Storyline:

- Effective Models
- Main Result (informal)
- Technicalities (SPT, algebra)
- Result (more formal)
Effective Models
Effective Models

Effective Models

Effective Models

\[ H = \sum_i J_{\text{intra}} S_{i,1} \cdot S_{i,2} + J_{\text{inter}} S_{i,2} \cdot S_{i+1,1}. \] (1)

Effective Models

- Cond. Mat.: connections to experiment
- Stat. Phys.: ladders interpolating between low and high spin physics
- Science. in general: toy models
- Phys. in general as search for effective models / descriptions

→ How to estimate the validity?
Main Result (informal)
Main Result (The Paradigm)

- When ‘zooming out’ sufficiently, an effective model should be indistinguishable from the original model.

- Minimal requirement: if one model $S_1$ effectively describes another model $S_2$ both are required to be in the same phase $'[S_1] = [S_2]'$. 
Main Result (Concrete Example)

\[
H(J_{AF}, J_F) = J_{AF} \sum_{\langle ij \rangle_{AF}} S_i \cdot S_j - J_F \sum_{\langle ij \rangle_F} S_i \cdot S_j
\]

\[
\equiv H_{AF} \quad \quad \quad \quad \equiv -H_F
\]
Main Result (Concrete Example)

\[ H(J_F) = H_{AF} + J_F \cdot H_F \]

Technicalities
Local Unitary Circuits
SPT Order

- Quantum Phases $\sim$ equivalence relations $[\psi]_{LUC}$:
  $$|\psi\rangle \sim |\phi\rangle \iff \exists \text{ constant depth LUC } U : U |\phi\rangle = |\psi\rangle$$

- With symmetries: Quantum Phases $\sim$ equivalence relations $[(|\psi\rangle, R)]_{LUC}$ of pairs of states and symmetries (group representations $R$):
  $$(|\psi\rangle, R_1) \sim (|\phi\rangle, R_2) \iff \exists \text{ constant depth LUC } U : U |\phi\rangle = |\psi\rangle \text{ and } UR_2 U^\dagger = R_1$$
SPT Order

- Quantum Phases $\sim$ equivalence relations $[\psi]_{LUC}$:
  \[ |\psi\rangle \sim |\phi\rangle \iff \exists \text{ constant depth } LUC \cdot |\phi\rangle = |\psi\rangle \]

- Quantum Phases $\rightarrow$ Topological Order

- Symmetric Quantum Phases $\rightarrow$ Symmetry Protected Topological (SPT) Order

In $d$ dimensions:

SPT Order wrt. group $G \Leftrightarrow H^{d+1}(G, U(1))$

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Cohomology and Phases in 1d

Given a uMPS $|\psi(M)\rangle$ symmetric wrt. $\Sigma$, then there exists a phase $\theta$ and a unitary $V$ such that

$$M \Sigma = e^{i\theta} V^\dagger M V$$

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Given a uMPS $|\psi(M)\rangle$ symmetric wrt. $\Sigma$, then there exists a phase $\theta$ and a unitary $V$ such that

$$M \Sigma = e^{i\theta} V^\dagger M V$$

Directly obtain $\omega \in H^2(G, U(1))$ via

$$\omega(g, h) \mathbb{1} = V_g V_h V_g^\dagger V_h^\dagger$$

SU(2) and SO(3)

Standard definitions and faithful representations on $\mathbb{C}^n$:

$$SO(3) \equiv \{ g \in \mathbb{R}^{3 \times 3} : g^T g = 1 \}$$
$$g_1(v) = \exp(i v \cdot s_1)$$

$$SU(2) \equiv \{ g \in \mathbb{C}^{2 \times 2} : g^\dagger g = 1 \}$$
$$g_{\frac{1}{2}}(v) = \exp(i v \cdot s_{\frac{1}{2}})$$

with $v = \theta n$ for an angle $\theta$ and a unit vector $n$
**SU(2) and SO(3)**

Group algebras, and why we should care:

\[ \mathcal{K}(SU(2)) \cong M(1, 1) \oplus M(2, 2) \oplus M(3, 3) \oplus \ldots \]

\[ \mathcal{K}(SO(3)) \cong M(1, 1) \oplus M(3, 3) \oplus M(5, 5) \oplus \ldots \]

Moreover it is equivalent:

- \( V \) is a faithful group representation of \( G \)
- for any irreducible representation \( W \) of \( G \) there exists an \( n \) such that \( V \otimes \cdots \otimes V \cong W \oplus \ldots \) \( n \) times
$SU(2)$ and $SO(3)$

Local (on site) description:

$\mathcal{G} \equiv \{g \otimes g : g \in SU(2)\} \cong SO(3)$

$\mathbb{Z}_2 \triangleleft SU(2)$ trivial under $g \mapsto g \otimes g$

$SU(2) \cong \text{Double-Cover}(SO(3))$
Results
The Model

\[ S = \frac{1}{2} \]

\[ H(J_{AF}, J_F) = J_{AF} \sum_{\langle ij \rangle_{AF}} S_i \cdot S_j - J_F \sum_{\langle ij \rangle_F} S_i \cdot S_j \]
The Model

\[ S = \frac{1}{2} \]

Effective description:

\[ S = 1 \]
Model vs. Effective Model

\[ S = \frac{1}{2} \]

\[ J_F \to \infty \]

\[ J_{AF} \]

Idea

\[ M = e^{i\theta} \]

Combine!

Ladder order parameter measuring validity of the effective \( S = 1 \) model!

\[ g \simeq SO(3) \]
Result

\[ H(J_F) = H_{AF} + J_F \cdot H_F \]

Thank you for your attention!
Small $J_F$ limit

\[ H(J_F) = H_{AF} + J_F \cdot H_F \]

For $S=1/2$:

$J_F \rightarrow 0$
Small $J_F$ limit

$$H(J_F) = H_{AF} + J_F \cdot H_F$$

$S=1/2$
Small $J_F$ limit

$H(J_F) = H_{AF} + J_F \cdot H_F$

$\Rightarrow$ Perturbation theory favors the parallel configurations!

Ground State Space
Large $J_F$ limit

\[ H(J_F) = H_{AF} + J_F \cdot H_F \]
Large $J_F$ limit

\[ H(J_F) = H_{AF} + J_F \cdot H_F \]

$S = 1/2$ \quad $J_F \rightarrow \infty$ \quad $J_{AF}$

$S = 1$
Large $J_F$ limit

$$H(J_F) = H_{AF} + J_F \cdot H_F$$

$\Rightarrow$ Is numerically found to be in a SPT Haldane phase. In particular, we find it to be in the SPT non-trivial phase wrt. $SO(3)$. 
Large $J_F$ limit

\[
\begin{align*}
H_F(J_F) &= H_{AF} + J_F \cdot H_{F} \\
S &= 1
\end{align*}
\]