SU(4) topological RVB spin liquid on the square lattice

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OG, S. Capponi and D. Poilblanc, arXiv:1901.05905

Overview

Resonating Valence Bond physics

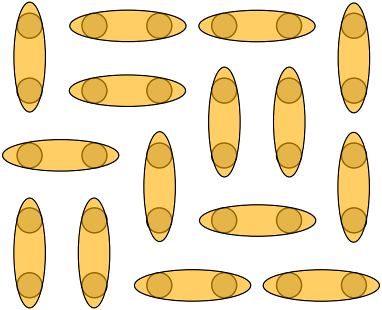
SU(N) symmetric tensors

Tensors Networks algorithms

Results on RVB SU(4) wavefunctions

Conclusion

Resonating Valence Bond state

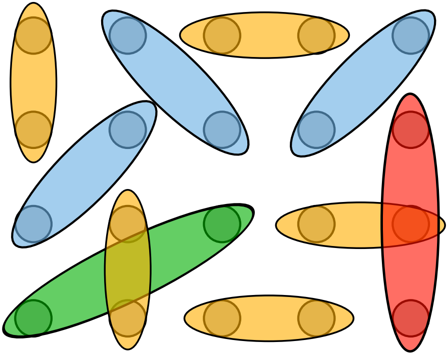


$$|RVB\rangle = \sum_C |C\rangle$$

- attempt to describe high-T_c superconductivity
- quantum spin liquid
- critical state with algebraic decaying of dimer-dimer correlations.

P. W. Anderson, Science **235**, 1196 (1987)

More general dimers covering states

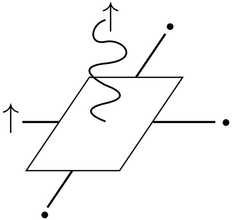


$$|\Psi\rangle = \sum_{C'} \alpha(C')|C'\rangle$$

- always a spin liquid
- can be either critical or gapped.

SU(N) symmetric tensors

- Choose physical variable S
- Choose virtual space V
- Take projector $\mathcal{P} : V^{\otimes 4} \to S$
- tensor of Clebsch-Gordan coefficients



Ex: RVB tensor

$$S = \mathbf{2}$$

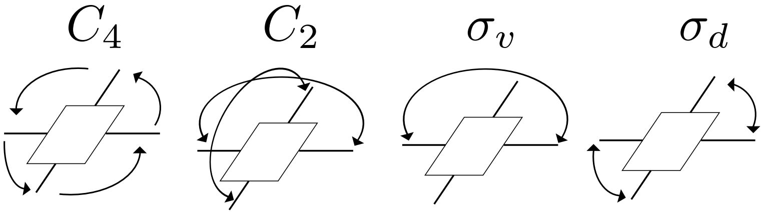
$$V = \mathbf{2} \oplus \mathbf{0}$$

$$A_{\uparrow}[\uparrow, 0, 0, 0] = 1$$

$$A_{\downarrow}[\downarrow, 0, 0, 0] = 1$$

M. Mambrini, R. Orús and D. Poilblanc, PRB **94**, 205124 (2016).

C_{4v} character table



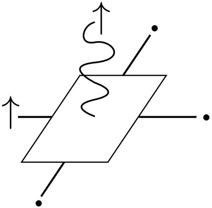
irrep	dim	$2\ C_4$	C_2	$2\sigma_v$	$2\sigma_d$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
B_1	1	-1	1	1	-1
B_2	1	-1	1	-1	1
E	2	0	-2	0	0

Simple cooking recipe: $D^4 \to d$

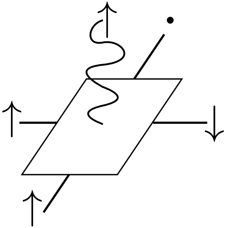
- take $SU(N)$ Hamiltonian, e.g. $\mathcal{H} = ||\sum_i \mathbf{S}_i||^2$.
- codiagonalize $\mathcal{H}$ with S^z and local symmetries operators σ
- if degeneracies, choose orthogonal highest weights and apply lowering operators
- take $(D^4 \times D^4)$ transfer matrix U
- select d desired columns according to $\mathcal{H}$ and σ eigenvalues: $(D^4 \times d)$ projector
- **ensure same eigenvectors phases**
- reshape projector as (d, D, D, D, D) tensor
- enjoy with vanilla ice cream

generalized SU(2) RVB tensors

$$(\mathbf{2} \oplus \mathbf{1})^4 = 12 \cdot \mathbf{2} \oplus \ldots$$



$1A_1$
$1B_1$
$1E$



$1A_1 \; 1A_2$
$1B_1 \; 1B_2$
$2E$

Transition critical $\rightarrow$ gapped state.

J.-Y. Chen and D. Poilblanc, PRB **97**, 161107 (2018)

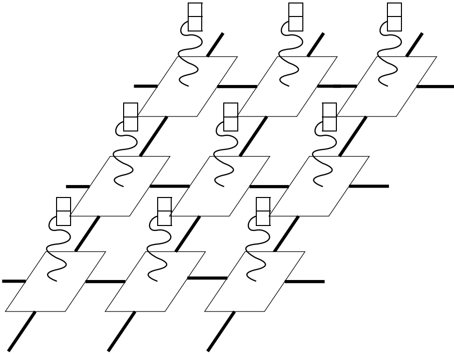
SU(4) irrep **6**

- irrep of two SU(4) fermions in the fundamental representation
- self-conjugate:

$$\ydiagram{1,1} \otimes \ydiagram{1,1} = \bullet \oplus \ydiagram{2,1,1} \oplus \ydiagram{2,2}$$

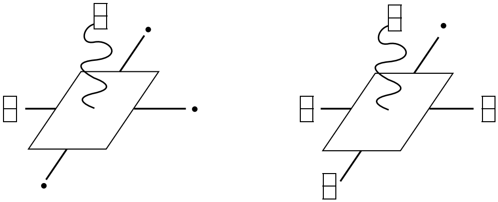
$$S = \ydiagram{1,1} \Rightarrow d = 6$$

$$V = \ydiagram{1,1} \oplus \bullet \Rightarrow D = 7$$



SU(4) RVB tensors

$$\left(\yng(1,1) \oplus \bullet \right)^{\otimes 4} = 16 \cdot \yng(1,1) \oplus \dots$$



$1A_1 : T_0$

$(+1B_1 + 1E)$

$2A_1 : T_1, T_2$

$1A_2 : T_3$

$(+2B_1 + 1B_2 + 3E)$

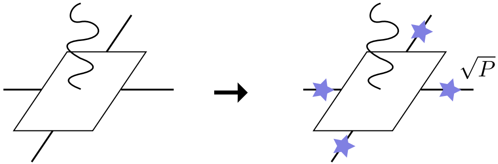
$$A = a_0 T_0 + a_1 T_1 + a_2 T_2 + i a_3 T_3$$

rotation invariant!

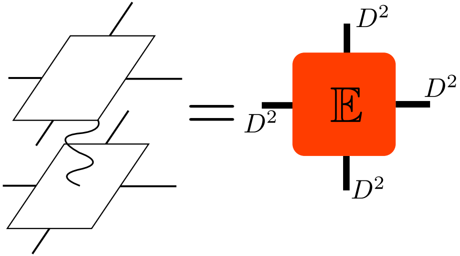
$U(1)$ symmetry breaks into $\mathbb{Z}_2$

Double-layer tensor

Add projector on the singlet on each virtual leg:

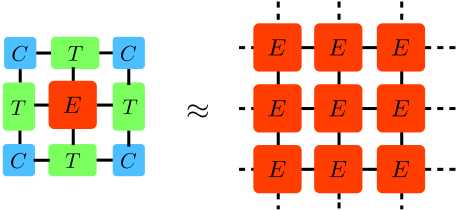


Contract physical index:

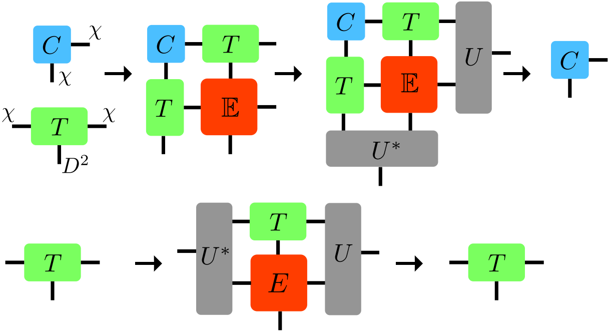


CTMRG algorithm

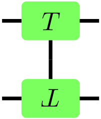
Simulate infinite environment for a tensor:



Renormalize only one corner:

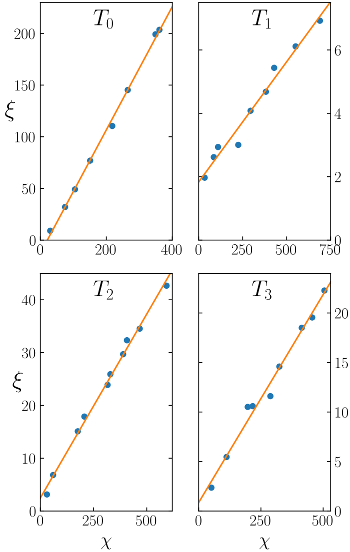


Correlation length

$\mathcal{T} =$ 

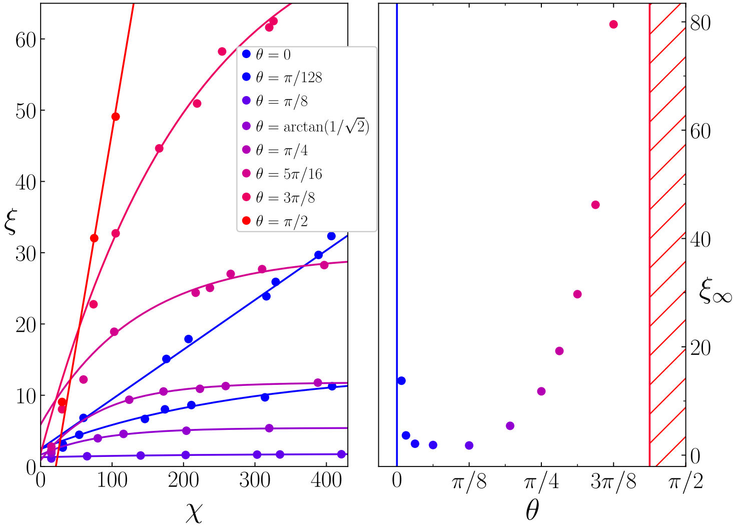
$$\xi = 1/\log(\lambda_1/\lambda_2)$$

$$\text{finite } \chi \Rightarrow \text{finite } \xi$$

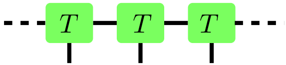


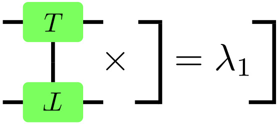
Gapped wavefunctions

$$A = \cos\theta\, T_2 + \sin\theta\, T_0$$

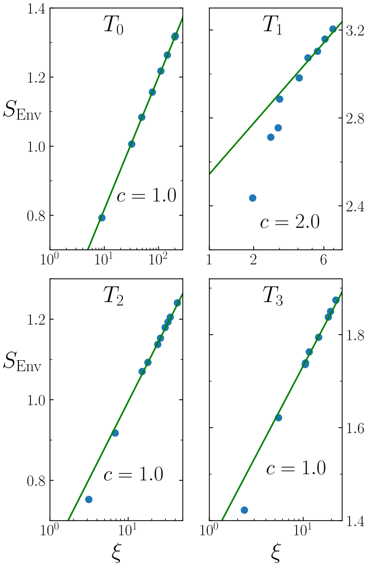


Environnement entropy and central charge

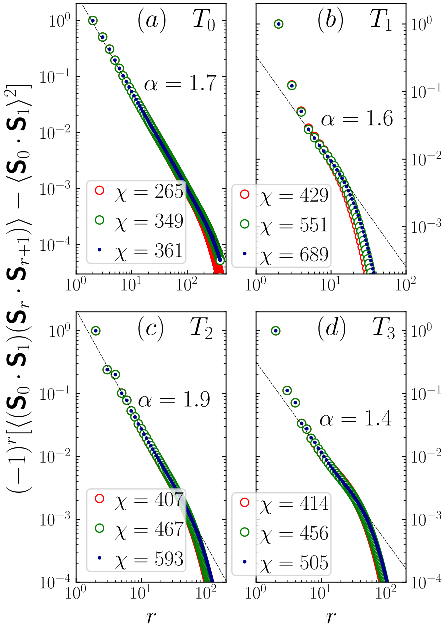
$|\Psi_{\mathrm{Env}}\rangle =$ 

$\Sigma :$ 

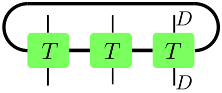
$$S_{\mathrm{Env}} = -\mathrm{Tr}\,\Sigma^2 \log \Sigma^2$$
$$= c/6 \log \xi + S_0$$



dimer-dimer correlator



Renyi entropies

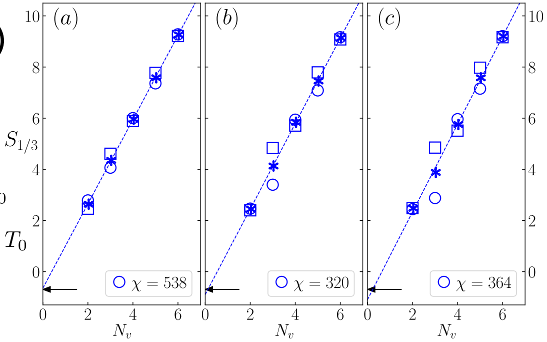
$\sigma_b =$ 

$$S_q = \frac{1}{1-q} \log \mathrm{Tr}(\sigma_b^2)^q$$

(a) $A = \cos(\pi/8) T_2 + \sin(\pi/8)\, T_0$

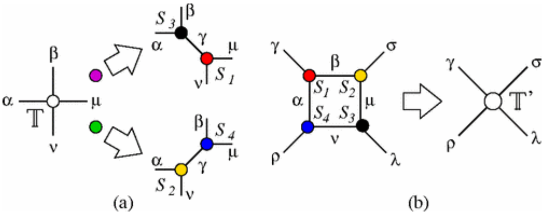
(b) $A = \cos(3\pi/8) T_2 + \sin(3\pi/8)\, T_0$

(c) $A = (T_0 + i T_3)/\sqrt{2}$



Modular matrices

Tensor Renormalization Group (TRG)

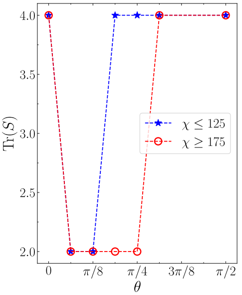


$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Figure from Z.-C. Gu et al, Phys. Rev. B **78**, 205116

$$A = \cos\theta\, T_2 + \sin\theta\, T_0$$



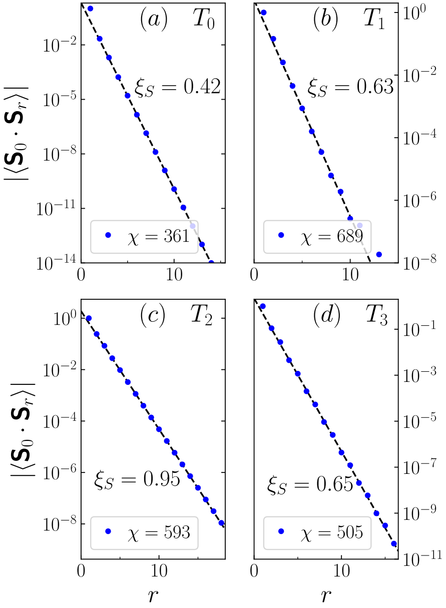
Conclusion

- We constructed $SU(N)$ symmetric tensors for a family of SU(4) RVB wavefunctions
- we used CTMRG and TRG algorithm to investigate the properties of those states
- critical states close to $U(1)$ points
- $\mathbb{Z}_2$ topological order away

Next goal: find a parent Hamiltonian and stabilize an SU(4) quantum spin liquid phase!

Thank you for your attention!

$\mathbf{S} \cdot \mathbf{S}$ correlator



Entanglement spectra