String-local Quantum Fields: An Overview

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Sinopsis

1. Why string-local fields?
2. Examples of string-local fields
3. The string-independence principle
4. Chirality of electroweak interactions
5. Afterword

(Based on joint work with José M. Gracia-Bondía and Jens Mund)

Origins: particles with localized fields

We deal here with quantum fields, built directly from positive-energy representations of the Poincaré group in the setting of Wigner's particle classification.

For massless particles ($p^2 = 0$) of helicity ± 1, the photon potential $A_\mu(x)$ "does not want to live on Hilbert space". The usual solution asks for an indefinite metric and gauge invariance.

Also, the "last" particle species: $p^2 = 0$, $w^2 < 0$, for "continuous spin" repns, does not allow for point-localized quantum fields[1] though "modular localization" in spacelike cones is possible.[2]

These objections were eventually overcome[3] by a string-local description of quantum fields which (a) "live on Hilbert space", and (b) apply to all particle types.

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[1] J. Yngvason: CMP 18 (1970), 195-203.

[2] R. Brunetti, D. Guido, R. Longo: RMaP 14 (2002), 759-785.

[3] J. Mund, B. Schroer, J. Yngvason, CMP 268 (2006), 621-672.

What are string-local fields?

We work in Minkowski space M^4. A (half-)"string" is actually a ray

$$S_{x,e} := \{ x + te : t \geq 0 \}$$

where the direction e is usually spacelike, $e^2 = -1$; but lightlike strings $S_{x,l}$, with $l^2 = 0$, are also useful.

A string-local (SL) field is an operator-valued distribution $\varphi_k(x,e)$ on the Hilbert space of a positive-energy irrep U of $\mathcal{P}_+^\uparrow$, satisfying

- covariance:

$$U(a,\Lambda)\varphi_k(x,e)U^\dagger(a,\Lambda) = \varphi_l(\Lambda x + a, \Lambda e)\, D(\Lambda)^l_k$$

  for a suitable matrix representation D of $\mathcal{L}_+^\uparrow$; and

- string-locality:

$$[\varphi_r(x,e), \varphi_r(x',e')] = 0$$

  when the rays $S_{x,e}$ and $S_{x',e'}$ are spacelike separated.

Tools: intertwiners and correlators

For now, take massive particles; $U_1 = U_1^{(m,s)}$ on 1-particle space:

$$[U_1(A,\Lambda)f](p) := e^{i(ap)} D^s(R(\Lambda,p))\, f(\Lambda^{-1}p)$$

with $(ap) \equiv a^\mu p_\mu$, where $R(\Lambda,p)$ is a "Wigner rotation".

Next, find a set of intertwiners $u_k(p,e)$ satisfying

$$D^s(R(\Lambda,p)) u_k(\Lambda^{-1}p, \Lambda^{-1}e) = u_l(p,e) D(\Lambda)^l_k\,.$$

Build a free field on the corresponding Fock space:

$$\varphi_k(x,e) := \int d\mu(p) \Big[e^{i(px)} u_k(p,e) a^\dagger(p) + e^{-i(px)} u_k(p,e)^* a(p) \Big].$$

The (Wightman) 2-point function $\langle 0 \,|\, \varphi_k(x,e)\, \psi_l(x',e') \,|\, 0 \rangle$ depends only on the correlator:

$$M^{\varphi\psi}_{kl}(p;e,e') := u^\varphi_k(p,e)^*\, u^\psi_l(p,e').$$

Example 1: vector bosons

We begin with the field strength $F_{\mu\nu}(x) \equiv F_{[\mu\nu]}(x)$ for a spin-1 particle, which is "point-local". Its intertwiners are given by

$$v_{k,\mu\nu}(p) := i\, p_\mu \varepsilon_{k,\nu}(p) - i\, p_\nu \varepsilon_{k,\mu}(p)$$

where ε is a polarization dreibein ($m > 0$) or zweibein ($m = 0$), satisfying $p^\mu \varepsilon_{k,\mu}(p) = 0$.

An integration along the ray now gives a string-local potential

$$A_\mu(x,e) := \int_0^\infty dt\, F_{\mu\nu}(x + te)\, e^\nu \equiv I_e F_{\mu\nu}(x)\, e^\nu$$

living on the same Hilbert space as $F_{\mu\nu}$. One finds $\partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}$. For covariance, we get the formula

$$U(a,\Lambda) A_\mu(x,e) U^\dagger(a,\Lambda) = \Lambda^\nu{}_\mu A_\nu(\Lambda x + a, \Lambda e)$$

so A_μ is a vector potential (no gauging needed when $m = 0$).

Two's company

The usual point-local Proca field $A^{\mathrm{p}}_\mu(x)$, for which $\partial_\mu A^{\mathrm{p}}_\nu - \partial_\nu A^{\mathrm{p}}_\mu = F_{\mu\nu}$ also, has bad UV behaviour:

$$M^{A^{\mathrm{p}}A^{\mathrm{p}}}_{\mu\nu}(p) = -g_{\mu\nu} + p_\mu p_\nu/m^2,$$

but that of A_μ is much better; with $(pe)_\pm := (pe) \pm i0$, its intertwiner is

$$u^A_{k,\mu}(p,e) = \varepsilon_{k,\mu}(p) - p_\mu(\varepsilon_k(p)\,e)/(pe)_+ \quad \text{and thus}$$

$$M^{AA}_{\mu\nu}(p;e,e') = -g_{\mu\nu} + \frac{p_\mu e_\nu}{(pe)_-} + \frac{p_\nu e'_\mu}{(pe')_+} - \frac{p_\mu p_\nu (ee')}{(pe)_-(pe')_+}$$

which is of order 0 as $p^2 \to \infty$.

When $m > 0$, $dA = dA^{\mathrm{p}} = F$ gives a scalar field $a(x,e)$ such that

$$A_\mu(x,e) =: A^{\mathrm{p}}_\mu(x) + \frac{1}{m}\partial_\mu a(x,e); \quad \text{and} \quad a(x,e) = -\frac{1}{m}\partial^\nu A_\nu(x,e).$$

This a we call the escort field for the vector potential A_μ. (It is somewhat analogous to the Stückelberg field of the usual formalism.)

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Example 2: Massless limits for spin 2

For spin two "massive gravitons", whose field strength is the linearized Riemann tensor $R_{\mu\kappa,\nu\lambda}(x) \equiv R_{[\mu\kappa],[\nu\lambda]}(x)$, we define:

$$A_{\mu\nu}(x,e) := l_e^2 R_{\mu\kappa,\nu\lambda}(x)\, e^\kappa e^\lambda .$$

As $m \to 0$, there is a van Dam-Veltman-Zakharov discontinuity:

$$\lim_{m\to 0} {}_m M^{A^p A^p}_{\mu\nu,\kappa\lambda} = \tfrac{1}{2}(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \tfrac{2}{3}g_{\mu\nu}g_{\kappa\lambda}), \quad \text{but}$$

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There are now two escort fields,[4] regular as $m \to 0$:

$$a^{(1)}_\mu(x,e) := -(1/m)\partial^\nu A_{\mu\nu}(x,e), \quad a^{(0)}(x,e) := -(1/m)\partial^\mu a^{(1)}_\mu(x,e).$$

The corrected potential

$$A^{(2)}_{\mu\nu}(x,e) := A_{\mu\nu}(x,e) + \tfrac{1}{2} M^{AA}_{\mu\nu}(p;-e,e)\, a^{(0)}(x,e)$$

decouples from $a^{(0)}$ and gives the helicity-(± 2) field as $m \to 0$.
The escorts carry away the rest: 5 graviton spin states fall to 2.

[4] J. Mund, K-H. Rehren, B. Schroer: PLB 773 (2017), 625 + NPB 924 (2017), 699.

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Interactions: perturbation theory setup

We work with the Epstein-Glaser approach to perturbation theory, where the scattering operator depends on a coupling function $g(x)$ and a string variable l (here taken to be lightlike, for simplicity):

$$\mathbb{S}[g;l] = 1 + \sum_{k=1}^{\infty} \frac{i^k}{k!} \int S_k(x_1,\dots,x_k,l)g(x_1)\cdots g(x_k)\,d^4x_1\cdots d^4x_k.$$

The interaction is displayed in the first-order vertex coupling $S_1(x,l)$.

In electroweak theory, vector bosons $A_a^\mu(x,l)$ are linked with matter fields $\psi(x)$ – ordinary fermions, not assumed to be chiral – through

$$S_1^F(x,l) = g(b^a A_{a\mu} J_V^\mu + \tilde{b}^a A_{a\mu} J_A^\mu); \quad J_V^\mu = \overline{\psi}\gamma^\mu\psi, \quad J_A^\mu = \overline{\psi}\gamma^\mu\gamma^5\psi,$$

with coefficients b^a and $\tilde{b}^a$, to be determined.

To $S_1^F(x,l)$ one must add interacting bosonic terms $S_1^B(x,l)$, too.

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The principle of string independence

For interacting SL fields, physical observables cannot depend on the string coordinates. This string-independence principle requires the existence of a (1-form in l) vector field $Q_\mu(x, l)$ such that

$$d_l S_1(x,l) \equiv (\partial S_1/\partial l^\sigma)\, dl^\sigma = \partial^\mu Q_\mu(x,l).$$

Next, the higher S_k are constructed as time-ordered products:

$$S_2(x,x',l) = S_1(x,l)S_1(x',l) \text{ or } S_1(x',l)S_1(x,l),$$

according as $\{x + tl\}$ is later or earlier than $\{x' + tl\}$; this fixes S_2 outside a nullset in $\mathbb{M}_4^2 \times \mathbb{S}^2$ where the two strings cannot be ordered. On that nullset, S_2 must be defined as an extension of distributions.

String independence now demands that (with T for time-ordering):

$$d_l \mathrm{T}[S_1(x,l)S_1(x',l)] = \partial^\mu \mathrm{T}[Q_\mu(x,l)S_1(x',l)] + \partial'^\mu \mathrm{T}[S_1(x,l)Q_\mu(x',l)]$$

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Application: electroweak sector of the SM

For interacting bosons, the general pattern is

$$S_1^B(x,l) = g \sum_{a,b,c} f_{abc} F_{a,\mu\nu}(x) A_b^\mu(x,l) A_c^\nu(x,l)$$

$$+ g \sum_{a,b,c}{}' f_{abc} M_{abc} \Big(A_{a,\mu}(x,l) A_b^\mu(x,l) \phi_c(x,l) - A_{a,\mu}(x,l)\, \partial^\mu \phi_b(x,l) \phi_c(x,l) \Big)$$

where $\sum'$ runs over massive fields only, $M_{abc} = m_a^2 - m_b^2 - m_c^2$, and the structure constants f_{abc} are completely skewsymmetric.

We now specialize these generic $A_{a,\mu}$ to the MVB $W_{\pm,\mu}(x,l)$ and $Z_\mu(x,l)$ with the known masses m_W, m_Z; and a massless $A_\mu(x,l)$. Massless particles do not have escort fields, but we add (only) one pointlike higgs scalar $\phi_4(x)$, needed for renormalizability.

Since $m_W \leq m_Z$, we can define the Weinberg angle Θ by $\cos\Theta := \dfrac{m_W}{m_Z}$.

The constants are $f_{123} = \frac{1}{2}\cos\Theta,\ f_{124} = \frac{1}{2}\sin\Theta,\ f_{134} = f_{234} = 0$.

SM-like couplings, at first order

We can find suitable $Q_\mu(x,l)$ so that $d_l S_1 = \partial^\mu Q_\mu$. This requirement constrains many of the coefficients. A typical summand of Q^B_μ is:

$$ig\cos\Theta(\partial_\mu Z_\lambda - \partial_\lambda Z_\mu)(W_+^\lambda\, d_l\phi_- - W_-^\lambda\, d_l\phi_+).$$

and here is the most general $S_1^F(x,l)$ at this stage:

$$\begin{aligned} g\Big(& b_1 W_{-\mu}\bar{e}\gamma^\mu\nu + \tilde{b}_1 W_{-\mu}\bar{e}\gamma^\mu\gamma^5\nu + b_1 W_{+\mu}\bar{\nu}\gamma^\mu e + \tilde{b}_1 W_{+\mu}\bar{\nu}\gamma^\mu\gamma^5 e \\ & + b_3 Z_\mu\bar{e}\gamma^\mu e + \tilde{b}_3 Z_\mu\bar{e}\gamma^\mu\gamma^5 e + b_4 Z_\mu\bar{\nu}\gamma^\mu\nu + \tilde{b}_4 Z_\mu\bar{\nu}\gamma^\mu\gamma^5\nu + b_5 A_\mu\bar{e}\gamma^\mu e \\ & + i(m_e - m_\nu)b_1\phi_-\bar{e}\nu + i(m_e + m_\nu)\tilde{b}_1\phi_-\bar{e}\gamma^5\nu - i(m_e - m_\nu)b_1\phi_+\bar{\nu}e \\ & + i(m_e + m_\nu)\tilde{b}_1\phi_+\bar{\nu}\gamma^5 e + 2im_e\tilde{b}_3\phi_Z\bar{e}\gamma^5 e + 2im_\nu\tilde{b}_4\phi_Z\bar{\nu}\gamma^5\nu \\ & + c_0\phi_4\bar{e}e + \tilde{c}_0\phi_4\bar{e}\gamma^5 e + c_5\phi_4\bar{\nu}\nu + \tilde{c}_5\phi_4\bar{\nu}\gamma^5\nu\Big). \end{aligned}$$

Notice the combination $b_1 + \tilde{b}_1\gamma^5$: if we could show that $\tilde{b}_1 = \pm b_1$, chirality (left or right) would follow.

Our claim is that string independence yields precisely that.

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Second-order conditions

Two-point functions are expected values of time-ordered products:

$$\langle\!\langle \mathsf{T}_0\, \varphi \chi' \rangle\!\rangle \equiv \langle 0 \,|\, \mathsf{T}_0[\varphi(x,l)\, \chi(x',l)] \,|\, 0 \rangle$$
$$= \frac{i}{(2\pi)^4} \int d^4p\, \frac{e^{-i(p(x-x'))}}{p^2 - m^2 + i0} \sum_r u_r^{\varphi}(p,l)^* u_r^{\chi}(p,l)$$

depending on the "intertwiners" $u_r(p,l)$ that specify φ and χ.
String independence at second order in g demands that the relation

$$d_l\, \mathsf{T}_0[S_1\, S_1'] = \partial^\mu\, \mathsf{T}_0[Q_\mu\, S_1'] + \partial'^\mu\, \mathsf{T}_0[S_1\, Q_\mu'],$$

which holds off the singular set of $(x - x', l)$, be valid everywhere (by adjusting T_0 to T). This means taming obstructions of the form $\langle\!\langle \mathsf{T}_0\, \partial_\mu \varphi \chi' \rangle\!\rangle - \partial_\mu \langle\!\langle \mathsf{T}_0\, \varphi \chi' \rangle\!\rangle$ in an overall crossing of Q_μ-terms with S_1-terms, which must vanish:

$$\sum_{\varphi,\chi'} \frac{\partial Q^\mu}{\partial \varphi} \Big(\langle\!\langle \mathsf{T}\, \partial_\mu \varphi \chi' \rangle\!\rangle - \partial_\mu \langle\!\langle \mathsf{T}\, \varphi \chi' \rangle\!\rangle \Big) \frac{\partial S_1'}{\partial \chi'} = 0.$$

Dealing with the obstructions

Fermions of the same kind give pointlike obstructions:

$$\langle\!\langle \mathsf{T}_0\, \gamma^\mu \partial_\mu \psi\, \bar{\psi}' \rangle\!\rangle - \gamma^\mu \partial_\mu \langle\!\langle \mathsf{T}_0\, \psi\, \bar{\psi}' \rangle\!\rangle = -\delta(x - x').$$

Bosonic obstructions, being string-like, can be trickier:

$$\langle\!\langle \mathsf{T}_0\, \partial^\mu A_\mu A'_\kappa \rangle\!\rangle - \partial^\mu \langle\!\langle \mathsf{T}_0\, A_\mu A'_\kappa \rangle\!\rangle = i l_\kappa\, \delta_l(x - x'),$$

where δ_l is a distribution supported on the singular set:

$$\delta_l(x) := \int_0^\infty \delta(x - sl)\, ds.$$

To make the overall crossing vanish, some derived fields require renormalizing T_0 to T; for instance:

$$\langle\!\langle \mathsf{T}\, \partial_\lambda A_\mu A'_\kappa \rangle\!\rangle := \langle\!\langle \mathsf{T}_0\, \partial_\lambda A_\mu A'_\kappa \rangle\!\rangle + c_{\lambda\mu\kappa}\, \delta_l$$

with yet-to-be-determined coefficients $c_{\lambda\mu\kappa}$.

Crossing the bar

There are many possible crossings, each resulting in some fields times $\delta(x-x')$ or $\delta_l(x-x')$. What happens is that they occur in pairs, one of type (Q^F, S_1^F) and one of type (Q^B, S_1^F). One such pair gives

$$(8im_e\tilde{b}_3^2 - ic_0 m_Z/\cos\Theta)\,\phi_Z(x,l)\,d_l\phi_Z(x,l)\bar{e}(x)e(x)\,\delta(x-x').$$

So string independence forces the relation

$$c_0 = 8\tilde{b}_3^2\,m_e\cos^2\Theta/m_W.$$

A different pair of crossings leads to $c_0 = m_e/2m_W$, and therefore

$$\tilde{b}_3 = \pm\frac{1}{4\cos\Theta} =: \varepsilon_1\frac{1}{4\cos\Theta}\,.$$

Two more such pairs of crossings, both involving c_5, yield

$$\tilde{b}_4 = \pm\frac{1}{4\cos\Theta} =: \varepsilon_2\frac{1}{4\cos\Theta}\,.$$

Here ε_1 and ε_2 are (so far) undetermined signs.

Matching the signs

We meet some "dangerous" crossings, that give expressions ending in $c_{[\lambda\mu]\kappa}\,\delta_l(x-x')$. String independence decrees that the coefficients be constrained by $c_{[\lambda\mu]\kappa}=0$.

Comparing terms of the tamer $\delta(x-x')$ type, we eventually reach

$$\tilde{b}_3\cos\Theta = 2b_1\tilde{b}_1 = -\tilde{b}_4\cos\Theta,$$

which implies $\varepsilon_2=-\varepsilon_1$. Using that, we find

$$i(m_e-m_\nu)b_1 = 2\tilde{b}_1(2im_e\tilde{b}_3+2im_\nu\tilde{b}_4)\cos\Theta = i(m_e-m_\nu)\varepsilon_1\tilde{b}_1$$

and so $\tilde{b}_1=\varepsilon_1 b_1$: chirality is not an input[5] to the model!
We are free to take $\varepsilon_1=-1$. ("Nature's choice".)

In the mopping up, we also come to the coefficient of $A_\mu\bar{e}\gamma^\mu e$, namely $gb_5 = g\sin\Theta$: the electric charge.

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[5] J. M. Gracia-Bondía, J. Mund, JCV: AHP 19 (2018), 843-874.

EW chirality from string independence

The final form of the interaction term S_1^F is:

$$g\Big\{ -\frac{1}{2\sqrt{2}}\, W_{-\mu}\bar{e}\gamma^\mu(1-\gamma^5)\nu - \frac{1}{2\sqrt{2}}\, W_{+\mu}\bar{\nu}\gamma^\mu(1-\gamma^5)e$$
$$+ \frac{1-4\sin^2\Theta}{4\cos\Theta}\, Z_\mu\bar{e}\gamma^\mu e - \frac{1}{4\cos\Theta}\, Z_\mu\bar{e}\gamma^\mu\gamma^5 e$$
$$- \frac{1}{4\cos\Theta}\, Z_\mu\bar{\nu}\gamma^\mu(1-\gamma^5)\nu + \sin\Theta\, A_\mu\bar{e}\gamma^\mu e$$
$$+ i\,\frac{m_e - m_\nu}{2\sqrt{2}}\,(\phi_-\bar{e}\nu - \phi_+\bar{\nu}e) - i\,\frac{m_e + m_\nu}{2\sqrt{2}}\,(\phi_-\bar{e}\gamma^5\nu + \phi_+\bar{\nu}\gamma^5 e)$$
$$- i\,\frac{m_e}{2\cos\Theta}\,\phi_Z\bar{e}\gamma^5 e + i\,\frac{m_\nu}{2\cos\Theta}\,\phi_Z\bar{\nu}\gamma^5\nu + \frac{m_e}{2m_W}\,\phi_4\bar{e}e + \frac{m_\nu}{2m_W}\phi_4\bar{\nu}\nu \Big\}.$$

In fact, one can write $S_1^F = S_1^{F,\mathrm{p}} + \partial^\mu V_\mu$, where the divergence term sweeps away the escort fields, $W_\pm \mapsto W_\pm^{\mathrm{p}}$ and $Z \mapsto Z^{\mathrm{p}}$ in $S_1^{F,\mathrm{p}}$; this is almost the standard formulation. However, A_μ remains string-like.

EW chirality from string independence

The final form of the interaction term S_1^F is:

$$
\begin{aligned}
g\Big\{ &-\frac{1}{2\sqrt{2}}\, W_{-\mu}\bar{e}\gamma^\mu(1-\gamma^5)\nu - \frac{1}{2\sqrt{2}}\, W_{+\mu}\bar{\nu}\gamma^\mu(1-\gamma^5)e \\
&+ \frac{1-4\sin^2\Theta}{4\cos\Theta}\, Z_\mu\bar{e}\gamma^\mu e - \frac{1}{4\cos\Theta}\, Z_\mu\bar{e}\gamma^\mu\gamma^5 e \\
&- \frac{1}{4\cos\Theta}\, Z_\mu\bar{\nu}\gamma^\mu(1-\gamma^5)\nu + \sin\Theta\, A_\mu\bar{e}\gamma^\mu e \\
&+ i\,\frac{m_e - m_\nu}{2\sqrt{2}}\,(\phi_-\bar{e}\nu - \phi_+\bar{\nu}e) - i\,\frac{m_e + m_\nu}{2\sqrt{2}}\,(\phi_-\bar{e}\gamma^5\nu + \phi_+\bar{\nu}\gamma^5 e) \\
&- i\,\frac{m_e}{2\cos\Theta}\,\phi_Z\bar{e}\gamma^5 e + i\,\frac{m_\nu}{2\cos\Theta}\,\phi_Z\bar{\nu}\gamma^5\nu + \frac{m_e}{2m_W}\,\phi_4\bar{e}e + \frac{m_\nu}{2m_W}\phi_4\bar{\nu}\nu \Big\}.
\end{aligned}
$$

In fact, one can write $S_1^F = S_1^{F,\mathrm{p}} + \partial^\mu V_\mu$, where the divergence term sweeps away the escort fields, $W_\pm \mapsto W_\pm^{\mathrm{p}}$ and $Z \mapsto Z^{\mathrm{p}}$ in $S_1^{F,\mathrm{p}}$; this is almost the standard formulation. However, A_μ remains string-like.

Other pros and cons of SL fields

For Wigner's last particle, a stress-energy tensor has been developed, allowing for coupling to gravity.[6]

The abelian Higgs model [Mund + Schroer, in progress]: the expected "Mexican hat" potential emerges from string independence. (The outcome coincides with the usual model in the unitary gauge, using "nonrenormalizable" Proca fields.)

The main remaining challenge is the construction of time-ordered products with SL fields (within an EG-framework), in order to confirm renormalizability.

The locality issue in that construction: two or more strings $S_{x,e}$ often cannot be causally separated; but it is always possible to chop them into segments that can.[7] Thus TO-products of "linear" fields can be defined. For Wick polynomials of such, the jury is still out.

[6] K-H. Rehren: JHEP 11:130 (2017).

[7] L. T. Cardoso, J. Mund, JCV: MPAG 21:3 (2018).

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