Oscillations and decay of superfluid currents in a one-dimensional Bose gas on a ring

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Motivation

Decay of persistent in a 1D ring

Results

Conclusions

Physics in a 1D ring trap

• Why ring geometries?

\[ \psi \propto e^{i\ell\theta} \text{ with } v_{\theta} = \frac{\hbar \ell}{mR} \]

Quantization of circulation

We are interested in the decay of persistent currents

• Atomtronics


Decay of persistent currents

In 3D/2D the decay is associated with vortex induced phase slips.

Which is the microscopic mechanism in 1D?

Experimental motivation

Phase imprinting method


- How is the barrier going to introduce phase slips?
- What is the role of finite temperature?

The barrier will couple different angular momentum states

The temperature should allow for thermally activated phase slips
Methods

**Weakly and Strongly interacting bosons**

- **Theoretical methods**
  - GPE
    - 1D GPE simulations at \( T=0 \)
    - Two-State model
    - Projected Gross-Pitaevskii equation (PGPE)
  - Tonks-Girardeau
    - Zero temperature
    - Finite temperatures
**Physical system and Protocol**

\[ T = 0 \]

i) Ground state ring + barrier
ii) Phase imprinting a current
iii) Start coherent evolution

\[ T > 0 \]

i) PGPE sampled \( \psi \) or Fermi-Dirac for TG
ii) Phase imprinting a current
iii) Start coherent evolution
GPE and PGPE

**Gross-Pitaevskii equation**

\[ i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g|\psi|^2 \right) \psi \]

**Stochastic Projected Gross-Pitaevskii equation (SPGPE)**

\[ i\hbar \frac{\partial \psi_C}{\partial t} = P_C \left[ (1 - i\alpha) \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + P_C \left[ V(x) + g|\psi_C|^2 \right] \right) \psi_C + \eta \right] \]

Where \( \eta \) is a centered white noise

\[ \langle \eta^*(x, t) \eta(x', t') \rangle = 2\hbar k_B T \alpha \delta(t - t') \delta(x - x') \]

- \( \alpha \) ensures relaxation towards an equilibrium state
- \( \psi \) are the macroscopically occupied modes
- only thermal fluctuations
- we simulate the coherent part, the incoherent is treated as bath (related to \( \eta \))

\[ \psi(x, t = 0) = 0 \rightarrow \text{thermal state with SPGPE} \rightarrow \text{Evolve with PGPE \( ((\eta, \alpha) = 0) \)} \]
Tonks-Girardeau

**Exact solution**

- For $g \to \infty$ the many-body wavefunction vanishes at $\Psi(..., x_i = x_j, ...) = 0$
- Bose-Fermi mapping $\psi_{TG}(x_1, ..., x_N, t) = \prod_{i \leq j \leq l \leq N} \text{sgn}(x_i - x_j) \det[\psi_k(x_i, t)]$ with $\psi_j$ being the single particle eigenstates
**Current**

**Current per particle** In the GPE regime

\[ J(t) = -i \frac{\hbar}{2m} \frac{1}{N} \int_{0}^{L} \frac{dx}{L} \langle \psi^* \partial_x \psi - \partial_x \psi^* \psi \rangle \]

In the TG regime

\[ J(t) = \frac{\hbar}{m} \int_{0}^{L} \text{Im} \left[ \sum_{n} f(\epsilon_n) \psi_n^*(x, t) \partial_x \psi_n(x, t) \right] \]

**Parameters GPE:**

- \( N = 2000 \)
- \( \gamma = \frac{mg}{\hbar^2 n} = 0.01 \)
- \( \mu_{GP} = \frac{gN}{L} \)

**Parameters TG:**

- \( N = 23 \)
- \( \gamma = \infty \)
- \( \mu_{TG} = E_F = \frac{\hbar^2 n^2 \pi^2}{2m} \)

**µ, k_B T ≤ \hbar \omega_z, \hbar \omega_⊥**

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**GPE: \( T = 0 \)**

- \( \lambda_{GP} \leq 1 \rightarrow \) self-trapping
- \( \lambda_{GP} > 1 \rightarrow \) oscillations (irregular)
- \( \lambda_{GP} \gg 1 \rightarrow \) oscillations (triangular)

**Self-trapping of current states**

- We construct a Josephson model for the current states
  - \( \psi_{TM} = \phi_1(t)\varphi_1(x) + \phi_2(t)\varphi_2(x) \)
  where \( \varphi_{1/2} \) are linear combinations of the first and second excited eigenstates
- Valid for \( \gamma \ll 1 \)
**GPE: \( T > 0 \)**

- \( \lambda_{GP} \leq 1 \rightarrow \) exponential decay
- \( \lambda_{GP} > 1 \rightarrow \) damped oscillations
- \( \lambda_{GP} \gg 1 \rightarrow \) oscillations + damping

**Fitting**

\[
J(t) = Ae^{-\Gamma_A t} + B \cos(\omega t + \phi)e^{\Gamma_B t}
\]

- For \( \lambda_{GP} > 1 \), \( \Gamma \) grows monotonically
- **What is the microscopic mechanism driving the decay?**
GPE: $T > 0$, Microscopic mechanism

$\lambda = 0.6, \ T = \mu_0$

Discrete jumps of the current

$\lambda = 2, \ T = \mu_0$

Dephased oscillations
GPE: $T > 0$, Microscopic mechanism

Phase slips associated to random soliton reflections
Luttinger: $T > 0$, Intermediate interactions

Luttinger liquid approach

- Valid for a low energy perturbation $\Rightarrow$ weak quench
- Weak barrier limit $\rightarrow \lambda < 1$ and $\gamma \gtrsim 5$

$$\hat{H} = \frac{E_Q J^2}{2} - E_J \cos (2\hat{\theta}_0) + \sum_{\mu \geq 1} \left[ \frac{\hat{P}_\mu^2}{2M_R} + \frac{1}{2} M_R \Omega_\mu^2 \hat{Q}_\mu^2 + \frac{4\sqrt{2\pi \hbar^2}}{M_R L} \hat{J}_\mu \hat{P}_\mu + \frac{16\pi^2 \hbar^2}{M_R L^2} \hat{J}^2 \right]$$

From these low energy theory we observe two main regimes:

- Josephson regime for $E_J > E_Q \rightarrow \omega = \sqrt{E_Q E_J / \hbar}$
- Rabi-like regime for $E_Q > E_J \rightarrow \omega = E_J / \hbar$

Damping or oscillations?:

- $\Delta E_b = E_Q / K$ and $\Delta E_p = \sqrt{E_Q E_J}$
- Josephson regime $\Rightarrow \Delta E_b \ll \Delta E_p$, i.e., **Damping**
- Rabi regime $\Rightarrow \Delta E_b \gg \Delta E_p$, i.e., **No damping**

For strong interactions $E_Q$ decreases. Within the weak barrier limit we find that the system is always in the Rabi-like regime.
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Tonks-Girardeau: $T = 0$

- $\lambda_{TG} < 1 \rightarrow$ Rabi-like oscillations
- $\lambda_{TG} \gtrsim 1 \rightarrow$ oscillations + envelope
- $\lambda_{TG} \gg 1 \rightarrow$ dephased oscillations

- Rabi-like oscillations associated to coherent phase slips
- We see more and more excitations involved in the dynamics for larger barriers

For very large barriers we see irregular oscillations with revivals
Tonks-Girardeau: $T > 0$

- Exponential decay of current
- Large number of excitations
- weighted by the Fermi-Dirac distribution

\[ J = \frac{\hbar}{Nm} \text{Im} \left[ \sum_{k} \sum_{j} A_{j,k} e^{-i(\epsilon_j - \epsilon_k)t/\hbar} \right] \]

\[ A_{j,k} = \frac{\hbar}{mL} \text{Im} \left[ \sum_{n} f(\epsilon_n) \langle \chi_n | \psi_k \rangle \langle \psi_j | \chi_n \rangle \int_0^L dx \, \psi_k^*(x) \partial_x \psi_j(x) \right] \]

- $\lambda_{TG} < 1 \rightarrow$ Rabi-like oscillations
- $\lambda_{TG} \gtrsim 1 \rightarrow$ damped-oscillations
- $\lambda_{TG} \gg 1 \rightarrow$ overdamped-oscillations

"Universal" dynamics with respect to $N$
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**Tonks-Girardeau: Excitations**

\[
J = \frac{\hbar}{Nm} \text{Im} \left[ \sum_{j} \sum_{k} A_{j,k} e^{-i(\epsilon_j - \epsilon_k) t / \hbar} \right]
\]

\[
A_{j,k} = \frac{\hbar}{mL} \text{Im} \left[ \sum_{n} f(\epsilon_n) \langle \chi_n | \psi_k \rangle \langle \psi_j | \chi_n \rangle \int_0^L dx \psi_k^*(x) \partial_x \psi_j(x) \right]
\]

- For \( \lambda_{TG} \ll 1 \rightarrow \omega_i \simeq \omega_j \quad \forall i, j \)
- For \( \lambda_{TG} > 1 \rightarrow \) many frequencies with different associated amplitudes
- Phase imprinting produces a highly excited state
- Multiple particle-hole excitations
- Incoherent phase slips

\[
\omega_{j,k} = \frac{\epsilon_j - \epsilon_k}{\hbar}
\]
Conclusions

- Josephson oscillations of the current in a 1D ring plus barrier after phase imprinting

- Self-trapping at weak interactions and weak barriers however at finite temperature the current decays

- For strong interactions coherent phase slips drive the dynamics for weak barriers. At finite temperatures, multiple particle-hole excitations induce the decay of persistent currents (incoherent phase slips)
Thank you for your attention