Numerical Relativity with Scalar Gauss-Bonnet Gravity

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Why NR in modified gravity theories?

• Conclusion of Kento’s talk: “True potential of GWs is limited by the lack of knowledge of the merger phase in non-GR theories”

• NR is the only tool we have to study the merger phase of compact binaries

• Early-inspiral phase carries information on negative-PN contributions (such as dipole radiation), late-inspiral and merger carry information on the strong-field, large curvature regime of gravity: complementary information

• PN approaches can not be accurate enough to test GR deviation even in late inspiral: phenomenological waveforms & EOB require calibration of parameters from NR simulations. Even few NR simulations of binary BH (BBH) mergers may be sufficient for this task!

• As discussed in Thomas’, Frans’, Luis’ talks, this is a very challenging task (well posedness, gauge choices, difficult numerical implementation). Very few results up to now:
  - most results in scalar-tensor theory (Palenzuela et al., Healy et al., Barausse et al.) but no-hair theorems!
  - dynamical Chern-Simons: well-posed only with EFT approach (Delsate et al. ’15)
    binary BH evolution with EFT approach in (Okounkova et al. ‘17), see Okounkova’s talk
Why sGB gravity?

SCALAR-TENSOR GRAVITY (one scalar field)

Horndeski

scalar Gauss-Bonnet

quadratic curvature terms

\[
S = \int d^4x \sqrt{-g} \left\{ K(\phi, X) - G_3(\phi, X) \Box \phi \\
+ G_4(\phi, X) R + G_{4,\phi}(\phi, X) \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)(\nabla_\mu \nabla_\nu \phi) \right] \\
+ G_5(\phi, X) \nabla_\mu \nabla_\nu \phi - \frac{G_{5,\phi}(\phi, X)}{6} \left[ (\Box \phi)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)(\nabla_\mu \nabla_\nu \phi) \right] + 2(\nabla_\mu \nabla_\nu \phi)(\nabla_\mu \nabla_\sigma \phi)(\nabla_\mu \nabla_\sigma \phi) \right\} + S_{\text{mat}}[\Psi, \gamma(\phi)g_{\mu\nu}]
\]

\[
S = \frac{1}{16\pi} \int \sqrt{-g} d^4x \left[ R - 2 \nabla_\mu \phi \nabla^\mu \phi - V(\phi) + f_1(\phi) R^2 \\
+ f_2(\phi) R_{\mu\nu} R^{\mu\nu} + f_3(\phi) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + f_4(\phi) T T \\
+ S_{\text{mat}}[\Psi, \gamma(\phi)g_{\mu\nu}] \right]
\]

\[
S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} \nabla_\alpha \varphi \nabla^\alpha \varphi + f(\varphi) \mathcal{G} \right] + S_m[g_{\mu\nu}, \psi]
\]
Why sGB gravity?

- $\mathcal{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ Gauss-Bonnet invariant (total derivative)
- second-order field equations $\Rightarrow$ no Ostrogradski instability, could exist beyond small coupling limit
- BHs have scalar hair
- GR deviations appear at large curvature $\Rightarrow$ no constraints from binary pulsars, need GW
- fundamental physics motivation: low-energy effective string theory (Gross & Sloan '87),
- first terms in polynomial curvature expansion of a possibly renormalizable theory (Stelle '77)

$$S = \frac{1}{2} \int d^4 x \sqrt{-g} \left[ R - \frac{1}{2} \nabla_\alpha \varphi \nabla^\alpha \varphi + f(\varphi) \mathcal{G} \right] + S_m[g_{\mu\nu}, \psi]$$

The simplest, well-behaved modification of the strong-field limit of gravity!

Different possible choices of the coupling function $f(\varphi)$:

- exponential: $f = e^{\alpha \varphi}$ Einstein-dilaton Gauss-Bonnet (EdGB) gravity [string-inspired]
  (Mignemi & Stewart '93, Kanti et al. '96, Pani & Cardoso '09, Yunes & Stein '11, etc.)
- linear: $f = \alpha \varphi$ shift-symmetric Gauss-Bonnet gravity
  (Sotiriou & Zhou '14a, '14b, Barausse & Yagi '15, Benkel et al. '16, '17)
- scalarized: $f = \alpha \varphi^2$ Gauss-Bonnet gravity with scalarization
  $f = \alpha (1 - \exp(-\beta \varphi^2))$ (Silva et al. '17, Doneva & Yazadjiev '17)
Why sGB gravity?

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  \((\text{Sotiriou & Zhou '14a, '14b, Barausse & Yagi '15, Benkel et al. '16, '17})\)

- \( \zeta = \alpha / M^2 \) dimensionless coupling

- stationary BH solutions (Kanti et al.'96): \( \zeta < \zeta_{\text{max}} \sim 1 \) we do not have to require \( \zeta \ll 1 \)

- scalar field profile \( \varphi \sim Q/r \) Q scalar charge: BHs have hair! \( Q/M \sim \zeta \)

- PN description of BH binary inspiral (Yagi et al. '12)

\[ \delta \dot{E}(\varphi) = \dot{E}_\text{GR} \frac{5}{96} \frac{\zeta}{\eta^4} \frac{\delta m^2}{m^2} v^{-2} \]
Is sGB gravity ruled out?

1) Is sGB gravity well-posed?  Maybe!
   Well-posedness: existence of a unique solution which depends continuously on initial data.
   It requires that \(|\delta u(t, \cdot)| \leq F(t)|\delta u(0, \cdot)|\)

   Strong hyperbolicity is a condition for well-posedness: the principal part is diagonalisable.
   For instance, DCS gravity (as a full theory) seems to be not strongly hyperbolic \((\text{Delsate et al.,'15})\)
   so it should be considered as a truncation of a more fundamental theory (EFT approach)

   Recently, it has been shown that sGB gravity is not strongly hyperbolic \((\text{Papallo & Reall '17, Papallo '17})\) …
   but this was only shown in generalised harmonic gauge
   \[ 0 = g^{\nu \rho} \nabla_\nu \nabla_\rho x^\mu = \frac{1}{\sqrt{-g}} \partial_\nu \left( \sqrt{-g} g^{\mu \nu} \right) \]
   there is no reason to believe that the same applies in a BSSN-like formulation!

2) Is sGB gravity ruled out by GW observations?  No!

   GW170817 has shown that GWs and light travel with the same speed.
   This sets bounds on several modified gravity theories, including sGB gravity
   \((\text{Ezquiaga & Zumalacarregui '17, Baker et al. '17, Sakstein & Jain '17, Creminelli & Vernizzi '17})\)

   \[ |\alpha_T| = \left| \frac{c^2 - c_T^2}{c^2} \right| \lesssim 10^{-15} \]

   “models such as Einstein-dilaton-Gauss-Bonnet that do not lead an accelerating universe, of interest for example for deviations detectable via black hole tests, are still allowed \((\text{Sakstein & Jain, '17})\)”
BH binaries in sGB gravity - first step: EFT approach

As a first step, we perform NR simulations of BBH coalescences in an EFT framework (analogue to that followed in Okounkova et al. ’17 for DCS gravity, see Okounkova’s talk)

We expand the action, and then the field equations, in powers of $\zeta = \alpha_{GB}/M^2 << 1$ (note that sGB viable also for $\zeta \sim 1$, see later). We also assume $\Phi^{(0)}=0$ (no-hair)

$$\Phi = \sum_{k=0}^{\infty} \frac{1}{k!} \zeta^k \Phi^{(k)} = \zeta \Phi^{(1)} + \ldots$$

$$g_{ab} = g_{ab}^{(0)} + \sum_{k=1}^{\infty} \frac{1}{k!} \zeta^k h_{ab}^{(k)} = g_{ab}^{(0)} + \frac{\zeta^2}{2} h_{ab}^{(2)}$$

The field equations

$$\Box \Phi = -\frac{\alpha_{GB}}{4} f'(\Phi) \mathcal{R}_{GB},$$

$$G_{ab} = -\alpha_{GB} g_{ab} + \frac{1}{2} T_{ab}^{(\Phi)},$$

are expanded as:

$$G_{ab}^{(0)} = 0, \quad \Box^{(0)} \Phi^{(1)} = -\mathcal{R}_{GB}^{(0)}.$$

We chose EdGB gravity, $f'=e^\Phi=1+\ldots$

Scalar field in a BBH background sourced by $R_{GB}$, without backreaction.
With this approach, well-posedness automatically guaranteed (at any order, it is inherited from well-posedness of equations at 0-th order).

We evolve the scalar field simultaneously with the BBH background in the same BSSN framework.
BH binaries in sGB gravity - first step: EFT approach

Scalar radiation extracted at $r=40M$:

$q = 1$

$q = 1/2$

- Excitation of scalar radiation sourced by curvature / orbital dynamics

- Post-merger ringdown: as expected, the scalar field oscillates with a combination of GR quasi-normal modes of scalar and gravitational radiation

  (preliminary: $l=m=1$ has the frequency of the scalar $l=m=1$ QNM,
  $l=m=2$ has both the frequencies of scalar and gravitational $l=m=2$ QNMs but caution: frequency extraction difficult due to dependency on initial ringdown time)
It should allow to estimate, even in this first-order computation, how much the GR deviation affects the orbital motion and thus the magnitude of the effects on the GW phase, and the detectability from interferometric detectors!
BH binaries in sGB gravity - second step: fully coupled equations

This is a long-term project, just at the beginning!

- We do not assume small coupling, we do not expand metric/scalar field, and evolve the fully coupled equations of metric + scalar field

- 3+1 decomposition:

\[ \gamma_{ab} = g_{ab} + n_a n_b \]

\[ ds^2 = -(\alpha^2 - \beta^a \beta_a) dt^2 + \beta_a dt dx^a + \gamma_{ab} dx^a dx^b \]

\[ \mathcal{L}_n \gamma_{ab} = -2 K_{ab} = -2 \left( A_{ab} - \frac{1}{3} \gamma_{ab} K \right) \]

variables: \( \Phi, \gamma_{ab}, K_\Phi, K, A_{ab} \)

\[ K_\Phi = -\mathcal{L}_n \Phi \]

- Constraints (energy and momentum) involve metric and scalar field

\[ \mathcal{H}^{GR} = R - K_{ab} K^{ab} + K^2 \]

\[ E^{GR}_{ab} = R_{<ab>} - A_{ac} A^c_b + \frac{1}{3} (K A_{ab} + \gamma_{ab} A^2) \]

\[ C_{ab} = D_a D_b f(\Phi) - K_{ab} \mathcal{L}_n f(\Phi) ; \quad C = \gamma_{ab} C_{ab} \]

This matrix has to be inverted to get a first-order in time formulation (work in progress…)

- Future steps: BSSN-like formulation, gauge choice, numerical implementation…
BH binaries in sGB gravity - second step: fully coupled equations

A brief comment on the well-posedness

\[
\begin{pmatrix}
1 & -2f'\mathcal{H}^{GR} & 8\alpha_{GB}f' E^{GR} \delta_{<ab>} \\
2\alpha_{GB} \mathcal{H}^{GR} & 6 - 16\alpha_{GB} C & 16\alpha_{GB} C^{<ab>} \\
8\alpha_{GB} f' E^{GR}_{cd} & -16\alpha_{GB} C_{<cd>} & \delta_{<c}^{b} \delta_{d>}^{a} (1 - 8\alpha_{GB} C) - 16\delta_{<c}^{a} \delta_{d>}^{b}
\end{pmatrix}
\begin{pmatrix}
\mathcal{L}_{n} K_{\Phi} \\
\frac{1}{6} \mathcal{L}_{n} K \\
\frac{1}{8} \mathcal{L}_{n} A_{<ab>}
\end{pmatrix}
= 
\begin{pmatrix}
S_{\Phi} \\
S_{K} \\
S_{<cd>}
\end{pmatrix}
\]

The result in Delsate et al., 15 is based on the fact that in the DCS case the matrix (more precisely, the part involving the time derivative of the traceless extrinsic curvature) is degenerate:

DCS: \[
\cdots + \alpha_{CS} \left( \delta_{<c}^{a} \epsilon_{d>}^{b e} D_{e} \Phi \right) \mathcal{L}_{n} X_{<ab>} = S_{<cd>}
\]

sGB: \[
\cdots + \left( \delta_{<c}^{a} \delta_{d>}^{b} (1 - 8\alpha_{GB} C) - 16\alpha_{GB} \delta_{<c}^{a} \delta_{d>}^{b} \right) \mathcal{L}_{n} A_{<ab>} = S_{<cd>}
\]

- The former is obviously degenerate due to the presence of the Levi-Civita tensor:

\[
\left( \delta_{<c}^{a} \epsilon_{d>}^{b e} D_{e} \Phi \right) D^{c} \Phi D_{d} \Phi \equiv 0
\]

- The second is non-degenerate (at least, for small enough coupling constant)

This is an indication that (non-EFT) sGB gravity has not an evident ill-posedness such as DCS gravity but of course all this is very, very preliminar!
Conclusions

- Developing NR techniques and performing NR simulations of modified gravity theories such as quadratic gravity theories is challenging but rewarding.

- sGB gravity is a natural candidate to perform these computations. Recent claims of it ill-posedness and of tension with GW observational data only refer to a specific gauge/formulation, and to specific applications of the theory. sGB still stands as a viable and promising GR modification.

- With the easiest approach one simply evolves a scalar field equation in a GR BBH background. This is a necessary first step before addressing the fully coupled problem. Still, this approach gives order-of-magnitude estimates of the orbital motion modifications and thus of the detectability.

- For the fully coupled problem we are just at the beginning. Some preliminary indication suggest that the mechanisms immediately leading to ill-posedness in DCS gravity are not present in sGB gravity. These are just preliminary and partial indications: a systematic study of well-posedness of sGB gravity is still to be done.