

# Edge state **dynamics** of a bosonic fractional Chern insulator

Adolfo G. Grushin, Néel Institute, CNRS

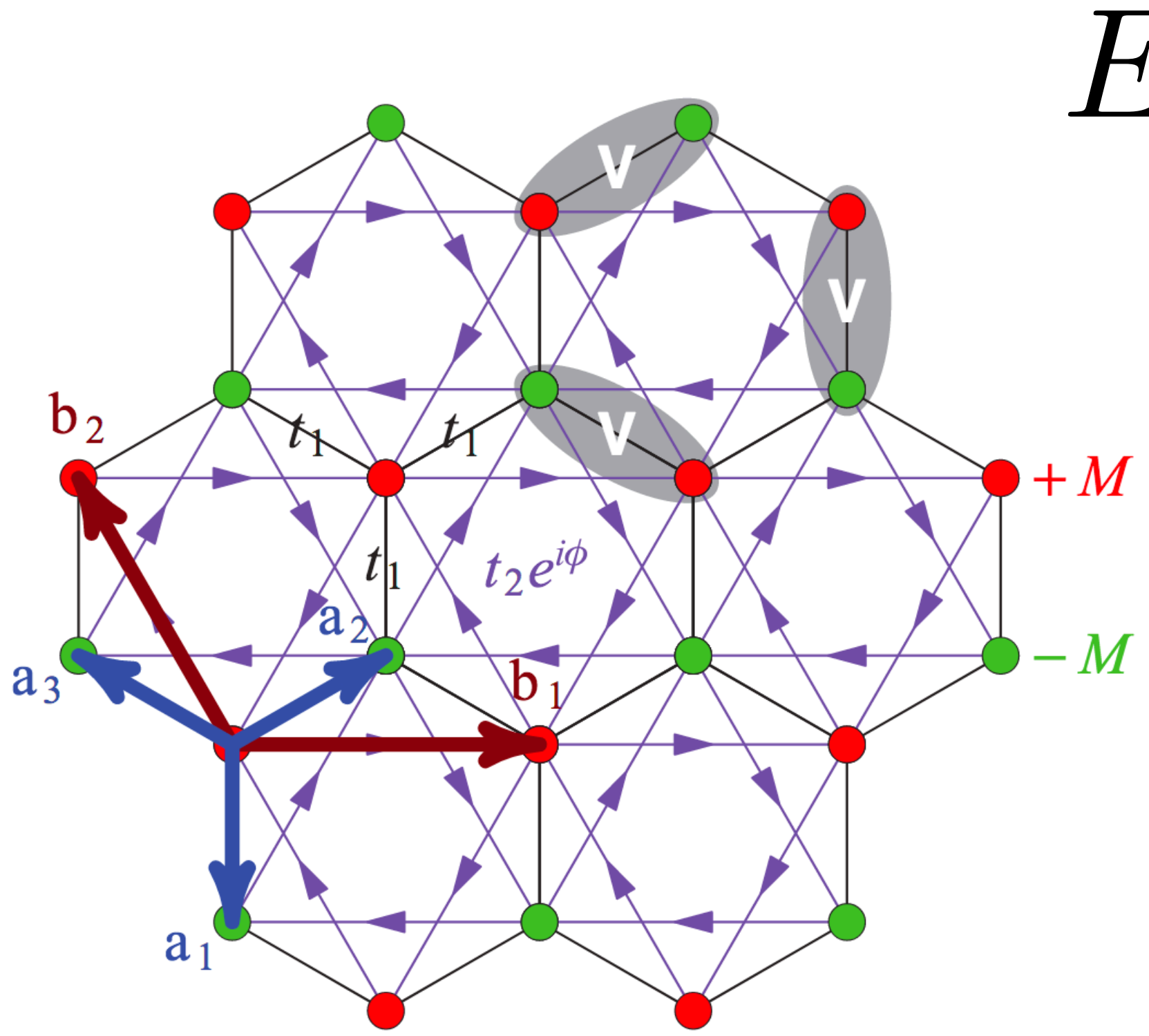
Benasque, 26/2/19



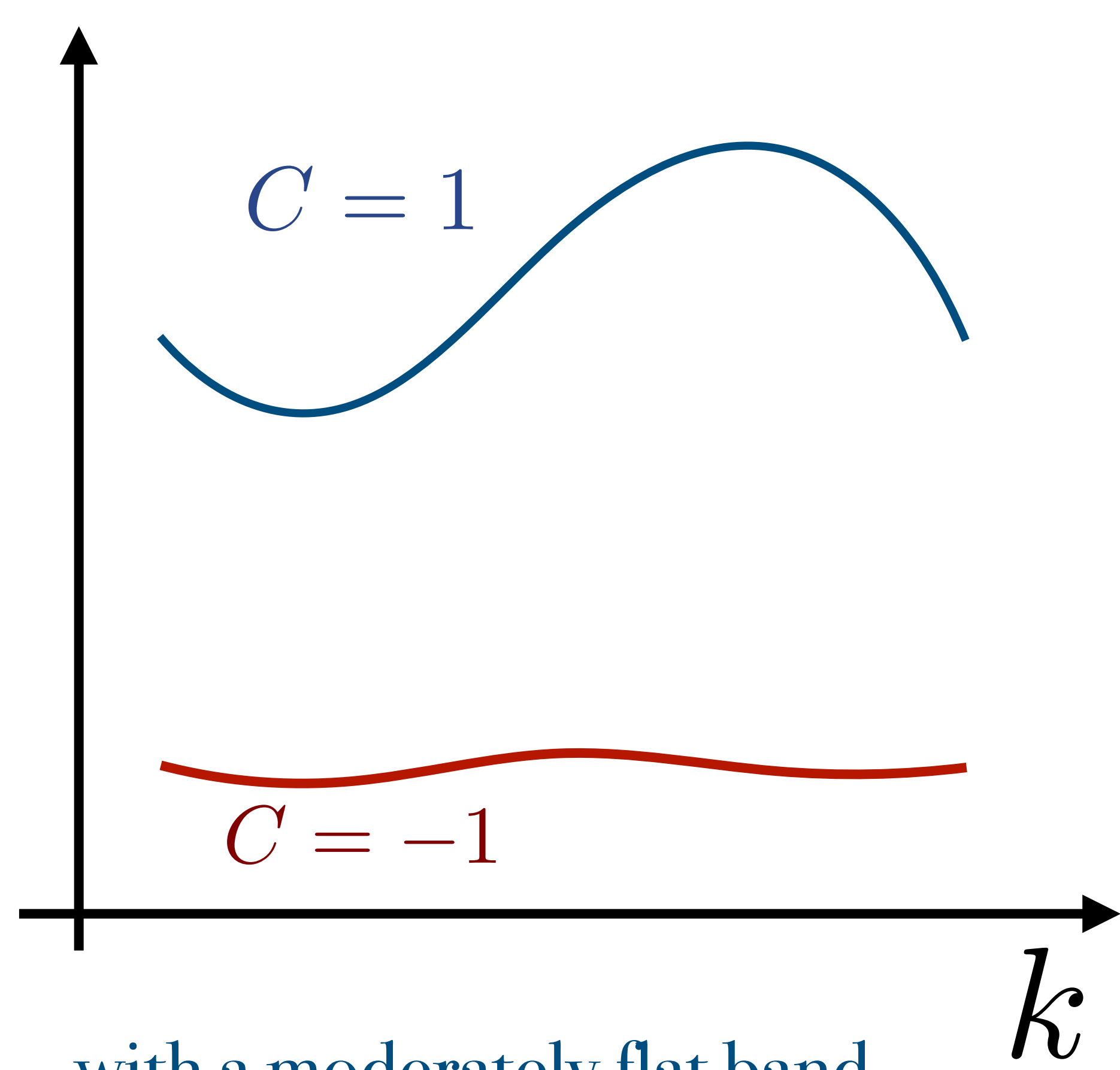
X. Y. Dong, AGG, J. Motruk, F. Pollmann, Phys Rev. Lett. (2018)



# The promise of a fractional quantum Hall effect without Landau levels



A Chern Insulator



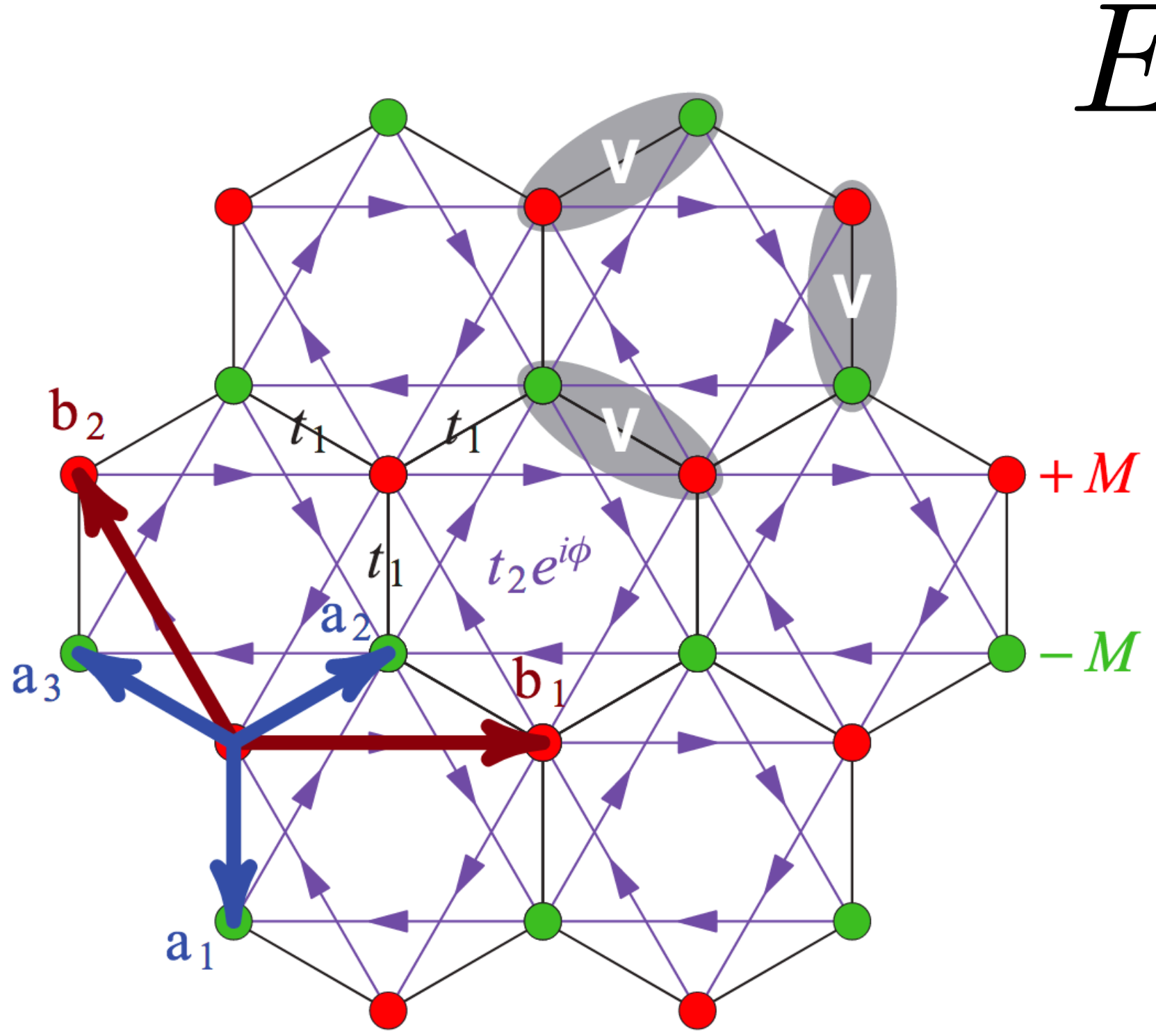
with a moderately flat band

Neupert et al. Phys. Scr. T164, 014005, (2015)

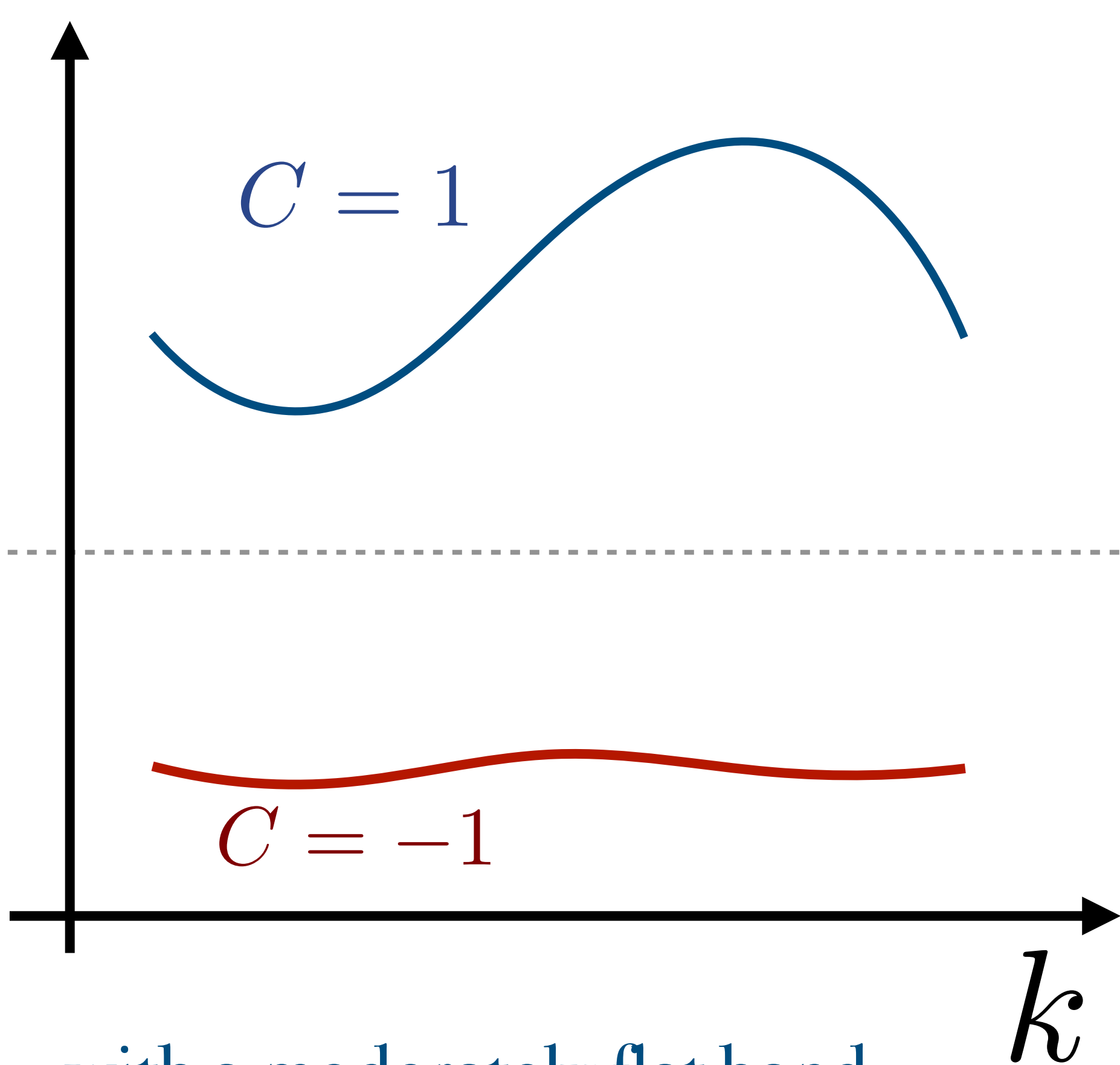
Y. L. Wu et al. Phys. Rev. B 85, 075116 (2012)

Emil J. Bergholtz, Zhao Liu Int. J. Mod. Phys. B 27, 1330017 (2013)

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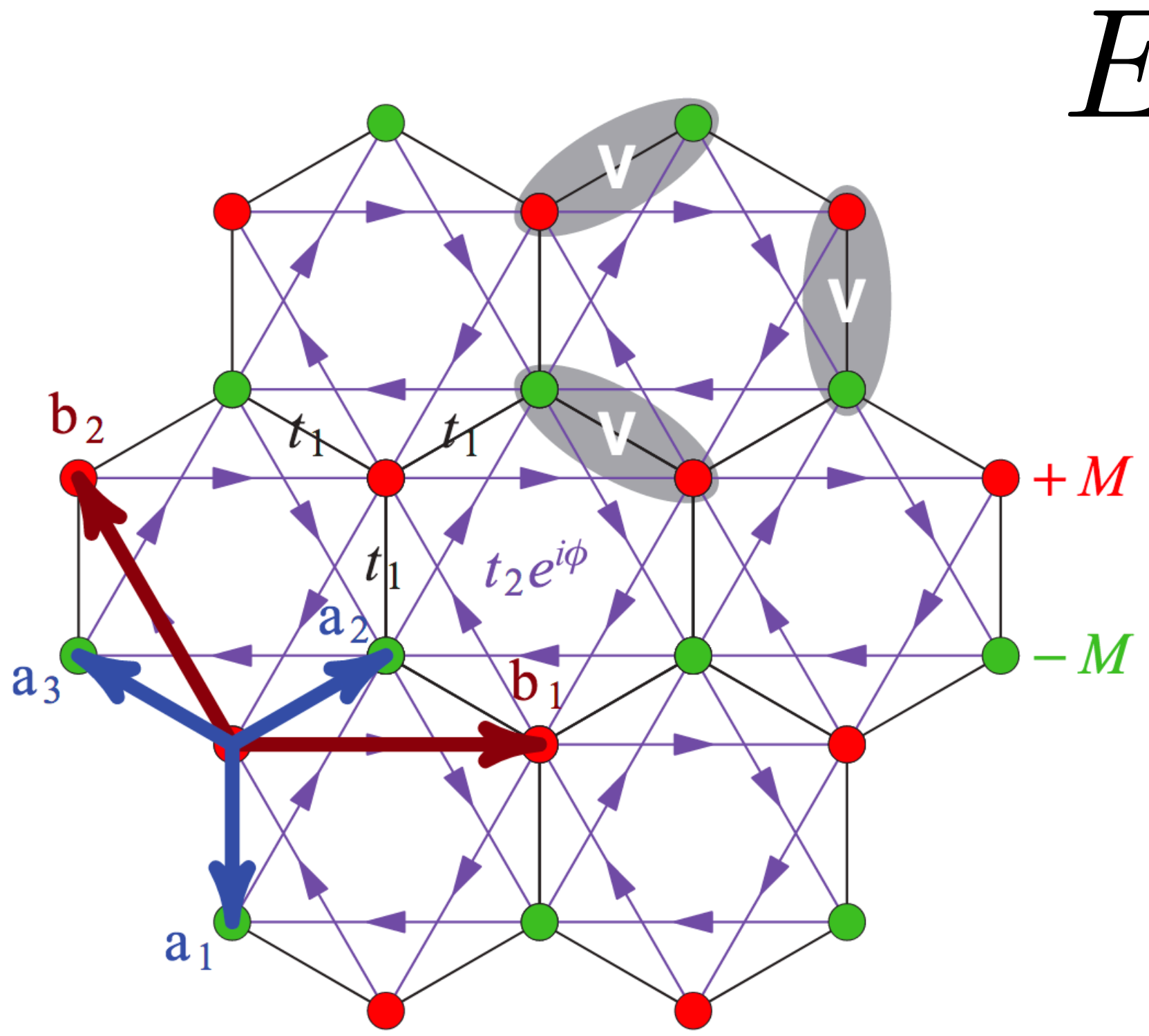
$$\sigma_{xy} = C \frac{e^2}{h}$$

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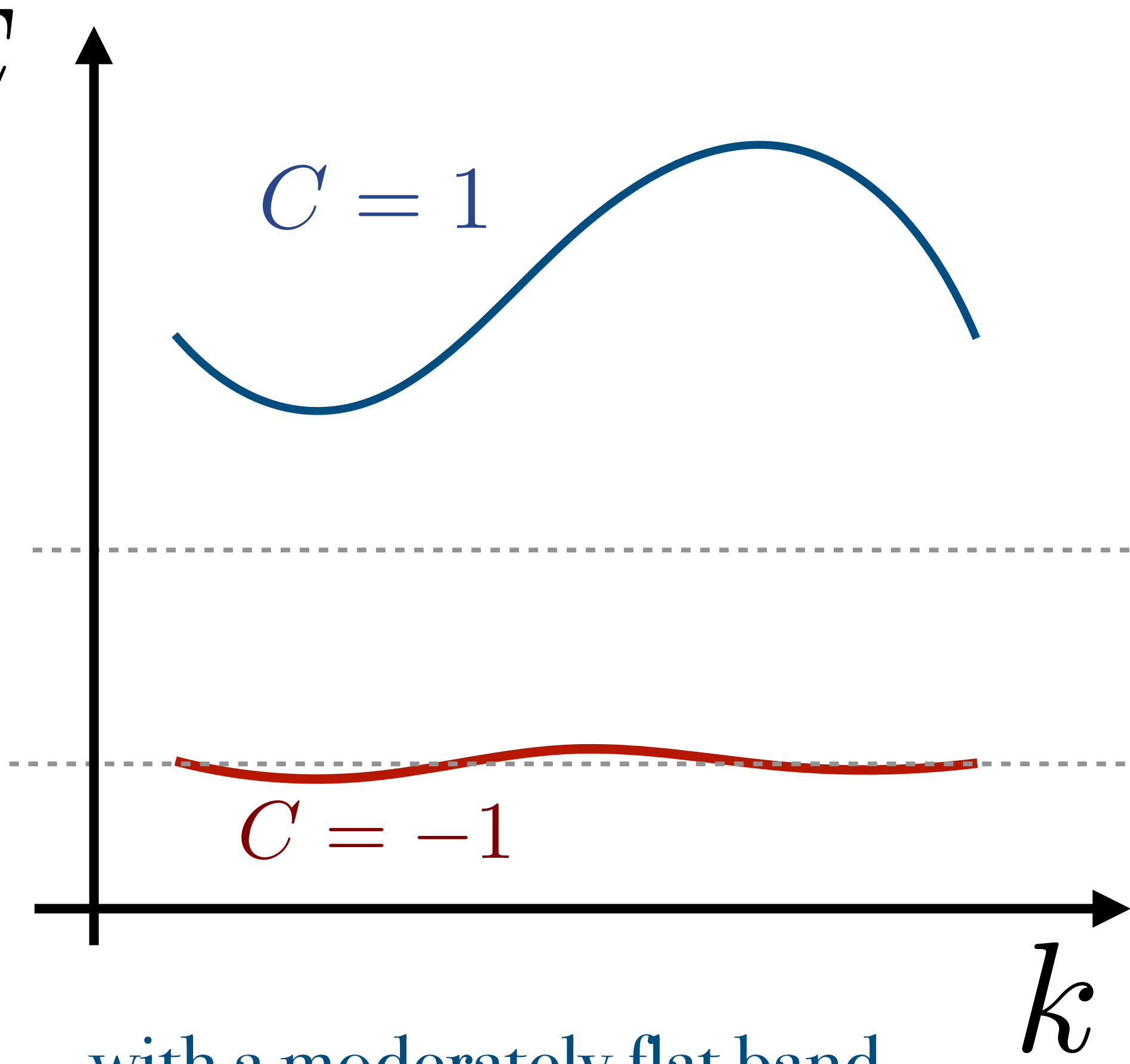
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# The promise of a fractional quantum Hall effect without Landau levels



A Chern Insulator



with a moderately flat band

$$\sigma_{xy} = C \frac{e^2}{h}$$

$$\sigma_{xy} = \frac{1}{m} \frac{e^2}{h}$$

at 1/m filling

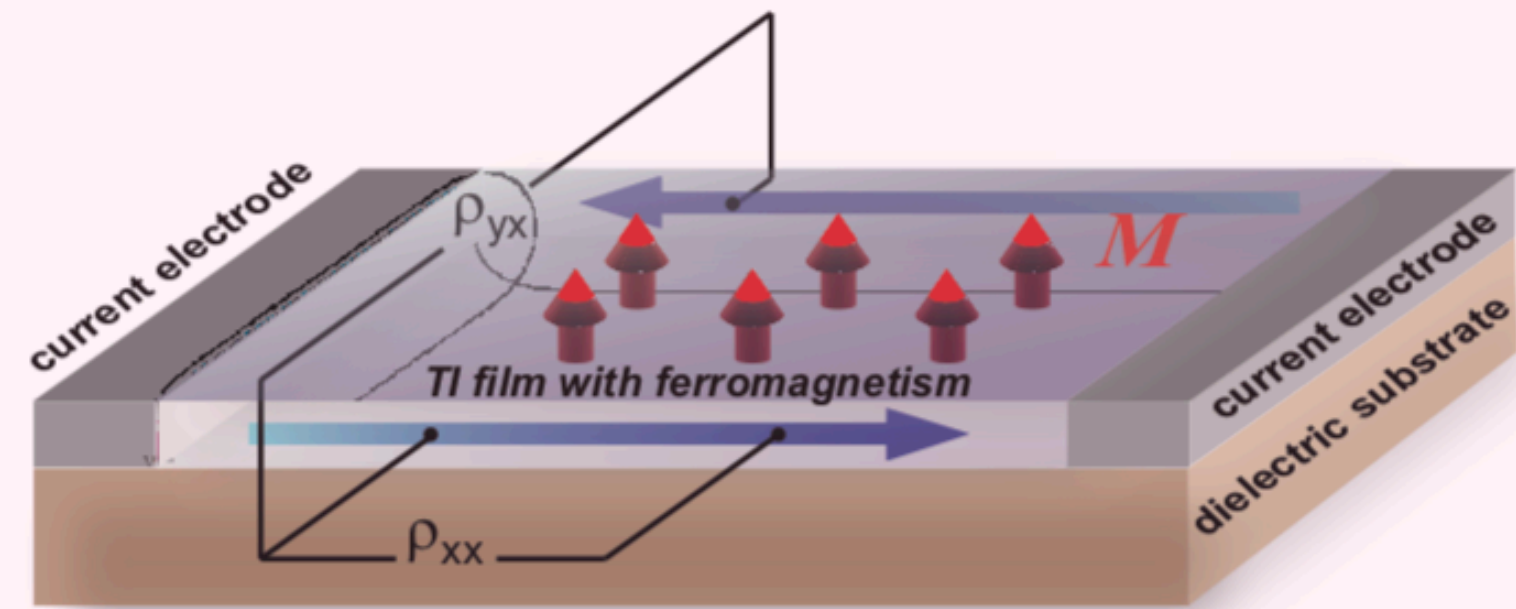
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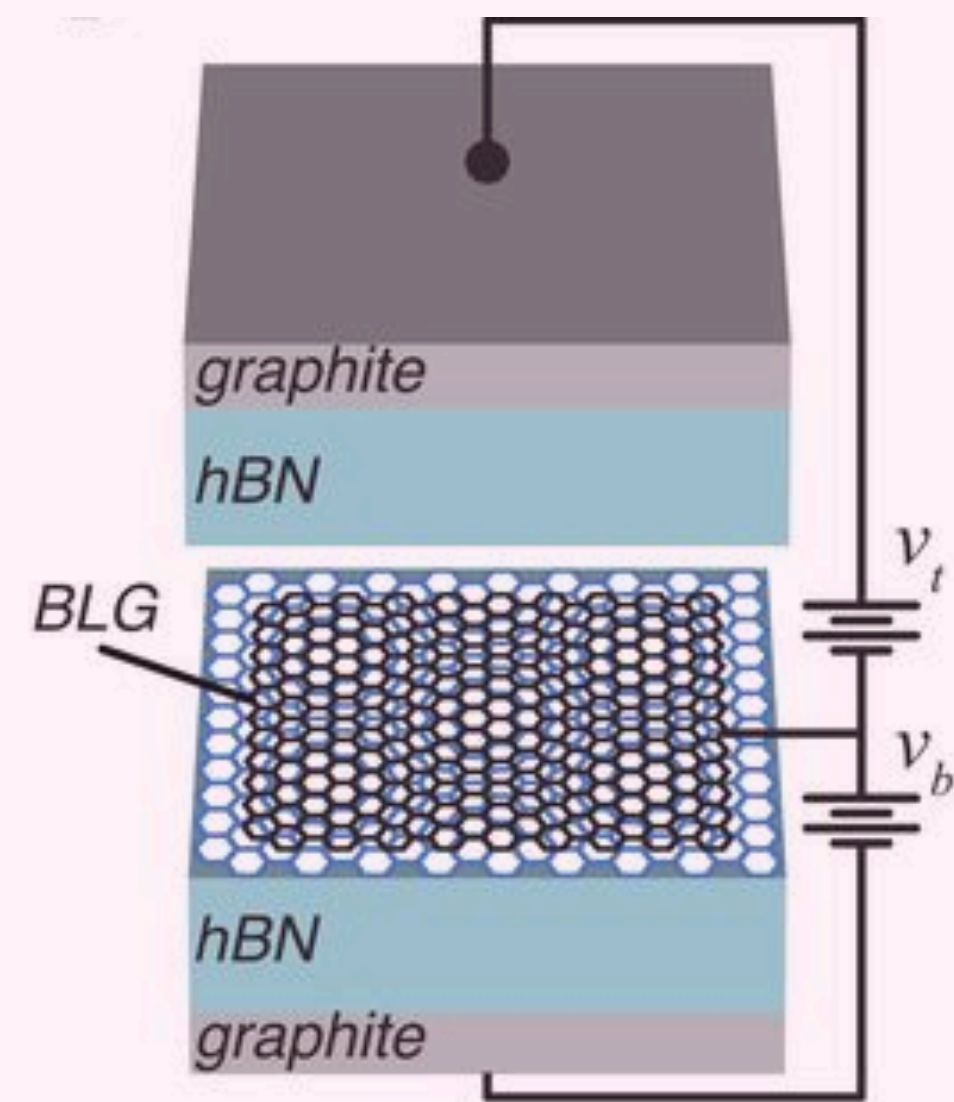


# Chern insulators in the solid state



## Doped topological insulators

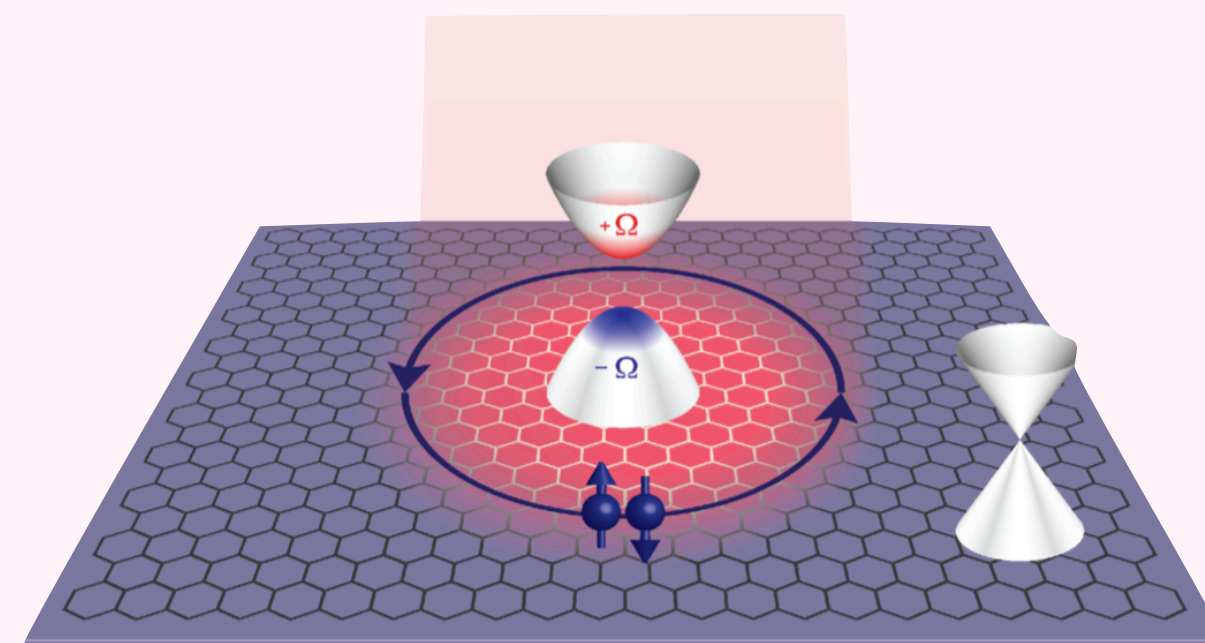
C. Z. Chang Science (2013)



## Moire Graphene

E. M. Spanton et. al Science (2018)

z

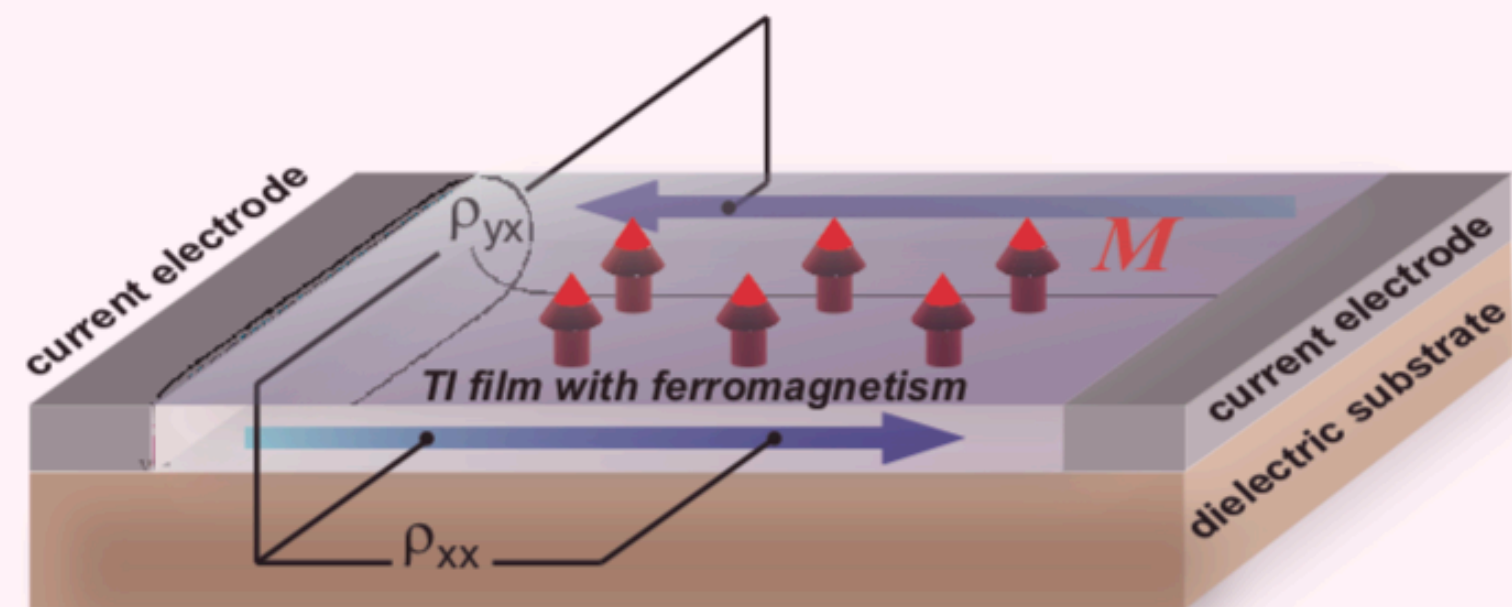


## Irradiated Graphene

J.W. McIver et al. 1811.03522

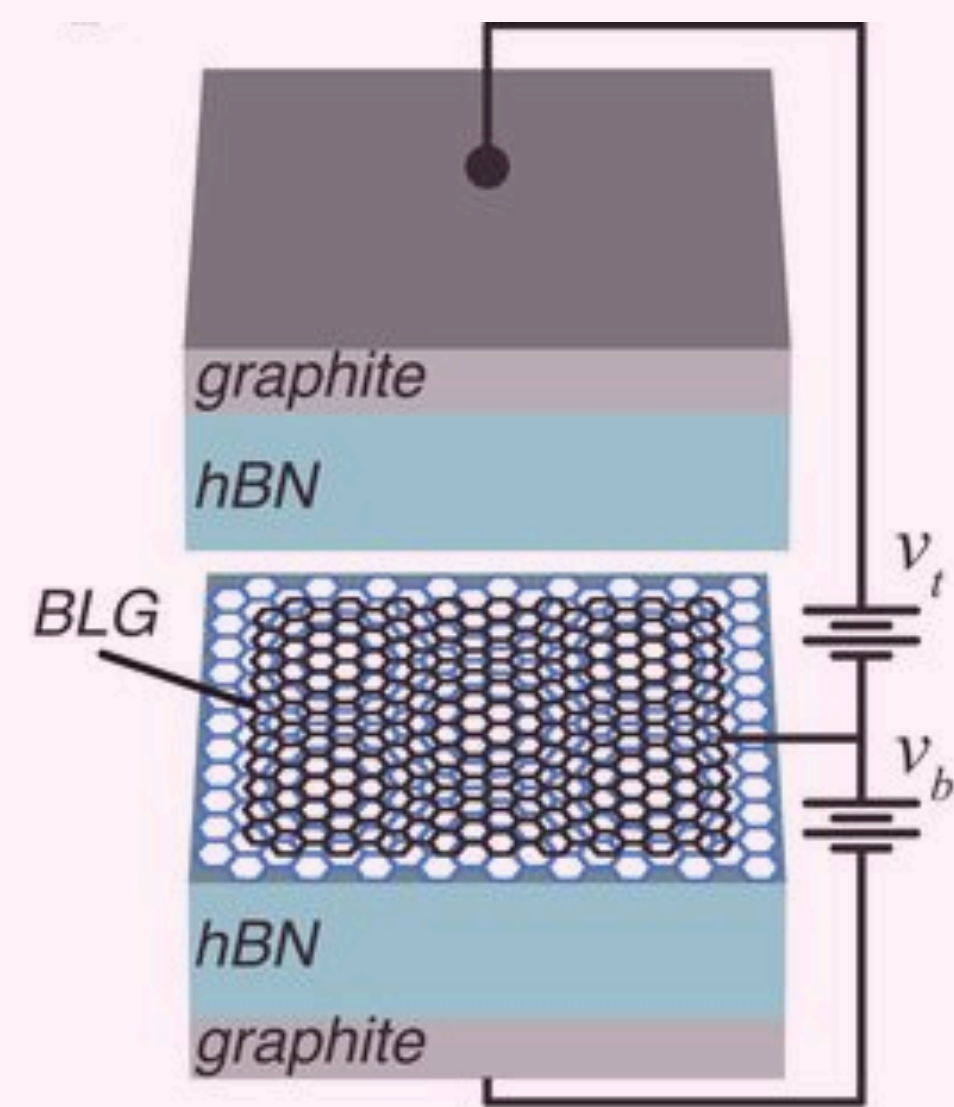


# Chern insulators in the solid state



## Doped topological insulators

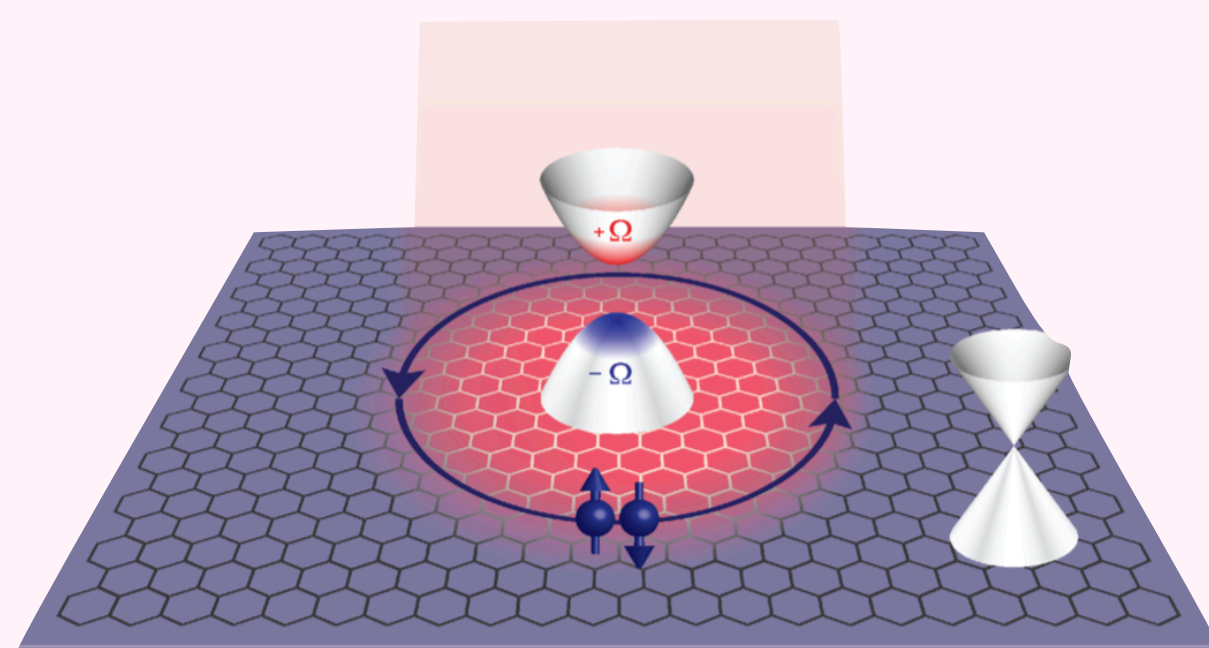
C. Z. Chang Science (2013)



## Moire Graphene

E. M. Spanton et. al Science (2018)

## Floquet

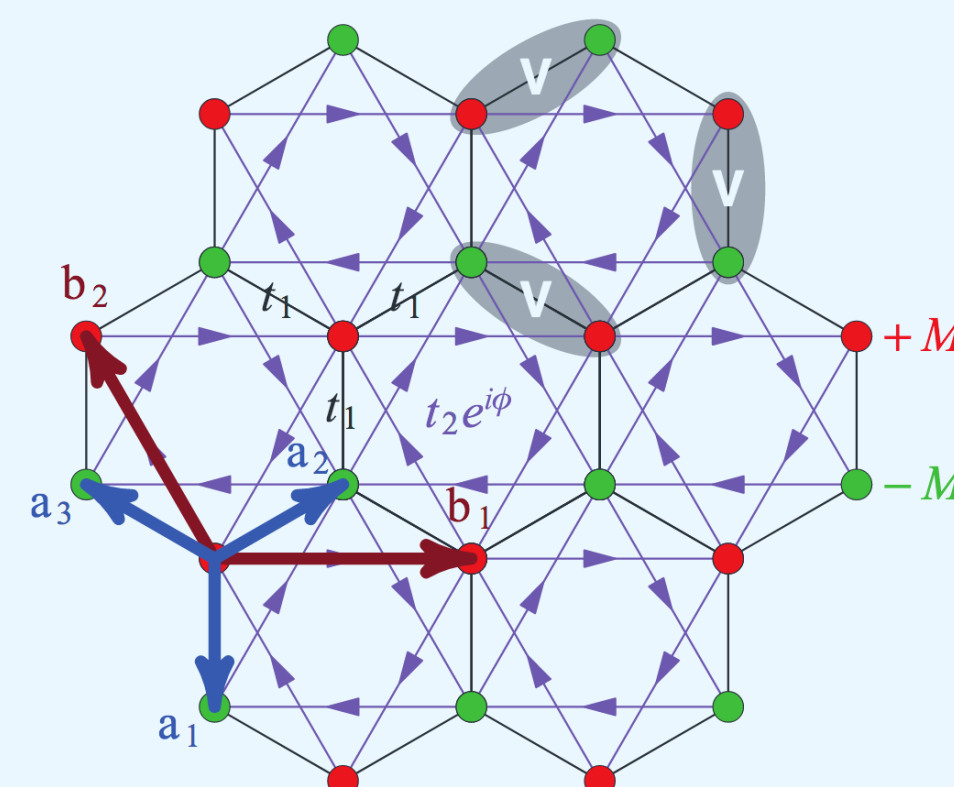


## Irradiated Graphene

J.W. McIver et al. 1811.03522

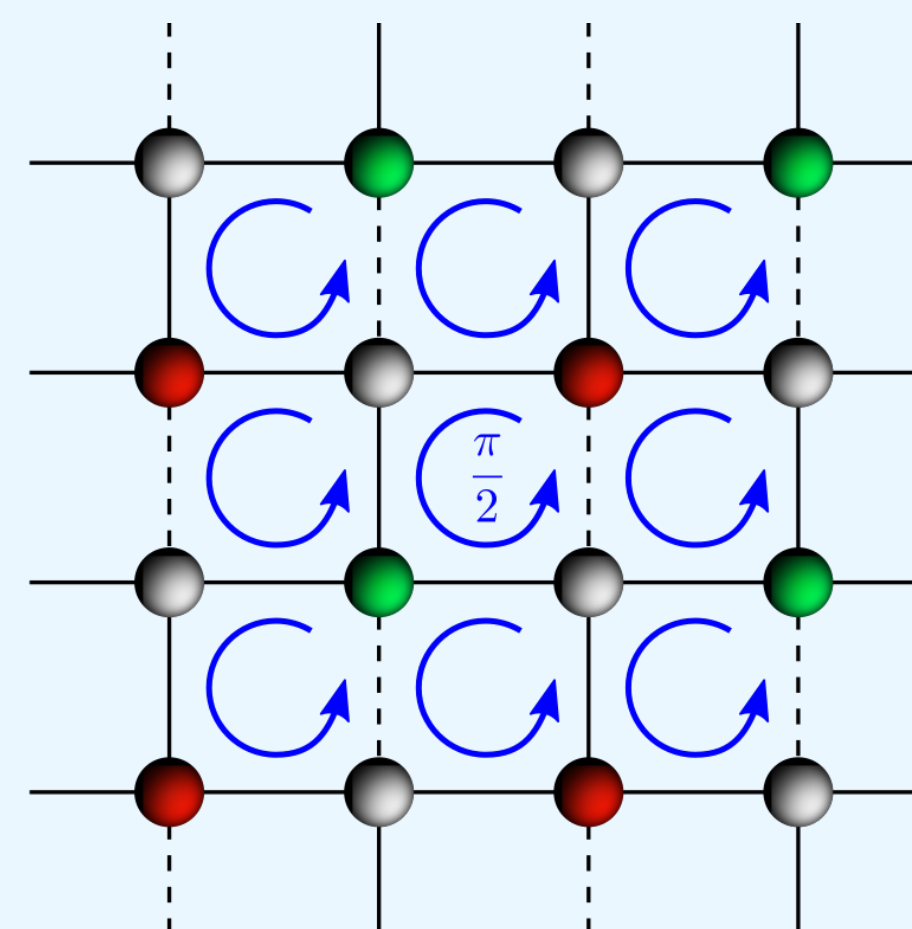
# Chern insulators in synthetic matter

## Ultra-cold fermions



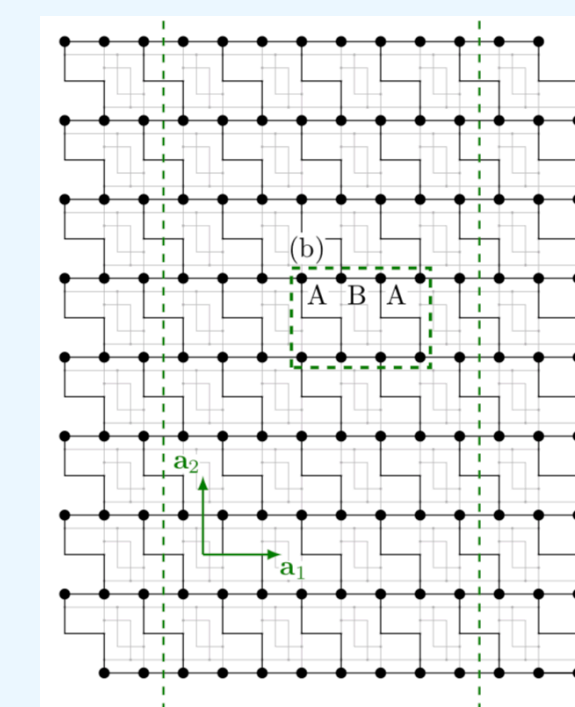
G. Jotzu et. al Nature 2014

## Ultra-cold bosons



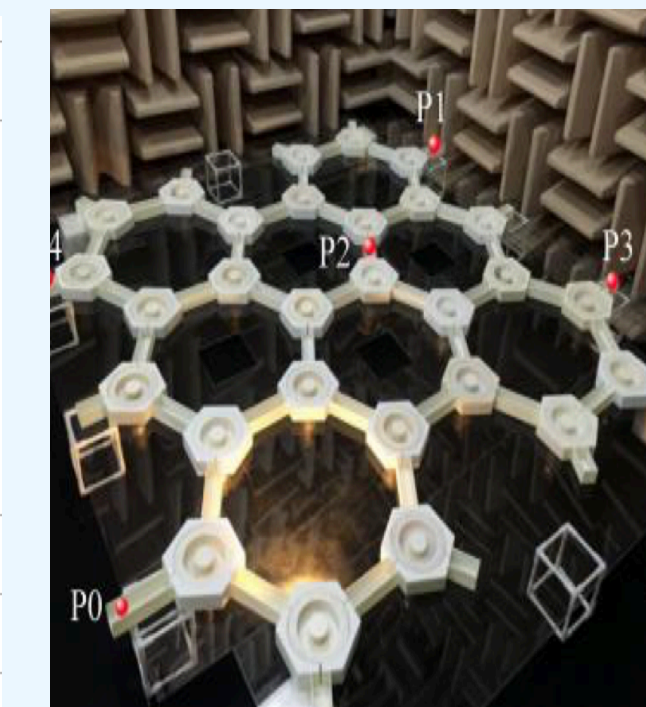
Aidelsburger et. al Nat. Phys (2013)

## Circuit



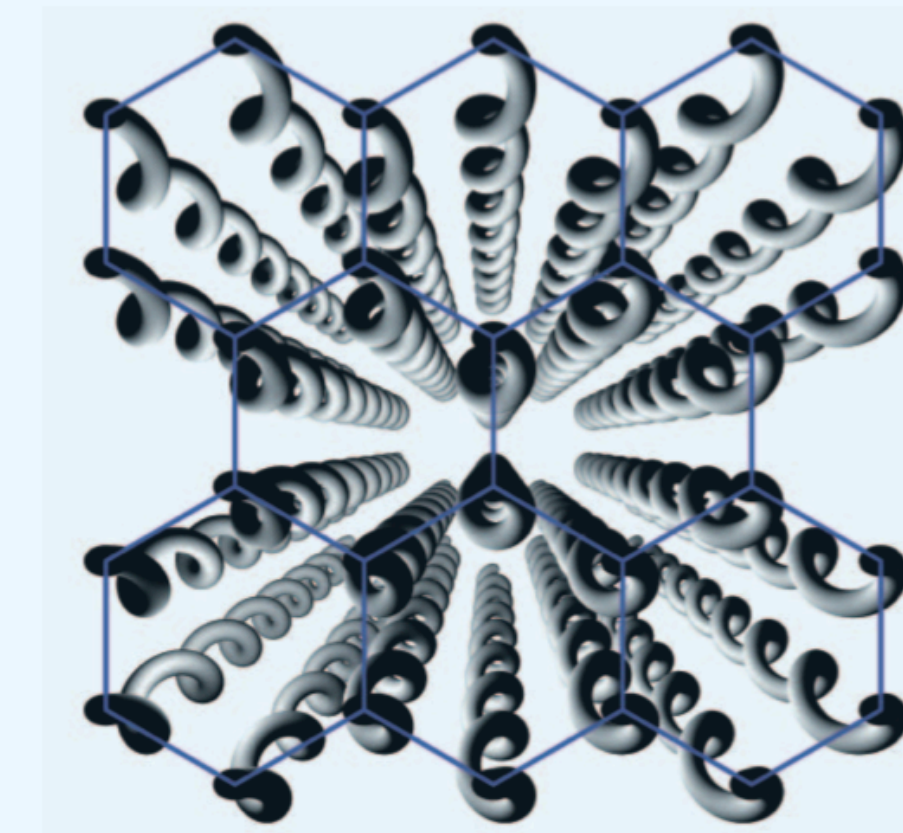
T. Hofmann et al.  
1801.07942

## Acoustic



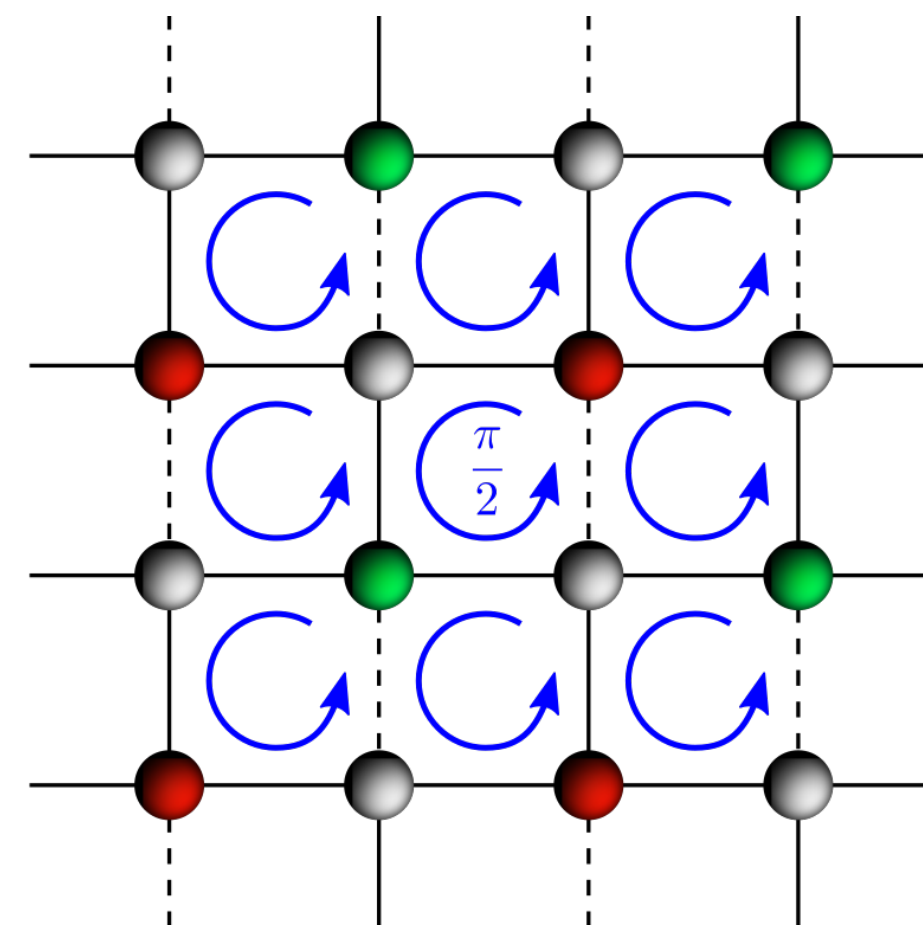
Y. Zhu et al.  
1801.07942

## Photonic



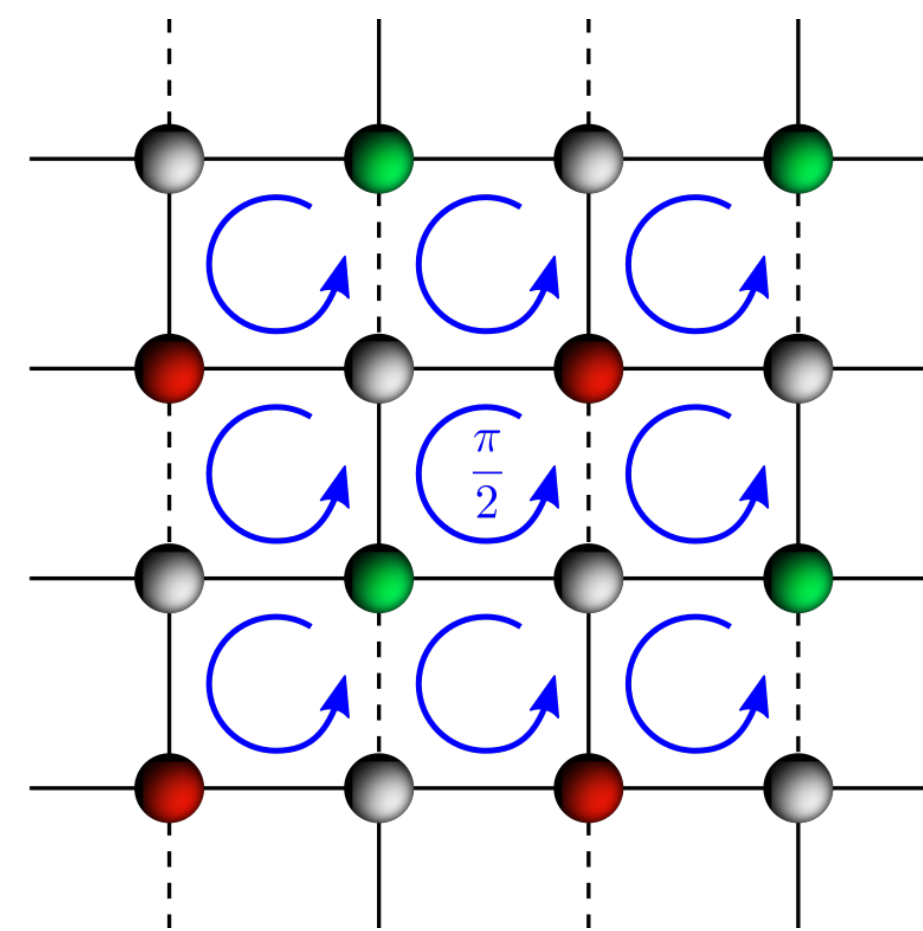
M. Rechtsman, Nature (2013)

## Ultra-cold bosons





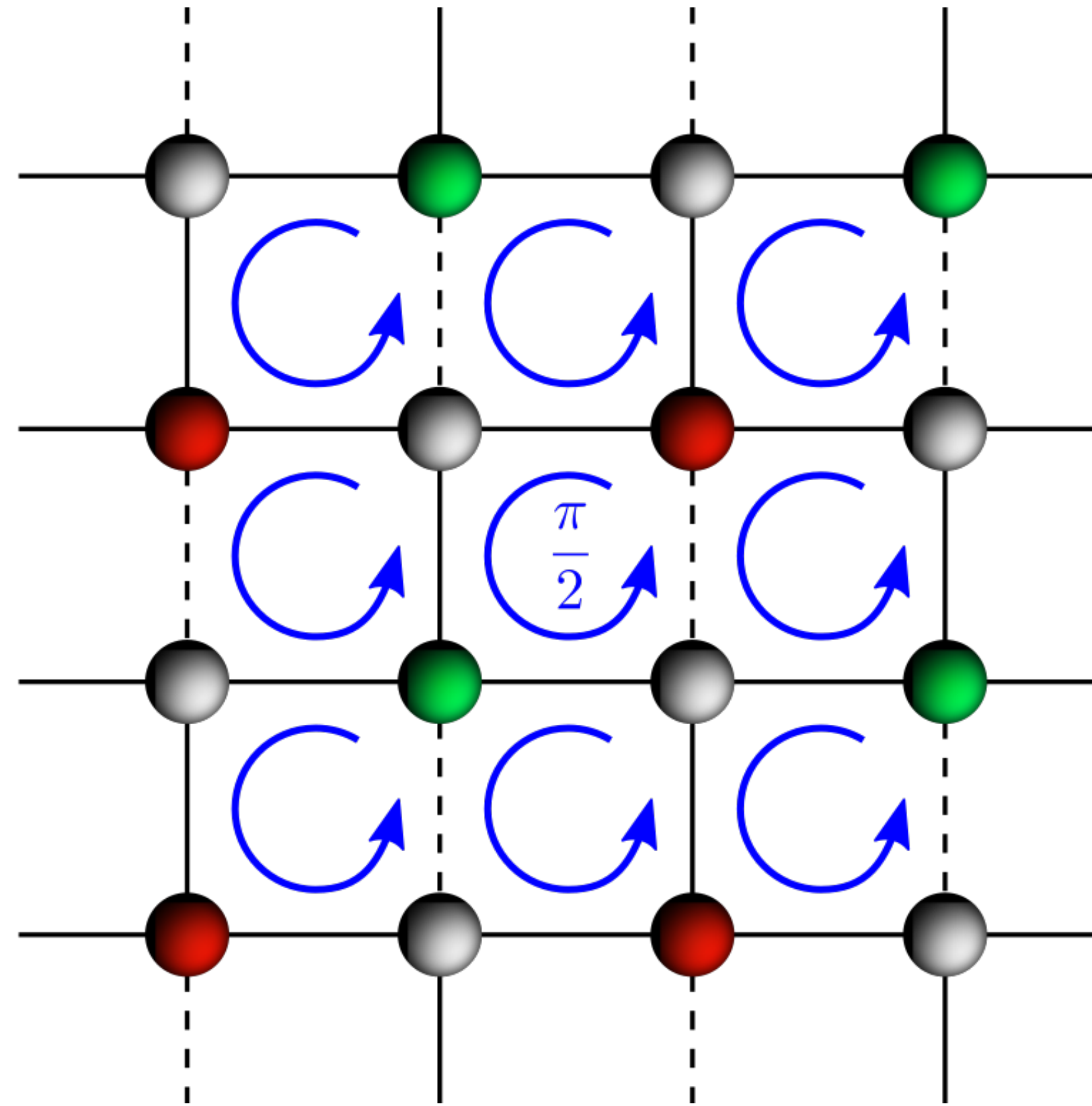
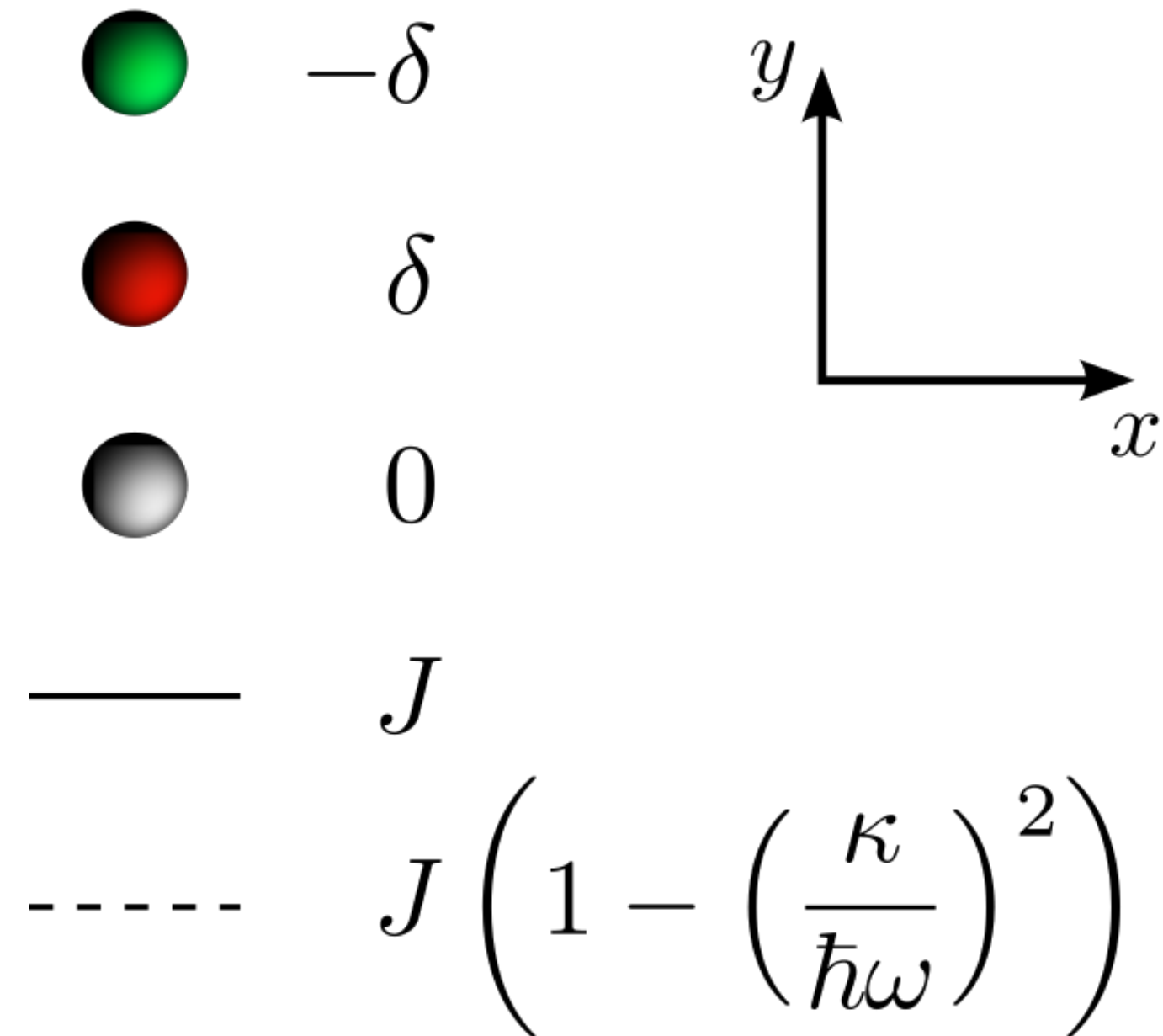
## Ultra-cold bosons



Aidelsburger et. al Nat. Phys (2013)



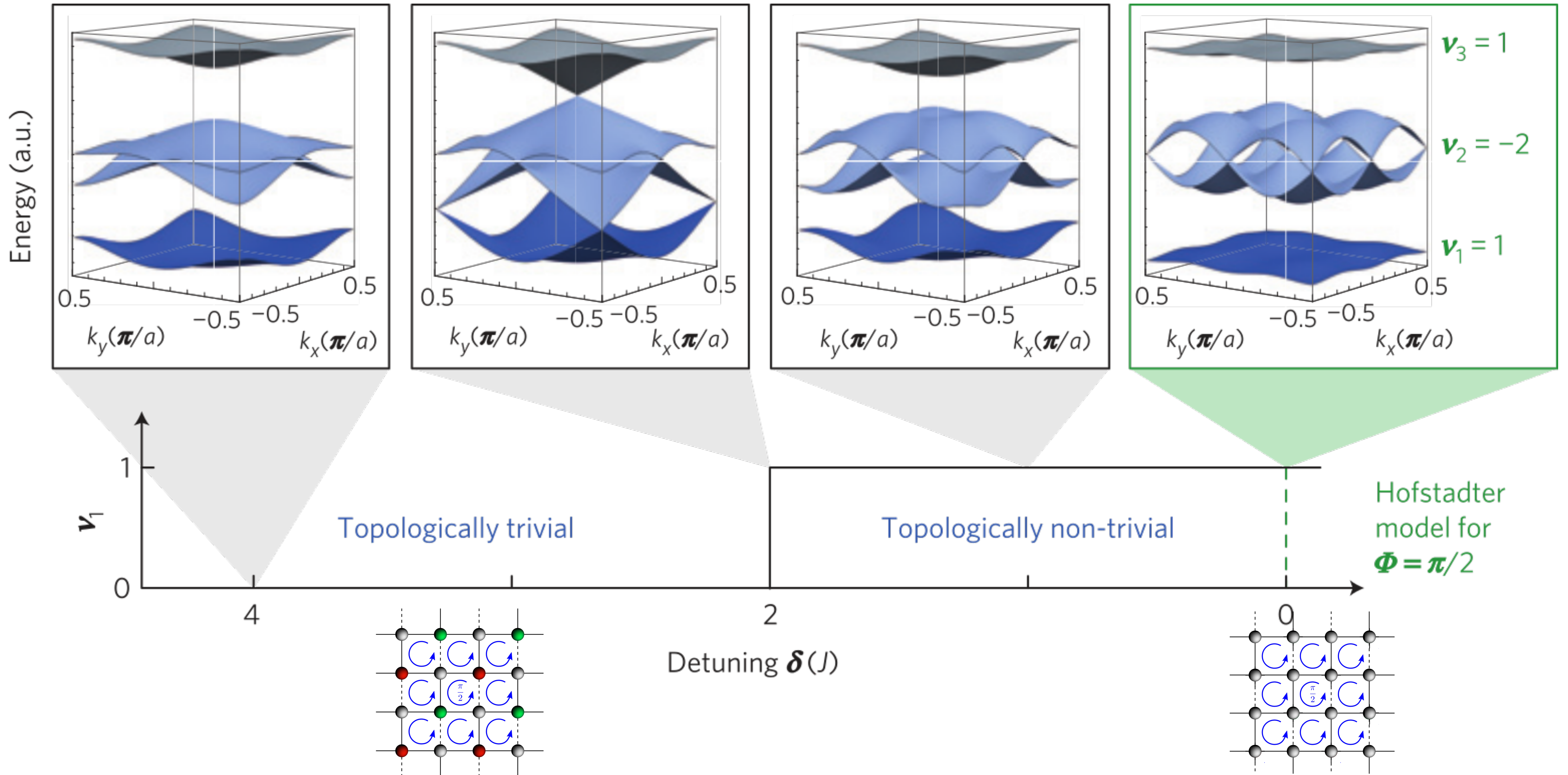
# (Floquet) Chern insulator with ultra cold bosons



$$H_{\text{eff}} = -J \sum_{m,n} \left\{ \hat{a}_{m+1,n}^\dagger \hat{a}_{m,n} e^{i[\pi/2(m+n) - \phi_0]} + (1 + f_{m,n}) \hat{a}_{m,n+1}^\dagger \hat{a}_{m,n} + \text{h.c.} \right\} + \frac{\delta}{2} \sum_{m,n} [(-1)^m + (-1)^n] \hat{n}_{m,n}$$

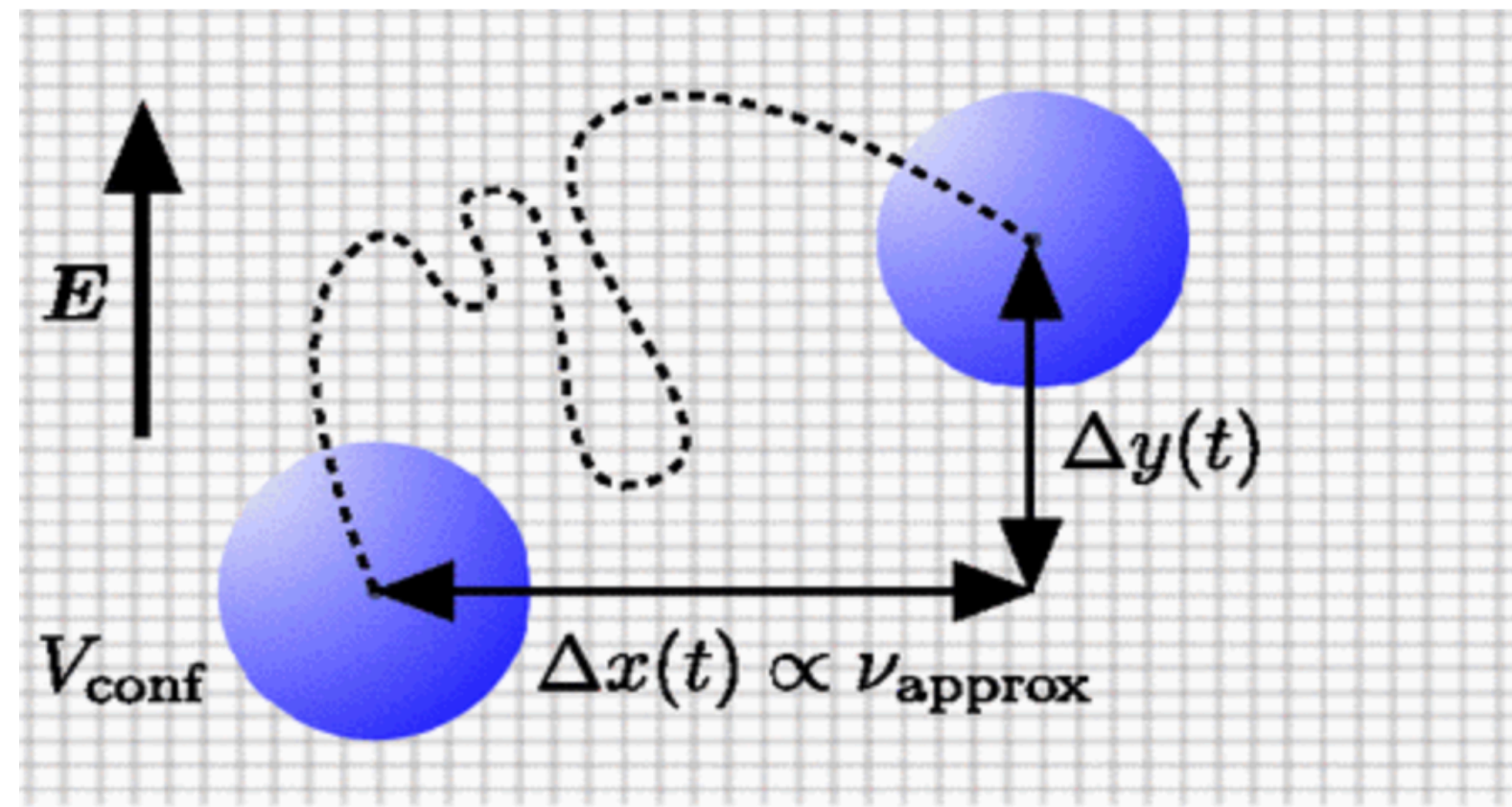
# (Floquet) Chern insulator with ultra cold bosons

M. Aidelsburger et al. Nat. Phys (2015)



# How does one know it is a Chern insulator?

Wave packet motion



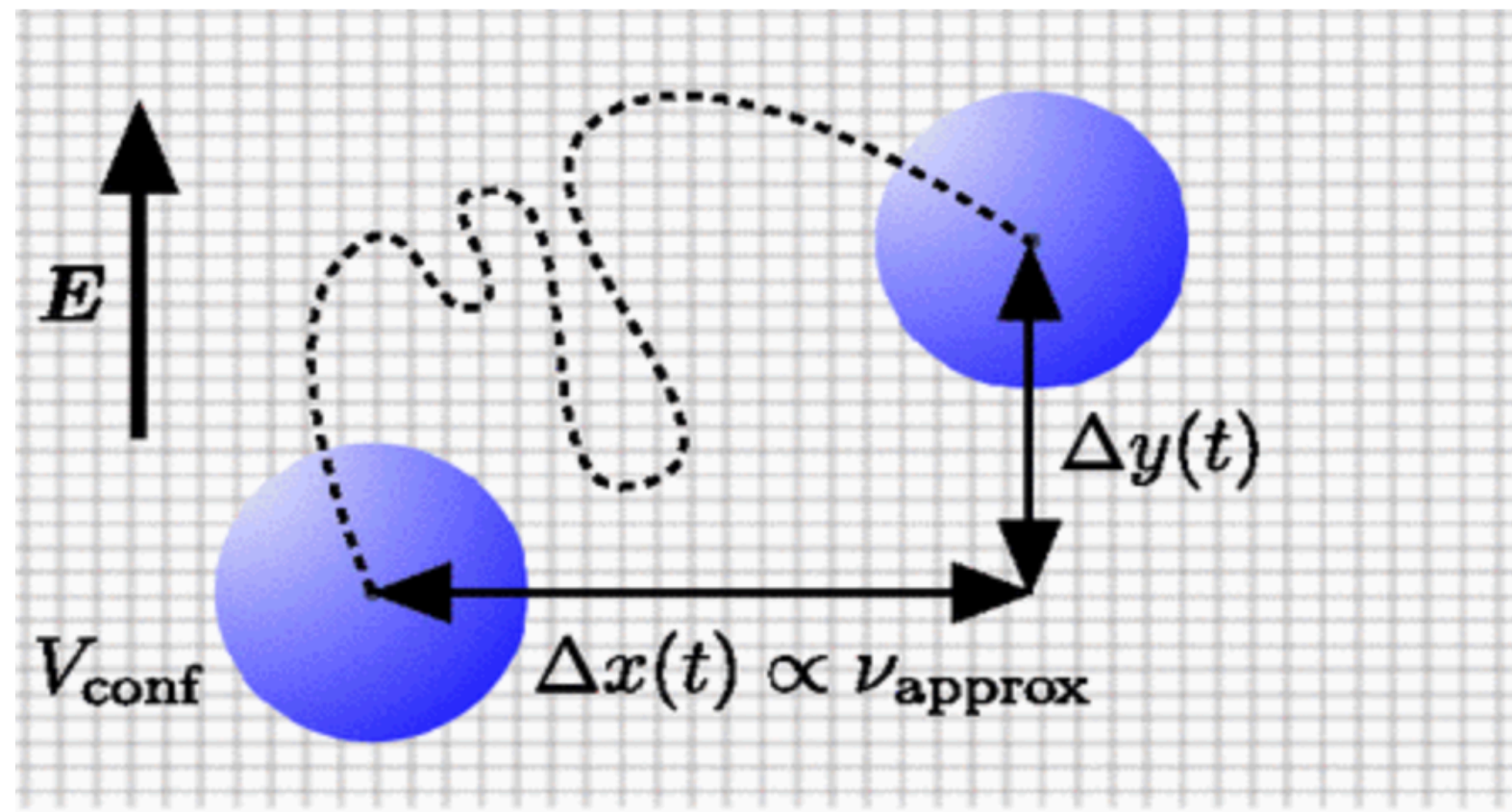
A. Dauphin and N. Goldman PRL 2013

$$x(t) = -(a^2 t E_y / \pi \hbar) \nu_{\text{approx}}$$

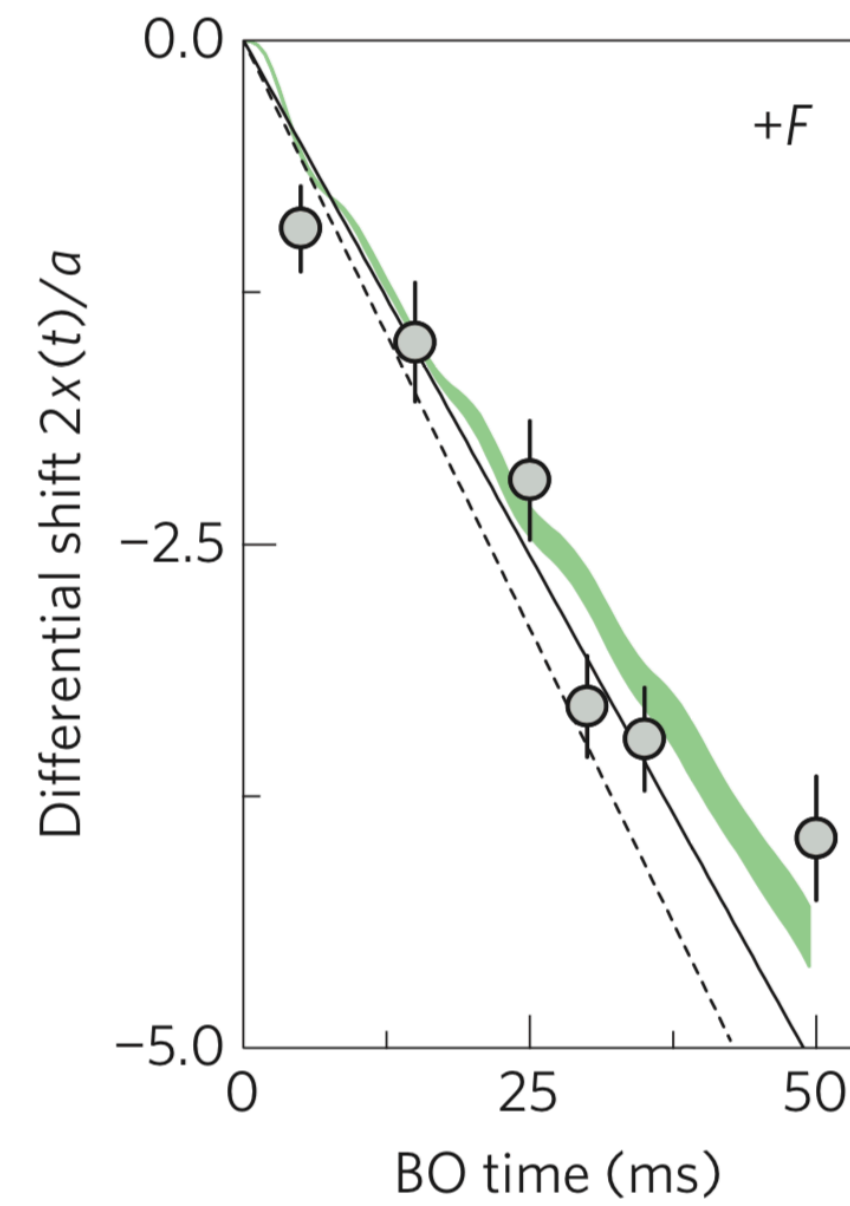


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## Wave packet motion



A. Dauphin and N. Goldman PRL 2013



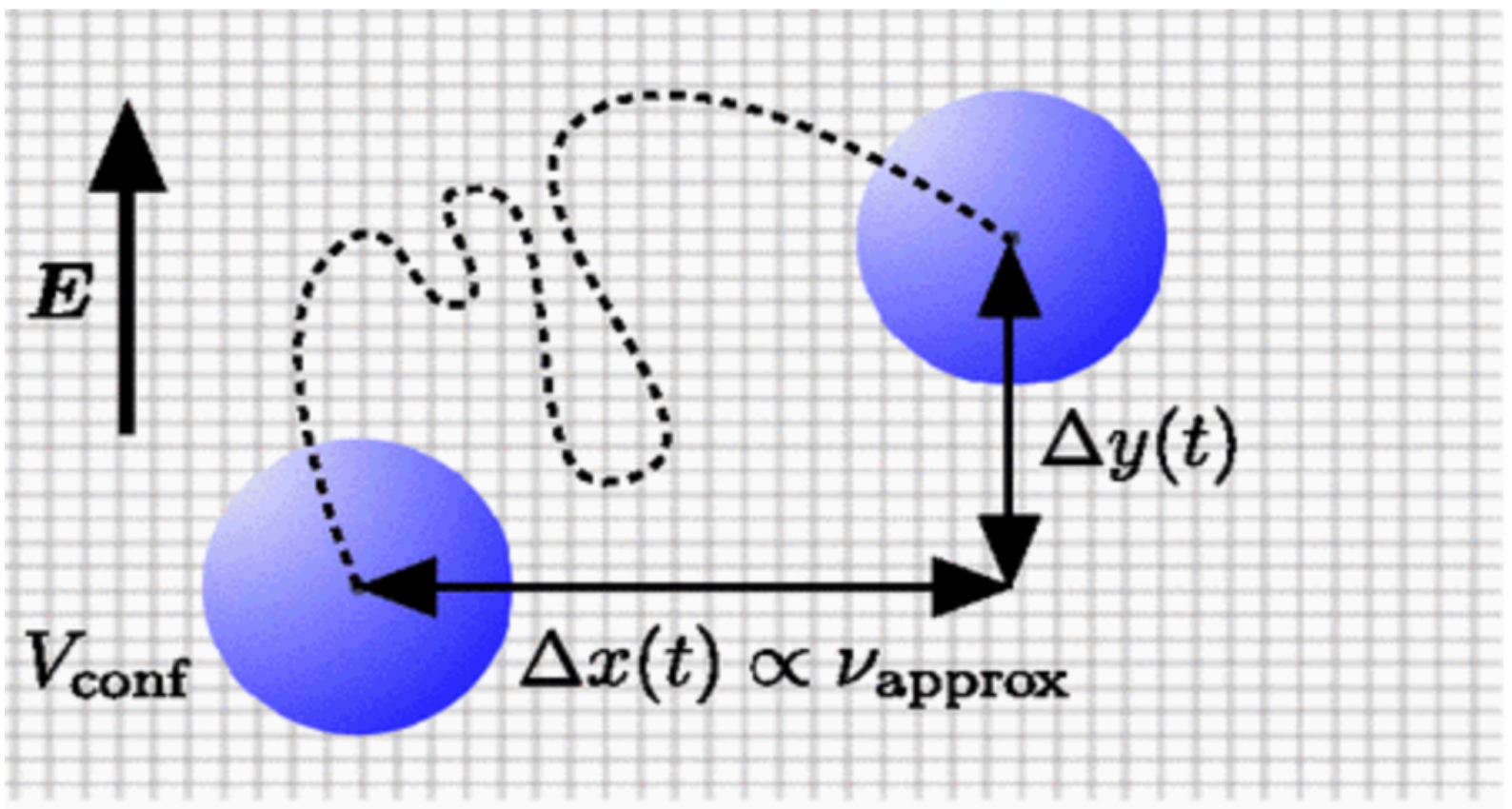
M. Aidelsburger et al. Nat. Phys (2015)

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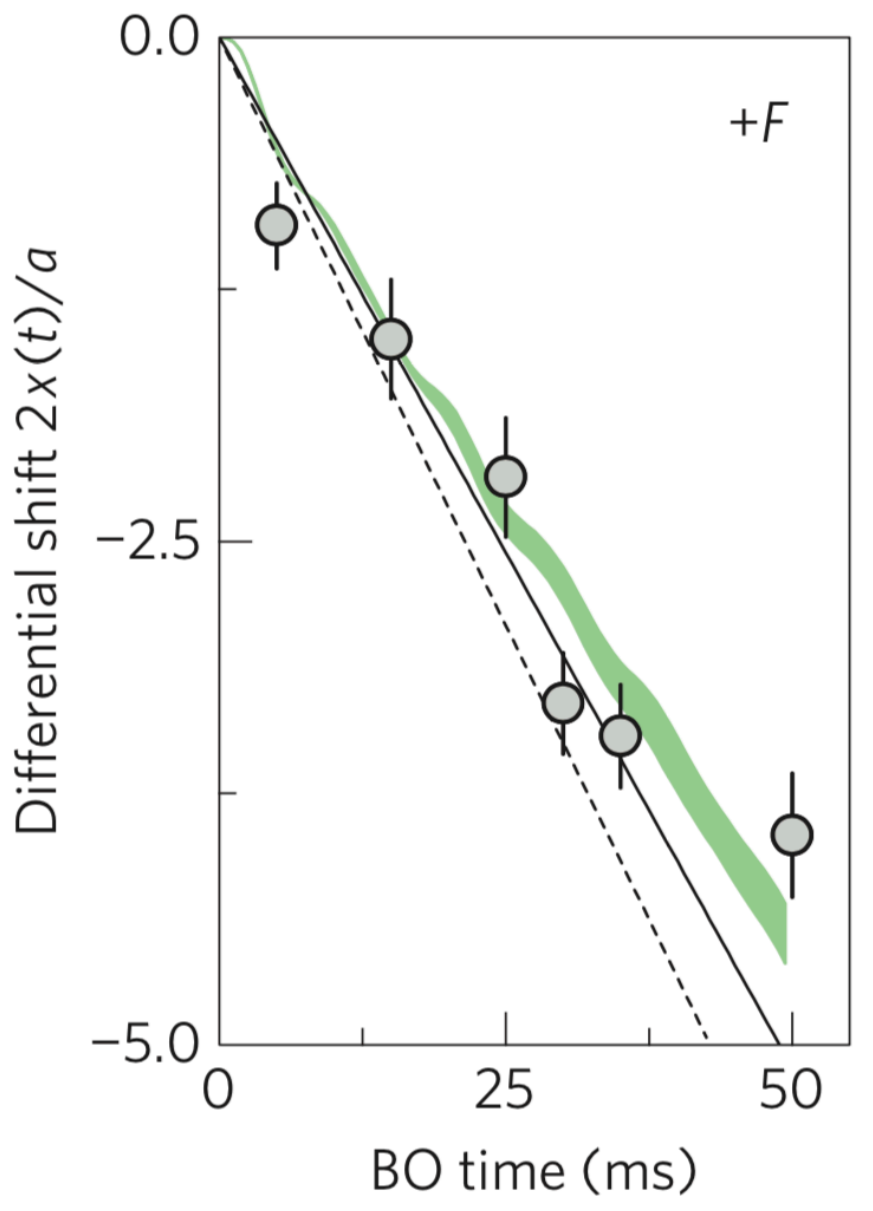


# How does one know it is a Chern insulator?

## Wave packet motion



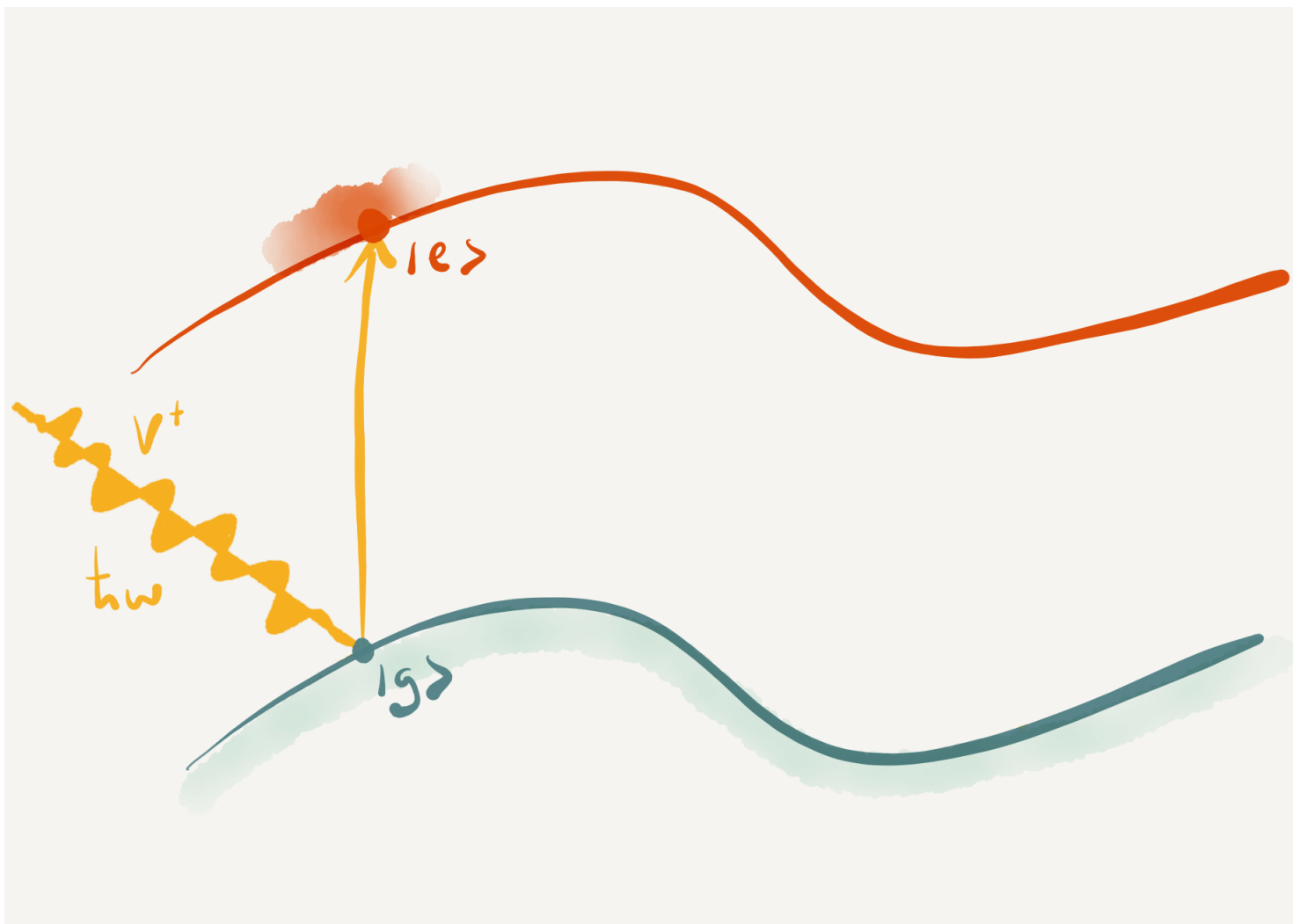
A. Dauphin and N. Goldman PRL 2013



M. Aidelsburger et al. Nat. Phys (2015)

$$x(t) = -(a^2 t E_y / \pi \hbar) \nu_{\text{approx}}$$

## Circular dichroism

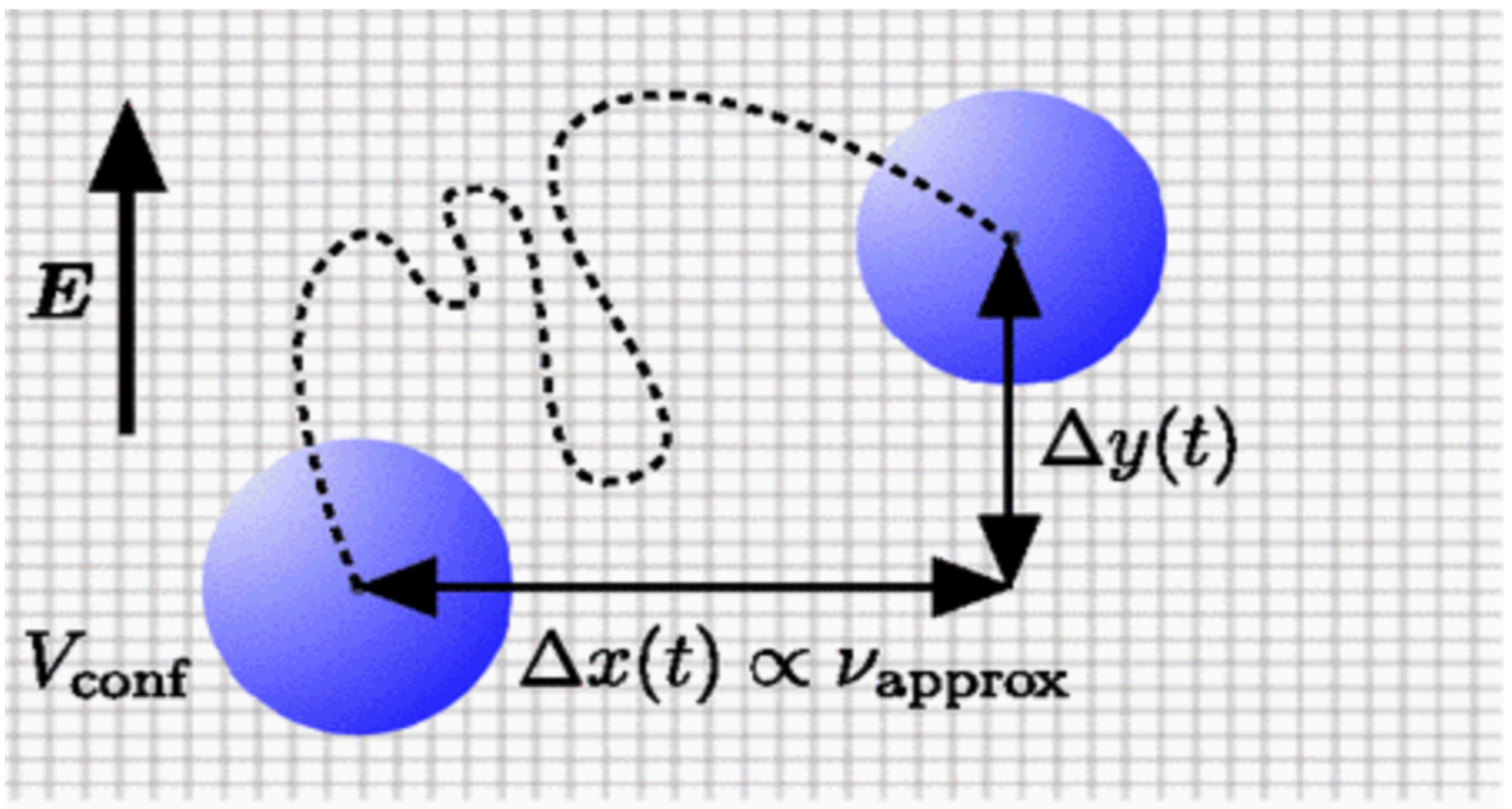


$$(\Gamma_+^{\text{int}} - \Gamma_-^{\text{int}}) / 2A_{\text{cell}} = (E_{\text{sp}} / \hbar)^2 C$$

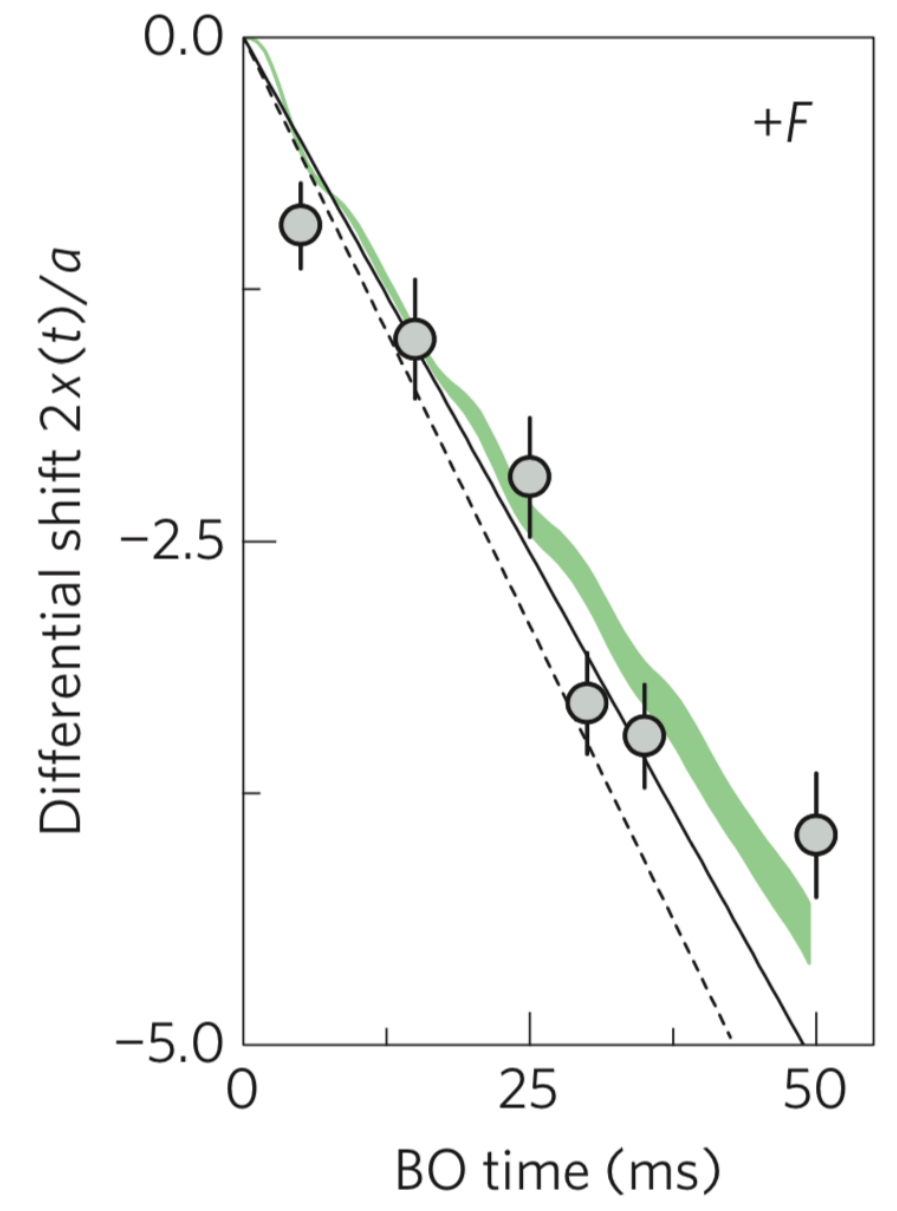
D. T. Tran, A. Dauphin, AGG, N. Goldman, P. Zoller Sci. Adv. (2018)

# How does one know it is a Chern insulator?

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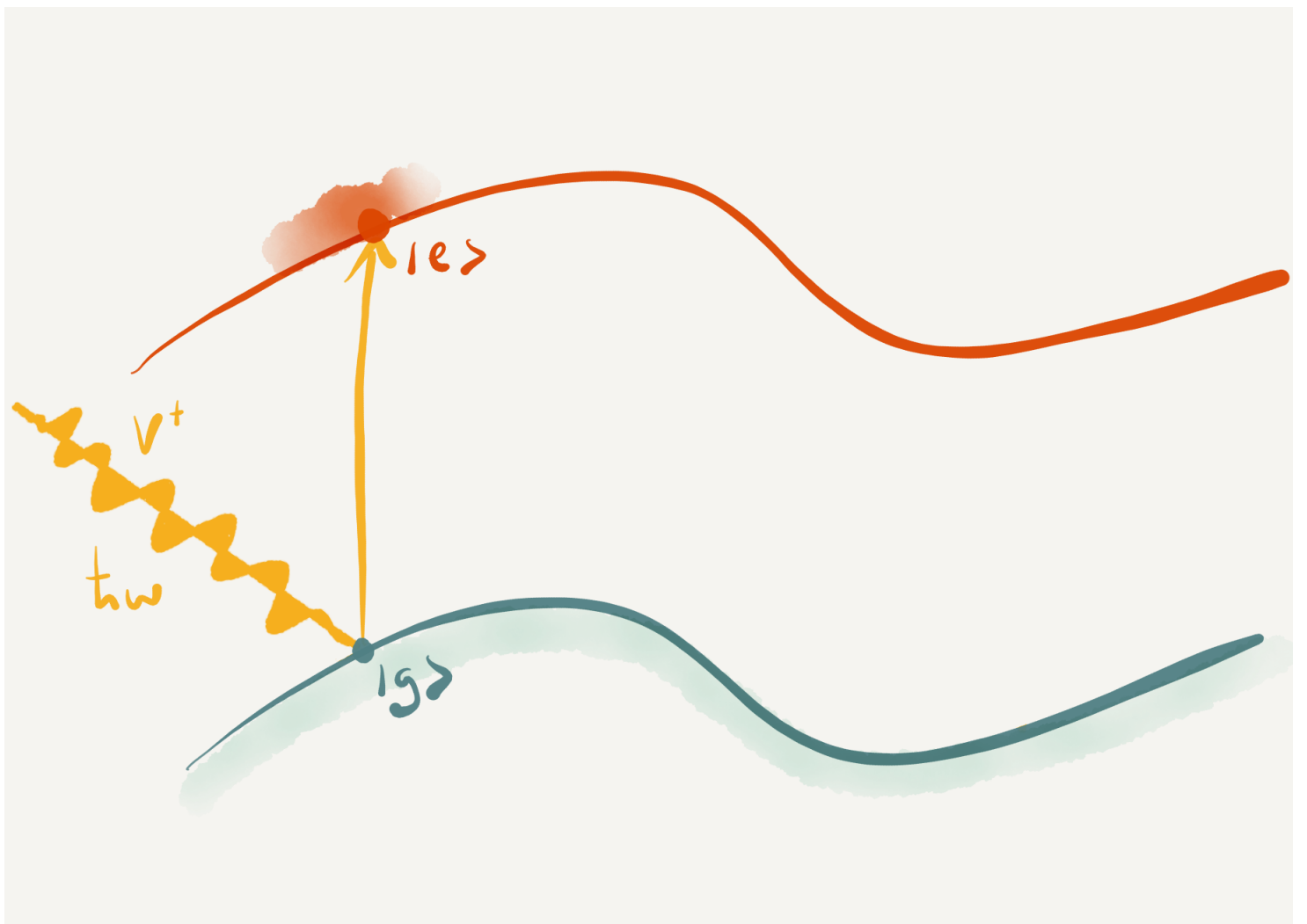
A. Dauphin and N. Goldman PRL 2013



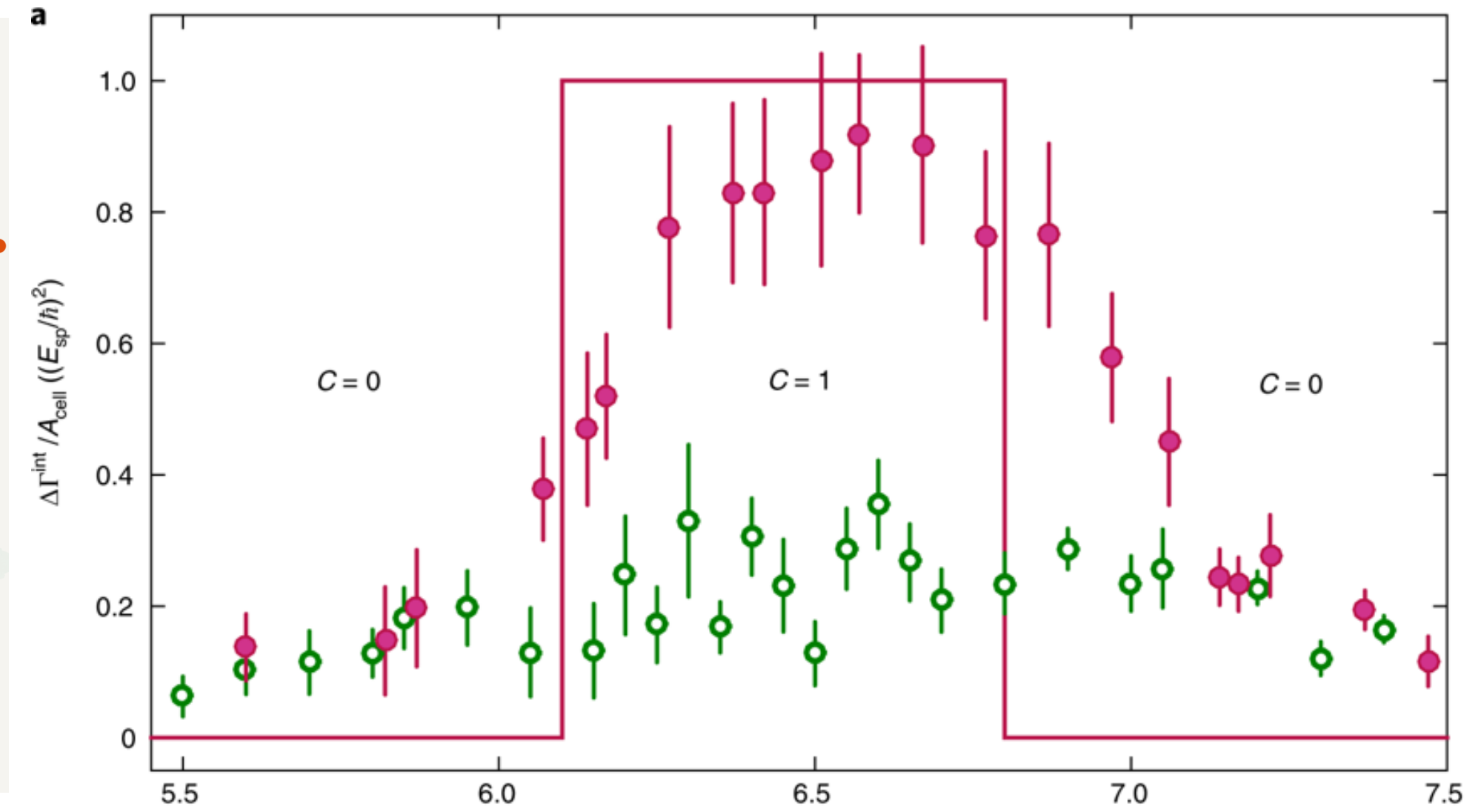
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## Circular dichroism



D. T. Tran, A. Dauphin, AGG, N. Goldman, P. Zoller Sci. Adv. (2018)



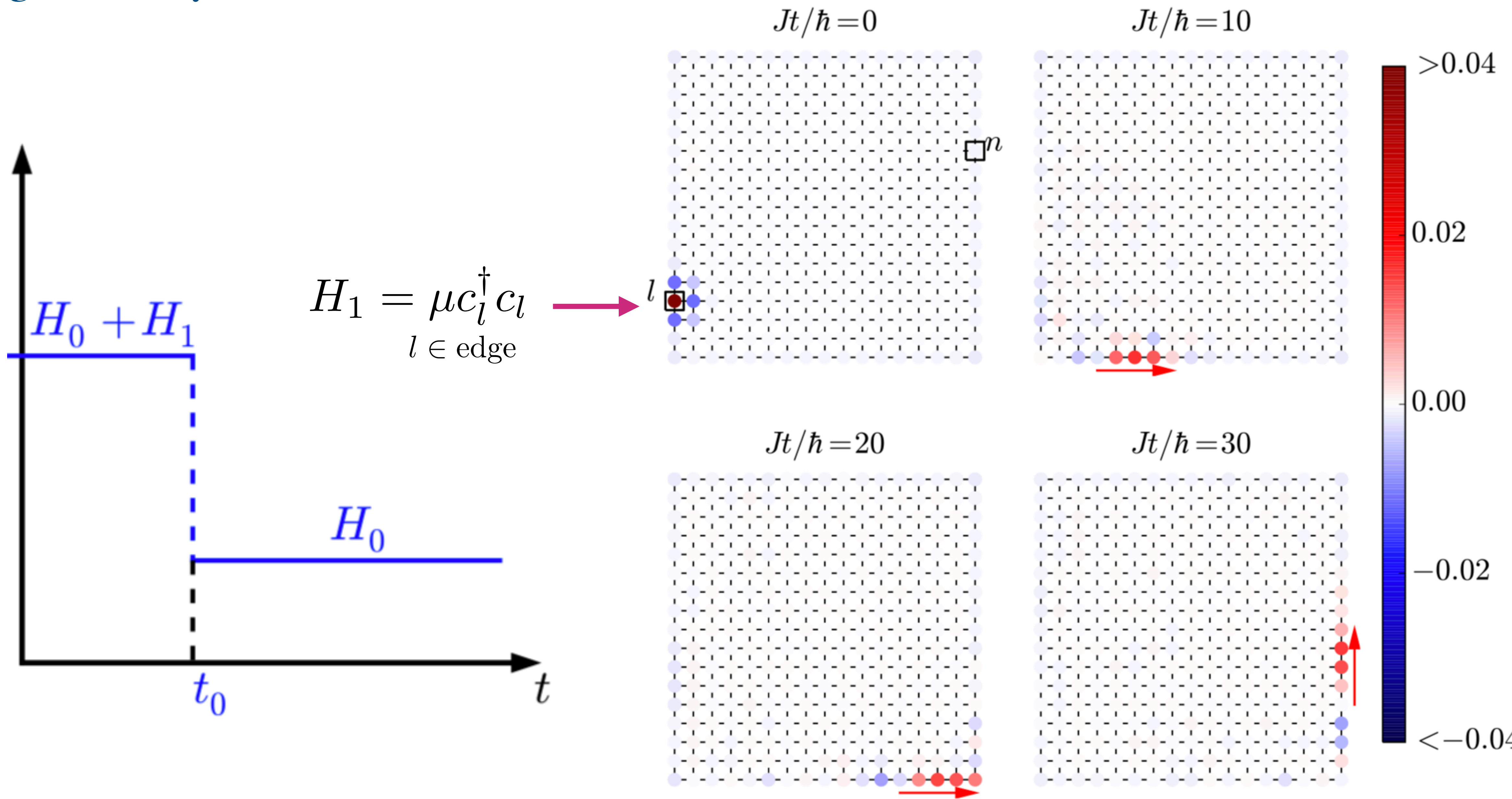
L. Asteria, et. al Nat. Phys, 2019

$$(\Gamma_+^{\text{int}} - \Gamma_-^{\text{int}}) / 2A_{\text{cell}} = (E_{\text{sp}} / \hbar)^2 C$$

Can we distinguish a fractional Chern insulator by looking at edge state dynamics?



# Edge state dynamics of a Chern insulator



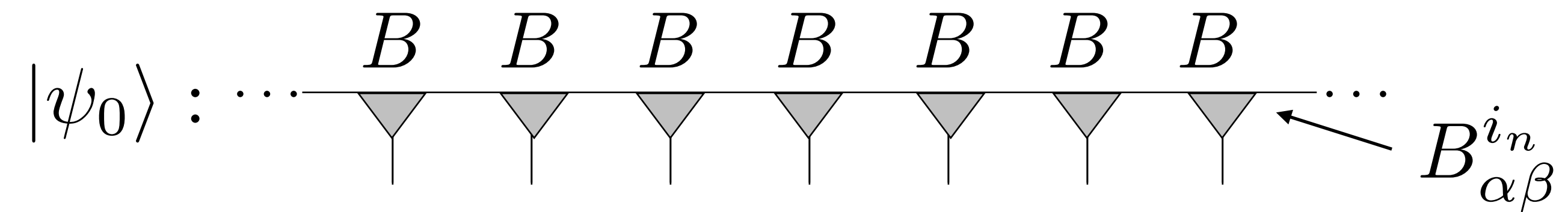


# Edge state dynamics of a bosonic fractional Chern insulator

# Density Matrix Renormalization Group

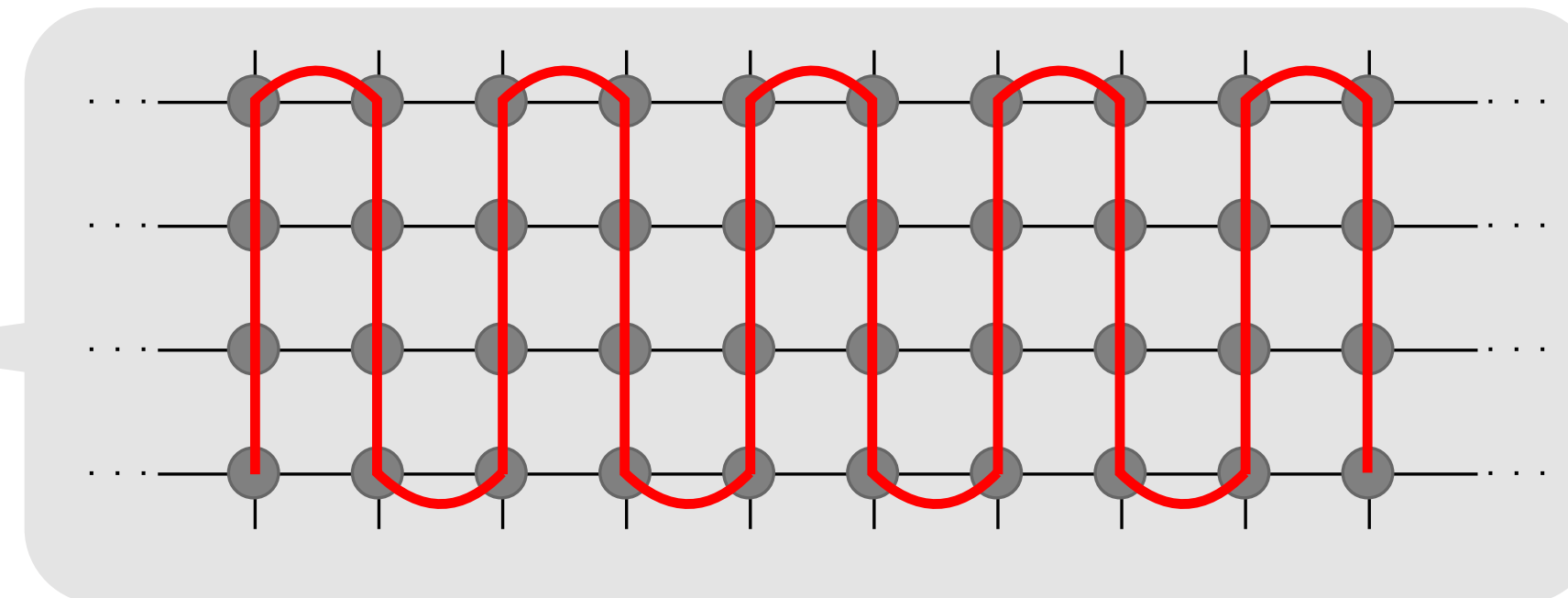
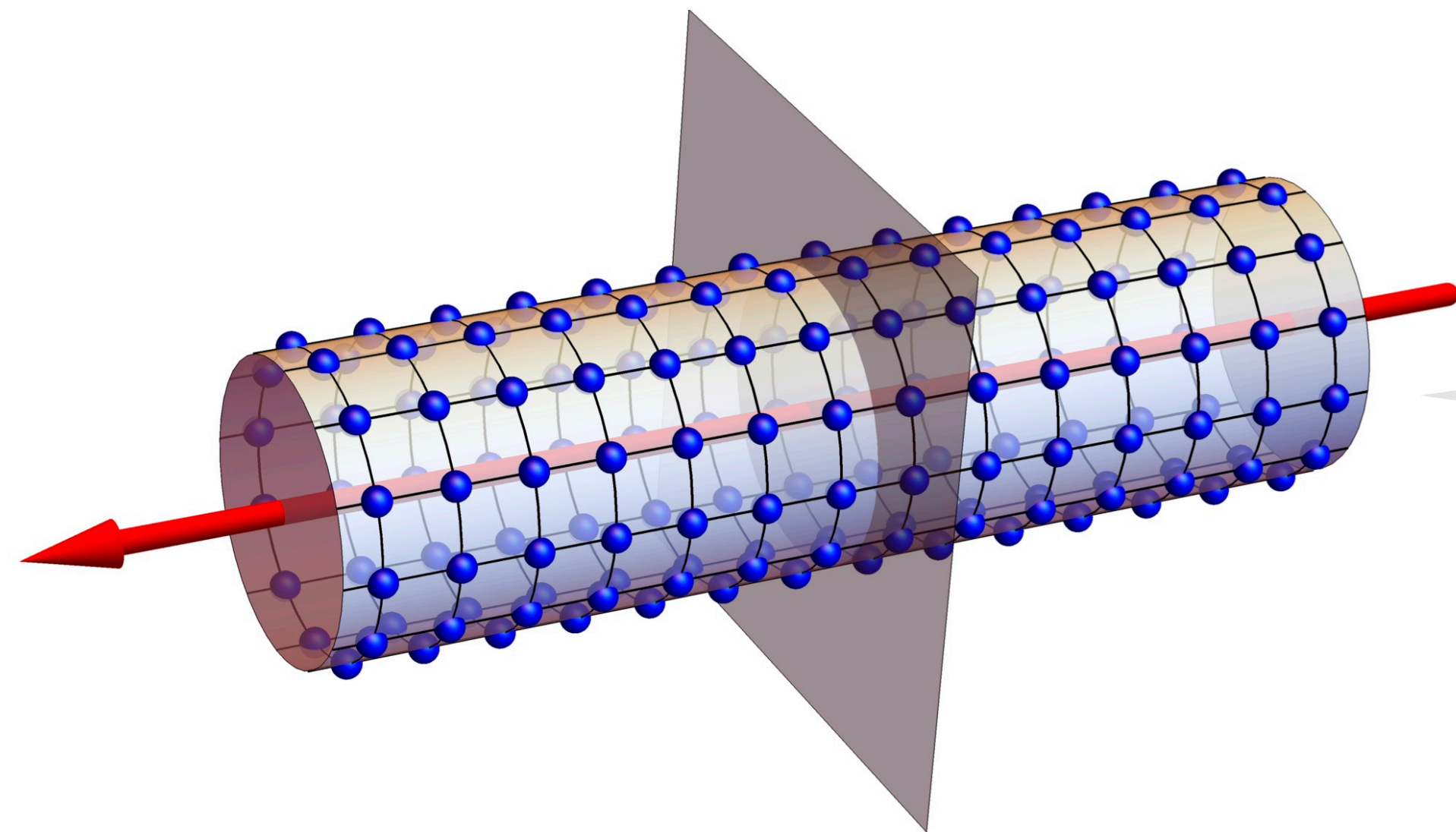
## Matrix-Product State representation of the ground state

M. Fannes et al Comm Math. Phys. '92, Schollwoeck Ann. Phys.'11



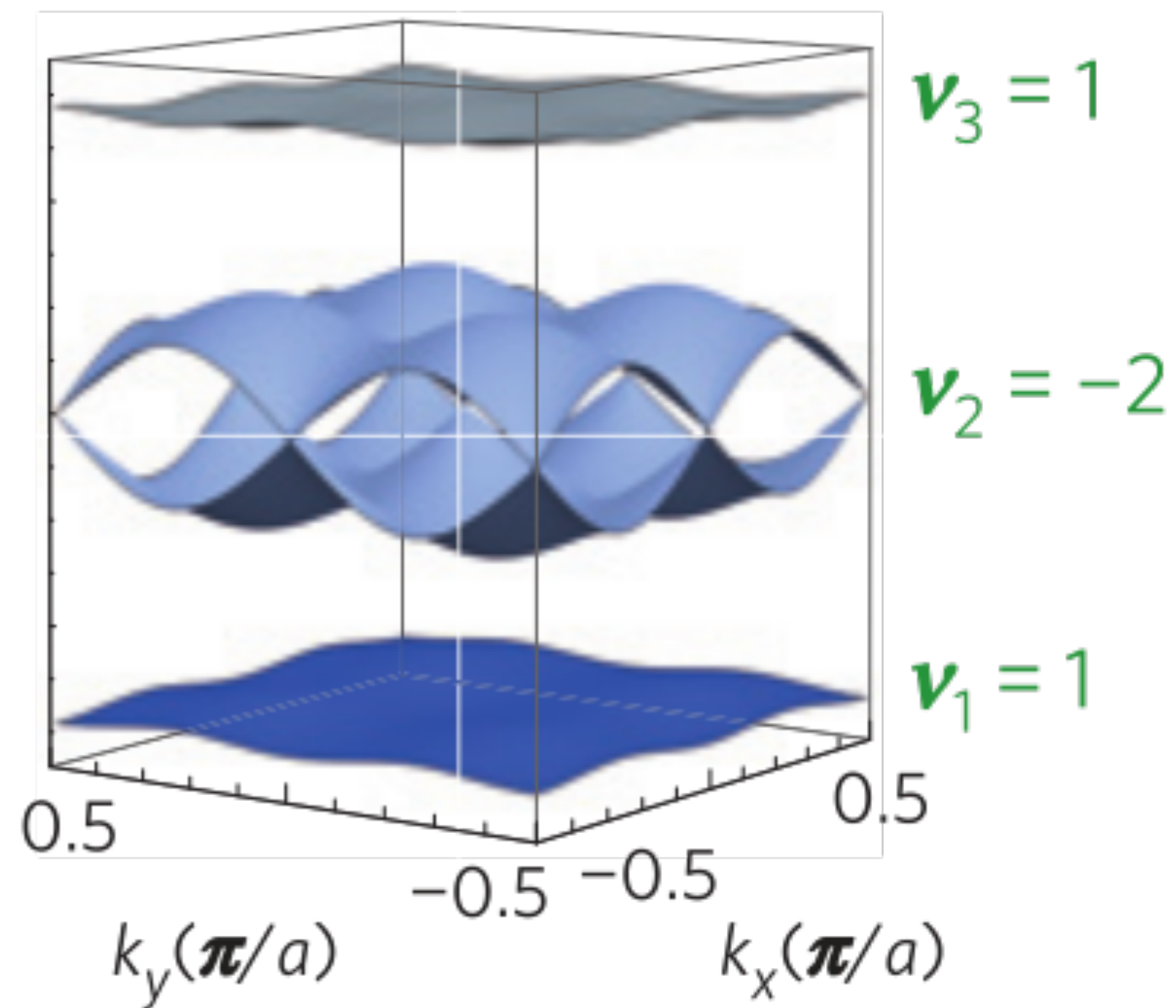
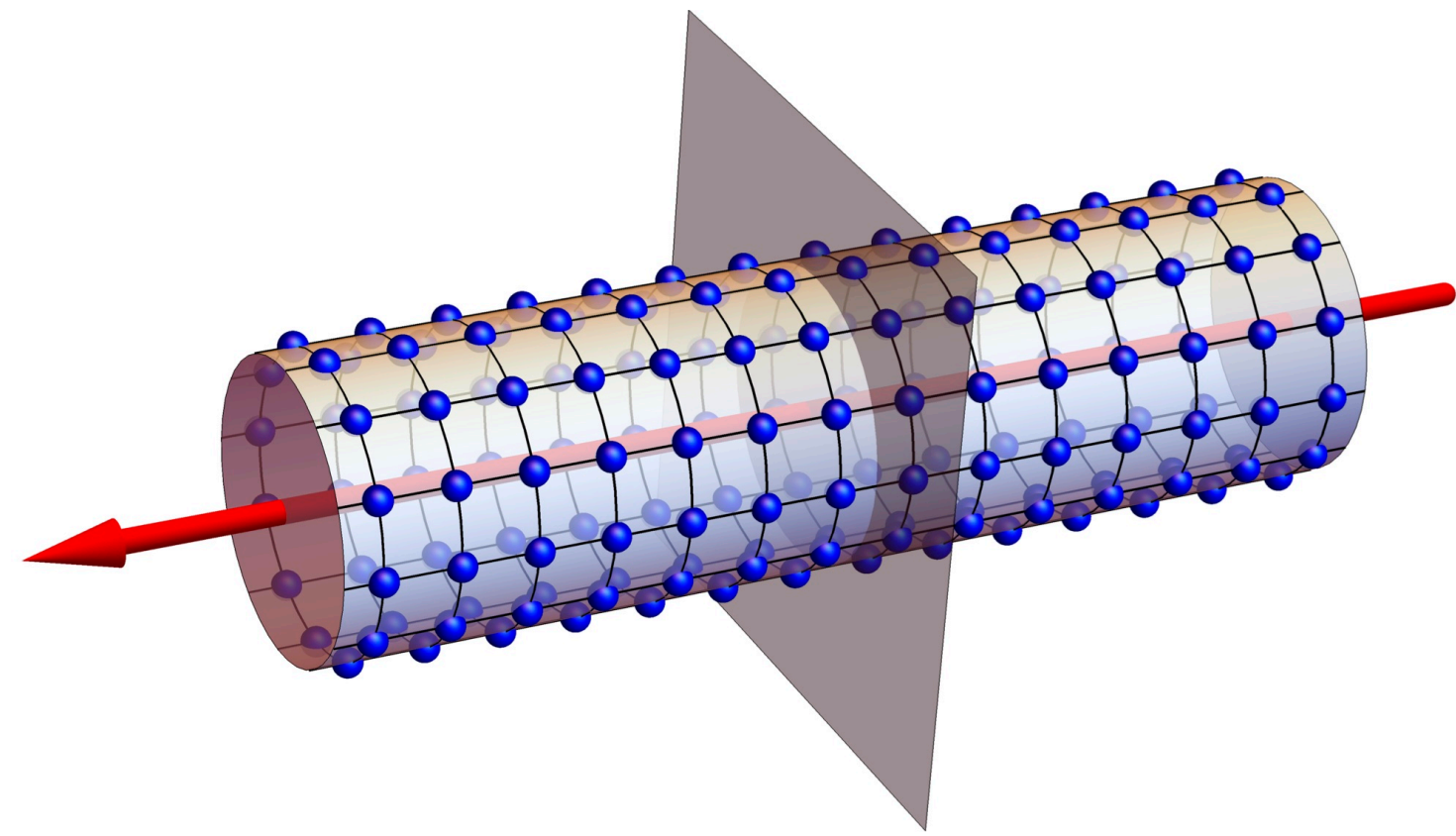
## DMRG on cylinders with circumference up to $L=12$

S. R. White PRL '92

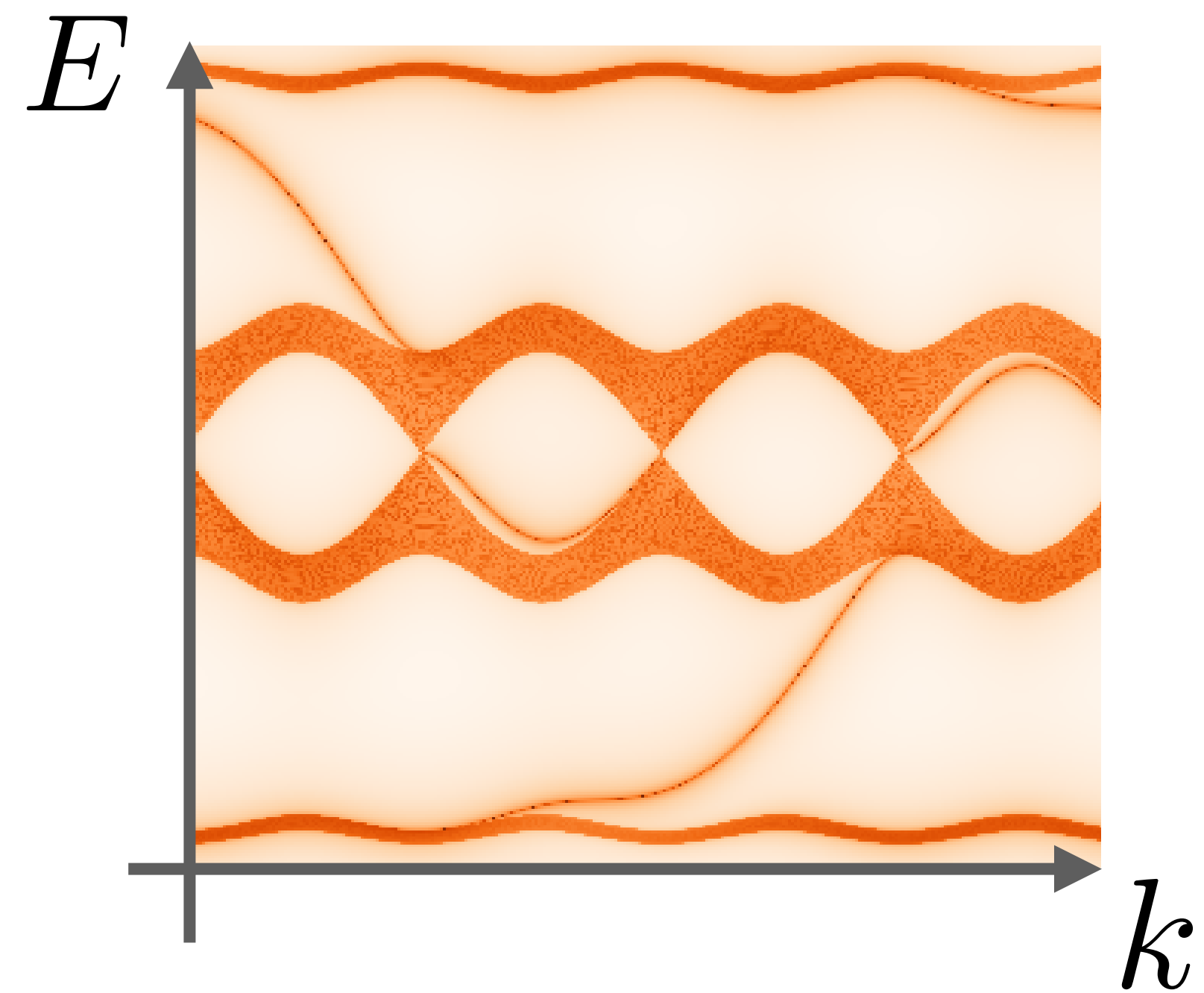
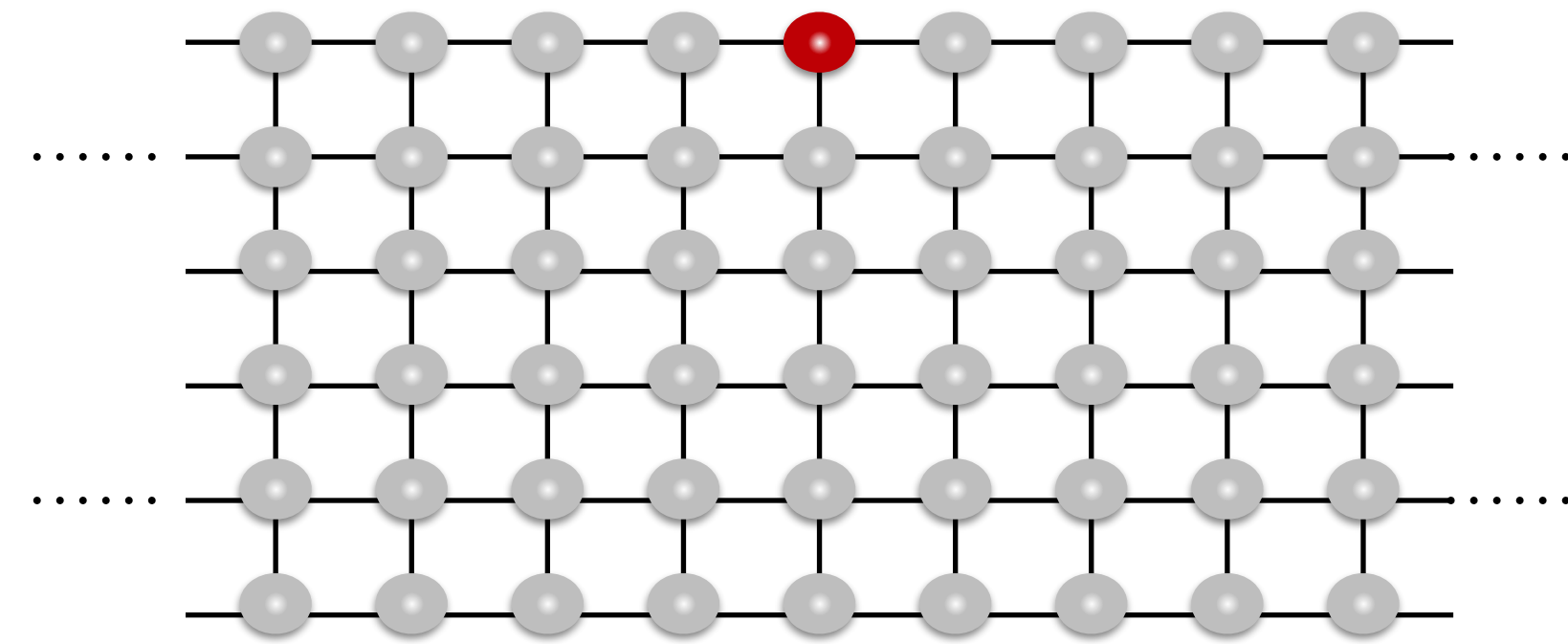


2D physics at cost of long-range interaction in 1D representation

# Infinite cylinder



# Finite cylinder



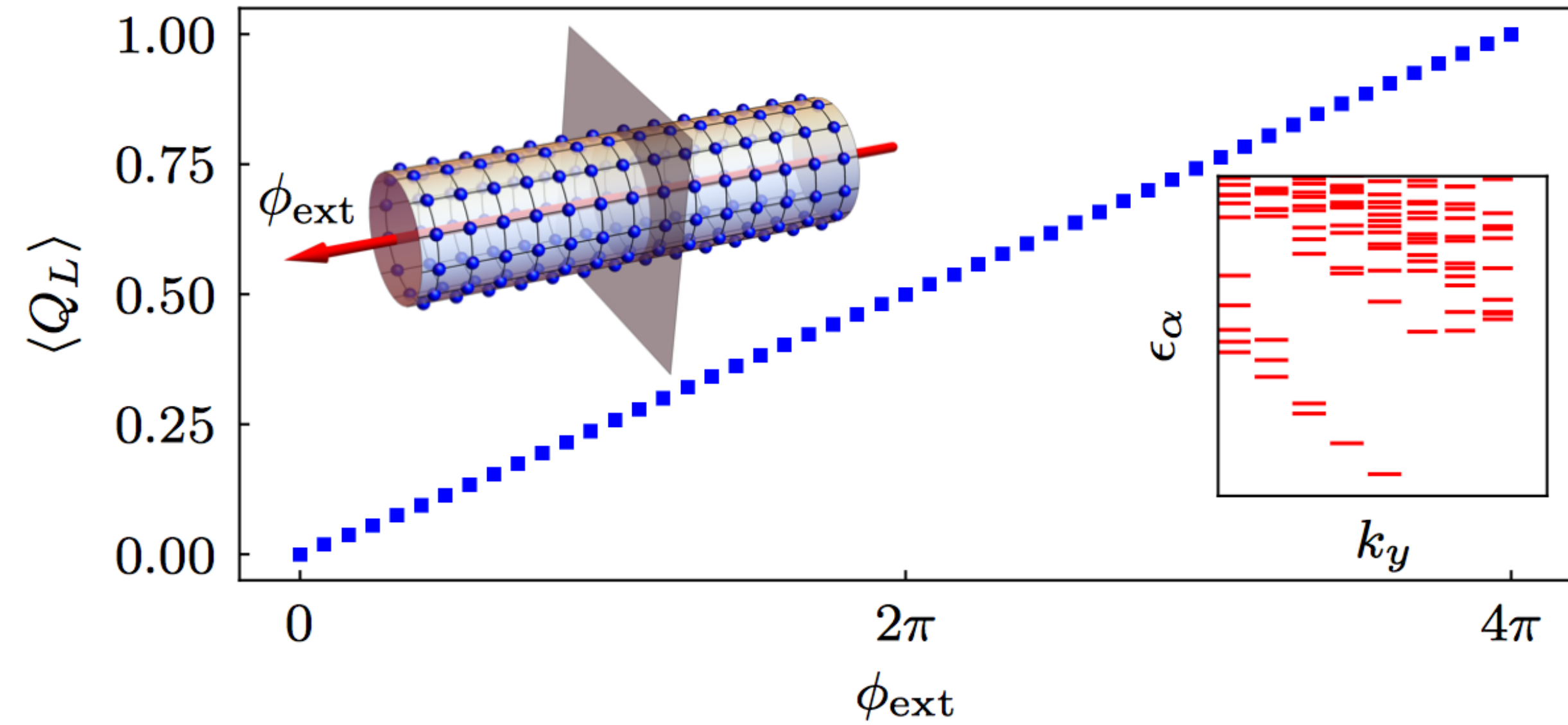
# Static ground state: fractional Chern insulator at 1/8 filling

Its been a while...

[Hafezi et al. '07; Möller and Cooper '09; ...]

Quantized Hall conductivity

$$\sigma_{xy} = 1/2$$





# Static ground state: fractional Chern insulator at 1/8 filling

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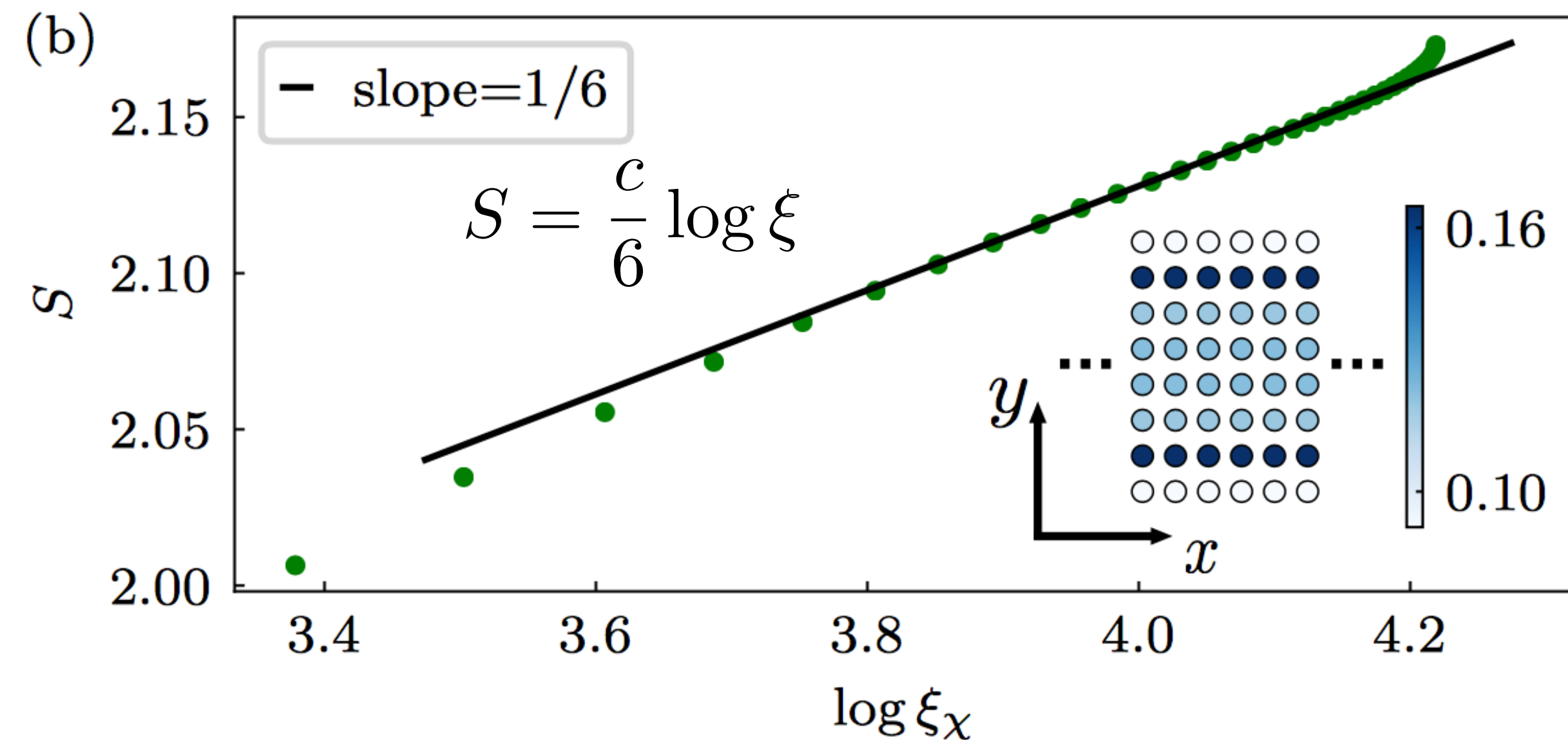
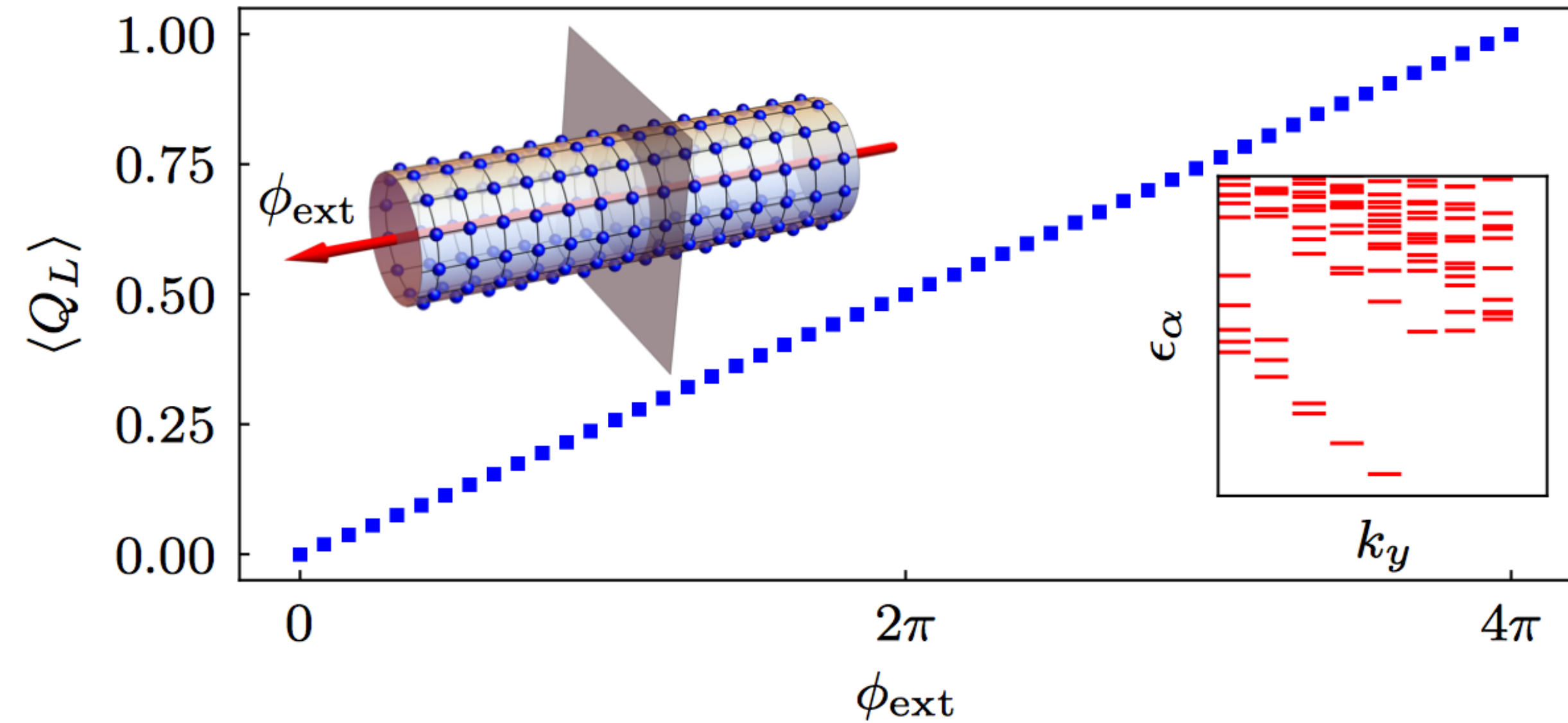
[Hafezi et al. '07; Möller and Cooper '09; ...]

Quantized Hall conductivity

$$\sigma_{xy} = 1/2$$

Gapless edge states

$$c = 1$$



# Time evolution

Hamiltonian expressed as a sum of local terms  $H = \sum_x H_x$

Expand  $U = \exp(-itH)$  for  $t \ll 1$ :

$$U(t) = 1 + t \sum_x H_x + \frac{1}{2} t^2 \sum_{x,y} H_x H_y + \dots$$

Compact Matrix Product Operator (MPO)

$$W_{\alpha\beta}^{[n]j_n j'_n} = \alpha \begin{array}{c} j'_n \\ | \\ \text{---} \diamond \text{---} \\ | \\ j_n \end{array} \beta$$

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$$\approx 1 + t \sum_x H_x + t^2 \underbrace{\sum_{x < y} H_x H_y}_{\epsilon \sim \underline{Lt^2}}$$

Neglect overlapping terms in expansion

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Compact Matrix Product Operator (MPO)

$$1 + t \underbrace{\sum_x H_x}_{\epsilon \sim \underline{L^2 t^2}} \rightarrow \prod_x \underbrace{(1 + t H_x)}_{\epsilon \sim \underline{Lt^2}}$$

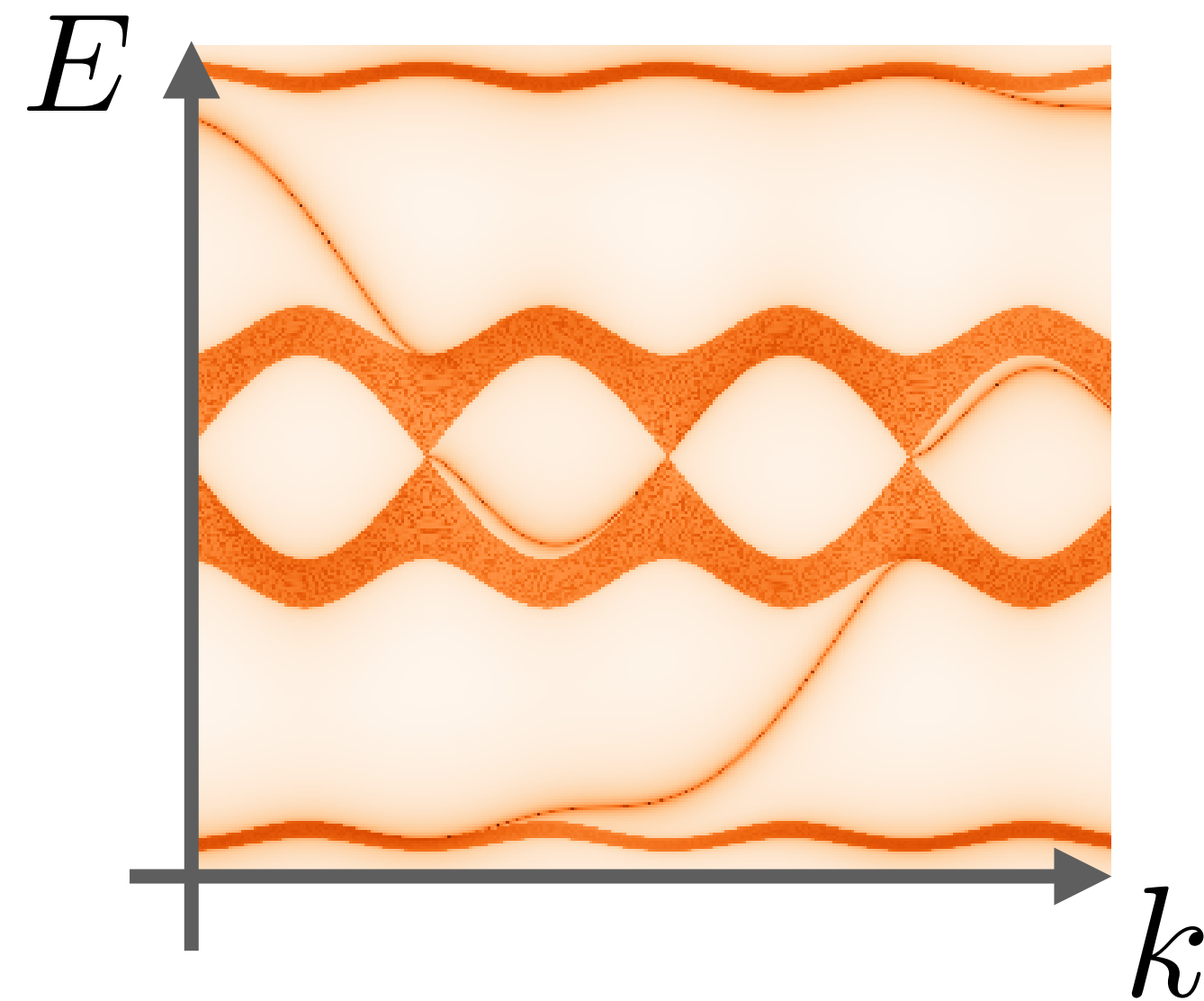
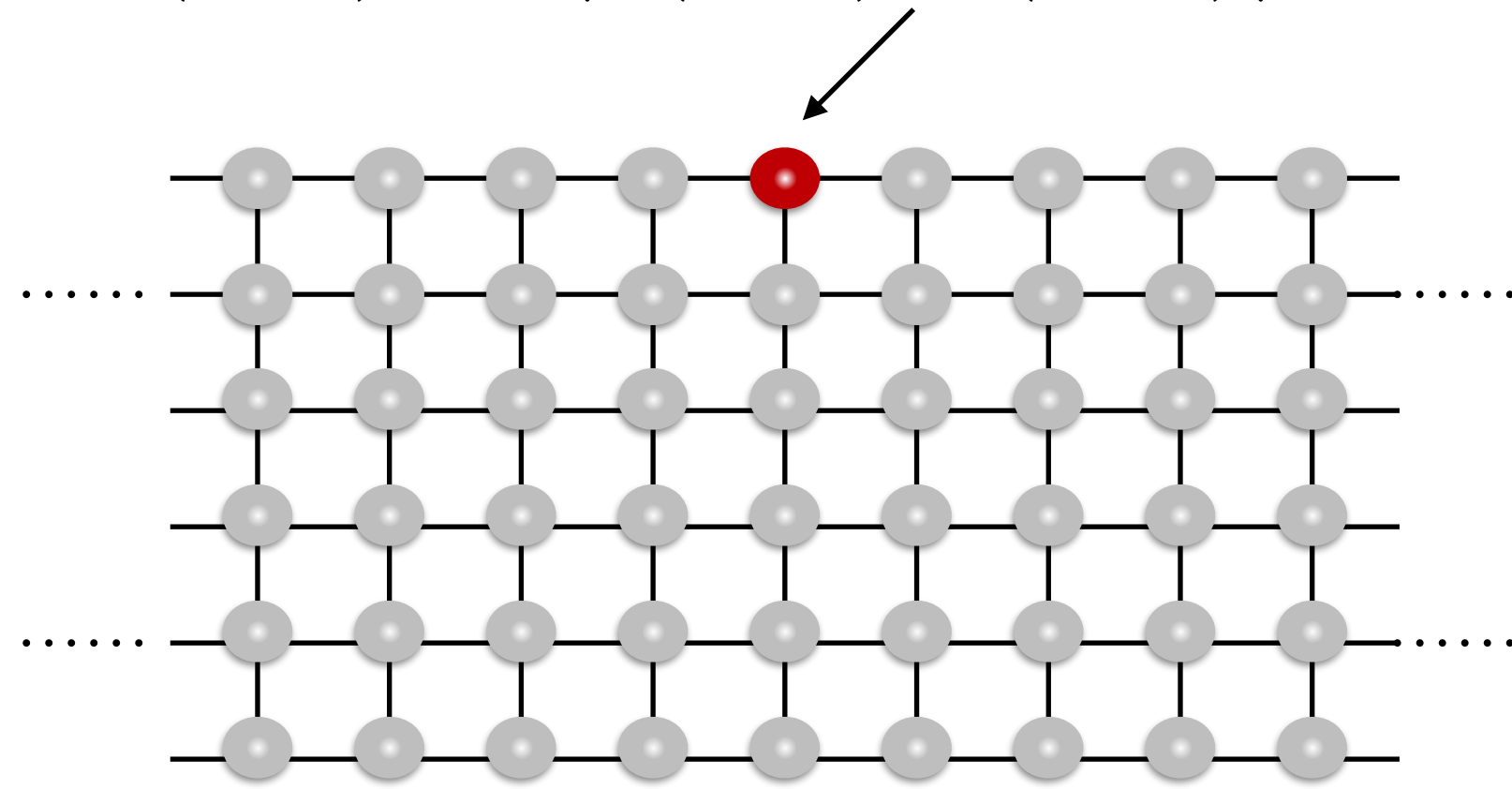
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# Dynamical signatures of the FCI phase

## Dynamical correlation function

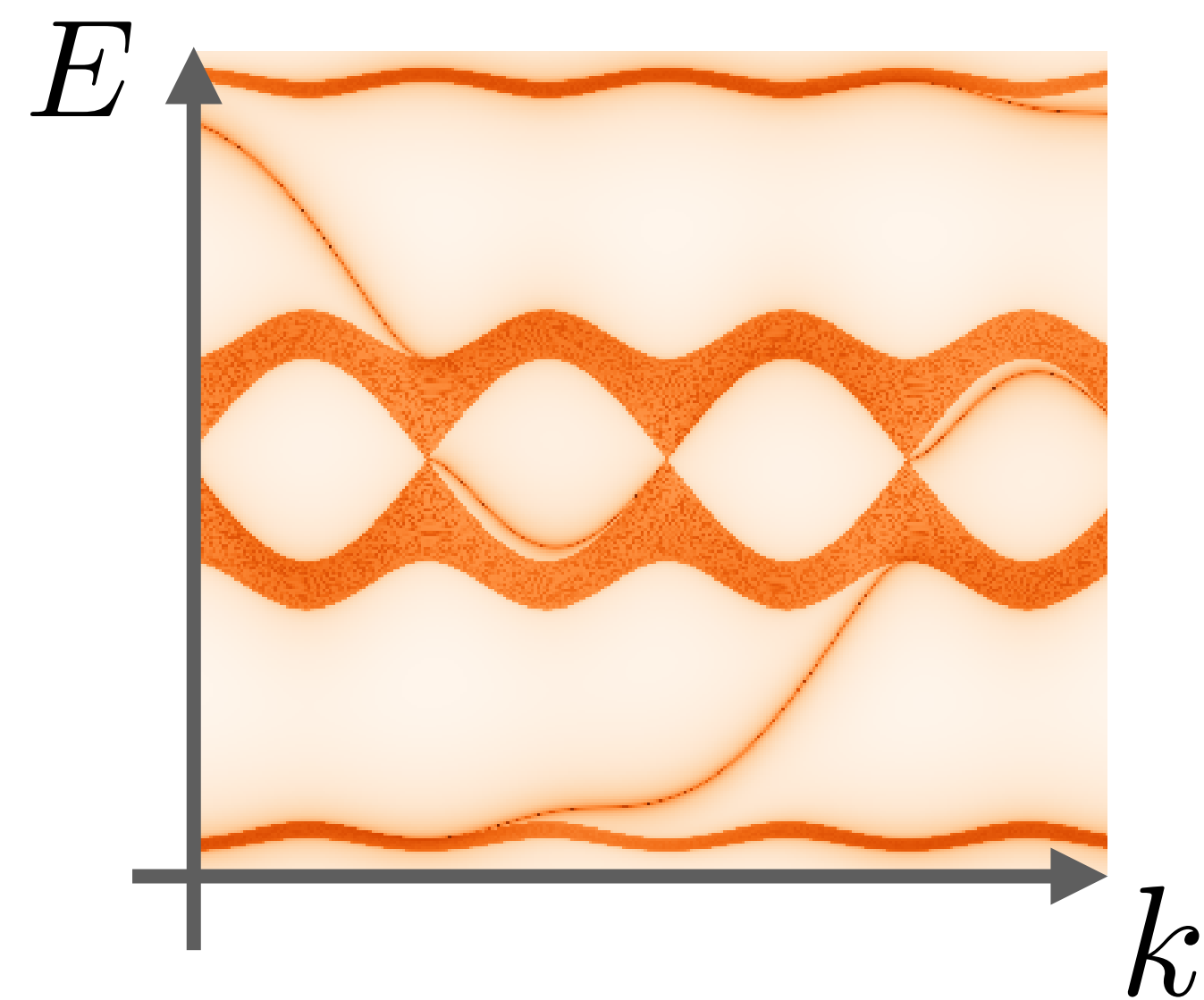
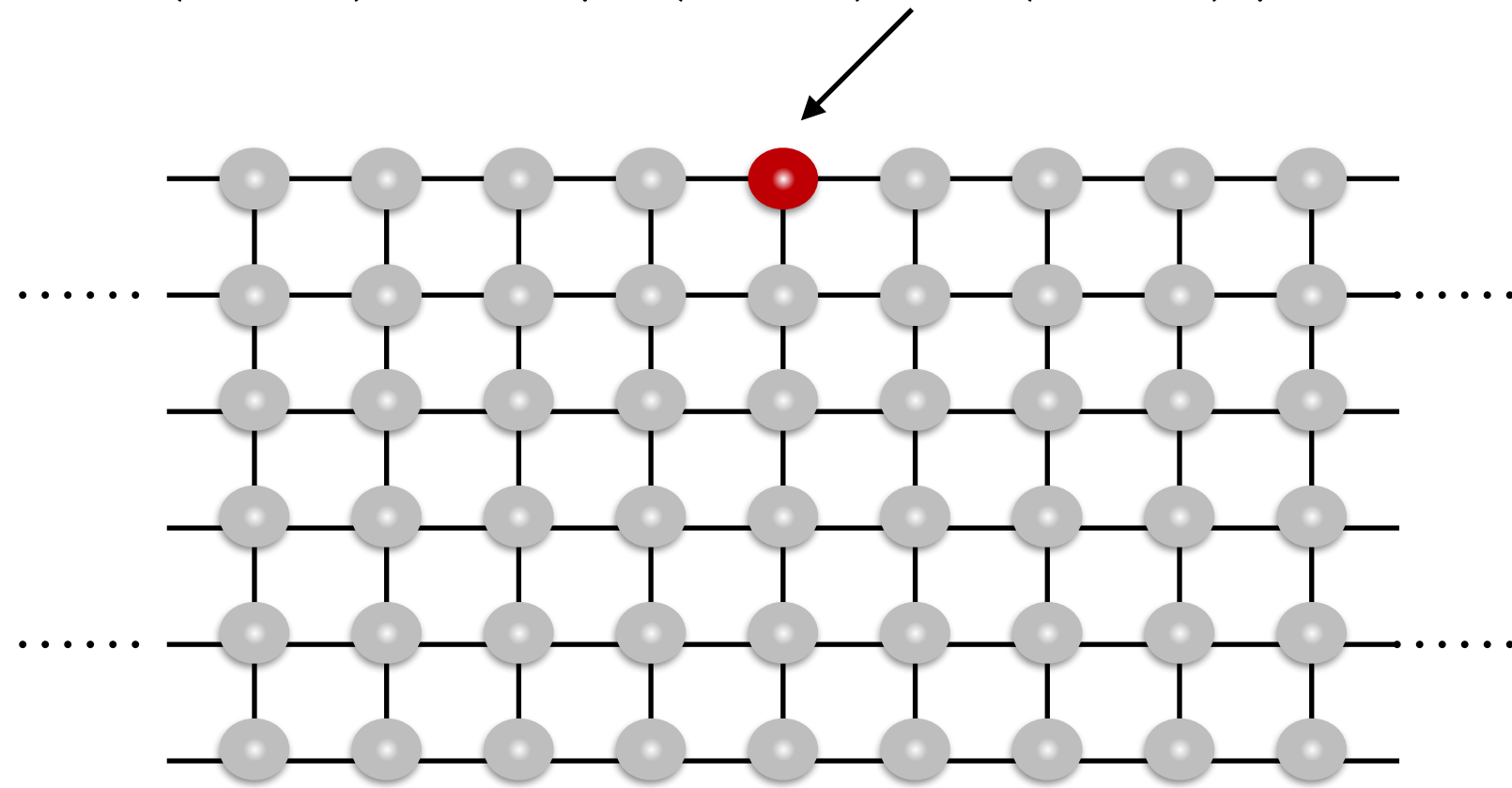
$$G(x, t) = \langle b(x, t) b^\dagger(0, 0) \rangle$$



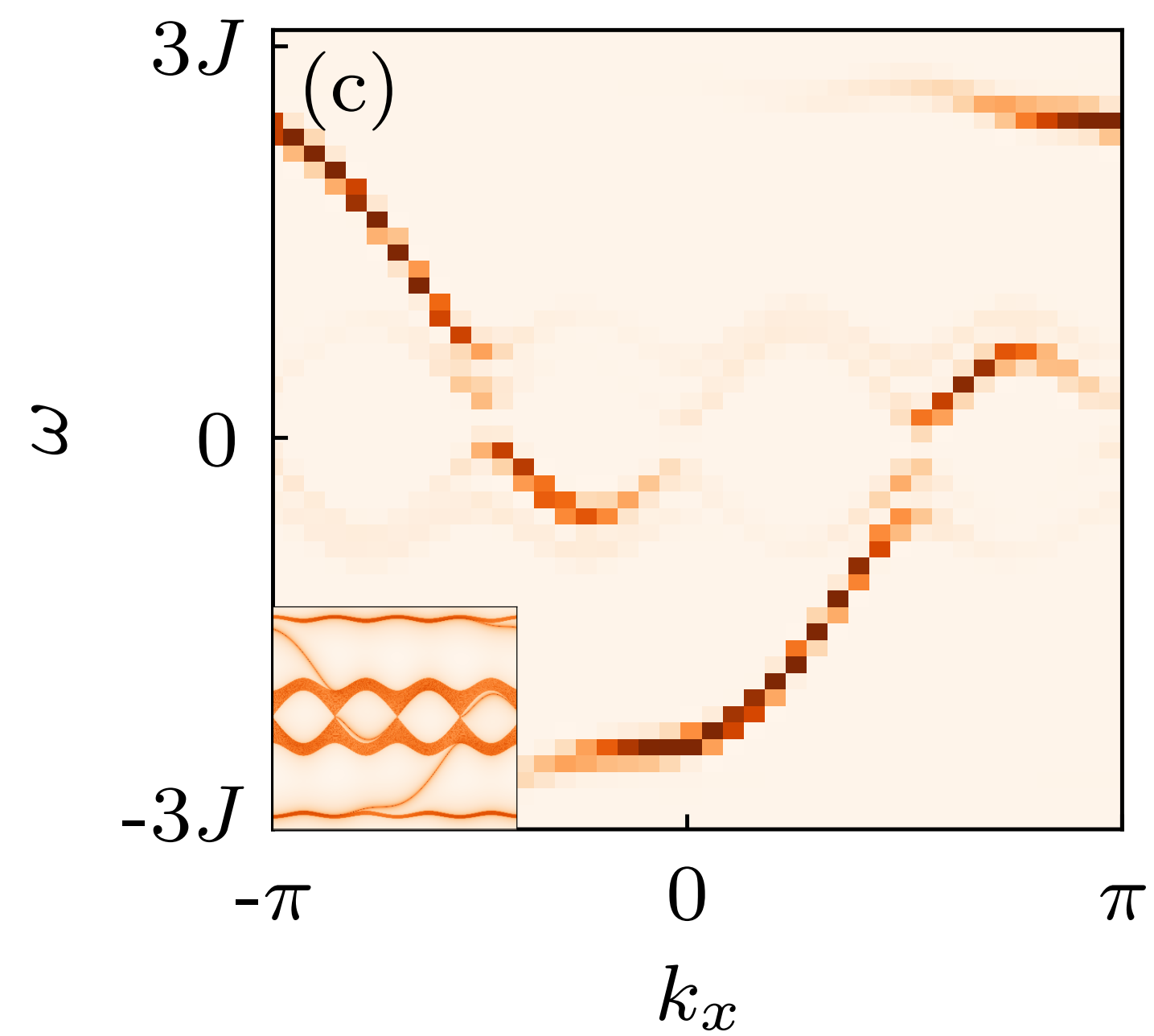
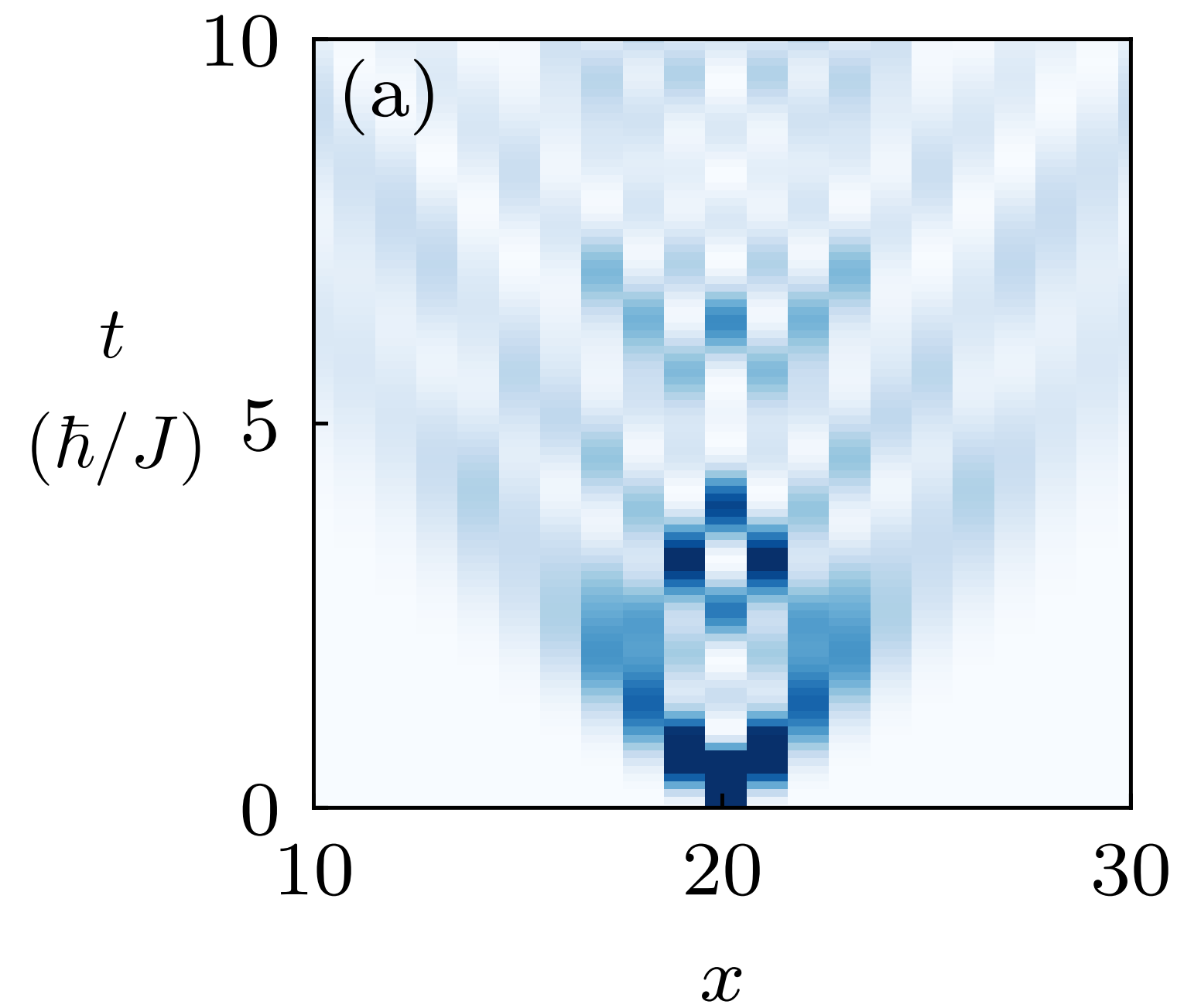
# Dynamical signatures of the FCI phase

## Dynamical correlation function

$$G(x, t) = \langle b(x, t) b^\dagger(0, 0) \rangle$$



Single particle



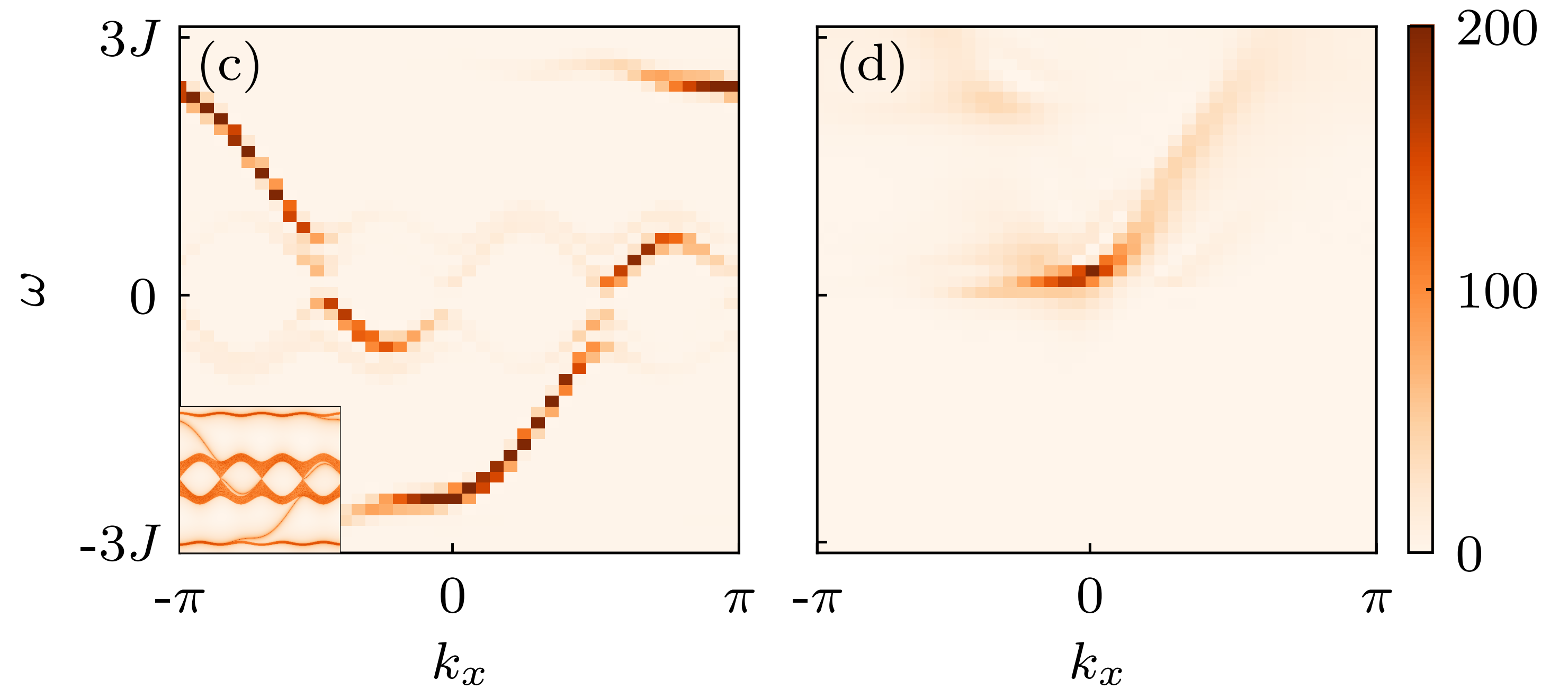
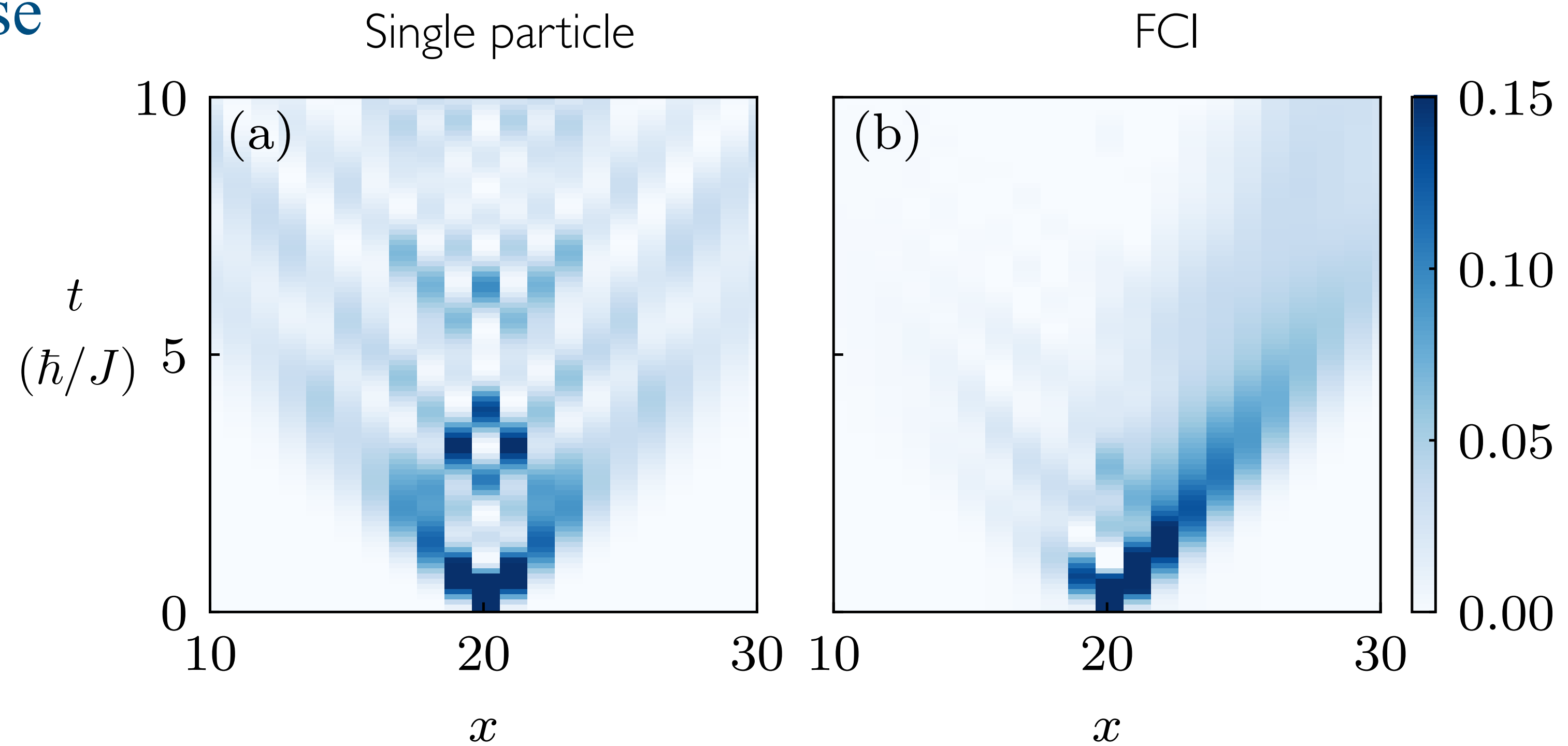
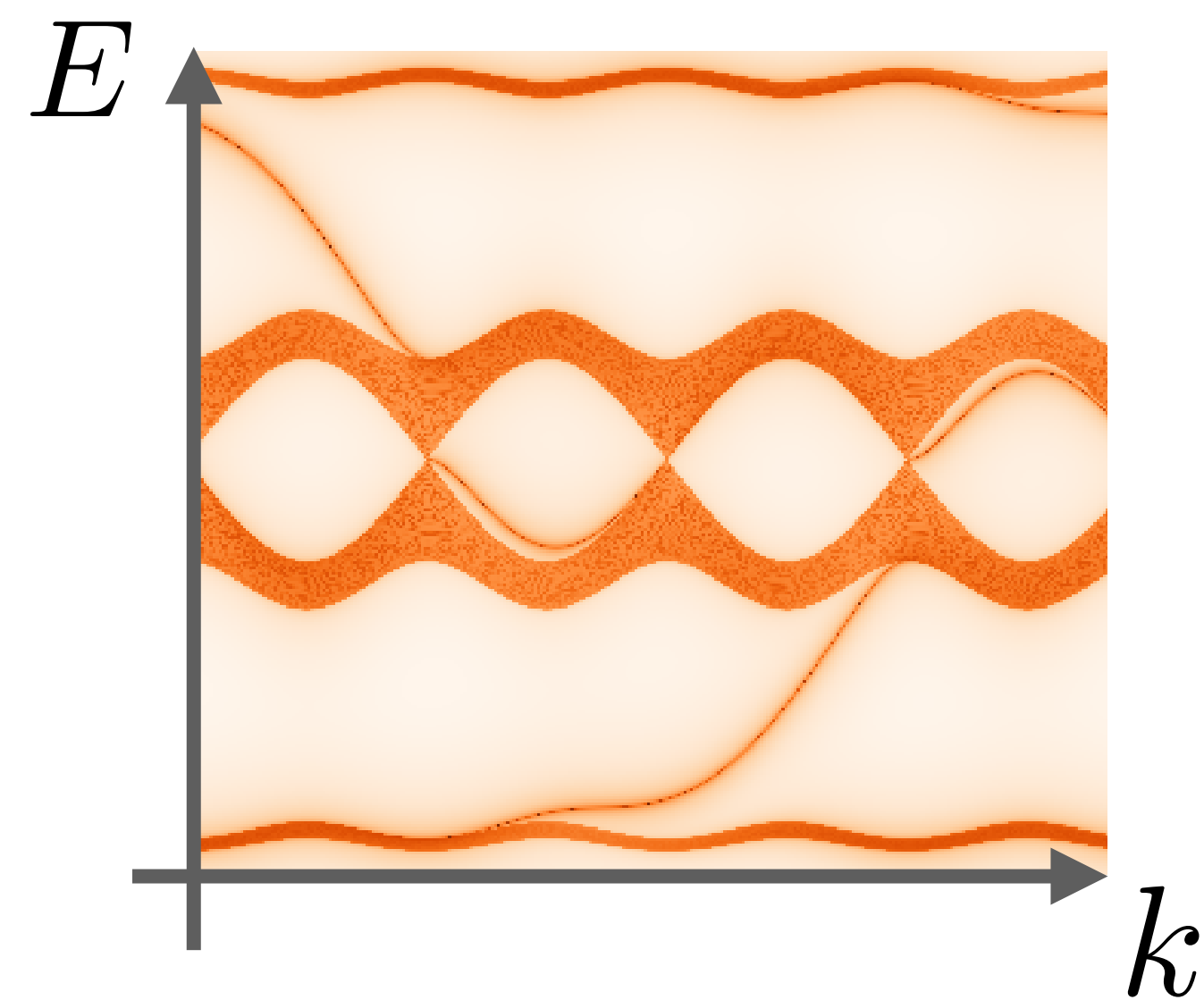
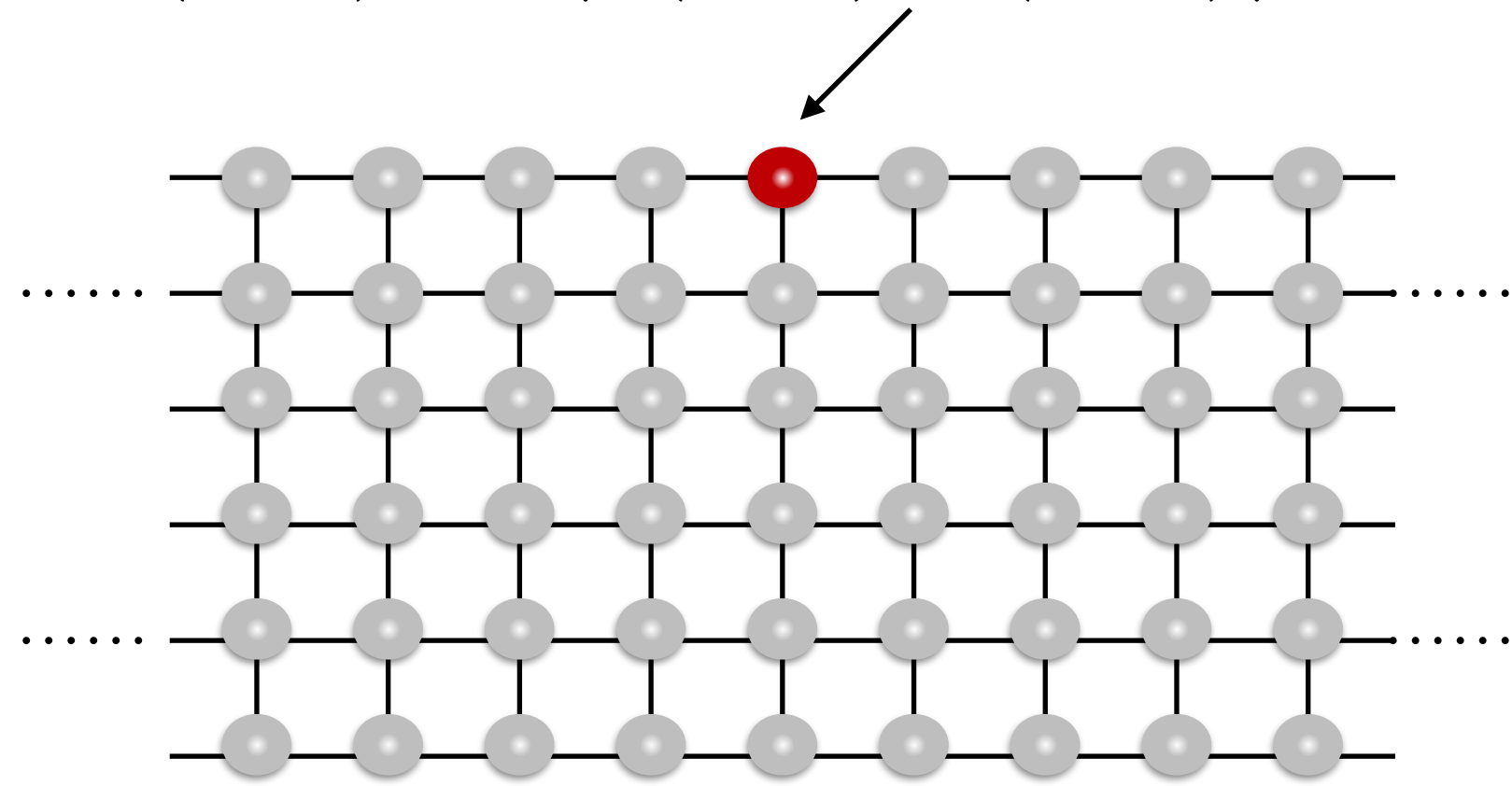
$k_x$



# Dynamical signatures of the FCI phase

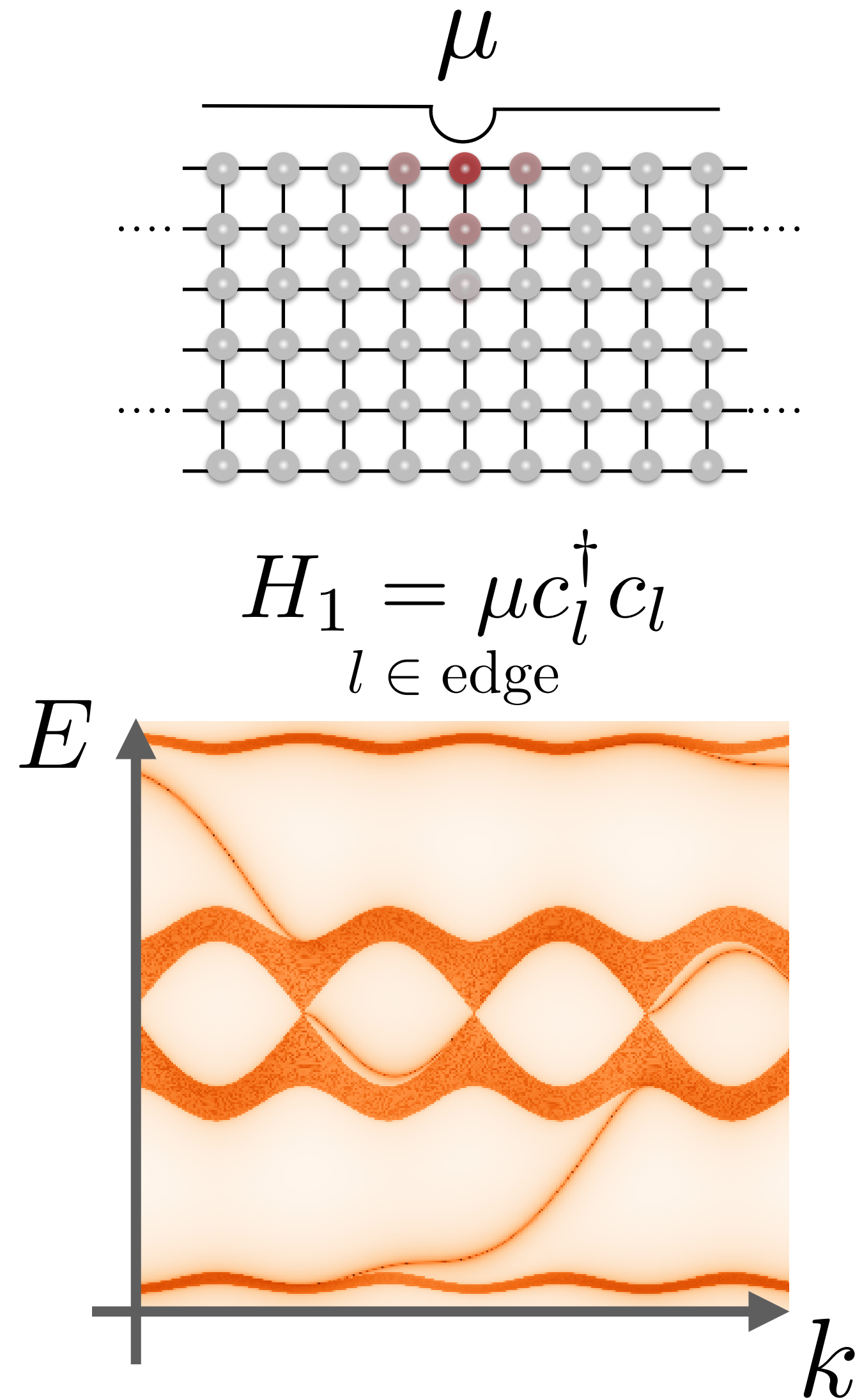
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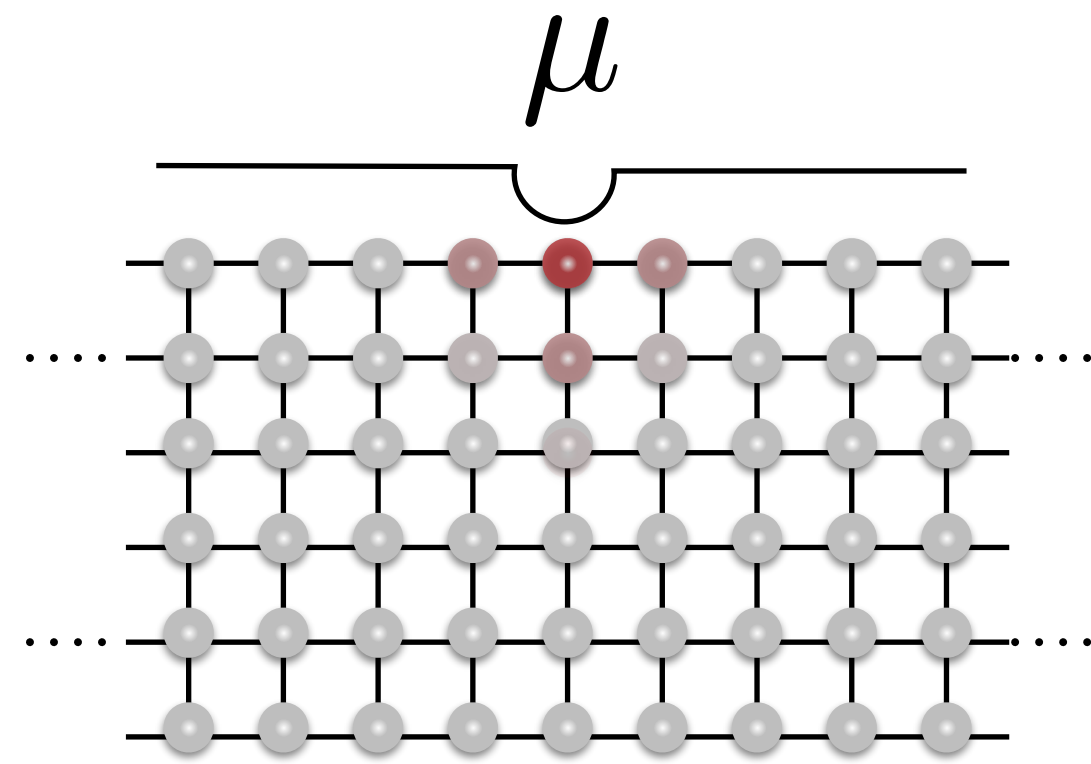
# Dynamical signatures of the FCI phase

Fourier transformation of the density evolution following local quench:



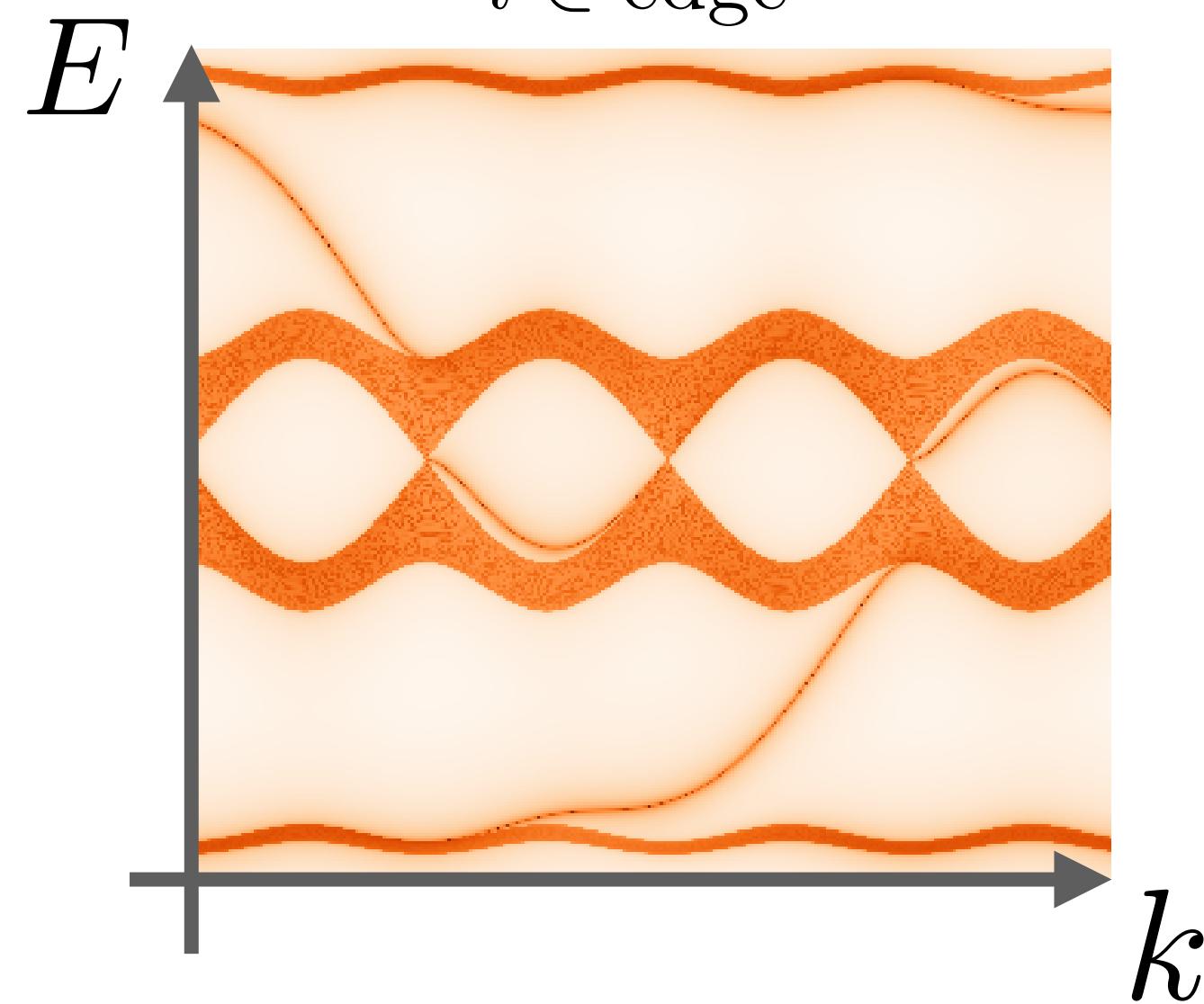
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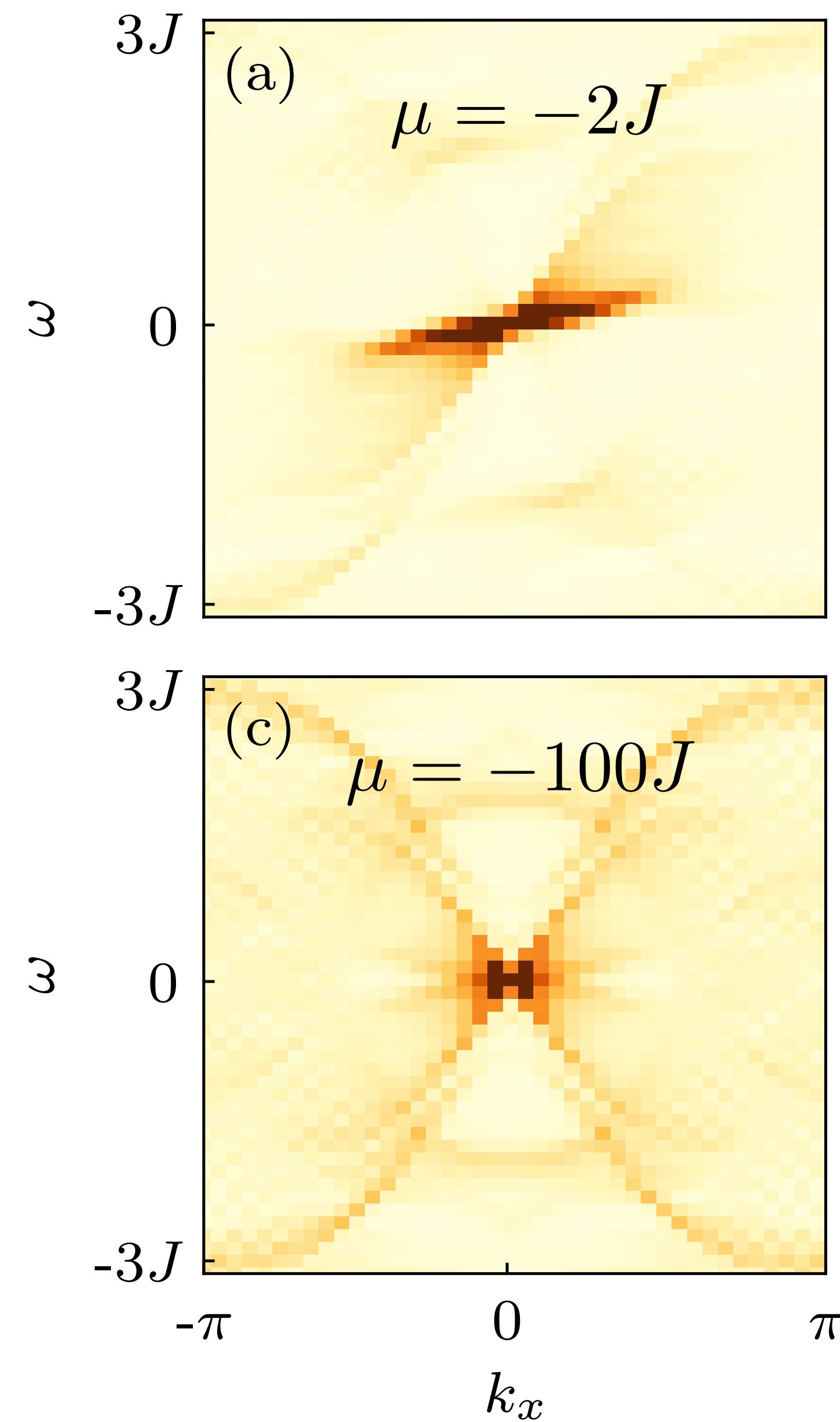


$$H_1 = \mu c_l^\dagger c_l$$

$l \in \text{edge}$



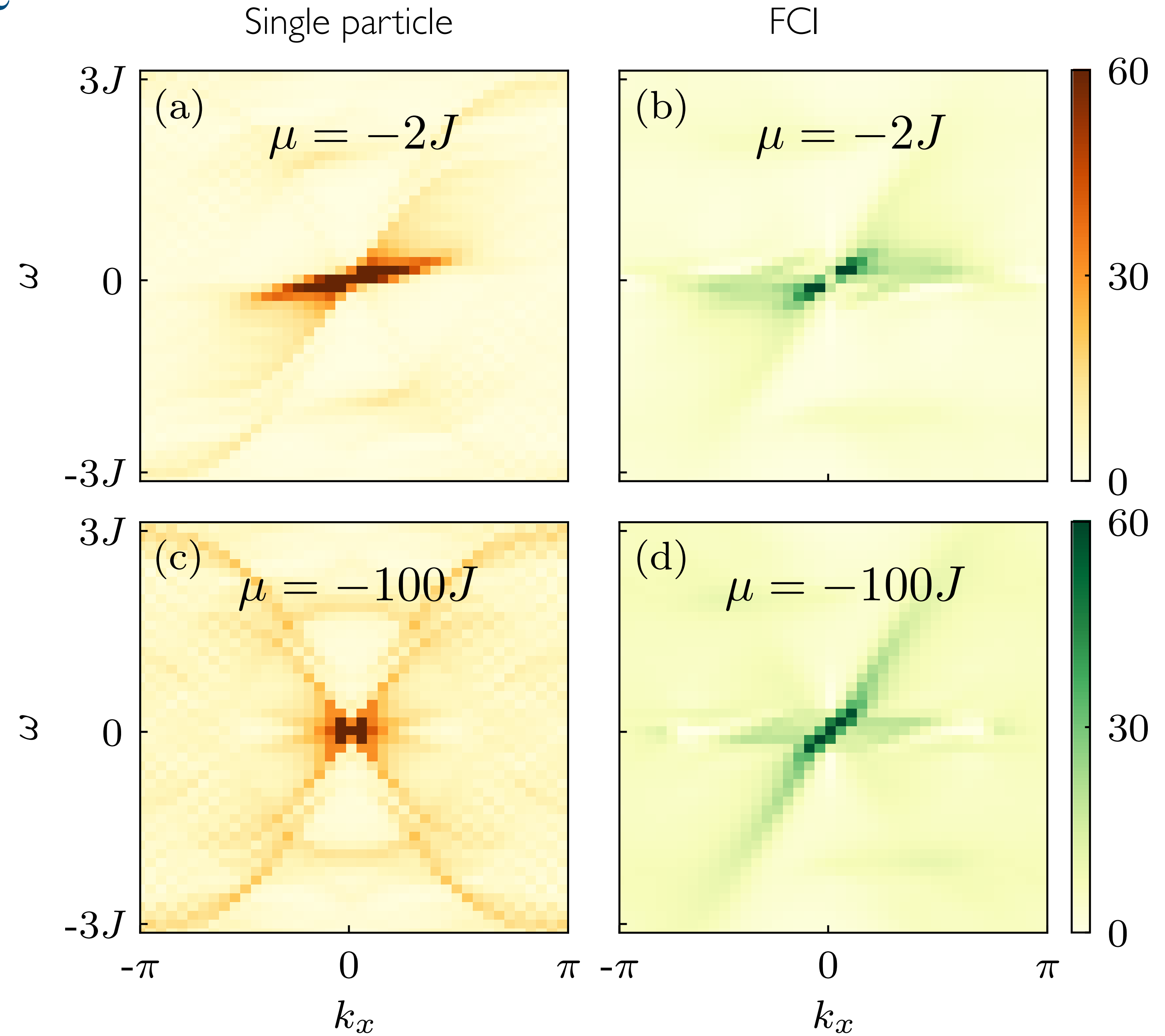
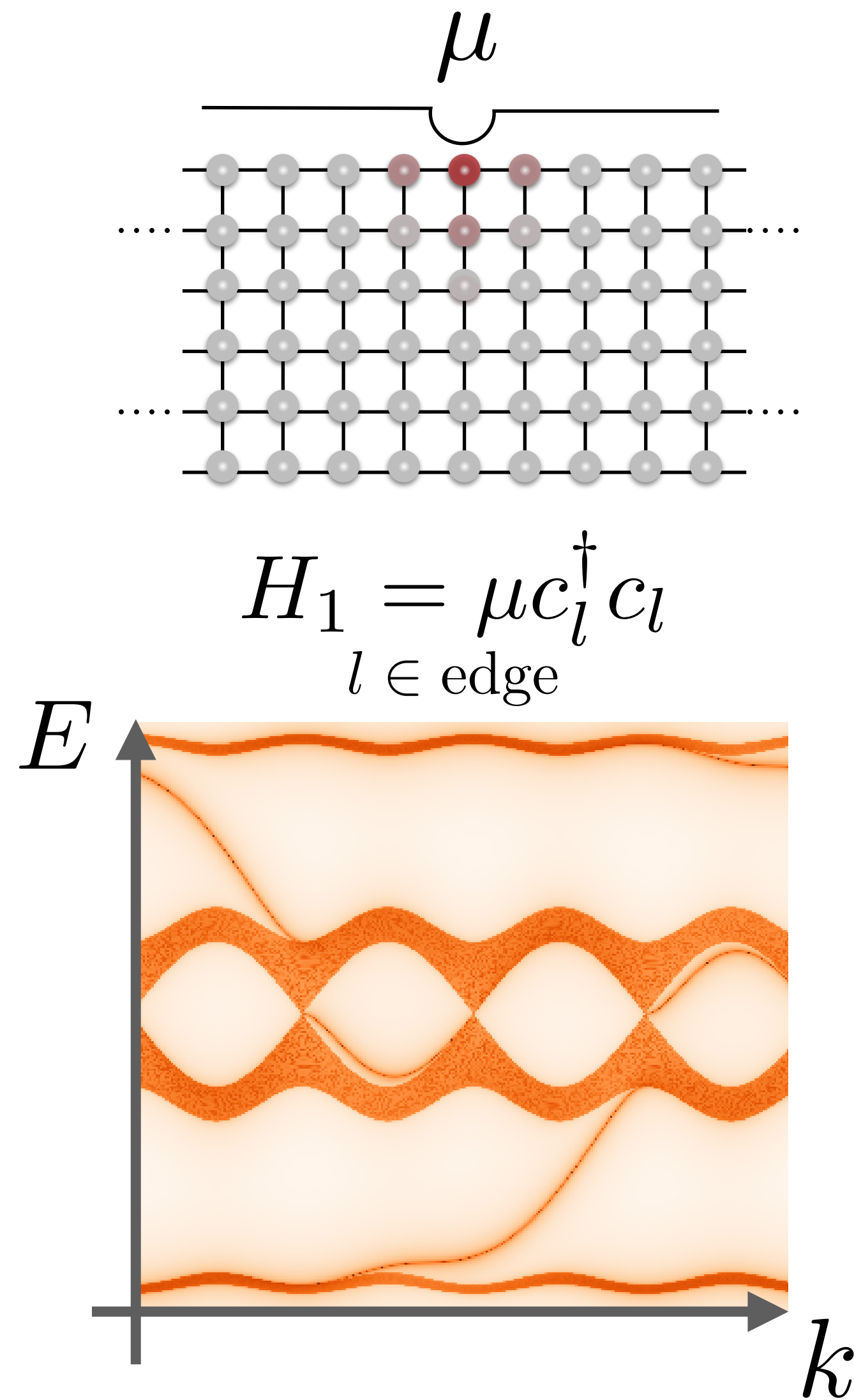
Single particle





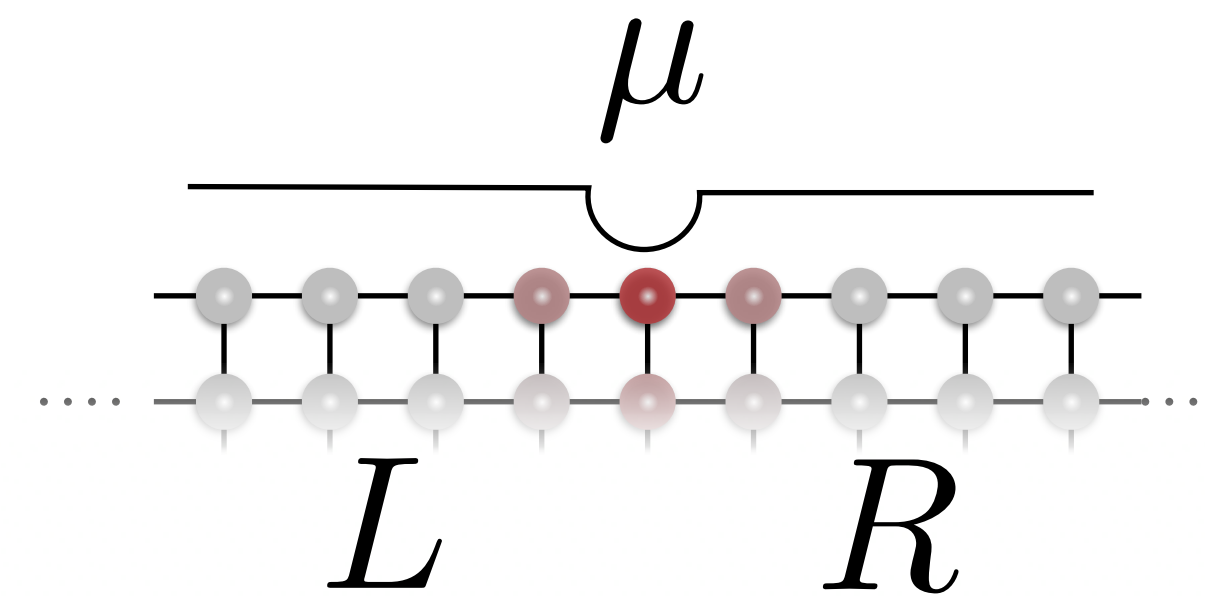
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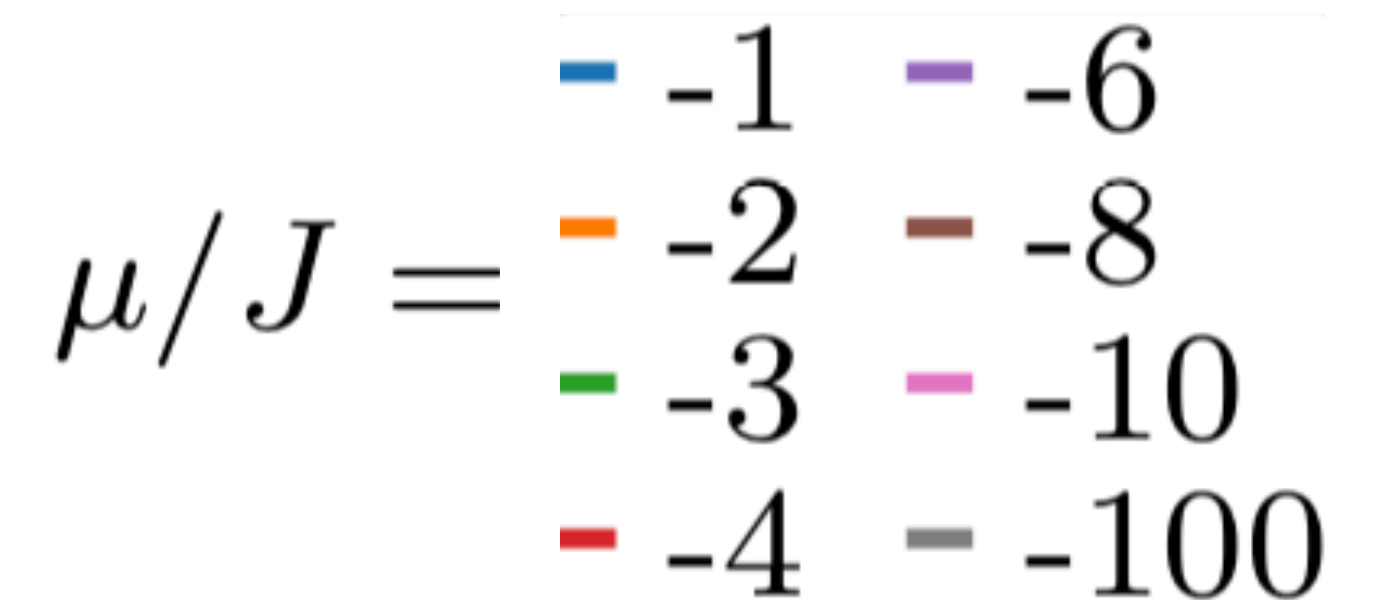
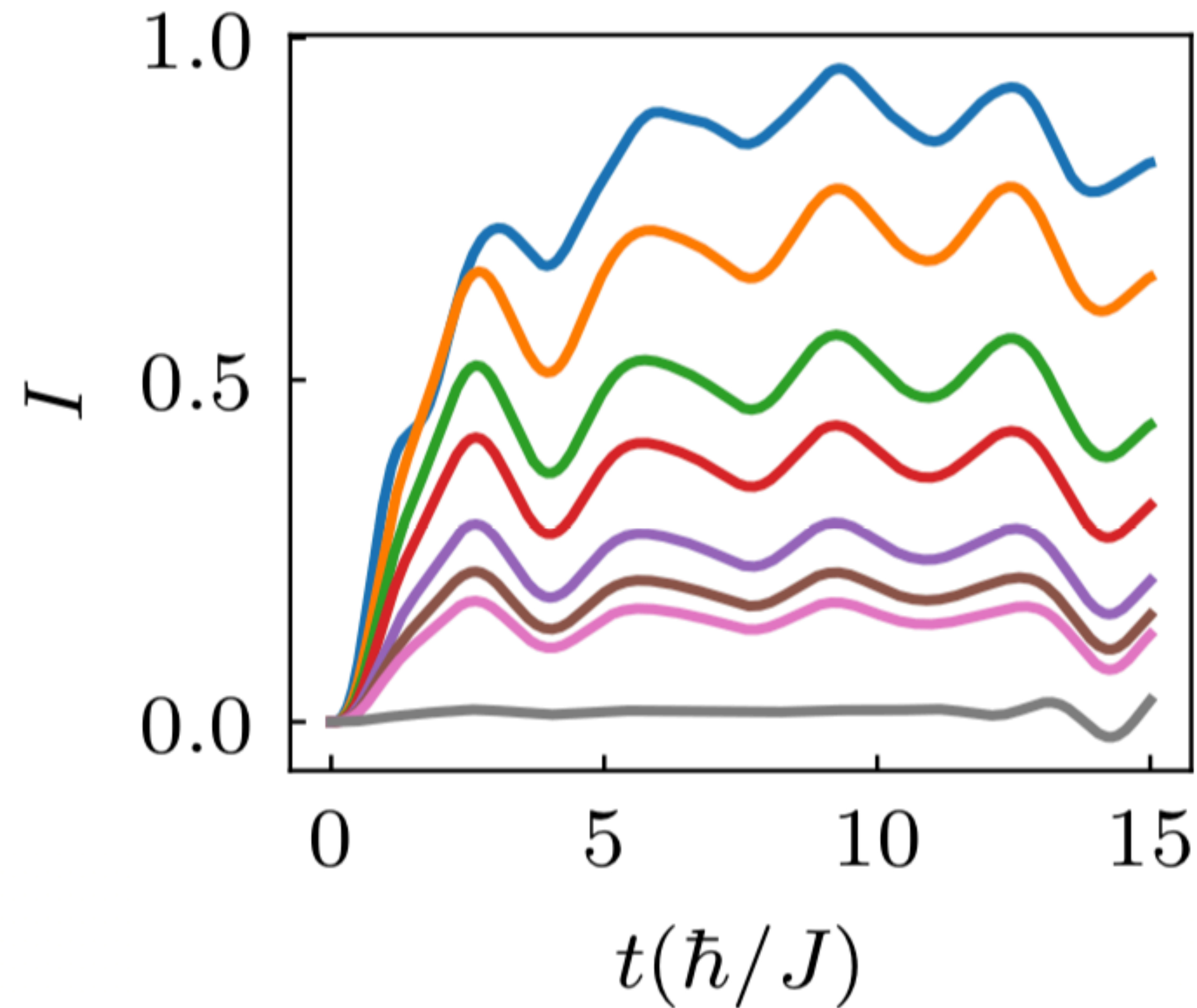


# Dynamical signatures of the FCI phase

The time evolution of the imbalance :  $I = N_R - N_L$

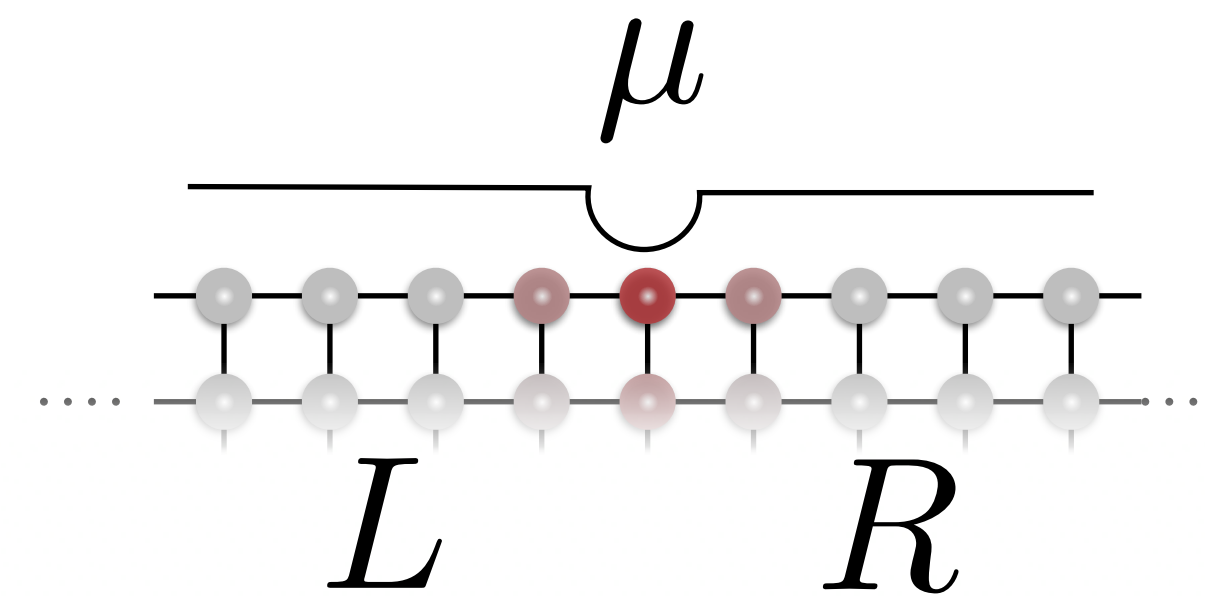


Single particle

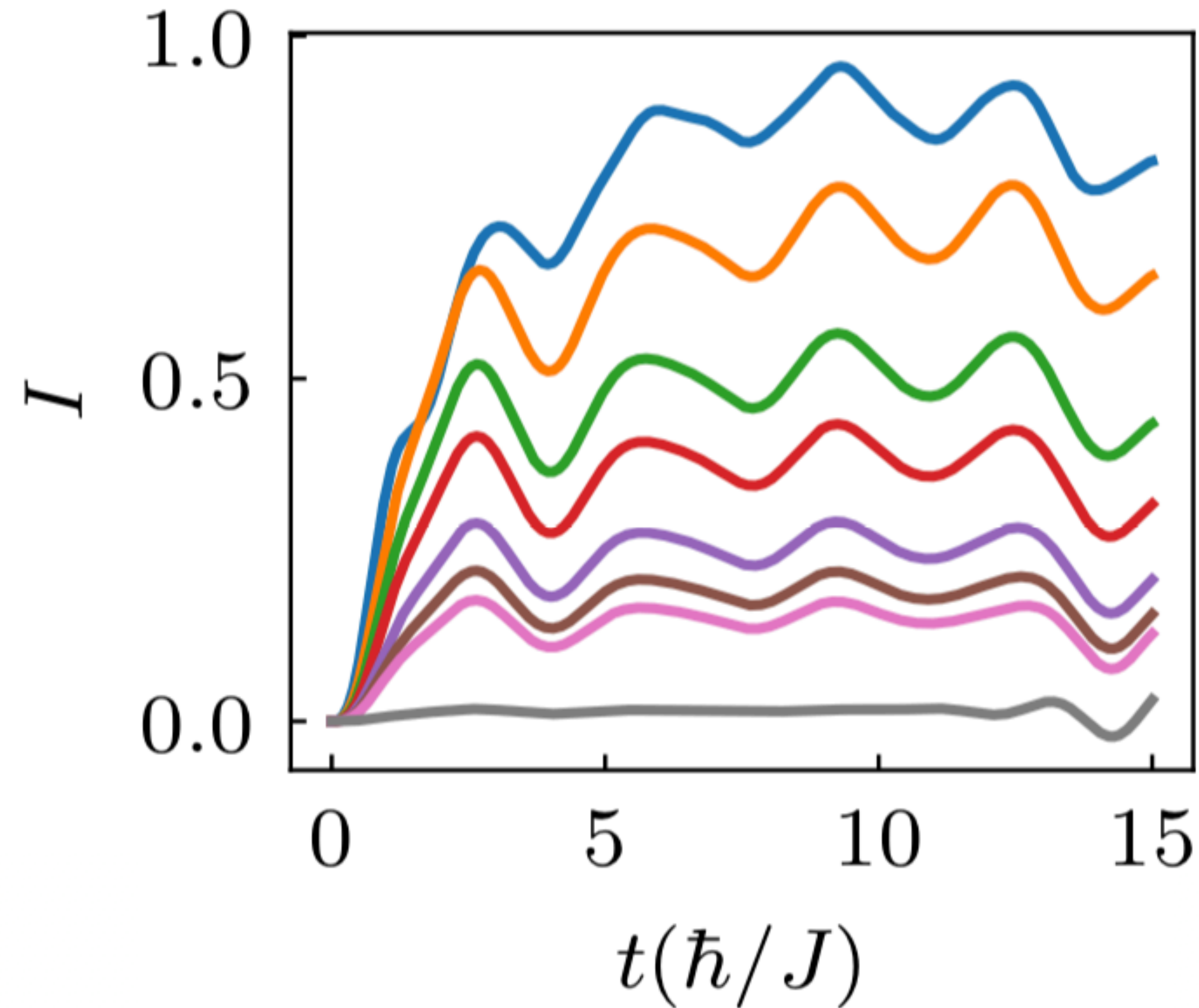


# Dynamical signatures of the FCI phase

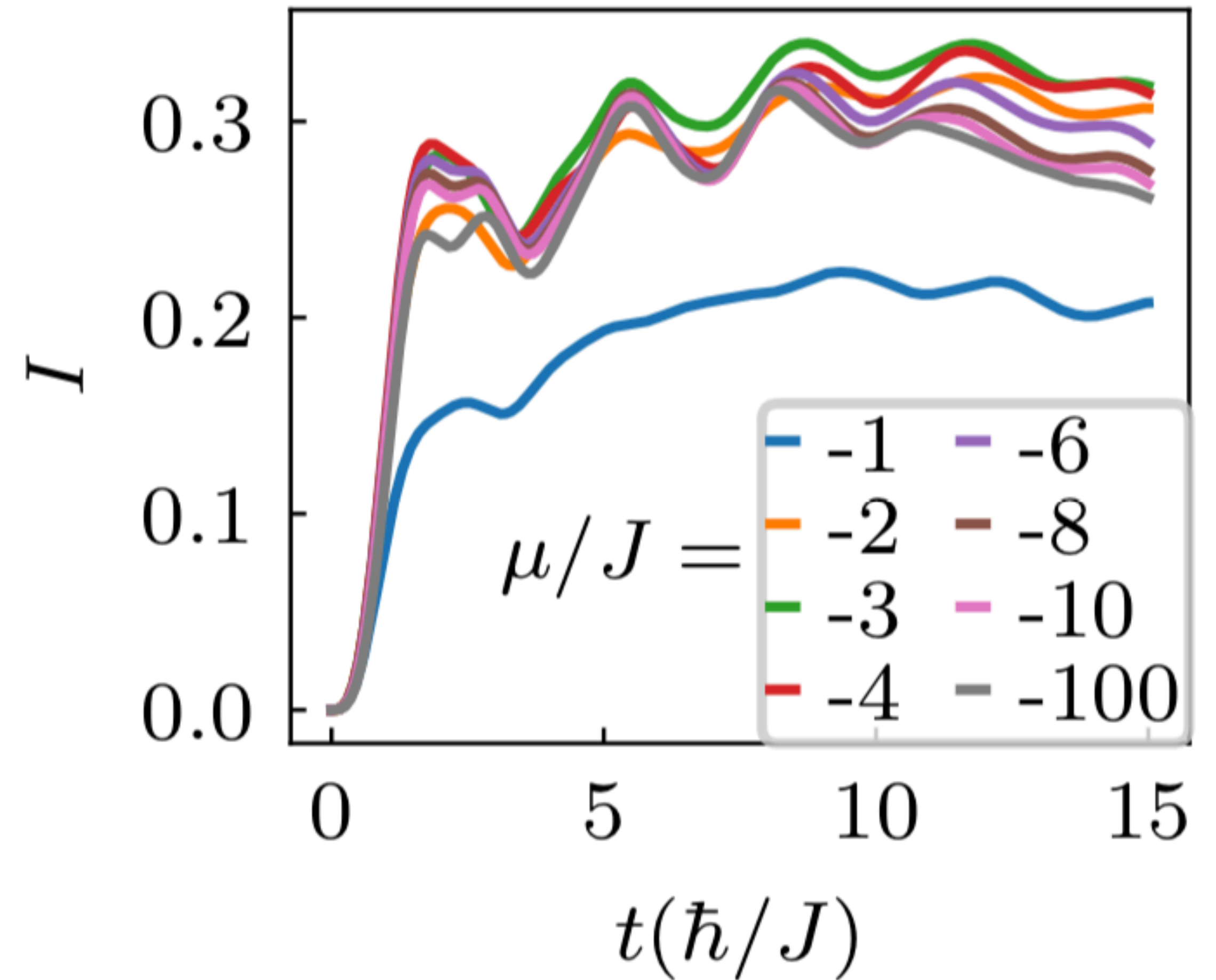
The time evolution of the imbalance :  $I = N_R - N_L$



Single particle



FCI





# Edge state dynamics of a quantum Hall edge

Edge state spectral function knows about fractional excitations

For a state with  $\sigma_{xy} = \frac{1}{m} \frac{e^2}{h}$

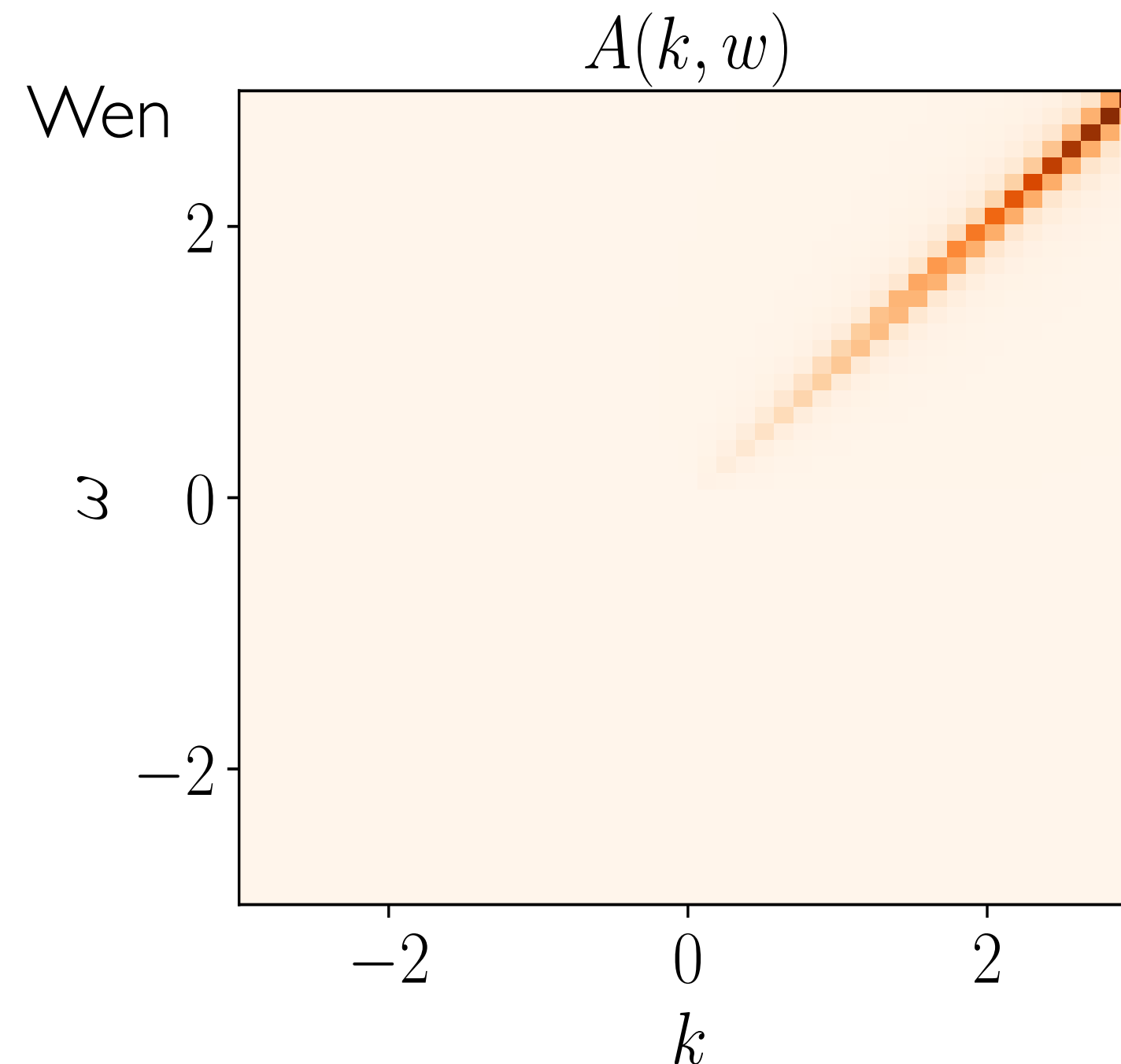
The spectral function is\*  $A(k, \omega) \propto (\omega + vk)^{m-1} \delta(\omega - vk)$

and the DOS is  $N(\omega) \propto \omega^{m-1}$

\*Assumptions: thermodynamic limit of a 1D isolated edge



X. G. Wen PRB (1990)



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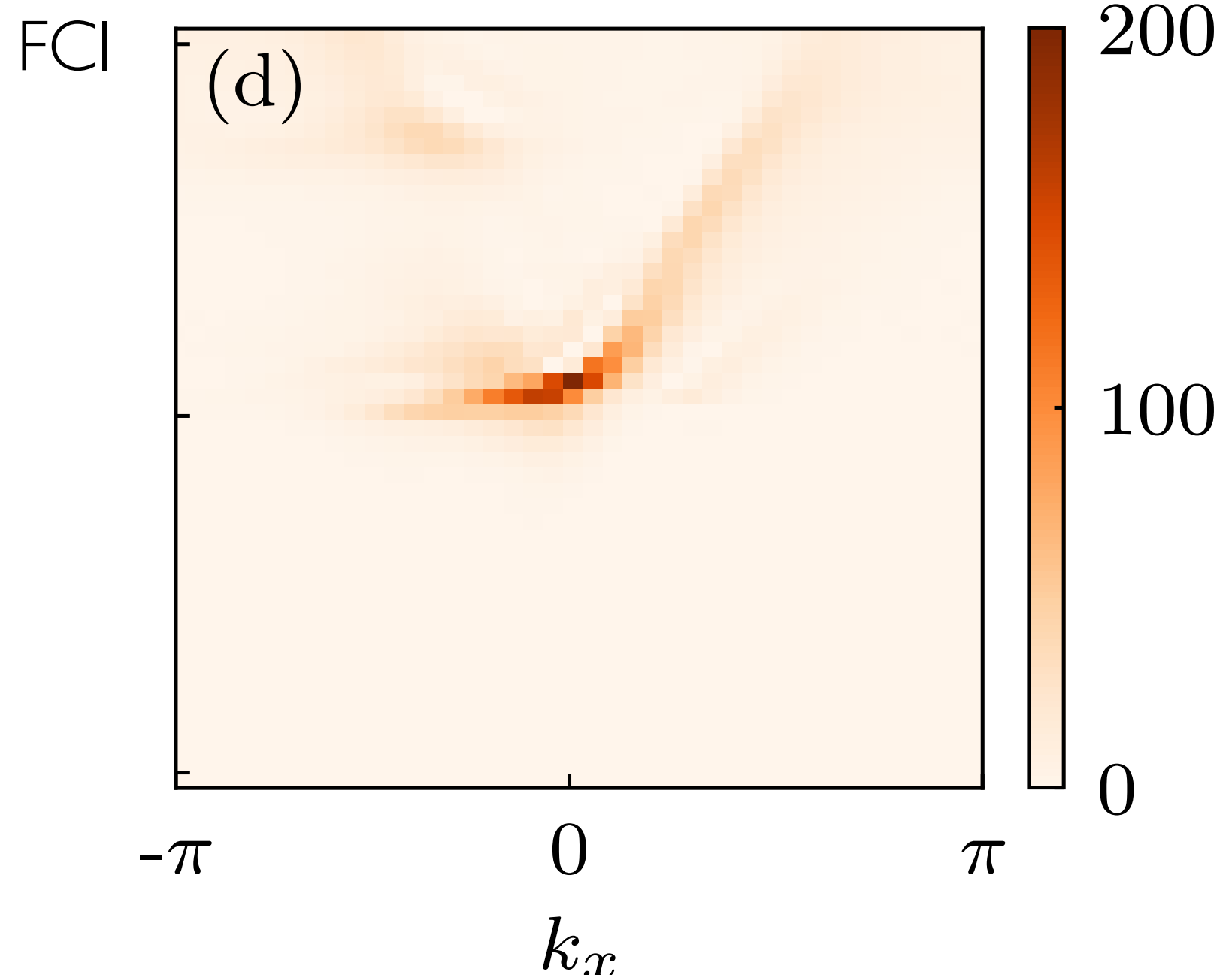
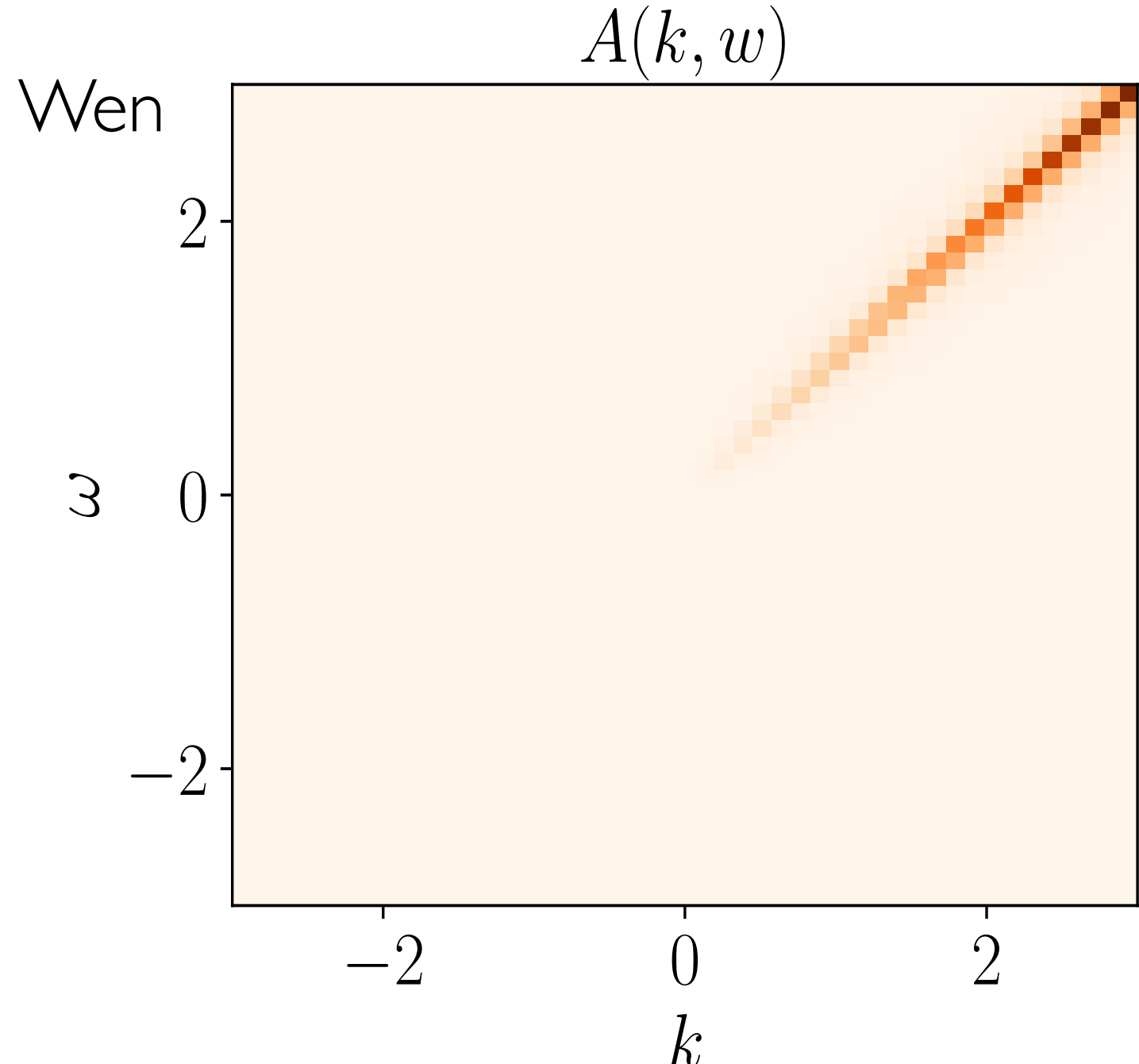
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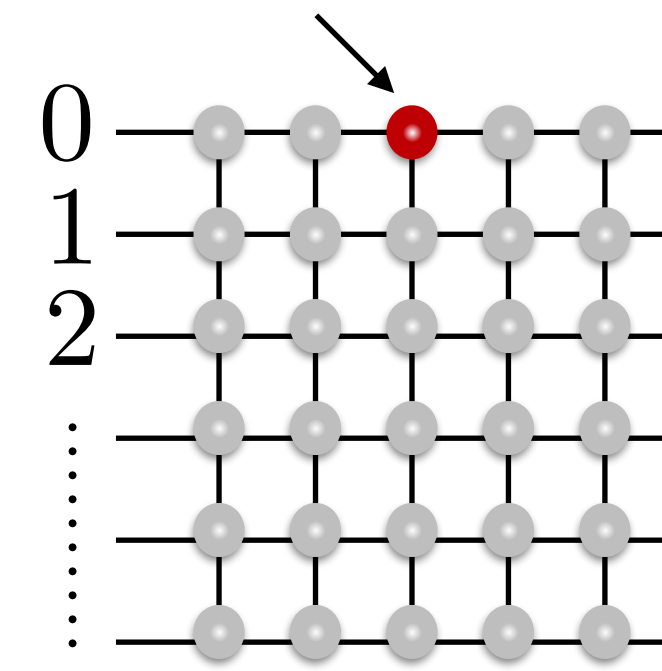


X. G. Wen PRB (1990)

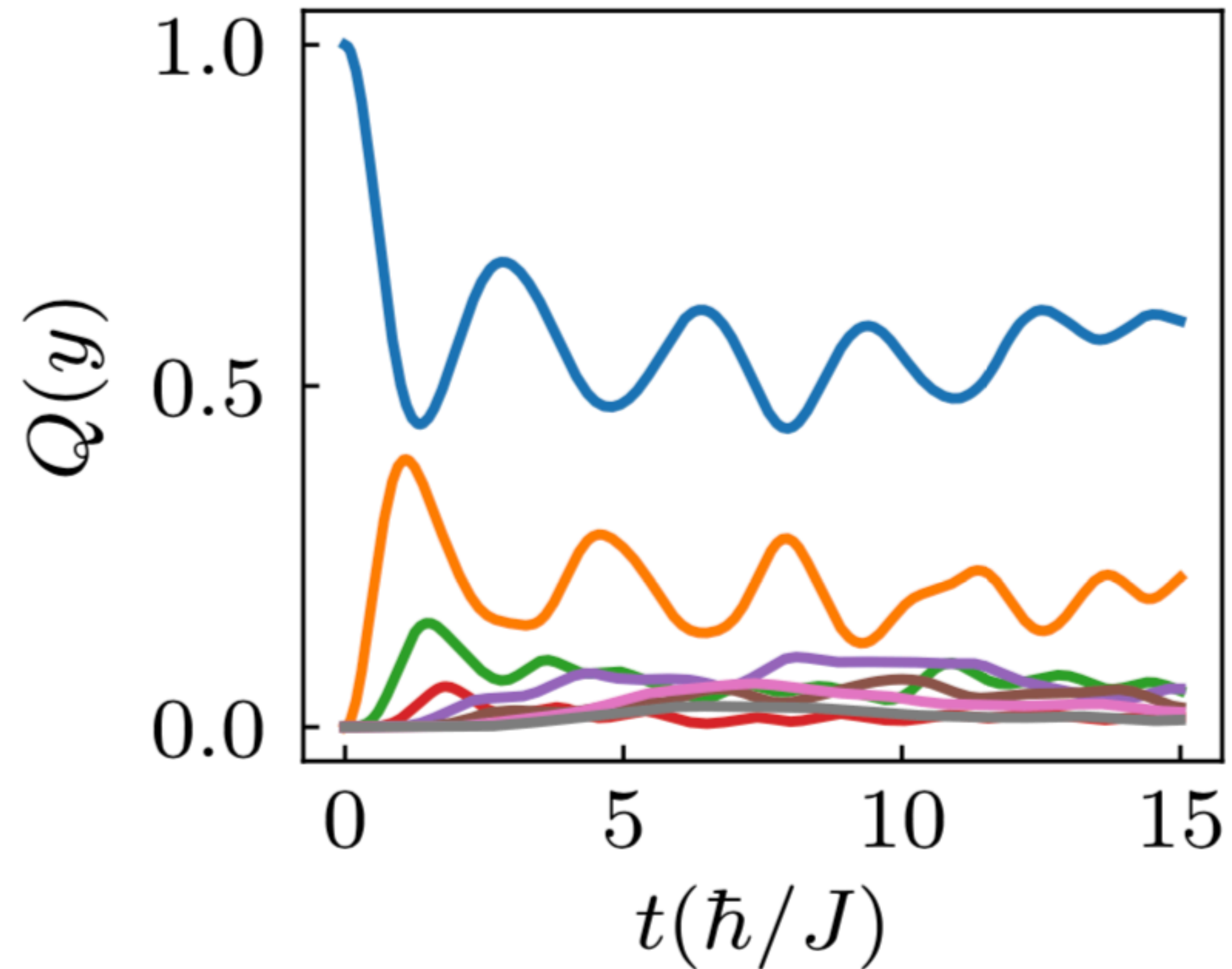


# Dynamical signatures of the FCI phase

The edge does not behave as an isolated Luttinger liquid:



Single particle



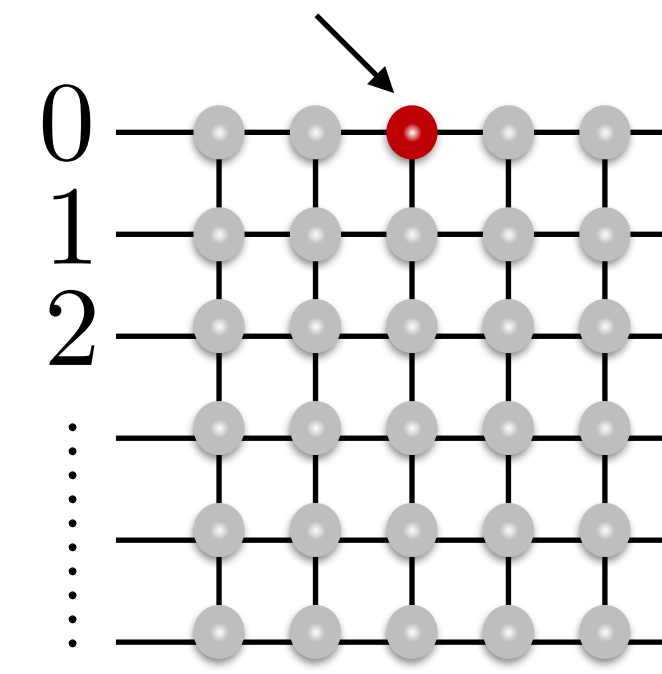
row =

— 0	— 2	— 4	— 6
— 1	— 3	— 5	— 7

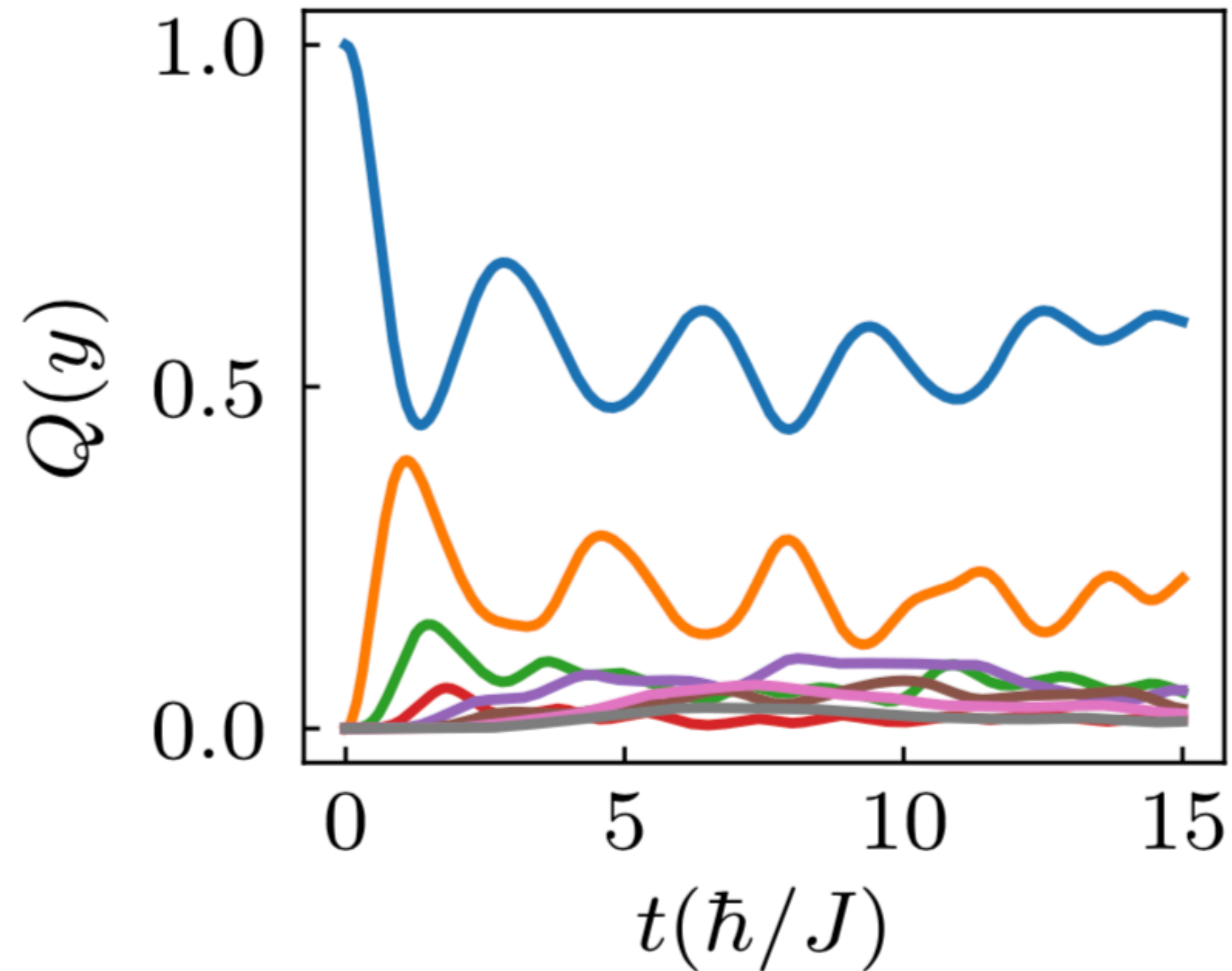


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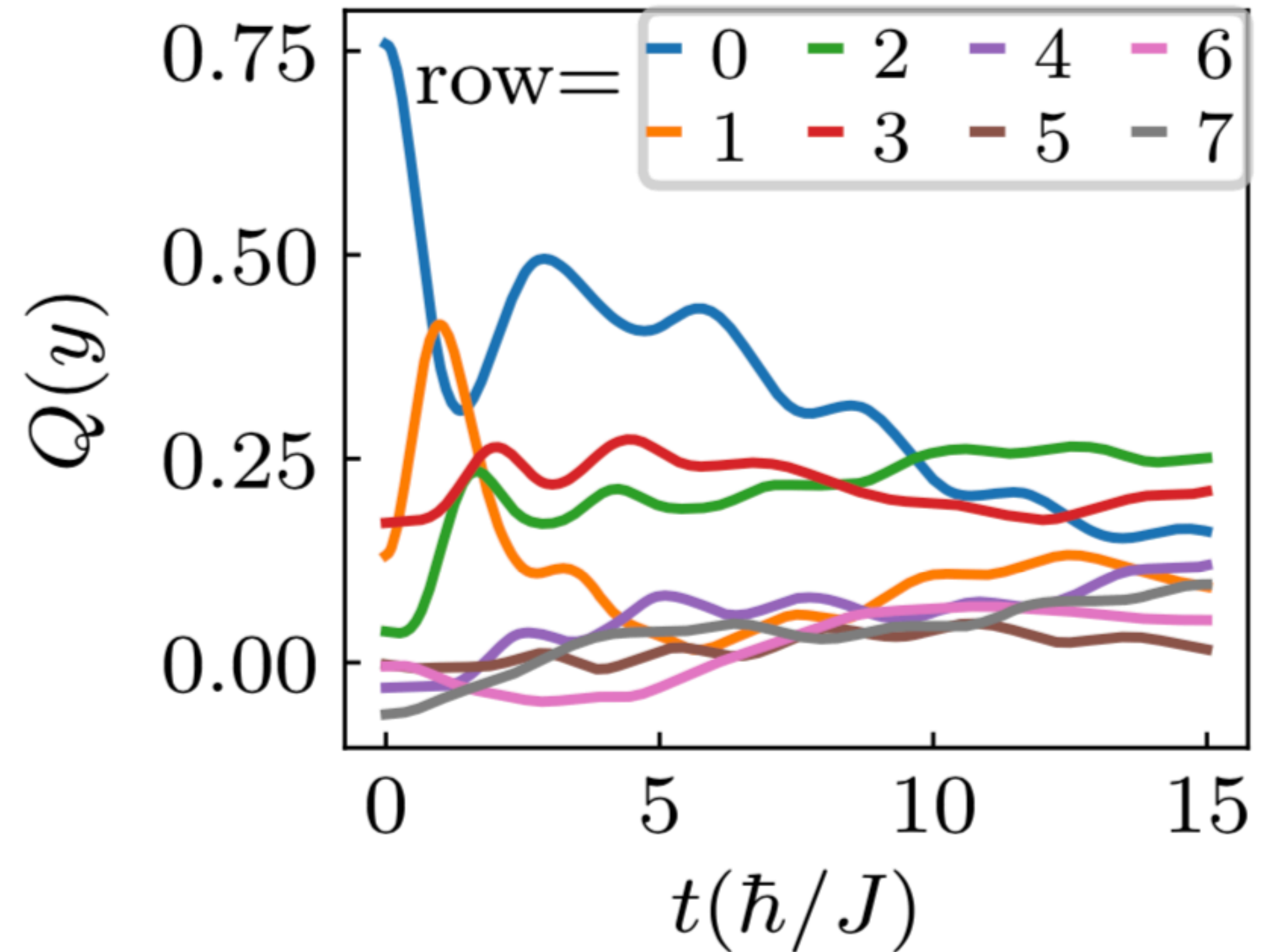
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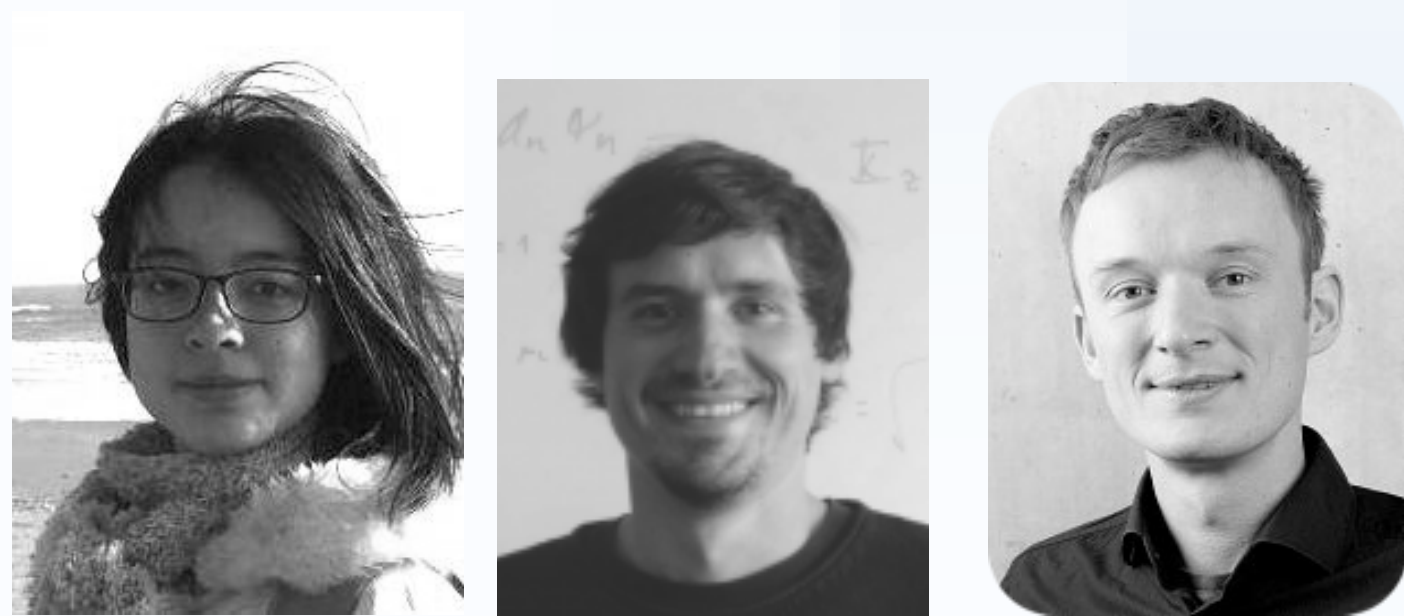


Harper-Hofstadter

# Edge state **dynamics** of a bosonic fractional Chern insulator

is chiral but not that of an isolated Luttinger liquid.

It is insensitive to the strength of a perturbation, unlike the Chern insulator



X. Y. Dong, AGG, J. Motruk, F. Pollmann, Phys Rev. Lett. (2018)





# Dynamical signatures of the Chern insulator

