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Physics Beyond the Standard Model

TAE 2018, Benasque | **Exercices** | **Date: 10.09.2018**

Exercice 1: On natural units

a) How long does it take for a photon to travel from your feet to your brain?
b)What is the maximal frequency at which the brain can function?
c) When is AI going to take over humans?
d) Muons have a lifetime $\tau \sim 2 \times 10^6$ s. They can be produced by reaction of cosmic rays with the atmosphere at 15-20 kms altitude and still can reach the surface of the Earth. Estimate the energy of the cosmic rays.
e) Using dimensional arguments, compute the distance between the two neutral plates for the Casimir pressure between them to be of the order of 1 atm (the exact computation of the Casimir pressure gives an extra numerical factor $\pi^2/240$ in front of what predicted by dimensional arguments).

Exercice 2: Anomaly cancellation

a) Show that in the SM, the gauge anomaly cancellations conditions reduce to

$$\mathrm{Tr}_L Y - \mathrm{Tr}_R Y = 0 \quad \text{and} \quad \mathrm{Tr}_L Y^3 - \mathrm{Tr}_R Y^3 = 0$$

.

b) With the particle content of the SM, these two conditions fix all the charges of the quarks and leptons. However, check that in the presence of right-handed neutrinos, the ratio of the electric charges of the electron over the up quark is not uniquely fixed. Still, verify that the Hydrogen atom remains electrically neutral.

Exercice 3: Goldstone equivalence theorem

a) Neglecting the bottom mass, calculate, in the unitary gauge, the decay rates for $t \to bW_T^+$ and $t \to bW_L^+$. Reproduce the latter result using the equivalence theorem, i.e. by computing the $t \to b\phi^+$, where ϕ^+ is the would-be Goldstone boson.
b) In the heavy Higgs mass limit, compute, in the unitary gauge, the decay widths of $h \to W_T^+ W_T^-$ and $h \to W_L^+ W_L^-$. Reproduce the latter result using the equivalence theorem, i.e. by computing the $h \to \phi^+\phi^-$, where $\phi^\pm$ are the would-be Goldstone bosons.

Exercice 4: Oblique parameters

Compute the S and T oblique parameters when the SM Lagrangian is supplemented by the following gauge invariant dimension-6 operators

$$\frac{c_T}{\Lambda^2}|H^\dagger D_\mu H|^2 \quad \text{and} \quad \frac{c_S}{\Lambda^2} H^\dagger W^a_{\mu\nu}\sigma^a H B_{\mu\nu}.$$

Exercice 5: Quadratic divergences. Coleman–Weinberg potential

The general expression of the 1-loop Coleman–Weinberg potential is given by:

$$V(h) = \int \frac{d_E^k}{2(2\pi)^4} \text{STr} \log(k_E^2 + M^2(h))$$

where STr denotes the trace over bosons minus the trace over fermions.

a) Regularize the integral with a UV cutoff and expand the potential in power of the cutoff to show that

$$V(h) = -\frac{\Lambda^4}{128\pi^2}\text{STr}\,\mathbb{1} + \frac{\Lambda^2}{64\pi^2}\text{STr}\,M^2(h) + \frac{1}{64\pi^2}\text{STr}\,M^4(h)\log\frac{M^2(h)}{\Lambda^2}$$

b) Compute $\text{STr}\mathbb{1}$ and $\text{STr}M^2(h)$ within the SM.

c) Show that the quadratically divergent part of the 1-loop potential is

$$V(h) = \left(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2\right)\frac{3G_F\Lambda^2}{32\sqrt{2}\pi^2}h^2,$$

in agreement with the usual computation of the divergent one-loop Feynman diagrams.

d) Show that quadratically divergent correction to the Higgs mass is gauge-independent.

Exercice 6: Proton decay

With the particle content of the SM, baryon number is an accidental symmetry when restricting to renormalisable interactions. What is the mass dimension of the interactions that can induce a decay of the proton? Given that the current experimental lower bound on the lifetime of the proton in 10^{34} years, find the lower bound on the scale of these interactions.

Exercice 7: Anti-de Sitter space-time. Randall–Sundrum geometry

Consider a conformally flat space-time: $ds^2 = \Omega^2(x)\eta_{MN}dx^M dx^N$. We recall that the Einstein tensor is given by (D is the total space-time dimension):

$$G_{MN} = (2-D)\frac{\partial_M\partial_N\Omega}{\Omega} + (D-2)\frac{\partial_P\partial^P\Omega}{\Omega}\eta_{MN} + 2(D-2)\frac{\partial_M\Omega\partial_N\Omega}{\Omega^2} + \frac{(D-5)(D-2)}{2}\frac{\partial_P\Omega\partial^P\Omega}{\Omega^2}\eta_{MN}$$

a) In the specific case of a 5D space-time when the conformal factor depends only on z:

$$ds2 = \Omega^2(z)(\eta_{\mu\nu}dx_\mu dx_\nu - dz^2)$$

show that the only non-vanishing component of the Einstein tensor are

$$G_{\mu\nu} = -3\frac{\Omega''}{\Omega}\eta_{\mu\nu} \quad \text{and} \quad G_{zz} = 6\left(\frac{\Omega'}{\Omega}\right)^2$$

.

b)Consider a 5D space-time with a (negative) vacuum energy, $\int d^5x\,\sqrt{g}\left(-M_5^3\mathcal{R} + \Lambda\right)$ Show that a solution of the Einstein equation is the anti-de Sitter metric

$$\Omega = \frac{R}{z} \quad \text{with} \quad R = \sqrt{12M_5^3/\Lambda}$$

c) If the space has two boundaries (at $z = R$ and $z = R'$), find the values of the vacuum energy localised at the two boundaries to satisfy the boundary conditions (continuity of the Einstein eqs. at the boundaries).

Exercice 8: Mass-dimensions, $\hbar$-dimensions

In natural units ($c = \hbar = 1$), the couplings are dimensionless quantities. But it is often helpful to remember that couplings are dimensionful quantities.
a) Compute the dimensions of spin-0, -1/2, -1 fields in 4D in units of $\hbar$ and length.
b) Compute the dimensions of a gauge coupling, a Yukawa coupling, a Higgs quartic coupling.
c) By dimensional arguments, derive the functional form of the $V_L V_L$ scattering amplitude.
d) In the MSSM, the Higgs quartic coupling is related to the gauge couplings. Using dimensional arguments, find the functional form of this relation.

Exercice 9: Black-holes

a) Assuming that the Earth is a perfect black-body, estimate the average temperature on Earth using the Stefan–Boltzmann law. (we recall that the average temperature of the Sun is 5778 K, the radius of the Sun is 696'000 kms and the distance between the Sun and Earth is 150'000'000 kms).
b) Compute the escape velocity at the surface of a Black Hole of size equal to its Schwarzschild radius.
c) S. Hawking understood that the laws of quantum mechanics imply that a BH is radiating particles, hence energy. Based on dimensional arguments, find the temperature of a BH of mass M (there is actually a numerical factor of $1/(8\pi)$ in front of what can be guessed by dimensional arguments).
d) Using the Stefan–Boltzmann law, compute the radiating power of a BH of mass M (you'll assume that the BH decays only into photons, and we recall that the Stefan–Boltzmann constant is equal to $\sigma = (\pi^2 k_B^4)/(60\hbar^3 c^2)$).
e) Compute the evaporation of a BH of mass M.
f) What is the lower mass of a BH to be as old as the Universe?