Optical momentum and angular momentum in complex media

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Assumptions

We consider:
– monochromatic optical fields
– in isotropic and lossless media.

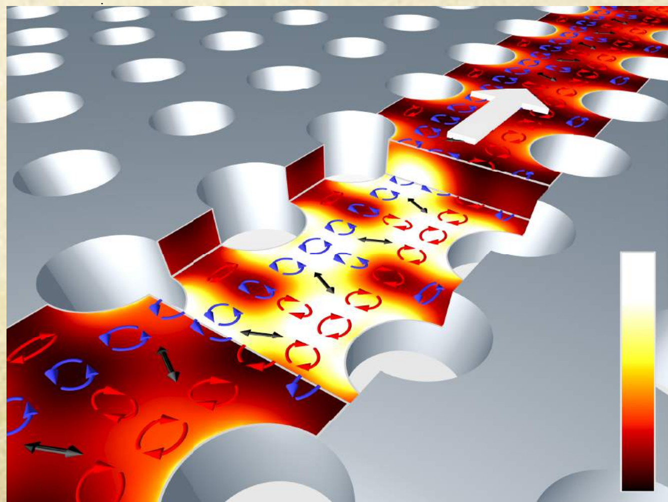
(a) The fields are generally structured (inhomogeneous):

$$\mathrm{Re}\left[\mathbf{E}(\mathbf{r})e^{-i\omega t}\right], \quad \mathrm{Re}\left[\mathbf{H}(\mathbf{r})e^{-i\omega t}\right];$$

(b,c) The media are inhomogeneous and dispersive:

$$\varepsilon(\mathbf{r},\omega), \quad \mu(\mathbf{r},\omega).$$

This includes "structured light in structured matter".



Abraham-Minkowski dilemma

Abraham–Minkowski dilemma

When mentioning optical momentum in a medium, the first question is: Abraham or Minkowski?

M. Abraham

H. Minkowski

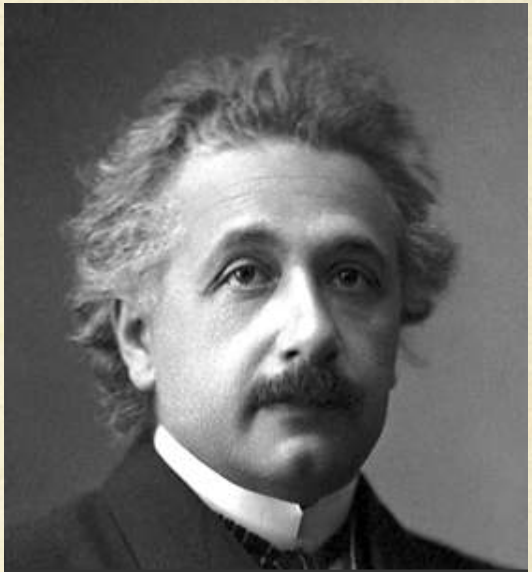
$$\boxed{\mathcal{P}_A = k_0\,\mathrm{Re}\left(\mathbf{E}^* \times \mathbf{H}\right)}$$

$$\boxed{\mathcal{P}_M = \varepsilon\mu\,\mathcal{P}_A}$$

Brevik *PR* (1979); Pfeifer *et al. RMP* (2007); Milloni, Boyd *AOP* (2010); Kemp *JAP* (2011)

Abraham–Minkowski dilemma

The “naïve” explanation of the Abraham momentum density is related to the relativity, velocity of photons, i.e., their kinetic properties:



A. Einstein

$$\boxed{\mathcal{P}_A = mv = \frac{E}{c^2}\frac{c}{\tilde{n}} = \frac{\hbar k_0}{\tilde{n}}}$$

$$n = \sqrt{\varepsilon\mu}\ , \quad \tilde{n} = n + \omega\frac{dn}{d\omega}$$

However, this makes sense only for plane–wave–like photons in homogeneous transparent media.

Abraham–Minkowski dilemma

In turn, the Minkowski momentum density is explained via the quantum–mechanical de Broglie relation with the wavevector, i.e., canonical property:

$$\boxed{\mathcal{P}_M = \hbar k = n \hbar k_0}$$

L. de Broglie

However, this works only for plane–wave–like photons in homogeneous, non–dispersive, and transparent media.

Abraham–Minkowski dilemma

Therefore, the Abaraham and Minkowski momenta are often associated with the kinetic (velocity) and canonical (wavevector) properties of plane-wave-like photons in homogeneous, transparent, and dispersionless media.

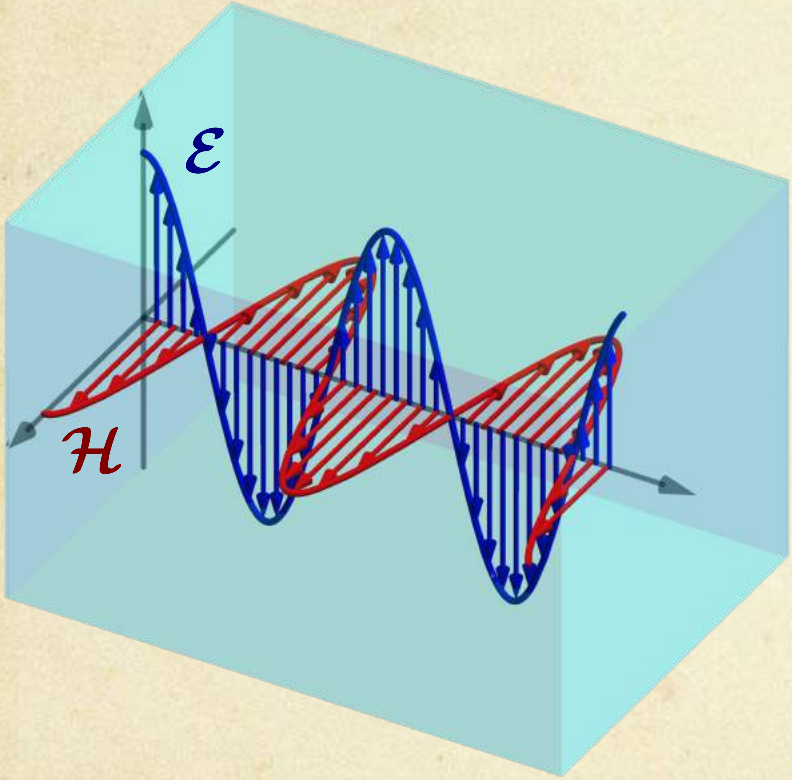
Dewar (1977); Nelson (1991); Garrison & Chiao (2004); Barnett (2010); Dodin & Fisch (2012).

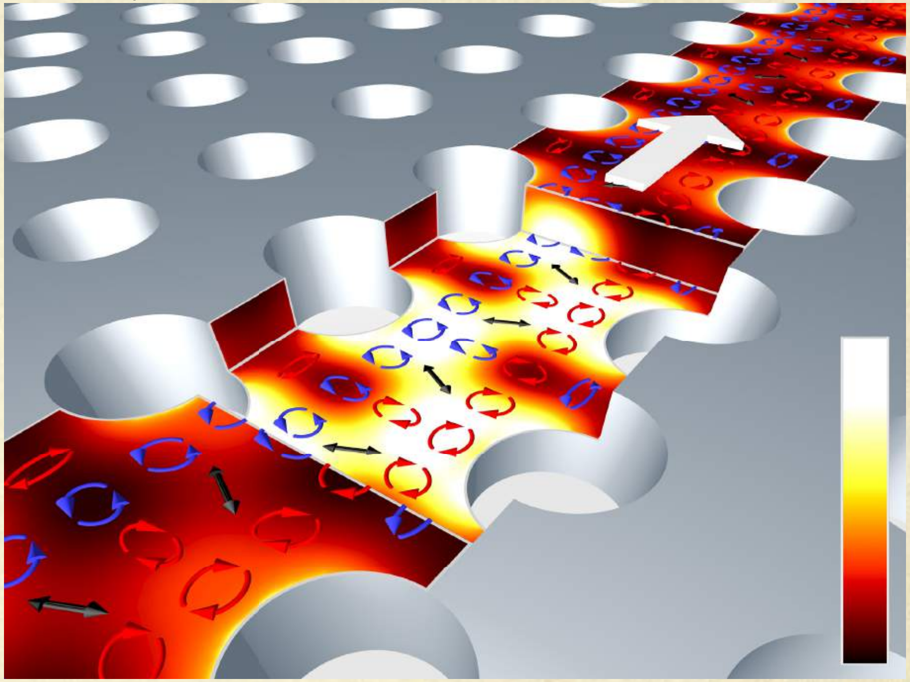
Even in simple dispersive media, one needs to modify the Minkowski momentum to get the canonical de Broglie result:

$$\tilde{\mathcal{P}}_M = \mathcal{P}_M + \{\text{disp.}\} = \hbar k$$

However, we need a theory working for structured light in structured media!

Abraham–Minkowski dilemma





homogeneous vs. structured

Kinetic and canonical pictures for free-space light

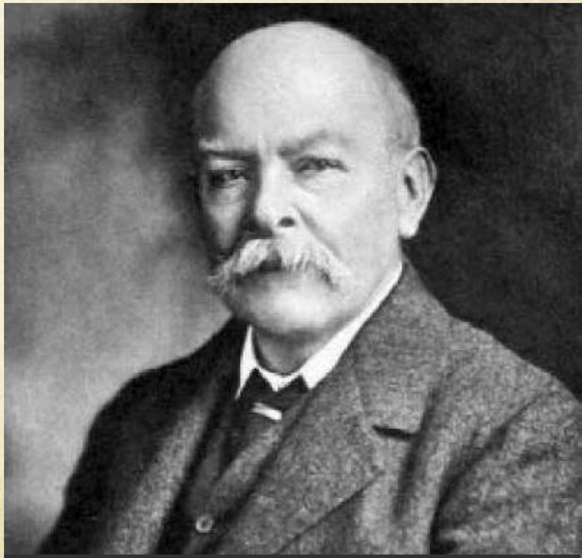
Structured light in free space

The momentum and angular momentum (AM) properties of structured light were recently studied in detail in free space.

In this case the Abraham and Minkowski momenta converge to the Poynting momentum:

$$\boxed{\mathcal{P} = k_0 \, \mathrm{Re}\left(\mathbf{E}^* \times \mathbf{H}\right)}$$

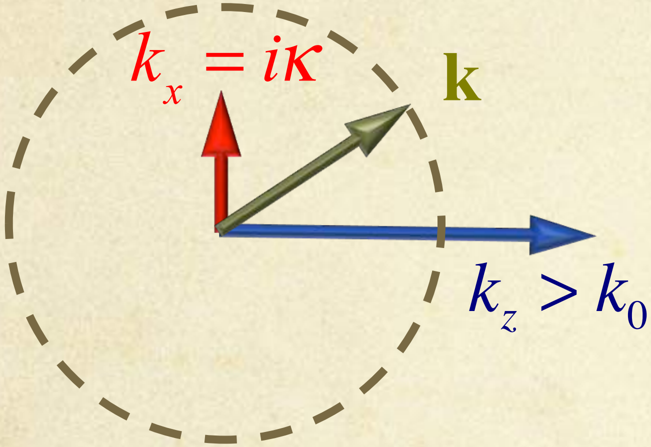
However, the Poynting vector is a kinetic (energy–flux) property, which cannot describe canonical (wavevector) momentum of structured light.

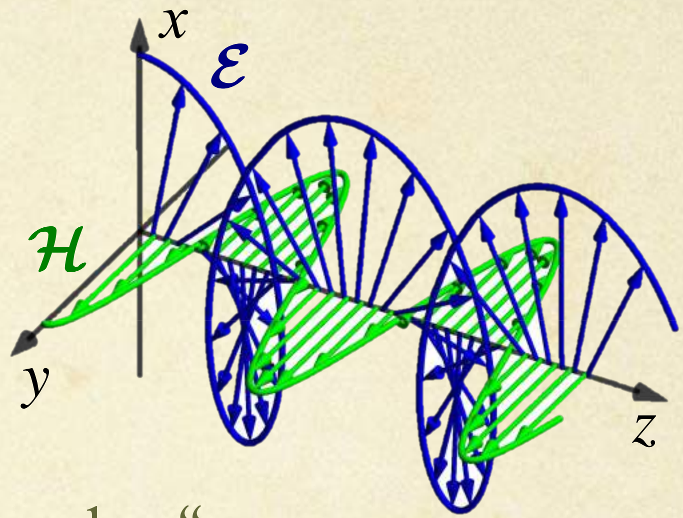


J. H. Poynting

Structured light in free space

Simple example: an evanescent wave $\propto \exp(ik_z z - \kappa x)$.





Its wavevector corresponds to the "supermomentum"

$$\hbar k_z > \hbar k_0 \,,$$

but the Poynting vector is always "subluminal":

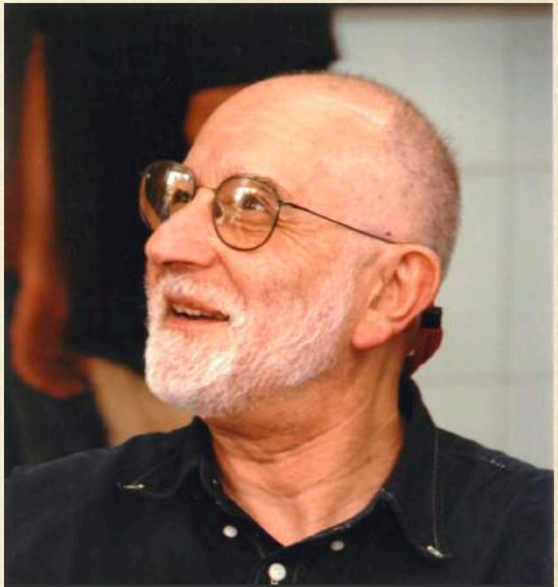
$$c^2 |\mathcal{P}| / W < c \,, \quad \text{i.e.,} \quad |\mathcal{P}| < \hbar k_0 \ \text{ per photon.}$$

Huard & Imbert (1978); Matsudo *et al.* (1998); Bliokh *et al.*; Barnett & Berry (2013)

Structured light in free space

The canonical (orbital) momentum density for structured free–space light was first written by M. V. Berry (2009):

$$\boxed{\mathbf{P} = \frac{1}{2}\mathrm{Im}\left[\mathbf{E}^{*}\cdot(\nabla)\mathbf{E} + \mathbf{H}^{*}\cdot(\nabla)\mathbf{H}\right]}$$


M. V. Berry

It describes the local wavevector properties of structured fields, Including “supermomentum”

$$c^{2}\left|\mathbf{P}\right|/W > c, \quad \text{i.e.,} \quad \left|\mathbf{P}\right| = \hbar k_{z} > \hbar k_{0}\,.$$

Berry *JOA* (2009); Bliokh *et al. NJP* (2013)

Structured light in free space

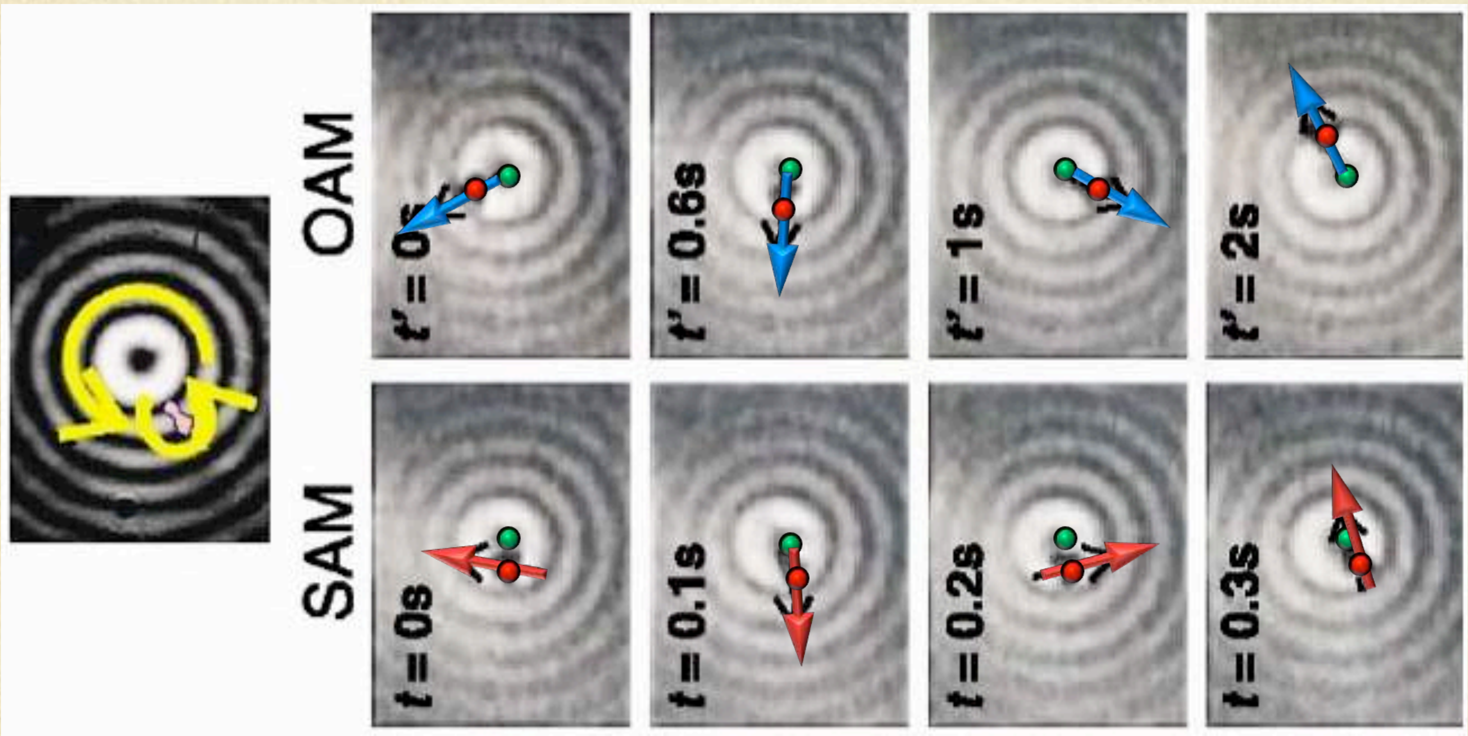
The difference between the kinetic (Poynting) and canonical pictures is closely related to the spin–orbital AM decomposition:

Kinetic (Poynting)	Canonical (spin–orbital)
$\boldsymbol{\mathcal{P}} = k_0 \, \mathrm{Re}\left(\mathbf{E}^* \times \mathbf{H}\right)$	$\mathbf{P} = \frac{1}{2} \mathrm{Im}\left[\mathbf{E}^* \cdot (\nabla)\mathbf{E} + \mathbf{H}^* \cdot (\nabla)\mathbf{H}\right]$
$\boldsymbol{\mathcal{J}} = \mathbf{r} \times \boldsymbol{\mathcal{P}}$	$\mathbf{S} = \frac{1}{2} \mathrm{Im}\left[\mathbf{E}^* \times \mathbf{E} + \mathbf{H}^* \times \mathbf{H}\right]$ $\mathbf{L} = \mathbf{r} \times \mathbf{P}$

Integral: $\langle \boldsymbol{\mathcal{P}} \rangle = \langle \mathbf{P} \rangle, \quad \langle \boldsymbol{\mathcal{J}} \rangle = \langle \mathbf{L} \rangle + \langle \mathbf{S} \rangle$

Structured light in free space

Canonical picture perfectly describes momentum, spin, and orbital AM properties of free–space light:



What are the canonical properties of light in media?

O'Neil *et al. PRL* (2002); Garces-Chavez *et al. PRL* (2003);
Bliokh *et al.*, *NC* (2014); *PRL* (2014); *PRX* (2015); *PR* (2015); *NP* (2016).

General theory:
Structured light in complex media

Brillouin energy density

The main known quantity that works perfectly for structured fields in complex media is the Brillouin energy density (1921):

$$\boxed{\tilde{W} = \frac{\omega}{2}\left(\tilde{\varepsilon}|\mathbf{E}|^2 + \tilde{\mu}|\mathbf{H}|^2\right)}$$

$$\left(\tilde{\varepsilon},\tilde{\mu}\right) = \left(\varepsilon,\mu\right) + \omega\frac{d\left(\varepsilon,\mu\right)}{d\omega}$$

Dispersive corrections are crucial!

L. Brillouin

Kinetic–Abraham energy flux

The Poynting vector also preserves its meaning for structured light in complex media as the kinetic–Abraham momentum:

$$\boxed{\mathcal{P}_A = k_0 \operatorname{Re}\left(\mathbf{E}^* \times \mathbf{H}\right)}$$

In fact, it describes the energy flux and **group velocity** of the wave rather than its momentum. For localized solutions in an inhomogeneous dispersive medium:

$$\boxed{\mathbf{v}_g = \frac{c^2 \left\langle \mathcal{P}_A \right\rangle}{\left\langle \tilde{W} \right\rangle}} \qquad v_g = \frac{\partial \omega}{\partial k} < c$$

Canonical–Abraham quantities

First, in 2012 we performed the spin–orbital decomposition of the the Poynting–Abraham vector, introducing the canonical–Abraham picture:

$$\mathbf{P}_A = \frac{1}{2}\mathrm{Im}\left[\frac{\mathbf{E}^* \cdot (\nabla)\mathbf{E}}{\mu} + \frac{\mathbf{H}^* \cdot (\nabla)\mathbf{H}}{\varepsilon}\right] + \{\mathrm{grad.}\}$$

$$\mathbf{S}_A = \frac{1}{2}\mathrm{Im}\left(\frac{\mathbf{E}^* \times \mathbf{E}}{\mu} + \frac{\mathbf{H}^* \times \mathbf{H}}{\varepsilon}\right), \quad \mathbf{L}_A = \mathbf{r} \times \mathbf{P}_A$$

However, these quantities involve gradient corrections and singularities at interfaces between media.

Bliokh & Nori *PRA* (2012); Bliokh *et al.* *NC* (2014)

Canonical–Abraham quantities

First, in 2012 we performed the spin–orbital decomposition of the the Poynting–Abraham vector, introducing the canonical–Abraham picture:

$$\{\mathrm{grad.}\} = -\frac{1}{4}\left[\nabla\mu^{-1}\times \mathrm{Im}\left(\mathbf{E}^{*}\times\mathbf{E}\right)+\nabla\varepsilon^{-1}\times \mathrm{Im}\left(\mathbf{H}^{*}\times\mathbf{H}\right)\right]\ldots$$

$$\mathbf{S}_{A} = \frac{1}{2}\mathrm{Im}\left(\frac{\mathbf{E}^{*}\times\mathbf{E}}{\mu}+\frac{\mathbf{H}^{*}\times\mathbf{H}}{\varepsilon}\right), \quad \mathbf{L}_{A} = \mathbf{r}\times\mathbf{P}_{A}$$

However, these quantities involve gradient corrections and singularities at interfaces between media.

Bliokh & Nori *PRA* (2012); Bliokh *et al. NC* (2014)

Kinetic–Minkowski quantities

Second, in 2011–2012, T. G. Philbin derived, using the phenomenological Lagrangian–Noether approach, the kinetic Minkowski–type momentum and AM of light in a dispersive medium:

$$\tilde{\mathcal{P}}_M = \mathcal{P}_M + \{\text{disp.}1\}$$

$$\tilde{\mathcal{J}}_M = \mathbf{r} \times \tilde{\mathcal{P}}_M + \{\text{disp.}2\}$$

However, these quantities involve cumbersome dispersive corrections.

T. G. Philbin

Philbin *PRA* (2011); Philbin & Allanson *PRA* (2012)

Kinetic–Minkowski quantities

Second, in 2011–2012, T. G. Philbin derived, using the phenomenological Lagrangian–Noether approach, the kinetic Minkowski–type momentum and AM of light in a dispersive medium:

$$\{\text{disp.1}\} = \frac{\omega}{2}\,\mathrm{Im}\left[\frac{d\varepsilon}{d\omega}\mathbf{E}^{*}\cdot(\nabla)\mathbf{E} + \frac{d\mu}{d\omega}\mathbf{H}^{*}\cdot(\nabla)\mathbf{H}\right]$$

$$\{\text{disp.2}\} = \frac{\omega}{2}\,\mathrm{Im}\left[\frac{d\varepsilon}{d\omega}\mathbf{E}^{*}\times\mathbf{E} + \frac{d\mu}{d\omega}\mathbf{H}^{*}\times\mathbf{H}\right]$$

These dispersive corrections have canonical–like forms.

Philbin *PRA* (2011); Philbin & Allanson *PRA* (2012)

Canonical–Minkowski quantities

Thus, both the canonical (spin–orbital) Abraham approach and kinetic (Poynting–like) Minkowski approach have considerable drawbacks and not entirely clear physical meaning.

To have a proper momentum and AM pictures for structured light in complex media, we developed the **canonical Minkowski–type approach**.
It corresponds to the kinetic Minkowski–type quantities derived by Philbin: $\tilde{\boldsymbol{\mathcal{P}}}_M = \tilde{\mathbf{P}}_M + \nabla \times \mathbf{S}_M / 2$

$$\left\langle \tilde{\boldsymbol{\mathcal{P}}}_M \right\rangle = \left\langle \tilde{\mathbf{P}}_M \right\rangle, \quad \left\langle \tilde{\boldsymbol{\mathcal{J}}}_M \right\rangle = \left\langle \tilde{\mathbf{L}}_M \right\rangle + \left\langle \tilde{\mathbf{S}}_M \right\rangle$$

Canonical–Minkowski quantities

Remarkably, the canonical Minkowski–type quantities take very natural forms similar to the Brillouin energy, without awkward gradient/dispersive corrections:

$$\tilde{W} = \frac{\omega}{2}\left(\tilde{\varepsilon}|\mathbf{E}|^2 + \tilde{\mu}|\mathbf{H}|^2\right)$$

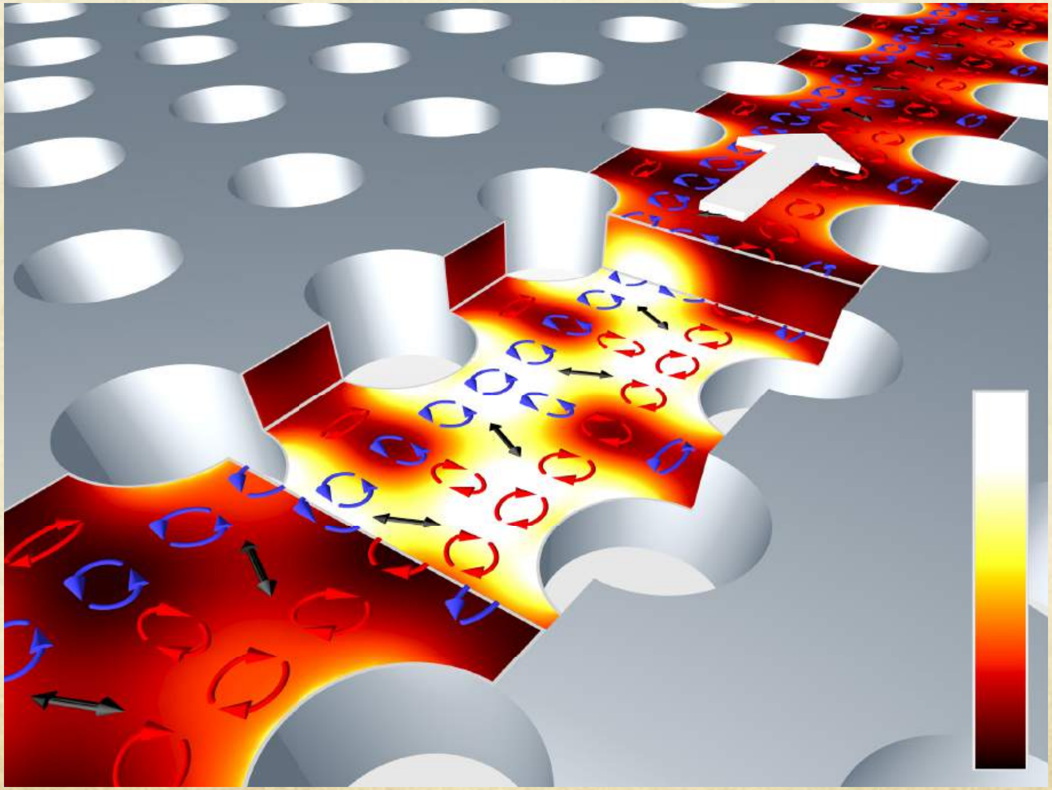
$$\tilde{\mathbf{P}}_M = \frac{1}{2}\mathrm{Im}\left[\tilde{\varepsilon}\,\mathbf{E}^*\cdot(\nabla)\mathbf{E} + \tilde{\mu}\,\mathbf{H}^*\cdot(\nabla)\mathbf{H}\right]$$

$$\tilde{\mathbf{S}}_M = \frac{1}{2}\mathrm{Im}\left(\tilde{\varepsilon}\,\mathbf{E}^*\times\mathbf{E} + \tilde{\mu}\,\mathbf{H}^*\times\mathbf{H}\right),\;\; \tilde{\mathbf{L}}_M = \mathbf{r}\times\tilde{\mathbf{P}}_M$$

Bliokh, Bekshaev, Nori *PRL* (2017), *NJP* (2017).

Canonical–Minkowski quantities

These expressions are valid for structured (mono-chromatic) optical fields in arbitrary inhomogeneous and dispersive (isotropic and lossless) media:



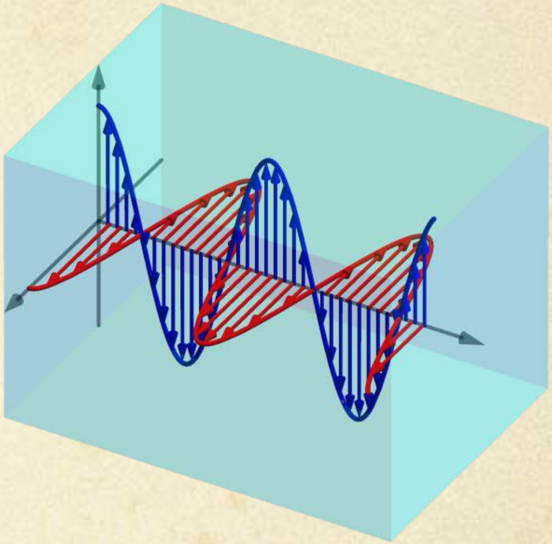
Bliokh, Bekshaev, Nori *PRL* (2017), *NJP* (2017).

Canonical–Minkowski quantities

In the simplest case of a plane wave in a homogeneous transparent medium, different momenta and spins yield the following values per photon:

$$\mathcal{P}_A = \mathbf{P}_A = \frac{1}{\tilde{n}n}\hbar\mathbf{k}, \quad \mathbf{S}_A = \frac{1}{\tilde{n}n}\hbar\sigma\bar{\mathbf{k}}$$

$$\boxed{\tilde{\mathcal{P}}_M = \tilde{\mathbf{P}}_M = \hbar\mathbf{k}, \qquad \tilde{\mathbf{S}}_M = \hbar\sigma\bar{\mathbf{k}}}$$



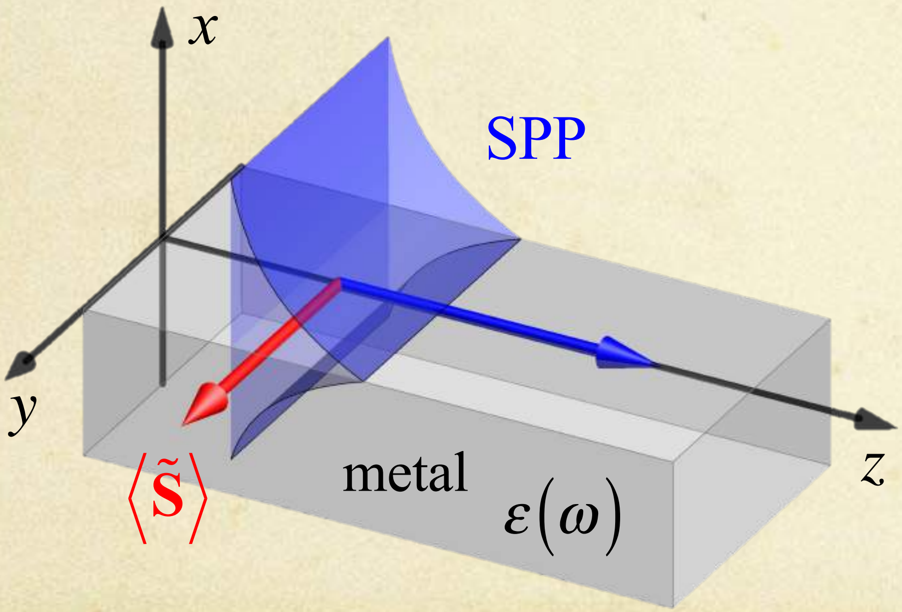
Importantly, the Minkowski-type quantities are conserved in media with proper symmetries (see, e.g., Snell's law and transverse beam shifts).

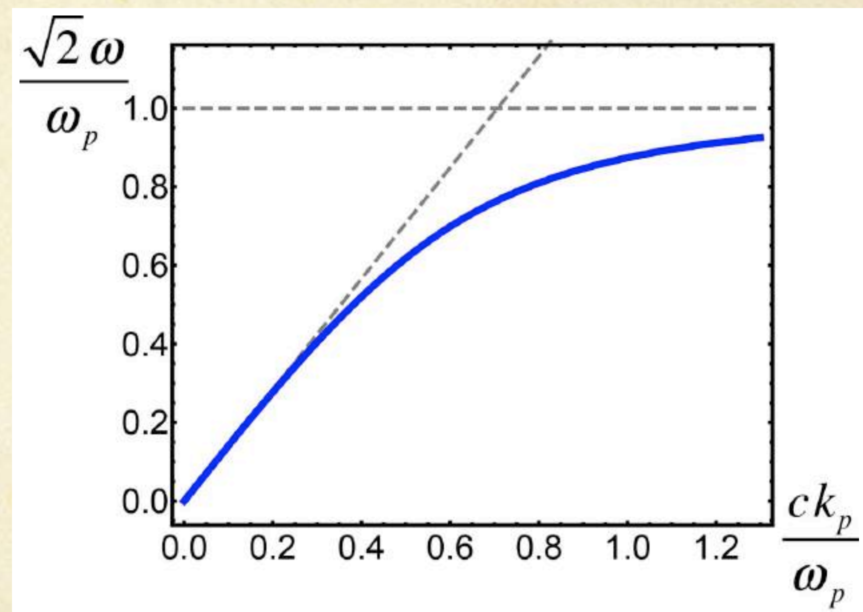
Fedoseyev (1988), Player (1987), Bliokh & Bliokh (2006,2007), ...

Application to surface plasmon-polaritons at metal-vacuum interfaces

Surface plasmon–polaritons

SPPs provide a simple but very nontrivial example:
- highly dispersive medium;
- highly inhomogeneous medium;
- structured field (with a well-defined wavevector).



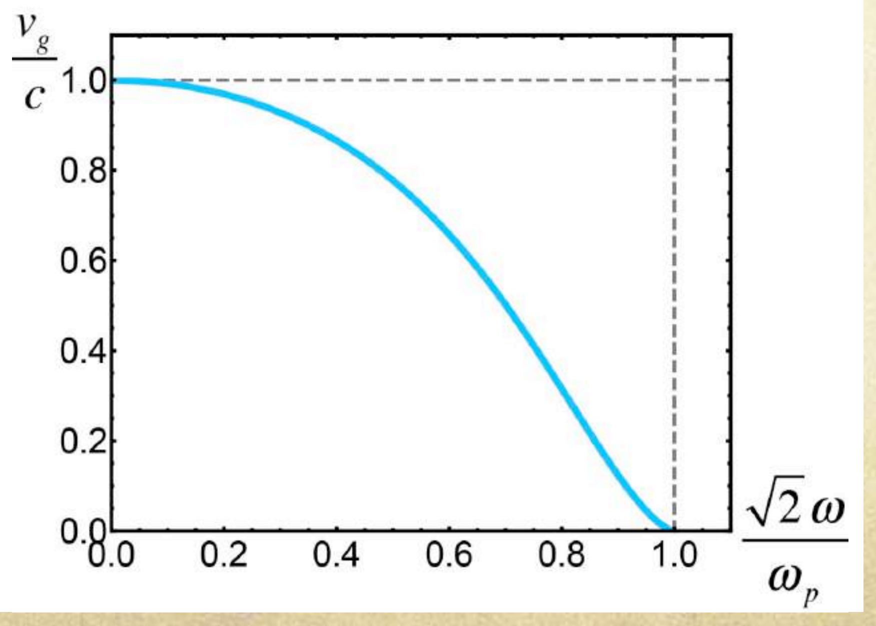


$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} < -1, \quad \tilde{\varepsilon} > 0, \quad \mu = 1, \quad k_p = \sqrt{\frac{\varepsilon}{1+\varepsilon}}\, k_0 > k_0$$

Abraham momentum and group velocity

The kinetic Abraham (Poynting) momentum provides the SPP group velocity (always subluminal, $v_g < c$):

$$\mathbf{v}_g = \frac{c^2 \langle \mathcal{P}_A \rangle}{\langle \tilde{W} \rangle} = c \frac{\sqrt{-\varepsilon}\left(-1-\varepsilon\right)^{3/2}}{1+\varepsilon^2} \overline{\mathbf{z}} = \frac{\partial \omega}{\partial k_p} \overline{\mathbf{z}}$$



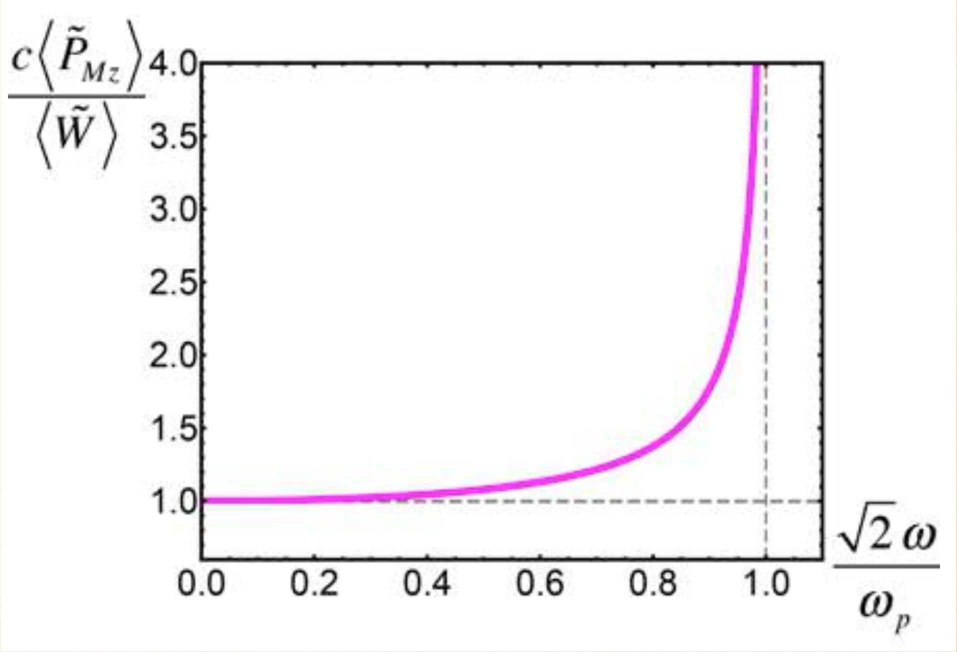
$$\langle \ldots \rangle = \int \ldots dx$$

Nkoma *et al.* *JPC* (1974)

Canonical super–momentum

The novel canonical Minkowski–type momentum corresponds to the SPP wave vector and, hence, super–momentum $\tilde{P}_M = \hbar k_p > \hbar k_0$ per polariton:

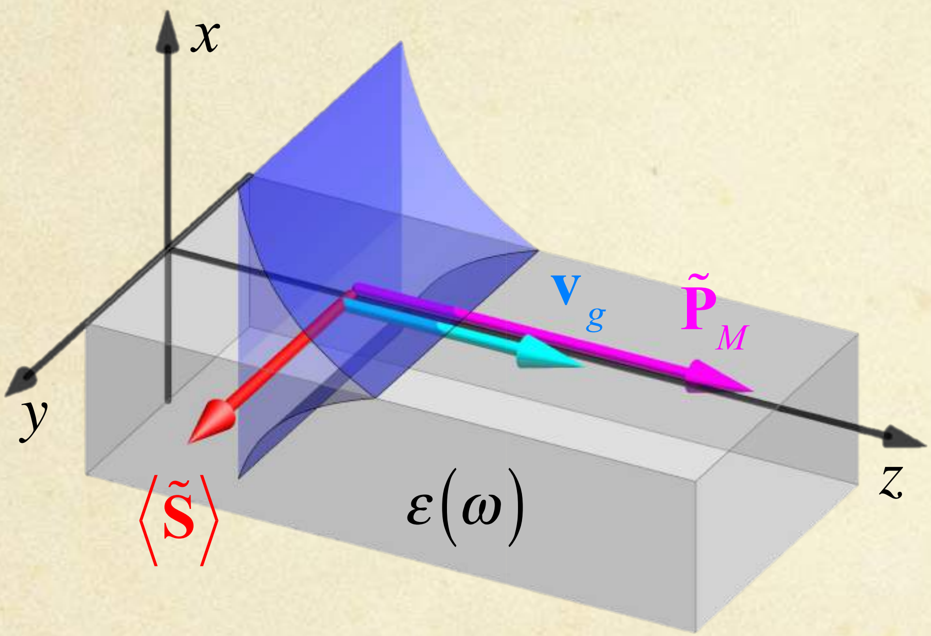
$$\boxed{\frac{\tilde{\mathbf{P}}_M}{\tilde{W}} = \frac{\langle \tilde{\mathbf{P}}_M \rangle}{\langle \tilde{W} \rangle} = \frac{k_p}{\omega}\bar{\mathbf{z}}}$$

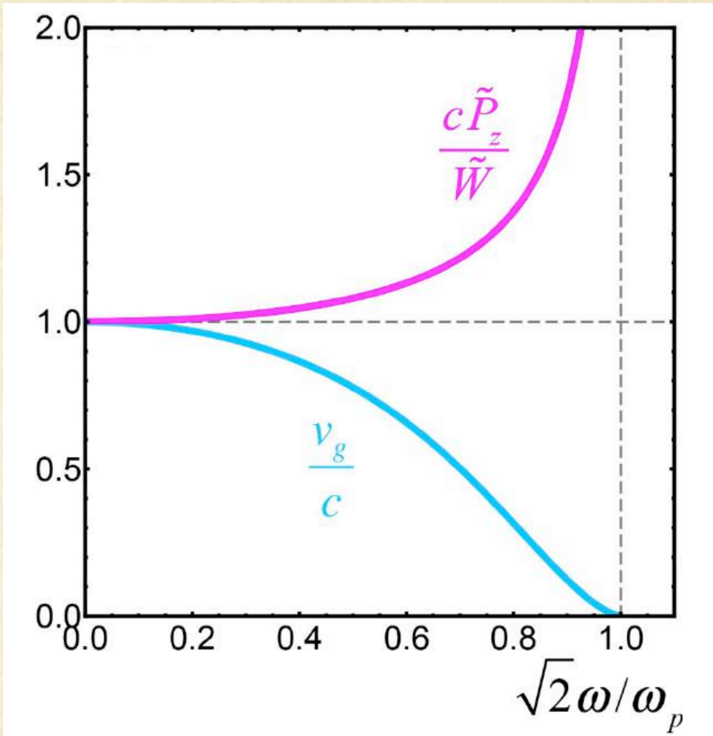


None of the previous approaches yield this simple result!

Canonical super–momentum

Thus, slow velocity of propagation is accompanied by high momentum carried by SPP:



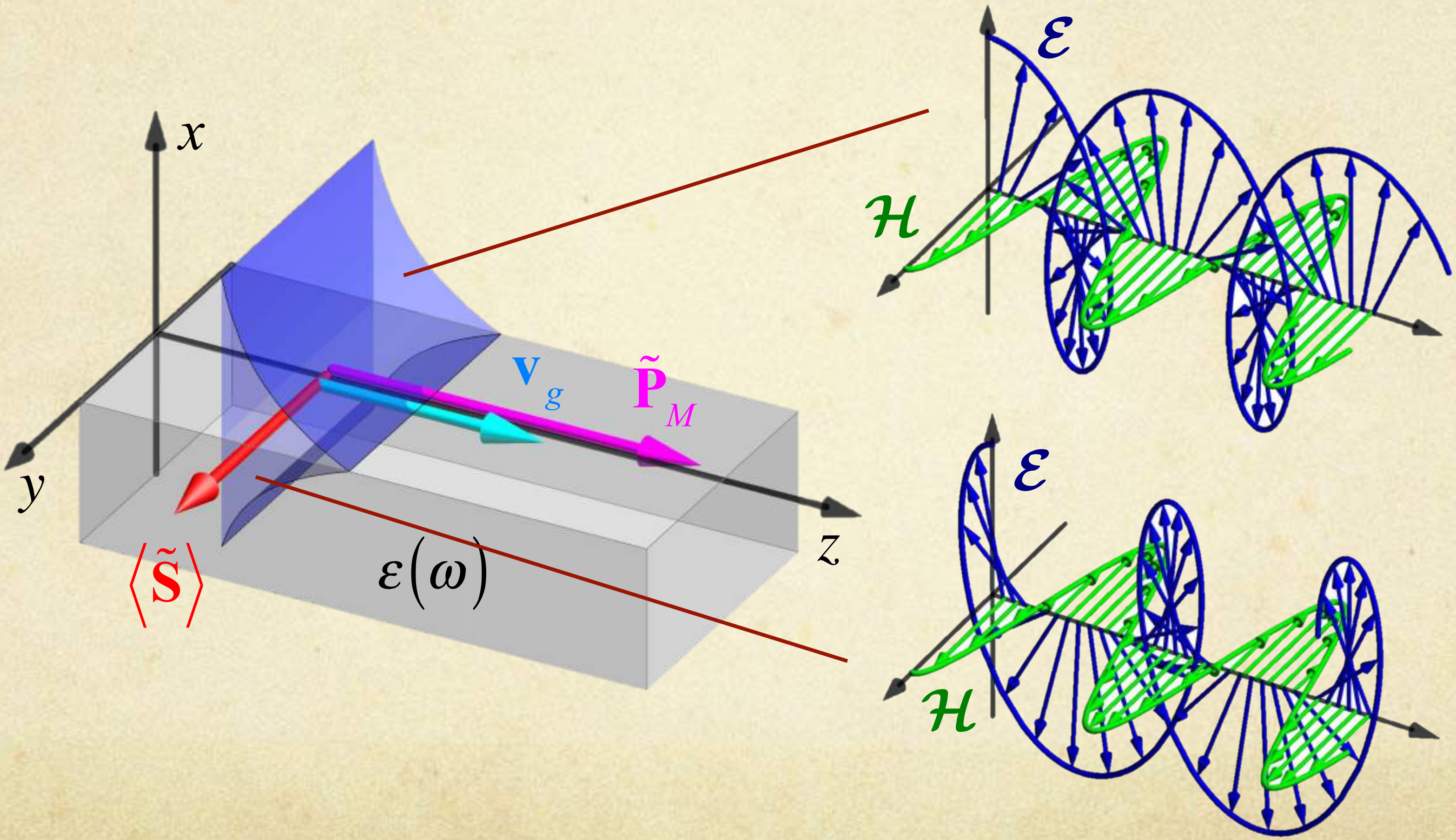


In vacuo, during an interaction between a moving atom and a surface wave of frequency ν, the exchanged momentum is greater than $h\nu/c$. First we show, using a semi-classical treatment, that this momentum is $\hbar k_x$ in agreement with De Broglie's relation $p = \hbar k$, but unlike the usual notion of wave momentum attached to the Poynting vector. We present experimental methods to measure this momentum and we give results for two atom speeds.

Huard & Imbert OC (1978); Matsudo *et al.* OC (1998)

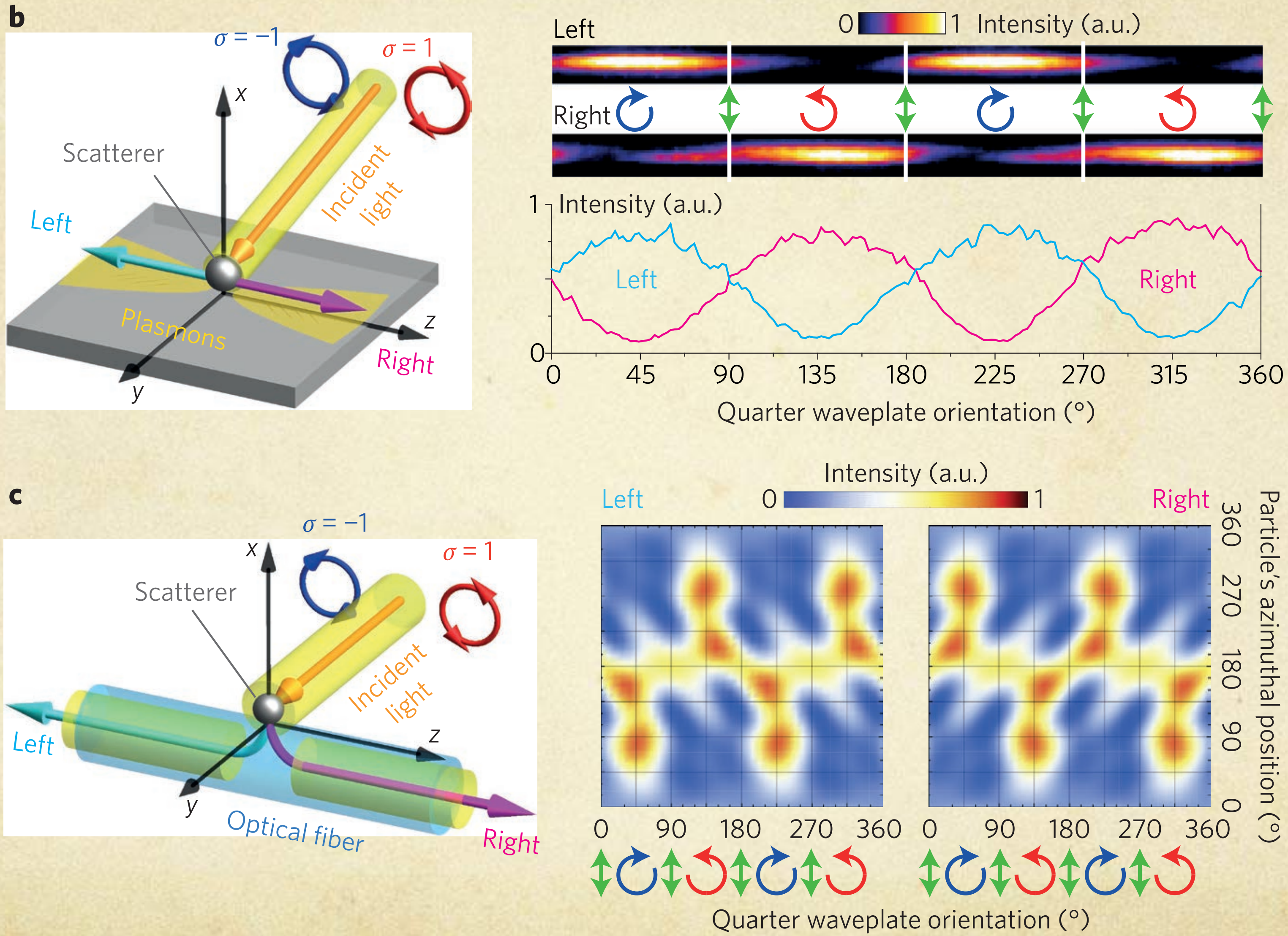
Transverse spin of a SPP

Another quantity of high interest is the transverse spin of a SPP:



Bliokh & Nori *PRA* (2012)

Transverse spin of a SPP



Rodriguez-Fortuno *et al.*, *Science* (2013); Petersen *et al.*, *Science* (2014);
le Feber *et al.*, *Nat. Commun.* (2014); Soller *et al.*, *Nat. Nanotechnology* (2015); ...

Transverse spin of a SPP

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Transverse and longitudinal angular momenta of light

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Spin–orbit interactions of light

K. Y. Bliokh[1,2*], F. J. Rodríguez-Fortuño[3], F. Nori[1,4] and A. V. Zayats[3]

nature photonics

From transverse angular momentum to photonic wheels

Andrea Aiello[1,2], Peter Banzer[1,2,3*], Martin Neugebauer[1,2] and Gerd Leuchs[1,2,3]

REVIEW

doi:10.1038/nature21037

Chiral quantum optics

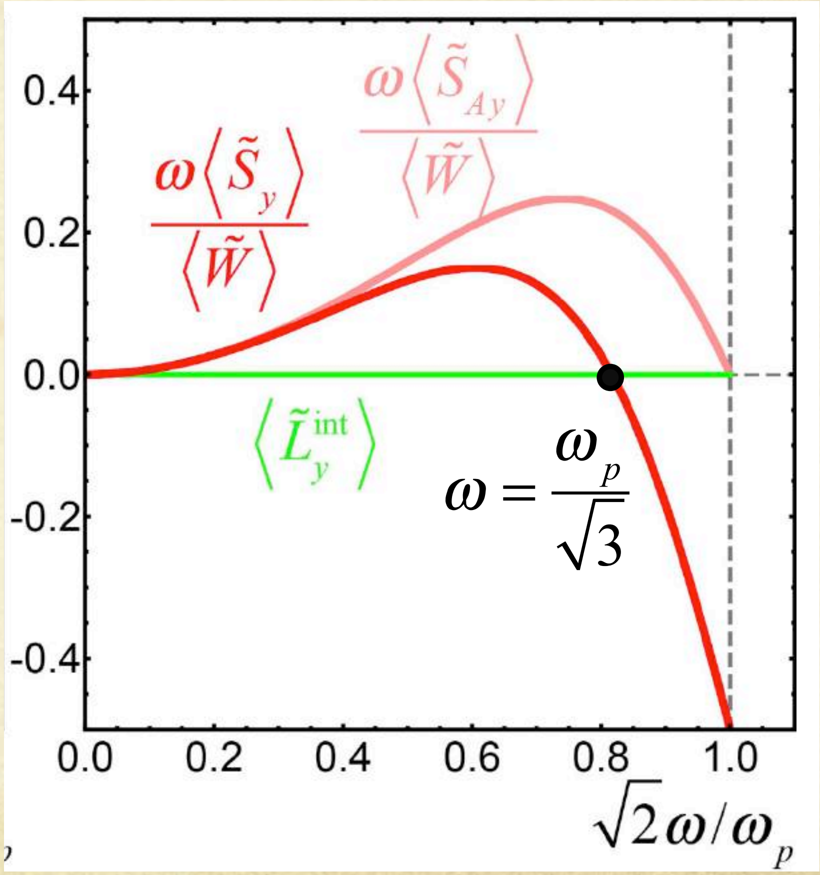
Peter Lodahl[1], Sahand Mahmoodian[1], Søren Stobbe[1], Arno Rauschenbeutel[2], Philipp Schneeweiss[2], Jürgen Volz[2], Hannes Pichler[3,4] & Peter Zoller[3,4]

Transverse spin of a SPP

Using our canonical definitions of the spin and orbital AM, we obtain the following values:

$$\boxed{\frac{\omega\left\langle \tilde{\mathbf{S}}_M \right\rangle}{\left\langle \tilde{W} \right\rangle} = \frac{\left(-2-\varepsilon\right)\sqrt{-\varepsilon}}{1+\varepsilon^2}\,\bar{\mathbf{y}}}$$

$$\left\langle \tilde{L}_M^{\mathrm{int}} \right\rangle = -\int \left(x - \left\langle x \right\rangle\right) \tilde{P}_{Mz}\, dx = 0$$

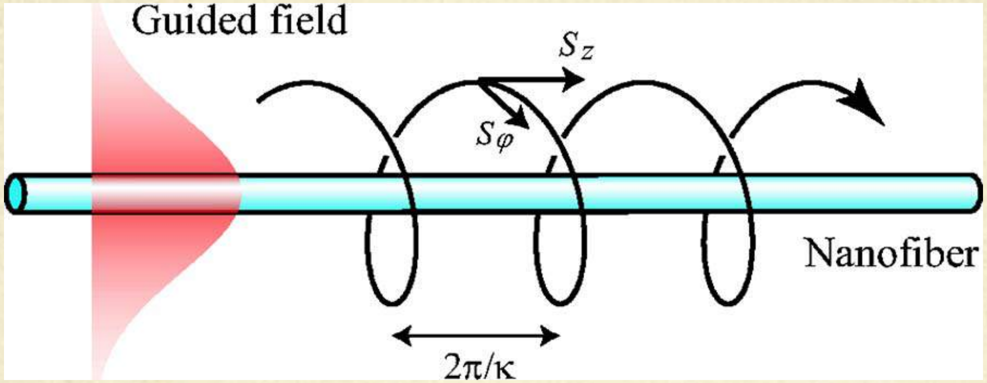


This is the first accurate calculation of the transverse spin of a SPP, 5 years later!

Application to modes of cylindrical waveguides

Angular momentum of guided modes

We apply our formalism to the modes of cylindrical waveguides: both dielectric and metallic (nanowires):



The Abraham (Poynting) and canonical momenta yield:

$$\mathbf{v}_g = \frac{c^2 \langle \mathcal{P}_A \rangle}{\langle \tilde{W} \rangle} = \frac{\partial \omega}{\partial k_z} \bar{\mathbf{z}}$$

$$\frac{\tilde{\mathbf{P}}_M}{\tilde{W}} = \frac{\langle \tilde{\mathbf{P}}_M \rangle}{\langle \tilde{W} \rangle} = \frac{k_z}{\omega} \bar{\mathbf{z}}$$

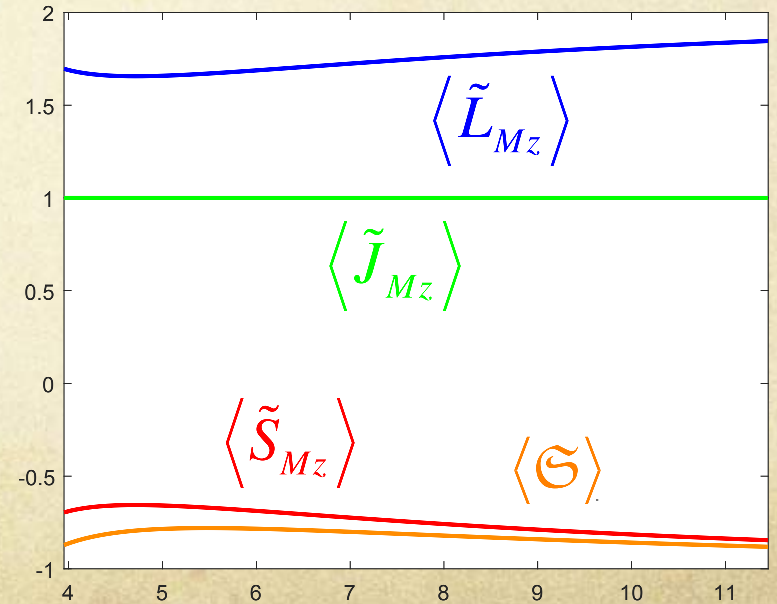
M.F. Picardi *et al.* (2018), in preparation

Angular momentum of guided modes

Most importantly, we obtain the quantization of the total AM of the cylindrical eigenmodes in inhomogeneous media:

$$\frac{\omega \tilde{L}_{Mz}}{\tilde{W}} = \frac{(\ell-1)\tilde{W}^{+} + (\ell+1)\tilde{W}^{-} + \ell \tilde{W}_{z}}{\tilde{W}} \qquad \frac{\omega \tilde{S}_{Mz}}{\tilde{W}} = \frac{\tilde{W}^{+} - \tilde{W}^{-}}{\tilde{W}}$$

$$\boxed{\frac{\omega \tilde{J}_{Mz}}{\tilde{W}} = \frac{\omega\left(\tilde{L}_{Mz} + \tilde{S}_{Mz}\right)}{\tilde{W}} = \ell}$$



M.F. Picardi *et al.* (2018), in preparation

Optical helcity
in complex media

Optical helicity in media

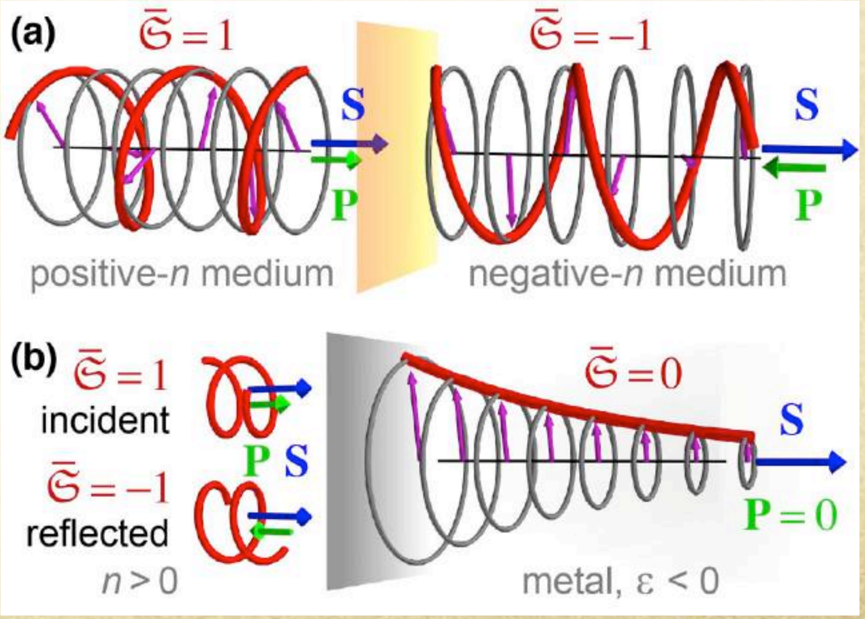
Extending the quantum-like operator approach to the canonical quantities, we derived helicity density in dispersive inhomogeneous media:

$$\boxed{\hat{\mathfrak{S}} = \frac{\hat{\mathbf{S}}\cdot\hat{\mathbf{P}}}{|n|\,k_0} = \begin{pmatrix} 0 & i\nu Z \\ -i\nu Z^{-1} & 0 \end{pmatrix}}$$

$$n = \pm\sqrt{\varepsilon\mu},\ Z = \pm\sqrt{\frac{\mu}{\varepsilon}},\ \nu = \frac{n}{|n|},$$

$$\boxed{\mathfrak{S} = \mathrm{Re}(\nu\tilde{n})\,\mathrm{Im}\left(\mathbf{H}^{*}\cdot\mathbf{E}\right)}$$

Plane wave: $\boxed{\dfrac{\omega\mathfrak{S}}{\tilde{W}} = \pm 1}$



F. Alpeggiani *et al.* (2018), arXiv:1802.09392

Microscopic calculations and other phenomena

Microscopic calculations

Importantly, we performed microscopic calculations (fields + electron plasma) of the SPP momentum and AM densities in the metal, and found these to be fully consistent with the kinetic (Philbin) and canonical (our) Minkowski–type quantities: $\tilde{W}, \tilde{\mathcal{P}}_M, \tilde{\mathcal{J}}_M, \tilde{\mathbf{P}}_M, \tilde{\mathbf{S}}_M$.

In particular, we showed that the electrons in the metal move along small ellipses, thereby providing the material dispersion contributions to the transverse spin AM:

$$\boxed{\mathbf{S}_{\text{mat}} = \frac{\omega}{2}\frac{d\varepsilon}{d\omega}\,\text{Im}\left(\mathbf{E}^* \times \mathbf{E}\right)}$$

Magnetization of the metal

Since electrons are charged particles, this motion also generates a magnetization of the metal:

$$\boxed{\mathbf{M} = \frac{e}{2mc}\mathbf{S}_{\text{mat}} = \frac{e\omega}{4mc}\frac{d\varepsilon}{d\omega}\,\mathrm{Im}\left(\mathbf{E}^{*}\times\mathbf{E}\right)}$$

This is a special case of the inverse Faraday effect.

Pitaevskii (1961), Kono *et al.* (1981), Hertel (2006),...

It means that a SPP carries not only transverse spin but also the transverse magnetic moment:

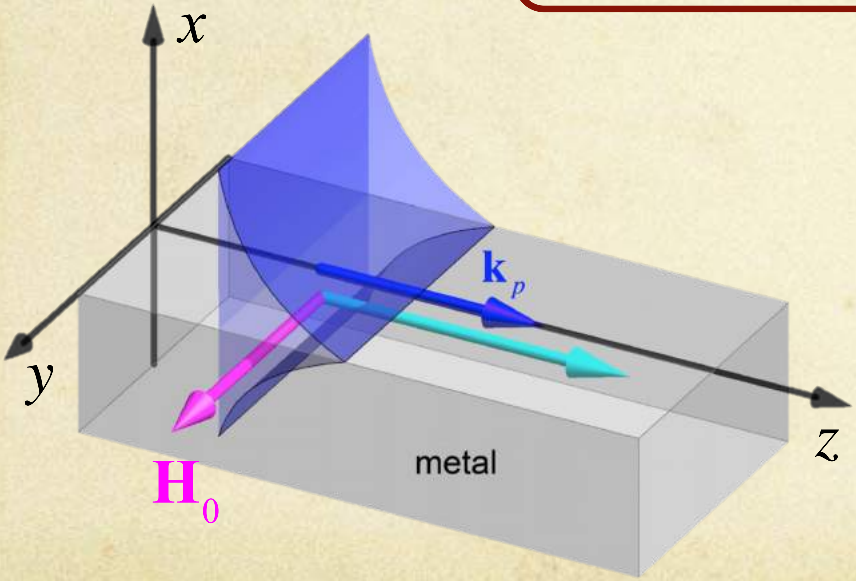
$$\boxed{\boldsymbol{\mu} = \frac{\hbar\omega\left\langle\mathbf{M}\right\rangle}{\left\langle\tilde{W}\right\rangle} = \frac{2\sqrt{-\varepsilon}}{1+\varepsilon^{2}}\,\mu_B\,\bar{\mathbf{y}}}$$

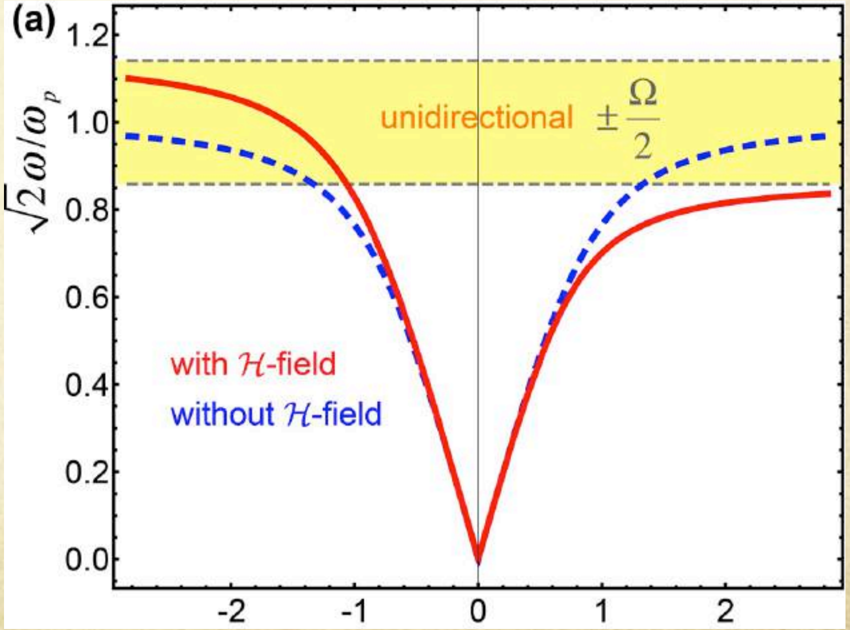
$$\mu_B = \frac{|e|\hbar}{2mc}$$

Magneto–plasmonic effects

The presence of the magnetic moment $\boldsymbol{\mu}$ immediately explains the nonreciprocical magneto–plasmonic spectrum in an applied magnetic field $\mathbf{H}_0 = H_0 \overline{\mathbf{y}}$:

$$\boxed{\delta\omega = -\hbar^{-1} \boldsymbol{\mu} \cdot \mathbf{H}_0}$$





Yu *et al.* *PRL* (2008), Bliokh *et al.* *OL* (2018)

Duality aspects

Finally, note that we used the dual–symmetric forms of all equations. For free–space fields, this is a matter of the convention. One can equally use the electric (or magnetic) biased canonical quantities:

$$\mathbf{P} \to 2\mathbf{P}^e = \mathrm{Im}\left[\mathbf{E}^* \cdot (\nabla)\mathbf{E}\right], \qquad \langle \mathbf{P} \rangle = 2\langle \mathbf{P}^e \rangle$$

$$\mathbf{S} \to 2\mathbf{S}^e = \mathrm{Im}\left[\mathbf{E}^* \times \mathbf{E}\right], \qquad \langle \mathbf{S} \rangle = 2\langle \mathbf{S}^e \rangle$$

Barnett *JMO* (2010), Berry *JOA* (2010), Bliokh *et al. NJP* (2013)

However, this is true only for localized free–space fields (not for evanescent waves).

Duality aspects

This is not the case for localized fields in media. For example, SPPs have purely-electric transverse spin:

$$\boxed{\left\langle \tilde{\mathbf{P}}_M \right\rangle \neq 2\left\langle \tilde{\mathbf{P}}_M^e \right\rangle, \quad \left\langle \tilde{\mathbf{S}}_M \right\rangle \neq 2\left\langle \tilde{\mathbf{S}}_M^e \right\rangle}$$

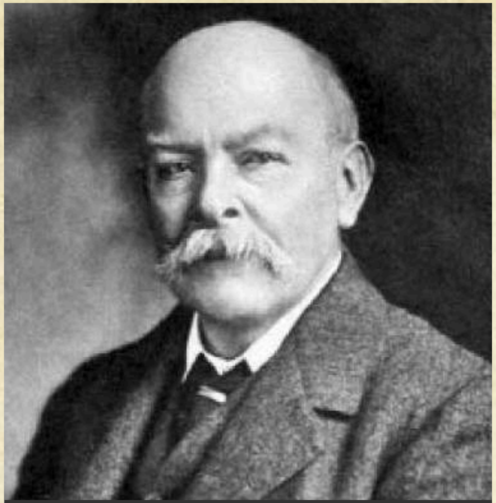
Moreover, the microscopic calculations are consistent only with the dual-symmetric form of the canonical quantities:

$$\mathbf{S}_{\text{mat}} = \frac{\omega}{2}\frac{d\varepsilon}{d\omega}\,\mathrm{Im}\left(\mathbf{E}^* \times \mathbf{E}\right)$$

$$\boxed{\tilde{\mathbf{S}}_M = \frac{1}{2}\mathrm{Im}\left(\tilde{\varepsilon}\,\mathbf{E}^* \times \mathbf{E} + \tilde{\mu}\,\mathbf{H}^* \times \mathbf{H}\right)} \qquad \boxed{\cancel{2\tilde{\mathbf{S}}_M^e = \mathrm{Im}\left(\tilde{\varepsilon}\,\mathbf{E}^* \times \mathbf{E}\right)}}$$

This supports the dual-symmetric theory (QED ???).

Conclusions

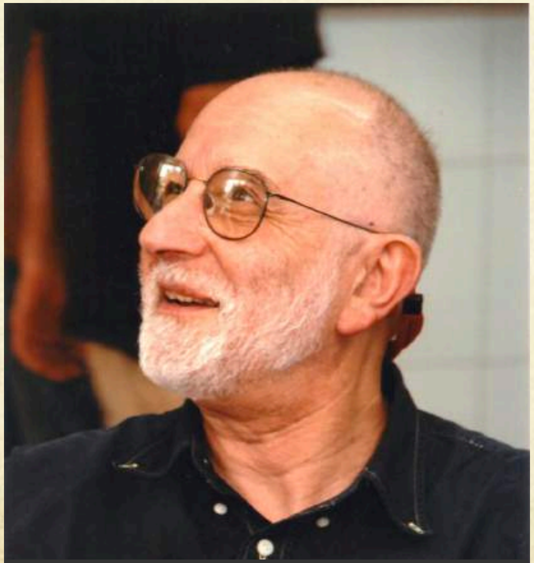


J. H. Poynting

H. Minkowski

M. Abraham

L. Brillouin



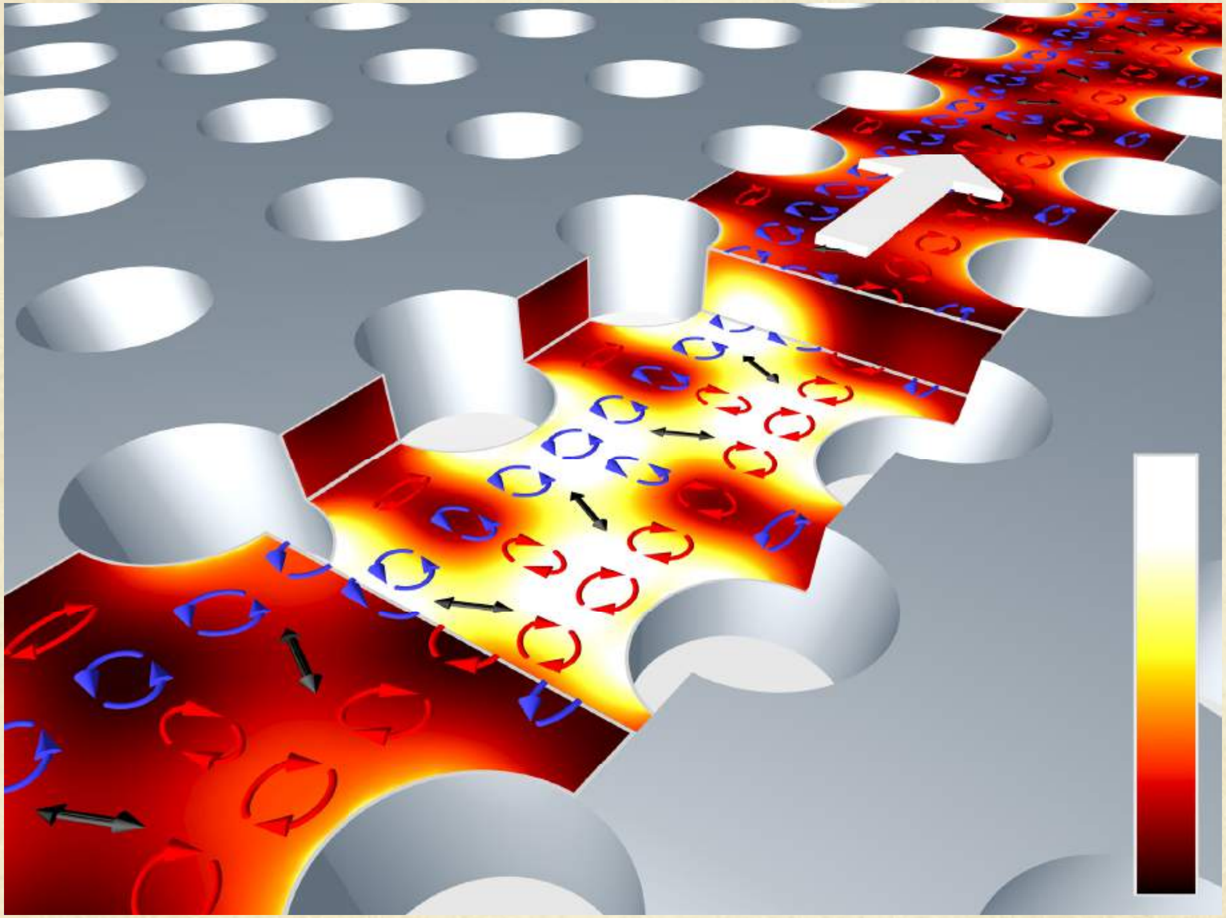
M. V. Berry

T. G. Philbin

$$\tilde{W} = \frac{\omega}{2}\left(\tilde{\varepsilon}\left|\mathbf{E}\right|^2 + \tilde{\mu}\left|\mathbf{H}\right|^2\right)$$

$$\tilde{\mathbf{P}}_M = \frac{1}{2}\mathrm{Im}\left[\tilde{\varepsilon}\,\mathbf{E}^*\cdot(\nabla)\mathbf{E} + \tilde{\mu}\,\mathbf{H}^*\cdot(\nabla)\mathbf{H}\right]$$

$$\tilde{\mathbf{S}}_M = \frac{1}{2}\mathrm{Im}\left(\tilde{\varepsilon}\,\mathbf{E}^*\times\mathbf{E} + \tilde{\mu}\,\mathbf{H}^*\times\mathbf{H}\right),\ \ \tilde{\mathbf{L}}_M = \mathbf{r}\times\tilde{\mathbf{P}}_M$$



Thank you!

It is impossible to study this remarkable theory without experiencing the strange feeling that the equations somehow have a proper life, that they are smarter than we.

H. Hertz

R. Feynman

You can recognize truth by its beauty and simplicity. When you get it right, it is obvious that it is right, because usually what happens is that more comes out than goes in.