

Proposed Problems on Flavour Physics

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Problem 1

a) The electromagnetic interaction of a fermion with electric charge Q is governed by the on-shell matrix element of the electromagnetic current [$q^\mu = (k - k')^\mu$]:

$$\begin{aligned} \langle f(k') | J_{\text{em}}^\mu(x) | f(k) \rangle &= \langle f(k') | e^{iPx} J_{\text{em}}^\mu(0) e^{-iPx} | f(k) \rangle \\ &= e^{-iqx} Q \bar{u}_f(k') \left[F_1(q^2) \gamma^\mu - i F_2(q^2) \sigma^{\mu\nu} q_\nu \right] u_f(k). \end{aligned}$$

Using the conservation of the electromagnetic current, show that $F_1(0) = 1$ to all orders in perturbation theory.

b) The QCD vector current $V_{ij}^\mu = \bar{q}_j \gamma^\mu q_i$ satisfies $\partial_\mu V_{ij}^\mu = i(m_{q_j} - m_{q_i}) \bar{q}_j q_i$. Show that in the isospin limit $\langle p | \bar{u} \gamma^\mu d | n \rangle = \bar{p} \gamma^\mu n$ at $q^2 = 0$.

c) The transition $\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$ is governed by the hadronic matrix element

$$\langle \pi^+(k') | \bar{u} \gamma^\mu s | \bar{K}^0(k) \rangle = f_+(q^2) (k + k')^\mu + f_-(q^2) (k - k')^\mu.$$

Show that the contribution to the decay amplitude from the form factor $f_-(q^2)$ is proportional to m_e . Show also that $f_+(0) = 1$ in the limit $m_s = m_u$.

Problem 2

Consider the mixing between a neutral meson P^0 and its antiparticle \bar{P}^0 , with mass eigenstates

$$|P_{\mp}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left[p |P^0\rangle \mp q |\bar{P}^0\rangle \right].$$

Show that the time evolution of a state which was originally produced as a P^0 or a \bar{P}^0 is given by

$$\begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} g_1(t) & \frac{q}{p} g_2(t) \\ \frac{p}{q} g_2(t) & g_1(t) \end{pmatrix} \begin{pmatrix} |P^0\rangle \\ |\bar{P}^0\rangle \end{pmatrix},$$

where

$$\begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix} = e^{-iMt} e^{-\Gamma t/2} \begin{pmatrix} \cos [(\Delta M - \frac{i}{2}\Delta\Gamma)t/2] \\ -i \sin [(\Delta M - \frac{i}{2}\Delta\Gamma)t/2] \end{pmatrix},$$

with $\Delta M \equiv M_{P_+} - M_{P_-}$, $\Delta\Gamma \equiv \Gamma_{P_+} - \Gamma_{P_-}$.

Problem 3

Consider the most general Yukawa Lagrangian with two Higgs doublets:

$$\mathcal{L}_Y = - \sum_{a=1}^2 \left\{ \bar{Q}'_L \left(\mathcal{Y}_d^{(a)'} \phi_a d'_R + \mathcal{Y}_u^{(a)'} \tilde{\phi}_a u'_R \right) + \bar{L}'_L \mathcal{Y}_l^{(a)'} \phi_a l'_R \right\} + \text{h.c.},$$

where $\phi_a(x)$ are the $Y = \frac{1}{2}$ scalar doublets, $\tilde{\phi}_a(x) \equiv i\tau_2 \phi_a^*$ their charge-conjugate fields, Q'_L and L'_L denote the left-handed quark and lepton doublets and d'_R , u'_R and l'_R the corresponding right-handed fermion singlets. All fermionic fields are written as N_G -dimensional flavour vectors, with N_G the number of fermion generations; the couplings $\mathcal{Y}_f^{(a)'}$ ($f = d, u, l$) are $N_G \times N_G$ complex matrices in flavour space.

a) Show that this Lagrangian leads to flavour-changing neutral current (FCNC) interactions of the physical scalars (for simplicity, work in the so-called ‘Higgs basis’ where only ϕ_1 acquires a vacuum expectation value).

b) Impose the alignment in flavour space of the Yukawa matrices $\mathcal{Y}_f^{(a)}$; i.e., the conditions $\mathcal{Y}_f^{(2)} = \zeta_f \mathcal{Y}_f^{(1)}$ with ζ_f ($f = u, d, s$) 3 arbitrary complex constants. Check that FCNC interactions are absent at tree level. Work out the form of the Yukawa Lagrangian in the fermion mass-eigenstate basis.