BEC @
Georgia Southern

Producing smooth flow in racetrack atom circuits by stirring at zero and non-zero temperatures

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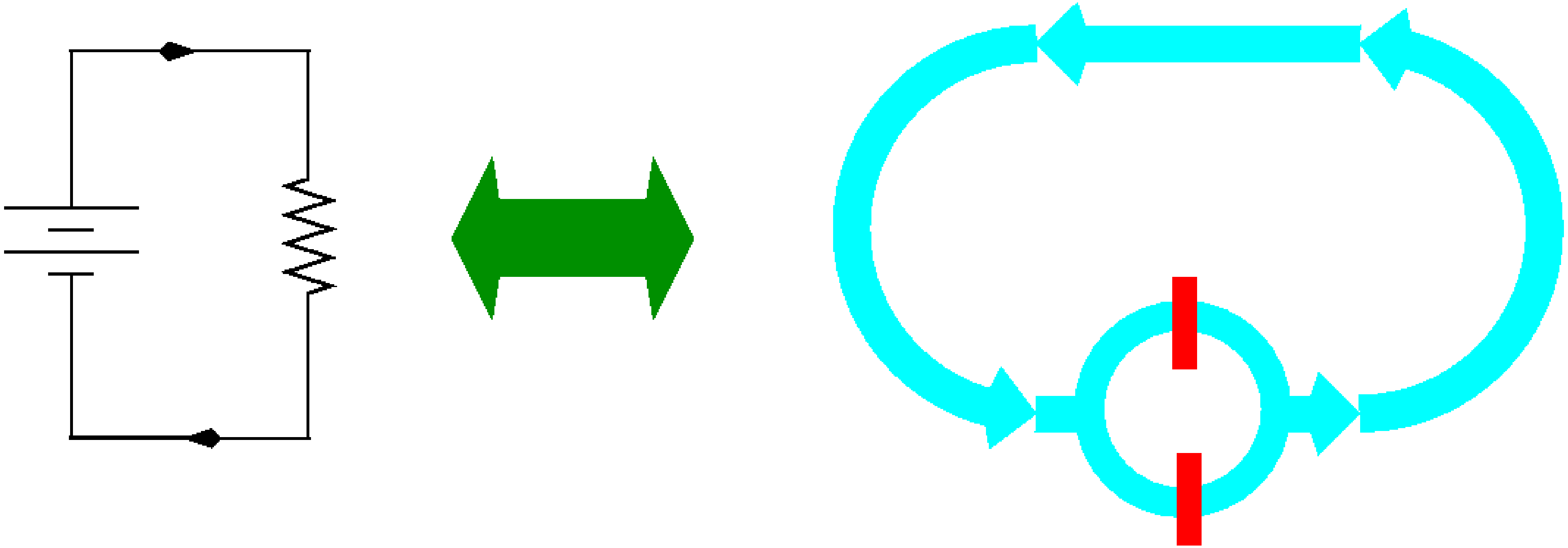
$$ = NSF, NIST

Electronic circuits need electron current flow for applications

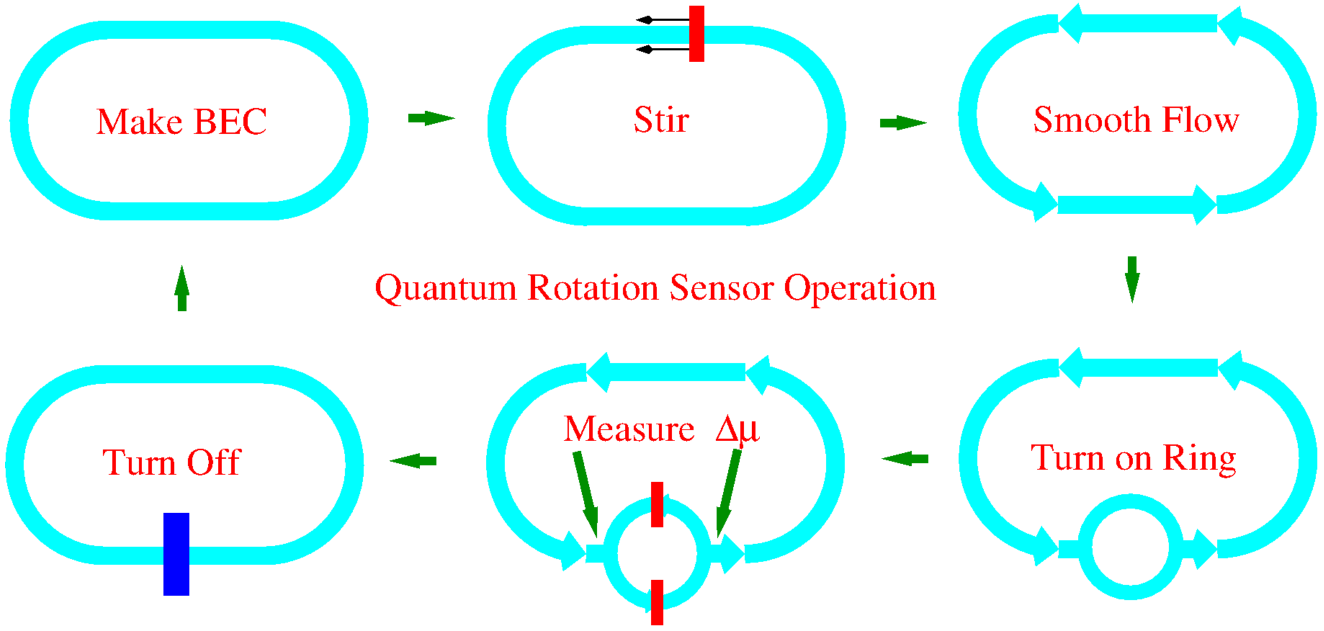


smooth neutral-atom current flow is necessary for atomtronic system applications

Can on-demand smooth flow be produced by stirring?

Racetrack Rotation Sensor Idea

1. Create BEC in racetrack potential
2. Stir with barrier to create smooth flow.
3. Morph on ring channel.
4. Morph on barriers in the ring.
5. Measure the chemical potential difference produced by system rotation.
6. Turn off flow to reset for another measurement.

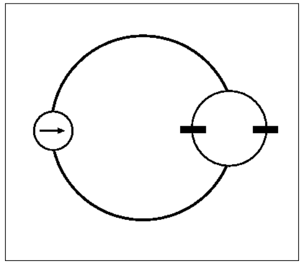


Need the ability to make on-demand smooth flow for this idea to work…we studied smooth-flow production by stirring a BEC in a racetrack potential

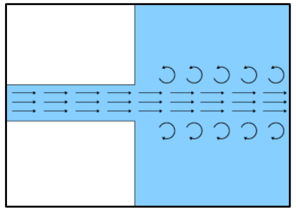
Can we make an atom circuit with smooth flow?

$$\partial \mathbf{v}(\mathbf{r},t)/\partial t \approx 0 \quad \mathbf{v}(\mathbf{r},t) \neq 0$$

- Ways to create flow:
  - reshaping and tilting the path of the atom circuit,
  - OAM transfer (Ryu, et al., PRL **99**, 260401 (2007))
  - phase imprint (Kumar, et al., PRA **97**, 043615 (2018))
  - stirring (Wright, et al., PRL **110**, 025302 (2013))
- Applications of smooth flow:
  - precision sensors for navigation and metrology



  - fundamental physics



- Can then design elements for applications.

Zero-Temperature (GPE) and Non-Zero Temperature (ZNG) Model

ZNG (Zaremba-Nikuni-Griffin) Model

- The ZNG theory describes the behavior of a thermal equilibrium system that is subject to a weak, external perturbation.
- Assumes that the ultracold atom system consists of a Bose-Einstein condensate plus a non-condensate part that is modeled as a classical gas.
- Accounts for time-dependent condensate and non-condensate behavior and allows for both time-dependent atom clouds to affect each other.
- Quantities that describe the system are the condensate wave function, called $\Phi(\mathbf{r}, t)$ and the single-particle distribution function, called $f(\mathbf{p}, \mathbf{r}, t)$.

The single-particle distribution function can be understood as:

$$dN(\mathbf{p}, \mathbf{r}, t) \equiv f(\mathbf{p}, \mathbf{r}, t) \frac{d^3p \, d^3r}{(2\pi\hbar)^3} =$$

the number of non-condensate atoms having momentum in a volume of d^3p around **p** and located within a volume d^3r of **r** .

Equation of motion for the condensate

Generalized Gross-Pitaevskii Equation

$$i\hbar\frac{\partial}{\partial t}\Phi(\mathbf{r},t) = \left(\frac{-\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}}(\mathbf{r},t) + gn_c(\mathbf{r},t) + 2g\tilde{n}(\mathbf{r},t) - iR(\mathbf{r},t)\right)\Phi(\mathbf{r},t)$$

$$\tilde{n}(\mathbf{r},t) = \int \frac{d^3p}{(2\pi\hbar)^3} f(\mathbf{p},\mathbf{r},t) = \text{non-condensate density}$$

$$R(\mathbf{r},t) = \text{particle exchange rate between BEC and non condensate}$$

Quantum Boltzmann equation

The single-particle distribution function satisfies the Quantum Boltzmann equation (QBE). We can understand this equation in the following way:

$$\frac{\partial f(\mathbf{p}, \mathbf{r}, t)}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{diff}} + \left(\frac{\partial f}{\partial t}\right)_{\mathrm{force}} + \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}$$

This equation says that the rate of change of the number of particle at phase space point (**r** , **p**) at time t is affected in three ways between times t and t + dt:

1. Diffusion – particles are already moving and so either change their position from **r** to something else or from something else to **r** .
2. Force – An external force can change a particle's momentum from **p** to something else or from something else to **p**.
3. Collisions – collisions between particles at **r** can have their momenta changed .

ZNG Equation of Motion for non-condensate

Quantum Boltzmann Equation (QBE)

$$\boxed{\frac{\partial f}{\partial t} - \boldsymbol{\nabla}_r U_{\rm eff} \cdot \boldsymbol{\nabla}_p f + \frac{\mathbf{p}}{m} \cdot \boldsymbol{\nabla}_r f = C_{12}[f, \Phi] + C_{22}[f]}$$

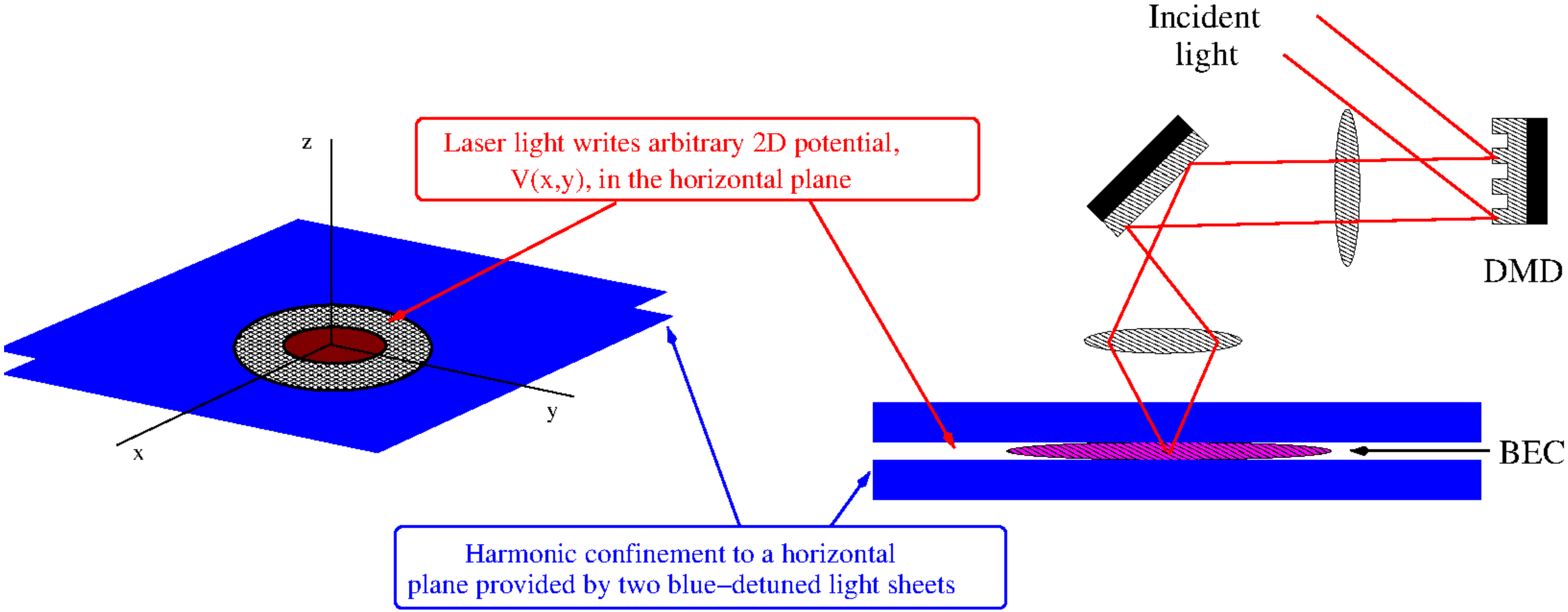
$U_{\rm eff}(\mathbf{r}, t) = V_{\rm trap}(\mathbf{r}, t) + 2g(n_c + \tilde{n}) =$ effective potential felt by non-condensate atoms.

$C_{12}[f, \Phi],\ C_{22}[f]$ are collision integrals.

Model System and Simulation Characteristics

Our atom-circuit setup

$$V_{\mathrm{trap}}(x,y,z) = \frac{1}{2}M\omega_z^2 z^2 + V(x,y)$$



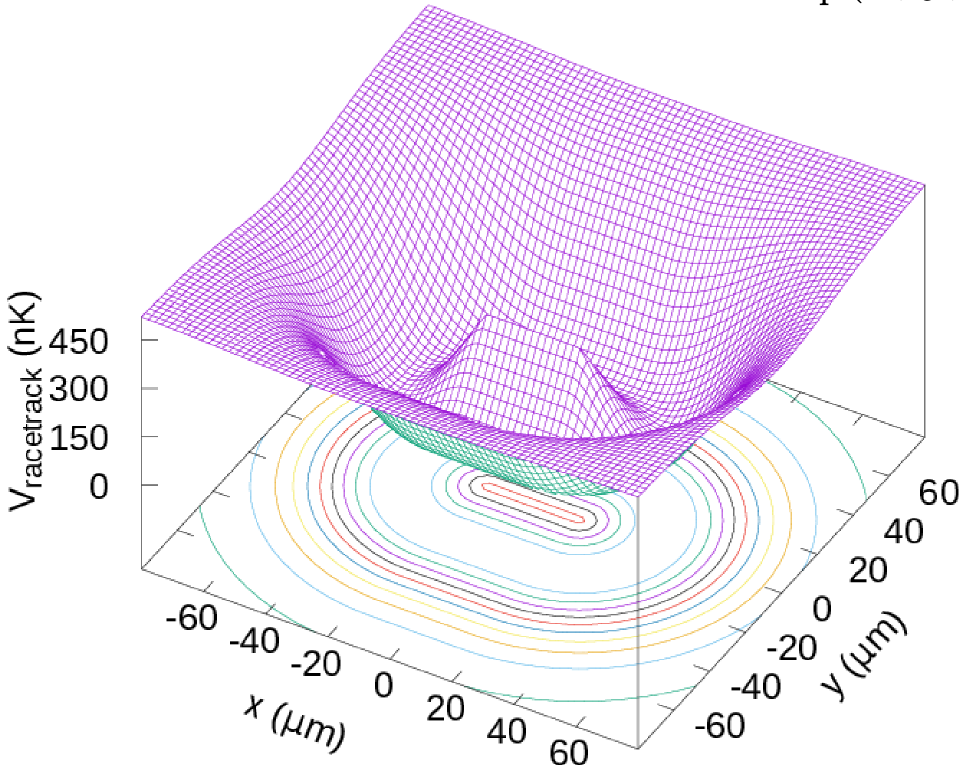
Laser light writes arbitrary 2D potential, V(x,y), in the horizontal plane

Harmonic confinement to a horizontal plane provided by two blue–detuned light sheets

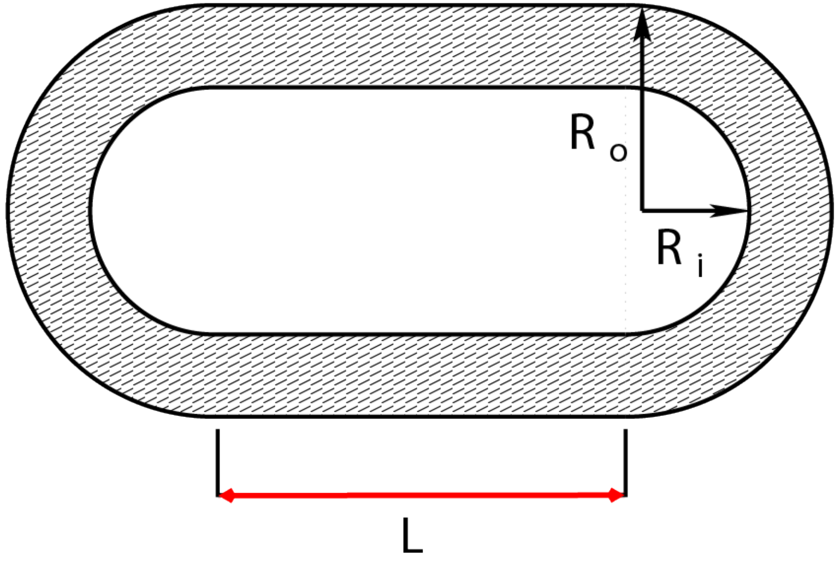
The full potential felt by the condensate atoms is a vertical harmonic potential plus an arbitrary 2D potential, V(x,y), in a horizontal plane:

Our atom-circuit setup

$$V_{\text{trap}}(x,y,z) = \frac{1}{2}M\omega_z^2 z^2 + V(x,y)$$



Racetrack potential



$$V_{\text{racetrack}}(x,y) = V_{\text{rt}}\left\{\frac{1}{2}\left[\tanh\left(\frac{\rho(x,y)-R_o}{s}\right)+\tanh\left(\frac{R_i-\rho(x,y)}{s}\right)\right]+\tanh\left(\frac{R_o-R_i}{2s}\right)\right\}$$

Systematic zero- and non-zero smooth-flow production studies

- We studied how much flow was produced by stirring a BEC in a racetrack potential using the GPE model for T=0.
- The ZNG model was used to study flow production at non-zero T.
- The goal is to develop a simple model to predict the final flow.
- Smooth-flow production simulations were performed where a racetrack condensate was stirred with a fixed-shape paddle to see how much flow could be produced by stirring.
- We varied the following parameters:
  - V_{pmax} , paddle strength
  - TR , paddle stir speed expressed as the number of "Total Revolutions" completed around the L=0 racetrack in 4 seconds
  - L, the length of the racetrack straightaway
  - T, the temperature of the initial equilibrium state.

Stirring schedule for all simulations

Barrier Characteristics:

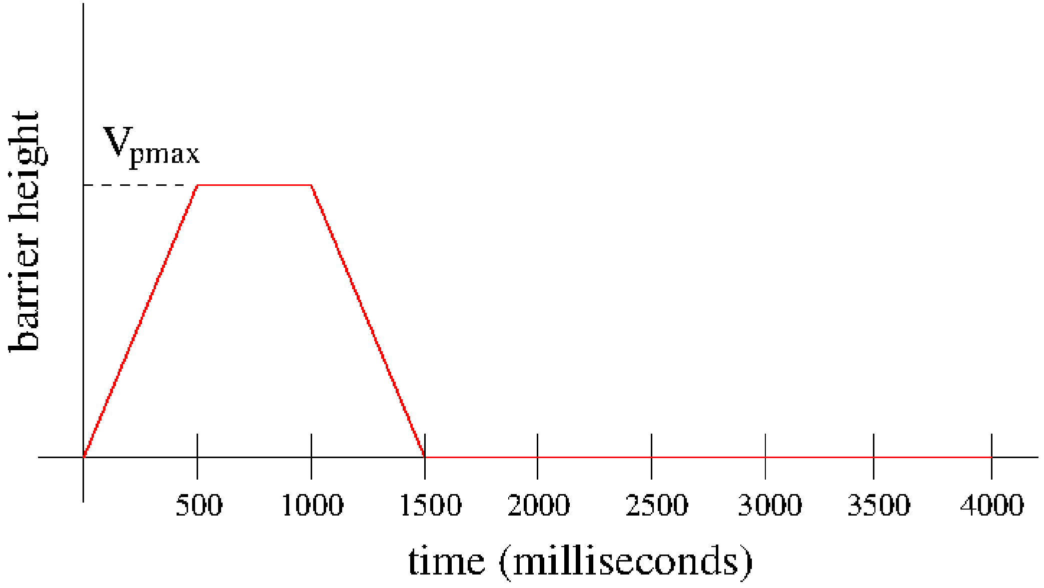
- Rectangular and perpendicular to the racetrack midline
- Perpendicular width is twice that of the racetrack
- Always moving at constant speed
- Energy height varies with time

Barrier energy height schedule :

- Ramp up to V_{pmax} over 500 ms
- Remain at V_{pmax} for 500 ms
- Ramp down to zero over 500 ms
- Simulation runs for a total of 4000 ms for T=0 and 2000 ms for non-zero T.

At 40 ms intervals of the system evolution we calculated:

- Optical density
- Phase distribution
- Velocity distribution x and y components
- Vorticity z component

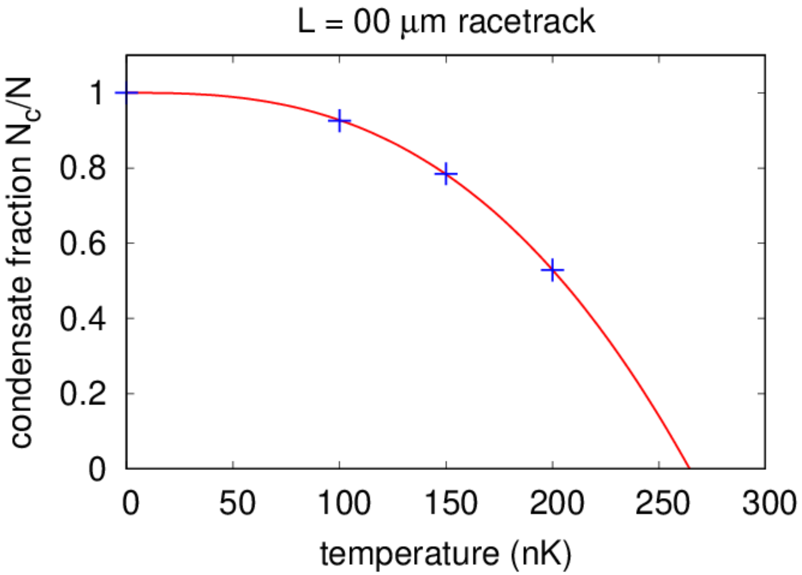


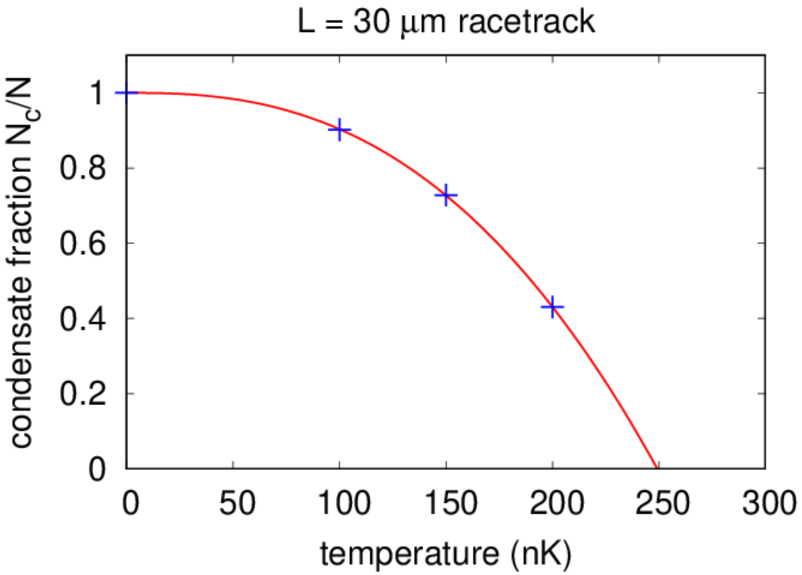
Smooth flow production study parameter space
(all for T=0 and T = 100, 150, 200 nK in blue)

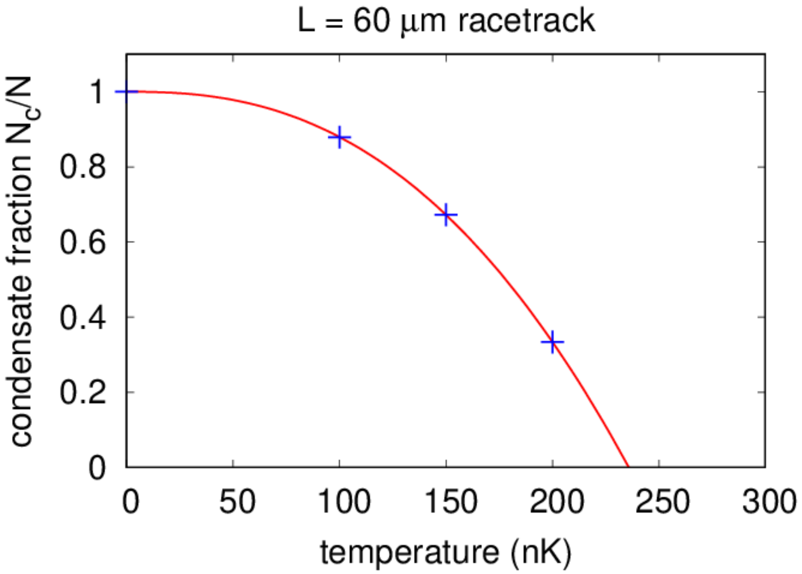
TR \ L	**L = 0 μm**	**L = 10 μm**	**L = 20 μm**	**L = 30 μm**	**L = 40 μm**	**L = 50 μm**	**L = 60 μm**
TR = 3 v = 111 μm/s	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ
TR = 6 v = 222 μm/s	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ
TR = 9 v = 333 μm/s	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ
TR = 12 v = 444 μm/s	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ	V_{pmax}=.50 μ, .52 μ ,…,2.0 μ

condensate fractions vs racetrack length

(Total # of atoms fixed at 500,000)







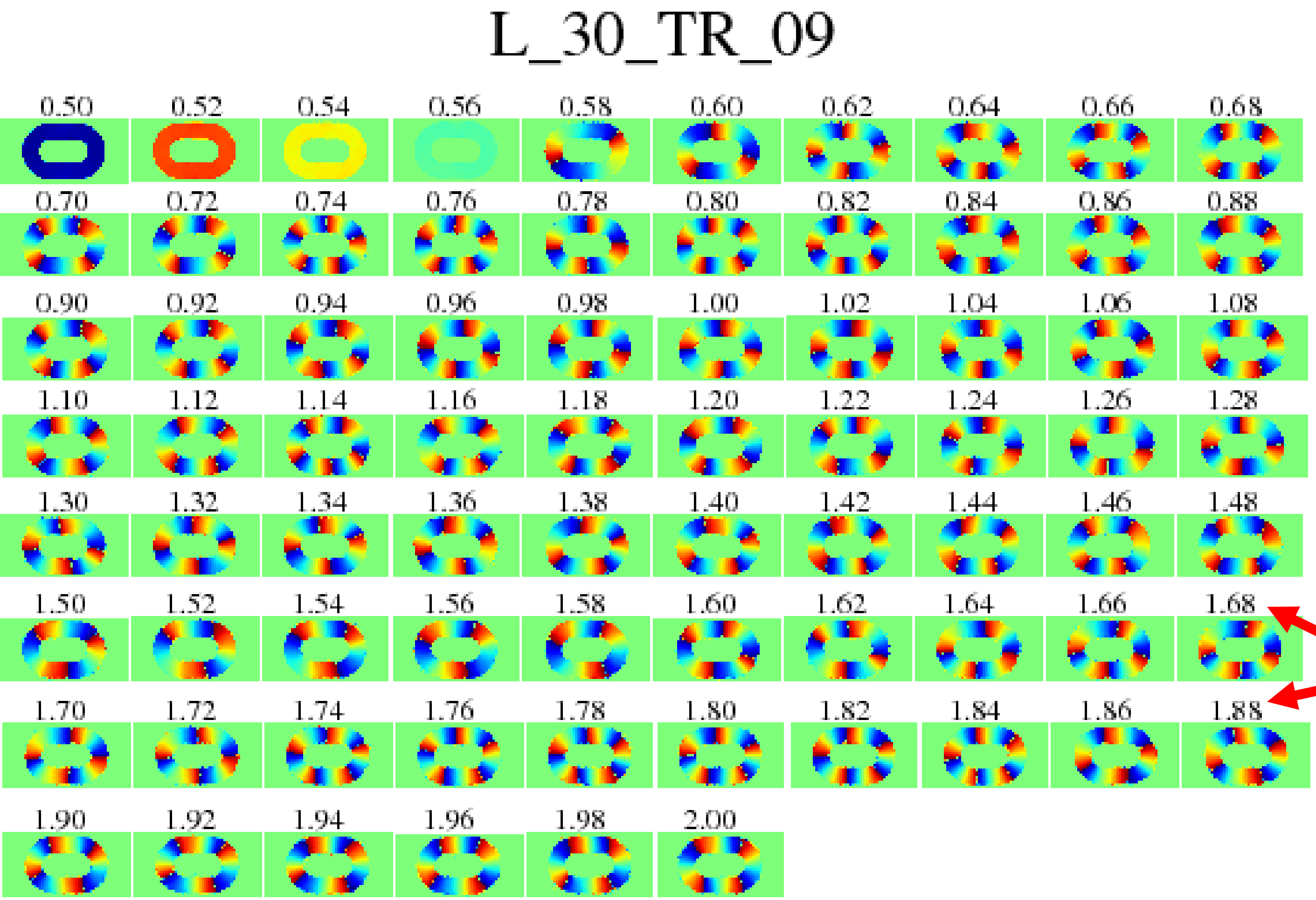
Overview of zero-T systematics study results and extension to non-zero-T

Final phase plots show how much flow is produced

Flow produced by stirring with fixed trap geometry and barrier speed:

- It is possible to make smooth flow by stirring!
- Flow produced is not monotonic with increasing barrier energy height.
- Flow is quantized but winding number can jump by more than one unit from one barrier height to the next.

Length (L) = 30 mm, Stir speed (TR) = 9x37=333 mm/s, Temperature (T) = 0 nK

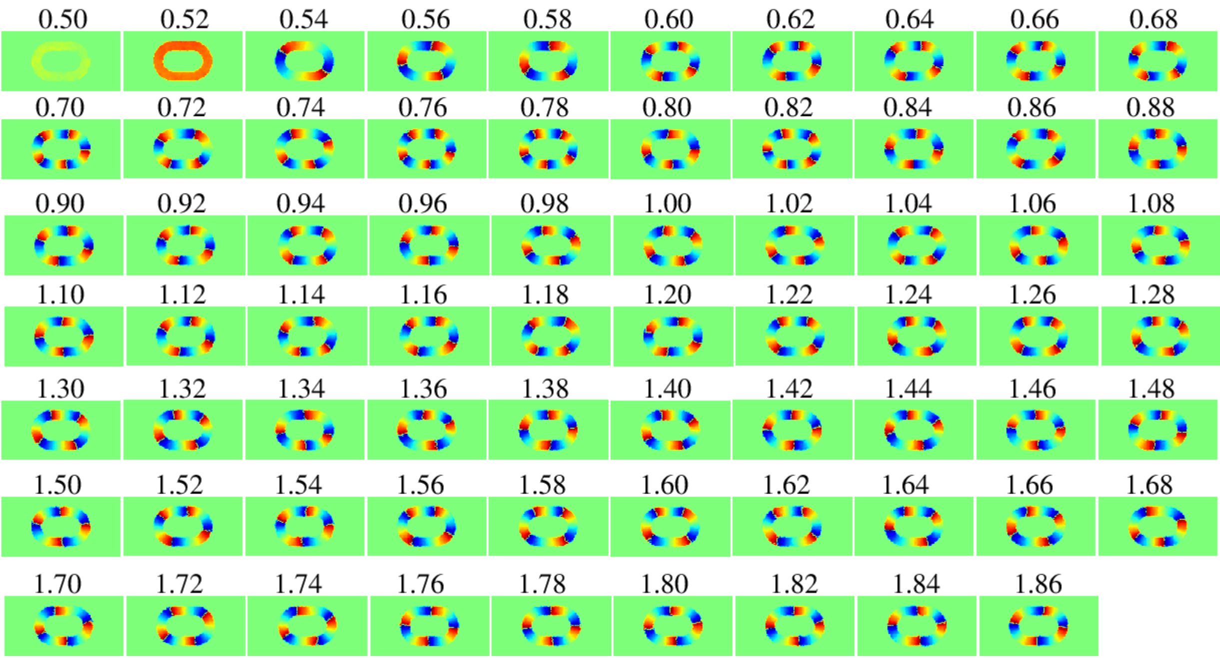


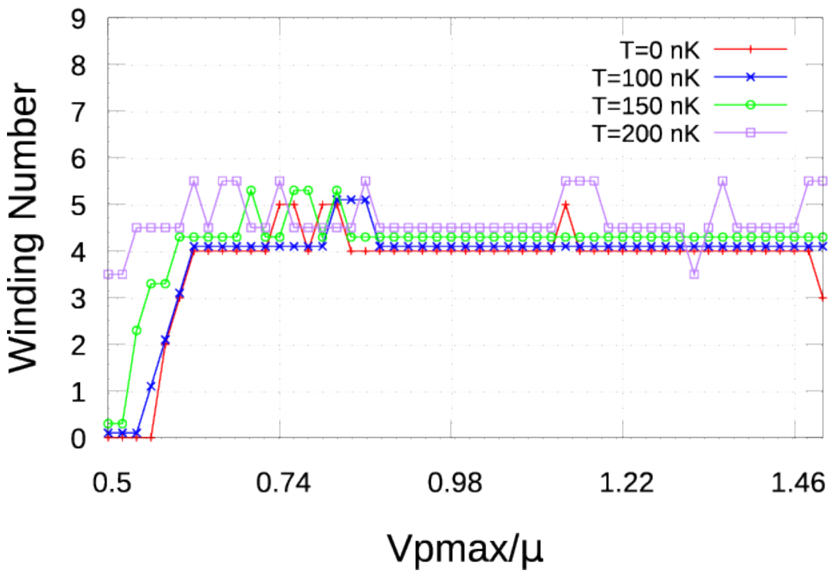
L = racetrack length (μm)
TR = stir speed
(1 TR = 37 μm/s)
T = temp (nK)

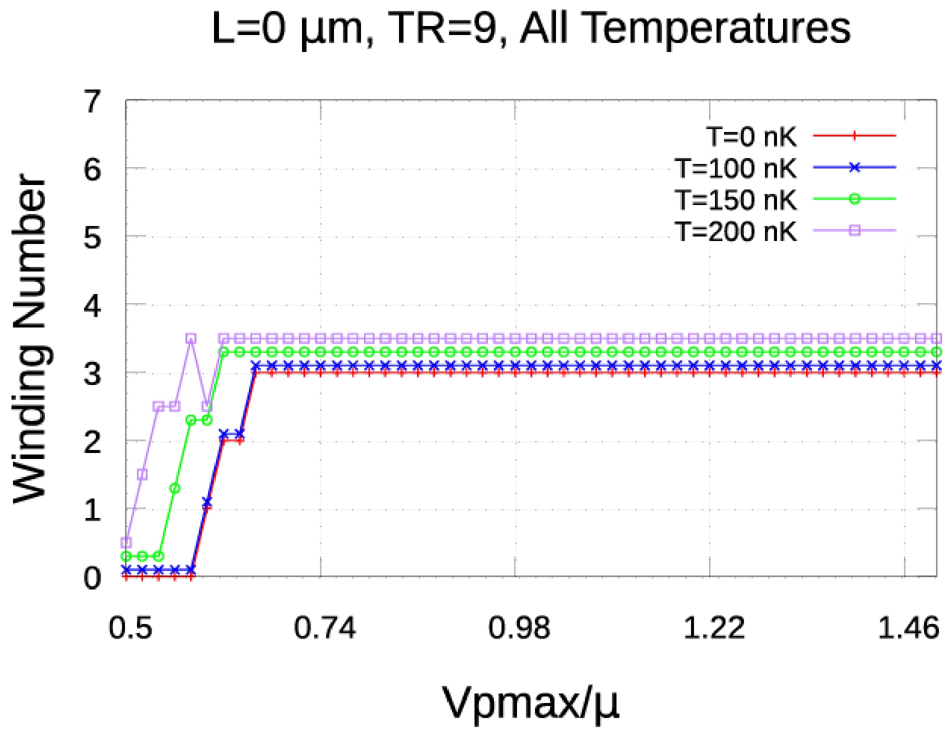
These numbers give the maximum barrier height, Vpmax, in units of the chemical potential, μ, of the initial condensate.

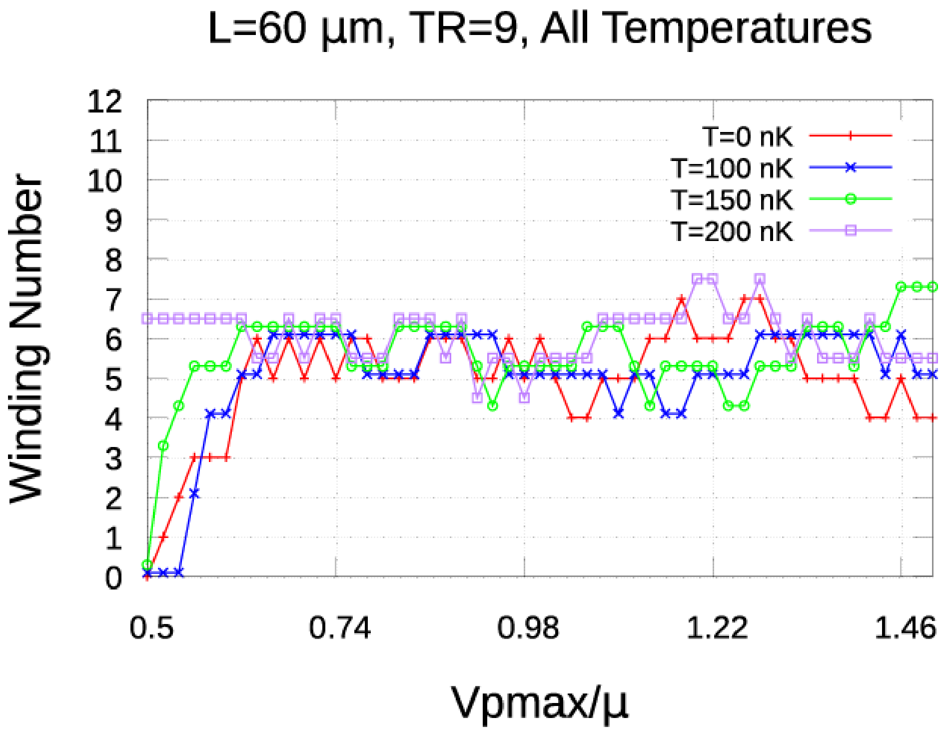
can also make flow at finite temperature

L = racetrack length (μm)
TR = stir speed (1 TR = 37 μm/s)
T = temp (nK)


L_30_TR_09_T_150


L=30 μm, TR=9, All Temperatures


L=0 µm, TR=9, All Temperatures


L=60 µm, TR=9, All Temperatures

- There are two general features of these plots for all cases considered:
  1. There is a critical max barrier strength, $V_{p\max,c}$, below which no flow is present at the end of the simulation
  2. As $V_{p\max}$ increases, the flow levels off to an average value, w_{avg}, around which it can oscillate.

Summary of Results

- Trends for $V_{\mathrm{pmax},c}$
  - For fixed length and stirring speed, with increasing temperature $V_{\mathrm{pmax},c}$ gets lower.
  - For fixed length and temperature, with increasing stirring speed $V_{\mathrm{pmax},c}$ gets lower.
  - For fixed stirring speed and temperature, with increasing racetrack length $V_{\mathrm{pmax},c}$ gets slightly lower (small effect).
- Trends for w_{avg}
  - For fixed length and stirring speed, increasing temperature has little effect on w_{avg}
  - For fixed length and temperature, with increasing stirring speed w_{avg} gets higher.
  - For fixed stirring speed and temperature, with increasing racetrack length w_{avg} gets higher.

Mechanism of zero-T flow production

Condensate flow dynamics

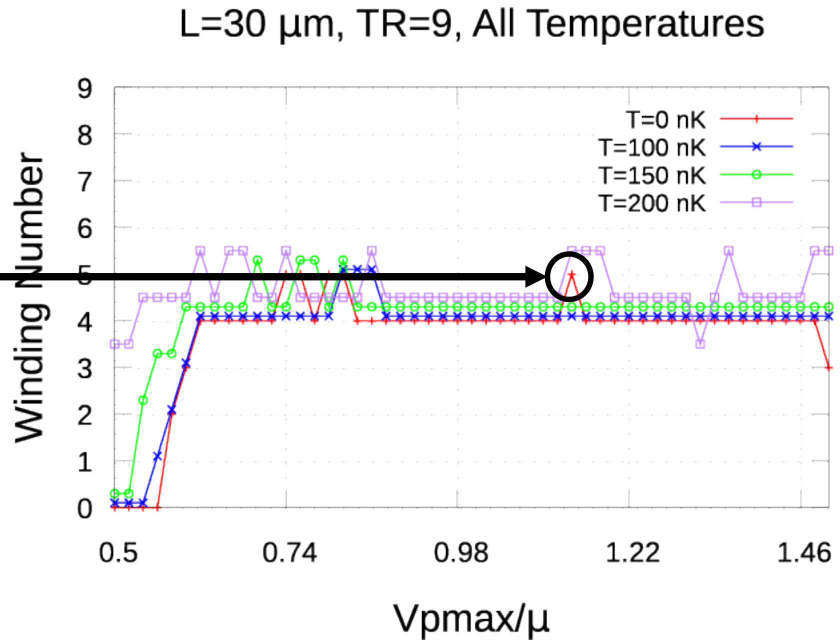
We found that the final flow depends on the following experimental conditions for fixed stirring schedule:

- Speed of stirring, v_s or TR
- Maximum barrier height, V_{pmax}
- Confinement geometry, L
- Temperature, T

What is the mechanism for producing the final flow in the GPE model?

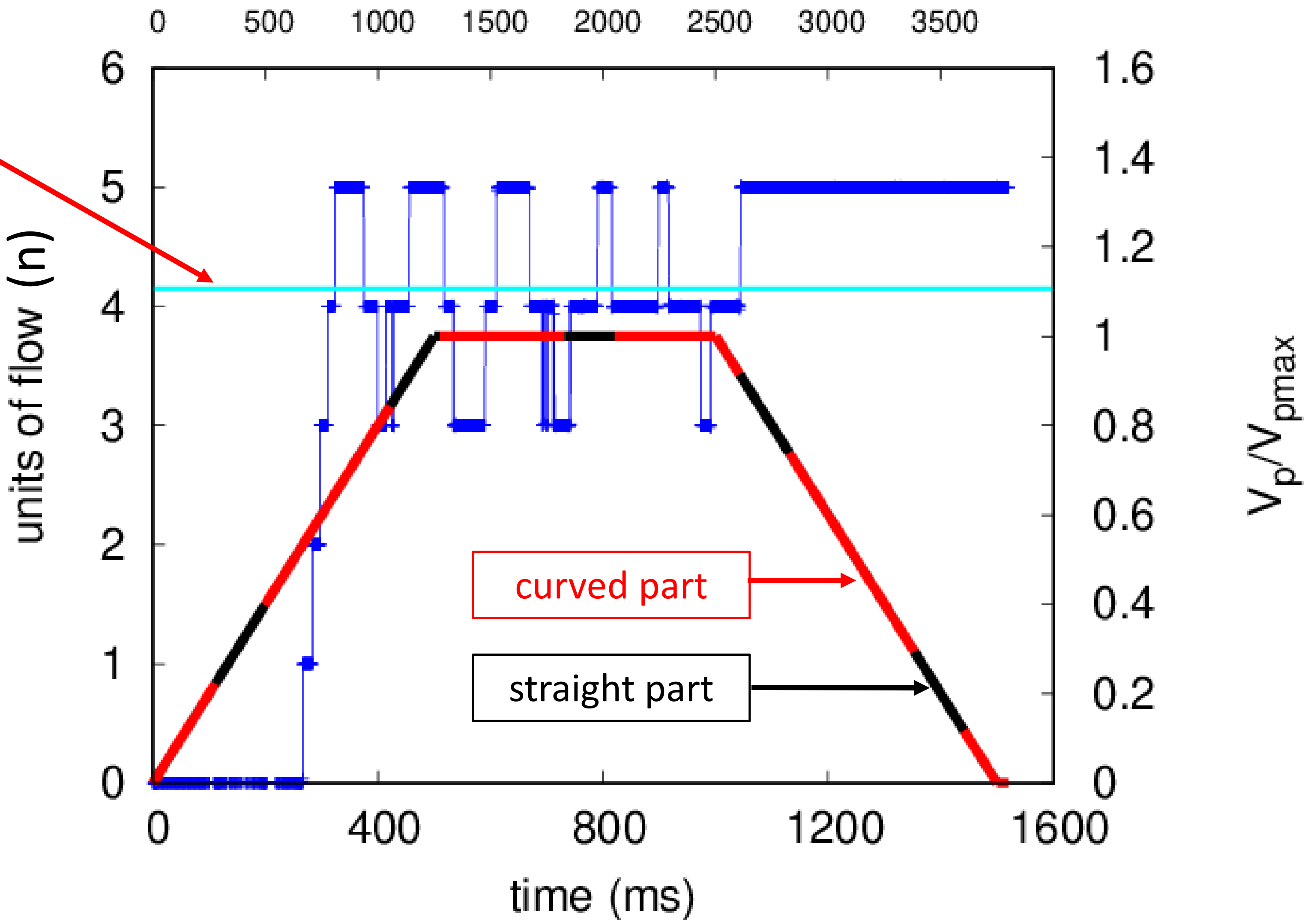
We performed a fine-timescale study of the condensate dynamics during the stirring process for the case with the following characteristics:

- T = 0 nK
- L = 30 μm
- V_{pmax} = 54 nK = 1.14 μ
- v_s = 339 μm/second (TR=09)



BEC midline circulation during stirring phase

(L_30_TR_09_T_000)



barrier stirring speed

curved part

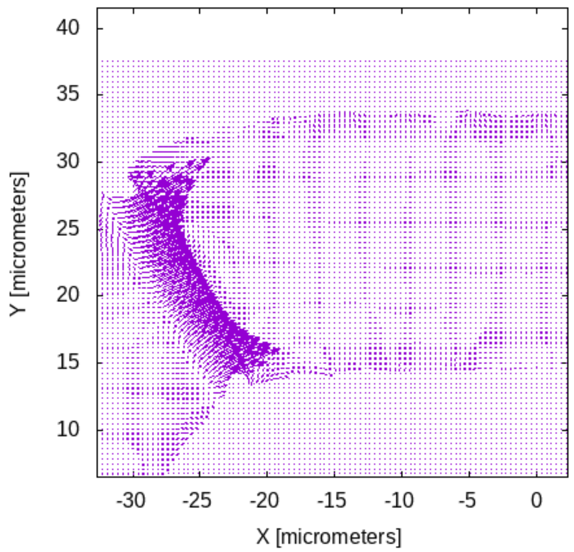
straight part

$$\oint_{\mathcal{C}} \mathbf{v} \cdot \mathbf{dl} = \frac{\hbar}{m} \times 2\pi n$$

$\mathcal{C} =$ midline track

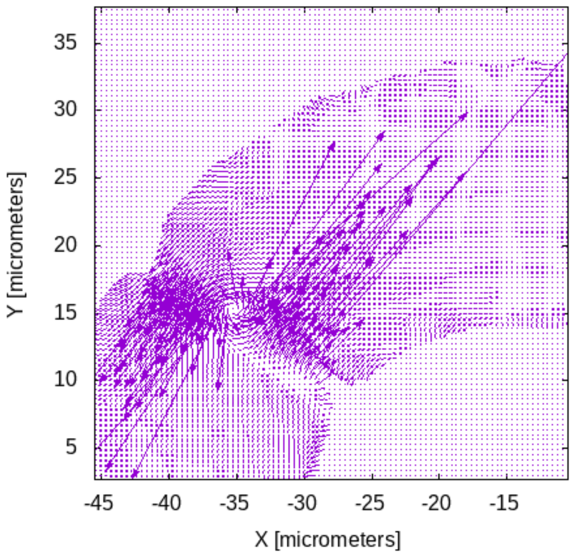
Velocity Distributions of Flow Production: Overview

velocity distribution frame 0590



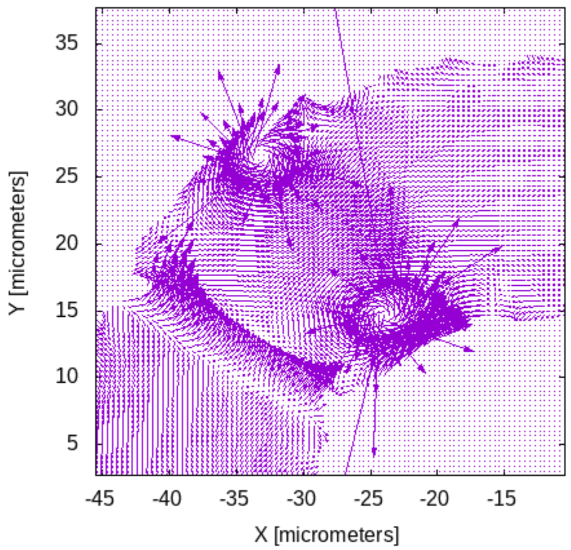
Backflow develops in the barrier region as the barrier height increases

velocity distribution frame 0665

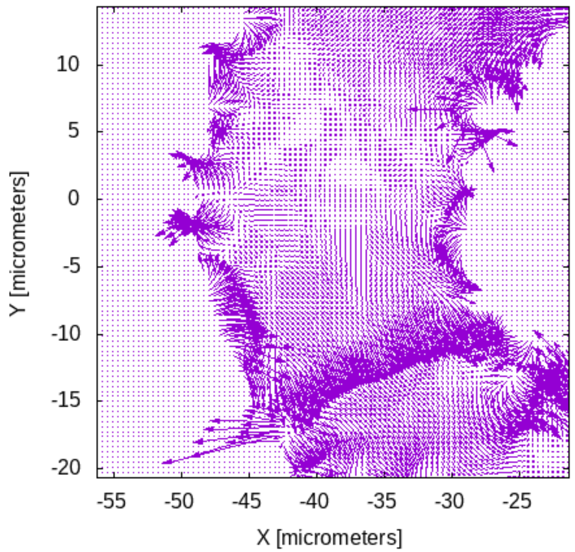


Above some critical flow speed in the barrier region a vortex forms on the outside end of the barrier and migrates into the middle. A vortex/anti-vortex pair is formed.

velocity distribution frame 0680



velocity distribution frame 0864



Vortices continue to be formed until the induced flow moves faster than the barrier. At this point a forward flow develops. Flow is lost/gained afterward by

- forward flow in the barrier
- straight-curved or curved to straight transitions

Research Group and Collaborators

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Summary and Future Work

- We investigated whether it is possible to create smooth flow in a “racetrack” atom circuit by stirring with a rectangular-shaped barrier at zero and non-zero temperature.
- We found that it is possible to make smooth flow in this way at all temperatures and other parameters considered and studied the dependence of the final flow produced on stirring speed, max energy height, racetrack length, and the temperature of the initial state.
- The final flow produced was not monotonic with increasing max barrier height for fixed stirring speed, racetrack length, and temperature.
- Our simulations suggest the creation of vortices by flow through the barrier region. Their transfer in and out of the interior of the BEC is responsible for the increase/decrease of overall circulation around the racetrack.
- Future research will focus on more in-depth study (dynamics) of non-zero temperature cases and the effect of the racetrack length on the circulation produced by stirring, as well as the effect of collisions between BEC and non-condensate atoms.