Generalized 2nd law under non-Markovian feedback control using a levitated particle

Quantum Engineering of Levitated Systems
Benasque Sep. 16 – 22

Maxime Debiossac, David Grass, Jose Joaquin Alonso, Eric Lutz & Nikolai Kiesel

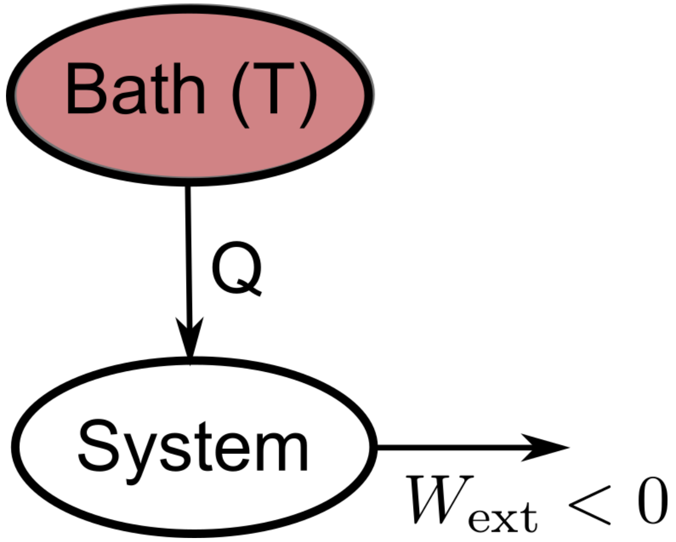
universität wien

Information on a system allows to extract more work

Isothermal process :

- Kelvin-Planck statement : ‘no work can be cyclically extracted from a system coupled to a single reservoir at temperature T’ :

$$\dot{W}_{\text{ext}} < 0$$

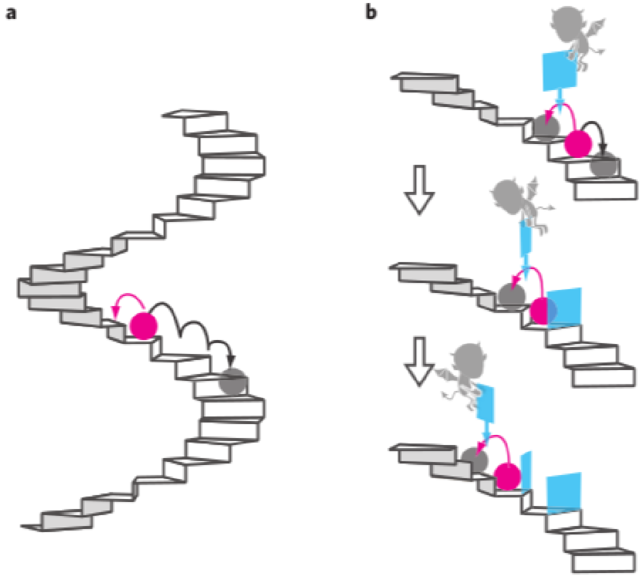


Information on a system allows to extract more work

Isothermal process :

- Kelvin-Planck statement : ‘no work can be cyclically extracted from a system coupled to a single reservoir at temperature T’ :

$$\dot{W}_{\text{ext}} < 0$$



Shoichi Toyabe *et al*, Nat. Phys., 2010

- Role of information : $\dot{W}_{\text{ext}} < k_B T \dot{I}$

Information on a system allows to extract more work

Isothermal process :

- Kelvin-Planck statement : ‘no work can be cyclically extracted from a system coupled to a single reservoir at temperature T’ :

$$\dot{W}_{\mathrm{ext}} < 0$$

- Role of information : $\dot{W}_{\mathrm{ext}} < k_B T \dot{I}$

$$\boxed{\dot{S}_{\mathrm{pump}} \leq \dot{I}}$$

- Pumping entropy : $\dot{W}_{\mathrm{ext}} < k_B T \dot{S}_{\mathrm{pump}}$

Rosinberg *et al*, PRE, 2015

Outline

1. Role of time delay in science
2. Information engine
3. Results
4. Conclusion

Outline

1. Role of time delay in science
2. Information engine
3. Results
4. Conclusion

Schrödinger: What is life?

Schrödinger (19**42**) : How does a living organism avoid decay?

→ It feeds on 'negative entropy'

Schrödinger: What is life?

Schrödinger (19**42**) : How does a living organism avoid decay?

→ It feeds on 'negative entropy'

‘Negative entropy’ or ‘pumping entropy’ (Kim & Qian, PRL, **2**00**4**)

VOLUME 93, NUMBER 12 PHYSICAL REVIEW LETTERS week ending 17 SEPTEMBER 2004

Entropy Production of Brownian Macromolecules with Inertia

Kyung Hyuk Kim*

Department of Physics, University of Washington, Seattle, Washington 98195, USA

Hong Qian

Department of Applied Mathematics, University of Washington, Seattle, Washington 98195, USA

(Received 19 July 2003; published 16 September 2004)

We investigate the nonequilibrium steady-state thermodynamics of single Brownian macromolecules with inertia under feedback control in an isothermal ambient fluid. With the control being represented by a velocity-dependent external force, we find such an open system can have a negative entropy production rate, and we develop a mesoscopic theory consistent with the second law. We propose an equilibrium condition and define a class of external force, which includes the transverse Lorentz force, leading to equilibrium.

Time delays are ubiquitous in science

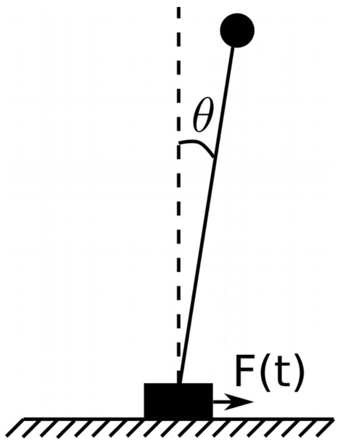
- Feedback is never instantaneous due to finite delay τ.

Time delays are ubiquitous in science

- Feedback is never instantaneous due to finite delay τ.
- Continuous time-delayed differential equation (biology, chemistry, physics,engineering)

Time delays are ubiquitous in science

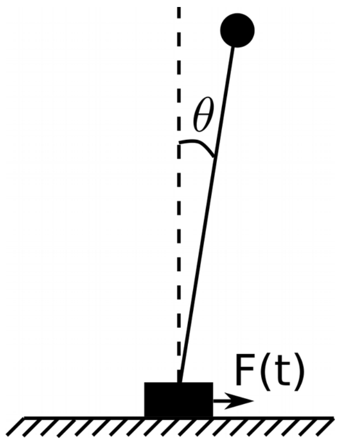
- Feedback is never instantaneous due to finite delay τ.
- Continuous time-delayed differential equation (biology, chemistry, physics,engineering)



Human balance
J. Milton *et al*, Chaos, 2009

Time delays are ubiquitous in science

- Feedback is never instantaneous due to finite delay τ.
- Continuous time-delayed differential equation (biology, chemistry, physics,engineering)

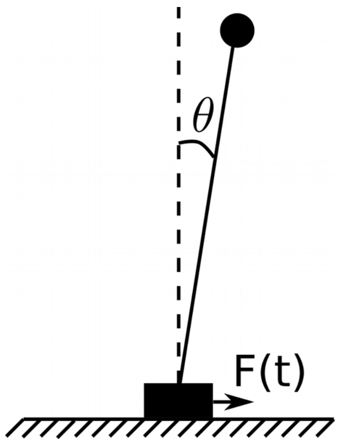


Human balance
J. Milton *et al*, Chaos, 2009

Stability of neural networks
CM Marcus & RM Westervelt, PRA, 1989

Time delays are ubiquitous in science

- Feedback is never instantaneous due to finite delay τ.
- Continuous time-delayed differential equation (biology, chemistry, physics,engineering)

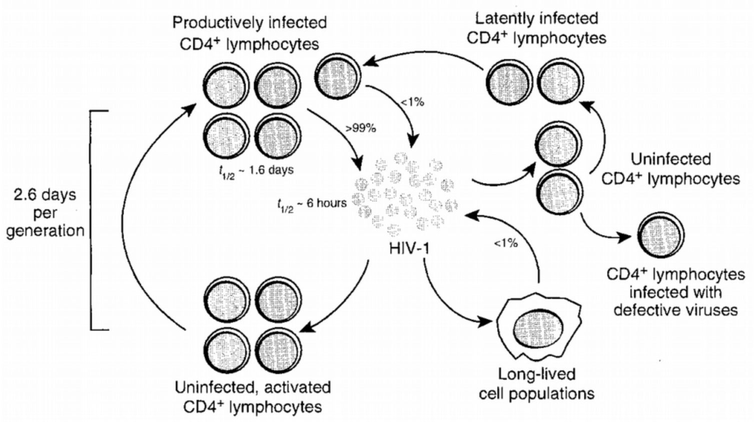


Human balance

J. Milton *et al*, Chaos, 2009

Stability of neural networks

CM Marcus & RM Westervelt, PRA, 1989



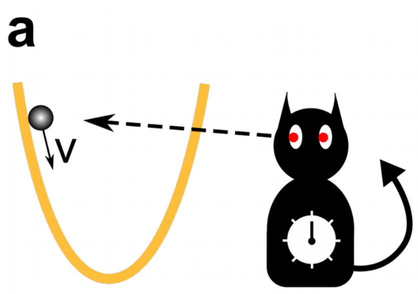
HIV infection

PW Nelson & AS Perelson, Math. Biosci., 2002

Outline

1. Role of time delay in science
2. Information engine
3. Results
4. Conclusion

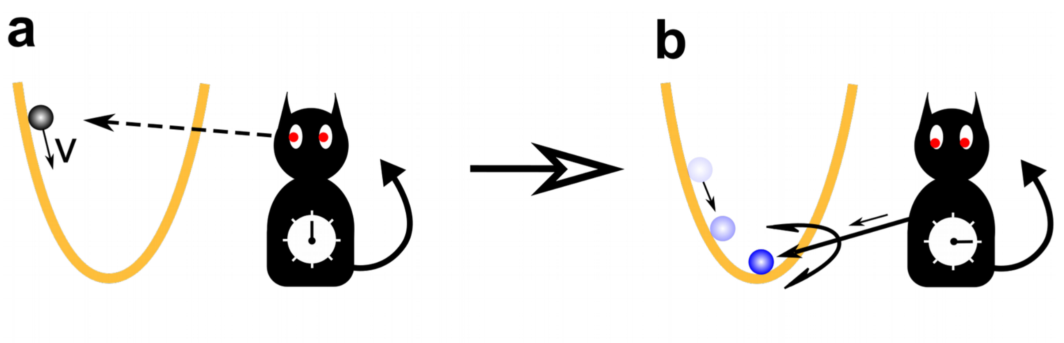
A maxwell's daemon that manipulates time



Protocol :

1) Measure position

A maxwell's daemon that manipulates time

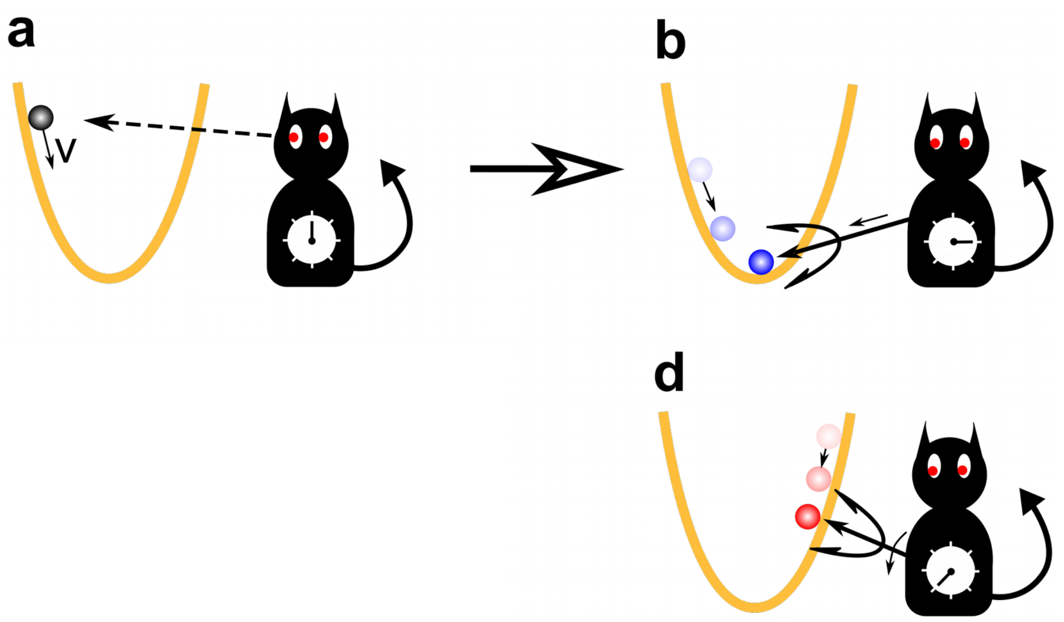


Protocol :

1) Measure position

2) Apply a feedback force $F_{\mathrm{fb}} \propto x_{t-\tau}$

A maxwell’s daemon that manipulates time

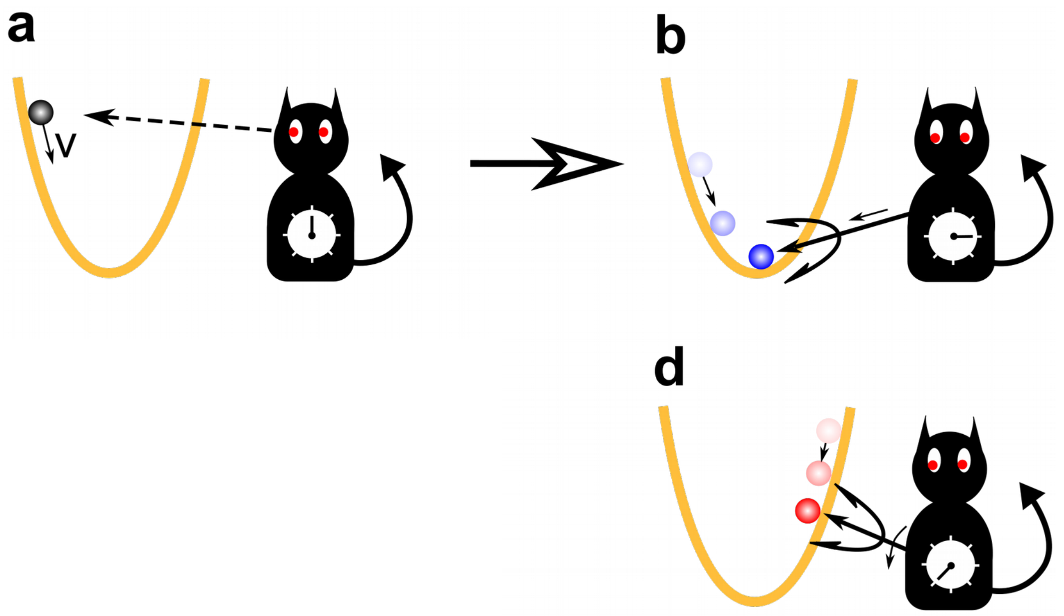


Protocol :

1) Measure position

2) Apply a feedback force $F_{\mathrm{fb}} \propto x_{t-\tau}$

A maxwell's daemon that manipulates time



Protocol :

1) Measure position

2) Apply a feedback force $F_{\rm fb} \propto x_{t-\tau}$

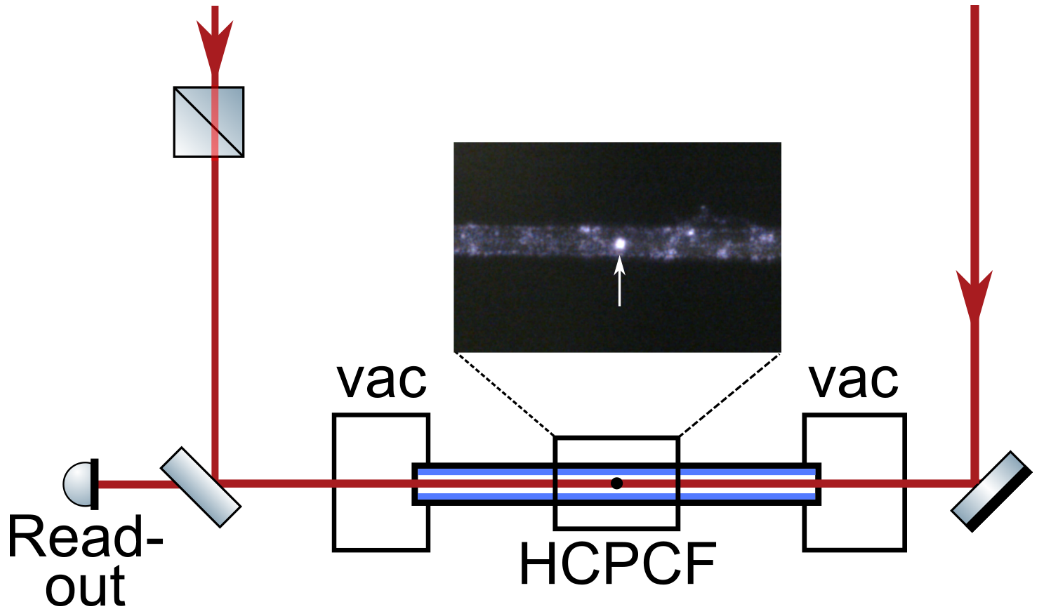
Weakly damped system : $x \propto \cos(\Omega_0 t)$

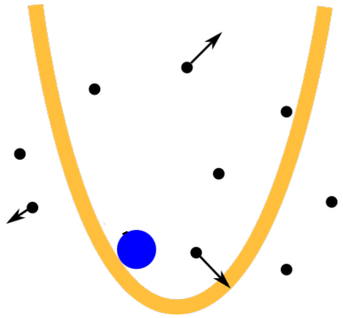
$$F_{\rm fb} \propto \cos(\Omega_0 (t-\tau))$$

$$= x \cos\tau - \boxed{\frac{1}{\Omega_0} v \sin\tau} \text{ friction}$$

Time delay → Memory effect → Non-Markovianity

Realization of the protocol with a levitated particle



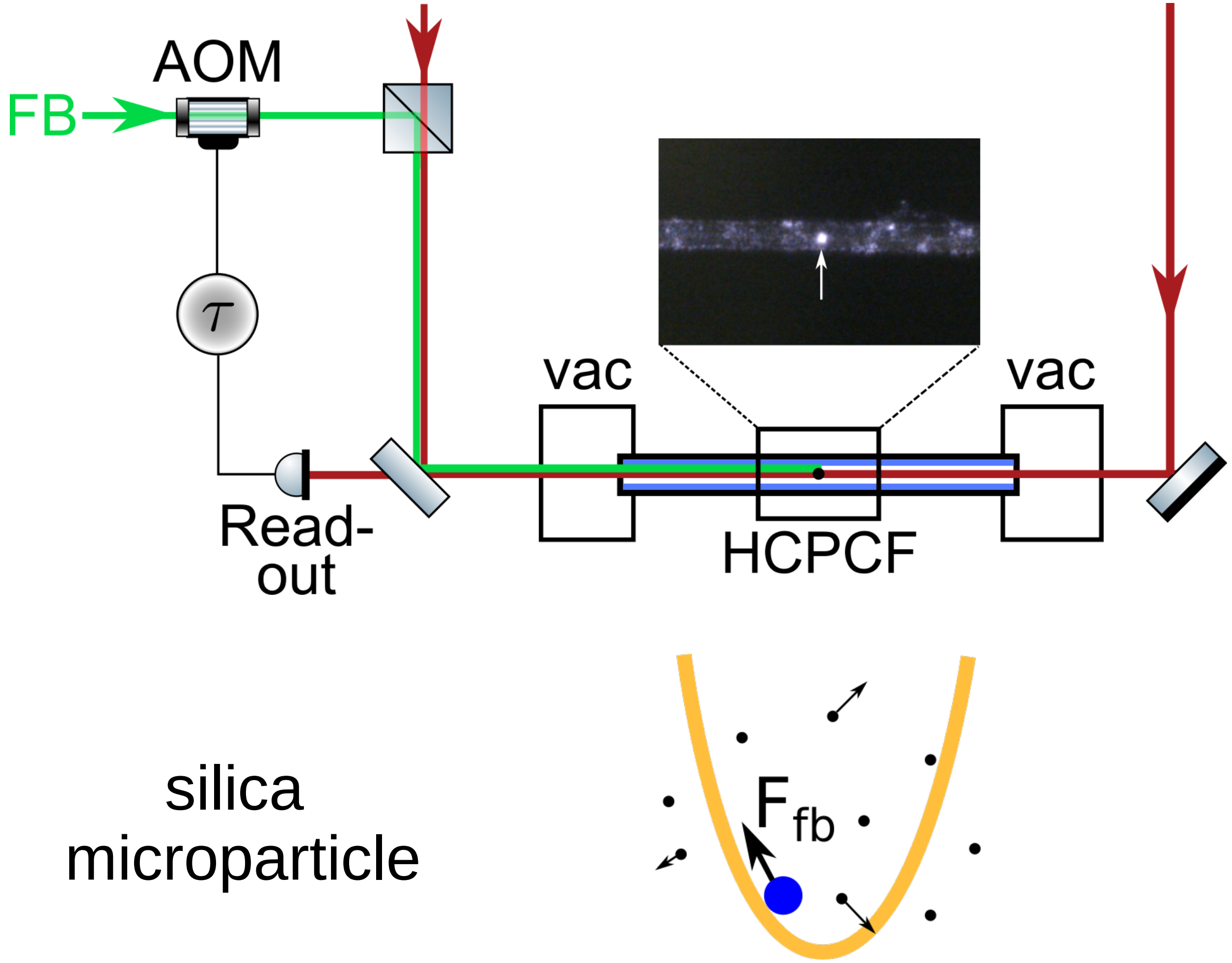


silica microparticle

Harmonic trap :

$$\left.\begin{array}{l}\nu_0 = 400\ \mathrm{kHz} \\ \Gamma_p \approx 1\ \mathrm{kHz}\end{array}\right\} Q_0 = \frac{\nu_0}{\Gamma_p} \approx 100$$

Realization of the protocol with a levitated particle



Harmonic trap :

$$\left.\begin{array}{l}\nu_0 = 400 \text{ kHz} \\ \Gamma_p \approx 1 \text{ kHz}\end{array}\right\} Q_0 = \frac{\nu_0}{\Gamma_p} \approx 100$$

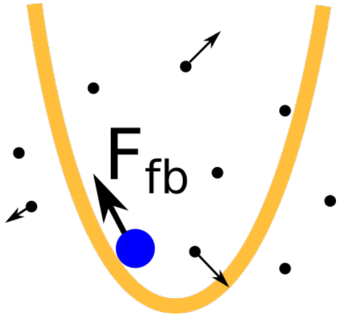
Feedback :

Feedback gain : $g = \dfrac{\Gamma_{\text{fb}}}{\Gamma_p} \approx 0.5$

Feedback delay : $\tau = 2\pi\nu_0 t$

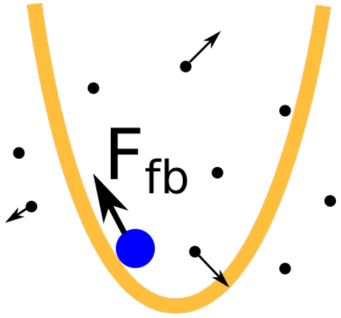
$$(g, Q_0, \tau)$$

From equation of motion to the tightest bound of the 2nd law



$$\ddot{x}_t + \Gamma_p \dot{x}_t + \Omega_0^2 x_t + \boxed{\frac{\Gamma_{\mathrm{fb}}}{\Omega_0} x_{t-\tau}} = \frac{F_{\mathrm{therm}}}{m}$$

From equation of motion to the tightest bound of the 2$^{\text{nd}}$ law

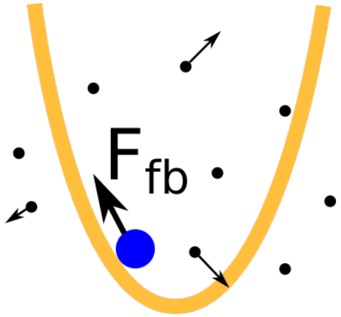


$$\ddot{x}_t + \Gamma_p \dot{x}_t + \Omega_0^2 x_t - \boxed{\frac{\Gamma_{\text{fb}}}{\Omega_0} x_{t-\tau}} = \frac{F_{\text{therm}}}{m}$$

Fokker-Plank equation
(Rosinberg *et al*, PRE, 2015)

$$\frac{dS}{dt} = \dot{S}_i - \dot{S}_{\text{pump}} - \frac{\dot{Q}}{T_0}$$

From equation of motion to the tightest bound of the 2nd law



$$\ddot{x}_t + \Gamma_p \dot{x}_t + \Omega_0^2 x_t + \boxed{\frac{\Gamma_{\mathrm{fb}}}{\Omega_0} x_{t-\tau}} = \frac{F_{\mathrm{therm}}}{m}$$

Fokker-Plank equation
(Rosinberg *et al*, PRE, 2015)

$$\frac{dS}{dt} = \dot{S}_i - \dot{S}_{\mathrm{pump}} - \frac{\dot{Q}}{T_0}$$

Steady state : $\dfrac{dS}{dt} = 0$

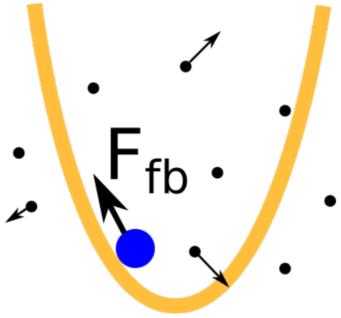
First law : $\dot{Q} = \dot{W}$

Non negativity : $\dot{S}_i \geq 0$

$$\boxed{\frac{\dot{W}_{\mathrm{ext}}}{T_0} \leq \dot{S}_{\mathrm{pump}}}$$

$$\dot{W}_{\mathrm{ext}} = -\dot{W} = \int F_{\mathrm{fb}} dx$$

From equation of motion to the tightest bound of the 2nd law



$$\ddot{x}_t + \Gamma_p \dot{x}_t + \Omega_0^2 x_t - \boxed{\frac{\Gamma_{\mathrm{fb}}}{\Omega_0} x_{t-\tau}} = \frac{F_{\mathrm{therm}}}{m}$$

Fokker-Plank equation
(Rosinberg *et al*, PRE, 2015)

$$\frac{dS}{dt} = \dot{S}_i - \dot{S}_{\mathrm{pump}} - \frac{\dot{Q}}{T_0}$$

Steady state : $\frac{dS}{dt} = 0$

First law : $\dot{Q} = \dot{W}$

Non negativity : $\dot{S}_i \geq 0$

$$\boxed{\frac{\dot{W}_{\mathrm{ext}}}{T_0} \leq \dot{S}_{\mathrm{pump}} \leq \dot{I}}$$

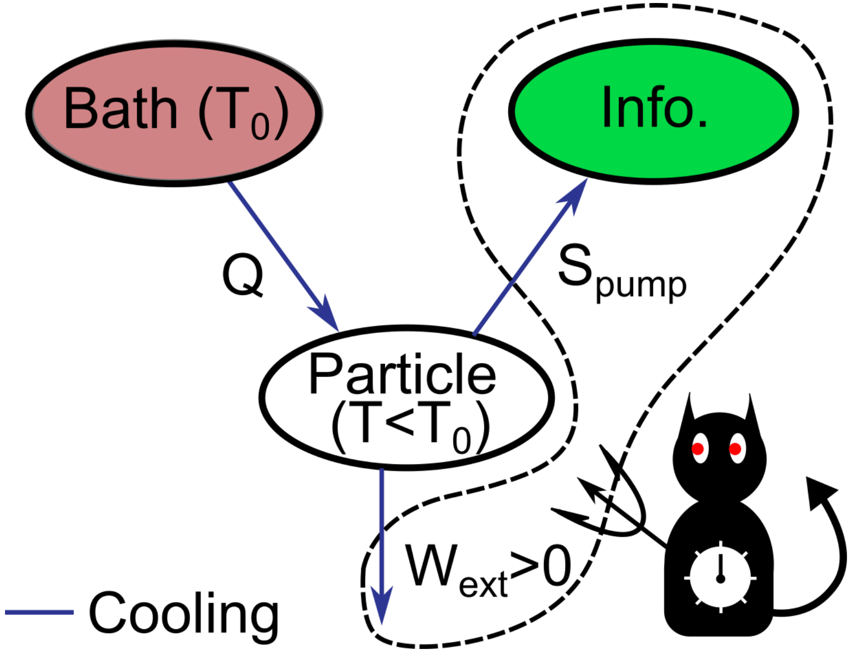
Tightest bound

$$\dot{W}_{\mathrm{ext}} = -\dot{W} = \int F_{\mathrm{fb}} dx$$

$\dot{I}$ = mutual information flow

The Information engine can cool or heat the particle motion

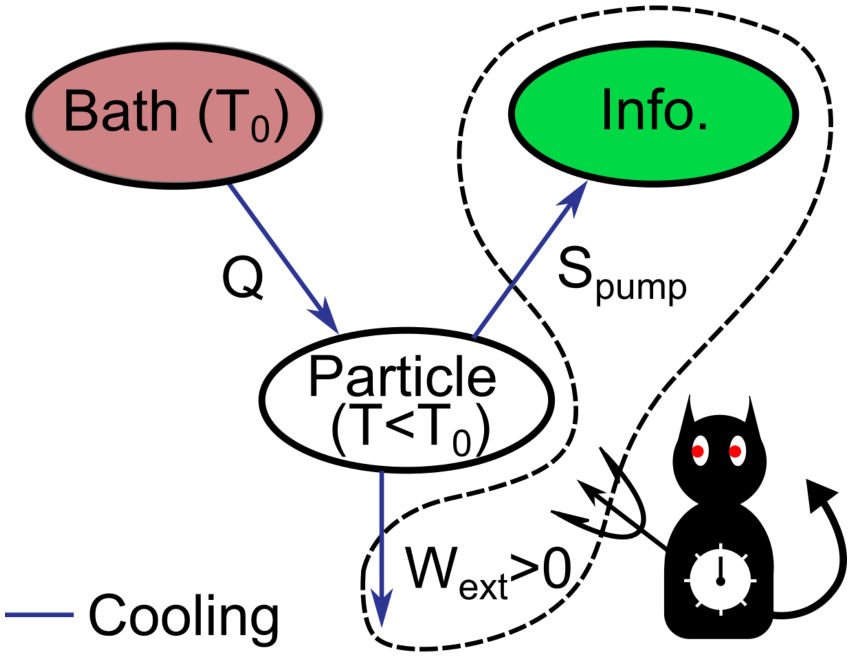
Cooling mode



$$0 < \tau < \pi$$

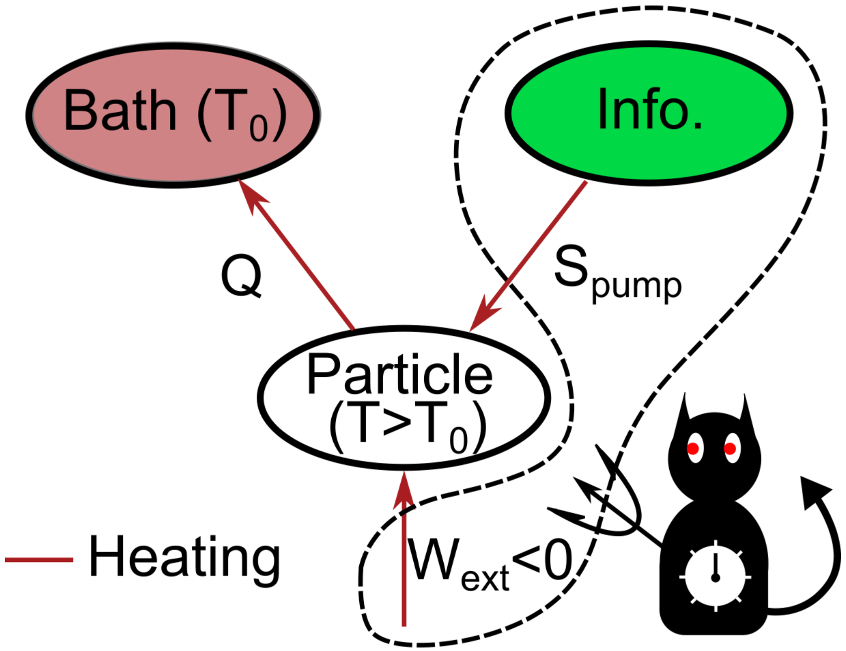
The Information engine can cool or heat the particle motion

Cooling mode



$$0 < \tau < \pi$$

Heating mode



$$\pi < \tau < 2\pi$$

Outline

1. Role of time delay in science
2. Information engine
3. Results
4. Conclusion

Thermodynamics quantities are measured using the velocity variance

$$\frac{\dot{W}_{\text{ext}}}{T_0} = \frac{1}{Q_0}\left(1 - \boxed{\sigma_v^2}\right)$$

$$\dot{S}_{\text{pump}} = \frac{1}{Q_0}\left(\frac{1 - \boxed{\sigma_v^2}}{\boxed{\sigma_v^2}}\right)$$

$$\sigma_v^2 = f(g, Q_0, \tau)$$

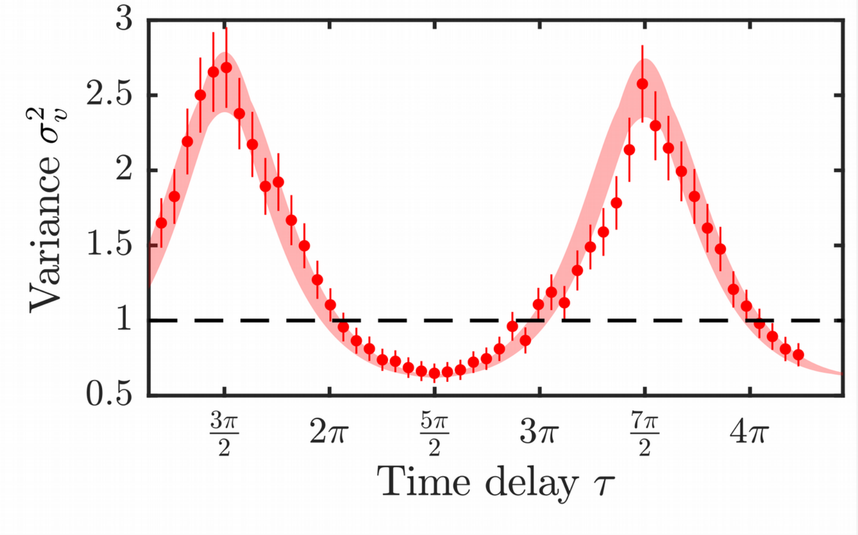
Thermodynamics quantities are measured using the velocity variance

$$\frac{\dot{W}_{\text{ext}}}{T_0} = \frac{1}{Q_0}\left(1-\sigma_v^2\right)$$

$$\dot{S}_{\text{pump}} = \frac{1}{Q_0}\left(\frac{1-\sigma_v^2}{\sigma_v^2}\right)$$

$$\sigma_v^2 = f(g, Q_0, \tau)$$

$$x(t)/x_{\text{therm}} \xrightarrow{\frac{d}{dt}} v(t)$$



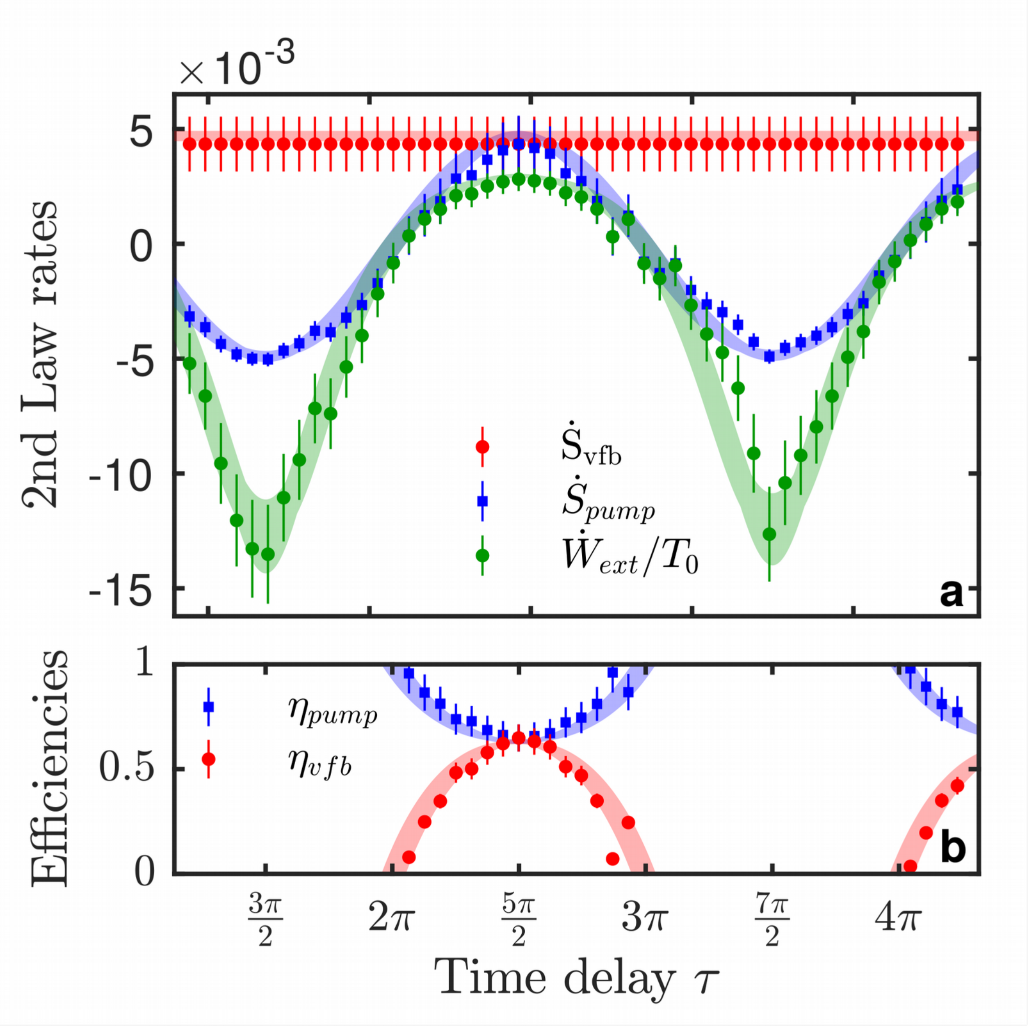
Cooling : $\sigma_v^2 < 1$

Heating : $\sigma_v^2 > 1$

Pumping entropy is the tightest bound to the extracted work

$$\boxed{\frac{\dot{W}_{\rm ext}}{T_0} \leq \dot{S}_{\rm pump}}$$

$$\begin{cases} Q_0 = 122 \\ g = 0.6 \end{cases}$$

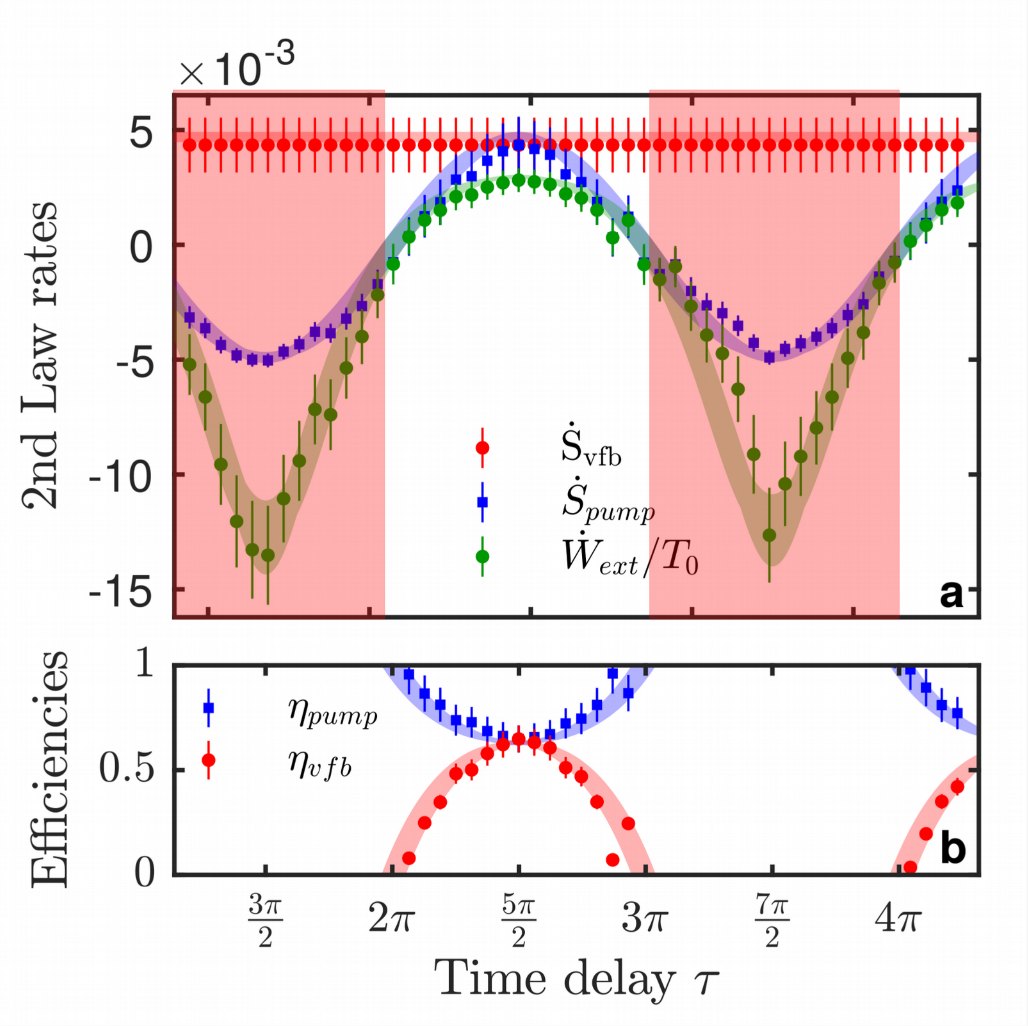


$\dot{S}_{\rm vfb} = \dot{S}_{\rm pump}$ for $\tau = \frac{\pi}{2}$

Pumping entropy is the tightest bound to the extracted work

$$\boxed{\frac{\dot{W}_{\text{ext}}}{T_0} \leq \dot{S}_{\text{pump}}}$$

$$\begin{cases} Q_0 = 122 \\ g = 0.6 \end{cases}$$

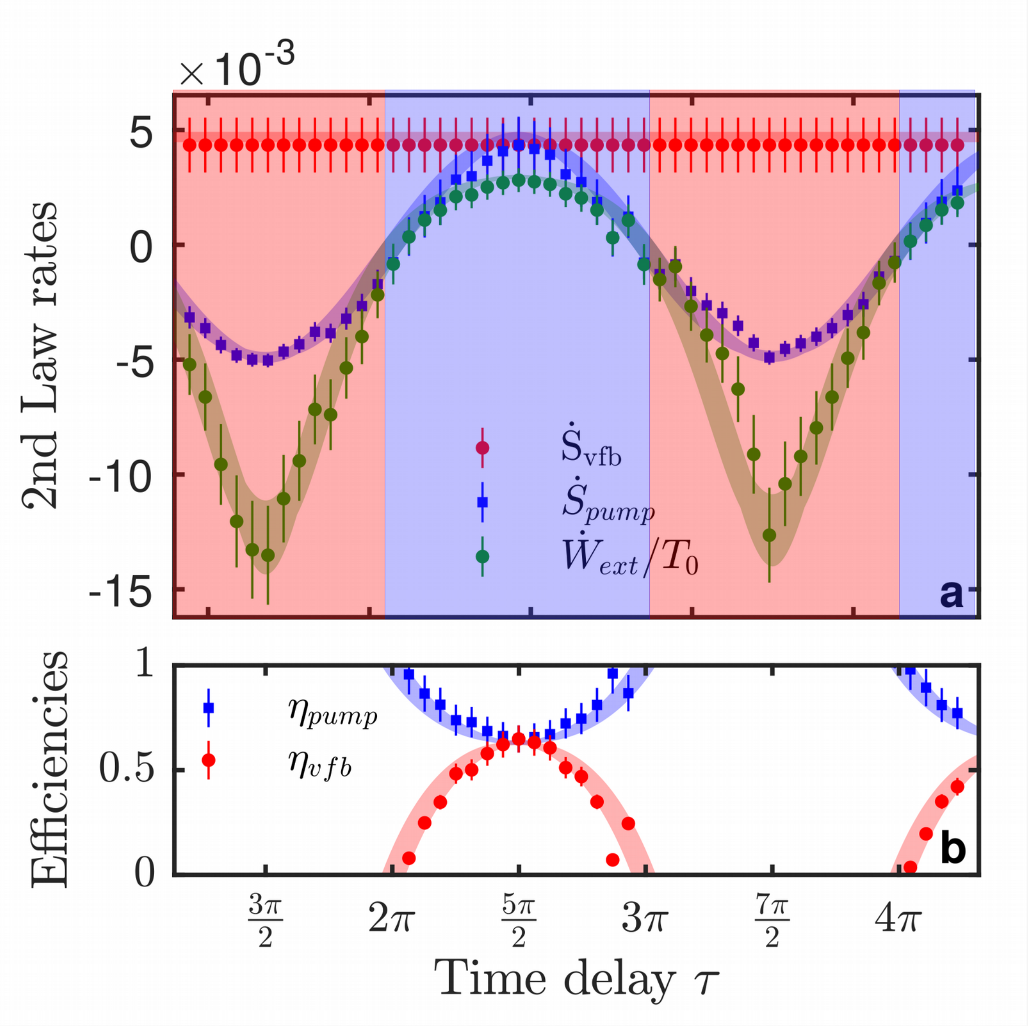


$\dot{S}_{\text{vfb}} = \dot{S}_{\text{pump}}$ for $\tau = \frac{\pi}{2}$

Pumping entropy is the tightest bound to the extracted work

$$\boxed{\frac{\dot{W}_{\rm ext}}{T_0} \leq \dot{S}_{\rm pump}}$$

$$\begin{cases} Q_0 = 122 \\ g = 0.6 \end{cases}$$



$$\dot{S}_{\rm vfb} = \dot{S}_{\rm pump} \text{ for } \tau = \frac{\pi}{2}$$

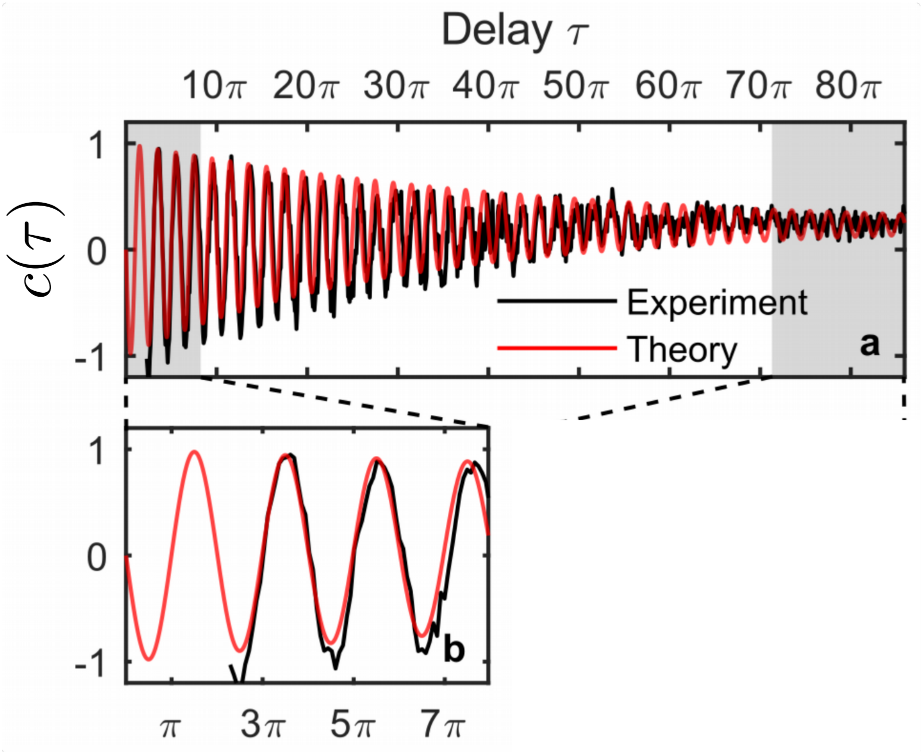
Efficiencies :

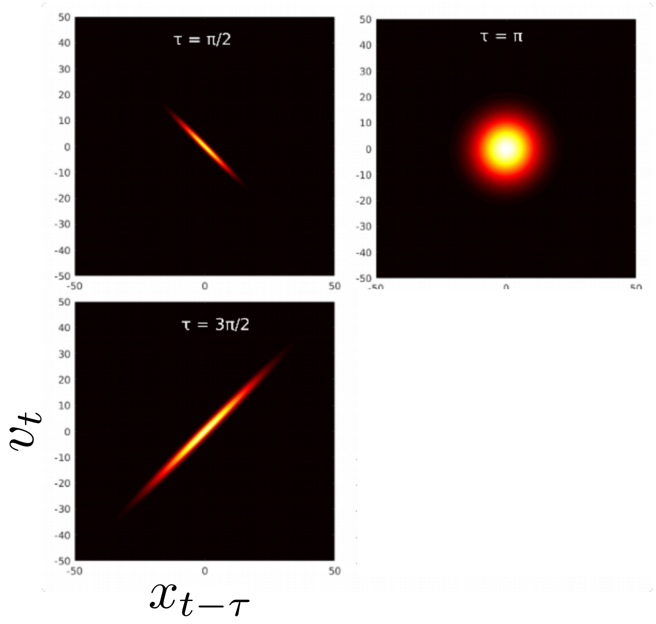
$$\boxed{\eta_{\rm pump}} = \frac{\dot{W}_{\rm ext}}{T_0 \dot{S}_{\rm pump}}$$

$$\boxed{\eta_{\rm vfb}} = \frac{\dot{W}_{\rm ext}}{T_0 \dot{S}_{\rm vfb}}$$

Very long delays lead to a colored noise

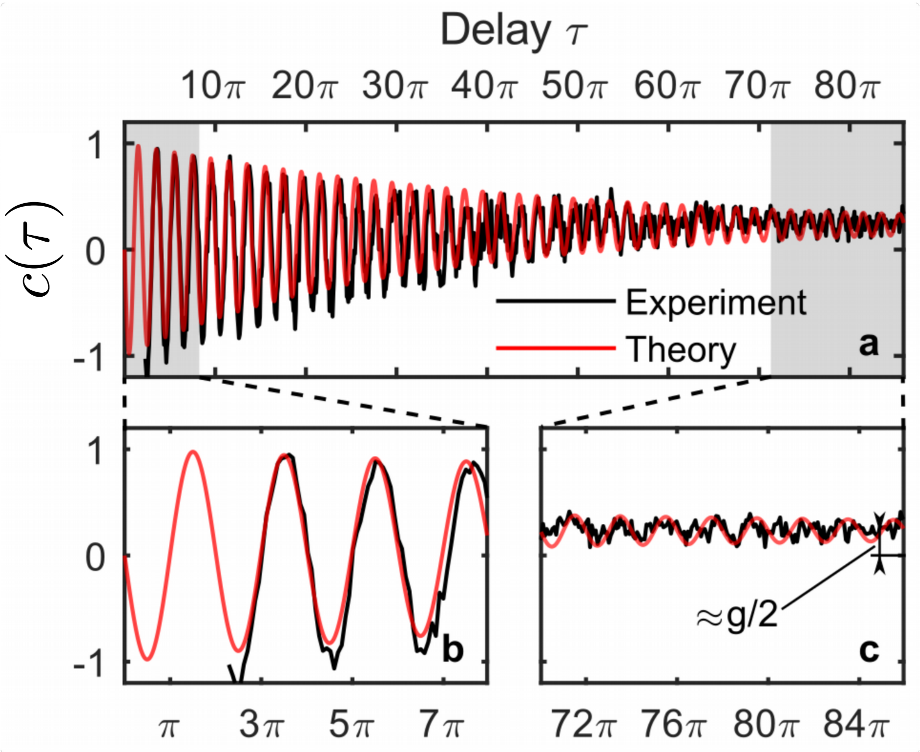
- Memory effect for $\tau \to \infty$?
- Correlation between $x_{t-\tau}$ and v_t : $c(\tau) = \frac{1}{g}\frac{1}{\sigma_x \sigma_v}\left(\sigma_v^2 - 1\right)$

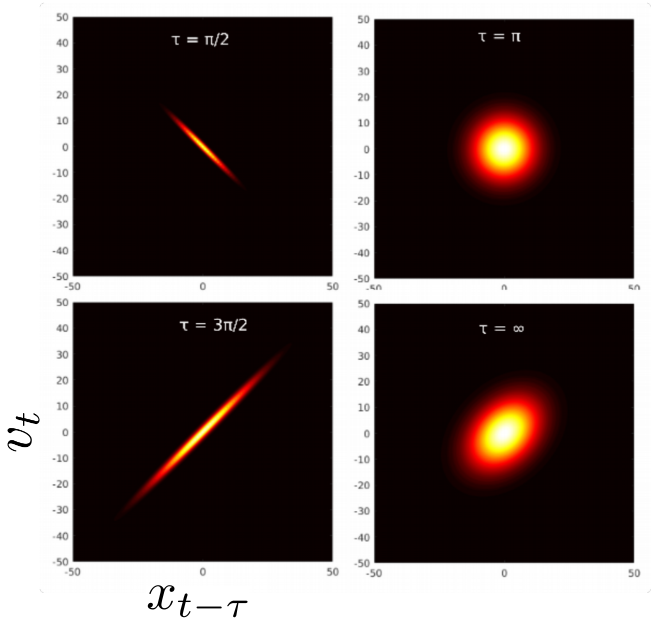




Very long delays lead to a colored noise

- Memory effect for $\tau \to \infty$?
- Correlation between $x_{t-\tau}$ and v_t : $c(\tau) = \frac{1}{g} \frac{1}{\sigma_x \sigma_v} \left(\sigma_v^2 - 1\right)$





Outline

1. Role of time delay in science
2. Information engine
3. Results
4. Conclusion

Conclusion

- Importance of time delays in science.
- Pumping entropy is the tightest bound to the 2^{nd} law.
- Memory effects blurred for very long delays.

Perspectives :

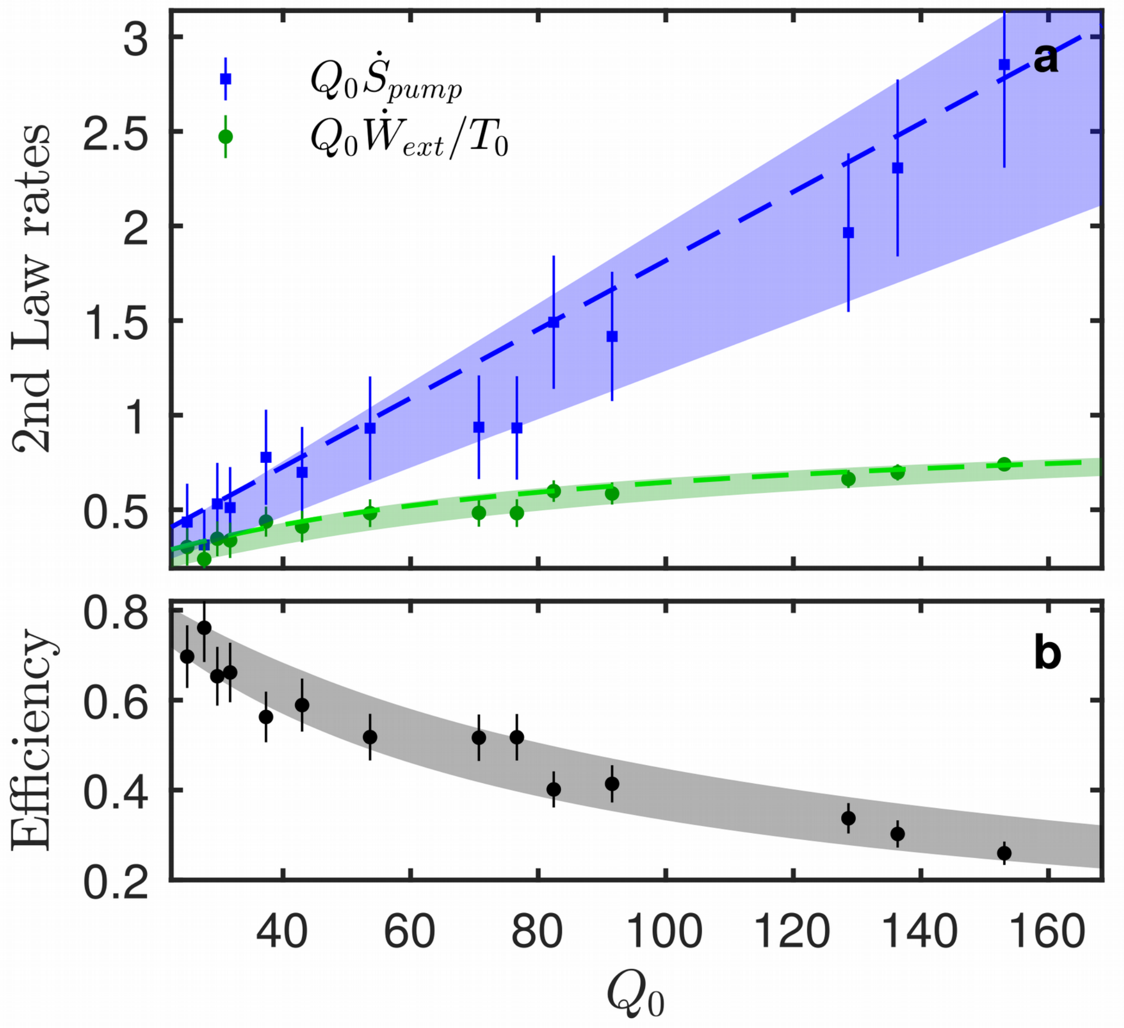
- Include noise
- A cyclic information engine

$$\sigma_v^2 = \frac{1}{Q_0} \frac{\omega_2 f(\omega_2) - \omega_1 f(\omega_1)}{\omega_2^2 - \omega_1^2}$$

$$\omega_{1,2} = \sqrt{1 - \frac{1}{2Q_0^2} \pm \frac{1}{Q_0}\sqrt{\Delta}}$$

$$\Delta = g^2 - 1 + \frac{1}{4Q_0^2}$$

$$f(\omega) = \frac{\omega + [Q_0(1-\omega^2) - g]\tan(\omega\tau/2)}{Q_0(1-\omega^2) - g - \omega\tan(\omega\tau/2)}$$



$Q_0\dot{S}_{pump}$
$Q_0\dot{W}_{ext}/T_0$
a
b
2nd Law rates
Efficiency
Q_0
3
2.5
2
1.5
1
0.5
0.8
0.6
0.4
0.2
40
60
80
100
120
140
160

Shrodinger : he asks himself how a living body manage to avoid decay. A obvious answer is by eating, drinking. But this causes the entropy in the living body to increase, therefore accelerating its death. This is why Shrodinger suggested that a living body feeds on negative entropy. But a living body is not a closed system where the entropy will have to increase. For open systems, the entropy can be reduced by exchanging heat and matter with the environement.

Neural network : Continuous-time analog neural networks with symmetric connections will always converge to fixed points when the neurons have infinitely fast response, but can oscillate when a small time delay is present. We analyze the dynamics of continuous-time analog networks with delay, and show that there is a critical delay above which a symmetrically connected network will oscillate. The results are useful as design criteria for building fast but stable electronic networks.

HIV infection : It is still an interesting exercise to determine how the intercellular delay affects overall disease progression. The stability of the steady infection state depends on the delay and even delay-induced oscillations could occur via instability.