Flavour Physics

2. CKM Structure

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V_{ij} DETERMINATIONS

CKM entry	Value	Source
$\lvert V_{ud} \rvert$	**0.97420 ± 0.00021** 0.9763 ± 0.0016 0.9749 ± 0.0026	Nuclear β decay $n \to p\, e^- \bar{\nu}_e$ $\pi^+ \to \pi^0 e^+ \nu_e$
$\lvert V_{us} \rvert$	**0.2231 ± 0.0007** 0.2253 ± 0.0007 0.2213 ± 0.0023	$K \to \pi\, e^- \bar{\nu}_e$ $K/\pi \to \mu\nu$, Lattice, V_{ud} τ decays
$\lvert V_{cd} \rvert$	0.230 ± 0.011 **0.216 ± 0.005**	$\nu\, d \to c\, X$ $D \to (\pi)\, l\, \nu$, Lattice
$\lvert V_{cs} \rvert$	**0.997 ± 0.017**	$D \to K\, l\, \nu$, $D_s \to l\, \nu$, Lattice
$\lvert V_{cb} \rvert$	0.0405 ± 0.0010 **0.0420 ± 0.0006**	$B \to D^* l\, \bar{\nu}_l$, $D\, l\, \bar{\nu}_l$ $b \to c\, l\, \bar{\nu}_l$
$\lvert V_{ub} \rvert$	0.00367 ± 0.00015 0.00451 ± 0.00020 **0.00398 ± 0.00040**	$B \to \pi\, l\, \bar{\nu}_l$ $b \to u\, l\, \bar{\nu}_l$
$\lvert V_{tb} \rvert / \sqrt{\sum_q \lvert V_{tq} \rvert^2}$ $\lvert V_{tb} \rvert$	> 0.975 (95% CL) **1.019 ± 0.025**	$t \to bW / t \to qW$ $p\bar{p} \to tb + X$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = \mathbf{0.9989 \pm 0.0005}$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = \mathbf{1.040 \pm 0.051}$$

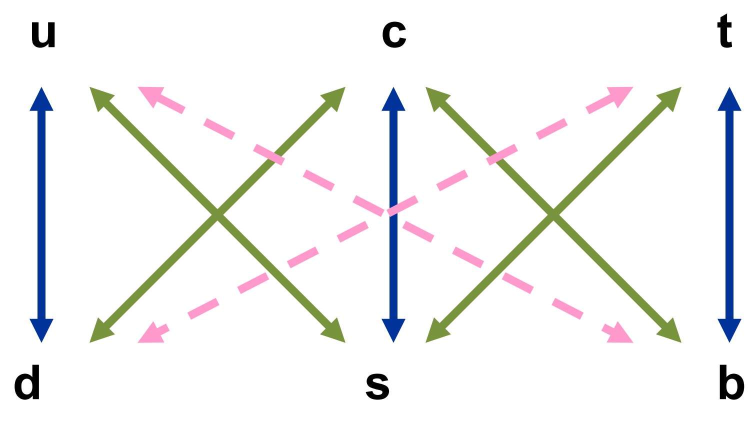
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = \mathbf{1.042 \pm 0.034}$$

$$\sum_j \left(|V_{uj}|^2 + |V_{cj}|^2 \right) = \mathbf{2.002 \pm 0.027} \quad \text{(LEP)}$$

Hierarchical Structure

$$\mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$$\lambda \approx \sin\theta_C \approx 0.223 \quad ; \quad A \approx 0.84 \quad ; \quad \sqrt{\rho^2+\eta^2} \approx 0.4$$



QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

$\frac{1}{2} N_G\,(N_G - 1)$ moduli, $\frac{1}{2} N_G\,(N_G + 1)$ phases

- $2\,N_G - 1$ **arbitrary phases:** $\bar{u}_i\, \mathbf{V}_{ij}\, d_j$

$$u_i \to e^{i\phi_i}\, u_i \quad ; \quad d_j \to e^{i\theta_j}\, d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)}\, \mathbf{V}_{ij}$$

$\Downarrow$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G\,(N_G - 1)$ **moduli** ; $\frac{1}{2}(N_G - 1)\,(N_G - 2)$ **phases**

- **$N_f = 2$: 1 angle, 0 phases** **(Cabibbo)**

$$\mathbf{V} = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \quad \Rightarrow \quad \text{No } \not{\mathcal{CP}}$$

- **$N_f = 3$:** 3 angles, 1 phase **(CKM)** $\quad c_{ij} \equiv \cos\theta_{ij} \;\; ; \;\; s_{ij} \equiv \sin\theta_{ij}$

$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda \approx \sin\theta_C \approx 0.223 \;\; ; \;\; A \approx 0.84 \;\; ; \;\; \sqrt{\rho^2 + \eta^2} \approx 0.4 \qquad \delta_{13} \neq 0 \quad (\eta \neq 0) \quad \Rightarrow \quad \not{\mathcal{CP}}$

PDG parametrization of the CKM matrix

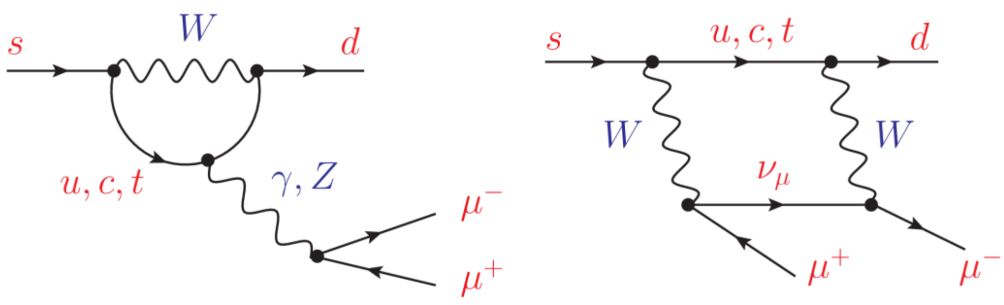
$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wolfenstein: $s_{12} \equiv \lambda \quad , \quad s_{23} \equiv A\lambda^2 \quad , \quad s_{13}\, e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$

$$\Rightarrow \quad \mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\,\lambda^2 \\ A\lambda^3\,(1-\rho-i\eta) & -A\,\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

GIM Mechanism



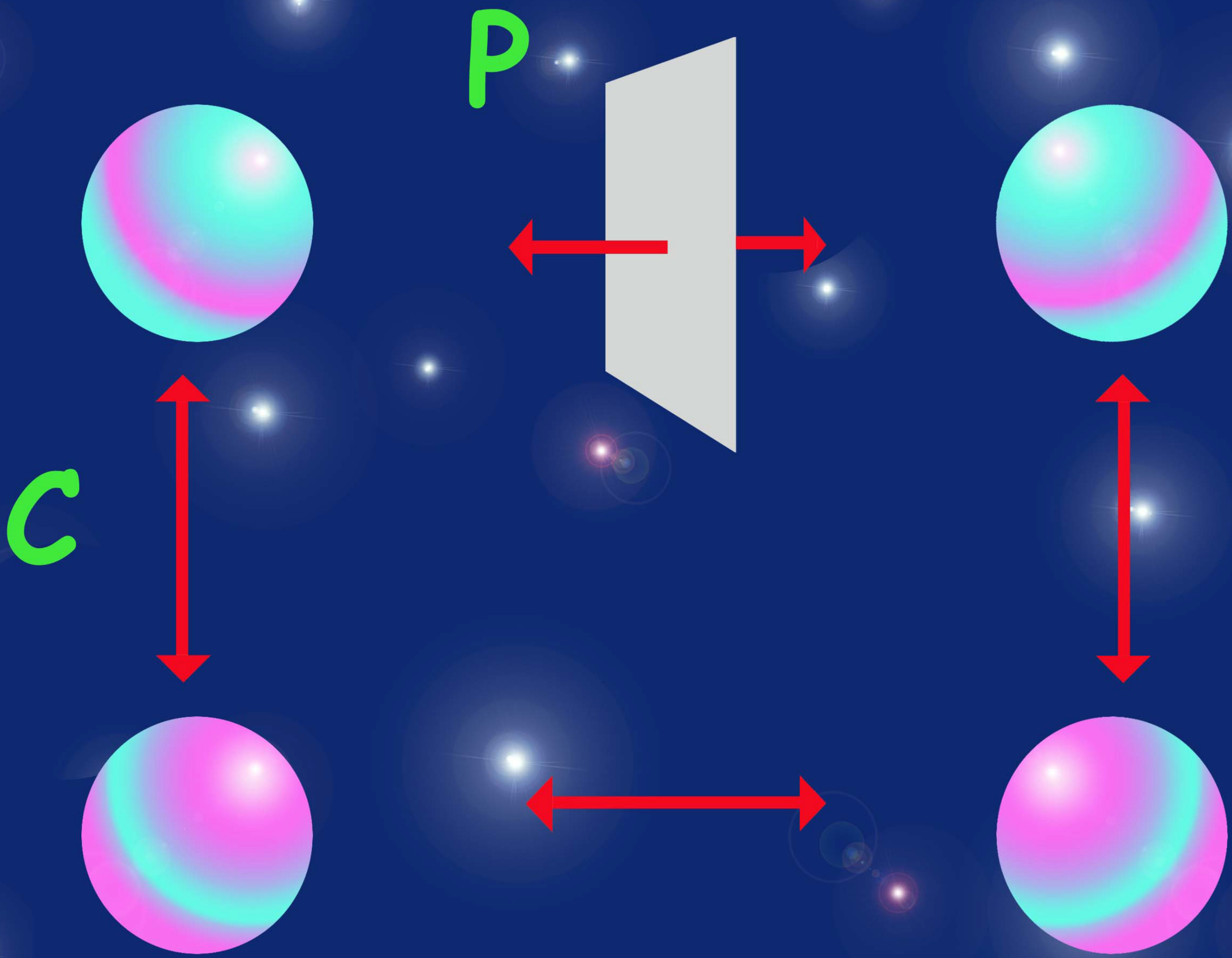
$$\mathcal{M} \propto \sum_{i=u,c,t} V_{is} V_{id}^* F(m_i^2/M_W^2)$$

$$\sum_{i=u,c,t} V_{is} V_{id}^* = 0 \quad \Rightarrow \quad \mathcal{M} = 0 \quad \text{if} \quad m_u = m_c = m_t$$

$\tilde{F}(x) \equiv F(x) - F(0)$ ⇒

$$\mathcal{M} \propto V_{cs} V_{cd}^* \tilde{F}(m_c^2/M_W^2) + V_{ts} V_{td}^* \tilde{F}(m_t^2/M_W^2)$$
$$\approx -\lambda\, \tilde{F}(m_c^2/M_W^2) - \lambda^5 A^2 (1-\rho+i\eta)\, \tilde{F}(m_t^2/M_W^2)$$

- **Top contribution dominates. Strong suppression:** $\mathcal{M} \propto \frac{g^4}{16\pi^2}\left[\lambda^5 A^2 \frac{m_t^2}{M_W^2}, \lambda \frac{m_c^2}{M_W^2}\right]$
- **CP effects fully governed by top contribution** $\left[\mathrm{Im}(V_{cs} V_{cd}^*) = -\,\mathrm{Im}(V_{ts} V_{td}^*)\right]$


P
C

- $\mathcal{C}$, $\mathcal{P}$: Violated maximally in weak interactions
- $\mathcal{CP}$: Symmetry of nearly all observed phenomena
- Slight (~ 0.2 %) $\not{\mathcal{CP}}$ in K^0 decays (1964)
- Sizeable $\not{\mathcal{CP}}$ in B^0 decays (2001)
- Huge Matter — Antimatter Asymmetry in our Universe → Baryogenesis

$\mathcal{CPT}$ Theorem: $\not{\mathcal{CP}} \Longleftrightarrow \not{\mathcal{T}}$

Thus, $\not{\mathcal{CP}}$ requires:

- Complex Phases
- Interferences

Standard Model $\not{CP}$: 3 fermion families needed

$$\not{CP} \quad \Longleftrightarrow \quad \mathbf{H}(M_u^2) \cdot \mathbf{H}(M_d^2) \cdot \mathbf{J} \neq 0$$

$$\mathbf{H}(M_u^2) \equiv (m_t^2 - m_c^2)\,(m_c^2 - m_u^2)\,(m_t^2 - m_u^2)$$

$$\mathbf{H}(M_d^2) \equiv (m_b^2 - m_s^2)\,(m_s^2 - m_d^2)\,(m_b^2 - m_d^2)$$

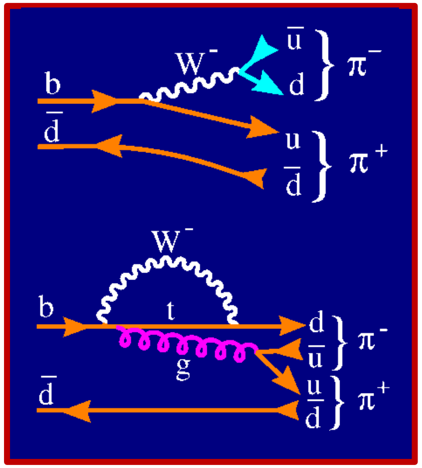
$$\mathbf{J} = c_{12}\, c_{13}^2\, c_{23}\, s_{12}\, s_{13}\, s_{23} \sin\delta_{13} = \left| A^2 \lambda^6 \eta \right| < 10^{-4}$$

- **Low-Energy Phenomena**
- **Small Effects ~ J**
- **Big Asymmetries ⟺ Suppressed Decays**
- **B Decays are an optimal place for $\not{CP}$ signals**

DIRECT $\not{\mathcal{CP}}$

$$\left|\mathbf{T}\left(\mathrm{P}\to\mathrm{f}\right)\right| \neq \left|\mathbf{T}\left(\overline{\mathrm{P}}\to\overline{\mathrm{f}}\right)\right|$$

$\overline{B}_d^0$



$$\mathbf{T}\left(\mathrm{P}\to\mathrm{f}\right) = \mathrm{T}_1\, e^{i\phi_1}\, e^{i\delta_1} + \mathrm{T}_2\, e^{i\phi_2}\, e^{i\delta_2}$$

$\mathcal{CP}$

$$\mathbf{T}\left(\overline{\mathrm{P}}\to\overline{\mathrm{f}}\right) = \mathrm{T}_1\, e^{-i\phi_1}\, e^{i\delta_1} + \mathrm{T}_2\, e^{-i\phi_2}\, e^{i\delta_2}$$

$$A^{\mathrm{CP}}_{\mathrm{P}\to\mathrm{f}} \equiv \frac{\Gamma\left(\mathrm{P}\to\mathrm{f}\right)-\Gamma\left(\overline{\mathrm{P}}\to\overline{\mathrm{f}}\right)}{\Gamma\left(\mathrm{P}\to\mathrm{f}\right)+\Gamma\left(\overline{\mathrm{P}}\to\overline{\mathrm{f}}\right)} = \frac{-2\,\mathrm{T}_1\,\mathrm{T}_2\,\sin(\phi_2-\phi_1)\,\sin(\delta_2-\delta_1)}{\mathrm{T}_1^{\,2}+\mathrm{T}_2^{\,2}+2\,\mathrm{T}_1\,\mathrm{T}_2\,\cos(\phi_2-\phi_1)\,\cos(\delta_2-\delta_1)}$$

One needs:

- **2 Interfering Amplitudes**
- **2 Different Weak Phases** $\left[\sin(\phi_2-\phi_1)\neq 0\right]$
- **2 Different FSI Phases** $\left[\sin(\delta_2-\delta_1)\neq 0\right]$

DIRECT $\not{CP}$

$$A_{CP}(B \to f) \equiv \frac{\mathrm{Br}(\bar{B} \to \bar{f}) - \mathrm{Br}(B \to f)}{\mathrm{Br}(\bar{B} \to \bar{f}) + \mathrm{Br}(B \to f)}$$

$$A_{CP}(B_d^0 \to \pi^- K^+) = -0.082 \pm 0.006 \qquad \textbf{(13.7 σ)}$$

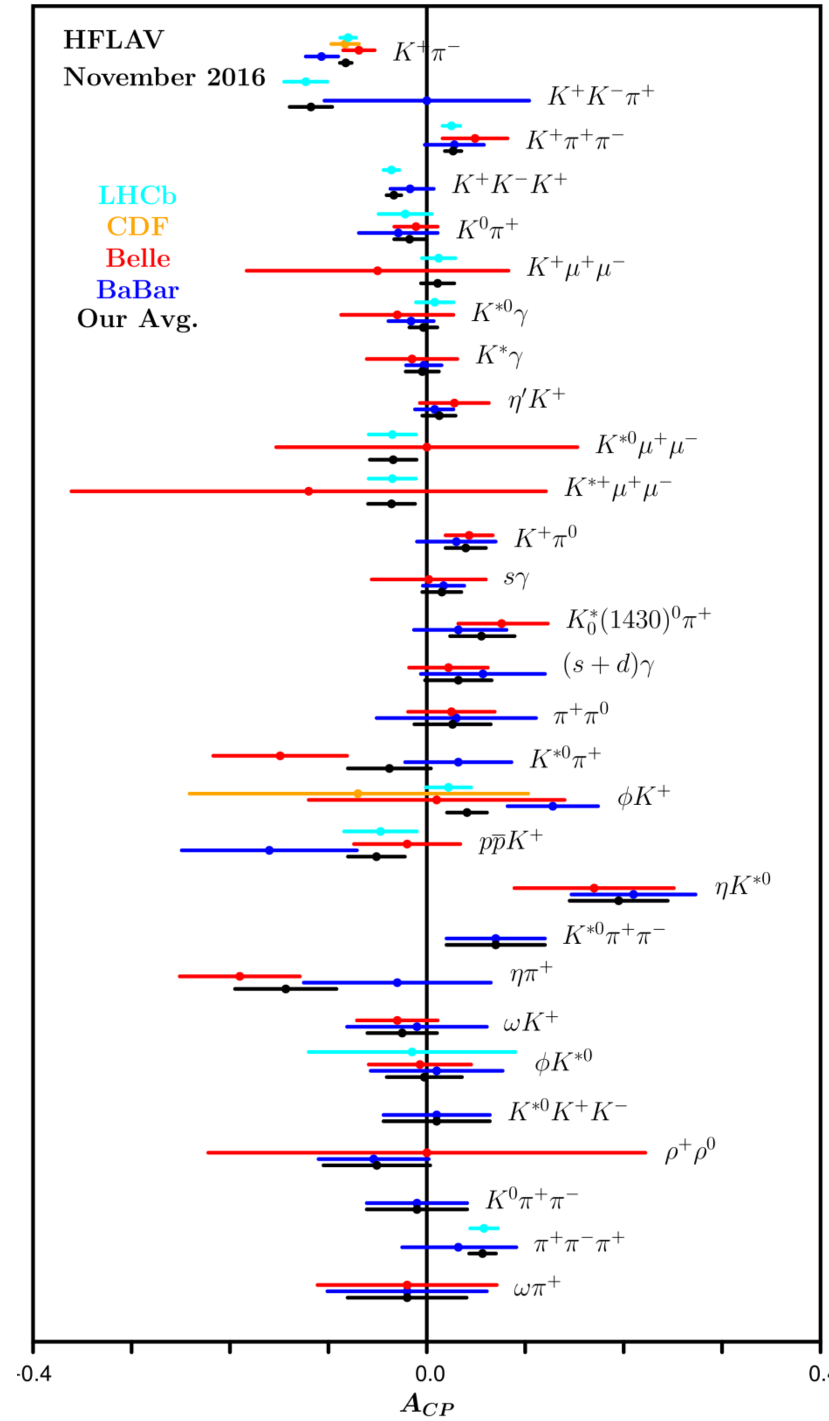
$$A(B_s^0 \to \pi^- K^+) = -0.26 \pm 0.04 \qquad \textbf{(6.5 σ)}$$

$$A_{CP}(B^+ \to K^+ K^- \pi^+) = -0.118 \pm 0.022 \qquad \textbf{(5.4 σ)}$$

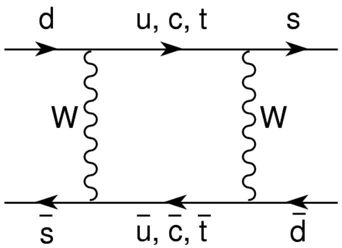
Large & Interesting Signals

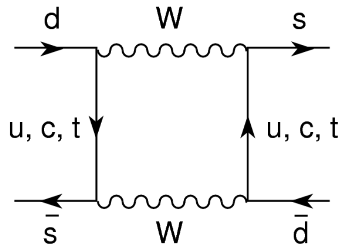
Big challenge: **Get reliable SM predictions**

Severe hadronic uncertainties



INDIRECT $\not{CP}$: $K^0 - \bar{K}^0$ MIXING





$$\left|K^0_{S,L}\right\rangle \sim p\,\left|K^0\right\rangle \mp q\,\left|\bar{K}^0\right\rangle$$

$$q/p \equiv \left(1-\bar{\varepsilon}_K\right)/\left(1+\bar{\varepsilon}_K\right)$$

$$\left\langle \bar{K}^0\right|\mathbf{H}\left|K^0\right\rangle \sim \sum_{ij} \lambda_i\,\lambda_j\; S(r_i,r_j)\;\eta_{ij}\;\;\left\langle O_{\Delta S=2}\right\rangle$$

$$\left\langle O_{\Delta S=2}\right\rangle = \alpha_s(\mu)^{-2/9}\left\langle \bar{K}^0\right|\left(\bar{s}_L\,\gamma^\alpha d_L\right)\left(\bar{s}_L\,\gamma_\alpha d_L\right)\left|K^0\right\rangle \equiv \left(\frac{4}{3}M_K^2\,f_K^2\right)\hat{B}_K$$

$$\lambda_i \equiv V_{id}\,V_{is}^* \qquad ; \qquad r_i \equiv m_i^2/M_W^2 \qquad (i=u,c,t)$$

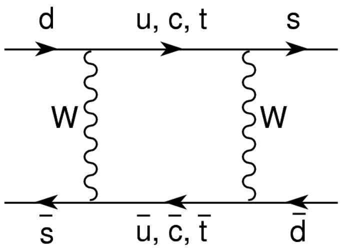
- **GIM Mechanism:** $\lambda_u + \lambda_c + \lambda_t = 0$

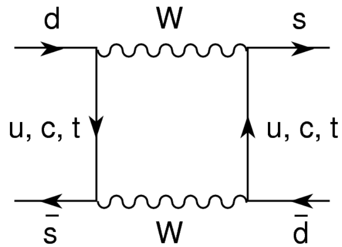
$$\left(M_{K_L} - M_{K_S}\right)/M_{K^0} = (7.00 \pm 0.01)\cdot 10^{-15}$$

- $\mathcal{CP}$ **:** $\mathrm{Im}\,\lambda_t = -\,\mathrm{Im}\,\lambda_c \simeq \eta\lambda^5 A^2$

- **Hard GIM Breaking:** $S(r_i,r_i) \sim r_i$ ➡ **t quark**

INDIRECT $\not{\mathcal{CP}}$: $K^0 - \bar{K}^0$ MIXING





$$\left|K^0_{S,L}\right\rangle \sim p\left|K^0\right\rangle \mp q\left|\bar{K}^0\right\rangle$$

$$q/p \equiv \left(1-\bar{\varepsilon}_K\right)/\left(1+\bar{\varepsilon}_K\right)$$

$$\left\langle\bar{K}^0\right|\mathbf{H}\left|K^0\right\rangle \sim \sum_{ij}\lambda_i\,\lambda_j\,S(r_i,r_j)\,\eta_{ij}\quad\left\langle O_{\Delta S=2}\right\rangle$$

$$\left\langle O_{\Delta S=2}\right\rangle = \alpha_s(\mu)^{-2/9}\left\langle\bar{K}^0\right|\left(\bar{s}_L\gamma^\alpha d_L\right)\left(\bar{s}_L\gamma_\alpha d_L\right)\left|K^0\right\rangle \equiv \left(\frac{4}{3}M_K^2 f_K^2\right)\hat{B}_K$$

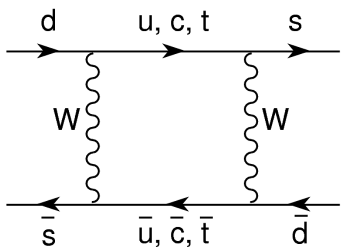
$$\lambda_i \equiv V_{id}\,V_{is}^* \quad ; \quad r_i \equiv m_i^2/M_W^2 \quad (i=u,c,t)$$

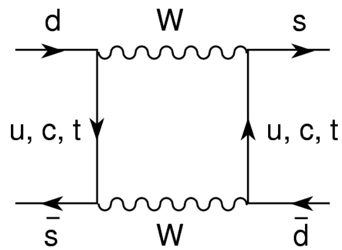
$$\mathcal{C}\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle \quad , \quad \mathcal{P}\left|K^0\right\rangle = -\left|K^0\right\rangle \quad , \quad \mathcal{CP}\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle$$

$$\left|K^0_{1,2}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle \mp \left|\bar{K}^0\right\rangle\right) \quad , \quad \mathcal{CP}\left|K^0_{1,2}\right\rangle = \pm\left|K^0_{1,2}\right\rangle$$

$$\left|K^0_S\right\rangle \simeq \left|K^0_1\right\rangle + \bar{\varepsilon}_K\left|K^0_2\right\rangle \quad , \quad \left|K^0_L\right\rangle \simeq \left|K^0_2\right\rangle + \bar{\varepsilon}_K\left|K^0_1\right\rangle$$

INDIRECT $\not{CP}$: $K^0 - \bar{K}^0$ MIXING





$$|K^0_{S,L}\rangle \sim p\,|K^0\rangle \mp q\,|\bar{K}^0\rangle$$

$$q/p \equiv (1-\bar{\varepsilon}_K)/(1+\bar{\varepsilon}_K)$$

$$K^0 \to \pi^- l^+ \nu_l \quad (\bar{s} \to \bar{u}) \qquad ; \qquad \bar{K}^0 \to \pi^+ l^- \bar{\nu}_l \quad (s \to u)$$

$$\frac{\Gamma(K^0_L \to \pi^- l^+ \nu_l) - \Gamma(K^0_L \to \pi^+ l^- \bar{\nu}_l)}{\Gamma(K^0_L \to \pi^- l^+ \nu_l) + \Gamma(K^0_L \to \pi^+ l^- \bar{\nu}_l)} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{2\,\mathrm{Re}(\bar{\varepsilon}_K)}{1+|\bar{\varepsilon}_K|^2} = (0.332 \pm 0.006)\%$$

$\Rightarrow$ $\mathrm{Re}(\bar{\varepsilon}_K) = (1.66 \pm 0.03) \cdot 10^{-3}$

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)} \approx \varepsilon_K \qquad \eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \approx \varepsilon_K$$

$$\varepsilon_K = (2.228 \pm 0.011) \cdot 10^{-3}\, e^{i\phi_\varepsilon}$$

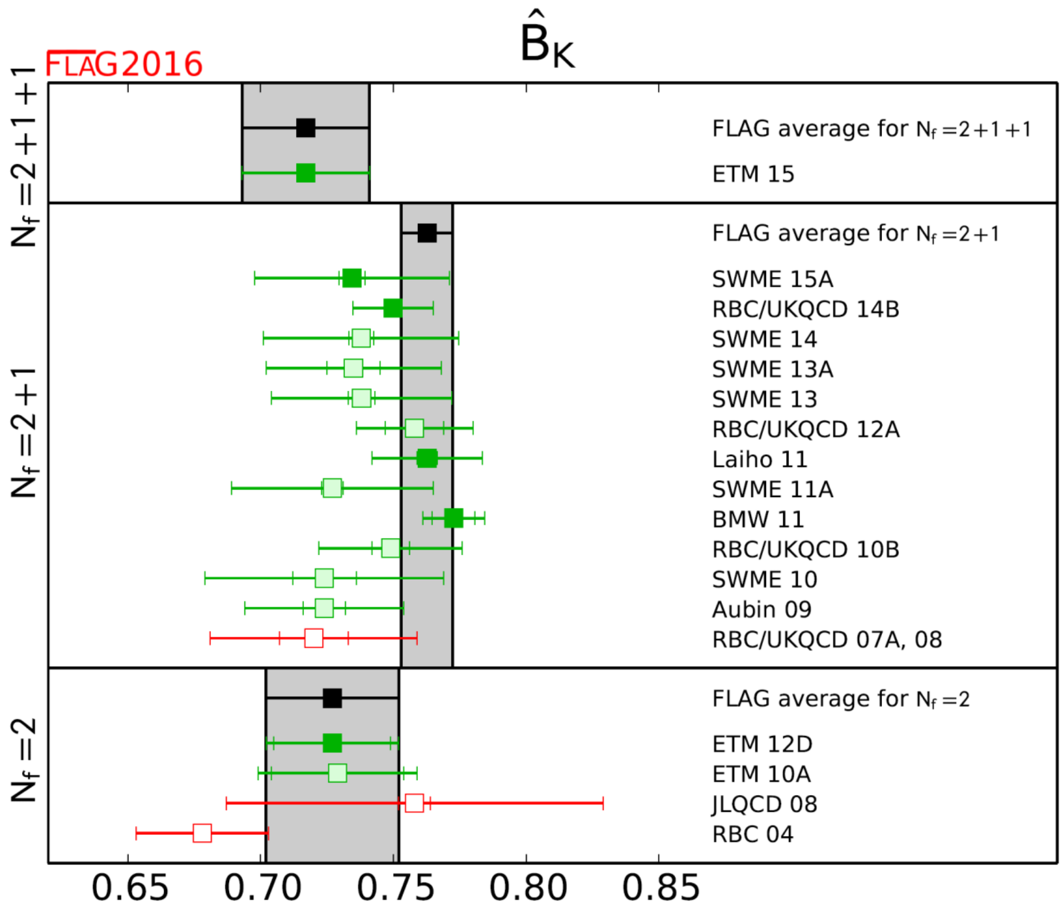
$$\phi_\varepsilon = (43.52 \pm 0.05)^\circ$$

$\Rightarrow$ Buras et al

$$\eta\left[(1-\rho)A^2 + 0.22\right] A^2\, \hat{B}_K = 0.143$$

Lattice Results for $\hat{B}_K$

$$B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.557 \pm 0.007 \qquad , \qquad \hat{B}_K = 0.763 \pm 0.010 \qquad (N_f = 2+1)$$



Flavianet Lattice Averaging Group

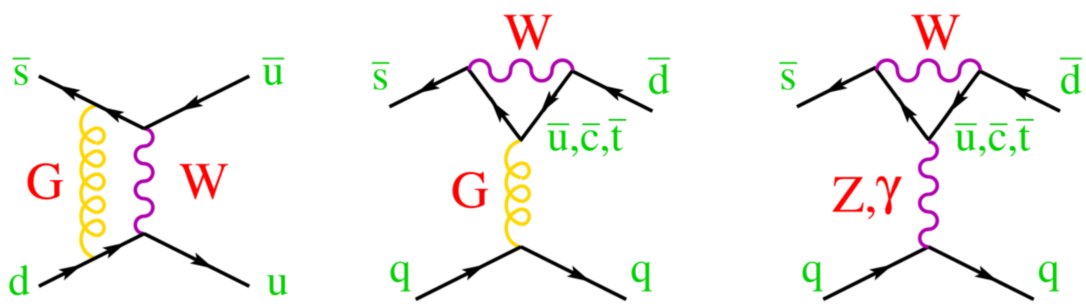
DIRECT $\not{C}\!\not{P}$ in $K \to \pi\,\pi$

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)} \approx \varepsilon_K + \varepsilon'_K \qquad\qquad \eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \approx \varepsilon_K - 2\,\varepsilon'_K$$

$$\mathrm{Re}\left(\varepsilon'_K/\varepsilon_K\right) \approx \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\} = (16.8 \pm 1.4)\cdot 10^{-4}$$

NA48, NA31

KTeV, E731



$$\mathrm{Re}\left(\varepsilon'_K/\varepsilon_K\right)_{\mathrm{Th}} = \left(19\,{}^{+11}_{-9}\right)\cdot 10^{-4}$$

- Short-distance OPE
  Ciuchini et al, Buras et al
- Long-distance χPT
  Pallante-Pich-Scimemi
  Cirigliano-Ecker-Neufeld-Pich

$(15 \pm 7)\times 10^{-4}$ **2017 update** Gisbert-Pich, 1712.06147