The Promise of Numerical Relativity

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Numerical Relativity Beyond General Relativity
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Outline

- "The Promise of Numerical Relativity" →

  "The Need for Solutions"

- Open questions/opportunities for future work

  - the LHS : modified gravity
  - the RHS : exotic matter within general relativity
  - mixing the LHS/RHS : modified couplings

(focusing here on ground-based GW driven science, which for now means stellar mass, compact object mergers)

Tie in to earlier talks

- *Theoretical motivation for beyond GR theories*
  - nothing in the data yet calling for beyond-GR, so why bother?
    - GWs are exploring a new regime of gravity not yet tested by experiment or other direct astronomical observations
    - certainly can do null/consistency tests; will get stronger with more data
    - but what if an "anomaly" is discovered? How can we interpret that as a modification to GR if we do not have predictions for how dynamical, strong-field gravity can differ from GR?

Tie in to earlier talks

- *Theoretical motivation for beyond GR theories*
  - why does GR seem so "special" in this regime? I.e., why the dearth of *non-trivial, viable* modified/alternative theories:
    - consistent with existing weak-field tests, yet allow for markedly different behavior in the dynamical, strong field
    - can be confronted with LIGO/Virgo data
      - must be solvable to make predictions, i.e. possess a mathematically well-posed, initial value problem

Tie in to earlier talks

- *Theoretical motivation for beyond GR theories*
  - note, the following are all acceptable in a well-posed theory
    - exponential sensitivity of the evolution to perturbations in the initial data, but bounded by an exponential with a constant growth rate independent of the perturbation
      - "usual" unstable physical systems, chaotic dynamics, etc.
    - solutions have a finite range of validity beyond which some "singular" behavior develops
      - formation of shocks, singularities, Cauchy horizons
      - predictability may be lost beyond this point, but at least the theory prior to this makes a unique, testable prediction to its realm of validity

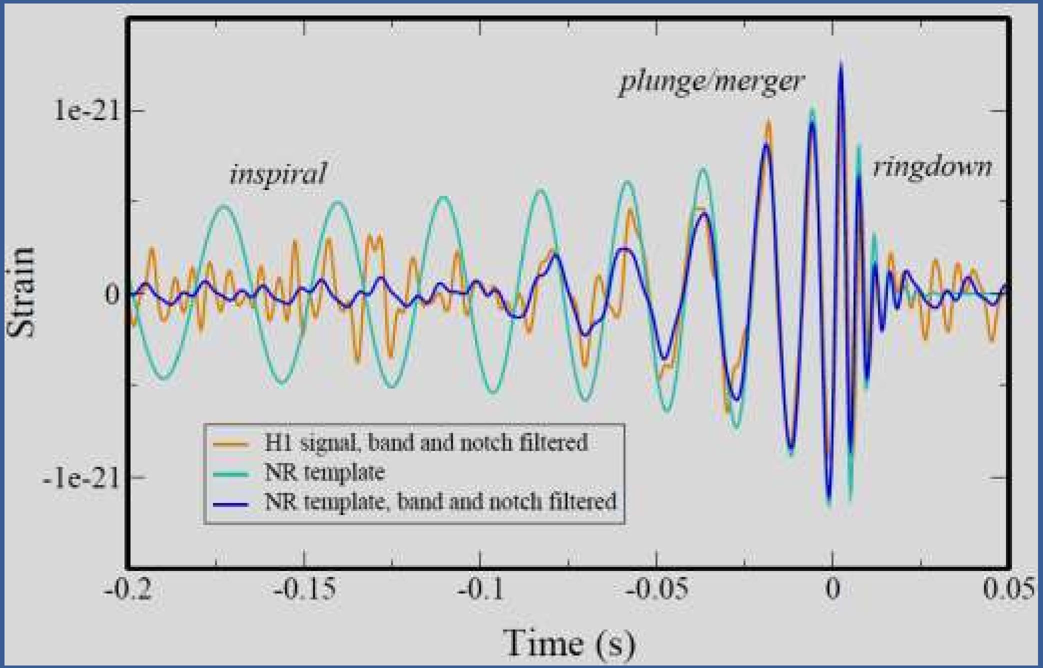
Tie in to earlier talks

- *Theory agnostic, or specific tests?*
  - Ideally, not an "or" question. Want both.
  - for specific tests, need solutions to a theory that can make predictions

Numerical Relativity

- NR is not a subfield of study within GR, it's simply the application of numerical PDE methods to solving the Einstein + matter equations

- The main problem with this part of beyond-GR are the lack of theories amenable to solution where LIGO/Virgo has so far given us the loudest signals :
  the merger regime



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- Likely (as with GR), numerical methods will be one of the crucial tools understanding the predictions of well-posed theories once they are identified, but until then the "promise" of NR applied to beyond-GR is unlikely to be realized

Numerical solution of PDEs

- Pros :
  - complexity of equations, initial conditions, and non-linearity not an issue, up to a point
  - numerics unforgiving to ill-posed problems
  - gain insight into problems by *looking* at *full* solutions

Numerical solution of PDEs

- Cons :
  - for any novel problem where an existing code cannot easily be adapted, its usually a very time-consuming enterprise to develop a new code
  - no “back-of-the-envelope” shortcuts for a first rough insight; you solve the full problem or you don’t
  - numerics unforgiving to ill-posed problems
  - difficult to gain insight into trends as a function of problem parameters by looking at individual solutions

Open Questions/Opportunities

- Break the discussion down into 3 broad classes
  - modified coupling between matter/geometry
  - modified geometry (LHS)
  - modified matter (RHS): “exotic” alternatives to black holes
- Focusing on issues related to numerics

Modified Coupling Between Geometry and Matter

- Prototypical example, scalar tensor theories

$$L \propto R(g_{ab}) + L_\theta(\theta, g_{ab}) + L_\varphi(\varphi, f(\varphi) g_{ab})$$

  - Einstein gravity with a scalar field θ, but other matter ϕ experiences "physical" geometry through a scalar field rescaled metric

- These are arguably the only class of modified gravity theories shown to be non-trivial and viable; however

  - with scalar tensor theories need "spontaneous scalarization" (Damour & Esposito Farese) to provide observationally interesting modifications, and also only relevant for binaries with neutron stars

  - with Einstein-Maxwell-Dilaton, need black holes with a significant charge

Modified Coupling Between Geometry and Matter

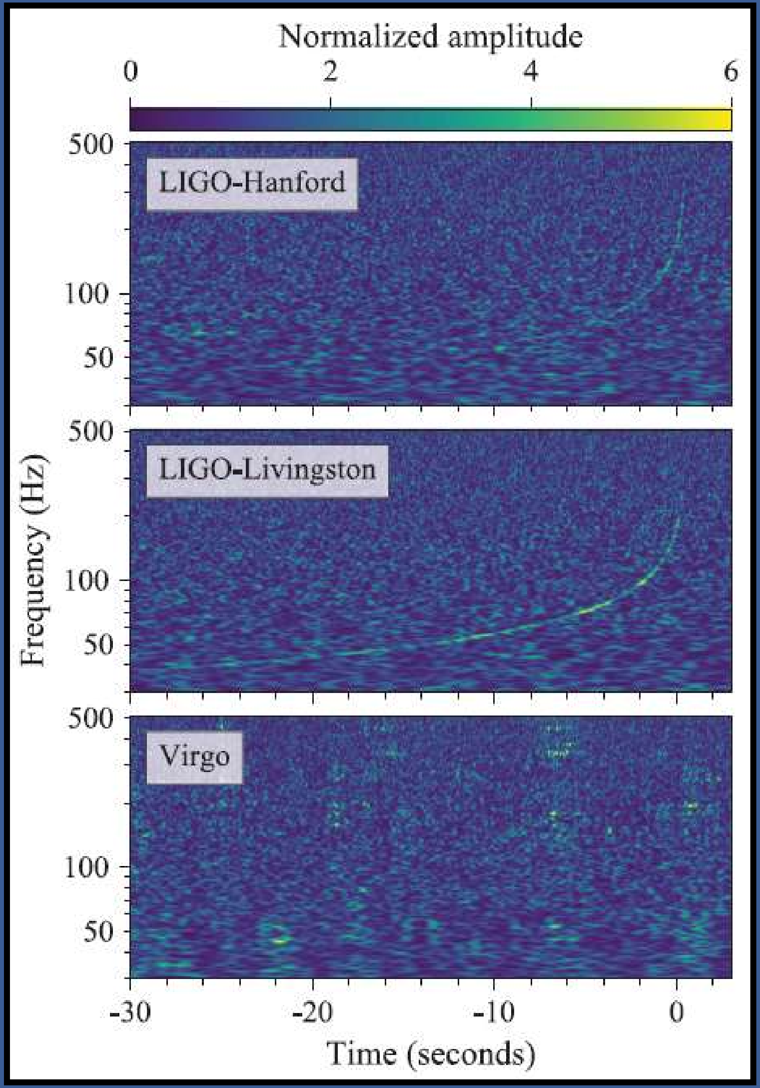
- Open questions/Opportunities
  - theories ready to explore parameter space and confront with data
  - several groups already have codes to study these systems

    Spontaneous/dynamical Scalarization :

    Palenzuela et al. PRD 89 (2014)
    Sperhake et al. PRL 119 (2017)

    Einstein-Maxwell-Dilaton :

    Hirschmann et al. PRD97 (2018)



LIGO/Virgo

Modified Geometry

- Two broad classes, (a) those that have higher than second derivatives, (b) those that are still governed by second order PDEs
  - (a) adding curvature invariants beyond R to the action higher generically introduces higher derivatives to the equations of motions, which generically are ill-posed (Ostrogradski)
    - some exceptions, notably degenerate higher order scalar tensor (DHOST) theories
  - one particularly interesting example that satisfies the "non-trivial" condition is dynamical Chern-Simons gravity, but is likely ill-posed

$$L \propto R + \alpha(\theta)[{}^*R_{abcd}R^{bacd}] + L(\theta)$$

  - Schwarzschild is a solution (trivially passes all solar system tests), but Kerr is not

Modified Geometry (a)

- Often argued that higher derivative terms are an artifact of some form of “truncation” of a well-posed theory
  - perhaps so, but that does not help us if we only have the truncated theory
- Argued that in some cases the truncated theory still has a healthy “sector” of solutions. How to find them, especially numerically? Two suggested approaches
  - treat the coupling parameter as small, and perform a perturbative expansion about the zero-coupling-parameter solution of interest, e.g. *Okounkova et al. PRD 96 (2017)*
  - use methods similar to the Israel-Stewart fix of relativistic hydrodynamics, *Cayuso et al. PRD 96 (2017)*

Modified Geometry (b) : 2nd order PDEs

- One example here which is "close" to being viable and non-trivial is EDGB gravity (which can be mapped to one of the general Horndeski class of theories)

$$L \propto R + \alpha(\theta)[R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}] + L(\theta)$$

  - of particularly interesting for compact objects is that it has the opposite affinity for horizons than spontaneous scalarization : NSs have negligible hair, while BHs acquire hair
  - *Papallo, PRD 96 (2017), Papallo and Reall PRD 96 (2017)* have recently shown that this theory in harmonic gauge is only weakly hyperbolic
    - not as bad as "not even weakly hyperbolic", but will still not allow for a well-posed initial value problem for generic scenarios (case-in-point the ADM formulation of the Einstein equations)

Modified Geometry (b)

- Questions and opportunities for EDGB/Horndeski (and any beyond-Horndeski/DHOST etc. where some mechanism prevents the higher derivatives from being fatal) :
  - can gauges, smart choice of variables, etc. be found where the equations can be put in strongly hyperbolic form?
    - may depend on the particular "background" solution
    - even if, methods used to control the constraints in pure GR may not carry over easily
  - assuming "yes" to the above, how do we numerically deal with the kinds of non-linearities that arise in these theories? :

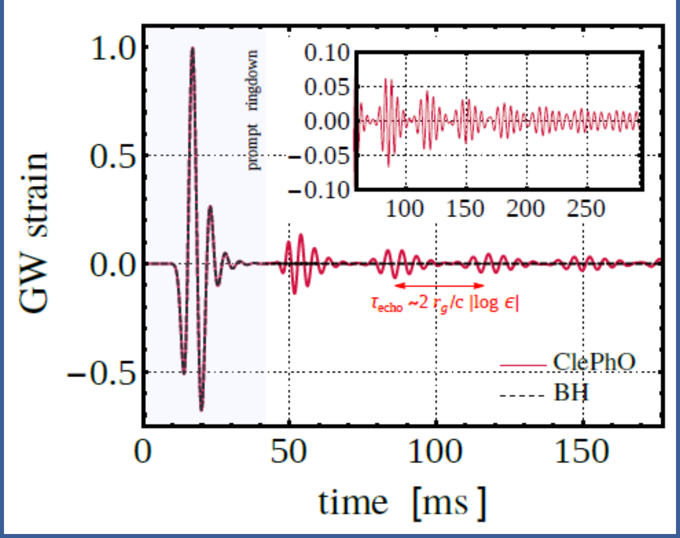
$$\Box\theta = -\partial V/\partial\theta + \lambda[(\Box\theta)^2 + \nabla\theta\cdot\Box\theta + \nabla_a\nabla_b\theta R^{ab} + \cdots]$$

Modified Geometry (b)

- Assuming difficulties on previous slide can be overcome, we still know there are nevertheless regions of solution space that will exhibit pathological behavior
  - are these “benign” like shocks in hydrodynamics?
    - if so, what are the analogues of the Rankine-Hugoniot conditions that allow us to deal with fluid shocks?
    - or are there aspects of some singularities, like pinch-off in an unstable fluid flow, where knowledge of the “microscopic” theory doesn’t really matter?
  - can some singularities, as with 4D GR, be swept behind an event horizon?

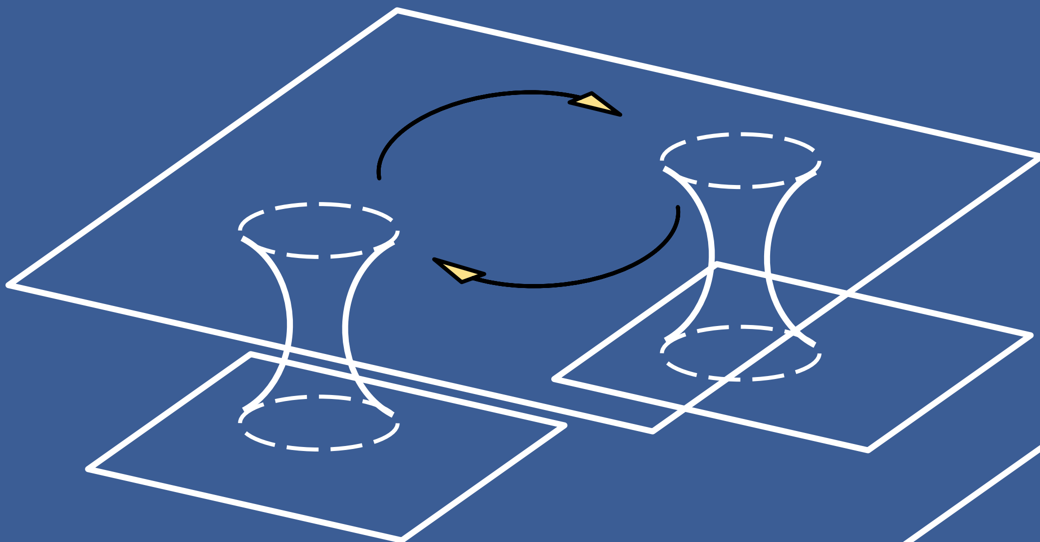
Modified matter

- "Exotic" matter giving new classes of compact object
  - either within GR, or in modified gravity, where the distinctions between which "sector" the modification is happening can be blurred
- Currently the only non-trivial, viable examples are boson stars and black holes with ultra-light particle hair
  - mergers of bosons already being explored, e.g. *Palenzuela et al. PRD 96 (2017); Helfer et al. 1802.06733;* and initial perturbative studies of hairy black holes *Baumann et al. 1804.03208*
- More interesting exotica (traversable wormholes, gravastars, fuzzballs) need much work to go from "intriguing idea" to solvable theory
  - though can argue this MUST be done, as how can we come up with the correct agnostic tests that will point to these objects without knowing what the merger regime looks like?
    - Case-in-point example, "echoes"

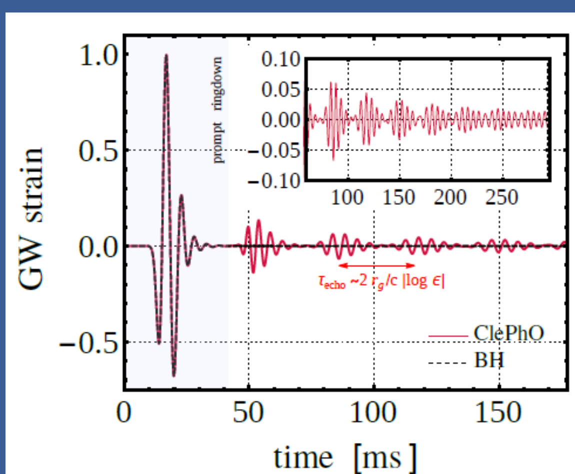


From Cardoso & Pani (2017)

Two-way traversable wormhole mergers



- What kind of “echo” will this produce?



Conclusions

- Need a mathematically well-posed theory to be able to bring numerical tools to bare to help understand the theory
- Many beyond-GR theories, mostly motivated by problems in cosmology
  - can borrow them to test gravity in compact object mergers, but only if well-posed (just as relevant a question in cosmology)
  - opportunity is to discover the subset of well-posed theories, and any new phenomenology arising from "extreme" non-linearity
- Many interesting ideas for alternatives to black holes, or new classes of compact objects in addition to black holes
  - if differences compared to black hole mergers subtle, would be crucial to have model, solvable theories implementing these ideas to discover them (or rule them out) as early as possible