Flat Bands Systems

Matteo Rizzi

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Benasque, Atomtronics, 10.05.2019
**Synthetic Quantum Matter**

*Bottom-up engineering*

trapped ions, superconducting qubits, quantum dots, NV centers, etc.
**Synthetic Quantum Matter**

**Bottom-up engineering**
trapped ions, superconducting qubits, quantum dots, NV centers, etc.

**Hamiltonian engineering**
cold atomic gases, photonic systems, …


A. Fetter. RMP 81, 647 (2009)

J. Dalibard, et al., RMP 83, 1523 (2011)

N. Goldman, et al., RPP 77, 126401 (2014)

C. Chin, et al., RMP 82, 1225 (2010)

Ramanathan et al., PRL 106, 130401 (2011)
**Synthetic Quantum Matter**

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trapped ions, superconducting qubits, quantum dots, NV centers, etc.

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Ramanathan et al., PRL 106, 130401 (2011);
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**Atomtronics**
Benasque 2019
General Picture

Geometrical constraints + Background gauge fields

Frustration, large degeneracy, single-particle topological character, *flat-band dispersion*

Interactions btw. constituents

Fractional topological phases *here fermions*

range $q \Rightarrow$ fraction $1/(q+1)$

Collective transport properties: *pair-transport in bosons, etc.*

see works by Tovmasyan, Peotta, Huber, Törma (& others) for enhanced fermionic pairing, too
General Picture

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From standard to flat bands

\[ \mathcal{H}_0 = \sum_k c_k^\dagger \left\{ -2t \cos(ak) \sigma_0 + \right\} c_k \]
From standard to flat bands

\[ \mathcal{H}_0 = \sum_k c_k^\dagger \left\{ \left[ + 2t \sin(ak) \right] \sigma_3 + \right\} c_k \]
Fractional topological phases in atomic two-leg ladders

From standard to flat bands

\[ \mathcal{H}_0 = \sum_k c_k^\dagger \left\{ \begin{array}{c} + 2t \sin(ak) \sigma_3 + [-2g \cos(ak)] \sigma_1 \end{array} \right\} c_k \]
Flat topological bands

\[ \mathcal{H}_0 = \sum_k c_k^\dagger \left\{ \begin{bmatrix} 2t \sin(ak) \end{bmatrix} \sigma_3 + \begin{bmatrix} -2g \cos(ak) \end{bmatrix} \sigma_1 \right\} c_k \]

\( g = t \) flat bands, with topological character!

constrained in \((B_1, B_3)\) plane by chiral symmetry:

\[ \sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k) \]
Flat topological bands

\[ \mathcal{H}_0 = \sum_k c_k^\dagger \left\{ [ + 2t \sin(ak)] \sigma_3 + [ - 2g \cos(ak)] \sigma_1 \right\} c_k \]

\( g = t \quad \text{flat bands, with topological character!} \)

\[ \mathcal{H}_0 = \sum_{k \in \text{BZ}} c_k^\dagger \left\{ \mathbf{B}(k) \cdot \sigma \right\} c_k \]

constrained in \((B_1, B_3)\) plane by chiral symmetry:

\[ \sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k) \]

**Symmetry Protected Topological (SPT) Order**

\( A_{\pm}(k) = \langle \varepsilon_{\pm}(k) | i\partial_k | \varepsilon_{\pm}(k) \rangle \quad \varphi_{\text{Zak}, \pm} = \int_{\text{BZ}} dk A_{\pm}(k) = \pi \)

\( \varphi_{\text{Zak}} \text{ measured in cold gases} \)

via Ramsey interferometry


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Fractional topological phases in atomic two-leg ladders
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SPT - Trivial transition

\( \mathcal{H}_0 = \sum_k c_k^\dagger \left\{ [ -2t \sin(ak) + 2m - 2g \cos(ak)] \sigma_3 \right\} c_k \)

\( g = t \) flat bands, with topological character! (SPT)

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constrained in \((B_1, B_3)\) plane by chiral symmetry:

\[ \sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k) \]

\( T : \sigma_1 \mathcal{H}_0^*(-k) \sigma_1 = +\mathcal{H}_0(k) \)

\( C : \sigma_3 \mathcal{H}_0^*(-k) \sigma_3 = -\mathcal{H}_0(k) \)

Class BDI of AZ table

Altland, Zirnbauer, PRB 55, 1142 (1997)
\( \mathcal{H}_0 = \sum_k c_k^{\dagger} \{ [2 t \sin(ak)] \sigma_3 + [2m - 2g \cos(ak)] \sigma_1 \} c_k \)

\( g = t \) flat bands, with topological character! (SPT)

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constrained in \((B_1, B_3)\) plane by chiral symmetry:

\( \sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k) \)

\( m = g = t \) isolated Dirac cone

[no Fermion doubling]

Creutz, Horváth, PRD 50, 2297 (1994)
Creutz, PRL 83, 2636 (1999)
\[
\mathcal{H}_0 = \sum_k c_k^\dagger \left\{ \left[ +2t \sin(ak) \right] \sigma_3 + \left[ 2m - 2g \cos(ak) \right] \sigma_1 \right\} c_k \\
g = t \text{ flat bands, with topological character! (SPT)} \\
\mathcal{H}_0 = \sum_{k \in BZ} c_k^\dagger \{ \mathbf{B}(k) \cdot \mathbf{\sigma} \} c_k \\
\text{constrained in } (B_1, B_3) \text{ plane by chiral symmetry:} \\
\sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k)
\]

\[\begin{align*}
m &= g = t \\
\text{isolated Dirac cone} \\
[\text{no Fermion doubling}]
\end{align*}\]

\[\begin{align*}
m > g = t \\
\text{transition to trivial gap} \\
\mathcal{W} = 0 \quad \varphi_{Zak} = 0 \quad \text{no more edge states}
\end{align*}\]
Fractional topological phases in atomic two-leg ladders

\[ \mathcal{H}_0 = \sum_k c_k^\dagger \left\{ [\Delta \epsilon/2 + 2t \sin(ak)] \sigma_3 + \left[ -2g \cos(ak) \right] \sigma_1 \right\} c_k \]

\( g = t \) flat bands, with topological character! (SPT)

\[ \mathcal{H}_0 = \sum_{k \in BZ} c_k^\dagger \{ \mathbf{B}(k) \cdot \sigma \} c_k \]

constrained in \((B_1, B_3)\) plane by chiral symmetry:

\[ \sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k) \]

Zeeman \( \Delta \epsilon \) breaks T & C symmetry!

\[ \mathcal{T}_U \] s.t. \( \mathcal{U}_\alpha \mathcal{H}_0(-k) \mathcal{U}_\alpha^\dagger = \pm \mathcal{H}_0(k) \)

Class AIII of AZ table

Altland, Zirnbauer, PRB 55, 1142 (1997)

[another scheme Velasco, Paredes, PRL 119, 115301 (2017)]

\[ \mathcal{AIII} SPT \text{ phase} \]
Aharanov-Bohm cages

\[ \mathcal{H}_0 = \sum_k c_k^\dagger \{ [ +2t \sin(ak)] \sigma_3 + [ -2g \cos(ak)] \sigma_1 \} c_k \]

\[ g = t \quad \text{flat bands, with topological character! (SPT)} \]

Basis of localized states \( w_{j,\pm}^\dagger \), a.k.a. Aharanov-Bohm cages

Vidal, Mosseri, & Doucot, PRL 81, 5888 (1998)

OBC: Mid-gap zero-energy edge states \((l^\dagger, r^\dagger)\), [bulk-edge correspondence]

P. Delplace, et al., PRB 84, 195452 (2011)
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P. Delplace, et al., PRB 84, 195452 (2011)

a workhorse for flat-band & SPT physics:

Tovmasyan, van Nieuwenburg, & Huber, PRB 88, 220510(R) (2013)
Takayoshi, Katsura, Watanabe, & Aoki, PRA 88, 063613 (2013)
Huber & Altman, PRB 82, 184502 (2010)
Tovmasyan, Peotta, Törmä, & Huber, PRB 94, 245149 (2016)
Sticlet, Seabra, Pollmann, & Cayssol, PRB 89, 115430 (2014)

Bragg techniques to measure edge states in ultracold cold atoms


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Fractional topological phases in atomic two-leg ladders
Effective model on the basis of Aharanov-Bohm cages

\[ t_{\text{imb}} = -i \Delta \epsilon / 4 \]

\[ \tilde{J} = -V_v / 4 \]

\[ T_d = V_v / 4 \]
Effective model

on the basis of Aharanov-Bohm cages

exotic Hubbard model

[without dipolar atoms or other “strange” schemes]

- imbalance induced hopping
- n.n. interactions
- pair tunnelling
- density-assisted tunnelling

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$\tilde{J} = -V_v/4 \quad T_d = V_v/4$

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Fractional topological phases
in atomic two-leg ladders
Effective model

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$\tilde{J} = -\frac{V_v}{4}$
$T_d = \frac{V_v}{4}$

$\tilde{J}$

$V_v/4$

$V_v/4$

$V_v/4$

$t_{imb} = -i\Delta\varepsilon/4$

$\varepsilon_l = \varepsilon_r$

$-t_{imb}\sqrt{2}$

$\varepsilon_+$

$\varepsilon_-$

$V_v/2$

$-t_{imb}$
Effective model

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*without dipolar atoms or other “strange” schemes*

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\[
\begin{align*}
\tilde{J} & = -\frac{V_v}{4} \\
T_d & = \frac{V_v}{4}
\end{align*}
\]

\[
t_{imb} = -i\Delta \epsilon/4
\]

\[
\epsilon_i = \epsilon_r
\]

\[
V_v/2
\]

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\[
\text{Atomtronics} \\
\text{Benasque 2019}
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Effective model

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e_i = e_r

\[ t_{imb} = -i\Delta e/4 \]

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**exotic Hubbard model**

[without dipolar atoms or other “strange” schemes]

- imbalance induced hopping
- n.n. interactions
- pair tunnelling
- density-assisted tunnelling

bulk-mediated [à la Fano-Anderson]

**edge-edge interactions**

J. Jünemann, et al., PRX 7, 031057 (2017)
Experimental schemes

An optical-lattice-based quantum simulator for relativistic field theories and topological insulators

A shaken lattice proposal for the Creutz ladder
J. Jünemann, et al., PRX 7, 031057 (2017)

Realization of a cross-linked chiral ladder by orbital-momentum coupling

Fractional topological phases in atomic two-leg ladders

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Half-filling $\nu = 1$ results

J. Jünemann, et al. (MR), PRX 7, 031057 (2017)
Can intrinsically interacting, gapped, SPT phases arise at $\nu = N/L \neq 1$ for which non-interacting system is gapless?

**Creutz-Hubbard ladder**

Projection on lowest band & approximate cage basis: 
... n.n. interactions ...
Can intrinsically interacting, gapped, SPT phases arise at $\nu = N/L \neq 1$ for which non-interacting system is gapless?

**Creutz-Hubbard ladder**

Projection on lowest band & approximate cage basis: … n.n. interactions …

Gapped phase at $\nu = \frac{1}{2}$ & critical $U/t$

$$\delta_{\text{charge}} = E_1(N) - \frac{1}{2} [E_1(N + 1) + E_1(N - 1)]$$
gapped phase at $\nu = \frac{1}{2}$ & critical $U/t$

$\delta_{\text{charge}} = E_1(N) - \frac{1}{2} [E_1(N + 1) + E_1(N - 1)]$

&

double degeneracy with PBC

$\delta_{\text{spin,} \alpha} = E_\alpha(N) - E_1(N)$

Twisted PBC $t \rightarrow t \exp(i\varphi/L)$
Topological CDW

generalized SPT invariant

destroyed by

\[ \hat{H}_{SB} = i M \sum_j (\hat{c}^\dagger_{j,\uparrow} \hat{c}_{j,\downarrow} - \text{H.c.}) \propto \sigma_y \]

explicitly breaking chiral symm.

\[ U_S = \sigma_y \]
\[ U_S H_0(k) U_S^\dagger = -H_0(k) \]
\[ U_S H_{SB} U_S^\dagger = +H_{SB} \]

gapped phase at \( \nu = \frac{1}{2} \) & critical \( U/t \)

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&

double degeneracy with PBC

\[ \delta_{\text{spin,} \alpha} = E_\alpha(N) - E_1(N) \]

Twisted PBC \( t \rightarrow t \exp(i\varphi/L) \)

\[ \mathcal{W} = \frac{i}{\pi} \int_0^{2\pi} d\varphi \ Tr[\langle \Psi_\alpha(\varphi)|\partial_\varphi|\Psi_\beta(\varphi)\rangle] \]

F. Wilczek & A. Zee, PRL 52, 2111 (1984)
generalized SPT invariant destroyed by
\[ \hat{H}_{SB} = iM \sum_j \left( \hat{c}^\dagger_{j,\uparrow} \hat{c}_{j,\downarrow} - \text{H.c.} \right) \propto \sigma_y \]

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gapped phase at \( \nu = \frac{1}{2} \) & critical \( U/t \)
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\&
double degeneracy with PBC
\[ \delta_{\text{spin},\alpha} = E_\alpha(N) - E_1(N) \]
Topological CDW

Reminiscent edge state physics, however not at zero energy…

$U = 0$

$\Delta \epsilon = 0$

$U = t$

$\Delta \epsilon = 10^{-4}$

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Fractional topological phases in atomic two-leg ladders
Open directions

- Bosonization (not shown) predicts a **devil’s staircase** at filling $1/(q+1)$ for int. range $q$
  
  e.g., $\nu = 1/3$ for $U = 2V = t$

- Up to now, BDI/AII/spatial inversion SPT’s: other classes?
  Extension to higher-dim. flat-band topological systems?

- apparent absence of bulk-edge correspondence at zero energy: why?

- experimental detection via mean chiral displacement or similar?
General Picture

Geometrical constraints + Background gauge fields

↓

Frustration, large degeneracy, single-particle topological character, flat-band dispersion + Interactions btw. constituents

Fractional topological phases
fermions
range q => fraction 1/(q+1)

Collective transport properties: here pair-transport in bosons, etc.

see works by Tovmasyan, Peotta, Huber, Törma (& others) for enhanced fermionic pairing, too
The Model: AB cages

\[ \hat{H}_0 = -J \sum_{j, \ell} \sum_{\alpha, \beta} T^{(\ell)}_{\alpha, \beta} \hat{b}^\dagger_{j+\ell, \alpha} \hat{b}_{j, \beta} \]
repulsion dominates. The case of no frustration has been examined extensively and as the ratio of the tunnelling parameter – 4 to the effective mass. For perfect frustration, when \( \tau = 0, \pm 1 \), single particle transport is absent and degeneracy of the low-lying entanglement spectrum levels reveal their fundamental di

\[ \hat{H}_0 = -J \sum_{j, \ell} \sum_{\alpha, \beta} T^{(\ell)}_{\alpha, \beta} \hat{b}_{j+\ell, \alpha}^\dagger \hat{b}_{j, \beta} \]

\[ T^{(0)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T^{(+1)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & e^{i\phi} \\ 0 & 0 & 0 \end{pmatrix}, \quad T^{(-1)} = (T^{(+1)})^\dagger \]

**Fully flat-bands (all!) at half-flux!**

Aharonov-Bohm interference


\[ E(k) = 2J\tau \sqrt{1 + \cos \left( k - \frac{\phi}{2} \right) \cos \left( \frac{\phi}{2} \right)} \]

\( \tau = 0, \pm 1 \)

Sub-lattice symmetry

\[ \Gamma H_0(k) \Gamma = -H_0(k) \]

\[ \Gamma = \text{diag}\{-1, +1, -1\} \]

No quantized topological invariant, though it appears in squared \( H \) ...

Kremer, et al., arXiv:1805.05209
Experimental realisations

Josephson Junction Arrays


Cold atoms !?

- real space: superlattices or DMD or synthetic dimensions (e.g., angular momentum!)
- shaking of single links or laser-assisted tunneling
- tunable interactions
- ... bosons / fermions ...

Pelegrí, et al., arXiv:1807.08096

Photonic Waveguides

Mukherjee, et al., arXiv:1805.03564

Kremer, et al., arXiv:1805.05209

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PRB 98, 184508 (2018)

Rhombi-Chain Bose-Hubbard: frustration & interactions
The Model: $\mathbb{Z}_2$ symmetry

Exotic Hubbard model with emergent local parity conserved!

If a gapless liquid exists, it should be made of pairs!


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PRB 98, 184508 (2018)

Rhombi-Chain Bose-Hubbard: frustration & interactions
The phase diagram

\[ \frac{\langle n_j \rangle}{\pi} = 1 \]

\[ J/U \]

<table>
<thead>
<tr>
<th></th>
<th>( \langle b_j^\dagger b_{j+r} \rangle )</th>
<th>( \langle (b_j^\dagger)^2 (b_{j+r})^2 \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI</td>
<td>( e^{-r/\xi} )</td>
<td>( e^{-2r/\xi} + \ldots )</td>
</tr>
<tr>
<td>LL</td>
<td>( r^{-\kappa_s} )</td>
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\( Q_1 \): how robust is the PLL phase?

\( Q_2 \): some Ising transition between PLL & LL?

\( b_j \sim \tilde{b}_j \sigma_j^z \)
The phase diagram

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**Q1:** how robust is the PLL phase?

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$b_j \sim \tilde{b}_j \sigma_j^z$
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Q1: how robust is the PLL phase?

Q2: some Ising transition between PLL & LL?

$b_j \simeq \tilde{b}_j \sigma_j^z$

Unfortunately compatible with some $|\pi - \phi_c| \simeq e^{-J/U}$

[or other fast decays…]

Higher filling helps a bit …

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PRB 98, 184508 (2018)

Rhombi-Chain Bose-Hubbard: frustration & interactions
Traditional observables

Correlation matrices

\[ \eta_{(s,i,i+r)}^{[s]} = \text{spec} \left( \langle \hat{b}_i^\dagger \hat{b}_{i+r,\beta} \rangle \right) \]

\[ \eta_{(p,i,i+r)}^{[p]} = \text{spec} \left( \langle \hat{b}_i^\dagger \hat{b}_{i+r,\beta} \rangle^2 \right) \]

Peaked structure factor at \( k = \pi \)

\[ \zeta_{(s/p)}^{[s/p]}(k) = \text{spec} \left( \text{FT} \left( \langle \ldots [s/p] \ldots \rangle \right) \right) \]
Traditional observables

correlation matrices

$$\eta_{\max}(r) \approx r^{-\kappa}$$
$$2\kappa = K$$

peaked structure factor at $k = \pi$

$$\zeta_{s/p}^{[s/p]}(k) = \text{spec} \left( \text{FT}(\langle \ldots [s/p] \ldots \rangle) \right)$$
Entanglement properties

\[ S_M(m) = \frac{c}{6} \ln \left[ \frac{M}{\pi} \sin \left( \frac{\pi m}{M} \right) \right] + A + O \left( \frac{1}{m} \right) \]

Calabrese & Cardy, JSTAT P06002 (2004)

same CFT central charge BUT different entanglement spectrum

working tool: Tensor Networks

entanglement quantities are gaining experimental relevance …

Open directions

• comparison of PLL robustness to Creutz-ladder flat bands: any relation to topological invariants? Ising model!? (we find no c=3/2 line)

  Tovmasyan, et al., PRB 88, 220510 (2013)
  Takayoshi, et al., PRA 88, 063613 (2013)
  Jünemann, et al. (MR), PRX 7, 031057 (2017)

• exploration of “square root TI” character: quantized pumping schemes?

  $H_{\phi \pi} \rightarrow \gamma_{Zak} \notin \{0, \pi\}$
  $H_{\phi \pi}^2 = H_{\text{trivial}} \oplus H_{\text{SSH}}$

  Kremer, et al., 1805.05209

  Tausendpfund, MR, unpublished

• revival of 2D AB-cages: glassy phase at hand!?

  MR, et al., PRB 73, 144511 (2006)

• many-body dynamics in presence of extensive local symmetries: MBL-like?


Atomtronics
Benasque 2019
matteo.rizzi@fz-juelich.de
PRB 98, 184508 (2018)
Rhombi-Chain Bose-Hubbard: frustration & interactions
DESIGNING ARTIFICIAL QUANTUM MATTER
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Palacio Miramar, Donostia-San Sebastián

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+ N. Tausendpfund & J. Nothhelfer

all of you for your attention!

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