Oscillations and decay of superfluid currents in a one-dimensional Bose gas on a ring

Atomtronics 2019

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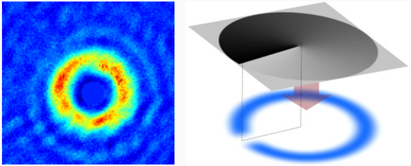
Physics in a 1D ring trap

- Why ring geometries?

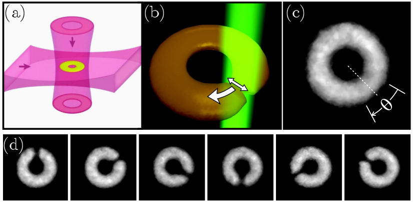
$$\psi \propto e^{i\ell\theta} \text{ with } v_\theta = \frac{\hbar\ell}{mR}$$

Quantization of circulation



* A. Kumar, *et. al.*, Phys. Rev. A **97**, 043615 (2018) (Perrin's group)
C. Ryu, *et. al.*, Phys. Rev. Lett. **99**, 260401 (2007)

- Atomtronics

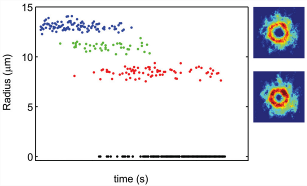


* K.C. Wright, *et. al.* Phys. Rev. Lett. **110**, 025302 (2013)
L. Amico, *et. al.*, New J. Phys. **19**, 020201 (2017)

We are interested in the decay of persistent currents

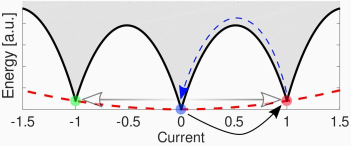
Decay of persistent currents

In 3D/2D the decay is associated with vortex induced phase slips



F. Piazza, *et. al.*, Phys. Rev. A **80**, 021601(R), (2009)
S. Moulder, *et. al.* Phys. Rev. A **86**, 013629, (2012)

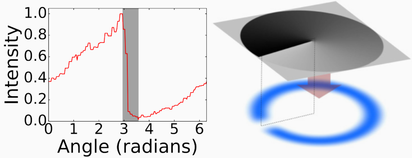
Which is the microscopic mechanism in 1D?



Experimental motivation

Phase imprinting method

inspired by Phys. Rev. A **97**, 043615 (2018)



- How is the barrier going to introduce phase slips?

The barrier will couple different angular momentum states

- What is the role of finite temperature?

The temperature should allow for thermally activated phase slips

Methods

Weakly and Strongly interacting bosons

- Theoretical methods
  - GPE
    - 1D GPE simualtions at T=0
    - Two-State model
    - Projected Gross-Pitaevskii equation (PGPE)
  - Tonks-Girardeau
    - Zero temperature
    - Finite temperatures

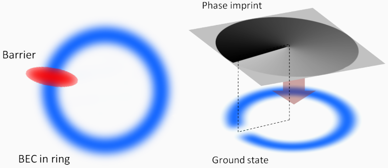
Physical system and Protocol

$T = 0$

i) Ground state ring + barrier

ii) Phase imprinting a current

iii) Start coherent evolution

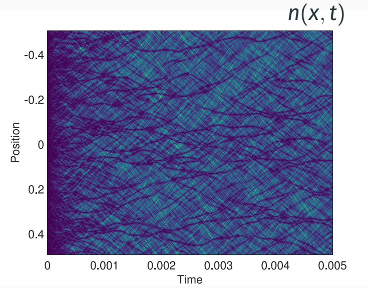


$T > 0$

i) PGPE sampled ψ or Fermi-Dirac for TG

ii) Phase imprinting a current

iii) Start coherent evolution



GPE and PGPE

Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) + g|\psi|^2\right)\psi$$

Stochastic Projected Gross-Pitaevskii equation (SPGPE)

$$i\hbar\frac{\partial\psi_C}{\partial t} = P_C\left[(1-i\alpha)\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + P_C\left[V(x) + g|\psi_C|^2\right]\right)\psi_C + \eta\right]$$

Where η is a centered white noise

$$\langle\eta^*(x,t)\eta(x',t') = 2\hbar k_B T\alpha\delta(t-t')\delta(x-x')\rangle$$

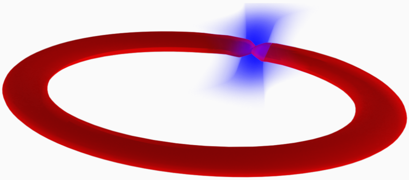
- α ensures relaxation towards an equilibrium state
- ψ are the macroscopically occupied modes
- only thermal fluctuations
- we simulate the coherent part, the incoherent is treated as bath (related to η)

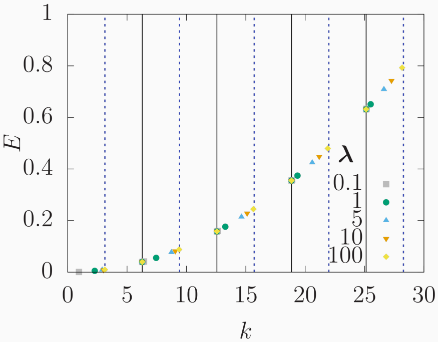
$\psi(x,t=0)=0 \longrightarrow$ thermal state with SPGPE $\longrightarrow$ Evolve with PGPE ($(\eta,\alpha)=0$)

Tonks-Girardeau

Exact solution

- For $g \to \infty$ the many-body wavefunction vanishes at $\Psi(..., x_i = x_j, ...) = 0$
- Bose-Fermi mapping $\Psi_{TG}(x_1, ..., x_N, t) = \Pi_{i \leq j \leq l \leq N} \mathrm{sgn}(x_i - x_j) \det[\psi_k(x_i, t)]$ with ψ_j being the single particle eigenstates





Current

$$\mu,\ k_BT \leq \hbar\omega_z, \hbar\omega_\perp$$

Current per particle In the GPE regime

$$J(t) = -i\frac{\hbar}{2m}\frac{1}{N}\int_0^L \frac{dx}{L}\ \langle \Psi^*\partial_x\Psi - \partial_x\Psi^*\Psi\rangle$$

In the TG regime

$$J(t) = \frac{\hbar}{m}\int_0^L \mathrm{Im}\left[\sum_n^{\infty} f(\epsilon_n)\psi_n^*(x,t)\partial_x\psi_n(x,t)\right]$$

Parameters GPE:

$$N = 2000 \qquad \lambda_{GP} = \frac{V_0}{\mu_{GP}}$$

$$\gamma = \frac{mg}{\hbar^2 n} = 0.01 \qquad k_BT < 2\mu_{GP}$$

$$\mu_{GP} = \frac{gN}{L} \qquad V(x) = V_0 e^{-\frac{x^2}{2\sigma^2}}$$

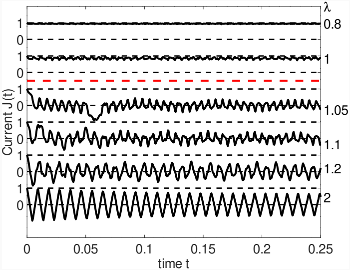
Parameters TG:

$$N = 23 \qquad \lambda_{TG} = \frac{\alpha n}{E_F}$$

$$\gamma = \infty \qquad V(x) = \alpha\delta(x)$$

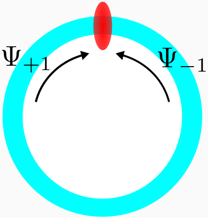
$$\mu_{TG} = E_F = \frac{\hbar^2 n^2 \pi^2}{2m}$$

GPE: $T = 0$



- $\lambda_{GP} \leq 1 \rightarrow$ self-trapping
- $\lambda_{GP} > 1 \rightarrow$ oscillations (irregular)
- $\lambda_{GP} \gg 1 \rightarrow$ oscillations (triangular)

Self-trapping of current states

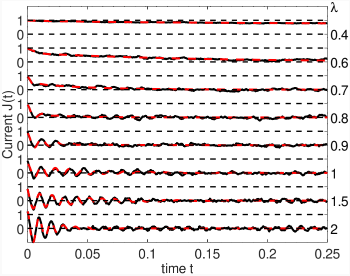


- We construct a Josephson model for the current states
- $\psi_{TM} = \phi_1(t)\varphi_1(x) + \phi_2(t)\varphi_2(x)$

where $\varphi_{1/2}$ are linear combinations of the first and second excited eigenstates

- Valid for $\gamma \ll 1$

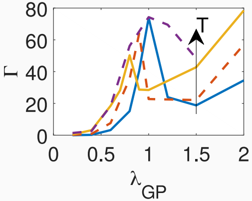
GPE: $T > 0$



- $\lambda_{GP} \leq 1 \rightarrow$ exponential decay
- $\lambda_{GP} > 1 \rightarrow$ damped oscillations
- $\lambda_{GP} \gg 1 \rightarrow$ oscillations + damping

Fitting

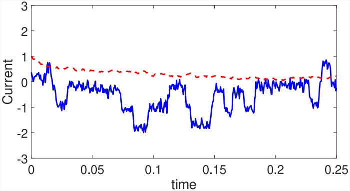
$$J(t) = Ae^{-\Gamma_A t} + B\cos(\omega t + \phi)e^{\Gamma_B t}$$



- For $\lambda_{GP} > 1$, Γ grows monotonically
- **What is the microscopic mechanism driving the decay?**

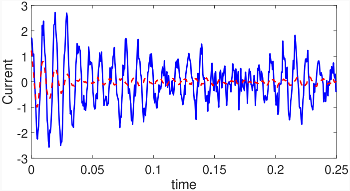
GPE: $T > 0$, Microscopic mechanism

$\lambda = 0.6, \;\; T = \mu_0$



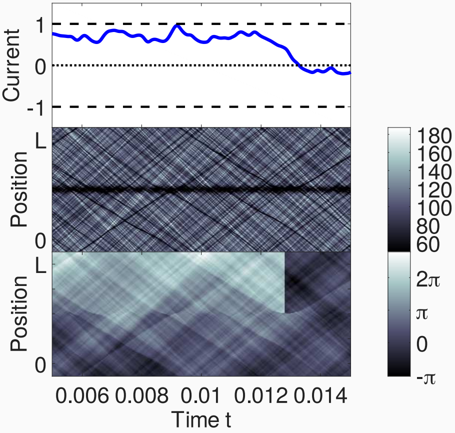
Discrete jumps of the current

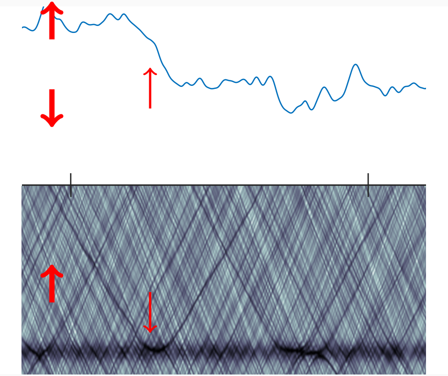
$\lambda = 2, \;\; T = \mu_0$



Dephased oscillations

GPE: $T > 0$, Microscopic mechanism





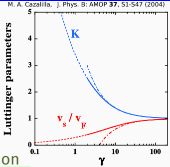
Phase slips associated to random soliton reflections

Luttinger: $T > 0$, Intermediate interactions

Luttinger liquid approach

- Valid for a low energy perturbation $\Rightarrow$ weak quench
- Weak barrier limit $\rightarrow \lambda < 1$ and $\gamma \gtrsim 5$

M. A. Cazalilla, J. Phys. B: AMOP **37**, S1-S47 (2004)



$$\hat{H} = \overbrace{\underbrace{\frac{E_Q}{2}\hat{J}^2}_{\text{“Potential term”}} - \underbrace{E_J \cos\left(2\hat{\theta}_0\right)}_{\text{“Nonlinear Kinetic term”}}}^{\text{Quantum particle}} + \overbrace{\sum_{\mu \geq 1}\left[\underbrace{\frac{\hat{P}_\mu^2}{2M_R} + \frac{1}{2}M_R\Omega_\mu^2\hat{Q}_\mu^2}_{\text{“Harmonic bath”}}\right.}^{\text{Bath}} + \overbrace{\underbrace{\frac{4\sqrt{2}\pi\hbar}{M_R L}\hat{J}\hat{P}_\mu}_{\text{“Coupling”}}}^{\text{Interaction}} + \left.\underbrace{\frac{16\pi^2\hbar^2}{M_R L^2}\hat{J}^2}_{\text{“Renormalization”}}\right]$$

where $\hat{J} \propto \partial_x \hat{\phi}$, $g_{eff} \propto \lambda$, $M_R = \frac{\hbar\pi}{v_J K}$, $\Omega_\mu = v k_\mu$ and we define $E_Q = \frac{\hbar^2(2\pi)^2}{M_R L^2}$ and $E_J = 2n_0 U_{eff}$

From these low energy theory we observe two main regimes:

- Josephson regime for $E_J > E_Q \longrightarrow \omega = \sqrt{E_Q E_J}/\hbar$
- Rabi-like regime for $E_Q > E_J \longrightarrow \omega = E_J/\hbar$

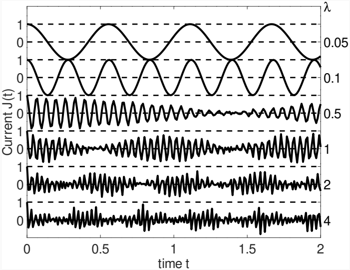
Damping or oscilaltions?:

- $\Delta E_b = E_Q/K$ and $\Delta E_p = \sqrt{E_Q E_J}$
- Josephson regime $\Rightarrow \Delta E_b \ll \Delta E_p$,i.e, **Damping**
- Rabi regime $\Rightarrow \Delta E_b \gg \Delta E_p$,i.e, **No damping**

Phys. Rev. Lett. **121**, 090404 (2018)

For strong interactions E_J decreases. Within the weak barrier limit we find that the system is always in the Rabi-like regime

Tonks-Girardeau: $T = 0$



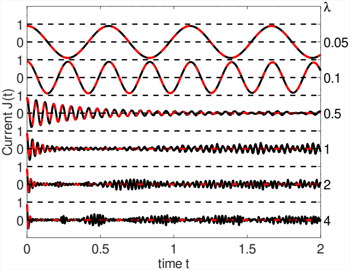
- $\lambda_{TG} < 1 \rightarrow$ Rabi-like oscillations
- $\lambda_{TG} \gtrsim 1 \rightarrow$ oscillations+envelope
- $\lambda_{TG} \gg 1 \rightarrow$ dephased oscillations

- Rabi-like oscillations associated to **coherent phase slips**

- We see more and more excitations involved in the dynamics for larger barriers

For very large barriers we see irregular oscillations with **revivals**

Tonks-Girardeau: $T > 0$



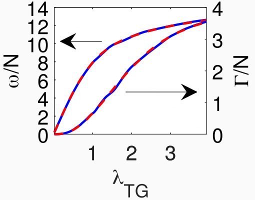
- $\lambda_{TG} < 1 \rightarrow$ Rabi-like oscillations
- $\lambda_{TG} \gtrsim 1 \rightarrow$ damped-oscillations
- $\lambda_{TG} \gg 1 \rightarrow$ overdamped-oscillations

- Exponential decay of current Large number of excitations
- weighted by the Fermi-Diract distribution

$$J = \frac{\hbar}{Nm} \mathrm{Im}\left[\sum_k^{\infty} \sum_j^{\infty} A_{j,k} e^{-i(\epsilon_j - \epsilon_k)t/\hbar} \right]$$

$$A_{j,k} = \frac{\hbar}{mL} \mathrm{Im}\left[\sum_n^{\infty} f(\epsilon_n) \langle \chi_n | \psi_k \rangle \langle \psi_j | \chi_n \rangle \int_0^L dx\, \psi_k^*(x) \partial_x \psi_j(x) \right]$$

- "Universal" dynamics with respect to N

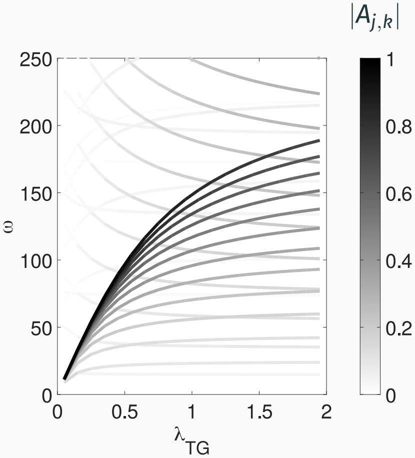


Tonks-Girardeau: Excitations

$$J = \frac{\hbar}{Nm}\mathrm{Im}\left[\sum_k^{\infty}\sum_j^{\infty} A_{j,k} e^{-i(\epsilon_j-\epsilon_k)t/\hbar}\right]$$

$$A_{j,k} = \frac{\hbar}{mL}\mathrm{Im}\left[\sum_n^{\infty} f(\epsilon_n)\langle\chi_n|\psi_k\rangle\langle\psi_j|\chi_n\rangle \int_0^L dx\, \psi_k^*(x)\partial_x\psi_j(x)\right]$$

- For $\lambda_{TG} \ll 1 \longrightarrow \omega_i \simeq \omega_j \quad \forall i,j$
- For $\lambda_{TG} > 1 \longrightarrow$ many frequencies with different associated amplitudes
- Phase imprinting produces a highly excited state
- Multiple particle-hole excitations
- **Incoherent phase slips**



$$\omega_{j,k} = \frac{\epsilon_j - \epsilon_k}{\hbar}$$

Summary

Conclusions

- Josephson oscillations of the current in a 1D ring plus barrier after phase imprinting
- Self-trapping at weak interactions and weak barriers however at finite temperature the current decays
- For strong interactions coherent phase slips drive the dynamics for weak barriers. At finite temperatures, multiple particle-hole excitations induce the decay of persistent currents (incoherent phase slips)

arXiv:1903.09229

Summary

Thank you for your attention