Generalized 2\textsuperscript{nd} law under non-Markovian feedback control using a levitated particle

Quantum Engineering of Levitated Systems
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Information on a system allows to extract more work

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- Kelvin-Planck statement: ‘no work can be cyclically extracted from a system coupled to a single reservoir at temperature T’:

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Shoichi Toyabe et al, Nat. Phys., 2010
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- Pumping entropy: \( \dot{W}_{\text{ext}} < k_B T \dot{S}_{\text{pump}} \)

Rosinberg et al, PRE, 2015
Outline

1. Role of time delay in science
2. Information engine
3. Results
4. Conclusion
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Schrödinger (1942) : How does a living organism avoid decay? 
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‘Negative entropy’ or ‘pumping entropy’ (Kim & Qian, PRL, 2004)
Time delays are ubiquitous in science

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Protocol:

1) Measure position
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Weakly damped system: 
\[
x \propto \cos(\Omega_0 t)
\]
\[
F_{fb} \propto \cos(\Omega_0 (t - \tau))
\]
\[
= x \cos \tau - \frac{1}{\Omega_0} v \sin \tau
\]

Time delay ➔ Memory effect ➔ Non-Markovianity
Realization of the protocol with a levitated particle

Harmonic trap:

\[ \nu_0 = 400 \text{ kHz} \]
\[ \Gamma_p \approx 1 \text{ kHz} \]

\[ Q_0 = \frac{\nu_0}{\Gamma_p} \approx 100 \]
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Feedback:

Feedback gain:

\[ g = \frac{\Gamma_{fb}}{\Gamma_p} \approx 0.5 \]

Feedback delay:

\[ \tau = 2\pi\nu_0 t \]

\( (g, Q_0, \tau) \)
From equation of motion to the tightest bound of the 2\textsuperscript{nd} law

\[ \ddot{x}_t + \Gamma_p \dot{x}_t + \Omega_0^2 x_t = \frac{\Gamma_{fb}}{\Omega_0} x_{t-\tau} - \frac{F_{\text{therm}}}{m} \]
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Fokker-Plank equation
(Rosinberg \textit{et al}, PRE, 2015)

\[ \frac{dS}{dt} = \dot{S}_i - \dot{S}_{\text{pump}} - \frac{\dot{Q}}{T_0} \]
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Steady state: \( \frac{dS}{dt} = 0 \)

First law: \( \dot{Q} = \dot{W} \)

Non negativity: \( \dot{S}_i \geq 0 \)
From equation of motion to the tightest bound of the 2\textsuperscript{nd} law

\[
\ddot{x}_t + \Gamma_p \dot{x}_t + \Omega^2_0 x_t - \frac{\Gamma_{fb}}{\Omega_0} x_{t-\tau} = \frac{F_{\text{therm}}}{m}
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Tightest bound

\[
\frac{\dot{W}_{\text{ext}}}{T_0} \leq \dot{S}_{\text{pump}} \leq \dot{I}
\]

\[
\dot{W}_{\text{ext}} = -\dot{W} = \int F_{fb} dx
\]

\(\dot{I} = \text{mutual information flow}\)
The Information engine can cool or heat the particle motion

Cooling mode

Bath \( (T_0) \)

\[ Q \]

Particle \( (T<T_0) \)

Info.

\[ S_{pump} \]

\[ W_{ext}>0 \]

Cooling

\[ 0 < \tau < \pi \]
The Information engine can cool or heat the particle motion

**Cooling mode**

- **Bath** ($T_0$)
- **Info.**
- **Particle** ($T<T_0$)

$Q$, $S_{\text{pump}}$

$W_{\text{ext}}>0$

$0 < \tau < \pi$

**Heating mode**

- **Bath** ($T_0$)
- **Info.**
- **Particle** ($T>T_0$)

$Q$, $S_{\text{pump}}$

$W_{\text{ext}}<0$

$\pi < \tau < 2\pi$
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Thermodynamics quantities are measured using the velocity variance

\[ \frac{\dot{W}_{\text{ext}}}{T_0} = \frac{1}{Q_0} \left( 1 - \sigma_v^2 \right) \]

\[ \dot{S}_{\text{pump}} = \frac{1}{Q_0} \left( 1 - \frac{\sigma_v^2}{\sigma_v^2} \right) \]

\[ \sigma_v^2 = f(g, Q_0, \tau) \]
Thermodynamics quantities are measured using the velocity variance

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\frac{\dot{W}_{\text{ext}}}{T_0} = \frac{1}{Q_0} \left( 1 - \sigma^2_v \right)
\]

\[
\dot{S}_{\text{pump}} = \frac{1}{Q_0} \left( \frac{1 - \sigma^2_v}{\sigma^2_v} \right)
\]

\[
\sigma^2_v = f(g, Q_0, \tau)
\]

\[
x(t)/x_{\text{therm}} \quad \frac{d}{dt} v(t)
\]

Cooling: \( \sigma^2_v < 1 \)

Heating: \( \sigma^2_v > 1 \)
Pumping entropy is the tightest bound to the extracted work

\[
\frac{\dot{W}_{\text{ext}}}{T_0} \leq \dot{S}_{\text{pump}}
\]

\[
\begin{cases}
Q_0 = 122 \\
g = 0.6
\end{cases}
\]

\[
\dot{S}_{\text{vfb}} = \dot{S}_{\text{pump}} \quad \text{for} \quad \tau = \frac{\pi}{2}
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Efficiencies:

\[ \eta_{\text{pump}} = \frac{\dot{W}_{\text{ext}}}{T_0 \dot{S}_{\text{pump}}} \]

\[ \eta_{\text{vfb}} = \frac{\dot{W}_{\text{ext}}}{T_0 \dot{S}_{\text{vfb}}} \]
Very long delays lead to a colored noise

- Memory effect for $\tau \rightarrow \infty$?
- Correlation between $x_{t-\tau}$ and $v_t$:
  $$c(\tau) = \frac{1}{g} \frac{1}{\sigma_x \sigma_v} \left( \sigma_v^2 - 1 \right)$$
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Conclusion

- Importance of time delays in science.
- Pumping entropy is the tightest bound to the 2\textsuperscript{nd} law.
- Memory effects blurred for very long delays.

Perspectives:

- Include noise
- A cyclic information engine
\[
\sigma_v^2 = \frac{1}{Q_0} \frac{\omega_2 f(\omega_2) - \omega_1 f(\omega_1)}{\omega_2^2 - \omega_1^2}
\]

\[
\omega_{1,2} = \sqrt{1 - \frac{1}{2Q_0^2}} \pm \frac{1}{Q_0} \sqrt{\Delta}
\]

\[
\Delta = g^2 - 1 + \frac{1}{4Q_0^2}
\]

\[
f(\omega) = \frac{\omega + [Q_0(1 - \omega^2) - g] \tan(\omega \tau/2)}{Q_0(1 - \omega^2) - g - \omega \tan(\omega \tau/2)}
\]
\( Q_0 \hat{S}_{\text{pump}} \)
\( Q_0 \hat{W}_{\text{ext}} / T_0 \)

(a)

(b)
**Shrodinger**: he asks himself how a living body manage to avoid decay. A obvious answer is by eating, drinking. But this causes the entropy in the living body to increase, therefore accelerating its death. This is why Shrodinger suggested that a living body feeds on negative entropy. But a living body is not a closed system where the entropy will have to increase. For open systems, the entropy can be reduced by exchanging heat and matter with the environment.

**Neural network**: Continuous-time analog neural networks with symmetric connections will always converge to fixed points when the neurons have infinitely fast response, but can oscillate when a small time delay is present. We analyze the dynamics of continuous-time analog networks with delay, and show that there is a critical delay above which a symmetrically connected network will oscillate. The results are useful as design criteria for building fast but stable electronic networks.

**HIV infection**: It is still an interesting exercise to determine how the intercellular delay affects overall disease progression. The stability of the steady infection state depends on the delay and even delay-induced oscillations could occur via instability.