Role of gravity in the pair creation induced by electric fields

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General comments

Quantum Field Theory in curved spacetimes

The presence of a time dependent gravitational field permits the spontaneous creation of particles out of the vacuum

Schwinger effect

Strong electric fields can create electron-positron pairs

$$E > E_c \sim 10^{18}\ V/m$$

→ Quantum Field Theory in presence of background electric fields

We study the pair production induced by a **classical** homogeneous and time dependent electric field.

In other words, the interaction between **quantum** particles and classical electromagnetic fields.

General comments

Semiclassical theory

Classical homogeneous background fields

Quantum scalar (fermionic) field

An *important problem* is the **backreaction** of the background field due to the created pairs

UV divergences in the sources appear

We must regularize these divergences:

Adiabatic regularization method

$$D_\mu \phi = (\nabla_\mu + iqA_\mu)\phi$$

$$(D_\mu D^\mu + m^2)\phi = 0$$

$$\nabla_\mu F^{\mu\nu} = \langle j^\nu \rangle$$

Vaccum expectation value of the current (given by QFT)

In most of the literature, it is assumed a specific implementation of the adiabatic renormalization program without realizing an underlying **inconsistency** when gravity is turned on

Framework

Two-dimensional scalar QED in Minkowski spacetime

Background electric field: $E(t) = -\dot{A}(t) \longrightarrow A_\mu = (0, -A(t))$

UV divergent!!

Maxwell equations

$$\nabla_\mu F^{\mu\nu} = \langle j^\nu \rangle_{ren}$$

Klein-Gordon equation

$$(D_\mu D^\mu + m^2)\phi = 0$$

Mode expansion

$$\ddot{A}(t) = q \int_{-\infty}^{\infty} \frac{dk}{2\pi} (k - qA) |h_k|^2$$

$$\ddot{h}_k + \left(m^2 + (k - qA)^2\right) h_k = 0$$

(*) $\phi(x) = \frac{1}{\sqrt{2(2\pi a)}} \int_{-\infty}^{\infty} dk [A_k e^{ikx} h_k(t) + B_k^\dagger e^{-ikx} h_{-k}^*(t)]$

Framework

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Maxwell equations

$$\nabla_\mu F^{\mu\nu} = \langle j^\nu \rangle_{ren}$$

Klein-Gordon equation

$$(D_\mu D^\mu + m^2)\phi = 0$$

$$\langle j^x \rangle_{ren} = q \int_{-\infty}^{\infty} \frac{dk}{2\pi}(k - qA)\left[|h_k|^2 - substractions\right]$$

$$\ddot{h}_k + \left(m^2 + (k - qA)^2\right) h_k = 0$$

(*) $\phi(x) = \frac{1}{\sqrt{2(2\pi a)}} \int_{-\infty}^{\infty} dk[A_k e^{ikx} h_k(t) + B_k^\dagger e^{-ikx} h_{-k}^*(t)]$

Adiabatic regularization

The main idea in dealing with scalar fields is to consider an adiabatic expansion of the mode function h_k based on the WKB-type ansatz, namely

$$h_k(t) = \frac{1}{\sqrt{\Omega_k(t)}} e^{-i\int^t \Omega_k(t')dt'} \qquad \Omega_k(t) = \omega_k^{(0)} + \omega_k^{(1)} + \omega_k^{(2)} + \cdots$$

The **order of the expansion** is based on the number of derivatives of the background fields.

A very *crucial point* to properly define the adiabatic expansion is to define the leading order, i.e., the zeroth order adiabatic term.

↕

Equivalent to choose the adiabatic order of A(t)

Once we have the adiabatic expansion we have to substract up to order **(n)** in the renormalizable observables ⇒

$$\langle j^x \rangle_{ren} = q\int_{-\infty}^{\infty} \frac{dk}{2\pi}(k - qA)\left[|h_k|^2 - \left|h_k^{(0)}\right|^2 - \left|h_k^{(1)}\right|^2 - \ldots\right]$$

Two possible choices

A(t) of order 0

$$\omega_k^{(0)} = \sqrt{(k-qA)^2+m^2}$$

A(t) of order 1

$$\omega_k^{(0)} = \sqrt{k^2+m^2} \equiv \omega$$

$$\langle j^x \rangle_{ren}^{I} = q\int_{-\infty}^{\infty} \frac{dk}{2\pi}\left[(k-qA)\,|h_k|^2 - \frac{(k-qA)}{\sqrt{(k-qA)^2+m^2}}\right]$$

$$\langle j^x \rangle_{ren}^{II} = q\int_{-\infty}^{\infty} \frac{dk}{2\pi}\left[(k-qA)\,|h_k|^2 - \frac{k}{\omega} + \frac{m^2 qA}{\omega^3}\right]$$

We have **two** different definitions

How can we determine which one is the correct one?

Two possible choices

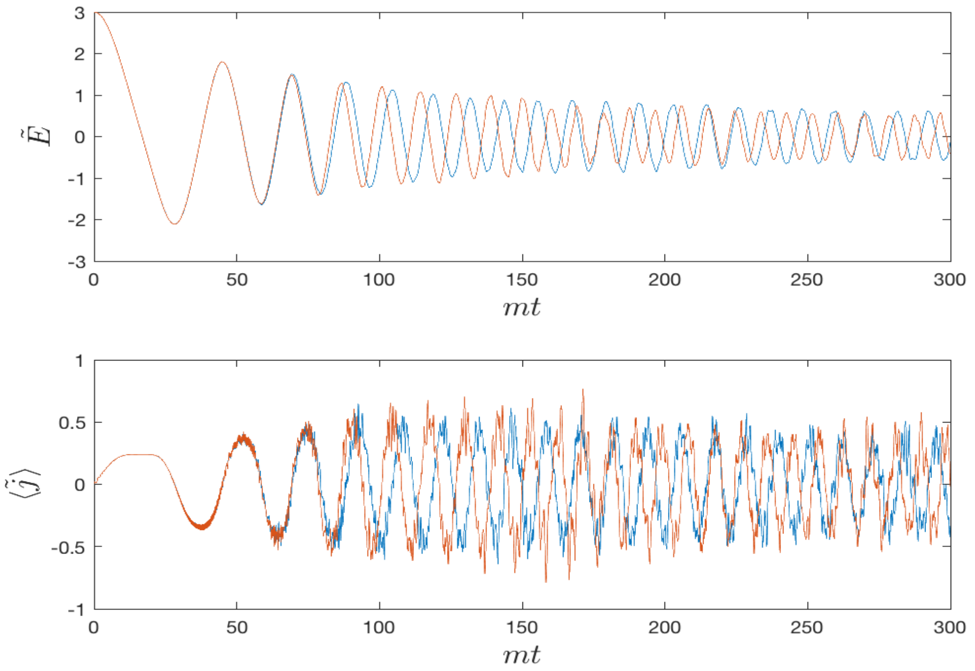
To quantify the difference between the two renormalization notions it is very convenient to define the following quantity:

$$\triangle\langle j^x\rangle_{ren} = \langle j^x\rangle_{ren}^{I} - \langle j^x\rangle_{ren}^{II} = q \lim_{\Lambda^\pm\to\pm\infty}\int_{\Lambda^-}^{\Lambda^+}\frac{dk}{2\pi}\left[\frac{k}{\omega} - \frac{m^2 qA}{\omega^3} - \frac{(k-qA)}{\sqrt{(k-qA)^2+m^2}}\right]$$

Difference between the subtraction terms

$$\triangle\langle j^x\rangle_{ren} = 0$$

The two possibilities are equivalent in Minkowski space



Role of Gravity

$$\ddot{h}_k + \left(m^2 + (k - qA)^2\right) h_k = 0 \quad \Rightarrow \quad \ddot{h}_k + \left(m^2 + \frac{1}{a^2}(k - qA)^2 + \frac{\dot{a}^2}{4a^2} - \frac{\ddot{a}}{2a}\right) h_k = 0$$

When we add gravity to the game, we have **two** background fields to consider

The adiabatic order of **a(t)** is 0, and the order of the adiabatic substractions is **fixed** by

Order of adiabatic subtractions	$D=4$	$D=3$	$D=2$
$\langle j^\mu \rangle$	3	2	1
$\langle T^{\mu\nu} \rangle$	4	3	2

The adiabatic order of **A(t)** could be

- 0 — In this case the background fields are of the same adiabatic order
- 1 — In this case we have a hierarchy between the background fields

Role of Gravity

$$\ddot{h}_k + \left(m^2 + (k - qA)^2\right) h_k = 0 \quad \Rightarrow \quad \ddot{h}_k + \left(m^2 + \frac{1}{a^2}(k - qA)^2 + \frac{\dot{a}^2}{4a^2} - \frac{\ddot{a}}{2a}\right) h_k = 0$$

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The adiabatic order of **A(t)** could be

0 ✗

1 ☺

Only the second choice (*A* of order 1) is compatible with energy conservation

$$\nabla_\mu \langle T^{\mu\nu} \rangle_{ren} + \nabla_\mu T^{\mu\nu}_{elec} = 0$$

Conclusions

Gravity plays a fundamental role to fix unambiguously the adiabatic rules in the renormalization of the source operators and hence in the correct expression for the backreaction equations.

The basic consistency check is the exact compatibility of the backreaction equations with the covariant conservation of the total stress-energy tensor.

In Minkowski space the two distinct notions for constructing the adiabatic subtraction terms turn out to be equivalent.