0 &= -\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \, \frac{1}{\Delta_3} \qquad [D=4]
\end{aligned}
$$

- Comparing with the definitions of the tensor coefficientes we have:

$$
\begin{aligned}
\frac{\mathrm{i}}{16\pi^2} C^\mu &= -2\mu^{4-D} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{A^\mu}{(q^2-\Delta_3)^3} \\
\Rightarrow \quad C_1 &= \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \, \frac{x}{\Delta_3} \qquad [D=4] \\
C_2 &= \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \, \frac{y}{\Delta_3} \qquad [D=4]
\end{aligned}
$$

Explicit calculation **Three-point functions**

$$\frac{\mathrm{i}}{16\pi^2}C^{\mu\nu} = 2\mu^{4-D}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y \int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{(q^2/D)g^{\mu\nu}+A^\mu A^\nu}{(q^2-\Delta_3)^3}$$

$$\Rightarrow \quad C_{11} = -\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{x^2}{\Delta_3} \qquad [D=4]$$

$$C_{22} = -\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{y^2}{\Delta_3} \qquad [D=4]$$

$$C_{12} = -\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \frac{xy}{\Delta_3} \qquad [D=4]$$

$$C_{00} = \frac{1}{4}\Delta_\epsilon - \frac{1}{2}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y\, \ln\frac{\Delta_3}{\mu^2} + \mathcal{O}(\epsilon) \qquad [D=4-\epsilon]$$

where $\Delta_\epsilon \equiv \dfrac{2}{\epsilon} - \gamma + \ln 4\pi$ and $q^\mu q^\nu$ was replaced by $(q^2/D)g^{\mu\nu}$ in the integrand

In C_{00} the Euler Gamma function was expanded around $x=0$ for $D=4-\epsilon$:

$$\mu^{4-D}\frac{\mathrm{i}\Gamma(2-D/2)}{(4\pi)^{D/2}\Gamma(3)}\left(\frac{1}{\Delta_3}\right)^{2-D/2} = \frac{\mathrm{i}}{16\pi^2}\frac{1}{2}\left(\Delta_\epsilon - \ln\frac{\Delta_3}{\mu^2}\right) + \mathcal{O}(\epsilon)$$

Note about Diracology in D dimensions

- Attention should be paid to the traces of Dirac matrices when working in D dimensions (dimensional regularization) since

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\mathbf{1}_{4\times 4}, \quad g^{\mu\nu}g_{\mu\nu} = \mathrm{Tr}\{g^{\mu\nu}\} = D$$

Thus, the following identities involving contractions of Lorentz indices can be proven:

$$\begin{aligned}
\gamma^\mu\gamma_\mu &= D \\
\gamma^\mu\gamma^\nu\gamma_\mu &= -(D-2)\gamma^\nu \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho} - (4-D)\gamma^\nu\gamma^\rho \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu + (4-D)\gamma^\nu\gamma^\rho\gamma^\sigma
\end{aligned}$$