Using the superconducting proximity effect to uncover topological materials: the case of Bismuth

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Murani et al, Phys.Rev. B 2017
Murani et al, Nature Comm. 2017
Schindler et al, Nature Phys. 2018
Murani et al, PRL 2019
Outline

- Bismuth as it is understood today:
  - a higher order topological insulator

- 3 proximity effect experiments in S/Bi nanowire/S junction
  - Field-controlled Interference probe supercurrent-carrying paths: edge states
  - (dc) Supercurrent versus Phase relation (CPR) probe Andreev spectrum: edge states are ballistic
  - High frequency (ac) susceptibility $\chi = dI/d\phi$ probes topological protection
Bismuth, a semi-metal

**Bulk Bi:** semi-metal with huge spin-orbit and $\lambda_F \approx 50$ nm

$\rightarrow$ No bulk states left in structures smaller than 50 nm

**Bi surfaces:** $\lambda_F \approx 1$ nm, $E_{SO} \sim E_F \sim 100$ meV, $g_{\text{eff}}: 1 \sim 100$

Photoemission shows that surface states are spin-split due to high spin-orbit

Better yet: Some surfaces are topological
(111) Bi bilayers are predicted to be 2D topological insulators

- (111) Surface = buckled honeycomb
  ≈ graphene with spin-orbit!
⇒ predicted 2D topological insulator

Murakami, 2006
Liu & Allen, 1991

3 edge states predicted

Whether these 1D states are topological is debated
Spin orbit interactions and Topological insulators

Depending on the crystal symmetry:
Possible electronic band inversions

In 2D:
Formation of 1D spin polarized helical edge states
Quantum Spin Hall state
Protected from non-magnetic disorder by SO
Forbidden back scattering without spin flip

\[ V_{SO} = \frac{\hbar}{4m^2c^2} s \cdot (\nabla V \times p) \]
Higher order Topological Insulators

3D topological insulator
3D insulating bulk
2D Conducting surfaces

2D topological insulator
2D insulating bulk
1D conducting edges

Second Order Topological Insulator
3D insulating bulk
2D insulating surfaces
1D conducting helical « hinges »
Bismuth: a High Order Topological Insulator

Conclusion: There is a helical edge state that winds around the crystal.

Topological Quantum chemistry

⇒ 6 hinge states (between six surfaces)

⇒ 3 Edge states on each free (111) surface

Schindler et al, Nature Phys. 2018
Our samples: Monocrystalline Bismuth nanowires

**Growth**: Sputtering on a hot surface, High resolution TEM

High quality single crystals
$\varnothing \sim 100 \text{ nm}$

Select desired orientation using EBSD

In general rhombic section

Select nanowires with (111) top surface
Diffusive surfaces states carry the normal current
We will see that all the supercurrent is carried by edge ballistic states
Proximity effect experiments in S/Bi nanowire/S junction

1. Field-controlled Interference probe supercurrent-carrying paths: edge states

2. (dc) Supercurrent versus Phase relation (CPR) probe Andreev spectrum: edge states are ballistic

3. High frequency (ac) susceptibility $\chi=\frac{dI}{d\phi}$ probes topological protection
Superconducting contacts to exploit macroscopic wavefunction (and its phase): Interference experiments will reveal supercurrent paths

Gauge invariant Josephson relation:

\[ I(\delta) = I_0 \sin \left( \delta - \frac{2e}{\hbar} \int \mathbf{A} \cdot d\mathbf{l} \right) \]

Critical current \( I_c(B) = \text{max of integral over all supercurrent paths: interference terms!} \)

\[ I_c(B) = \left| \int_{-W/2}^{W/2} J(x) \cdot e^{2\pi i LBx/\Phi_0} dx \right| \]

Critical current \( I_c(B) = |\text{Fourier transform of supercurrent distribution } J(x)| \)
Critical supercurrent reveals paths taken by pairs (via interference)

Many diffusive paths
Gaussian decay

Many paths WIDE (ballistic or diffusive)

Only 2 paths (edges)
ballistic

Several diffusive paths
Squid-like

« Fraunhoffer pattern »

« Squid-like »

Many paths NARROW, diffusive

\( I_c^{\text{max}}(B) \)

\( \Phi_0 / \text{sample area} \)

\( \Phi_0 / \text{edge state area} \)

\( \Phi_0 / \text{sample area} \)

S/Non topological HgTe QW/S, Hart 2014

S/Topological/S HgTe QW, Hart 2014

Chiodi 2012

S/Au wire/S
Contacting our Bi(111) wires with focused ion beam-assisted deposition to induce superconductivity

Kasumov 2005

Superconducting electrodes:
- C and Ga-doped amorphous W
- ~ 200 nm thick and wide
- Great superconducting properties: $T_c \approx 4$ K, $\Delta \approx 0.8$ meV, $H_c \approx 12$ Tesla!
Field-dependence of critical supercurrent reveals paths taken by pairs

- Oscillations with field: very few states
- Field direction dependence and period: supercurrent travels at the two acute wire edges
- High field decay scale (oscillations up to 10 Tesla in some samples): narrow channels (nm!).
- High critical current: well transmitted channels.
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Usual two contact SNS configuration

\[ I_c = \max I(\varphi), \varphi \text{ not controlled} \]

Better than critical current: supercurrent versus phase relation

\[ I(\varphi) = ? \]

Better: Ring geometry allows «phase biasing»

\[ \varphi = \frac{-2\pi \Phi}{\Phi_0} \]

\[ \Phi = B S \]

\[ \varphi \text{ controlled, proportional to applied magnetic flux} \]

\[ I(\varphi) = ? \]

\[ I(\varphi) \text{ depends on the transport regime in the N (diffusive, ballistic)} \]
Andreev Bound States in a phase-biased SNS junction

Resonance condition on accumulated phase:
Andreev Bound States with eigenenergies $\epsilon_m$.

Andreev bound states carry the supercurrent. Spectra and supercurrent depend on the transport regime in N.
Andreev spectrum and supercurrent in short ballistic junction

\[ I = \sum_{\epsilon_n} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n) \]

\[ \epsilon_n(\varphi) \sim \text{branches of } \cos(\varphi/2) \]

\[ \varphi \]

\[ \varphi = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi \]

\[ \frac{2\arccos \frac{\epsilon}{\Delta_0} \pm \Delta\varphi}{2\pi m} \]

\[ \text{supercurrent} \]

\[ \text{I} \sim \text{branches of } \sin(\varphi) \text{ with jump at } \pi \]
Andreev spectrum and supercurrent in long ballistic junction

\[ L \gg \xi_s = \frac{hV_F}{\Delta} \]

Andreev reflection

\[ 2\epsilon L_N \frac{\epsilon}{hv_F} - 2 \arccos \frac{\epsilon}{\Delta_0} \pm \Delta \phi = 2\pi m \]

\( \xi_n(\phi) \sim \phi: \text{linear segments} \)

\[ I(\phi) \sim \text{linear segments with jumps at } \pi \]

\[ I = \sum_{-\infty}^{0} \frac{\partial \epsilon_n}{\partial \phi} f(\epsilon_n) \]

Sawtooth \( I(\phi) \) characteristic of long ballistic junction
Disorder softens the proximity effect

Short ballistic SNS junction (perfect Andreev reflection)

Long ballistic SNS junction

Disordered I = I_0 (l_e^2/L^2)

Ballistic I_0 = M e \nu_F / L

Short disordered 1 - τ proba to backscatter

Long disordered (diffusive)

1 − τ proba to backscatter

 nuevos resultados
Supercurrent Vs phase relation can pinpoint transport regime

Our goal is to measure such a « Current-Phase Relation »
Current-phase measurement with an asymmetric SQUID

Della Rocca et al 2007

\[ \Phi = B \cdot S \]

\[ I_c = I_{c1} \sin \varphi_1 + I_{c2} f(\varphi_2) \]

\[ \varphi_1 - \varphi_2 = -2\pi \Phi / \Phi_0 \]

\[ I_c \text{ achieved for } \varphi_1 = \pi / 2 \]

Critical current of asymmetric SQUID yields current-phase relation of junction with smallest critical current
Measurement of current-phase relation to test channels that carry the supercurrent (on very same sample)

Add superconducting constriction in parallel

Build an asymmetric SQUID to measure the $I(\phi)$ relation
Check method with the Current Phase Relation of a SIS tunnel junction

Tunnel junction has a sinusoidal Current Phase relation ✔
Current Phase relation of S/Bi/S: switching current as a function of magnetic flux

\[ B \propto \varphi \]

\[ I_c (\mu A) \]

\[ \varphi = -2\pi BS/\Phi_0 \]

Sawtooth-shaped current phase relation: long ballistic!
Two sawtooths?

Two ballistic edges!
How ballistic are the two paths?

$I(\varphi)$ can be fit with:

$$
\sum \frac{(-1)^n}{n} \sin n\varphi \ e^{-0.15n} + 0.25 \sum \frac{(-1)^n}{n} \sin(1.1 \times n\varphi) \ e^{-0.45n}
$$

$$
\sum \frac{(-1)^n}{n} \sin n\varphi \ e^{-\alpha n} \sim \sum \frac{(-1)^n}{n} \sin n\varphi \ t^{2n}
$$

Channel transmission

Inner edge: channels with $t \approx 0.9$
Outer edge: channels with $t \approx 0.7$
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Consequence of parity protection on current phase relation?

\[ I = \sum_{-\infty}^{0} \frac{\partial \epsilon_n}{\partial \phi} f(\epsilon_n) \]

Difference easy to see?
Supercurrent through QSH edge should be $4\pi$ periodic, whereas $2\pi$ periodicity if ballistic non topological.
But poisoning can return periodicity to $2\pi$

Need to go beyond dc current phase measurements:
Measure high frequency response (especially near crossings) to beat poisoning/relaxation rate: measure at $\omega \gg \gamma_p$!
How to determine the topological character of the Bi SNS junction: finite frequency driving

Finite frequency driving:

$$\varphi(t) = \varphi_{dc} + \varphi_{ac} \cos \omega t$$

Linear response

$$\delta l(\omega) = \chi(\omega) (\varphi_{ac} \exp^{-i\omega t})$$

$$\chi = \chi' + i\chi''$$

Probing dynamics of SNS junctions close to equilibrium
ac susceptibility could distinguish topo/non topological states

...Measure absorption and T dependence

Diagonal absorption

(Imaginary part)

\[ \chi''(\omega) = \frac{\omega^2 \tau_{\text{in}}}{1 + (\omega \tau_{\text{in}})^2} \sum_n \left( \frac{\partial_n f_n}{\tau_n} \right) i_n^2 \]

absorption decreases as T increases
Periodic absorption peaks at $2n+1 \phi_0$ observed in a wide range of frequency: $f_{\text{min}} = 280 \text{MHz}$ to $f_{\text{max}} = 6.8 \text{ GHz}$.

$$d(1/Q) = -\frac{\delta Q}{Q^2} = \frac{L_c^2}{L_R \chi''}$$

**Coupling inductance**
$L_c \sim 100 \text{pH}$

**Resonator inductance**
$L_R \sim 1 \mu\text{H}$

**Resonator inductance**
$L_R \sim 1 \mu\text{H}$

$$L = (2n+1)\lambda_n / 4$$

High frequency experiments: SQUID in a multimode resonator.
Signature of zero energy Andreev level crossing

\[ \chi''_D = \frac{-\omega \tau_{im}}{(1 + \omega^2 \tau_{im}^2)} \frac{i^2(\varphi) df}{d\epsilon} \]

\[ i^\pm = \partial \varepsilon^\pm(\phi) / \partial \phi \]

\[ i^2 \text{ finite} = (ev_F/L)^2 \]

\[ \alpha = ev_F/4L\pi \]

Only adjustable parameter
\[ v_F = 4 \times 10^5 \text{ m/s} \]

Compatible with dc measurements
Compare ac susceptibility of S/Bi/S and S/diffusive Au/S

S/Bi/S

A. Murani, B. Dassonneville

In S/Bi/S:
max absorption at \( \pi \)!

\[ \chi''(\omega) = \frac{1}{1 + (\omega \tau)^2} \sum_n (\partial_\xi f_n)^2 \]

\[ \chi''(\omega) = \frac{1}{1 + (\omega \tau)^2} \sum_n (\partial_\xi f_n)^2 \]

S/diffusive Au/S

In SNS:
zero absorption at \( \pi \)!

6.7 GHz

B. Dassonneville PRL 2013
Experimental conclusion: Probing edge states in bismuth nanowires with mesoscopic superconductivity

Edge states revealed in Bismuth nanowires with (111) surfaces

« Edge » revealed by interference pattern of critical current
« Ballistic edge » revealed by sawtooth-shaped current-phase relation
« Topologically protected edge state » suggested by shape of dissipation peaks in ac response measurement.

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