Ultracold atoms with orbital angular momentum
A single ring for quantum sensing and a lattice of rings for quantum simulation

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Atomtronics Workshop, Benasque, May 8, 2019
Ultracold atoms
Quantum transport
Atomtronics in ring traps
Orbital angular momentum states
Complex tunneling and edge states

Laser-matter interaction
Sub-wavelength localization and nanoscopy
Atomic frequency combs
Spin-orbit coupling

Light propagation in coupled optical waveguides
Dark and bright OAM modes
SUSY techniques for mode filtering

Conical Refraction
Fundamentals: theory and experiment
Applications: trapping microparticles and BECs

Ultracold Atoms in Optical Lattices
Simulating Quantum Many-Body Systems
Maciej Lewenstein, Anna Sanpera, and Verónica Ahufinger
Oxford University Press (2012)

Applied Bohmian Mechanics
From Nanoscale Systems to Cosmology
Eds: Xavier Oriols and Jordi Mompart
Pan Stanford Publishing (2012)
BEC + OAM + Ring trap

Quantum sensing using imbalanced counter-rotating BEC modes

- **Magnetic fields with BECs** are measured by using stimulated Raman transitions [1], performing Bragg interferometry after free fall [2], measuring Larmor precession in spinor BECs [3], or looking at density fluctuations [4].

- **Rotations with BECs** can be measured taking profit of the Sagnac effect [5], with ring geometries being specially well suited for this purpose [6].

Ultracold atoms + OAM + Two rings + Tunneling

Geometrically induced complex tunneling with OAM states

Suitable forcing of the optical lattice

Combination of radio frequency and Raman fields that couple to the internal states of the atom

See also the talk by David Guéry-Odelin in Atomtronics 2019
Ultracold atoms + OAM + Lattice of rings + Tunneling

Topological edge states with ultracold atoms carrying OAM

Aharanov-Bohm caging with ultracold atoms carrying OAM

Complex tunnelings play a key role in quantum simulation. To cite a few examples, the realization of the Hofstader [1], XY spin [2], and Haldane [3] models. Through the synthetic dimension approach [4], demonstration of chiral edge states in bosonic [5] and fermion [6] ladders.

See also the talks by Roberta Citro and by Matteo Rizzi, and the poster by T. Haug et al., in Atomtronics 2019

A single ring for quantum sensing

- Two-dimensional BEC with $N$ atoms in a ring trap

OAM states:  \( \langle \tilde{r} | l, \pm \rangle = \phi_{l\pm}(\tilde{r}) = \phi_{l\pm}(r, \varphi) = f(r)e^{\pm il\varphi} \)

Initial state: imbalanced superposition of $l = \pm 1$ states

\[
\Psi(\tilde{r}, t = 0) = \sqrt{p_{1+}}\phi_{1+}(\tilde{r}) + \sqrt{p_{1-}}\phi_{1-}(\tilde{r}) \\
= f(r)(\sqrt{p_{1+}}e^{i\varphi} + \sqrt{p_{1-}}e^{-i\varphi})
\]

The density profile has a minimal density line due to quantum interference between the counter-rotating modes.
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- Numerical integration of the 2D GPE

Ring potential in the x-y plane: \( V(r) = \frac{1}{2} m \omega^2 (r - R)^2 \)

Harmonic potential in z: \( \omega_z \gg \omega \)

Time and space units: \( 1/\omega \) and \( \sigma = \sqrt{\frac{\hbar}{m\omega}} \)

Dimensionless 2D GPE (mean-field regime):

\[
i \frac{\partial \psi}{\partial t} = H \psi = \left[ -\frac{\nabla^2}{2} + V(r) + g_{2d} |\psi|^2 \right] \psi
\]

with \( g_{2d} = Na_s \sqrt{\frac{8\pi m\omega_z}{\hbar}} \)

Example: \( R = 5, g_{2d} = 1, p_{1+} = 0.7 \)
Example: $R = 5, \ g_{2d} = 1, \ p_{1+} = 0.7$

The minimal density line rotates at a constant speed, which depends on $g_{2d}$, and the populations of the OAM modes remain almost constant.
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• Expansion of the BEC wavefunction in OAM modes

Ansatz: \( \Psi = \sum_m a_m(t) \phi_m(r, \varphi) \)

Non-linear coupled equations:
\[
\frac{i}{\hbar} \frac{da_l}{dt} = \mu_l a_l + U \sum_{m \neq m'} a_m a_{m'}^* a_{(l+m' - m)}
\]

with \( U = g_{2d} \int |f(r)|^4 dr \equiv g_{2d} \mathcal{I} \) and \( H \phi_l(\vec{r}) = \mu_l \phi_l(\vec{r}) \)

The dynamics does not couple odd with even OAM modes

For small \( g_{2d} \) values, a four state model (FSM) with \( |l|=1,3 \) is enough to reproduce the previously shown 2D GPE simulations
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Example: $R = 5, g_{2d} = 1, \rho_{1+} = 0.7$

In the regime $U \ll (\mu_3 - \mu_1)$ assuming $|a_{1+}|^2 = \rho_{1+} = ct, |a_{1-}|^2 = \rho_{1-} = ct$
and neglecting $\mathcal{O}(a_{3\pm}^2)$

Rotation frequency of the minimal density line
$\Omega_{FSM} = \frac{U(p_{1+} - p_{1-})}{2(1 + \frac{U}{\mu_3 - \mu_1})}$

$U = g_{2d} \int |f(r)|^4 d\vec{r} \equiv g_{2d} \mathcal{I}$
$H \phi_l(\vec{r}) = \mu_1 \phi_l(\vec{r})$
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- Sensing of two-body interactions

Rotation frequency of the minimal density line

\[ \Omega_{\text{FSM}} = \frac{U(p_{1+} - p_{1-})}{2(1 + \frac{U}{\mu_3 - \mu_1})} \]

\[ U = g_{2d} \int |f(r)|^4 d\vec{r} = g_{2d} \mathcal{I} \]

\[ H_{\phi_l}(\vec{r}) = \mu_l \phi_l(\vec{r}) \]

\[ g_{2d} = \frac{1}{\mathcal{I}} \left( \frac{2\Omega}{(p_{1+} - p_{1-}) - 2\frac{\Omega}{\mu_3 - \mu_1}} \right) \]

All the quantities on the right hand side can be measured by imaging the density profile of the BEC. Experimental protocol presented in Pelegrí et al., NJP 20, 103001 (2018).
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Comparative between 2D GPE simulations and the FSM
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• Sensing of magnetic fields

Assume that the scattering length $a_s$ can be manipulated with an external magnetic field $B$, e.g., close to a Feshbach resonance.

Then, $g_{2d} = Na_s \sqrt{\frac{8\pi m \omega_z}{\hbar}}$ and $U = g_{2d} \int |f(r)|^4 d\mathbf{r} \equiv g_{2d} \mathcal{I}$ will be also $B$-dependent.

Recalling that $\Omega_{\text{FSM}} = \frac{U(p_{1+} - p_{1-})}{2(1 + \frac{U}{\mu_3 - \mu_1})}$

Then:

\[
\frac{d\Omega_{\text{FSM}}}{dB} = \frac{(p_{1+} - p_{1-})\mathcal{I}N \sqrt{\frac{8\pi m \omega_z}{\hbar}}}{2(1 + \frac{U(B)}{\mu_3 - \mu_1})^2} \frac{da_s}{dB}
\]

\[
\Delta B_{\text{th}} = \frac{\sqrt{\frac{8\hbar}{\pi m \omega_z}}}{(p_{1+} - p_{1-})\mathcal{I}N} \frac{1}{\frac{da_s}{dB}} \Delta \Omega.
\]
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- Sensing of rotations

In a frame rotating at an angular speed $\Omega_{\text{ext}}$, the dimensionless 2D GPE reads:

$$i \frac{\partial \Psi}{\partial t} = \left[ -\frac{\nabla^2}{2} + V(r) + g_{2d} |\Psi|^2 + i\Omega_{\text{ext}} \hbar \frac{\partial}{\partial \varphi} \right] \Psi$$

$\Omega_{\text{ext}}$ can be measured as the difference between the measured speed, $\Omega$, and the one expected from the FSM expression, $\Omega_{\text{FSM}}$

$$\Omega_{\text{ext}} = \Omega - \Omega_{\text{FSM}}$$
Two rings: complex tunneling from OAM states

- One atom with localized OAM in two tunnel-coupled identical cylindrically symmetric potentials

$$\Psi_j^\ell(r_j, \phi_j) = \langle \vec{r} | j, \ell \rangle = \psi(r_j)e^{i\ell(\phi_j - \phi_0)}$$

Example: single atom in the left trap with $\ell = 1$

$$\Psi_L^1(r_L, \phi_L) = \Psi_L^1(\phi_0 = 0)e^{-i\phi_0}$$
Two rings: complex tunneling from OAM states

Symmetries for two tunnel-coupled identical cylindrically symmetric potentials

Two symmetries for \( V(x,y) \):

- **x-Mirror**  \( M_x : y \leftrightarrow -y \)
- **y-Mirror**  \( M_y : x \leftrightarrow -x \)

The trapping potential and, therefore, the Hamiltonian are invariant under the \( x \)- and \( y \)-mirror transformations

\[
H \Psi(x, y, t) = \left[ -\frac{\hbar^2}{2m} \nabla_{\perp}^2 + V(x, y) \right] \Psi(x, y, t)
\]

\[
[H, M_x] = [H, M_y] = 0
\]

Tunneling amplitudes

\[
J_{j,n}^{k,p} = \langle k, p | H | j, n \rangle = \langle k, p | M_y^{-1} M_y H | j, n \rangle = \langle k, p | M_y^{-1} H M_y | j, n \rangle
\]

\( j, k = L, R \quad n, p = \text{winding number} \quad \text{How do they transform?} \)
Two rings: complex tunneling from OAM states

Example: single atom in the left trap with $m = 1$, i.e., $|L,1\rangle$

\[ y\text{-mirror: } x \leftrightarrow -x \]
\[ M_y |L,1\rangle = |R,-1\rangle e^{i(\pi - \phi_0)} e^{-i\phi_0} = -|R,-1\rangle e^{-2i\phi_0} \]

\[ x\text{-Mirror: } y \leftrightarrow -y \]
\[ M_x |L,1\rangle = |L,-1\rangle e^{-2i\phi_0} \]

\[ j,k = L,R \quad n,p = \text{winding number} \]
\[ J_{j,n}^{k,p} = \langle k, p | H | j, n \rangle = \langle k, p | M_y^{-1} HM_y | j, n \rangle = \langle k, p | M_x^{-1} HM_x | j, n \rangle \]
\[ J_{j,n}^{k,p} = (J_{j,n}^{k,p})^* \quad \text{Hermiticity} \]

For $n,p = \pm \ell$
\[ J_{L,\ell}^{L,-\ell} = |J_{L,\ell}^{L,-\ell}| e^{2i\ell\phi_0} \quad \rightarrow \quad J_1 \]
\[ J_{L,\ell}^{R,\ell} \quad \rightarrow \quad J_2 \]
\[ J_{L,\ell}^{R,-\ell} = |J_{L,\ell}^{R,-\ell}| e^{2i\ell\phi_0} \quad \rightarrow \quad J_3 \]
Two rings: complex tunneling from OAM states

Tunneling amplitudes as a function of the traps separation $d$ (in harmonic oscillator units)

Each trap potential: $V(r) = \frac{1}{2} m \omega^2 (r - R)^2$

For $|\ell| = 1$

(a) $R = 0$ (2D harmonic potentials)

(b) $R = 5 \sigma$
Three rings: complex tunnelings from OAM states

For $|\ell| = 1$

$$\hat{H} = h \begin{pmatrix}
0 & J_1 & J_2 & J_3 \\
J_1 & 0 & J_3 & 0 \\
J_2 & J_3 & 0 & J_2 \\
0 & 0 & J_2 & 0 \\
0 & 0 & J_3 e^{2i\Theta} & 0 \\
0 & 0 & J_3 e^{-2i\Theta} & J_2 \\
0 & 0 & J_1 e^{-2i\Theta} & 0 \\
J_1 (1 + e^{-2i\Theta}) & J_2 & J_3 e^{2i\Theta} & J_2 \\
J_3 e^{-2i\Theta} & 0 & J_1 e^{2i\Theta} & 0 \\
J_2 & J_1 e^{2i\Theta} & 0 & J_2 \\
J_2 & J_1 e^{-2i\Theta} & 0 & J_2 \\
J_3 e^{2i\Theta} & 0 & J_1 e^{-2i\Theta} & 0 \\
J_3 e^{-2i\Theta} & 0 & J_1 e^{2i\Theta} & 0 \\
0 & 0 & J_2 & J_2 \\
0 & 0 & J_3 e^{2i\Theta} & 0 \\
0 & 0 & J_3 e^{-2i\Theta} & J_2 \\
0 & 0 & J_1 e^{-2i\Theta} & 0 \\
\end{pmatrix}$$

where

$$J_1 = J_{L,1}^{L,-1}$$
$$J_2 = J_{L,1}^{C,1}$$
$$J_3 = J_{L,1}^{C,-1}$$

and

$|L+\rangle$
$|L-\rangle$
$|C+\rangle$
$|C-\rangle$
$|R+\rangle$
$|R-\rangle$
A lattice of rings for quantum simulation

Ultracold gas of non-interacting particles in a quasi-1D optical lattice with a diamond chain geometry. Atoms loaded into the manifold of $|l| = 1$ OAM states.

\[
\hat{\Psi} = \sum_{i} \sum_{\alpha=\pm} \phi^a_i(r_i, \phi_{a_i}) \hat{a}^i + \phi^b_i(r_i, \phi_{b_i}) \hat{b}^i + \phi^c_i(r_i, \phi_{c_i}) \hat{c}^i
\]

with \( \phi^j_i(r_j, \phi_{j_i}) = \langle \vec{r}|j_i, \pm \rangle \)
A lattice of rings for quantum simulation

\[ \hat{H} = J_1 \sum_{\alpha=\pm} e^{i\pi} \hat{b}_{\alpha}^{1\dagger} \hat{b}_{-\alpha}^{1} + \hat{c}_{\alpha}^{1\dagger} \hat{c}_{-\alpha}^{1} \]

\[ + J_2 \sum_{i=1}^{N_c} \sum_{\alpha=\pm} \left[ \hat{a}_{\alpha}^{i\dagger} \left( \hat{b}_{\alpha}^{i} + \hat{b}_{\alpha}^{i+1} + \hat{c}_{\alpha}^{i} + \hat{c}_{\alpha}^{i+1} \right) \right] + \text{h.c.} \]

\[ + J_3 \sum_{i=1}^{N_c} \sum_{\alpha=\pm} \left[ \hat{a}_{\alpha}^{i\dagger} \left( e^{i\pi} \hat{b}_{-\alpha}^{i} + \hat{b}_{-\alpha}^{i+1} + \hat{c}_{-\alpha}^{i} + e^{i\pi} \hat{c}_{-\alpha}^{i+1} \right) \right] + \text{h.c.} \]
A lattice of rings for quantum simulation

- Band structure

Six states per unit cell ➡️ six energy bands, which appear in degenerate pairs

\[ E_1(k) = E_2(k) \]
\[ E_3(k) = E_4(k) \]
\[ E_5(k) = E_6(k) \]

\( l = 1 \) manifold. \( d = 3.5 \) (h.o. units).

\( l = 1 \) manifold. \( d = 6.0 \) (h.o. units).

Gap of size \( 2\sqrt{2}J_2 \) and all bands dispersionless in the limit \( J_2 = J_3 \)
A lattice of rings for quantum simulation

- **Exact diagonalization**

20 unit cells. $d = 6.0$ (h.o. units).

Flat band states, with no population on the A states.

In-gap states, localized at the right edge and topologically non-trivial.
A lattice of rings for quantum simulation

A basis rotation decouples the model into two identical diamond chains with one state per site

\[ |D_i\pm\rangle = \frac{1}{\sqrt{2}} (|C_i+\rangle \pm |B_i+\rangle) \]

\[ |F_i\pm\rangle = \frac{1}{\sqrt{2}} (|C_i-\rangle \pm |B_i-\rangle) \]

- The decoupling into two identical chains explains the degeneracy in the original model.
- The net \( \pi \) flux through the plaquettes accounts for the gap opening [1].

A lattice of rings for quantum simulation

By performing a second basis rotation, the system is further mapped into a modified Su-Schrieffer-Hegger (SSH) model (similar mapping for the A' chain)

\[ |G_i+\rangle = \frac{1}{\sqrt{J_2^2 + J_3^2}} (J_2 |D_i+\rangle + J_3 |F_i-\rangle) \]
\[ |G_i-\rangle = \frac{1}{\sqrt{J_2^2 + J_3^2}} (J_3 |D_i+\rangle - J_2 |F_i-\rangle) \]

Example with 3 cells

\[ \sqrt{2} \sqrt{J_2^2 + J_3^2} \]
\[ 2\sqrt{2} \frac{J_2 J_3}{\sqrt{J_2^2 + J_3^2}} \]
\[ \sqrt{2} \frac{J_2^2 - J_3^2}{\sqrt{J_2^2 + J_3^2}} \]
A lattice of rings for quantum simulation

The topological nature of the edge states can be shown by performing a third mapping to a diamond chain with alternating hoppings [1].

Topological edge states persist in the entire $J_2 \neq J_3$ domain (except for the gap closing points $J_2/J_3 = 0$).

Zero-energy states can also be constructed in the case.

In the limit $J_2 = J_3$, there is Aharonov-Bohm caging [2].

This system is an example of square-root topological insulator [3].

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**Example with 3 cells**

Limit $J_2 = J_3 \equiv J$

Zero-energy bulk state:

$$\frac{1}{\sqrt{2}}(|G_2+\rangle - |G_3-\rangle)$$

Topological edge states:

$$\frac{1}{\sqrt{2}}(|G_3+\rangle + |A_3+\rangle) \rightarrow E = 2J$$

$$\frac{1}{\sqrt{2}}(|G_3+\rangle - |A_3+\rangle) \rightarrow E = -2J$$

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A lattice of rings for quantum simulation

- Aharanov-Bohm caging

Spatial confinement of initial wave packets composed of states $|A_j, \pm\rangle$ due to quantum interference

For $J_2 = J_3$
CONCLUSIONS

Ultracold atoms carrying OAM in ring traps constitute a very interesting platform for quantum sensing and quantum simulation (topology).

Examples:

Quantum sensing of non-linear interactions, magnetic fields, and rotations with an imbalanced superposition of the OAM modes of a BEC.

Complex tunnelings due to OAM states gives rise to non-trivial topology and dynamics (edge states and Aharanov-Bohm caging) in lattices of rings.

On progress:

- Interacting bosons in the Mott regime in a quasi 1D diamond lattice
- Corner states in 2D optical lattices
Thank you for your attention!