Coherent quantum phase slips in spatially inhomogeneous Josephson junction chains

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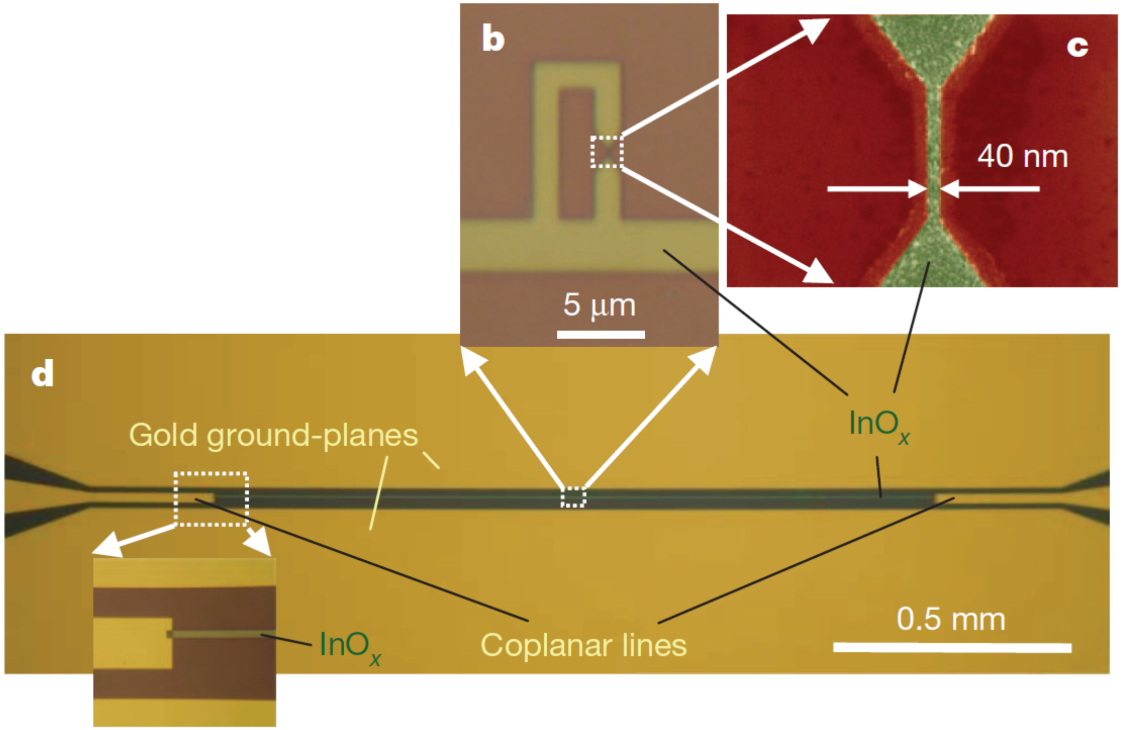
A. E. Svetogorov

Outline

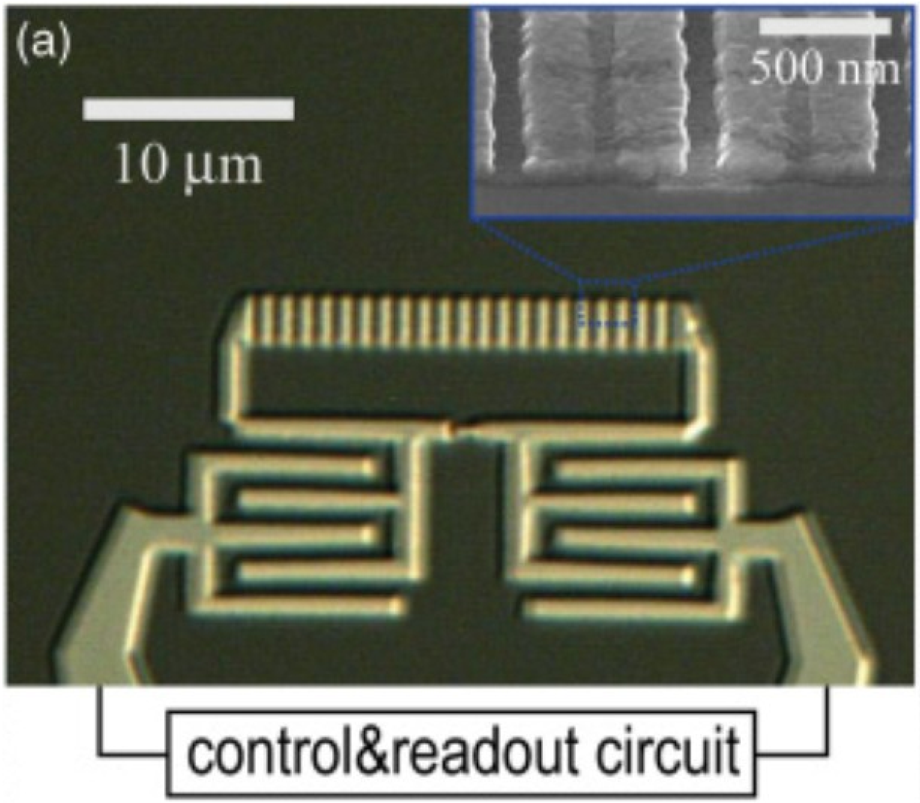
- Superconducting nanowires and Josephson junction chains
- Goldstone modes in a superconductor (= plasma oscillations)
- Coherent quantum phase slips in a Josephson junction
- Coherent QPS in a JJ chain/ring
- Phase normal modes of JJ chains
- Disorder in JJ chains and localization of normal modes
- Effect of spatial modulation on QPS

Superconducting nanowires

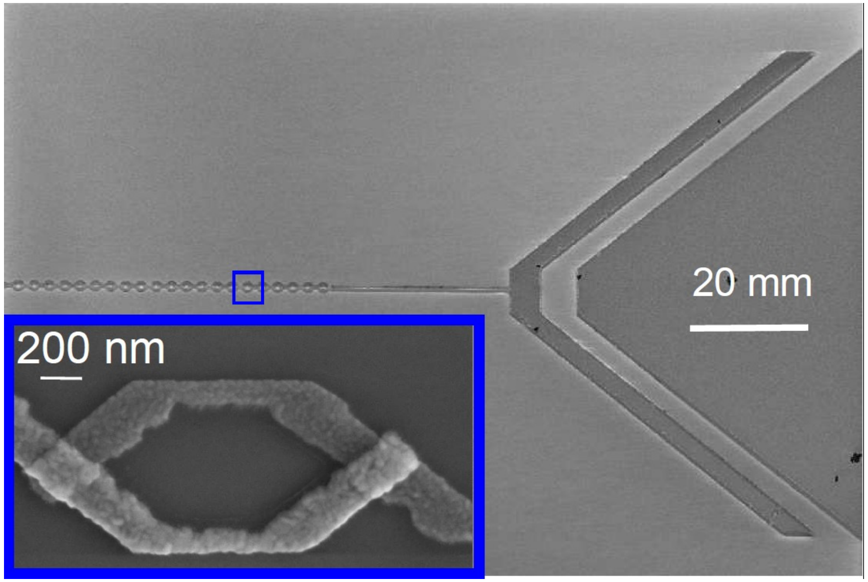


O. V. Astafiev *et al*., *Nature* **484**, 355 (2012)

Josephson junction chains



V. Manucharyan *et al*., PRB **85**, 024521 (2012)



T. Weiβl *et al*., *PRB* **92**, 104508 (2015)

Goldstone modes in a superconductor

Superconductor → complex order parameter $\Delta = \Delta_0 e^{i\varphi}$

- changing $|\Delta|$ costs energy
- changing φ uniformly costs **nothing** [spontaneously broken U(1)]

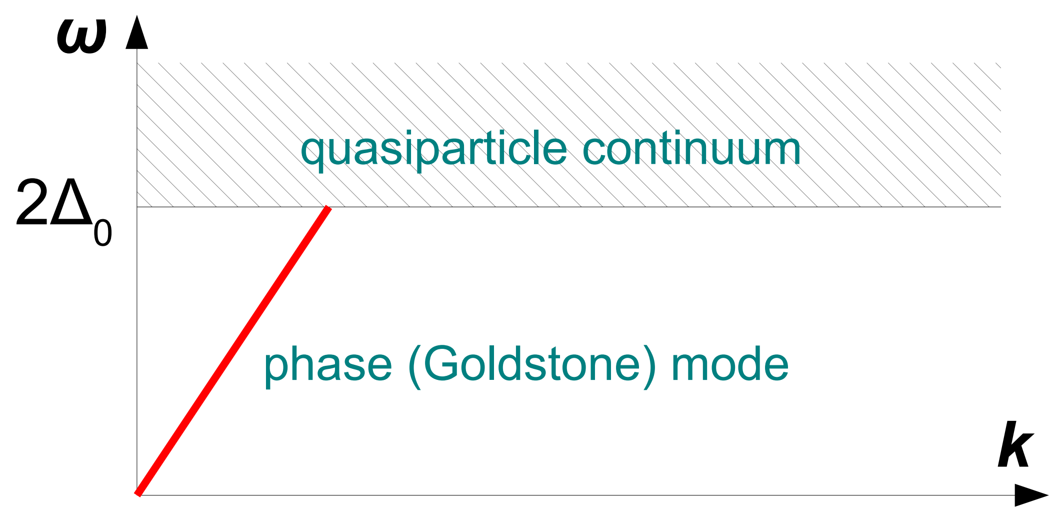
- changing φ **almost** uniformly should cost **little:**

$$\varphi = \varphi_0 e^{i\mathbf{k}\mathbf{r} - i\omega t} \quad \Rightarrow \quad \omega(\mathbf{k}) = \frac{v_F}{\sqrt{3}} |\mathbf{k}|$$

vanishes @ $k \to 0$

Goldstone mode frequency
in a bulk BCS superconductor
with neutral Cooper pairs
Littlewood & Varma, PRB **26**, 4883 (1982)

Excitations in a superconductor
with **neutral** Cooper pairs:



Goldstone modes in a superconductor

$$\frac{\partial\varphi}{\partial t} = 2(\mu - \epsilon_F)$$ chemical potential shift (by gauge invariance)

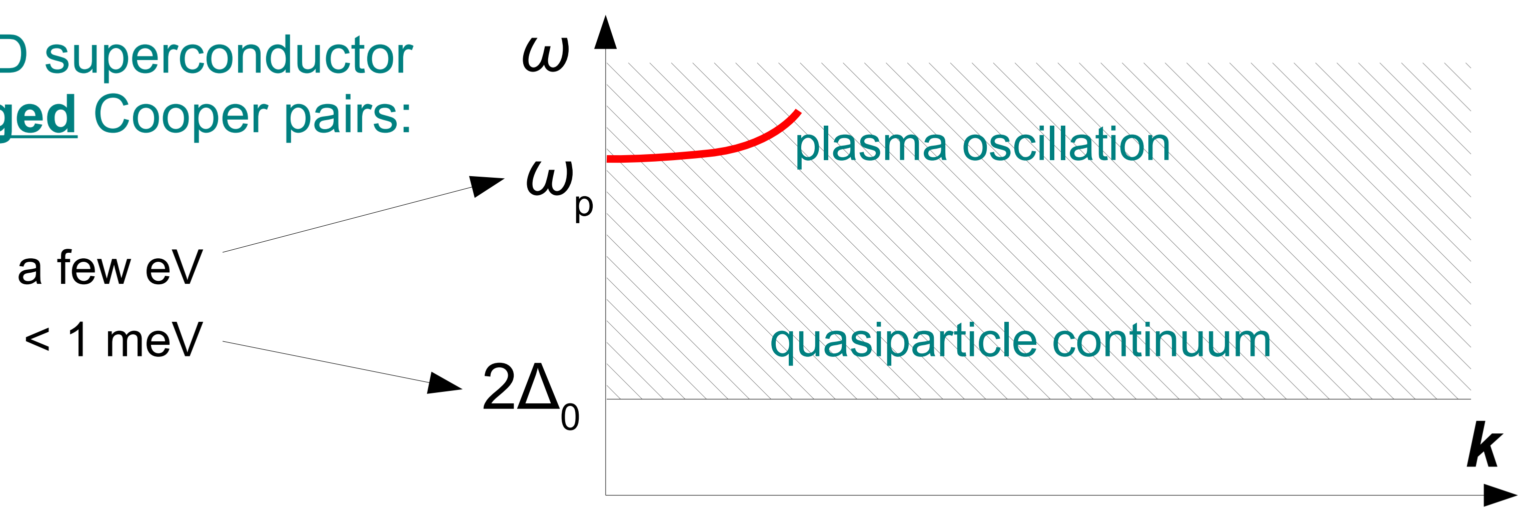
$$e\nu\,\frac{1}{2}\frac{\partial\varphi}{\partial t} = \rho$$ charge density → produces long-range electric field

normal density of states/volume

Electron density oscillations
with long-range Coulomb interaction
are called **plasma oscillations**

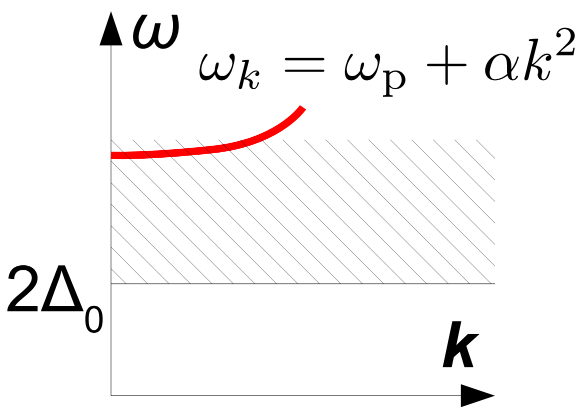
Goldstone modes in a 3D superconductor are gapped because of the Coulomb interaction

Excitations in a 3D superconductor
with **charged** Cooper pairs:

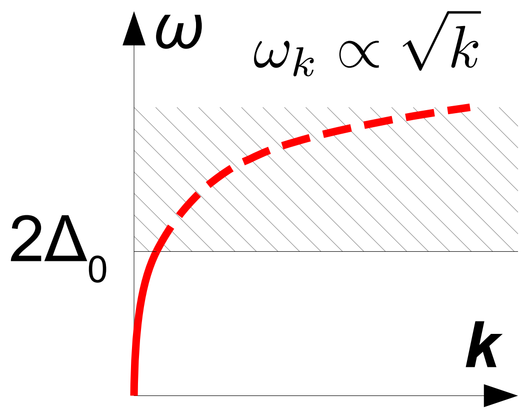


Plasma oscillations in nanostructures

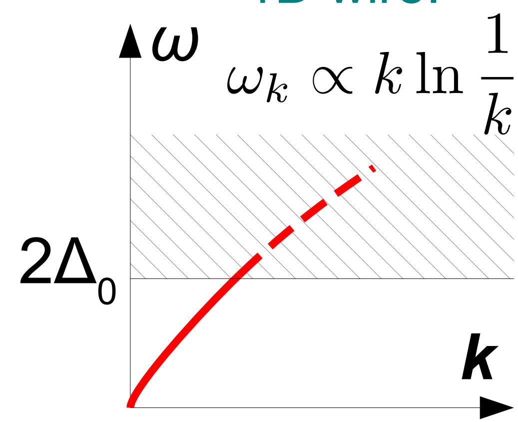
3D material:



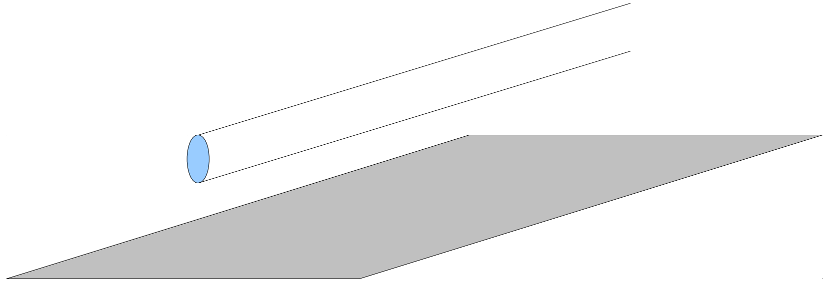
2D sheet:

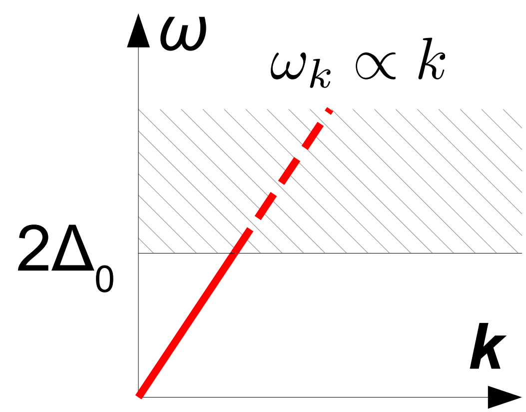


1D wire:



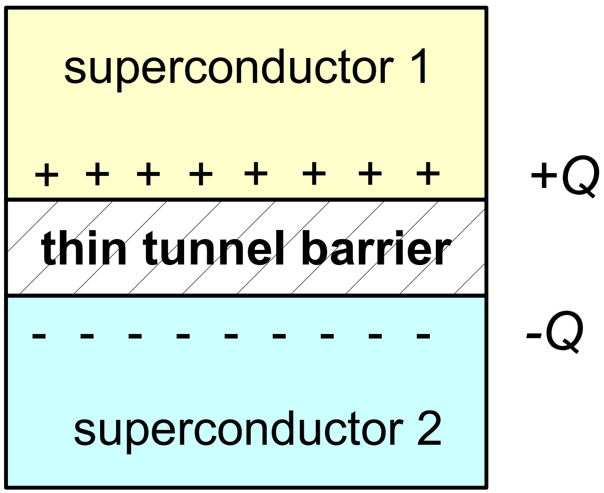
Superconducting wire above a ground plane
which screens the long-range Coulomb:





Low dimensions → Goldstone (Mooij-Schön) modes ≡ plasmons are gapless
Low temperatures → plasmons dominate the physics

Josephson junction



Superconducting phase difference $\theta = \phi_1 - \phi_2$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + E_J(1 - \cos\hat{\theta})$$

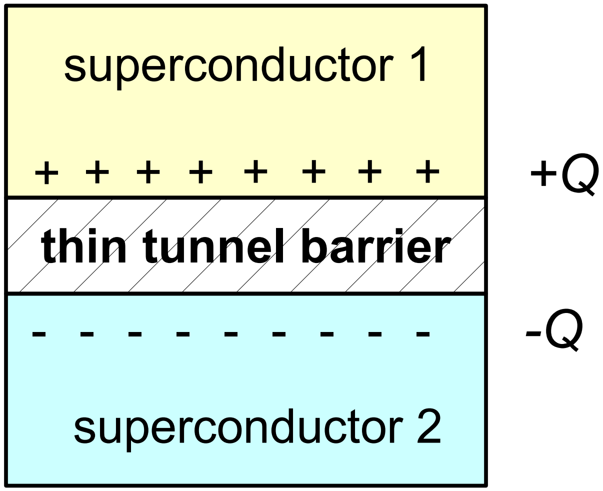
$$[\hat{Q}, \hat{\theta}] = 2ie$$

$$E_J \gg E_C \equiv \frac{(2e)^2}{C} \quad \Rightarrow \quad 1 - \cos\hat{\theta} \approx \frac{\hat{\theta}^2}{2}$$

harmonic oscillator

junction plasma frequency $\omega_{\mathrm{p}} = \sqrt{E_J E_C}$

Josephson junction



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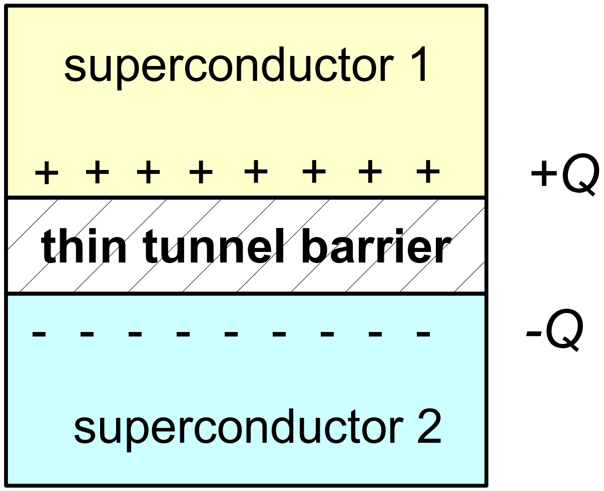
A pendulum
or
a particle
in a periodic potential?

Josephson junction



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A pendulum
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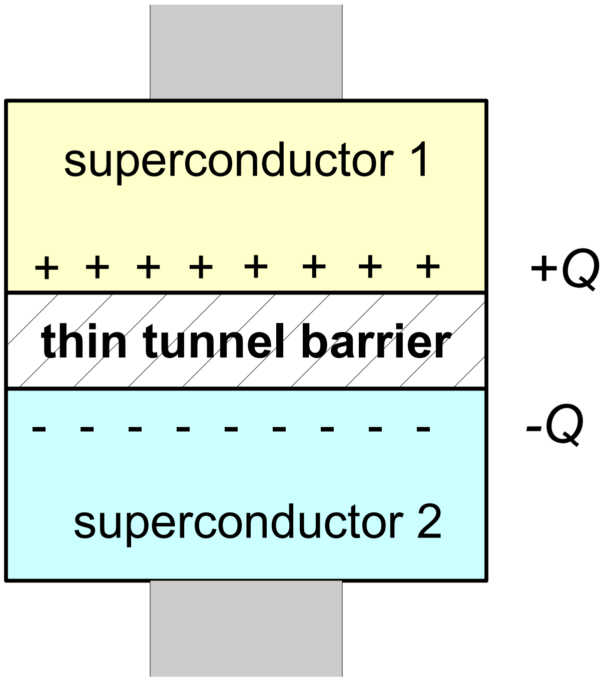
in a periodic potential?

Closed system

- compact phase
  (θ=0 and θ=2π are the same state)
- discrete charge eigenvalues

pendulum

Josephson junction



Superconducting phase difference $\theta = \phi_1 - \phi_2$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + E_J(1 - \cos\hat{\theta})$$

$$[\hat{Q}, \hat{\theta}] = 2ie$$

A pendulum
or
a particle
in a periodic potential?

System attached to leads

- non-compact phase
  (the circuit keeps track of the winding)
- continuous charge
  (some charge remains in the leads)

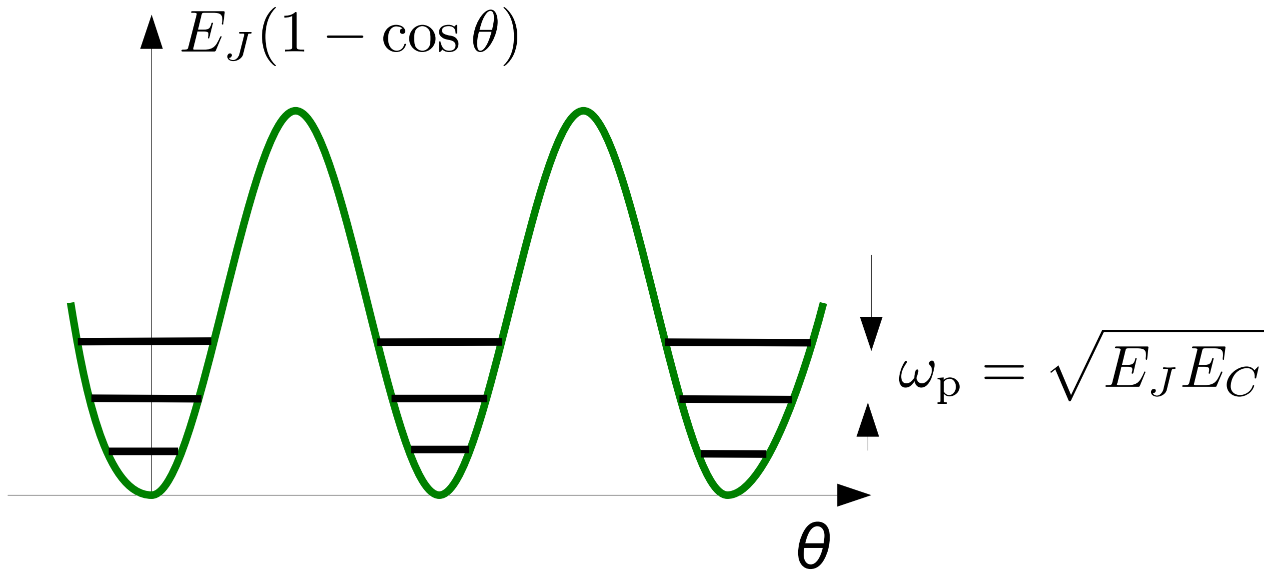
particle in a periodic potential

Closed system

- compact phase
  (θ=0 and θ=2π are the same state)
- discrete charge eigenvalues

pendulum

Quantum phase slips in a JJ



Quantum tunneling:

Harmonic oscillator levels → Bloch bands

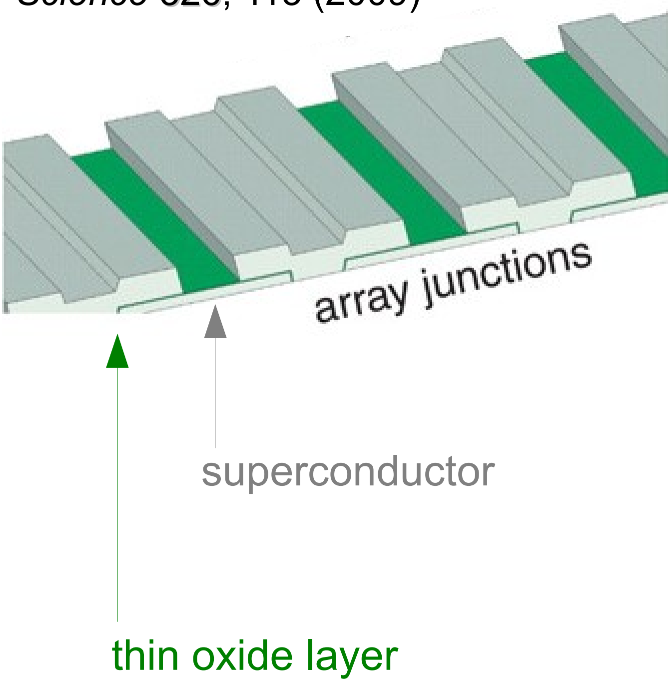
$$E_n(q) \approx \left(n + \frac{1}{2}\right)\omega_p - 2W_n \cos\frac{2\pi q}{2e}$$

quasicharge

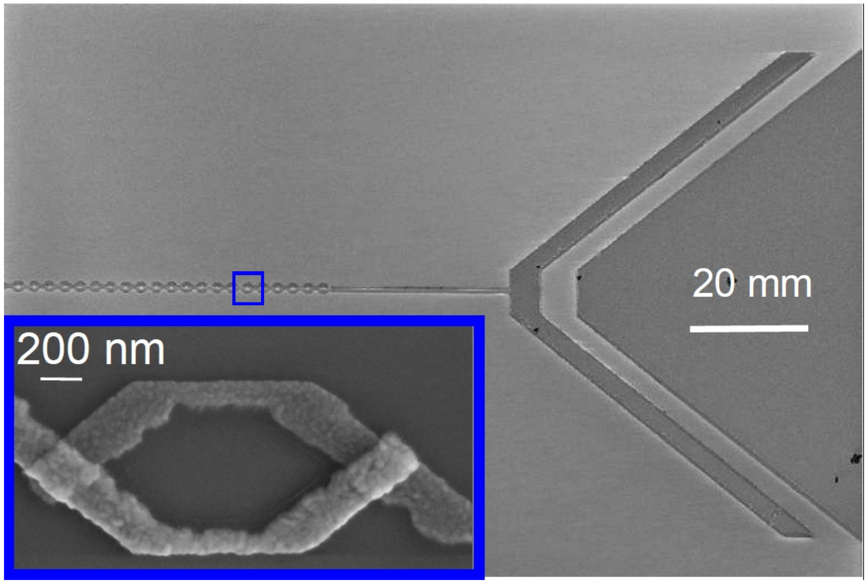
QPS amplitude

Josephson junction chains

V. Manucharyan *et al.*,
Science 326, 113 (2009)



SQUID chain [T. Weiβl *et al.*, *PRB* **92**, 104508 (2015)]

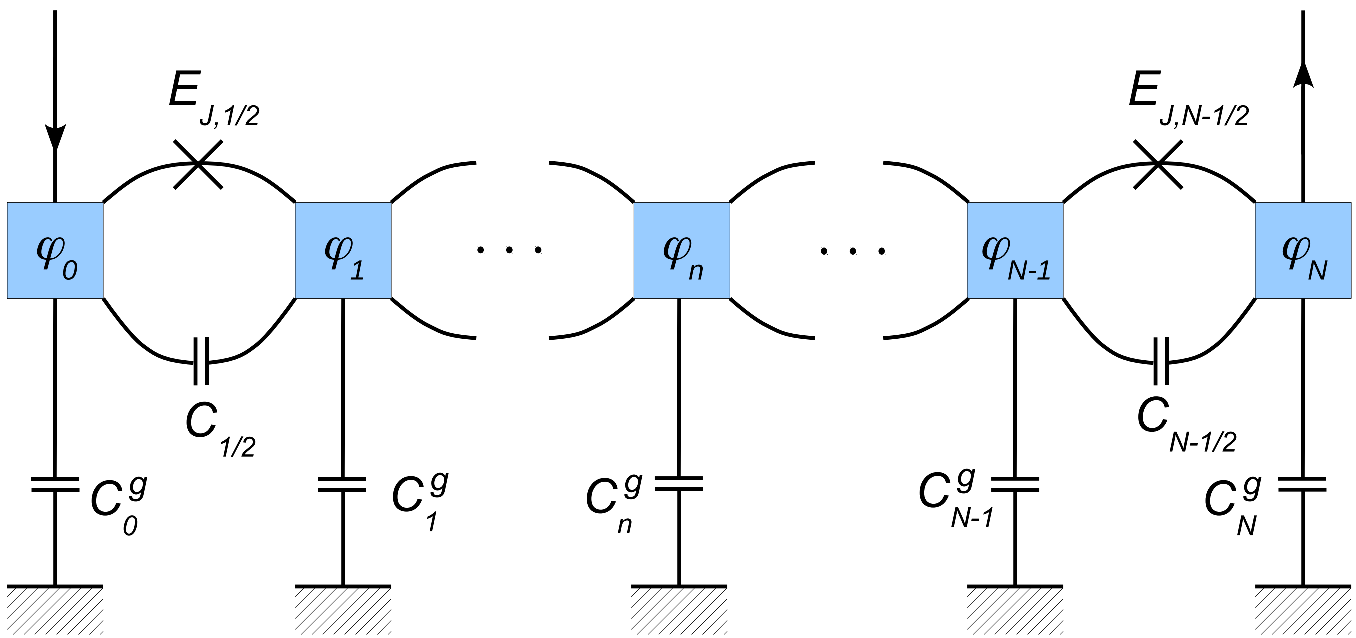


- large impedance with little dissipation Maslyuk *et al.*, PRL **109**, 137002 (2012)
- control over quantum coherence (phase slips) Pop *et al.*, Nature Phys. **6**, 589 (2010)

Josephson junction chain

external current (**dc** or **ac**)

external current



a small capacitance between the islands and the ground

Typically, $C \gg C^g$

$$\hat{H} = \frac{1}{2}\sum_{n,n'} C^{-1}_{nn'}\hat{q}_n\hat{q}_{n'} - \sum_{n=0}^{N-1} E_{J,n+1/2}\cos\left(\hat{\phi}_{n+1} - \hat{\phi}_n\right)$$

$$\left[\hat{q}_n, \hat{\phi}_{n'}\right] = 2ie\delta_{nn'}$$

Screened Coulomb interaction

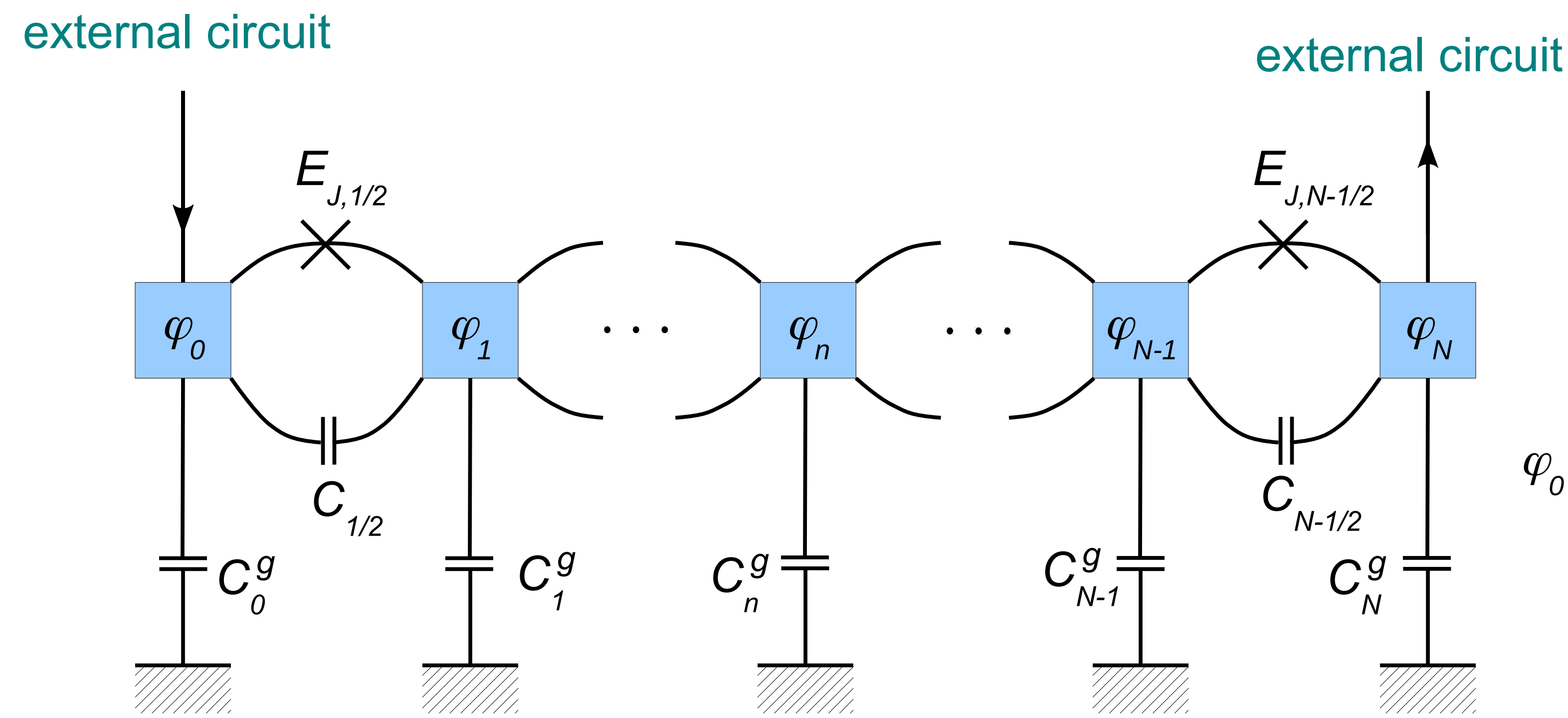
Capacitance matrix: $$C_{nn'} = \begin{pmatrix} C_g + C & -C & 0 & \dots \\ -C & C_g + 2C & -C & \dots \\ 0 & -C & C_g + 2C & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

Inverse capacitance matrix: $$C^{-1}_{nn'} \approx \frac{e^{-\sqrt{C_g/C}\,|n-n'|}}{2\sqrt{CC_g}}$$

Screening length: $\ell_s = \sqrt{C/C_g} \gg 1$ (typically, 5−10)

$\ell_s = 45$ J. Puertas *et al.*, npj Quantum Information **5**, 19 (2019)

QPS in open JJ chains



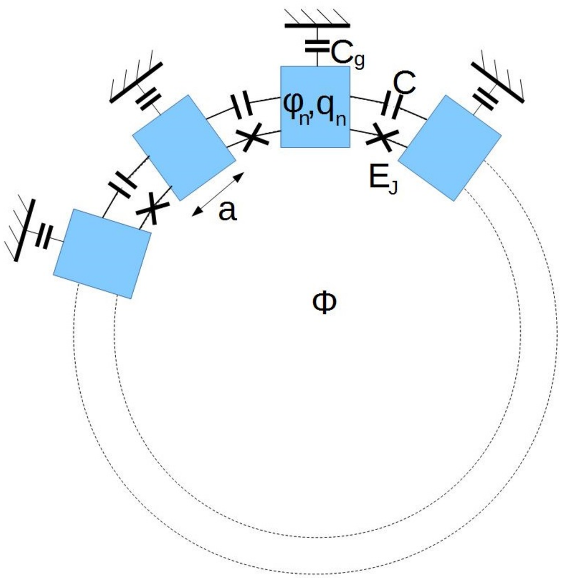
φ_0 and φ_N non-compact
$\varphi_1 \ldots \varphi_{N-1}$ compact

- 2 non-compact phases and $N-1$ compact phases
- global phase shift → conjugate to conserved total charge → fix $Q_{tot} = 0$
- non-compact $\varphi_0 - \varphi_N$ → periodic potential with minima
- N harmonic oscillators in each minimum → 1d Bloch bands

Quantum phase slips: tunneling between minima → Bloch band width

QPS in JJ rings

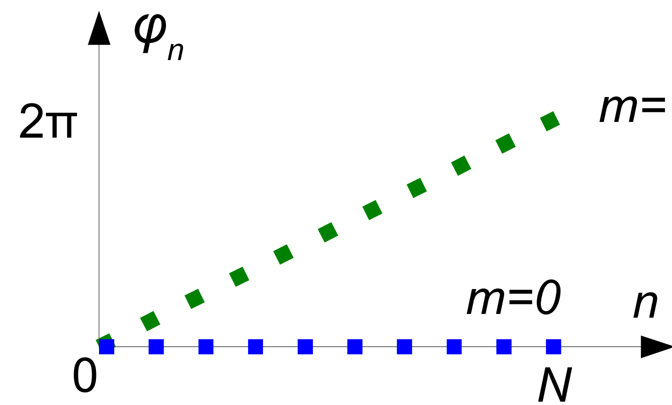
JJ ring pierced by a flux Φ:

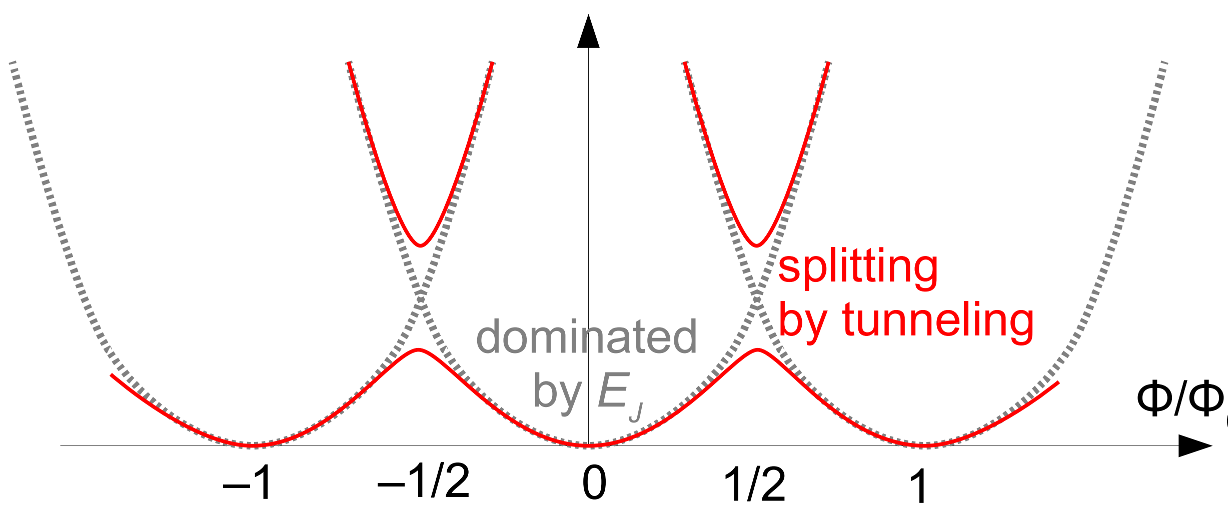


$$E^{(m)}(\Phi) \approx \frac{E_J}{2N}\left(\frac{2\pi\Phi}{\Phi_0} - 2\pi m\right)^2$$

All phases compact → no Bloch bands

Local potential minima: $\varphi_n^{(m)} = m\,\frac{2\pi n}{N}$





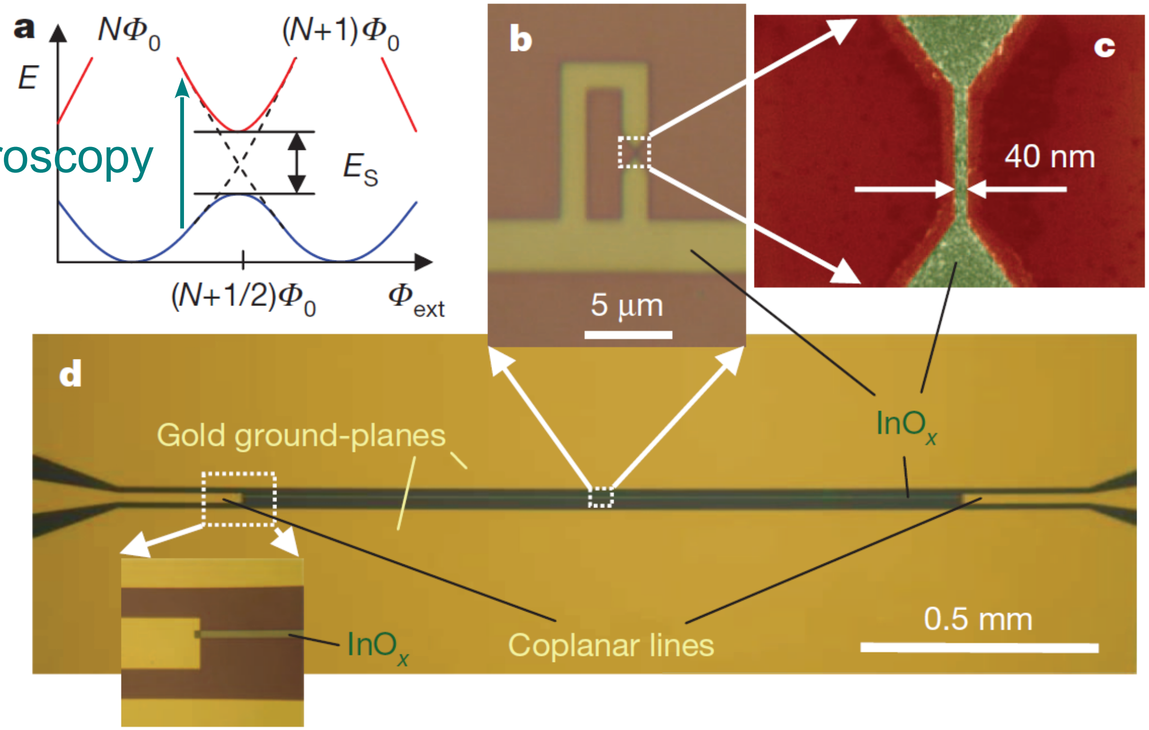
Quantum phase slips:
tunneling between minima → splitting at $\Phi = \Phi_0 / 2$

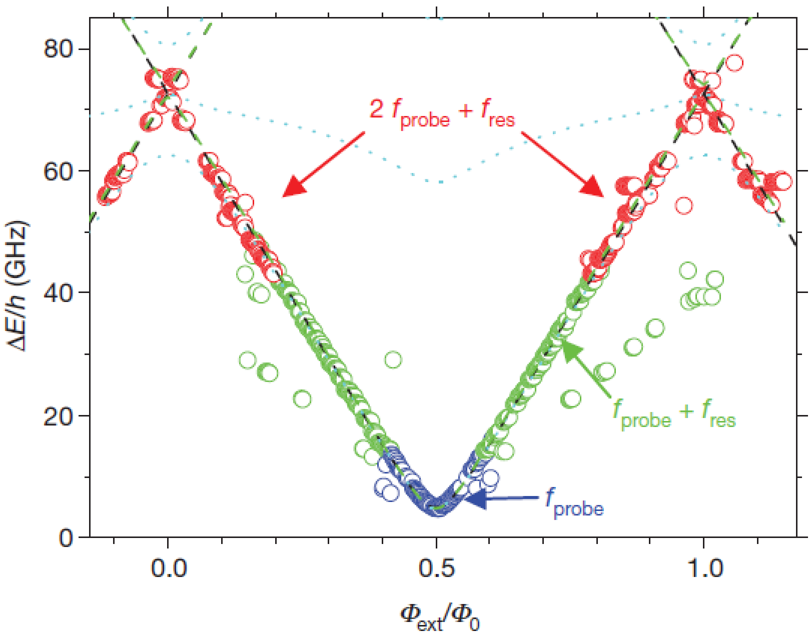
Ivanov *et al.*, PRB **65**, 024509 (2001)
Matveev *et al.*, PRL **89**, 096802 (2002)

Experimental observation in a wire

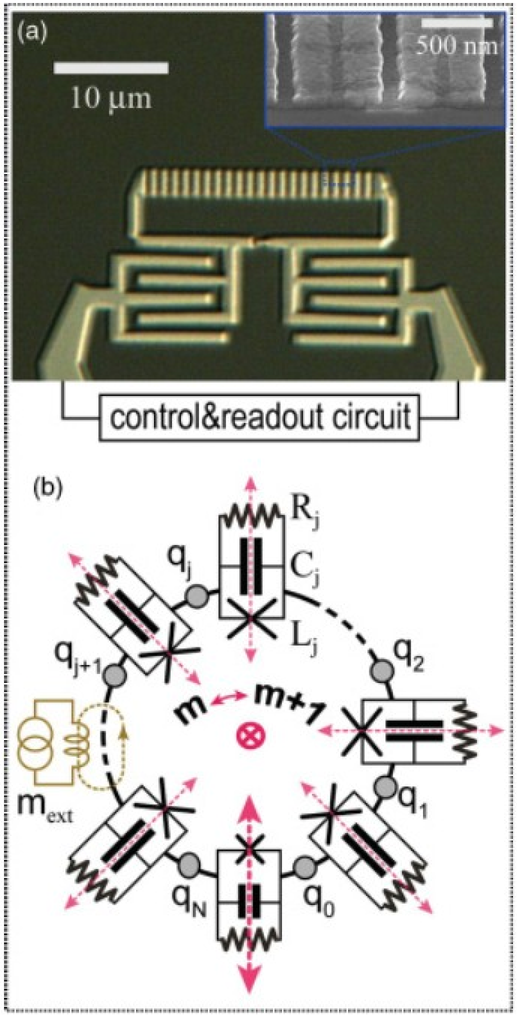
O. V. Astafiev *et al.*, *Nature* **484**, 355 (2012)

transition probed by microwave spectroscopy



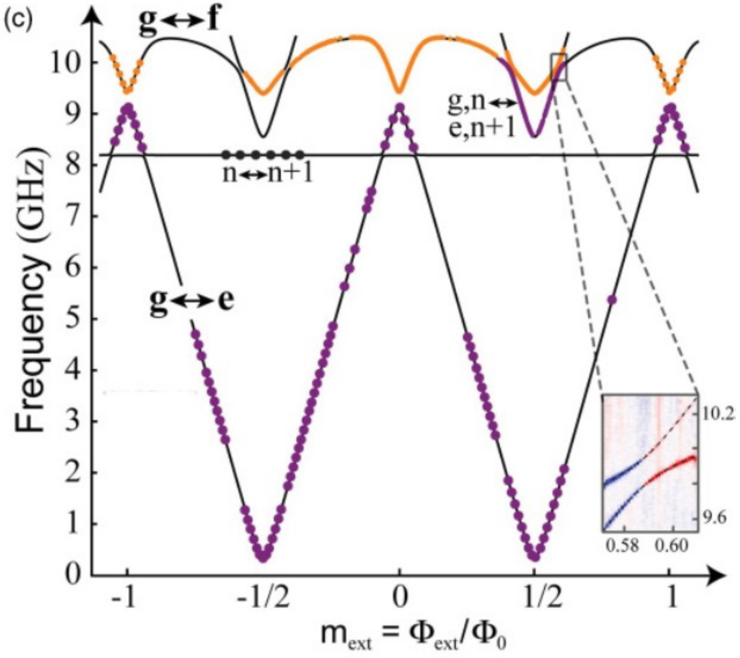


QPS in a fluxonium loop



fluxonium qubit = 1 small junction embedded in a loop of large junctions

V. Manucharyan *et al*., *Science* 326, 113 (2009)



microwave spectroscopy

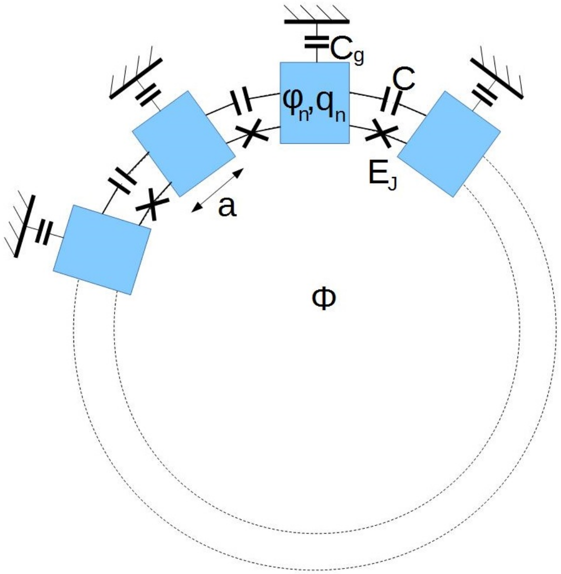
V. Manucharyan *et al*., PRB **85**, 024521 (2012)

Phase slips occur predominantly on the small junction

Coherent quantum phase slips

Matveev, Larkin, Glazman, PRL **89**, 096802 (2002)
Rastelli, Pop, Hekking, PRB **87**, 174513 (2013)

JJ ring pierced by magnetic flux:



Tunneling amplitude from an instanton calculation

Imaginary-time Lagrangian:

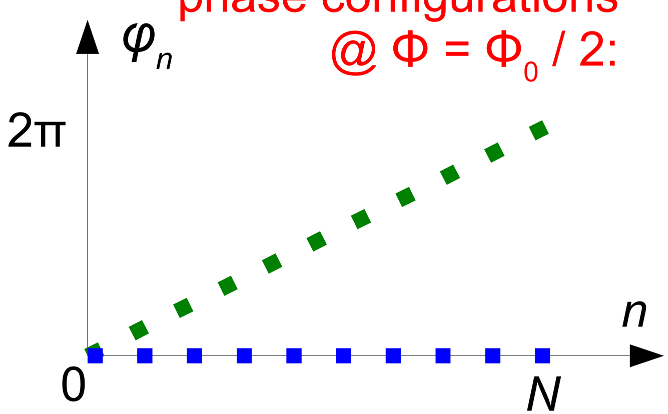
$$\mathcal{L} = \sum_n \left[\frac{C_g}{8e^2}\dot{\varphi}_n^2 + \frac{C}{8e^2}(\dot{\varphi}_{n+1} - \dot{\varphi}_n)^2 \right]$$

kinetic energy

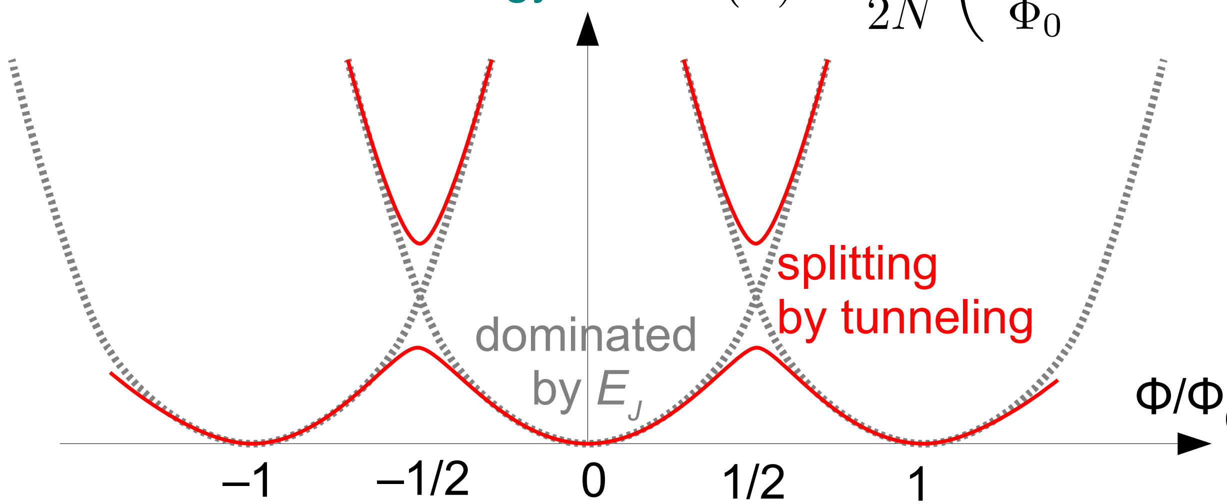
$$- \sum_n E_J \cos\left(\varphi_{n+1} - \varphi_n + \frac{2\pi\Phi}{N\Phi_0} \right)$$

potential energy

Tunneling between two degenerate classical phase configurations @ $\Phi = \Phi_0 / 2$:



Ground state energy $E^{(m)}(\Phi) \approx \frac{E_J}{2N}\left(\frac{2\pi\Phi}{\Phi_0} - 2\pi m\right)^2$



QPS in a spatially uniform ring

1. Phase winding on one of the junctions
2. Phase readjustment on the length ~ ℓ_s
3. Phase readjustment in the rest of the ring

sensitive to the phase normal modes

QPS amplitude:

amplitudes on different junctions added coherently (assuming no offset charges)

$$W = \frac{4N}{\sqrt{\pi}}\left(E_J^3 E_C\right)^{1/4} \exp\left\{-8\sqrt{\frac{E_J}{E_C}} - g\left[\ln\frac{N}{\ell_s} - 2.43 + O(1/\ell_s)\right]\right\}$$

Matveev, Larkin, Glazman, PRL **89**, 096802 (2002)

Hekking & Glazman, PRB **55**, 6551 (1997); Rastelli, Pop, Hekking, PRB 87, 174513 (2013)

Svetogorov *et al.*, PRB **97**,104514 (2018)

$$E_C = \frac{(2e)^2}{C}$$ junction charging energy << E_J

$$\ell_s = \sqrt{C/C_g} \gg 1$$

$$g \equiv \pi\sqrt{\frac{E_J}{(2e)^2/C_g}} > 2$$ otherwise insulator

Bradley & Doniach, PRB **30**, 1138 (1984)
Korshunov, JETP **68**, 610 (1989)

dimensionless admittance of the chain

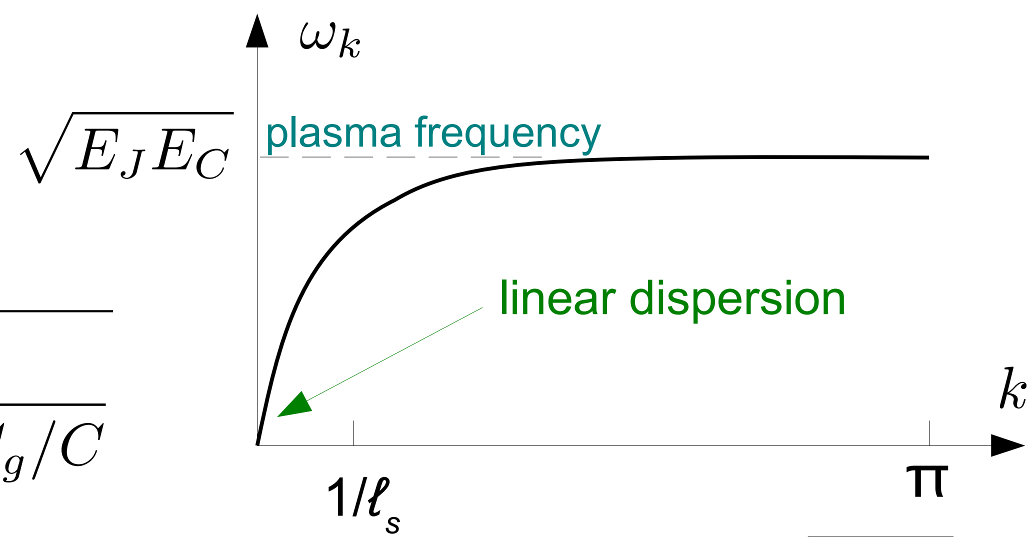
Phase normal modes

N−1 harmonic oscillators in each potential minimum:
plasma oscillations, Mooij-Schön modes

$$\sum_{n'} C_{nn'} \frac{d^2\phi_{n'}}{dt^2} = \left(\frac{2e}{\hbar}\right)^2 E_J(\phi_{n+1}+\phi_{n-1}-2\phi_n)$$

Infinitely long chain: $\phi_n \propto e^{ikn}$

$$\omega_k = \sqrt{E_J E_C}\sqrt{\frac{4\sin^2(k/2)}{4\sin^2(k/2)+C_g/C}}$$



screening length $\ell_s = \sqrt{C/C_g} \gg 1$

Phase normal modes

N−1 harmonic oscillators in each potential minimum:
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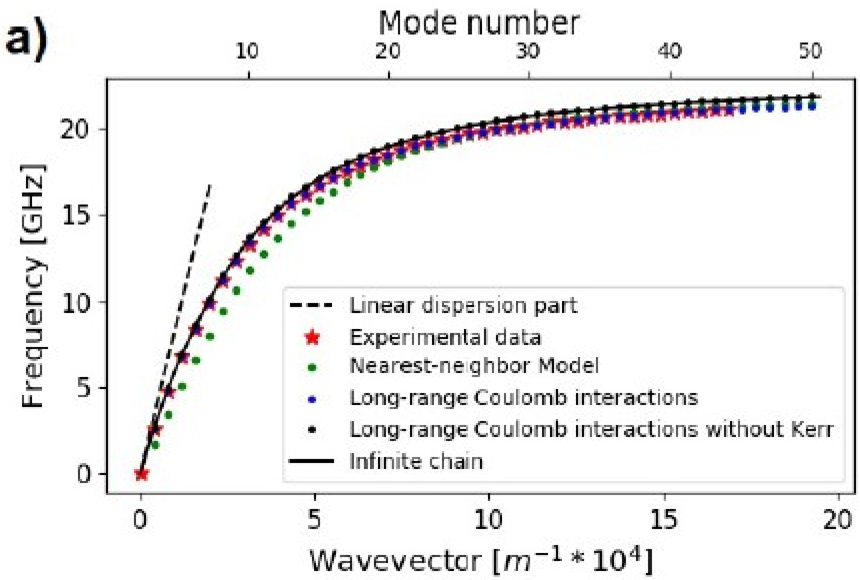
Finite length, N junctions:

$$k = 0, \frac{2\pi}{N}, \frac{4\pi}{N}, \ldots, \frac{(N-1)2\pi}{N}$$

Measured resonances
in the microwave transmission coefficient
of a 200-junction chain
Yu. Krupko et al., PRB **98**, 094516 (2018)



Random spatial modulation of areas

Josephson energy, junction capacitance ∝ **junction area**

$$E_{J,n} = E_J(1+\zeta_n), \quad C_n = C(1+\zeta_n)$$

weak relative modulation of the junction areas $\langle \zeta_n^2 \rangle = \sigma_S^2 \ll 1$

$$C_{g,n} = C_g(1+\eta_n)$$

weak relative modulation of the ground capacitances $\langle \eta_n^2 \rangle = \sigma_g^2$

All normal modes are localized

Long chains: localization length from the DMPK equation

$$\xi = \frac{2}{\sigma_S^2 + \sigma_g^2} \frac{4}{k^2}$$

diverges at $k \to 0$ (standard for Goldstone modes)

Short chains $N \ll \xi$: random perturbative shifts of the discrete frequencies

$$\frac{\langle \delta\omega_k^2 \rangle}{\omega_k^2} = \frac{\sigma_S^2 + \sigma_g^2}{N} \frac{3/8}{(1+k^2\ell_s^2)^2}$$

Basko & Hekking, PRB **88**, 094507 (2013)

Random offset charges

$$\hat{H} = \frac{1}{2} \sum_{n,n'} C^{-1}_{nn'} \left(\hat{q}_n - 2e\kappa_n\right)\left(\hat{q}_{n'} - 2e\kappa_{n'}\right) - \sum_{n=0}^{N-1} E_{Jn} \cos(\phi_{n+1} - \phi_n)$$

Classically: **no effect** on normal modes

Quantum-mechanically: energy levels sensititve
to the fractional part of κ_n

Effect of a periodic spatial modulation

Josephson energy, junction capacitance ∝ **junction area**

$$E_{J,n} = E_J(1+\zeta_n), \quad C_n = C(1+\zeta_n)$$

weak relative modulation of the junction areas

$$\zeta_n = t_\zeta \cos\frac{2\pi n}{a}$$

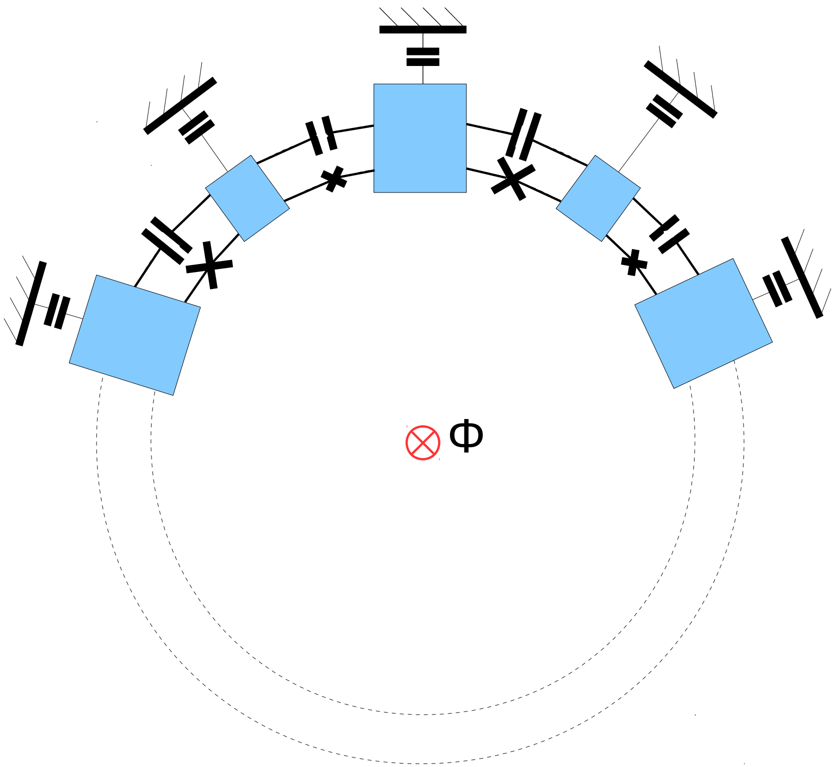
$$C_{g,n} = C_g(1+\eta_n)$$

weak relative modulation of the ground capacitances

$$\eta_n = t_\eta \cos\frac{2\pi n}{a}$$

modulation depth << 1

modulation period >> 1



Mooij-Schön modes are modified

structured environment

Effect on the QPS amplitude?

Effect of a periodic spatial modulation

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weak relative modulation of the ground capacitances

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modulation depth << 1

modulation period >> 1

wave equation with modulation

↓

correction to mode wave functions

↓

correction to the QPS amplitude on junction *n*:

local admittance at the QPS position

$$W_n = \frac{4}{\sqrt{\pi}}\left(E_J^3 E_C\right)^{1/4} \exp\left\{-8\sqrt{\frac{E_{J,n}}{E_{C,n}}} - \bar{g}\left(\ln\frac{N}{\ell_s} - 2.43\right) - \delta g_n \ln\frac{0.10\,a}{\ell_s}\right\}$$

purely local

determined by modes $1/N < k < 1/\ell_s$

If $a \gg \ell_s$ only; otherwise $\sim a^2/\ell_s^2$

determined by modes $1/a < k < 1/\ell_s$

Svetogorov *et al.*, PRB **97**,104514 (2018)

Effect of a random spatial modulation

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$$C_{g,n} = C_g(1+\eta_n)$$

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QPS amplitude on junction *n*: $W_n \propto e^{-S_n - 2\pi i(\kappa_1 + \ldots + \kappa_n)}$ random offset charges

Ivanov *et al*., PRB **65**, 024509 (2001)
Matveev et al., PRL **89**, 096802 (2002)

Effect of a random spatial modulation

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QPS amplitude on junction *n*: $W_n \propto e^{-S_n - 2\pi i(\kappa_1 + \ldots + \kappa_n)}$ random offset charges

Homogeneous chain: $S = 8\sqrt{E_J/E_C} + g\left(\ln\frac{N}{\ell_s} - 2.43\right)$

determined by the slipping junction

determined by modes with $1/N < k < 1/\ell_s$

Disordered chain: $\langle \delta S_n^2 \rangle = \left\langle \delta\left(8\sqrt{E_J/E_C}\right)^2 \right\rangle + \frac{\sim 1}{\ell_s}\langle \delta g^2 \rangle$

determined by modes with $k \sim 1/\ell_s$

Coherent QPS amplitude is **NOT** sensitive to Anderson localization of the normal modes

Svetogorov & Basko, PRB **98**, 054513 (2018)

Mesoscopic fluctuations

Corrections to the QPS amplitude are determined by the local values of the slipping junction parameters

$$W = \frac{W_{\mathrm{hom}}}{N} \sum_n e^{-\delta S_n - i\theta_n}$$

Mesoscopic fluctuations

Corrections to the QPS amplitude are determined
by the local values of the slipping junction parameters

$$\boxed{W = \frac{W_{\text{hom}}}{N} \sum_n e^{-\delta S_n - i\theta_n}}$$

1. No random charges: $\theta_n = 0$

$\langle \delta S_n \rangle = 0, \quad \langle \delta S_n \delta S_{n'} \rangle = \frac{64 E_J}{E_C} \left\langle \zeta_n^2 \right\rangle \delta_{nn'} \equiv \sigma^2 \delta_{nn'}$ Gaussian, uncorrelated

$\delta S_n \ll S_{\text{hom}}$ but may be $\delta S_n \gtrsim 1$ $\Rightarrow$ Strong mesoscopic fluctuations of the QPS amplitude

Central limit theorem for $N \gg e^{\sigma^2} - 1$, otherwise long tail in the distribution

Weakest junction dominates when $N \lesssim e^{0.6\,\sigma^{2/3}}$

Sum of log-normals ~ a log-normal ???

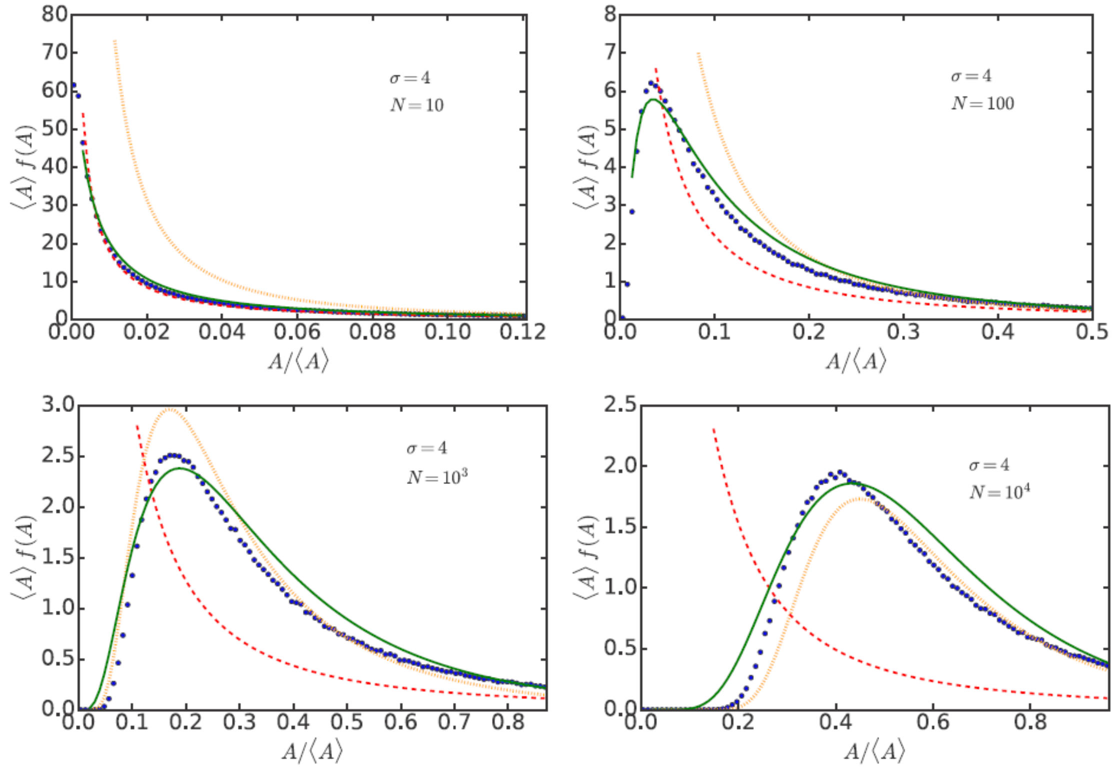
Mesoscopic fluctuations

Corrections to the QPS amplitude are determined
by the local values of the slipping junction parameters

$$W = \frac{W_{\text{hom}}}{N} \sum_n e^{-\delta S_n - i\theta_n}$$

1. No random charges:

Direct numerical sampling
Saddle point approximation
Weakest junction approximation
Lognormal fit



Sum of log-normals ~ a log-normal

Mesoscopic fluctuations

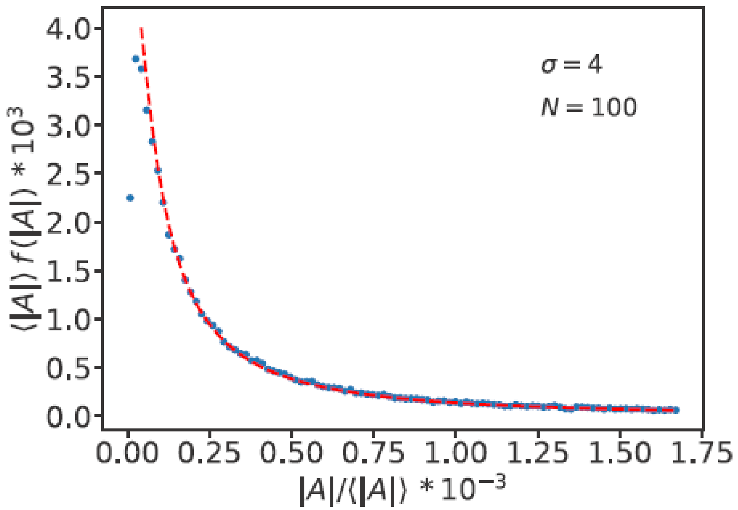
Corrections to the QPS amplitude are determined
by the local values of the slipping junction parameters

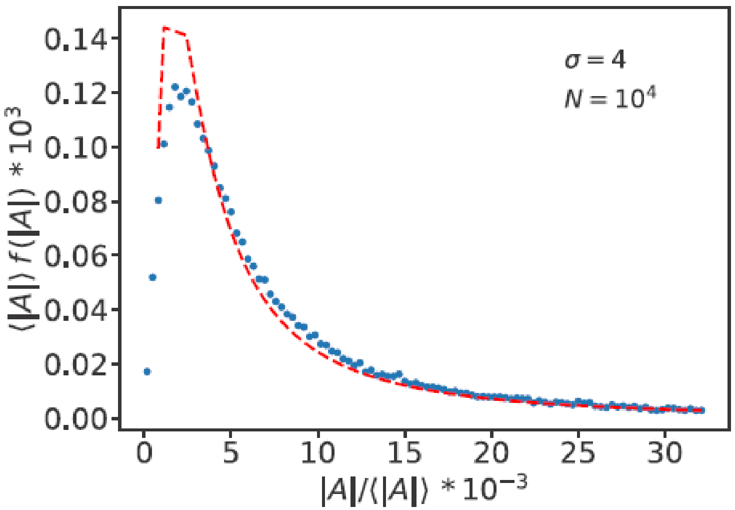
$$W = \frac{W_{\rm hom}}{N} \sum_n e^{-\delta S_n - i\theta_n}$$

2. Strong random charges: $\theta_n \in [0, 2\pi)$ uniformly and independently

Central limit theorem for $N \gg (e^{4\sigma^2} - 1)/2$

Weakest junction dominates when $N \lesssim e^{\sigma^{2/3}}$





Conclusions

Theory of coherent QPSs in spatially inhomogeneous JJ chains:

spatial inhomogeneity

$\downarrow$

Mooij-Schön modes

$\downarrow$

hydrodynamic part of the QPS action

homogeneous chain: $g \ln \frac{N}{\ell_s}$

periodic spatial modulation: $g \ln \frac{N}{\ell_s} + \delta g \ln \frac{a}{\ell_s}$

random spatial modulation: $g \ln \frac{N}{\ell_s} + \frac{\delta g}{\sqrt{\ell_s}}$

Mesoscopic fluctuations of the QPS amplitude are dominated by the local values of the parameters